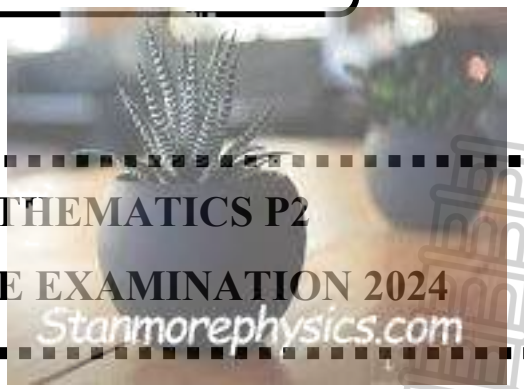
LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12



MATHEMATICS P2

PRE – JUNE EXAMINATION 2024

MARKS: **150**

TIME: **3 Hours**

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 Questions.
2. Answer **ALL** the questions.
3. Number your answers correctly according to the numbering system used in this question paper.
4. Clearly show **ALL** calculations, diagrams and graphs that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
7. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

Fifty motorists were asked to record the number of kilometres travelled in one week.
The following table shows the results:

Number of kilometres	Number of motorists	Cumulative frequency
$10 < x \leq 20$	2	
$20 < x \leq 30$	7	
$30 < x \leq 40$	4	
$40 < x \leq 50$	13	
$50 < x \leq 60$	16	
$60 < x \leq 70$	8	

1.1 Complete the Cumulative frequency column. (2)
1.2 Draw the cumulative frequency curve (ogive curve) (4)
1.3 Use your graph to estimate the median number of kilometres travelled per week. (2)
1.4 What percentage of motorists travelled more than 50km in one week? (2)

[10]

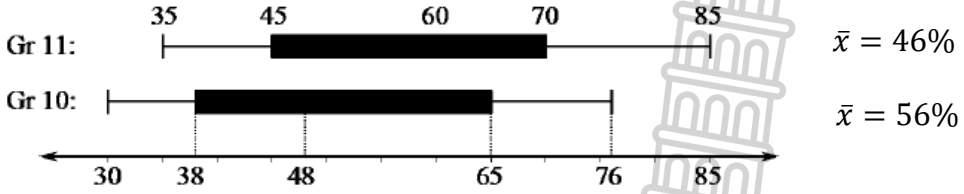
QUESTION 2

2.1 The maximum daily temperatures in degrees Celsius for Polokwane for the first 10 days in July were recorded in the following table:

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
24	25	22	28	27	21	18	17	24	25

2.1.1 Calculate the mean temperature for this data. (2)
2.1.2 Calculate the standard deviation from the mean for this data. (2)
2.1.3 How many days did the temperature lie outside one standard deviation of the mean? (4)

2.2 The box and whisker plots given below represent the Mathematics marks of the same 25 students at the end of their Grade 10 and 11 years at school.

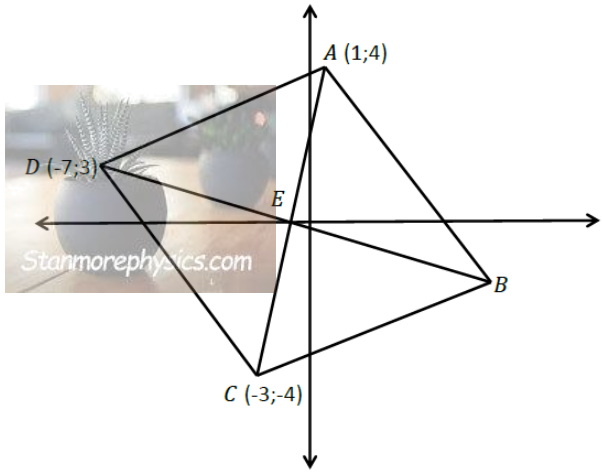


2.2.1 Calculate the inter-quartile range of each set of data. (4)
2.2.2 Comment on the way in which the distribution of the marks changes from the end of the Grade 10 year to the end of the Grade 11 year. (3)

[15]

QUESTION 3

In the diagram below $ABCD$ is a parallelogram.



3.1 Give the co-ordinates of point B. (2)

3.2 Find E, the midpoint of AC. (3)

3.3 Show that $AC \perp DB$. (3)

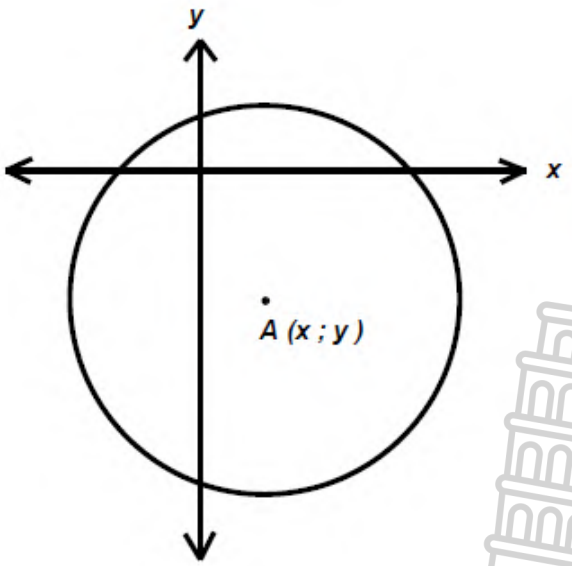
3.4 Hence give a reason why $ABCD$ is a rhombus. (1)

3.5 Find the size of angle $A\hat{D}C$. (6)

[15]

QUESTION 4

4.1 The equation of a circle is $x^2 + y^2 - 2x + 4y - 4 = 0$.



4.1.1 Determine the coordinates of A, the centre of the circle and the length of the radius, r. (5)

4.1.2 Calculate the value of p if N (1; p) with $p > 0$ is a point on the circle. (1)

4.1.3 Determine the equation of the tangent to the circle at N. (2)

4.2 A second circle, centre B, with equation $(x-4)^2 + y^2 = k^2$ cuts the circle given in (4.1) twice. Determine the values of k for which point A will be inside the circle B. (6)

[14]

QUESTION 5

5.1 If $sin\,\alpha = \frac{-2}{3}$ and $cos\,\alpha > 0$, calculate the values of the following without the use of a calculator:

5.1.1 $tan\,\alpha$ (3)

5.1.2 $2sin\,\alpha.cos\,\alpha$ (3)

5.1.3 $sin^2\alpha + cos^2\alpha$ (4)

5.2 Given that $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$, derive the identity for $\sin(\alpha-\beta)$ (4)

5.3 Determine, without the use of a calculator, the value of :

$\cos 35^\circ.\sin 25^\circ - \cos(-205^\circ).\cos 55^\circ$ (5)

5.4 Consider the following identity:

$$\frac{2\sin^3 x+\sin 2x\cos x}{\cos x}=2\tan x$$

5.4.1 Prove the identity. (4)

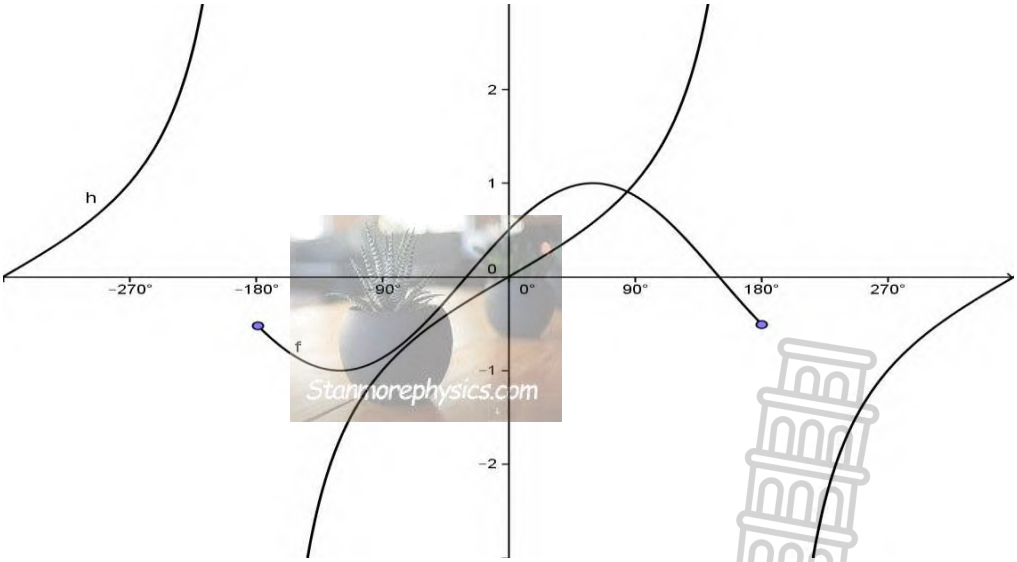
5.4.2 For which values of x, $x \in (-180°; 180°)$, is this identity not valid? (2)

