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PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

VHEMBE WEST DISTRICT

GRADE 12

MATHEMATICS P1

PRE-MIDYEAR EXAMINATION 2024

Stanmorephysics.com

MARKS: 120
TIME.   : 2, 5 *HOURS*

This question paper consists of 7 pages **including the cover page**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 11 questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

8.   Diagrams are NOT necessarily drawn to scale.

9.   An information sheet with formulae is included at the end of the question paper.

10.   Write neatly and legibly.

QUESTION 1

1.1   Solve for x:

1.1.1 $x^2 - 5x - 6 = 0$ (2)

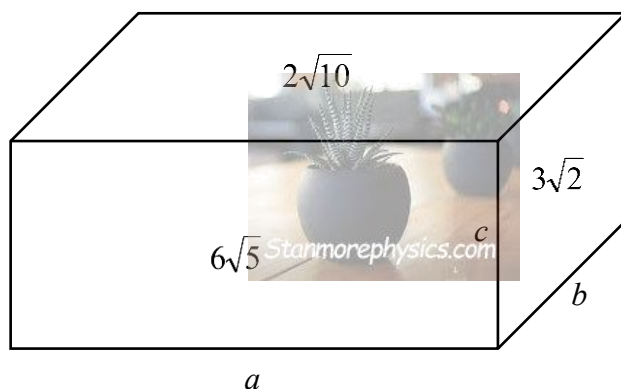1.1.2 $(3x-1)(x-4) = 16$ (correct to TWO decimal places) (4)

1.1.3 $4x - x^2 \geq 0$ (3)

1.1.4 $\dfrac{5^{2x} - 1}{5^x + 1} = 4$ (3)

1.2   Solve simultaneously for x and y:

$x + 3y = 2$ and $x^2 + 4xy - 5 = 0$ (5)

1.3   A rectangular box has dimensions a, b and c. The area of the surfaces are $2\sqrt{10}$; $3\sqrt{2}$ and $6\sqrt{5}$, as shown in the diagram below.

Calculate, **without using a calculator**, the volume of the rectangular box. (5)

[22]

QUESTION 2

2.1   The first FOUR terms of a quadratic pattern are: 15 ; 29 ; 41 ; 51

2.1.1   Write down the value of the 5th term. (1)

2.1.2   Determine an expression for the nth term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.3   Determine the value of T_{27} (2)

2.2 Given a geometric sequence: $36 \; ; \; -18 \; ; \; 9 \; ; \; \ldots$

    2.2.1   Determine the value of r, the common ratio.   (1)

    2.2.2   Calculate n if $T_n = \dfrac{9}{4\,096}$ (3)

    2.2.3   Calculate S_∞ (2)

    2.2.4   Calculate the value of $\dfrac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$ (4)

[17]

QUESTION 3

3.1 The first three terms of an arithmetic sequence are: $2p + 3 \; ; \; p + 6$ and $p - 2$.
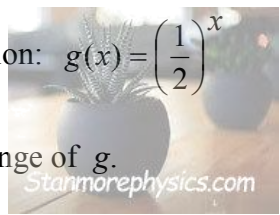
    3.1.1   Show that $p = 11$.   (2)

    3.1.2   Calculate the smallest value of n for which $T_n < -55$.   (3)

3.2 Given that $\displaystyle\sum_{k=1}^{6}(x - 3k) = \sum_{k=1}^{9}(x - 3k)$, prove that $\displaystyle\sum_{k=1}^{15}(x - 3k) = 0$.   (5)

[10]

QUESTION 4

Given the exponential function: $g(x) = \left(\dfrac{1}{2}\right)^x$

4.1 Write down the range of g.   (1)

4.2 Determine the equation of g^{-1} in the form $y = \ldots$ (2)

4.3 Is g^{-1} a function? Justify your answer.   (2)

4.4 The point $M(a \; ; 2)$ lies on g^{-1}.

    4.4.1   Calculate the value of a.   (2)

    4.4.2 $M^{/}$, the image of M, lies on g. Write down the coordinates of $M^{/}$.   (1)

4.5 If $h(x) = g(x + 3) + 2$, write down the coordinates of the image of $M^{/}$ on h.   (3)

[11]

