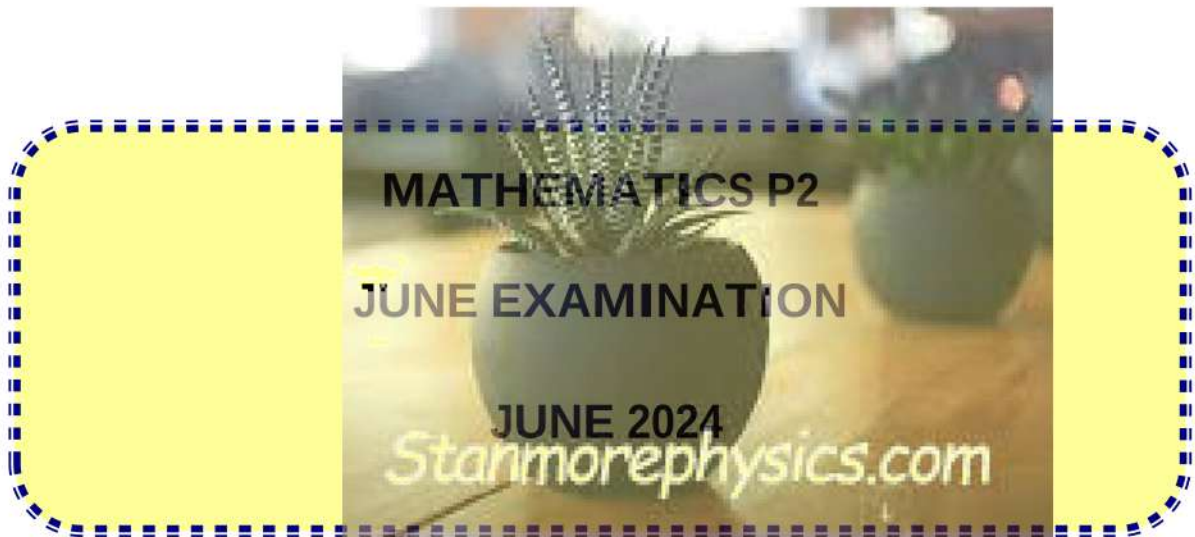




# education

Department of  
Education  
FREE STATE PROVINCE

**GRADE 12**



**MARKS: 150**

**TIME: 3 hours**

This question paper consists of 12 pages

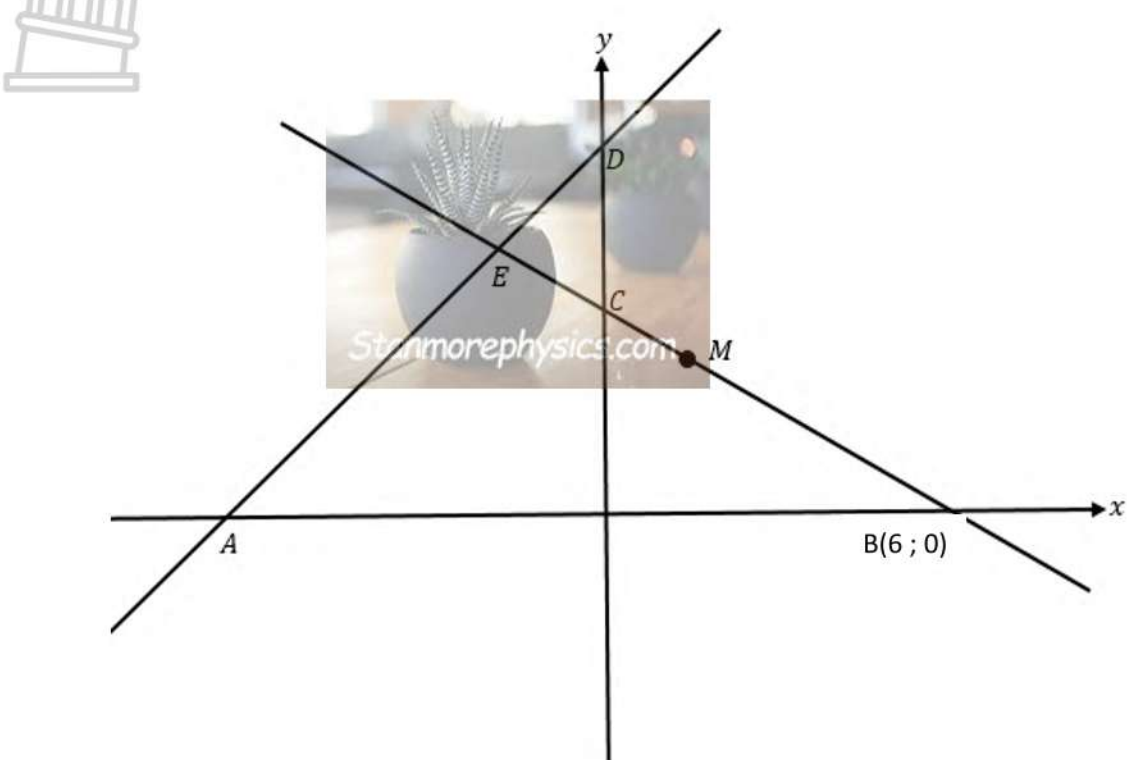
**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions:

1. This question paper consists of 8 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used to determine the answer.
4. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
5. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
6. An INFORMATION SHEET with formulae is included at the end of the question paper.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.
9. Answers only will NOT necessarily be awarded full marks.

**QUESTION 1**

In the diagram, A and D are the  $x$ - and  $y$ -coordinates of line AD respectively. Line BE cuts the  $x$ -axis at B and the  $y$ -axis at C. The equations of AD and EB are  $y = 2x + 10$  and  $y = -x + 6$  respectively. M is the midpoint of line BE,  $AE = \frac{11}{3}\sqrt{5}$ . CB and AD intersect at  $E(-4; 8)$ .