5.5 Determine the general solution for $cos\,2x - cos\,x = 2$ (7)

[32]

QUESTION 6

The diagram shows the graphs of $f(x) = sin(x + 30°)$ and $h(x) = tan\frac{1}{2}x$.



6.1 Write down the domain of (x). (2)

6.2 Write down the period of $h(x)$. (2)

6.3 Determine the equation of the function, (x), obtained when $h(x)$ is translated 45° to the right and then 2 units downwards. (2)

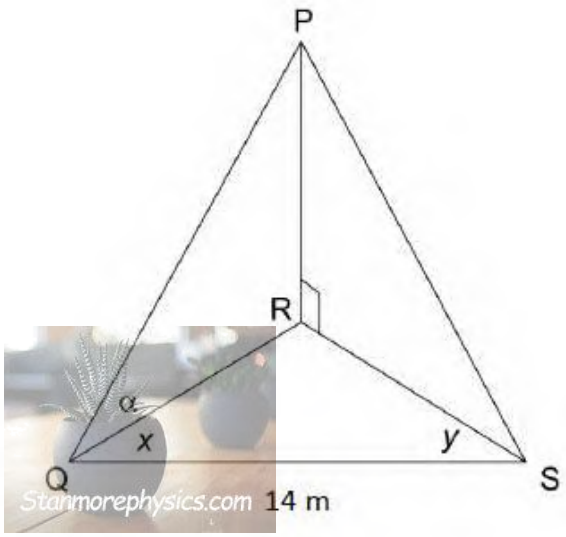
6.4 Determine a value of x for which $f(x) - h(x) = \frac{1}{2}$. (2)

6.5 Determine the distance between (x) and $h(x)$ if $x = -135°$. (3)

[11]

QUESTION 7

The diagram below shows a vertical tower PR, with points Q, R and S all on horizontal ground. The angle of elevation of P from Q is ∝, the length QS = 14 m. $R\hat{Q}S = x$ and $R\hat{S}Q = y$.



7.1 Express PR in terms of QR and ∝ (2)

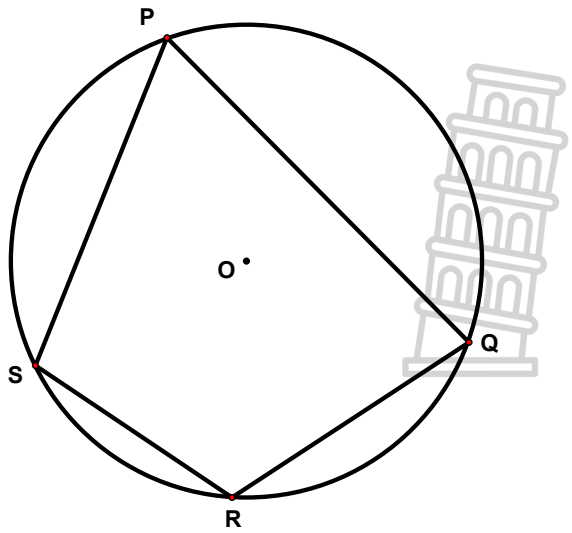
7.2 Show that $QR = \frac{14 \sin y}{\cos y}$ (3)

7.3 If $x = y$, show that $PR = \frac{7 \tan \propto}{\cos y}$ (4)

[9]

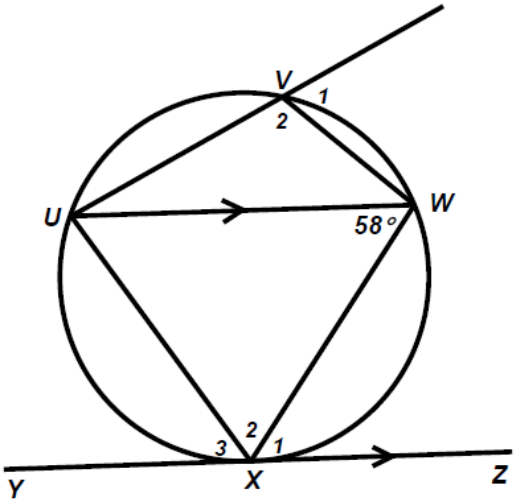
QUESTION 8

8.1 In the diagram below, P, Q, R and S are points that lie on the circumference of the circle with centre O. Given below is the partially completed proof of the theorem that states that $\hat{P} + \hat{R} = 180°$.



(7)

8.2 In the figure below, UVWX is a cyclic quadrilateral, with $UW||YZ$ and tangent YXZ touching the circle at X. $U\widehat{W}X = 58°$.



Determine the values of the following angles, showing all steps and reasons:

8.2.1 $\hat{X}_1$ (2)

8.2.2 $\hat{X}_3$ (2)

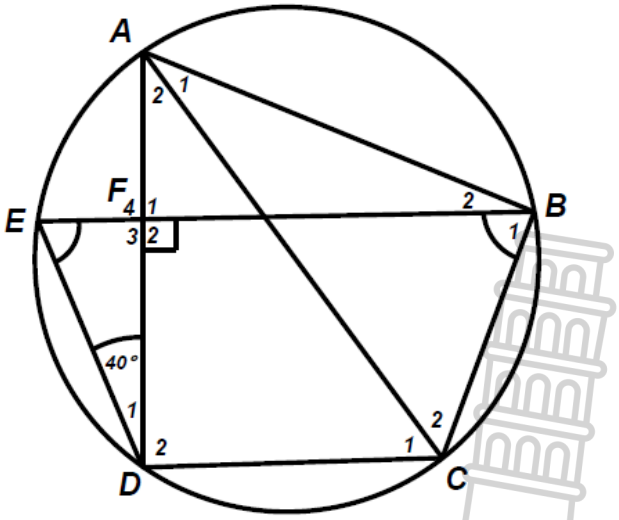
8.2.3 $\hat{X}_2$ (2)

8.2.4 $\hat{V}_1$ (2)

[15]

QUESTION 9

In the diagram below, AC is a chord of circle ABCDE. $AFD \perp EFB$, $\angle D1 = 40°$ and $\angle E = \angle B1$.