QUESTION 5

5.1 Given: $f(x) = \dfrac{1}{x+2} + 3$.
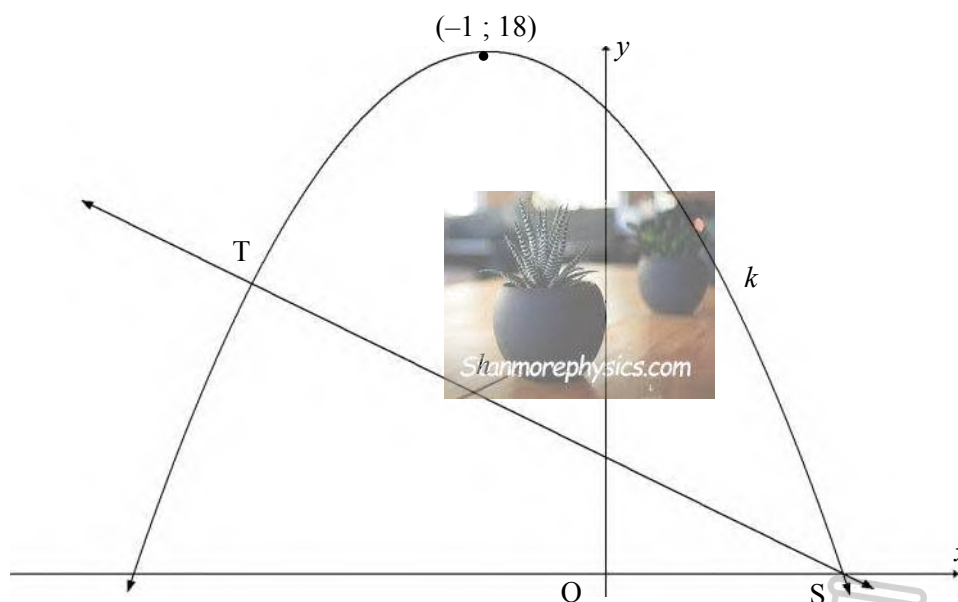
       5.1.1   Determine the equations of the asymptotes of f.   (2)

       5.1.2   Write down the y-intercept of f.   (1)

       5.1.3   Calculate the x-intercept of f.   (2)

       5.1.4   Sketch the graph of f. Clearly label ALL intercepts with the axes and any asymptotes.   (3)

5.2 Sketched below are the graphs of $k(x) = ax^2 + bx + c$ and $h(x) = -2x + 4$. Graph k has a turning point at (-1 ; 18). S is the x-intercept of h and k. Graphs h and k also intersect at T.

       5.2.1   Calculate the coordinates of S.   (2)

       5.2.2   Determine the equation of k in the form $y = a(x + p)^2 + q$ (3)

       5.2.3   If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T.   (4)

       5.2.4   Determine the value(s) of x for which $k(x) < h(x)$.   (2)

       5.2.5   It is further given that k is the graph of $g^{/}(x)$.

           (a)   For which values of x will the graph of g be concave up?   (2)

           (b)   Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection.   (3)

           [24]

QUESTION 6

6.1   Given $f(x) = x^2 + 2$.

Determine $f'(x)$ from first principles.   (5)

6.2   Determine $\dfrac{dy}{dx}$ if:

6.2.1 $y = 4x^3 + \dfrac{2}{x}$ (3)

6.2.2 $y = 4.\sqrt[3]{x} + (3x^3)^2$ (4)

6.3   If g is a linear function with $g(1) = 5$ and $g'(3) = 2$, determine the equation of g
in the form $y = \dots$ (3)

[15]

QUESTION 7

A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x-axis at $(-3\ ;\ 0)$; $\left(-\dfrac{3}{2}\ ;\ 0\right)$ and $(1\ ;\ 0)$.

7.1   Show that $h(x) = -2x^3 - 7x^2 + 9$.   (3)

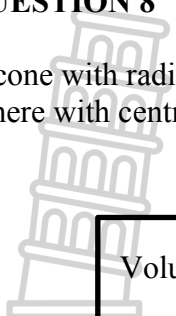7.2   Calculate the x-coordinates of the turning points of h.   (3)

7.3   Determine the value(s) of x for which h will be decreasing.   (2)
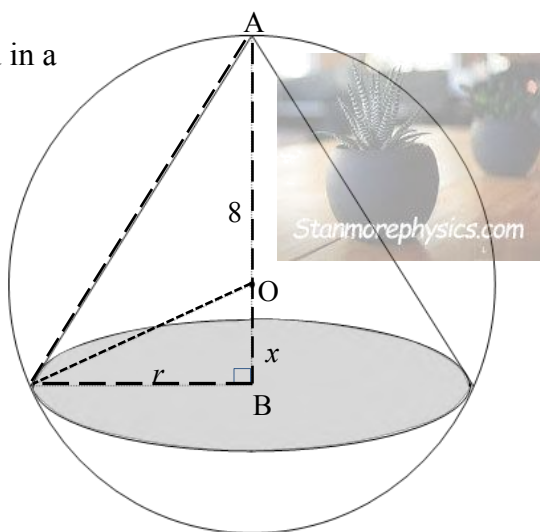
7.4   For which value(s) of x will there be a tangent to the curve of h that is parallel to
the line $y - 4x = 7$.   (4)

[12]