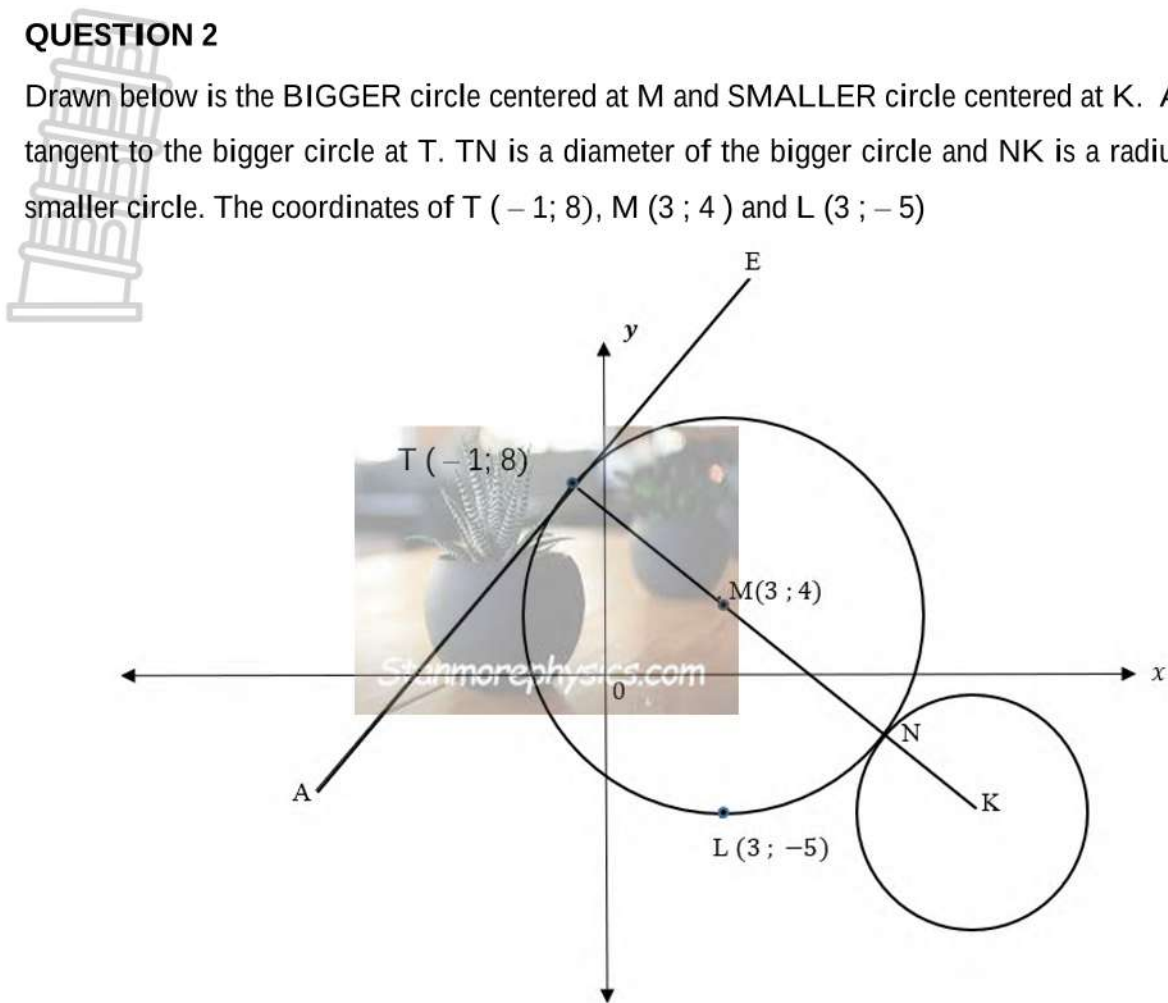


- 1.1 Write down the coordinates of C (1)
- 1.2 Calculate the angle of inclination of line BC. (2)
- 1.3 Calculate the length of line CD. (2)
- 1.4 Determine the equation of line through M, parallel to AD, in the form  $y = \dots$  (4)
- 1.5 Calculate the size of  $\widehat{AEB}$  (4)
- 1.6 Calculate the coordinates of G, such that ABGE is a parallelogram. (4)
- 1.7 Calculate the area of parallelogram ABGE (5)
- 1.8 If it is given that CD is a diameter of a circle passing through C and D, determine how many units must M be translated so that it becomes the center of the new circle. (4)

**[26]**

**QUESTION 2**

Drawn below is the BIGGER circle centered at M and SMALLER circle centered at K. ATE is a tangent to the bigger circle at T. TN is a diameter of the bigger circle and NK is a radius of the smaller circle. The coordinates of T ( - 1; 8), M (3 ; 4 ) and L (3 ; - 5)



- 2.1 Determine the equation of the circle in the form  $(x - a)^2 + (y - b)^2 = r^2$ . (2)
- 2.2 Determine the equation of the tangent through point T. (5)
- 2.3 Does point P(7; 3) lie inside, outside or on the circle. Show all calculations. (4)
- 2.4 If it is further given that KL is a tangent at L, to the circle centered at M. Determine the coordinates of K, the center of the smaller circle. (5)

**[16]**

**QUESTION 3**

3.1 Given :  $\sin \beta = \frac{1}{3}$  where  $\beta \in (90^\circ; 270^\circ)$ , determine the following by using a sketch and without the use of a calculator:

3.1.1  $\tan \beta$  (3)

3.1.2  $\cos 2\beta$  (2)

3.1.3  $\cos(-\beta - 450^\circ)$  (2)

3.2 Simplify the following to a single trigonometric ratio:

$$\frac{4 \cos(-x) \cdot \cos(90^\circ + x)}{\sin(30^\circ - x) \cdot \cos x + \cos(30^\circ - x) \cdot \sin x} \quad (6)$$

3.3 If  $\cos 23^\circ = a$ , express the following in terms of a:

3.3.1  $\tan 203^\circ$  (3)

3.3.2  $\sin 46^\circ$  (3)

3.4 Determine the values of the following, without using a calculator:

3.4.1  $\sin 105^\circ$  (4)

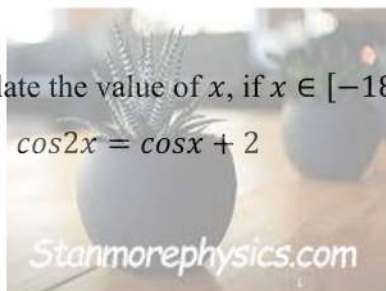
3.4.2  $\cos 69^\circ \cdot \cos 9^\circ + \cos 81^\circ \cdot \cos 21^\circ$  (3)

3.5 Prove the following identity:  $\frac{\sin 2x - \cos x}{1 - \cos 2x - \sin x} = \frac{1}{\tan x}$  (5)

3.6 Calculate the value of  $x$ , if  $x \in [-180^\circ; 360^\circ]$

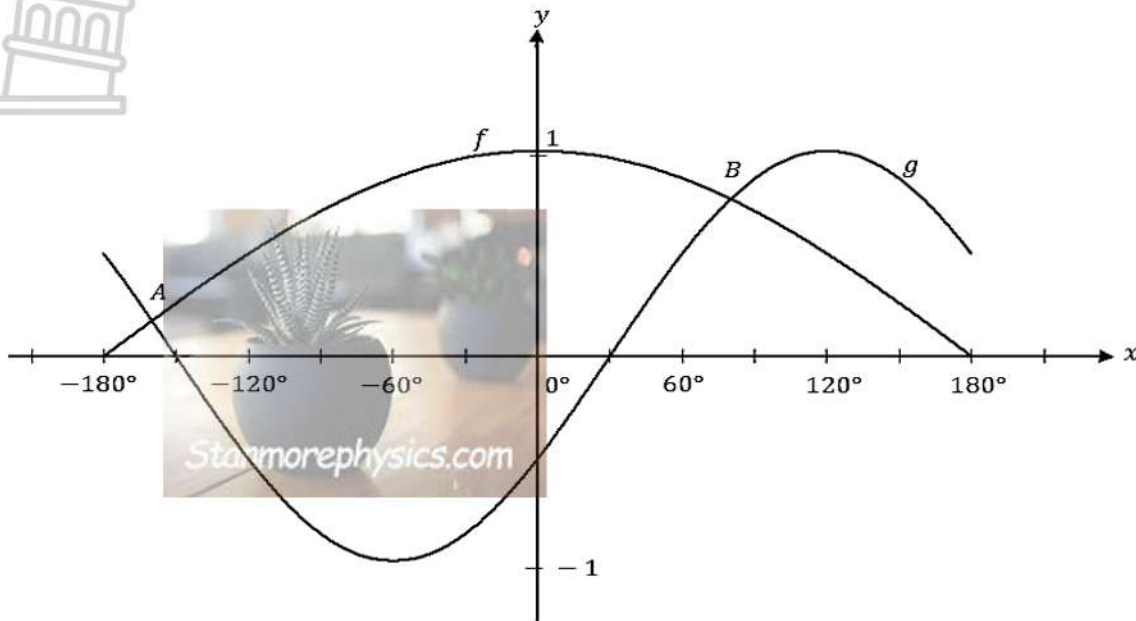
$\cos 2x = \cos x + 2$  (7)

**[38]**



**QUESTION 4**

The graphs of  $f(x) = \cos \frac{x}{2}$  and  $g(x) = \sin(x - 30^\circ)$  for  $x \in [-180^\circ; 180^\circ]$  are drawn below. The graphs intersect at points A and B.