In the following questions, give a reason for each statement:

9.1 Name THREE angles each equal to 50°. (3)

9.2 Calculate the size of $\angle DCB$. (3)

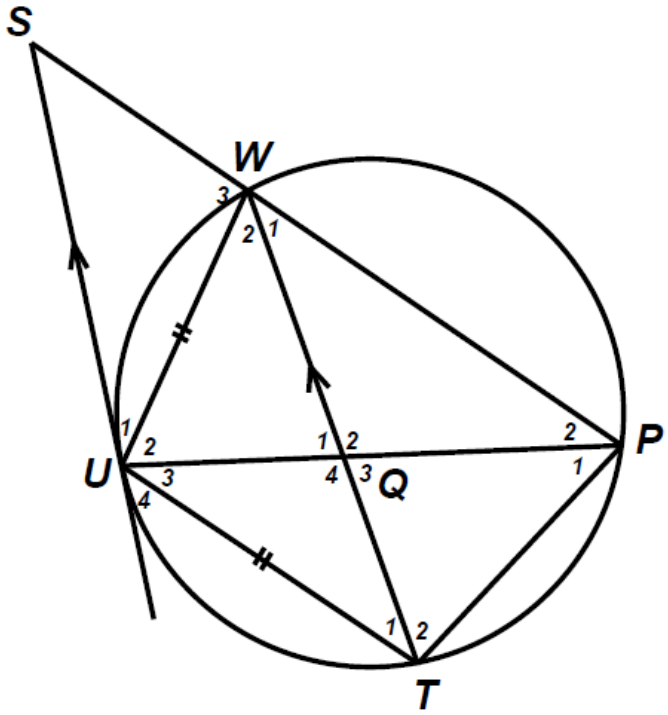
9.3 Prove that EB||DC. (3)

9.4 Prove that AC is a diameter of the circle. (4)

[13]

QUESTION 10

In the diagram, WPTU is a cyclic quadrilateral with $UW = UT$. Chords WT and PU intersect at Q. PW extends to S such that $SU \| WT$.



Prove that:

10.1 US is a tangent to circle PWUT at U. (6)

10.2 $\Delta SPU \,|||\, \Delta SUW$ (4)

10.3 $SU^2 = SP.SW$ (2)

10.4 $SU^2.QU = PU.SW^2$ (4)

[16]

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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DEPARTMENT OF
EDUCATION

VHEMBE WEST DISTRICT

GRADE 12

MATHEMATICS P2

PRE-MIDYEAR EXAMINATION 2024

MEMORANDUM

MARKS: 140

This question paper consists of 21 pages **including the cover page**

NOTE:

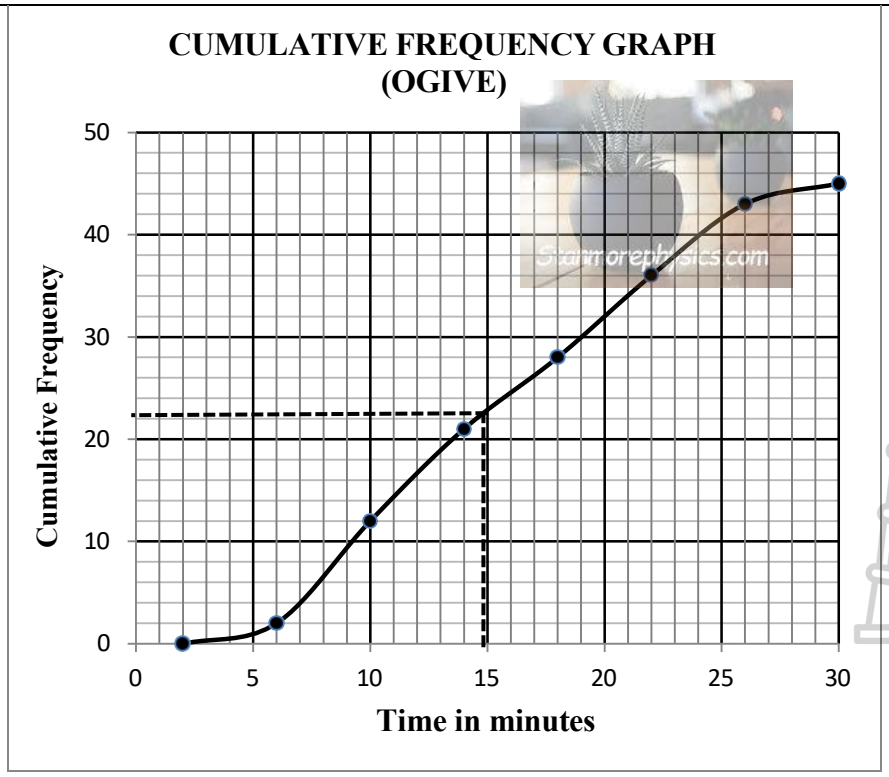
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

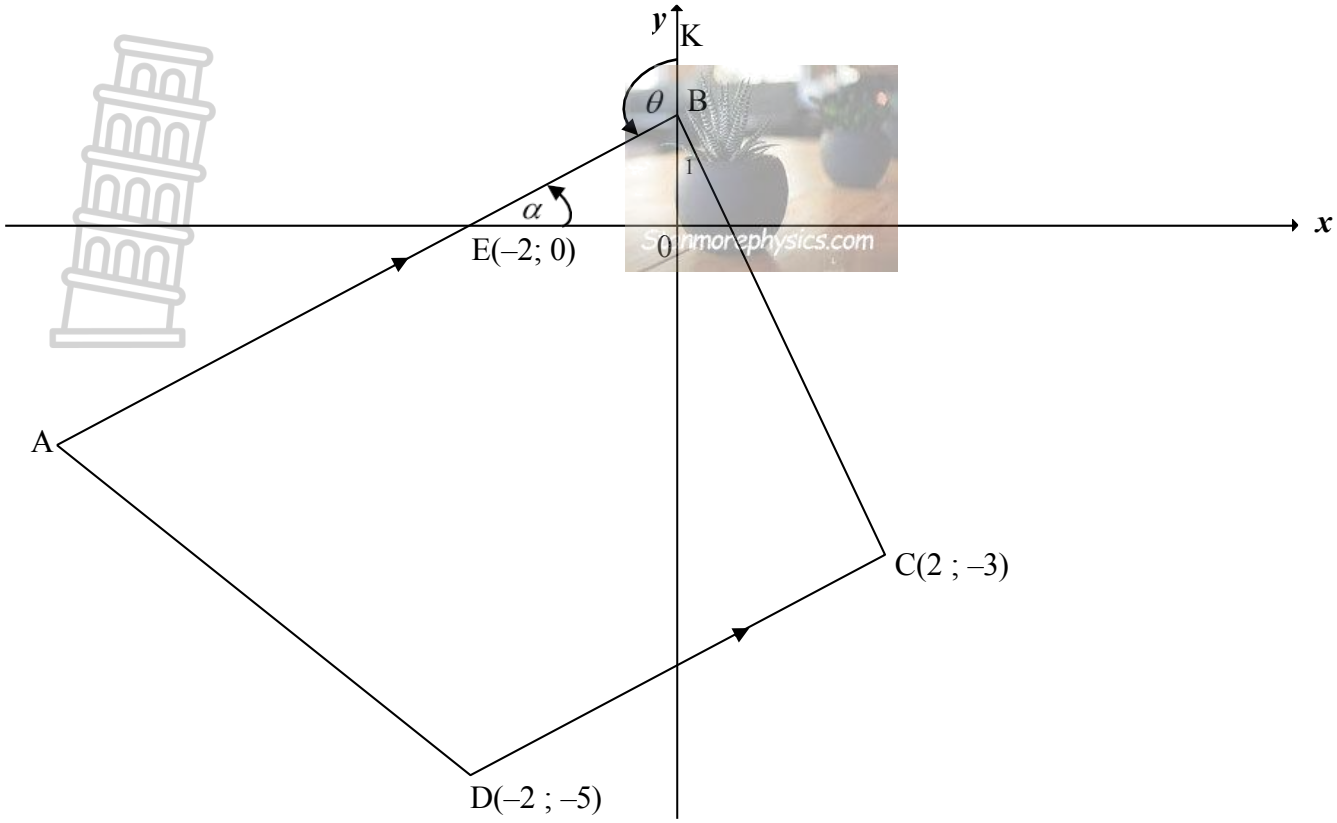
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	45 children	✓ answer (1)
1.2	$\bar{x} = \frac{\sum fx}{n} = \frac{(4\times2)+(8\times10)+(12\times9)+(16\times7)+(20\times8)+(24\times7)+(28\times2)}{45}$ $\bar{x} = \frac{692}{45}$ OR $\bar{x} = 15{,}38$ minutes Answer only: full marks	✓ 692 ✓ answer (2)
1.3	<table><tr><th>Time taken (t) (in minutes)</th><th>Number of children</th><th>Cumulative frequency</th></tr><tr><td>$2 < t \leq 6$</td><td>2</td><td>2</td></tr><tr><td>$6 < t \leq 10$</td><td>10</td><td>12</td></tr><tr><td>$10 < t \leq 14$</td><td>9</td><td>21</td></tr><tr><td>$14 < t \leq 18$</td><td>7</td><td>28</td></tr><tr><td>$18 < t \leq 22$</td><td>8</td><td>36</td></tr><tr><td>$22 < t \leq 26$</td><td>7</td><td>43</td></tr><tr><td>$26 < t \leq 30$</td><td>2</td><td>45</td></tr></table>	✓ first 4 cum freq correct ✓ last 3 cum freq correct (2)
1.4		✓ plotting cum freq at upper limits correctly (all points) ✓ shape (smooth) ✓ grounding (2;0) (3)
1.5	On graph at the y-value of 22,5 or 23 Median = ± 15 minutes. Answer only: full marks	graph answer (2)
		[10]