QUESTION 8

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. $OB = x$.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$
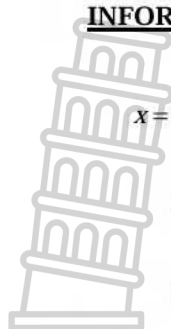
$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

8.1 Calculate the volume of the sphere. (1)

8.2 Show that $r^2 = 64 - x^2$. (1)

8.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)

[9]

TOTAL: **120**

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n$$

$$A = P(1 + i)^n \qquad \sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d) \qquad T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2 \qquad \text{In } \Delta ABC: \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area}\Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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Mathematics III

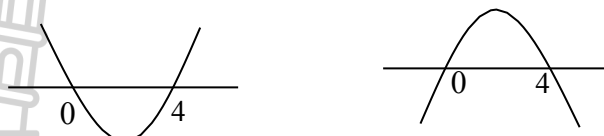Marking Guidelines/*Nasienriglyne*

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
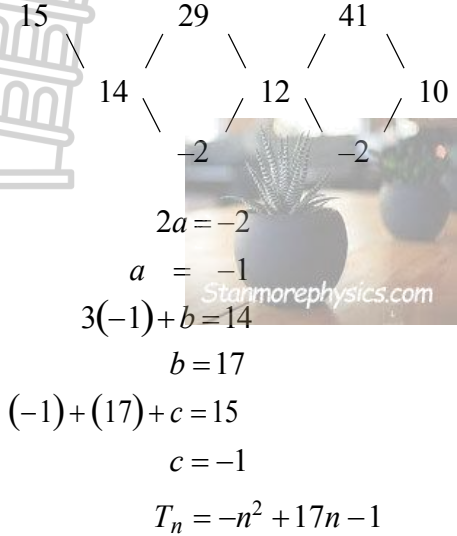- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die memorandum van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$x^2 - 5x - 6 = 0$ $(x - 6)(x + 1) = 0$ $x = 6$ or $x = -1$ **OR/*OF*** $x^2 - 5x - 6 = 0$ $x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$ $x = \dfrac{5 \pm \sqrt{49}}{2}$ $x = 6$ or $x = -1$	✓ factors ✓ both answers (2) **OR/*OF*** ✓ correct subst into correct formula ✓ both answers (2)
1.1.2	$(3x - 1)(x - 4) = 16$ $3x^2 - 13x - 12 = 0$ $x = \dfrac{13 \pm \sqrt{(-13)^2 - 4(3)(-12)}}{2(3)}$ $x = \dfrac{13 \pm \sqrt{313}}{6}$ $x = 5,12$ or $x = -0,78$ **OR/*OF*** $3x^2 - 13x - 12 = 0$ $x^2 - \dfrac{13}{3}x = 4$ $x^2 - \dfrac{13}{3}x + \left(-\dfrac{13}{6}\right)^2 = 4 + \left(-\dfrac{13}{6}\right)^2$ $\left(x - \dfrac{13}{6}\right)^2 = \dfrac{313}{36}$ $x = \dfrac{13 \pm \sqrt{313}}{6}$ $x = 5,12$ or $x = -0,78$	✓ standard form ✓ correct subst into correct formula ✓ ✓ answers (4) **OR/*OF*** ✓ standard form ✓ adding $\left(-\dfrac{13}{6}\right)^2$ both sides ✓ ✓ answers (4)

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Mathematics III

Marking Guidelines/Nasienriglyne

1.1.3	$x(4-x) \geq 0$ $-x(x-4) \geq 0$ or $-x(x-4) \geq 0$ $x(x-4) \leq 0$ $0 \leq x \leq 4$ or $x \in [0\,;4]$	✓ factorisation ✓✓ $0 \leq x \leq 4$ (3)
1.1.4	$\dfrac{5^{2x}-1}{5^{x}+1} = 4$ $\dfrac{(5^{x}+1)(5^{x}-1)}{5^{x}+1} = 4$ $5^{x}-1 = 4$ $5^{x} = 5$ $x = 1$ **OR/*OF*** $\dfrac{5^{2x}-1}{5^{x}+1} = 4$ $5^{2x}-1 = 4.5^{x}+4$ $5^{2x}-4.5^{x}-5 = 0$ $(5^{x}-5)(5^{x}+1) = 0$ $5^{x} = 5$ or $5^{x} \neq -1$ $x = 1$	✓ factors in numerator ✓ $5^{x}-1 = 4$ ✓ answer (3) **OR/*OF*** ✓ standard form ✓ factors ✓ answer (3)