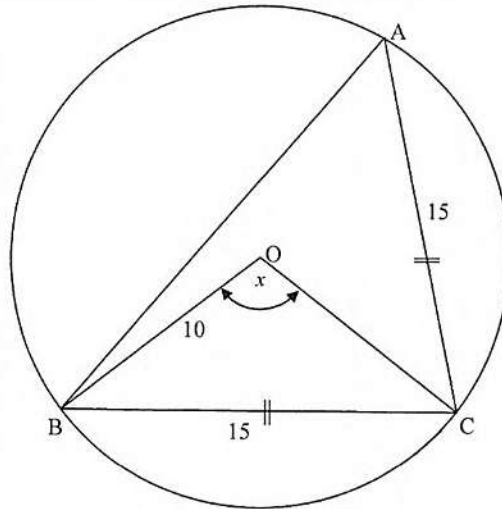
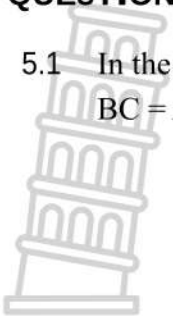


- 4.1 Write down the value of  $f(0^\circ) - g(0^\circ)$  (1)
- 4.2 Give the period of  $f(4x)$  (2)
- 4.3 Write down the range of  $4g(x)$  (2)
- 4.4 Given that the general solution of  $f(x) = g(x)$  is:  $x = 80^\circ - k \cdot 240^\circ, k \in \mathbb{Z}$ .  
 Determine the  $x$  values of A and B. (2)
- 4.5 For which value(s) of  $x$  will.
- 4.5.1  $f(x) > g(x)$  (2)
- 4.5.2  $f'(x) \cdot g(x) > 0$  where  $x > 0^\circ$  (2)

**[11]**

**QUESTION 5**

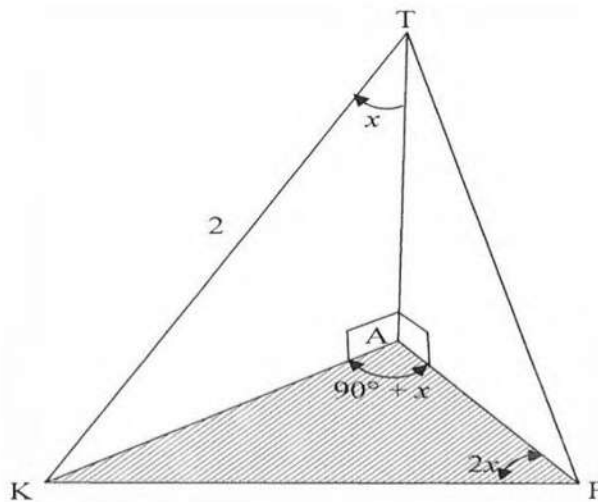
5.1 In the diagram below, a circle with centre O passes through A, B and C.  
 $BC = AC = 15$  units. BO and OC are joined.  $OB = 10$  units and  $\widehat{BOC} = x$



5.1.1 Calculate the size of  $x$ . (3)

5.1.2 Calculate the area of triangle ABC. (4)

5.2 In the figure, points K, A and F lie in the same horizontal plane and TA represents a vertical tower.  $\widehat{ATK} = x$ ,  $\widehat{KAF} = 90^\circ + x$  and  $\widehat{KFA} = 2x$  where  $0^\circ < x < 30^\circ$  and  $TK = 2$  units.



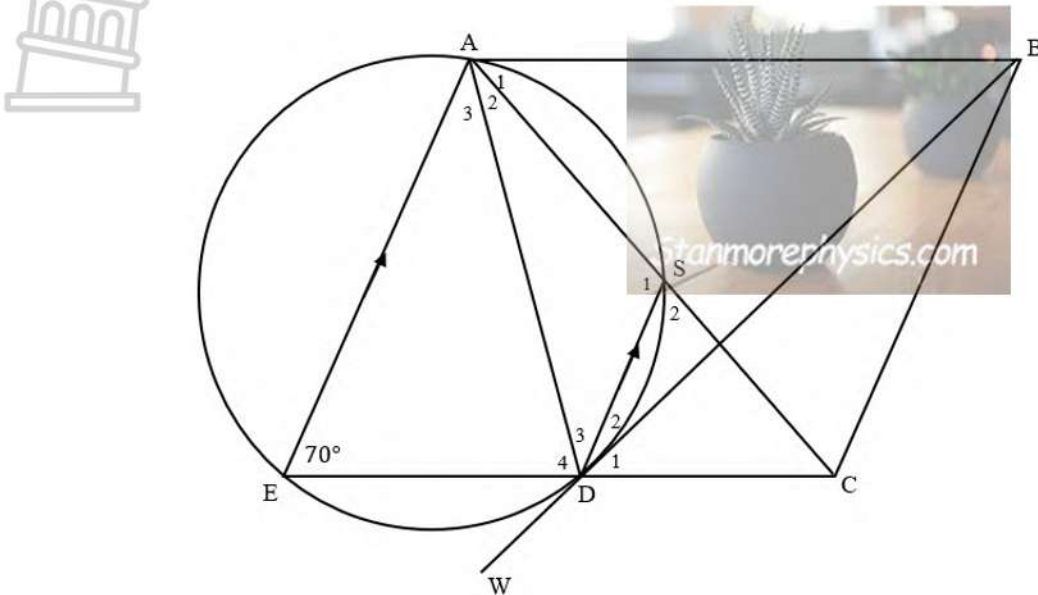
5.2.1 Express AK in terms of  $\sin x$ . (2)

5.2.2 Determine the value of KF (5)

**[14]**

**QUESTION 6**

AB is a tangent to circle ASDE at point A with  $AE \parallel SD$ . Chords AS and ED produced meet at C, such that  $ED = DC$ . BDW is a straight line and in parallelogram ABCE,  $\hat{E} = 70^\circ$  and  $BC = 16\text{cm}$ .



6.1 Determine with reasons:

- 6.1.1  $\hat{S}_2$  (2)
- 6.1.2  $\hat{A}_1$  (3)
- 6.1.3  $\hat{D}_3$  (2)
- 6.1.4 The length of SD. (3)

6.2 If it is further given that  $\hat{ADS} = \hat{BDC}$ , prove with reasons:

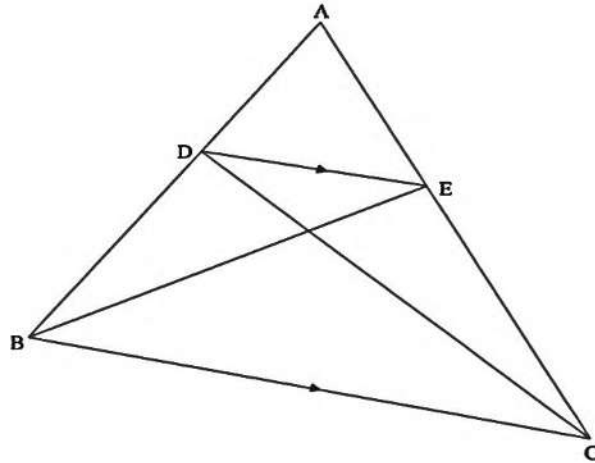
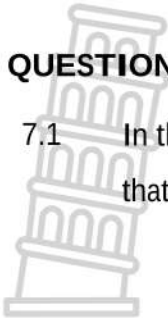
- 6.2.1 ABCD is a cyclic quadrilateral (2)
- 6.2.2 DB is a tangent to circle through A, S and D. (5)

**[17]**



**QUESTION 7**

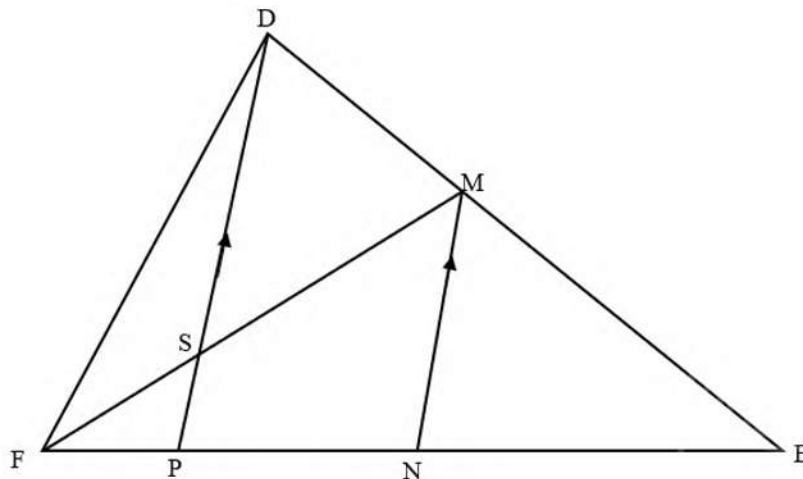
7.1 In the diagram below,  $\triangle ABC$  is drawn. D is a point on AB and E is a point on AC such that  $DE \parallel BC$ . BE and DC are drawn.