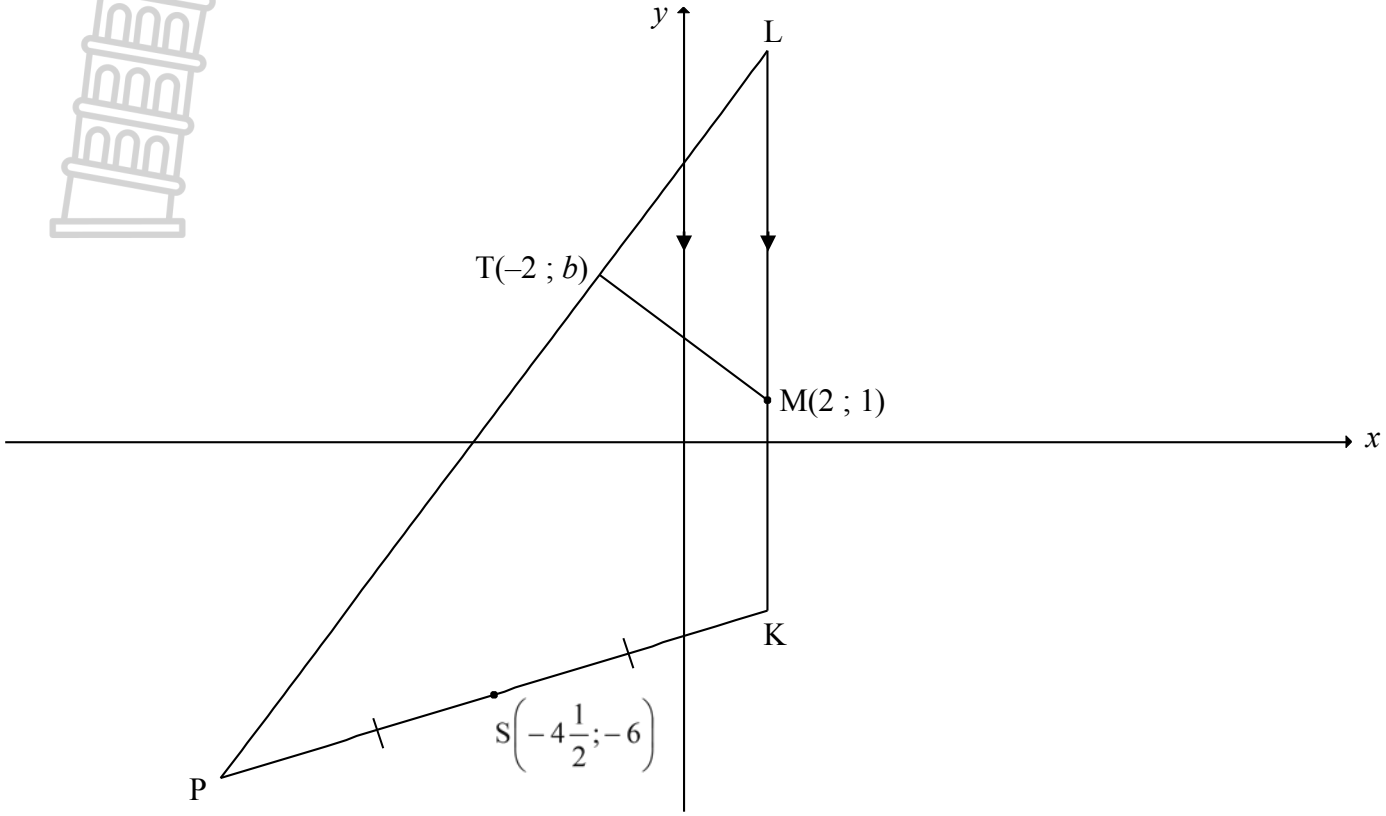
QUESTION/*VRAAG* 2



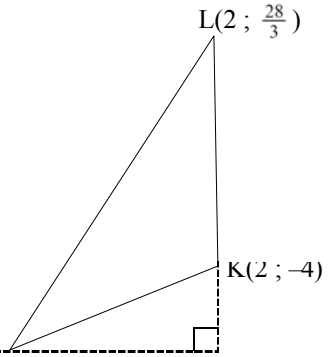
2.1.1	Midpoint of EC: $=\left(\frac{-2+2}{2}\ ;\ \frac{0+(-3)}{2}\right)=\left(0\ ;\ \frac{-3}{2}\right)$	✓*x* value ✓*y* value (2)
2.1.2	$m_{DC}=\frac{-3-(-5)}{2-(-2)}$ OR $\frac{-5-(-3)}{-2-2}$ $=\frac{2}{4}=\frac{1}{2}$ Answer only: full marks	✓ substitution ✓ answer (2)
2.1.3	$m_{AB}=\frac{1}{2}$ [AB ∥ DC] $y=\frac{1}{2}x+c$ $0=\frac{1}{2}(-2)+c$ $c=1$ $\therefore y=\frac{1}{2}x+1$ **OR** $y-y_1=\frac{1}{2}(x-x_1)$ $y-0=\frac{1}{2}(x-(-2))$	✓ $m_{AB}=\frac{1}{2}$ ✓substitution of $(-2;0)$ ✓equation (3)
2.1.4	$\tan\alpha=m_{AB}=\frac{1}{2}$ $\alpha=26{,}57°$ $\theta=90°+26{,}57°$ [ext ∠of Δ] $=116{,}57°$	✓ $\tan\alpha=\frac{1}{2}$ ✓value of α ✓value of θ (3)