1.2	$x = 2 - 3y$ .... (1)
	 $x^{2} + 4xy - 5 = 0$ ....(2)
	 Substitute (1) in (2):
	      $(2-3y)^{2} + 4y(2-3y) - 5 = 0$
	 $4 - 12y + 9y^{2} + 8y - 12y^{2} - 5 = 0$
	             $-3y^{2} - 4y - 1 = 0$
	              $3y^{2} + 4y + 1 = 0$
	            $(3y+1)(y+1) = 0$
	        $y = -\dfrac{1}{3}$ or $y = -1$
	          $x = 3$ or $x = 5$
	
	 OR/*OF*
		✓ $x = 2 - 3y$
		
		 ✓ correct subst into correct formula
		
		 ✓ either standard form
		
		 ✓ y – values
		 ✓ x – values
		                (5)
		
		 OR/*OF*

Mathematics III

Marking Guidelines/Nasienriglyne

	$y = \dfrac{2}{3} - \dfrac{x}{3}$ .... (1) $x^2 + 4xy - 5 = 0$ ....(2) Substitute (1) in (2): $x^2 + 4x\left(\dfrac{2}{3} - \dfrac{x}{3}\right) - 5 = 0$ $3x^2 + 8x - 4x^2 - 15 = 0$ $-x^2 + 8x - 15 = 0$ $x^2 - 8x + 15 = 0$ $(x-5)(x-3) = 0$ $x = 3$ or $x = 5$ $y = -\dfrac{1}{3}$ or $y = -1$	$y = \dfrac{2}{3} - \dfrac{x}{3}$ ✓ correct subst into correct formula ✓ either standard form ✓ x – values ✓ y – values (5)
1.3	$ab = 2\sqrt{10}$ $bc = 3\sqrt{2}$ $ac = 6\sqrt{5}$ $ab.bc.ac = 2\sqrt{10}.6\sqrt{5}.3\sqrt{2}$ $(abc)^2 = 36\sqrt{100}$ $abc = \sqrt{360} = 6\sqrt{10}$ **OR/OF** $ac = 6\sqrt{5}$ ∴ $a = \dfrac{6\sqrt{5}}{c}$ $bc = 3\sqrt{2}$ ∴ $b = \dfrac{3\sqrt{2}}{c}$ $ab = 2\sqrt{10}$ $\left(\dfrac{6\sqrt{5}}{c}\right)\left(\dfrac{3\sqrt{2}}{c}\right) = 2\sqrt{10}$ $18\sqrt{10} = 2\sqrt{10}.c^2$ $c^2 = 9$ $c = 3$ Volume $= abc = 2\sqrt{10}.3 = \sqrt{360} = 6\sqrt{10}$	☐ volume $= abc$ ☐ $ab.bc.ac = 2\sqrt{10}.6\sqrt{5}.3\sqrt{2}$ ☐ $(abc)^2 = 36\sqrt{100}$ ☐ answer (5) **OR/OF** ☐ $a = \dfrac{6\sqrt{5}}{c}$ ☐ $b = \dfrac{3\sqrt{2}}{c}$ ☐ value of c ☐ Volume $= abc$ ☐ answer (5) **[22]**

Mathematics/P1

QUESTION/*VRAAG* 2

2.1.1	59	answer (1)
2.1.2	$$\begin{array}{ccccccc} 15 & & 29 & & 41 & & 51 \\ & 14 & & 12 & & 10 \\ & & -2 & & -2 \end{array}$$ $$2a = -2$$ $$a = -1$$ $$3(-1) + b = 14$$ $$b = 17$$ $$(-1) + (17) + c = 15$$ $$c = -1$$ $$T_n = -n^2 + 17n - 1$$	$\checkmark$ second difference of -2 $\checkmark$ a $\checkmark$ b $\checkmark$ c (4)
2.1.3	$$T_{27} = -(27)^2 + 17(27) - 1$$ $$= -271$$	substitution answer (2)
2.2.1	$$r = \frac{-18}{36} = -\frac{1}{2}$$	$\checkmark$ answer (1)
2.2.2	$$T_n = 36\left(-\frac{1}{2}\right)^{n-1}$$ $$\frac{9}{4096} = 36\left(-\frac{1}{2}\right)^{n-1}$$ $$\frac{1}{16384} = \left(-\frac{1}{2}\right)^{n-1}$$ $$\left(-\frac{1}{2}\right)^{14} = \left(-\frac{1}{2}\right)^{n-1}$$ $$14 = n - 1$$ $$n = 15$$ **OR/*OF*** $36 \, ; -18 \, ; 9 \, ; \dfrac{-9}{2} \, ; \dfrac{9}{4} \, ; \dfrac{-9}{8} \, ; \dots ; \dfrac{9}{4096}$ If you look only at the denominator: $2 \, ; 4 \, ; 8 \, ; \dots \, ; 4096$ $$2^k = 4096$$ $$2^k = 2^{12}$$ $$k = 12$$ $\therefore n = 15$ terms	$T_n = 36\left(-\frac{1}{2}\right)^{n-1}$ $\dfrac{1}{16384} = \left(-\frac{1}{2}\right)^{n-1}$ answer (3) **OR/*OF*** $2^k = 4096$ $k = 12$ answer (3)