Use the diagram above to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove

that  $\frac{AD}{DB} = \frac{AE}{EC}$  (5)

7.2 In  $\triangle DEF$  below,  $DM : ME = 2 : 3$



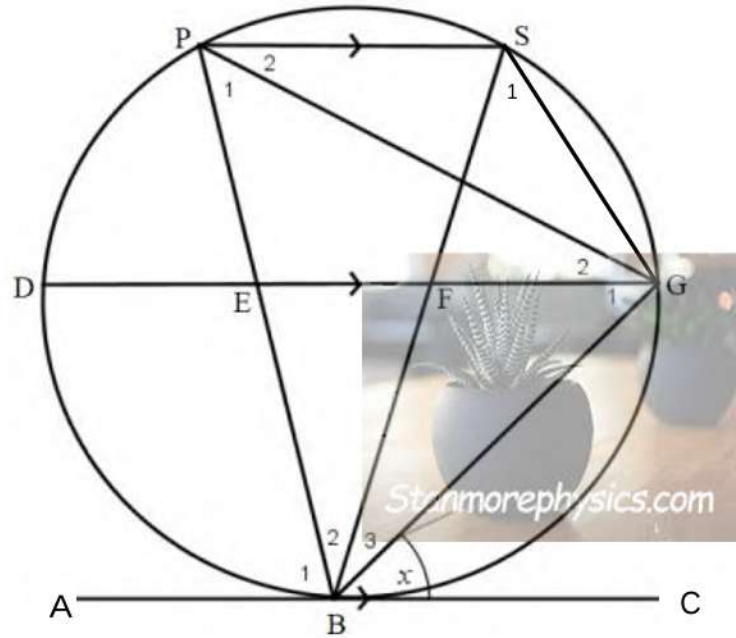
Given  $PE = 35$  cm and  $2FP = PN$ .

Determine:

7.2.1 PN (2)

7.2.2 FS (3)

7.3 In the diagram, P, S, G, B and D are points on the circumference of the circle such that  $PS \parallel DG \parallel AC$ . ABC is a tangent to the circle at B.  $\widehat{GBC} = x$ .



Prove that:

7.3.1  $\triangle PGB \parallel \triangle GEB$  (4)

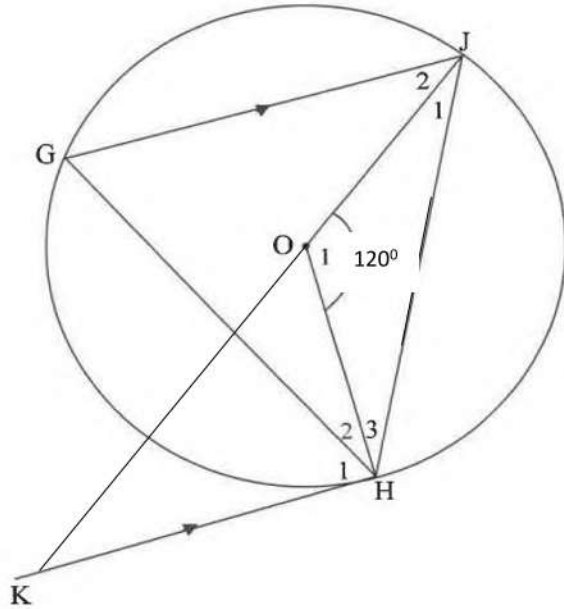
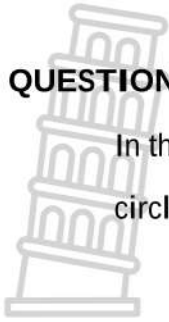
7.3.2  $SB \cdot FB = EB \cdot PB$  (3)

7.3.3 If  $GB = GE = 9\text{cm}$  and  $EB = \frac{3}{5} PG$ , determine the length of PG. (3)

**[20]**

**QUESTION 8**

In the diagram, O is the centre of the circle. GJ, JH and GH are chords of the circle.  $GJ \parallel KH$ .  $\hat{O}_1 = 120^\circ$ . JOK is a straight line.



8.1 Determine, with reasons, the size of the following angles:

8.1.1  $\hat{G}$  (2)

8.1.2  $\hat{H}_3$  (3)

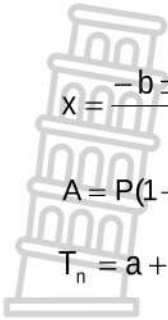
8.2 If KH is a tangent to the circle at H.

Prove:  $\cos 120^\circ = -\frac{OJ}{OK}$  (3)

[8]

**TOTAL 150**

INFORMATION SHEET/INLIGTINGSBLAD



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



# education

Department of  
Education  
FREE STATE PROVINCE

**GRADE 12**

**MATHEMATICS P2**

**JUNE EXAMINATION  
2024**


**MARKING GUIDELINES**

**MARKS: 150**

**This Marking Guidelines consists of 19 pages**

**QUESTION/ VRAAG 1**

1.1	$C(0; 6)$	$C(0; 6) \checkmark$	(1)	
1.2	$\tan C\hat{X} = m$ $\tan C\hat{B}X = -1$ $C\hat{B}X = 135^\circ$	Substitution $\checkmark$  $135^\circ \checkmark$	(2)	
1.3	$D(0; 10)$ $CD = 10 - 6$ $CD = 4$  <b>OR</b> $CD = \sqrt{(0 - 0)^2 + (6 - 10)^2}$ $CD = 4$	$CD = 10 - 6 \checkmark$ $CD = 4 \checkmark$  <b>OR</b> $CD = \sqrt{(0 - 0)^2 + (6 - 10)^2}$ $\checkmark$ $CD = 4 \checkmark$	(2)	
1.4	$E(-4; 8)$  $M\left(\frac{-4+6}{2}; \frac{8+0}{2}\right)$  $M(1; 4)$  $y - y_1 = m(x - x_1)$ $y - 4 = 2(x - 1)$ $y = 2x + 2$	$E\left(-\frac{4}{3}; \frac{22}{3}\right)$  $M\left(\frac{-4+6}{2}; \frac{22+0}{2}\right)$  $M\left(\frac{7}{3}; \frac{11}{3}\right)$  $-\frac{11}{3} = 2\left(-\frac{7}{3}\right) + c$ $y = 2x + 1$	Correct substitution $\checkmark$ answer $\checkmark$      Correct substitution $\checkmark$ equation $\checkmark$	(3)
1.5	$\tan E\hat{A}B = 2$ $E\hat{A}B = 63.43^\circ$  $A\hat{E}B = 135^\circ - 63.43^\circ$ $A\hat{E}B = 71.57^\circ$	$\tan E\hat{A}B = 2 \checkmark$ $E\hat{A}B = 63.43^\circ \checkmark$  $A\hat{E}B = 135^\circ - 63.43^\circ \checkmark$  $71.57^\circ \checkmark$	(4)	
1.6	$2x = -10$ $x = -5$ $A(-5; 0)$	$A(-5; 0) \checkmark$		