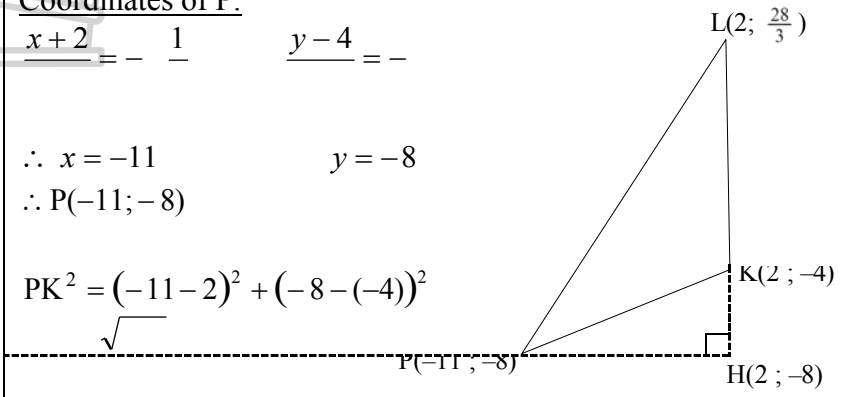
2.2	B(0 ; 1) $m_{BC} = \frac{1-(-3)}{0-2}$ **OR** $m_{BC} = \frac{(-3)-1}{2-0}$ $= -2$ $= -2$ $m_{AB} \times m_{BC} = \frac{1}{2} \times -2$ $= -1$ $\therefore AB \perp BC$	✓ coordinates of B ✓ $m_{BC} = -2$ ✓ product of gradients = –1 (3)
2.3.1	$A\hat{B}C = 90°$ $\therefore$ EC is diameter [converse: $\angle$ in semi circle] $\therefore$ centre of circle $= \left(0 ; -\frac{3}{2}\right)$	✓ answer (1)
2.3.2	$(x-0)^2 + \left(y+\frac{3}{2}\right)^2 = r^2$ $(-2-0)^2 + \left(0+\frac{3}{2}\right)^2 = r^2$ **OR** $(2-0)^2 + \left(-3-\left(\frac{-3}{2}\right)\right)^2 = r^2$ **OR** $(0-0)^2 + \left(1-\left(\frac{-3}{2}\right)\right)^2 = r^2$ **OR** $r = \frac{EC}{2} = \frac{\sqrt{(-2-2)^2 + (0-(-3))^2}}{2}$ **OR** $r = 1-(-\frac{3}{2})$ $\therefore r^2 = \frac{25}{4}$ or $r = \frac{5}{2}$ $x^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{4}$	✓ substitution of centre ✓ correct substitution of E(–1 ; 0), B(0 ; 1) or C(2 ; –3) to calculate r^2 or r ✓ value of r^2 or r ✓ equation (4)
		[18]

QUESTION/*VRAAG* 3



3.1	$(x-2)^2+(y-1)^2=25$ $(-2-2)^2+(b-1)^2=25$ $(b-1)^2=9$ $b-1=\pm 3$ $\therefore b=4$ or $b\neq -2$ **OF** $(x-2)^2+(y-1)^2=25$ $(-2-2)^2+(b-1)^2=25$ $16+b^2-2b+1=25$ $b^2-2b-8=0$ $\therefore b=4$ or $b\neq -2$	✓ equation of the circle ✓ substitution of point T ✓ simplification ✓ answer (4)
3.2.1	K(2 ; 1 – 5) ∴ K(2 ; –4) Answer only: full marks	✓ x value ✓ y value (2)
3.2.2	$m_{MT}=\frac{4-1}{-2-2}=-\frac{3}{4}$ $m_{PL}=\frac{4}{3}$ [radius ⊥ tangent] $y=\frac{4}{3}x+c$ $4=\frac{4}{3}(-2)+c$ $c=\frac{20}{3}$ $y=\frac{4}{3}x+\frac{20}{3}$	✓ m_{MT} ✓ $m_{PL}=\frac{4}{3}$ ✓ substitution of m_{PL} and the point T ✓ equation (4)

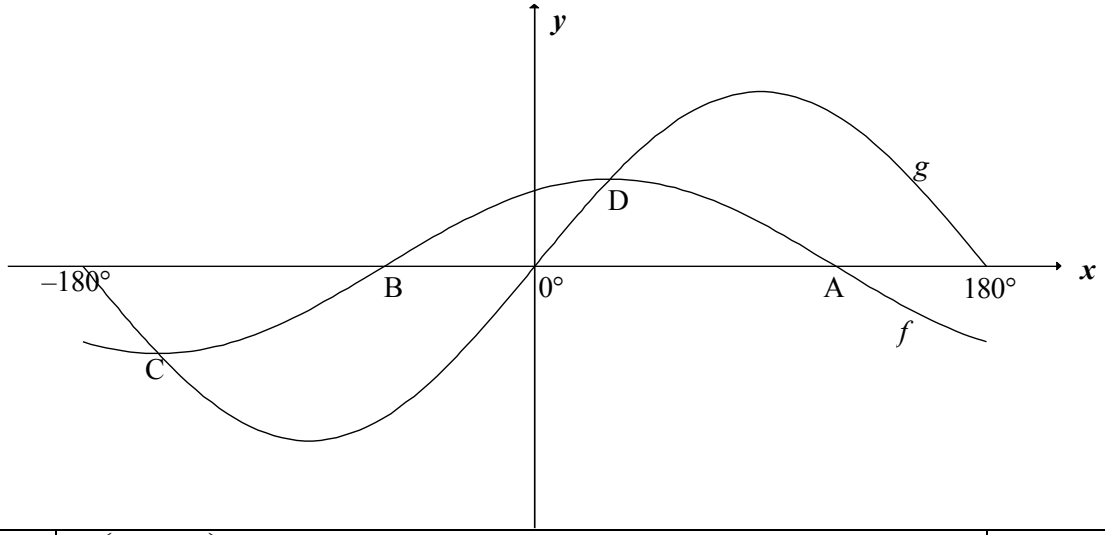
	OR $m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3}$ [radius ⊥ tangent] $y - y_1 = \frac{4}{3}(x - x_1)$ $y - 4 = \frac{4}{3}(x + 2)$ $y = \frac{4}{3}x + \frac{20}{3}$ **OR** P(–11 ; –8) $m_{PL} = \frac{4-(-8)}{-2-(-11)}$ $= \frac{4}{3}$ $y = \frac{4}{3}x + c$ $-8 = \frac{4}{3}(-11) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$	m_{MT} $m_{PL} = \frac{4}{3}$ substitution of m_{PL} and the point T equation (4) coordinates of P $m_{PL} = \frac{4}{3}$ substitution of m_{PL} and the point P or T equation (4)
3.2.3	$y = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ $\left(2 ; \frac{28}{3}\right)$ and K(2 ; –4): $LK = \frac{28}{3} - (-4) = \frac{40}{3}$ Coordinates of P: $\frac{x+2}{3} = -4\frac{1}{3}$ and $\frac{y-4}{3} = -6$ $\therefore x = -11$ $y = -8$ $\therefore$ P(–11; –8) ⊥height (PH) = 2 – (–11) = 13 Area Δ PKL = $\frac{1}{2}$(LK)(PH) $= \frac{1}{2}\left\|\frac{40}{3}\right\|13$ $= \frac{260}{3}$ **OR** 86,67 square units  	$y = \frac{28}{3}$ length of LK x_P y_P length of ⊥ height substitution into the area formula answer (7)

3.2.3	**OR** $y = \frac{4}{\quad}(2) + \frac{20}{\quad} = \frac{28}{\quad}$ $\left(2;\frac{28}{\quad}\right)$ and $K(2;-4)$: $LK = \frac{28}{\quad} - (-4) = \frac{40}{\quad}$ Coordinates of P: $\frac{x+2}{\quad} = -\frac{1}{\quad}$ $\frac{y-4}{\quad} = -$ $\therefore\ x = -11$ $y = -8$ $\therefore P(-11;-8)$ $PK^2 = (-11-2)^2 + (-8-(-4))^2$ $\sqrt{\quad}$ PK $\overline{-11-2}$ $\overline{13}$ $\tan\theta = \frac{4}{\quad}$ $\therefore \theta = 17{,}1027...°$ $\therefore PK\hat{}L = 90° + 17{,}1027...° = 107{,}1°$ Area $\Delta PKL = \frac{1}{\quad}(PK)(LK).\sin P\hat{K}L$ $= \frac{1}{\quad}\left(\sqrt{185}\right)\left(\frac{40}{\quad}\right)$ n107,10° = 86,67 square units  	✓$y = \frac{28}{\quad}$ ✓ length of LK ✓ x_P ✓ y_P ✓$P\hat{K}L$ ✓ substitution into the area rule ✓ answer (7)
3.3	The centres of the two circles lie on the same vertical line $x = 2$. and the sum of the radii = 10 $n - 1 = 10$ or $1 - n = 10$ $n = 11$ $n = -9$ Answer only: full marks	✓correct method ✓ sum of radii = 10 ✓ $n = 11$ ✓ $n = -9$ (4)
		[21]