Marking Guidelines/Nasienriglyne

2.2.3	$S_\infty = \dfrac{a}{1-r}$ $= \dfrac{36}{1-\left(-\dfrac{1}{2}\right)}$ $= 24$	✓ correct subst into correct formula with $-1 < r < 1$ ✓ answer if $-1 < r < 1$ (2)
2.2.4	$S_{250\,\text{even}} = \dfrac{-18\left(\left(\dfrac{1}{4}\right)^{250}-1\right)}{\dfrac{1}{4}-1}$ $= -24$ $S_{250\,\text{odd}} = \dfrac{36\left(\left(\dfrac{1}{4}\right)^{250}-1\right)}{\dfrac{1}{4}-1}$ $= 48$ $\dfrac{S_{odd}}{S_{even}} = \dfrac{48}{-24}$ $= -2$ **OR/OF** $\dfrac{T_1 + T_3 + T_5 + T_7 + ... + T_{499}}{T_2 + T_4 + T_6 + T_8 + ... + T_{500}}$ $= \dfrac{a + ar^2 + ar^4 + ... + ar^{498}}{ar + ar^3 + ar^5 + ... + ar^{499}}$ $= \dfrac{a + ar^2 + ar^4 + ... + ar^{498}}{r\left(a + ar^2 + ar^4 + ... + ar^{498}\right)}$ $= \dfrac{1}{r}$ $= -2$	✓ $r = \dfrac{1}{4}$ and $n = 250$ ✓ $S_{250\,\text{even}} = -24$ ✓ $S_{250\,\text{odd}} = 48$ ✓ answer (4) **OR/OF** ✓ $a + ar^2 + ar^4 + ... + ar^{498}$ ✓ $ar + ar^3 + ar^5 + ... + ar^{499}$ ✓ $r\left(a + ar^2 + ar^4 + ... + ar^{498}\right)$ ✓ answer (4) **[17]**

Mathematics/P1

QUESTION/VRAAG 3

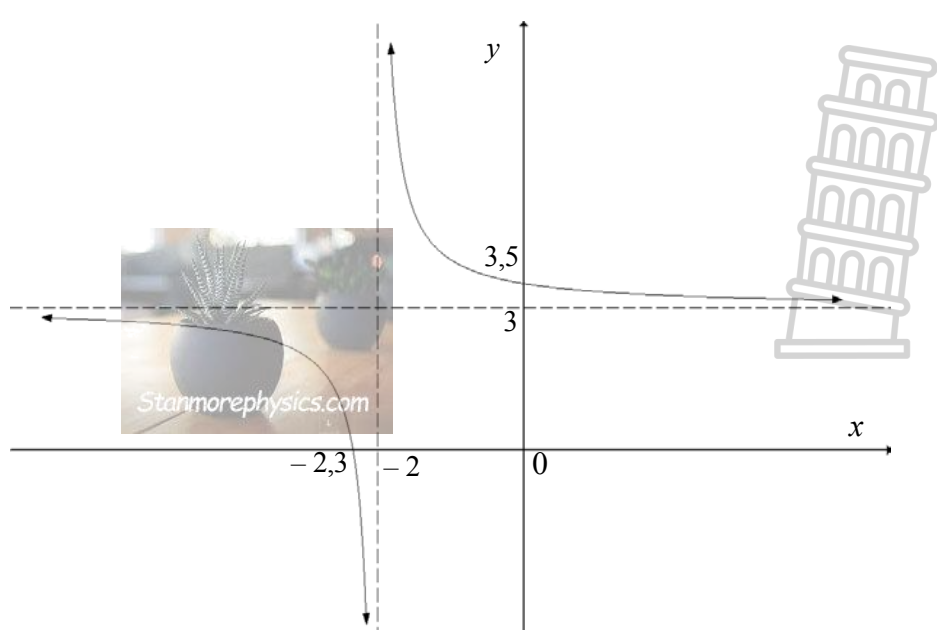| 3.1.1	$p + 6 - (2p + 3) = p - 2 - (p + 6)$ $-p + 3 = -8$ $p = 11$	✓ equating i.t.o p ✓ simplifying (2)
3.1.2	$T_n = 25 + (n-1)(-8) = 33 - 8n$ $33 - 8n < -55$ $-8n < -88$ $n > 11$ ∴Term 12 will be the first term smaller than -55 ∴*Term 12 sal die eerste term kleiner as -55 wees.*	✓ subst into T_n formula ✓ $n > 11$ ✓ $n = 12$ (3)
3.2	$S_6 = \dfrac{n}{2}[a+l] = \dfrac{6}{2}[(x-3)+(x-18)]$ $\qquad = 6x - 63$ $S_9 = \dfrac{n}{2}[a+l] = \dfrac{9}{2}[(x-3)+(x-27)]$ $\qquad = 9x - 135$ $6x - 63 = 9x - 135$ $3x = 72$ $x = 24$ $\therefore S_{15} = \dfrac{n}{2}[a+l] = \dfrac{15}{2}[(x-3)+(x-45)]$ $\qquad = \dfrac{15}{2}[2x - 48]$ $\qquad = \dfrac{15}{2}[2(24)-48] = 0 = $ RHS **OR/OF** $\displaystyle\sum_{k=7}^{9}(x-3k)=0$ $(x-21)+(x-24)+(x-27)=0$ $\therefore 3x - 72 = 0$ $3x = 72$ $x = 24$ $\displaystyle\sum_{k=1}^{15}(24-3k)$ $= 21+18+15+.....+-21.$ $S_n = \dfrac{n}{2}[a+l]$ $= \dfrac{15}{2}[21-21]$ $= 0 = $ RHS **OR/OF**	✓ $6x - 63$ ✓ $9x - 135$ ✓ 24 ✓ $\dfrac{15}{2}[(x-3)+(x-45)]$ ✓ substitution of x (5) **OR/OF** ✓ expansion ✓ $3x - 72 = 0$ ✓ 24 ✓ substitution of x ✓ sum of 15 terms (5) **OR/OF**