<p><b>Using E(-4 ; 8)</b>  midpoint EB = midpoint of AG.  <math>1 = \frac{-5 + x}{2}</math>          <math>4 = \frac{0 + y}{2}</math>  <math>x = 7</math>                                  <math>y = 8</math>  <b>G(7; 8)</b></p> <p style="text-align: center;"><b>OR</b></p> <p><math>2x = -10</math>  <math>x = -5</math>  <b>A(-5; 0)</b></p> <p>By using translation geometry :</p> <p>A → E  <math>(x ; y) \rightarrow (x + 1 ; y + 8)</math>  ∴ B → G                                  <b>B(6 ; 0) → G(6 + 1 ; 0 + 8)</b>    <b>∴ G(7 ; 8)</b></p> <p><b>Using E</b><math>\left(\frac{7}{3} ; \frac{11}{3}\right)</math>  midpoint EB = midpoint of AG.  <math>\frac{7}{3} = \frac{-5 + x}{2}</math>          <math>\frac{11}{3} = \frac{0 + y}{2}</math>  <math>x = \frac{19}{3}</math>                  <math>y = \frac{22}{3}</math>  <b>G</b><math>\left(\frac{19}{3} ; \frac{22}{3}\right)</math></p> <p style="text-align: center;"><b>OR</b></p> <p><math>2x = -10</math>  <math>x = -5</math>  <b>A(-5; 0)</b></p> <p>By using translation geometry :</p> <p>A → E  <math>(x ; y) \rightarrow (x + 0,33 ; y + 7,33)</math>  ∴ B → G  <b>B(6 ; 0) → G(6 + 0,33 ; 0 + 7,33)</b>    <b>G</b><math>\left(\frac{19}{3} ; \frac{22}{3}\right)</math></p>	<p>Method ✓  Midpoint of diagonals  x – Co-ordinate. ✓  y – Co-ordinate. ✓</p> <p><b>OR</b></p> <p><b>A(-5; 0) ✓</b></p> <p>Method ✓  x – Co-ordinate. ✓  y – Co-ordinate. ✓</p> <p>(4)</p> 
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<p>1.7</p>	<p><b>Use G(7 ; 8)</b>                  In parm ABGE                  Perp h = 8 units                  Base = <math>x_B - x_A = 6 - (-5) = 11</math> units                  Area parm ABGE = (perp h)(base)  <math>= (8)(11) = 88</math> units<sup>2</sup></p> <p style="text-align: center;">OR</p> <p>In <math>\Delta AEB</math>                  Perp h = 8 units                  Base = <math>x_B - x_A = 6 - (-5) = 11</math> units                  Area <math>\Delta AEB = \frac{1}{2}(b)(h)</math>  <math>= \frac{1}{2}(11)(8)</math>  <math>= 44</math> units<sup>2</sup>                  Area parm ABGE = (2)(Area <math>\Delta AEB</math>)  <math>= (2)44 = 88</math>units<sup>2</sup></p> <p><b>Use G(<math>\frac{19}{3}</math> ; <math>\frac{22}{3}</math>)</b>                  In parm ABGE                  Perp h = <math>\frac{22}{3}</math> units                  Base = <math>x_B - x_A = 6 - (-5) = 11</math> units                  Area parm ABGE = (perp h)(base)  <math>= 11 \times \frac{22}{3} = 80,67</math> units<sup>2</sup></p>	<p>Perp h = 8 ✓                  Base = 11 units ✓                  Substitution ✓                  Area ✓</p> <p>OR</p> <p>Perp h = <math>\frac{22}{3}</math> ✓                  Base = 11 units ✓                  Substitution ✓</p> <p>Area parm ABGE ✓ (4)</p>
<p>1.8</p>	<p>Midpoint of CD (0 ; 8)</p> <p>M(3 ; 4) → C(0; 8)</p> <p>M must be translated 3 units left and 4 units up.</p>	<p>midpoint ✓</p> <p>✓✓3 units left                  ✓✓4 units up (5)</p>
<p><b>[24]</b></p>		



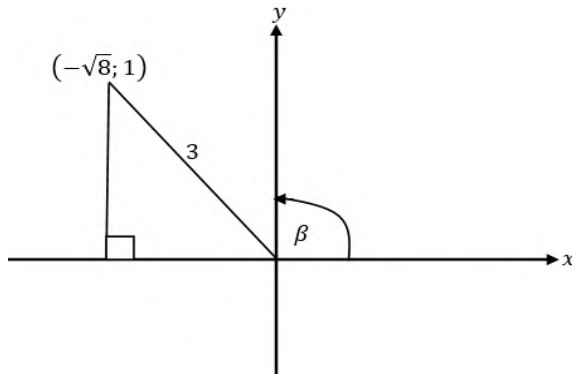
**QUESTION/ VRAAG 2**

<p>2.1</p>	<p><math>ML = 4 - (-5) = 9</math> units.  <math>(x - 3)^2 + (y - 4)^2 = 9^2</math>  <math>(x - 3)^2 + (y - 4)^2 = 81</math></p>	<p>Substitution ✓                  Answer ✓ (2)</p>
<p>2.2</p>	<p><math>T(-1; 8)</math>                      <math>T(-4; 8)</math>  <math>m_r = \frac{8-4}{-1-3} = -1</math>              <math>m_r = \frac{8-4}{-4-3} = -\frac{4}{7}</math>  <math>m_{rad} \times m_{tan} = -1</math>              <math>m_{rad} \times m_{tan} = -1</math>  <math>m_{tan} = 1</math>                                      <math>m_{tan} = \frac{7}{4}</math>   <math>y - y_1 = m(x - x_1)</math>  <math>y - (8) = (x - (-1))</math>              <math>8 = \frac{7}{4}(-4) + c</math>   <math>y = x + 7</math>                                      <math>y = \frac{7}{4}x + 15</math>   <math>T(-1; 4 + \sqrt{65})</math>   <math>m = \frac{4 + \sqrt{65} - 4}{-1 - 3}</math>  <math>m = -2,02</math>  <math>m_{rad} \times m_{tan} = -1</math>  <math>m = \frac{4}{\sqrt{65}}</math>  <math>y = \frac{4}{\sqrt{65}}x + 8 + \sqrt{65}</math></p>	<p><math>m_{rad} \checkmark</math>  <math>m_{rad} \times m_{tan} = -1 \checkmark</math>  <math>m_{tan} \checkmark</math>                   Substitution ✓                   equation ✓ (5)</p>
<p>2.3</p>	<p><math>(x - 3)^2 + (y - 4)^2 = r_2^2</math>  <math>(7 - 3)^2 + (3 - 4)^2 = 17</math>  <math>r_2^2 &lt; r_1^2</math>  <math>17 &lt; 81</math>   <math>\therefore</math> Inside the circle   <b>OR</b></p>	<p>Substitution ✓  <math>r_2^2 = 17 \checkmark</math>  <math>17 &lt; 81 \checkmark</math>                   Inside ✓ (4)   <b>OR</b></p>


	$d = \sqrt{(3 - 7)^2 + (4 - 3)^2}$ $d = \sqrt{17} = 4.12$ $d < r$ $\therefore \text{Inside the circle}$	Substitution ✓ $d = \sqrt{17}$ ✓ $d < r$ ✓ Inside ✓	(4)
2.4	$T(-1; 8)$ $T(-4; 8)$ $y_k = y_l = -5$ $y_k = y_l = -5$  $m_{rad} = m_{MK} = -1$ (collinear points)  $-1 = \frac{4 - (-5)}{3 - x_k}$ $-\frac{4}{7} = \frac{4 - (-5)}{3 - x}$  $-3 + x = 9$ $-12 + 4x = 36$ $12 = x_k$ $12 = x_k$  $T(-1; 4 + \sqrt{65})$ $y_k = y_l = -5$ $m_{rad} = m_{MK} = -1$  $-\frac{\sqrt{65}}{4} = \frac{9}{3 - x}$ $-3\sqrt{65} + \sqrt{65}x = 36$ $7,47 = x_k$	$y_k = -5$ ✓  $m_{rad} = m_{MK}$ ✓✓ substitution ✓      $x\text{-value}$ ✓	(5)
			[14]

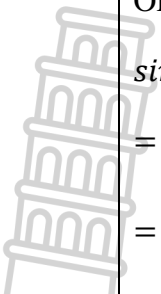


QUESTION/VRAAG 3



3.1.1	$r^2 = x^2 + y^2$ pyth $3^2 = x^2 + 1^2$ $x = -\sqrt{8}$ $\tan\beta = -\frac{1}{\sqrt{8}}$	Method ✓ $x = -\sqrt{8}$ ✓ answer ✓ (3)
3.1.2	$\cos 2\beta = 1 - 2\sin^2 \beta$ $= 1 - 2\left(\frac{1}{3}\right)^2$ $= \frac{7}{9}$	double angle identity ✓ answer ✓ (2)
3.1.3	$\cos(-\beta - 450^\circ) = \cos(-\beta - 90^\circ)$ $= -\cos(90^\circ + \beta)$ $= \sin \beta = \frac{1}{3}$	$-\cos(90^\circ + \beta)$ ✓ Answer ✓ (2)
3.2	$\frac{4 \cos(-x) \cdot \cos(90^\circ + x)}{\sin(30^\circ - x) \cdot \cos x + \cos(30^\circ - x) \cdot \sin x}$ $= \frac{-4 \cos x \sin x}{\frac{1}{2}(\cos^2 x + \sin^2 x)}$ $= \frac{-4 \cos x \sin x}{\frac{1}{2}(1 - \sin^2 x + \sin^2 x)}$ $= -8 \sin x \cos x = -4 \sin 2x$	$4 \cos x$ ✓ $-\sin x$ ✓ $(\cos^2 x + \sin^2 x)$ ✓ $\frac{1}{2}$ ✓ $-8 \sin x \cos x$ ✓ $-4 \sin 2x$ ✓ (6)
3.3.1	$\tan 203^\circ$	$\tan 23^\circ$ ✓