QUESTION/*VRAAG* 4

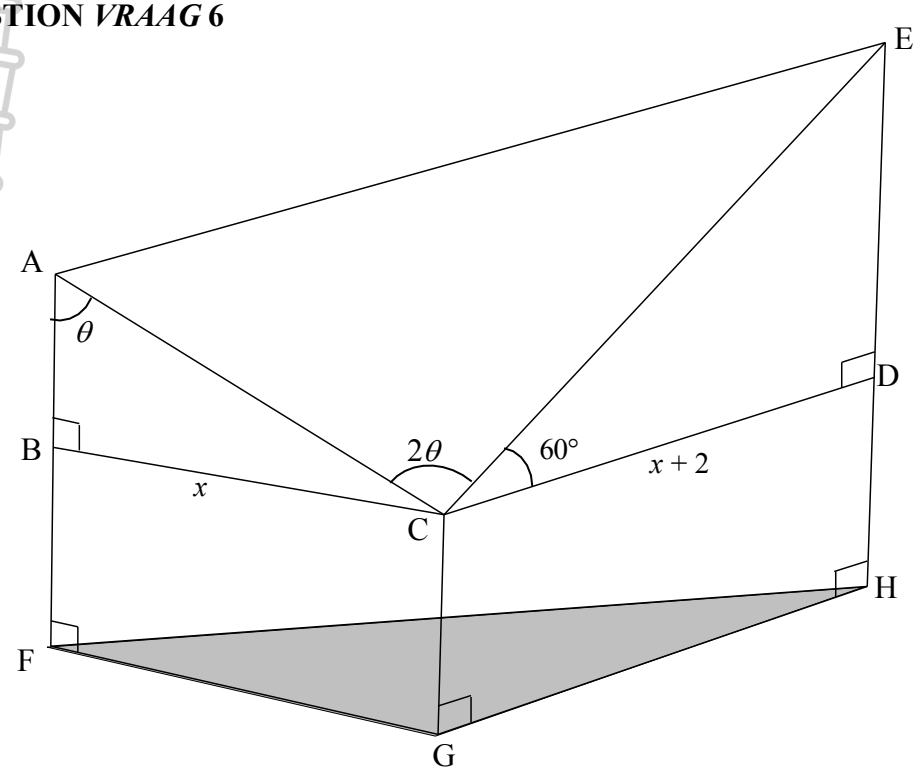
4.1.1	$\sin 191°$ $= -\sin 11°$	$-\sin 11°$ (1)
4.1.2	$\cos 22°$ $= \cos(2 \times 11°)$ $= 1 - 2\sin^2 11°$	answer (1)
4.2	$\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ $= -\cos x + \sqrt{2}(\sin x \cos 45° + \cos x \sin 45°)$ $= -\cos x + \sqrt{2}\left(\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x\left(\frac{1}{\sqrt{2}}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$ **OR** $\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ $= -\cos x + \sqrt{2}(\sin x \cos 45° + \cos x \sin 45°)$ $= -\cos x + \sqrt{2}\left(\sin x\left(\frac{\sqrt{2}}{2}\right) + \cos x\left(\frac{\sqrt{2}}{2}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$	$-\cos x$ expansion special angle ratios simplification of last 2 terms answer (5) $-\cos x$ expansion special angle ratios simplification of last 2 terms answer (5)
4.3	$\sin P + \sin Q = \sin P + \cos P$ $(\sin P + \cos P)^2 = \left(\frac{7}{\sqrt{5}}\right)^2$ $\sin^2 P + 2\sin P \cos P + \cos^2 P = \frac{49}{25}$ $2\sin P \cos P = \frac{49}{25} - 1$ $\sin 2P = \frac{49}{25} - \frac{25}{25}$ $= \frac{24}{25}$	$\sin Q = \cos P$ squaring expansion $\sin^2 P + \cos^2 P = 1$ answer (5)
		[12]

5.1	$\cos(x-30°) = 2\sin x$ $\cos x\cos 30° + \sin x\sin 30° = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x = \frac{3}{2}\sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30° + k.180°; \quad k \in Z$ **OR** $x = 30° + k.360°$ or $x = 210° + k.360°; \quad k \in Z$	✓ expansion ✓ special ∠s ✓ simplification ✓ equation in tan ✓ $30°$ ✓ $k.180°; k \in Z$ **OR** ✓ $30°$ and $210°$ ✓ $k.360°; \quad k \in Z$ (6)



5.2.1(a)	$A(120° ; 0)$	✓ answer (1)
5.2.1(b)	$C(-150° ; -1)$	✓ x value ✓ y value (2)
5.2.2(a)	$x \in (-90° ; 30°)$ OR $-90° < x < 30°$	✓ endpoints ✓ correct interval (2)
5.2.2(b)	$x \in (-160° ; 20°)$ OR $-160° < x < 20°$	✓ endpoints ✓ correct interval (2)
5.2.3	$y = 2^{2\sin x + 3}$ Range of $y = 2\sin x$: $y \in [-2 ; 2]$ **OR** $-2 \le y \le 2$ Range of $y = 2\sin x + 3$: $y \in [1 ; 5]$ **OR** $1 \le y \le 5$ Range: $y = 2^{2\sin x + 3}$: $y \in [2 ; 32]$ **OR** $2 \le y \le 32$ Answer only: full marks	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)

/ QUESTION *VRAAG* 6

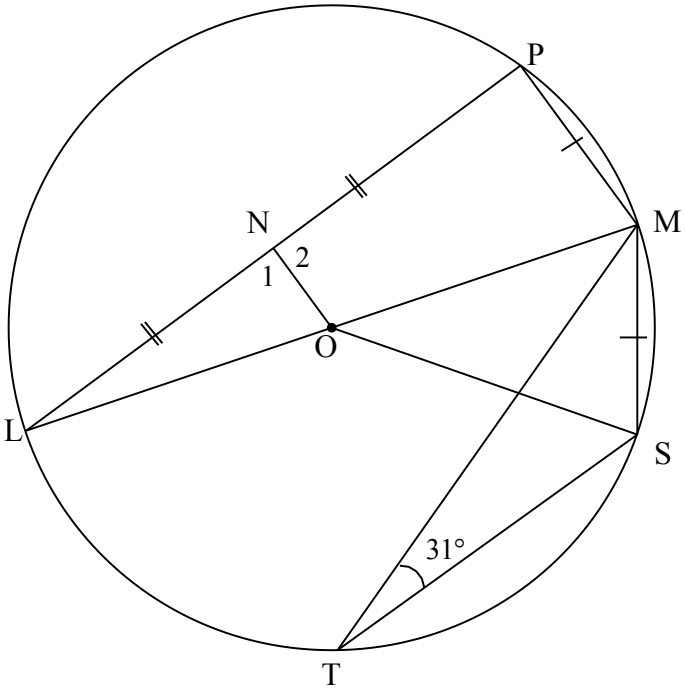


6.1.1	$\sin\theta = \frac{x}{AC}$ **OR** $\frac{\sin\theta}{x} = \frac{\sin 90°}{AC}$ $AC = \frac{x}{\sin\theta}$ $AC = \frac{x}{\sin\theta}$	✓ trig ratio ✓ simplification (2)
6.1.2	$\cos 60° = \frac{x+2}{CE}$ **OR** $\frac{\sin 30}{x+2} = \frac{\sin 90°}{CE}$ $CE = \frac{x+2}{\cos 60°}$ $CE = \frac{x+2}{\sin 30°}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2)$ $= 2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
6.2	$\text{Area } \Delta ACE = \frac{1}{2}AC.EC.\sin A\hat{C}E$ $= \frac{1}{2}\left(\frac{x}{\sin\theta}\right)(2(x+2))\sin 2\theta$ $= \frac{x(x+2)\times 2\sin\theta\cos\theta}{\sin\theta}$ $= 2x(x+2)\cos\theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin\theta}(2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