Mathematics/P1

Marking Guidelines/Nasienriglyne

	$(x-3)+(x-6)+(x-9)+(x-12)+(x-15)+(x-18)$	✓ expansion
	$=(x-3)+(x-6)+(x-9)+(x-12)+(x-15)+(x-18)$	
	$\qquad +(x-21)+(x-24)+(x-27)$	
	$\therefore 3x-72=0$	✓ $3x-72=0$
	$\qquad 3x=72$	
	$\qquad x=24$	✓ 24
	$\sum\limits_{k=1}^{15}(24-3k)$	
	$=21+18+15+.....+-21.$	✓ substitution of x
	$S_n=\dfrac{n}{2}\big[a+l\big]$	
	$\qquad =\dfrac{15}{2}\big[21-21\big]$	✓ sum of 15 terms (5)
	$\qquad =0=\text{RHS}$	**[10]**
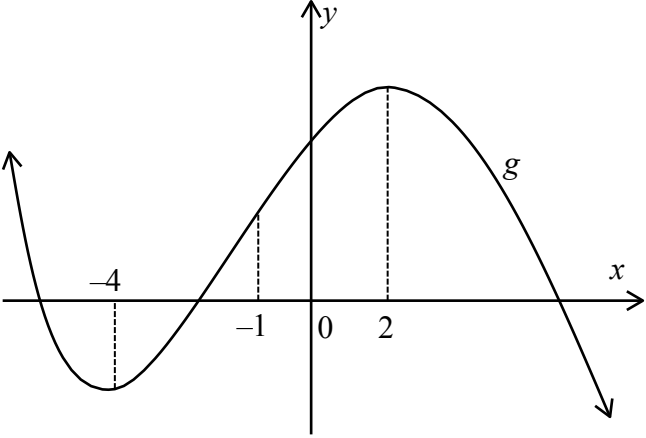
QUESTION/*VRAAG* 4

4.1	$y>0$	✓ answer
		(1)
	OR/*OF*	**OR/*OF***
	$y\in\left(0\;;\;\infty\right)$	✓ answer
		(1)
4.2	$g:y=\left(\dfrac{1}{2}\right)^x$	
	$g^{-1}:x=\left(\dfrac{1}{2}\right)^y$	✓ $x=\left(\dfrac{1}{2}\right)^y$
	$y=\log_{\frac{1}{2}}x$ or $y=-\log_2 x$ or $y=\log_2\dfrac{1}{x}$	✓ equation (2)
4.3	Yes. The vertical line test cuts g^{-1} once	✓ yes
	Ja. Die vertikale lyn toets sny g^{-1} slegs eenkeer.	✓ valid reason (2)
	OR/*OF*	**OR/*OF***
	Yes. For every x-value there is a unique y-value	✓ yes
	Ja. Vir elke x-waarde is daar 'n unieke y-waarde	✓ valid reason
		(2)
	OR/*OF*	**OR/*OF***
	Yes. g is a one-to-one function / *Ja. g is 'n een-tot-een funksie*	✓ yes
		✓ valid reason
		(2)
	OR/*OF*	**OR/*OF***
	Yes. The horizontal line cuts g only once	✓ yes
	Ja. Die horisontale lyn sny g slegs een keer	✓ valid reason
		(2)

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Mathematics/P...