	$\tan 23^\circ = \frac{\sqrt{1-a^2}}{a}$	$\sqrt{1-a^2} \checkmark$ Answer $\checkmark$ (3)
3.3.2	$\begin{aligned} \sin 46^\circ &= \sin 2(23^\circ) \\ &= 2\sin 23^\circ \cos 23^\circ \\ &= 2a\sqrt{1-a^2} \end{aligned}$	double angle $\checkmark$ Expansion $\checkmark$ Answer $\checkmark$ (3)
3.4.1	$\begin{aligned} \sin(60^\circ + 45^\circ) &= \sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \quad \text{OR} \quad \frac{\sqrt{6}+\sqrt{2}}{4} \end{aligned}$ <p>OR</p> $\begin{aligned} &= \sin(180^\circ - 75^\circ) \\ &= \sin 75^\circ \\ &= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ \\ &= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \quad \text{OR} \quad \frac{\sqrt{6}+\sqrt{2}}{4} \end{aligned}$	compound angles $\checkmark$ expansion $\checkmark$ substitution $\checkmark$ answer $\checkmark$ (4)  OR $\sin 75^\circ \checkmark$ expansion $\checkmark$ substitution $\checkmark$ answer $\checkmark$
3.4.2	$\begin{aligned} \cos 69^\circ \cdot \cos 9^\circ + \cos 81^\circ \cdot \cos 21^\circ &= \cos 69^\circ \cdot \cos 9^\circ + \sin 9^\circ \cdot \sin 69^\circ \\ &= \cos 60^\circ \\ &= \frac{1}{2} \end{aligned}$	co-function : $\cos 81^\circ \cdot \cos 21^\circ = \sin 9^\circ \cdot \sin 69^\circ$ $\checkmark$ $\cos 60^\circ \checkmark$ answer $\checkmark$  OR

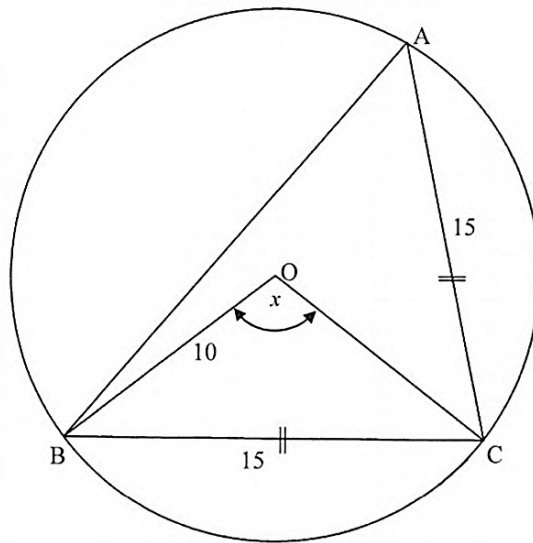
	<p>OR</p> $\sin 21^\circ \cdot \cos 9^\circ + \cos 21^\circ \cdot \sin 9^\circ$ $= \sin 30^\circ$ $= \frac{1}{2}$	<p>co-function</p> $\cos 69^\circ \cdot \cos 9^\circ = \sin 21^\circ \cdot \cos 9^\circ \checkmark$ $\sin 30^\circ \checkmark$ <p>answer <math>\checkmark</math> (3)</p>
<p>3.5</p>	$LHS = \frac{2\sin x \cdot \cos x - \cos x}{1 - (1 - 2\sin^2 x) - \sin x}$ $= \frac{2\sin x \cdot \cos x - \cos x}{2\sin^2 x - \sin x}$ $= \frac{\cos x(2\sin x - 1)}{\sin x(2\sin x - 1)}$ $= \frac{\cos x}{\sin x}$ $= \frac{1}{\frac{\sin x}{\cos x}}$ $\frac{1}{\tan x} = RHS$	<p><math>2\sin x \cdot \cos x \checkmark</math></p> <p><math>1 - 2\sin^2 x \checkmark</math></p> <p>factorization <math>\checkmark</math></p> <p>simplification <math>\checkmark</math></p> <p>identity for <math>\tan x \checkmark</math> (5)</p>
<p>3.6</p>	$\cos 2x = \cos x + 2$ $2\cos^2 x - 1 - \cos x - 2 = 0$ $2\cos^2 x - \cos x - 3 = 0$ $(2\cos x - 3)(\cos x + 1) = 0$ $\cos x \neq \frac{3}{2} \text{ or } \cos x = -1$ $x = 180^\circ$	<p><math>\checkmark</math> expansion</p> <p><math>\checkmark</math> standard form</p> <p><math>\checkmark</math> factors</p> <p><math>\checkmark \checkmark</math> answers</p> <p><math>\checkmark</math> rejection</p> <p><math>\checkmark x = 180^\circ</math> (7)</p>
		<p>[38]</p>



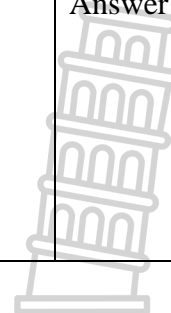
**QUESTION/VRAAG 4**

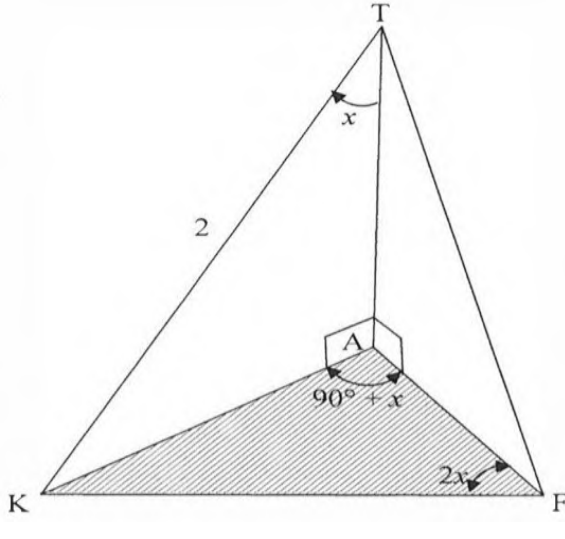
4		
4.1	$f(0^\circ) - g(0^\circ) = 1 - \left(-\frac{1}{2}\right) = \frac{3}{2}$ units	✓ Answer ✓ (1)
4.2	$\text{Period of } f(4x) = \frac{360^\circ}{4\left(\frac{1}{2}\right)}$ $= 180^\circ$	✓ substitution ✓ answer (2)
4.3	Range : $y \in [-4 ; 4]$ or $-4 \leq y \leq 4$	✓ ✓ answer (2)
4.4	$x_A = -160^\circ$ $x_B = 80^\circ$	$x_A = -160^\circ$ ✓ $x_B = 80^\circ$ ✓ (2)
4.5.1	$-160^\circ < x < 80^\circ$	✓ ✓ answer (2)
4.5.2	$0^\circ < x < 30^\circ$	✓ ✓ answer (2)
		<b>[11]</b>

QUESTION/VRAAG 5



5.1.1	<p>In <math>\triangle BOG</math> :</p> $15^2 = 10^2 + 10^2 - 2(10)(10)\cos x$ $225 = 100 + 100 - 200\cos x$ $\cos x = -\frac{25}{200}$ $x = 97,18^\circ$	<p>Substitution into cosine rule ✓                      simplification ✓                      Answer ✓ (3)</p>
5.1.2	$\hat{A} = \frac{1}{2}(\hat{O}) = 48,59^\circ \quad \angle \text{center} = 2 \angle \text{circumference}$ $\hat{C} = 180^\circ - 2(48,59^\circ) = 82,82^\circ$ $\text{Area} = \frac{1}{2}(15)(15)\sin 82,82^\circ$ $= 111,62 \text{ units}^2$	$\hat{A} = 48,59^\circ \checkmark$ $\hat{C} = 82,82^\circ \checkmark$ <p>Area rule ✓                      Answer ✓ (4)</p>