6.3	$EC = 2(12 + 2) = 28$ $AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos AC\hat{}E$ $= \left(\frac{12}{\sin 55°}\right)^2 + 28^2 - 2\left(\frac{12}{\sin 55°}\right)(28)\cos 110°$ $AE = 35,77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		[11]

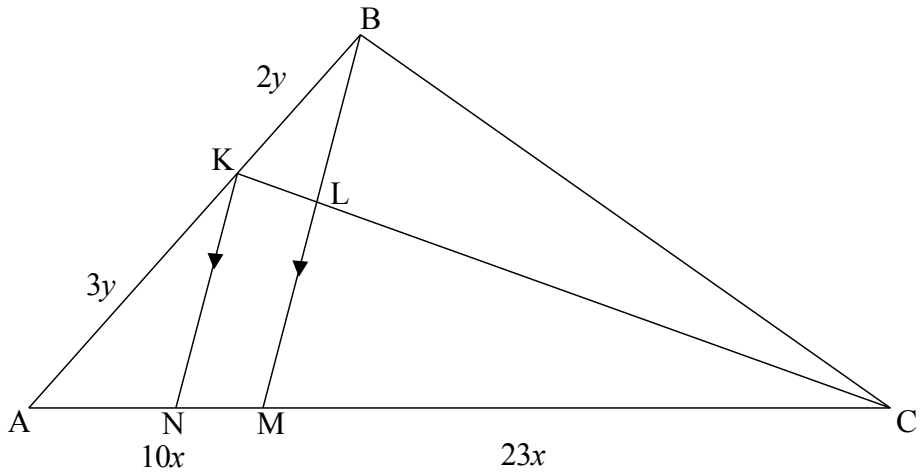
QUESTION/*VRAAG* 7

7.1



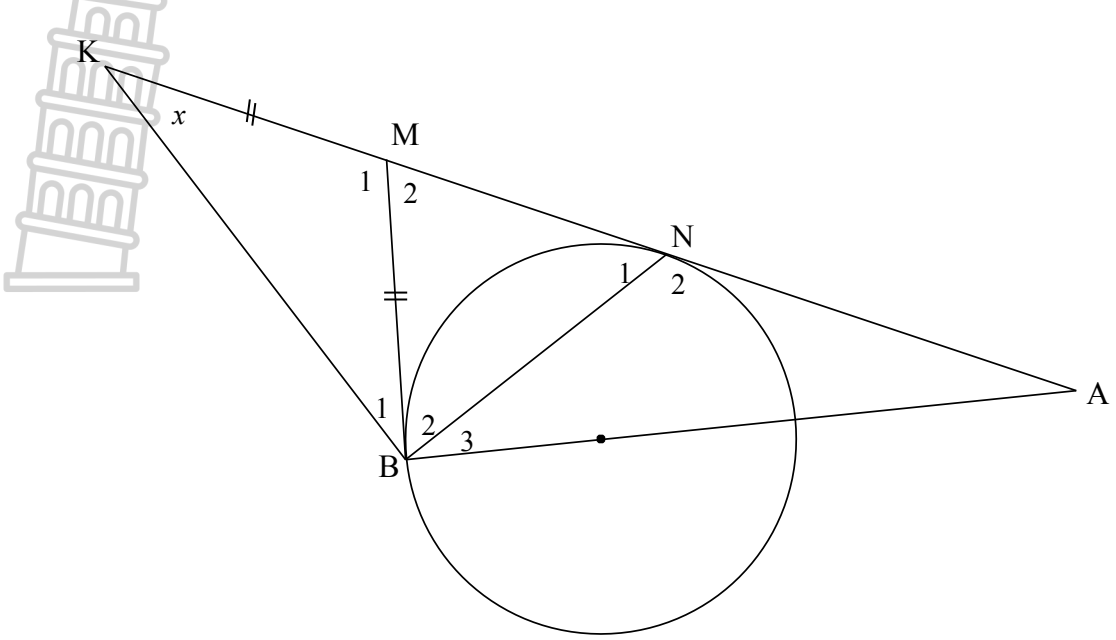
7.1.1(a)	$M\hat{O}S = 62°$ [∠ at centre = 2× ∠ at circumf/*middelpts∠=2omtreks∠*]	✓ S ✓ R (2)
7.1.1(b)	$\hat{L} = 31°$ [equal chords; equal ∠s /= *koorde; = ∠e*]	✓ S ✓ R (2)
7.1.2	LN = NP and LO = OM $\therefore ON = \frac{1}{2}PM$ [midpoint theorem/*middelpuntstelling*] $\therefore ON = \frac{1}{2}MS$ [PM = MS] **OR** $\hat{N}_1 = 90°$ [line from centre to midpt chord/*lyn v midpt na midpt kd*] $\hat{P} = 90°$ [∠ in semi-circle/*∠ in halfsirkel*] $\hat{L}$ is common/*gemeen* $\therefore \Delta NLO \,|||\, \Delta PLM$ (∠∠∠) $\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}$ $\therefore ON = \frac{1}{2}PM$ $\therefore ON = \frac{1}{2}MS$ [PM = MS]	✓ LO = OM ✓ S ✓ R ✓ S (4) ✓ S R ✓ S/R ✓ S ✓ S (4)

7.2



7.2.1	$\frac{AN}{AM}=\frac{AK}{AB}$ [line || one side of Δ **OR** prop theorem; KN ||BM/ *lyn || sy van Δ **OR** eweredigheidst; KN||BM*] $\frac{AN}{AM}=\frac{3y}{5y}=\frac{3}{5}$	✓ R ✓ S (2)
7.2.2	$\frac{AM}{MC}=\frac{10x}{23x}$ [given] $AM = 5y = 10x \quad \therefore \quad y = 2x$ $\frac{LC}{KL}=\frac{MC}{NM}$ [line || one side of Δ **OR** prop theorem; KN ||LM/ *lyn || sy van Δ **OR** eweredigheidst; KN||BM*] $=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$ **OR** $\frac{AM}{MC}=\frac{10x}{23x}$ [given] $\frac{AN}{MN}=\frac{3y}{2y}=\frac{6x}{4x}$ $\frac{LC}{KL}=\frac{MC}{NM}$ [line || one side of Δ **OR** prop theorem; KN ||LM/ *lyn || sy van Δ **OR** eweredigheidst; KN||BM*] $=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$	✓ S ✓ R ✓ S (3) ✓ S ✓ R ✓ S (3)
		[13]