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4.4.1	$y = -\log_2 x$ $2 = -\log_2 a$ $a = 2^{-2} = \dfrac{1}{4}$ or $a = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$	✓ correct subst into correct formula $(a\,;2)$ ✓ answer (2)
4.4.2	$M'\left(2\,;\dfrac{1}{4}\right)$ or $M'(2\,;a)$	✓ answer (1)
4.5	$M''\left(-1\,;\dfrac{9}{4}\right)$	✓ -1 ✓✓ $\dfrac{9}{4}$ (3)
		[11]

QUESTION/*VRAAG* 5

5.1.1	$x = -2$ $y = 3$	✓ answer ✓ answer (2)
5.1.2	$\left(0\,;\dfrac{7}{2}\right)$	✓ answer (1)
5.1.3	$\dfrac{1}{x+2} + 3 = 0$ $1 + 3(x+2) = 0$ $3x = -7$ $x = -\dfrac{7}{3}$ x-intercept $\left(-\dfrac{7}{3}\,;0\right)$	✓ $y = 0$ ✓ answer (2)
5.1.4		✓ asymptotes at $y = 3$ and $x = -2$ ✓ intercepts at $y = 3,5$ and $x = -2,3$ ✓ shape (reasonable representation in correct quadrants) (3)

Mathematics III

Marking Guidelines/*Nasienriglyne*

5.2.1	$-2x+4=0$ $\qquad 2x=4$ $\qquad x=2$ $\therefore S(2\,;\,0)$	$\checkmark$ $y=0$ $\checkmark$ $x=2$ (2)
5.2.2	Equation of k: $y=a(x+1)^2+18$ $\quad 0=a(2+1)^2+18 \qquad$ or $\quad 0=a(-4+1)^2+18$ $9a=-18$ $a=-2$ $y=-2(x+1)^2+18$	$\checkmark$ $y=a(x+1)^2+18$ $\checkmark$ substitute $(2\,;\,0)$ or $(-4\,;\,0)$ $\checkmark$ a (3)
5.2.3	$-2x^2-4x+16=-2x+4$ $-2x^2-2x+12=0$ $\qquad x^2+x-6=0$ $\qquad (x+3)(x-2)=0$ $\qquad\qquad x=-3 \text{ or } x=2$ $y=-2(-3)+4=10$ $T(-3\,;\,10)$	$\checkmark$ equating $\checkmark$ standard form $\checkmark$ choosing $x=-3$ $\checkmark$ answer (4)
5.2.4	$x<-3 \text{ or } x>2$ **OR/*OF*** $(-\infty\,;\,-3)\cup(2\,;\,\infty)$	$\checkmark\checkmark$ answer (2) **OR/*OF*** $\checkmark\checkmark$ answer (2)
5.2.5(a)	$x<-1$ **OR/*OF*** $(-\infty\,;\,-1)$	$\checkmark$ answer (2) **OR/*OF*** $\checkmark$ answer (2)
5.2.5(b)		▢ shape of cubic with local min tp moving to local max tp ▢ turning points at $x=2$ and $x=-4$ ▢ point of inflection at $x=-1$ (3) **[24]**

Mathematics III

QUESTION/*VRAAG* 6

6.1		
$f(x) = x^2 + 2$		
$f(x + h) = (x + h)^2 + 2$		
$\quad\quad\quad = x^2 + 2xh + h^2 + 2$		
$f(x + h) - f(x) = x^2 + 2xh + h^2 + 2 - (x^2 + 2)$	✓ $x^2 + 2xh + h^2 + 2$	
$\quad\quad\quad = 2xh + h^2$		
$f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$		
$\quad = \lim_{h \to 0} \dfrac{2xh + h^2}{h}$	✓ $\lim_{h \to 0} \dfrac{2xh + h^2}{h}$	
$\quad = \lim_{h \to 0} \dfrac{h(2x + h)}{h}$	✓ $\lim_{h \to 0} \dfrac{h(2x + h)}{h}$	
$\quad = \lim_{h \to 0}(2x + h) \;\checkmark$		
$\quad = 2x$	✓ answer	
		(5)
OR/*OF*	**OR/*OF***	
$f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$		
$= \lim_{h \to 0} \dfrac{x^2 + 2xh + h^2 + 2 - (x^2 + 2)}{h}$	✓ $x^2 + 2xh + h^2 + 2$	
$= \lim_{h \to 0} \dfrac{2xh + h^2}{h}$	✓ $\lim_{h \to 0} \dfrac{2xh + h^2}{h}$	
$= \lim_{h \to 0} \dfrac{h(2x + h)}{h}$	✓ $\lim_{h \to 0} \dfrac{h(2x + h)}{h}$	
$= \lim_{h \to 0}(2x + h)$		
$= 2x$	✓ answer	(5)