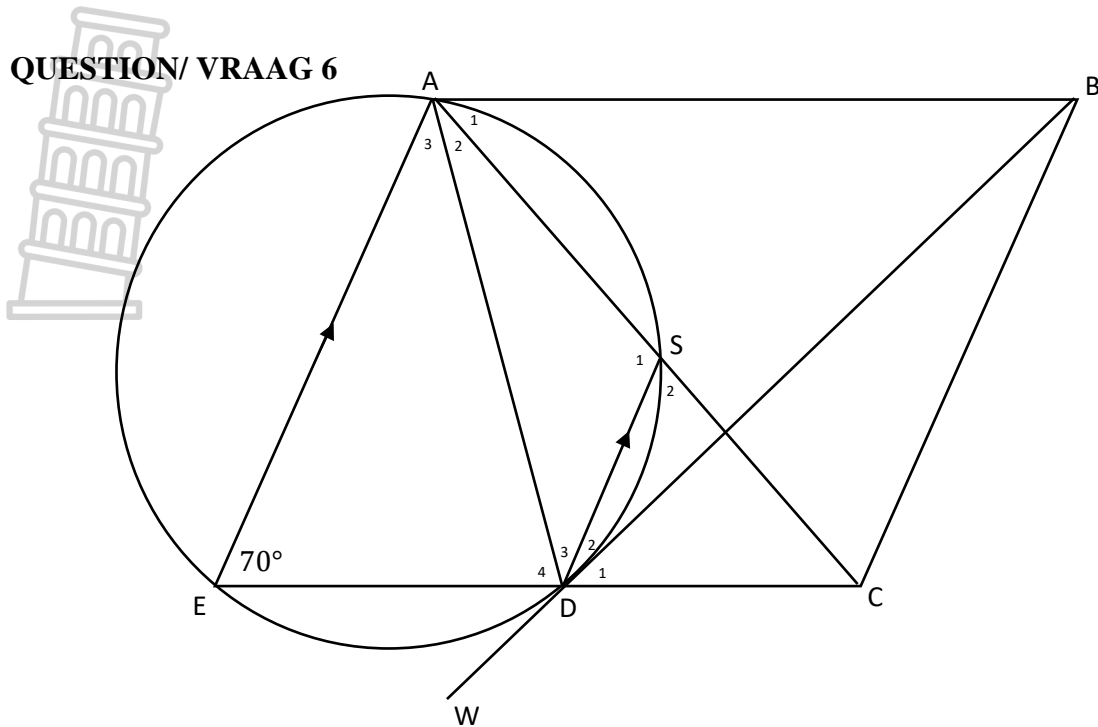


<p>5.2</p> 		
<p>5.2.1</p>	$\frac{AK}{2} = \sin x$ $AK = 2 \sin x$	$\frac{AK}{2} = \sin x \checkmark$ <p>Answer <math>\checkmark</math> (2)</p>
<p>5.2.2</p>	$\frac{KF}{\sin(90^\circ + x)} = \frac{AK}{\sin 2x}$ $\frac{KF}{\cos x} = \frac{2 \sin x}{2 \sin x \cos x}$ $\frac{KF}{\cos x} = \frac{1}{\cos x}$ $KF = 1 \text{ unit}$	<p>Substitution into sine rule <math>\checkmark</math></p> <p><math>\cos x \checkmark</math></p> <p><math>2 \sin x \cos x \checkmark</math></p> <p>Simplification <math>\checkmark</math></p> <p>Answer <math>\checkmark</math> (5)</p>
		<p><b>[14]</b></p>





**QUESTION/ VRAAG 6**



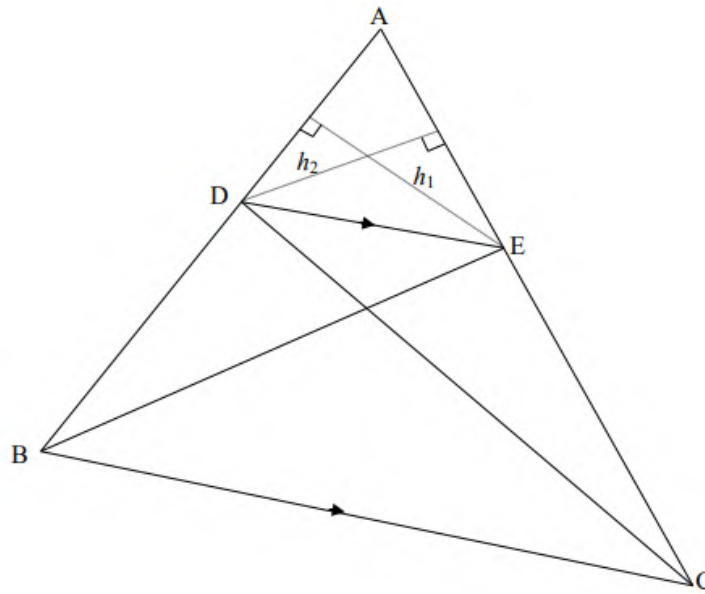
6.1.1	$\hat{S}_2 = \hat{E} = 70^\circ$	ext $\angle$ of cyclic quad / buite $\angle$ van kvh	✓ S ✓ R  (2)
6.1.2	$\hat{S}_2 = \hat{A}_2 + \hat{A}_3 = 70^\circ$  $\hat{A} + \hat{E} = 180^\circ$  $\therefore \hat{A}_1 = 40^\circ$	corresp $\angle$ s; AE // DS / ooreenk. $\angle$ e ; AE // DS  co-int $\angle$ s; AB // EC / ko-binne $\angle$ e ; AB // EC	✓ S/R  ✓ S/R  ✓ S  (3)
6.1.3	$\hat{D}_3 = \hat{A}_1 = 40^\circ$	tan chord theorem / raakl koord stelling	✓ S ✓ R  (2)
6.1.4	AE = BC  ED = DC and AE // SD  $\therefore AS = SC$  $\therefore SD = \frac{1}{2} AE$  SD = 8 cm	Opp sides of parm / Teenoorst. Sye parm  Given / gegee  Converse midpt th. / Omgekeerde middelpt st.	✓ S  ✓ S  ✓ 8 cm  (3)

6.2.1	$\widehat{D}_3 = \widehat{D}_1 = 40^\circ$ $\therefore \widehat{D}_1 = \widehat{A}_1$ $\therefore ABCD$ is cq / kvh	Given / Gegee  converse $\angle$ s in the same seg  omgekeerde $\angle$ e in dies. $\odot$ segm	$\checkmark$ S   $\checkmark$ R  (2)
6.2.2	$\widehat{B} = \widehat{E} = 70^\circ$ $A\widehat{D}C + \widehat{B} = 180^\circ$ $\therefore \widehat{D}_2 = 30^\circ$  $\widehat{S}_2 = \widehat{A}_2 + \widehat{D}_3$ $\therefore \widehat{A}_2 = 30^\circ$  $\therefore \widehat{D}_2 = \widehat{A}_2$ $\therefore DB$ is a tangent / $DB$ is 'n raaklyn	opp $\angle$ s of parm / teenorst. $\angle$ e van parm.  opp $\angle$ s of cyclic quad / teenorst. $\angle$ e van kvh   ext $\angle$ of $\Delta$ / buite $\angle$ van $\Delta$    converse tan chord theorem / omgekeerde raakl. koord stelling	$\checkmark$ S/R   $\checkmark$ S $30^\circ$   $\checkmark$ S / R   $\checkmark$ S  $\checkmark$ R  (5) <b>[17]</b>