QUESTION/*VRAAG* 8

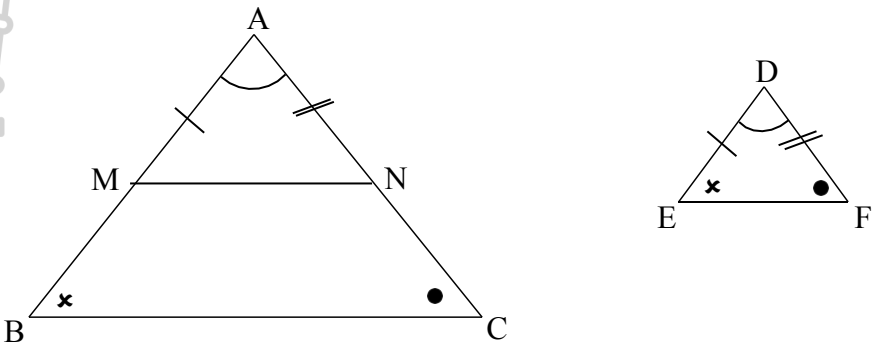


8.1	$\hat{B}_1 = x$ [∠'s opp = sides/*∠e teenoor = sye*] $\hat{M}_2 = 2x$ [ext ∠ of Δ] **OR** $\hat{M}_1 = 180° - 2x$ [∠s of Δ] BM = MN [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{N}_1 = \frac{180° - 2x}{2} = 90° - x$ [∠'s opp = sides/*∠e teenoor = sye*] **OR** NM = BM [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{B}_2 = \hat{N}_1$ [∠'s opp = sides/*∠e teenoor = sye*] $\hat{B}_1 = x$ [∠'s opp = sides/*∠e teenoor = sye*] In Δ KBN: $x + x + \hat{B}_2 + \hat{N}_1 = 180°$ [sum of ∠'s of Δ] $2x + 2\hat{N}_1 = 180°$ $x + \hat{N}_1 = 90°$ $\hat{N}_1 = 90° - x$	✓S ✓S ✓R ✓S ✓R ✓answer (6) ✓S ✓R ✓S ✓R ✓S ✓answer (6)
8.2	$M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent⊥diameter/*raaklyn⊥middellyn*] $\hat{B}_3 = 90° - \hat{B}_2$ $= 90° - (90° - x) = x$ $\hat{B}_3 = \hat{K} = x$ ∴ AB is a tangent/*raaklyn* converse tan-chord theorem/*omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ S ✓ R (5)

	OR $\hat{B}_2 = \hat{N}_1$ $\hat{B}_1 + \hat{B}_2 = x + (90° - x) = 90°$ $\therefore$ KN is diameter/*middellyn* [converse $\angle$ in semi-circle/ *omgekeerde $\angle$ in halfsirkel*] $M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent $\perp$diameter] $\therefore$ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ R ✓ R (5)
		[11]

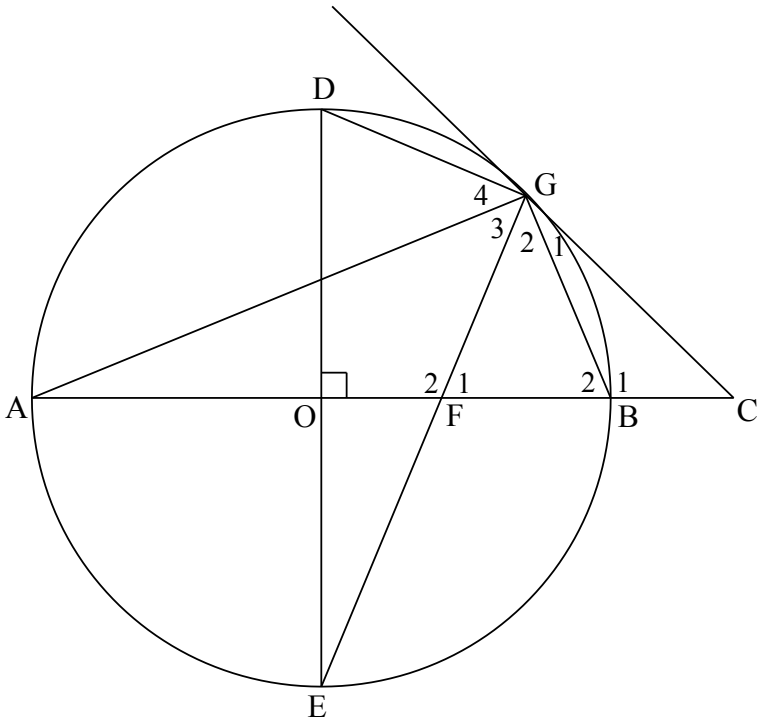
QUESTION/*VRAAG* 9

9.1



9.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. *Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.* Proof: In Δ AMN and Δ DEF AM = DE [Constr] AN = DF [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore \Delta AMN \equiv \Delta DEF$(SAS) $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ‖ BC [corresp ∠'s are equal/*ooreenkomstige ∠e* =] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ‖ one side of Δ **OR** prop theorem; MN ‖BC] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM = DE and AN = DF]	✓ Constr / *Konstr* ✓ $\Delta AMN \equiv \Delta DEF$ ✓ SAS ✓ MN ‖ BC and R ✓ $\frac{AB}{AM} = \frac{AC}{AN}$ ✓R (6)

9.2



9.2.1(a)	$D\hat{O}B=90°$ $D\hat{G}F=\hat{G}_3+\hat{G}_4=90°$ [∠ in semi-circle/∠ *in halfsirkel*] $D\hat{O}B+D\hat{G}F=180°$ ∴ DGFO is a cyclic quad. [converse: opp ∠s of cyclic quad/ *omgekeerde teenoorst ∠e v koordevh*] OR ∠s of quad = 180°/*∠e van koordevh = 180°*] **OR** $E\hat{O}B=90°$ $D\hat{G}F=\hat{G}_3+\hat{G}_4=90°$ [∠ in semi-circle/∠ *in halfsirkel*] $E\hat{O}B=D\hat{G}F$ ∴ DGFO is a cyclic quad. . [converse: ext ∠ = opp int ∠/ *omgekeerde buite∠ = teenoorst ∠*] OR ext∠ of quad = opp int ∠/*buite∠ v vh = teenoorst ∠*]	✓ S ✓R ✓ R (3) ✓ S ✓R ✓ R (3)
9.2.1(b)	$\hat{F}_1=\hat{D}$ [ext ∠of cyclic quad/buite∠ v koordevh] $\hat{G}_1+\hat{G}_2=\hat{D}$ [tan-chord theorem/raakl koordst] $\therefore \hat{F}_1=\hat{G}_1+\hat{G}_2$ ∴ GC = CF [sides opp equal ∠s/sye teenoor = ∠e]	✓ S ✓ R ✓ S ✓ R ✓ R (5)

9.2.2(a)	$AB = DE = 14$ [diameters/*middellyne*] $\therefore OB = 7$ units $\therefore BC = OC - OB = 11 - 7$ $= 4$ units Answer only: full marks	✓ S ✓ S ✓ S (3)
9.2.2(b)	In ΔCGB and ΔCAG $\hat{G}_1 = \hat{A} = x$ [tan-chord theorem/*raakl koordst*] $\hat{C} = \hat{C}$ [common] $\Delta CGB \text{ ||| } \Delta CAG$ [$\angle, \angle, \angle$] $\frac{CG}{CA} = \frac{CB}{CG}$ $\frac{CG}{18} = \frac{4}{CG}$ $CG^2 = 72$ $CG = \sqrt{72}$ or $6\sqrt{2}$ *or* 8,49 units	✓ S/R ✓ S ✓ S ✓ CA = 18 ✓ answer (5)
9.2.2(c)	$OF = OC - FC$ $= 11 - \sqrt{72}$ $\tan E = \frac{OF}{OE}$ $= \frac{11 - \sqrt{72}}{7} = 0,36$ $\hat{E} = 19,76°$ **OR** $OF = OC - FC$ $= 11 - \sqrt{72}$ $FE^2 = OE^2 + OF^2$ $= 7^2 + (11 - \sqrt{72})^2$ $FE = 7,437.. = 7,44$ $\cos E = \frac{OE}{FE}$ *OR* $\sin E = \frac{OF}{FE}$ $= \frac{7}{7,44} = 0,94$ $= \frac{11 - \sqrt{72}}{7,44} = 0,338$ $\hat{E} = 19,76°$ $\hat{E} = 19,76°$	✓ OF ✓ trig ratio ✓ substitution ✓ answer (4) ✓ OF ✓ trig ratio ✓ substitution ✓ answer (4)
		[26]

	TOTAL/*TOTAAL*:	**140**