6.2.1	$y = 4x^3 + 2x^{-1}$ $\dfrac{dy}{dx} = 12x^2 - 2x^{-2}$	✓ $+ 2x^{-1}$ ✓ $12x^2$ ✓ $-2x^{-2}$ (3)
6.2.2	$y = 4\sqrt[3]{x} + (3x^3)^2$ $= 4x^{\frac{1}{3}} + 9x^6$ $\dfrac{dy}{dx} = \dfrac{4}{3}x^{-\frac{2}{3}} + 54x^5$	✓ $4x^{\frac{1}{3}}$ ✓ $9x^6$ ✓ $\dfrac{4}{3}x^{-\frac{2}{3}}$ ✓ $54x^5$ (4)
6.3	Point of contact: (1 ; 5) $m = 2$ $y - y_1 = m(x - x_1)$ or $y = 2x + c$ $y - 5 = 2(x - 1)$ $5 = 2 + c$ $c = 3$ $y = 2x + 3$ $y = 2x + 3$	✓ $m = 2$ ✓ substitution of (1 ; 5) ✓ answer (3) **[14]**

QUESTION/VRAAG 7

7.1	$h(x) = -2(x + \dfrac{3}{2})(x - 1)(x + 3)$ $h(x) = -(2x + 3)(x^2 + 2x - 3)$ $h(x) = -2x^3 - 7x^2 + 9$ **OR/OF** $h(x) = -(2x + 3)(x - 1)(x + 3)$ $h(x) = -(2x + 3)(x^2 + 2x - 3)$ $h(x) = -2x^3 - 7x^2 + 9$	✓✓ $-2(x + \dfrac{3}{2})(x - 1)(x + 3)$ ✓ correct simplification (3) **OR/OF** ✓✓ $-(2x + 3)(x - 1)(x + 3)$ ✓ correct simplification (3)
7.2	$h'(x) = -6x^2 - 14x$ $-6x^2 - 14x = 0$ $-2x(3x + 7) = 0$ $x = 0$ or $x = -\dfrac{7}{3}$	✓ first derivative ✓ $= 0$ ✓ both answers (3)
7.3	$x < -\dfrac{7}{3}$ or $x > 0$ **OR/OF** $x \in \left(-\infty ; -\dfrac{7}{3} \right) \cup \left(0 ; \infty \right)$	✓✓ answer (2) **OR/OF** ✓✓ answer (2)

| 7.4 | $y = 4x + 7$

 $-6x^2 - 14x = 4$

 $0 = 6x^2 + 14x + 4$

 $0 = 3x^2 + 7x + 2$

 $0 = (3x+1)(x+2)$

 $x = -\dfrac{1}{3}$ or $x = -2$ | ✓ $y = 4x + 7$
 ✓ $h^{/}(x) = 4$

 ☐ standard form

 ✓ both answers

 (5)
 [12] |

QUESTION/*VRAAG* 9

8.1	Volume of Sphere $= \dfrac{4}{3}\pi(8)^3$ or $= \dfrac{2048\pi}{3}$ or $= 2144,66$	✓ answer (1)
8.2	$r^2 + x^2 = 8^2$ (Pythagoras) $r^2 = 64 - x^2$	✓ substitution or reason Pythagoras (1)
8.3	$V_{cone} = \dfrac{1}{3}\pi r^2 h$ $= \dfrac{1}{3}\pi(64 - x^2)(8 + x)$ $= \dfrac{\pi}{3}(512 + 64x - 8x^2 - x^3)$ $\dfrac{dV}{dx} = \dfrac{64\pi}{3} - \dfrac{16\pi}{3}x - \dfrac{3\pi}{3}x^2$ $0 = 64 - 16x - 3x^2$ $0 = (8 - 3x)(x + 8)$ $x = \dfrac{8}{3}$ $x \neq -8$ $\dfrac{V_{cone}}{V_{sphere}} = \dfrac{\dfrac{1}{3}\pi\left(\dfrac{512}{9}\right)\left(\dfrac{32}{3}\right)}{\dfrac{2048\pi}{3}}$ $= \dfrac{8}{27} = 0,3$	✓ $h = 8 + x$ ✓ $\dfrac{1}{3}\pi(64 - x^2)(8 + x)$ ✓ expansion ✓ $\dfrac{dV}{dx} = \dfrac{64\pi}{3} - \dfrac{16\pi}{3}x - \dfrac{3\pi}{3}x^2$ ✓ $x = \dfrac{8}{3}$ ✓ volume of the cone ✓ $\dfrac{8}{27}$ or $0,3$ (7) **[9]**

TOTAL/*TOTAAL*: 120