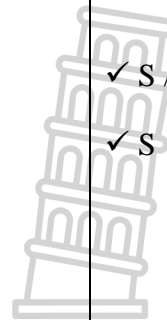


**QUESTION / VRAAG 7**

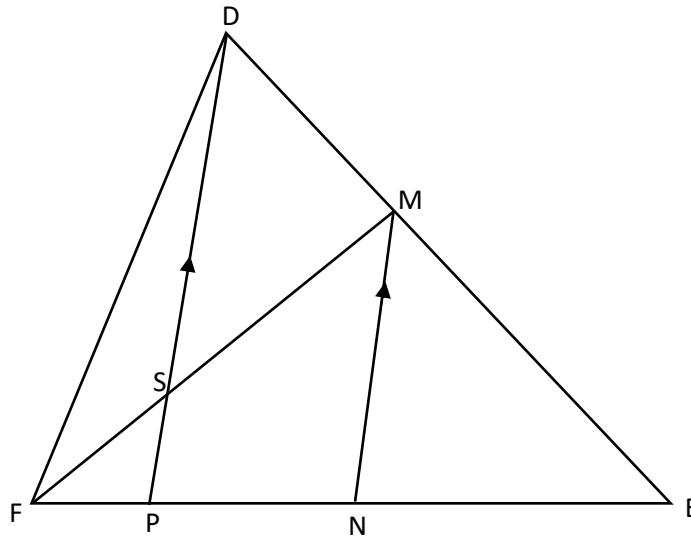
7.1



<p>Constr: Draw <math>h_1</math> from E <math>\perp</math> AD and <math>h_2</math> from D <math>\perp</math> AE                  Konstr: Trek <math>h_1</math> vanaf E <math>\perp</math> AD en <math>h_2</math> vanaf D <math>\perp</math> AE</p> <p>Proof/Bewys:</p> $\frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$ $\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$ <p>But area <math>\triangle BDE = \text{area } \triangle DEC</math> [same base &amp; height or DE <math>\parallel</math> BC/  <i>dies basis &amp; hoogte; of DE <math>\parallel</math> BC]</i></p> $\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	<p>✓ constr/konstr <b>OR</b>                  reason: common                  vertex or                  same height</p> $\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1}$ $\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{AE}{EC}$ <p>✓ S / R                  ✓ S</p>
<p>(5)</p>	



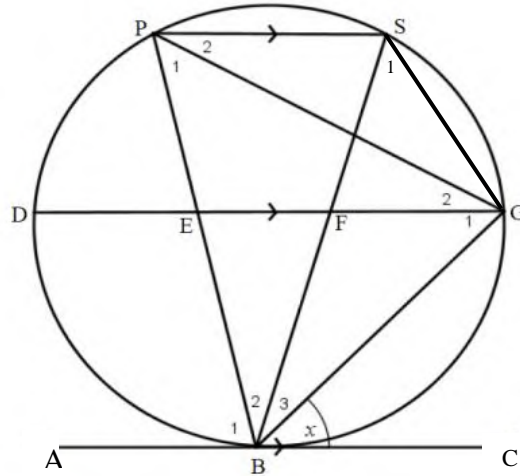
7.2



<p>7.2.1</p>	$\frac{PN}{NE} = \frac{DM}{ME}$ $\frac{PN}{NE} = \frac{2}{3}$ $\therefore PN = 35 \times \frac{2}{5}$ $PN = 14 \text{ cm}$	<p>line <math>\parallel</math> one side of <math>\Delta</math> / lyn <math>\parallel</math> een sy van <math>\Delta</math></p> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p><b>or / of</b></p> <p>prop theorem, DP//MN</p> <p>eweredighst. DP//MN</p> </div>	<p>✓ S / R</p> <p>✓ 14 cm</p> <p>(2)</p>
<p>7.2.2</p>	$\frac{FP}{FN} = \frac{FS}{FM}$ $FP = \frac{14}{2}$ $FP = 7$	<p>line <math>\parallel</math> one side of <math>\Delta</math> / lyn <math>\parallel</math> een sy van <math>\Delta</math></p> <p>_____</p>	<p>✓ S / R</p> <p>✓ S</p> <p>✓ A (3)</p> <p><b>[10]</b></p>



7.3



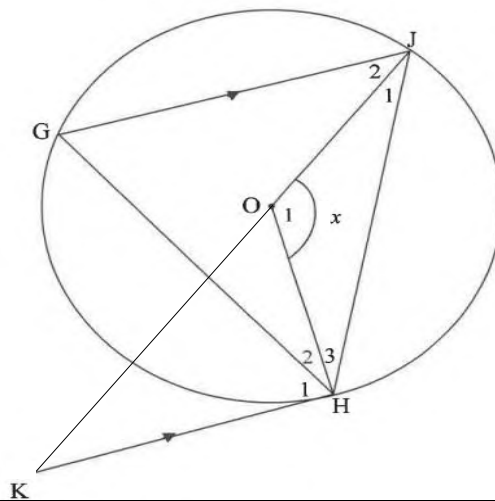
<p>7.3.1</p>	<p>In <math>\triangle PGB</math> and <math>\triangle GEB</math></p> <ul style="list-style-type: none"> <li>• <math>\hat{B}</math> is common / gemeenskaplik</li> <li>- <math>\hat{G}_1 = x</math> alt <math>\angle s</math>; <math>DG \parallel AC</math> / verw. <math>\angle e</math>; <math>DG \parallel AC</math></li> <li>- <math>\hat{P}_1 = x</math> tan chord theorem / raakl. koord stelling</li> <li>• <math>\therefore \hat{P}_1 = \hat{G}_1</math></li> <li>• <math>P\hat{G}B = G\hat{E}B</math> 3<sup>rd</sup> <math>\angle s</math> / 3de <math>\angle e</math></li> </ul> <p><math>\therefore \triangle PGB \parallel \triangle GEB</math> (<math>\angle\angle\angle</math>)</p>	<p>✓R                  ✓R                  ✓R                  ✓R <math>\angle\angle\angle</math>                  (4)</p>
<p>7.3.2</p>	<p><math>\frac{PG}{GE} = \frac{GB}{EB} = \frac{PB}{GB}</math></p> <p><math>\therefore GB^2 = PB \cdot EB</math></p> <p>In <math>\triangle SGB</math> and <math>\triangle GFB</math></p> <ul style="list-style-type: none"> <li>• <math>\hat{B}_3</math> is common gemeenskaplik</li> <li>- <math>\hat{S}_1 = \hat{P}_1 = x</math> <math>\angle s</math> in same segm. / <math>\angle e</math> in dies. segm.</li> <li>• <math>\therefore \hat{S}_1 = \hat{G}_1</math></li> <li>• <math>B\hat{F}G = S\hat{G}B</math> 3<sup>rd</sup> <math>\angle s</math> / 3de <math>\angle e</math></li> </ul> <p><math>\therefore \triangle SGB \parallel \triangle GFB</math> (<math>\angle\angle\angle</math>)</p> <p><math>\therefore GB^2 = SB \cdot FB</math>                      <math>\therefore SB \cdot FB = EB \cdot PB</math></p>	<p><math>GB^2 = PB \cdot EB</math> ✓</p> <p>S/R ✓</p> <p><math>GB^2 = SB \cdot FB</math> ✓</p> <p>(3)</p>



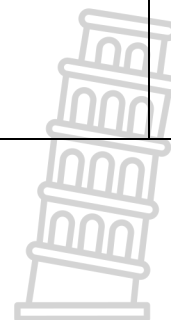
7.3.3	$\frac{PG}{GE} = \frac{GB}{EB}$ $\therefore \frac{PG}{9} = \frac{9}{0,6PG}$ $0,6PG^2 = 81$ $PG = \sqrt{135}$ $PG = 11,62cm$	sub 9 ✓ sub 0,6PG ✓  11,62cm ✓  (3) <b>[10]</b>
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QUESTION/ VRAAG 8



8.1.1	$2\hat{G} = \hat{O}_1$ $\hat{G} = 60^\circ$	$\angle$ at centre = $2 \times \angle$ at circumference Midpts $\angle = 2 \times$ Omtreks $\angle$	✓ R ✓ answer (2)
8.1.2	$\hat{H}_3 = \hat{J}_1$ $\hat{H}_3 + \hat{J}_1 + \hat{O}_1 = 180^\circ$ $\hat{H}_3 = 30^\circ$	$\angle$ s opp equal radii / $\angle$ e teenoor gelyke rad $\angle$ sum in $\Delta$ / binne $\angle$ e van $\Delta$	✓S/R ✓S ✓answer (3)
8.2	$\hat{K} = 120 - 90^\circ$ $\sin(120 - 90^\circ) = \frac{OH}{OK}$ $OH = OJ$ $\cos x = -\frac{OJ}{OK}$	ext $\angle$ of $\Delta KOH$ / buite $\angle$ van $\Delta KOH$ $\Delta OKH$ Radii / radiuses	✓S ✓trig ratio ✓S (3) [8]



TOTAL 150