



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**SENIOR CERTIFICATE/
NATIONAL SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

SEPTEMBER 2021(2)

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages and 1 information sheet.



INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

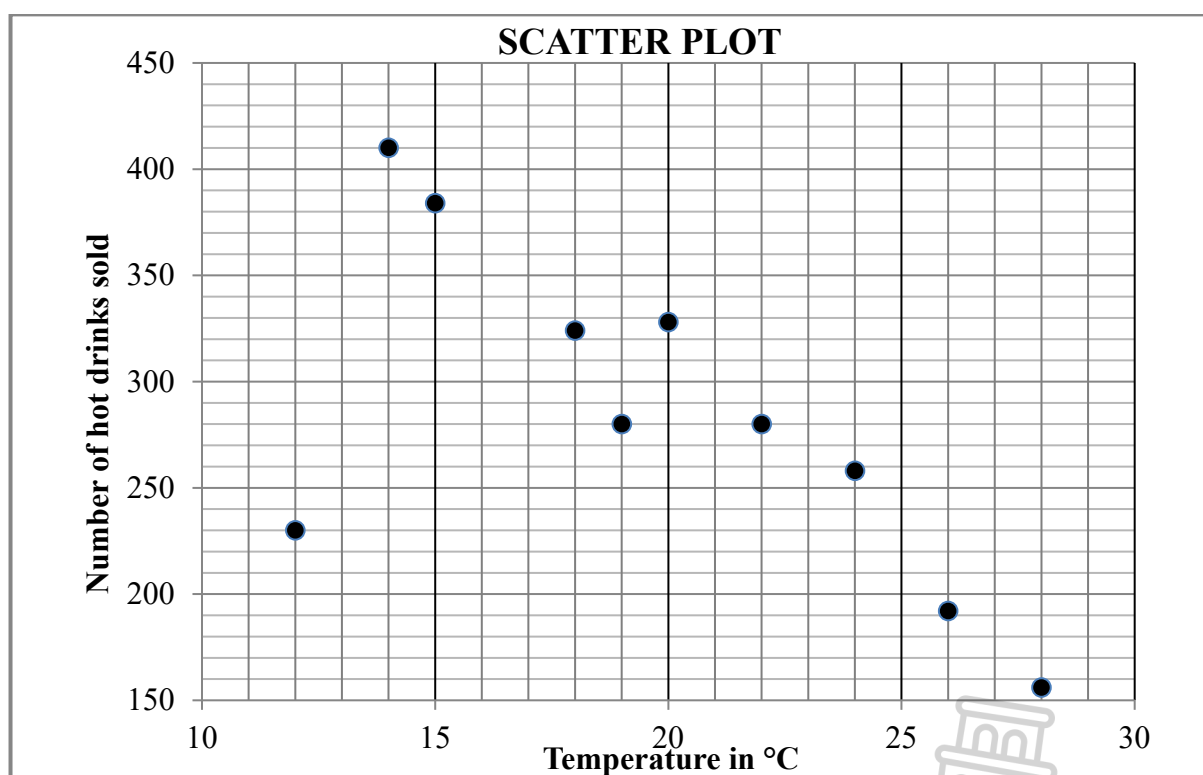
1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.



QUESTION 1

An annual sports festival is held over a period of 11 days. A tuckshop sells hot drinks at this festival. On each of the first 10 days, the owner of the tuckshop recorded the temperature at 13:00 and the number of cups of hot drinks sold. This information is presented in the table and scatter plot below.

Temperature (in °C)	14	24	26	18	20	28	22	15	12	19
Number of hot drinks sold	410	258	192	324	328	156	280	384	230	280



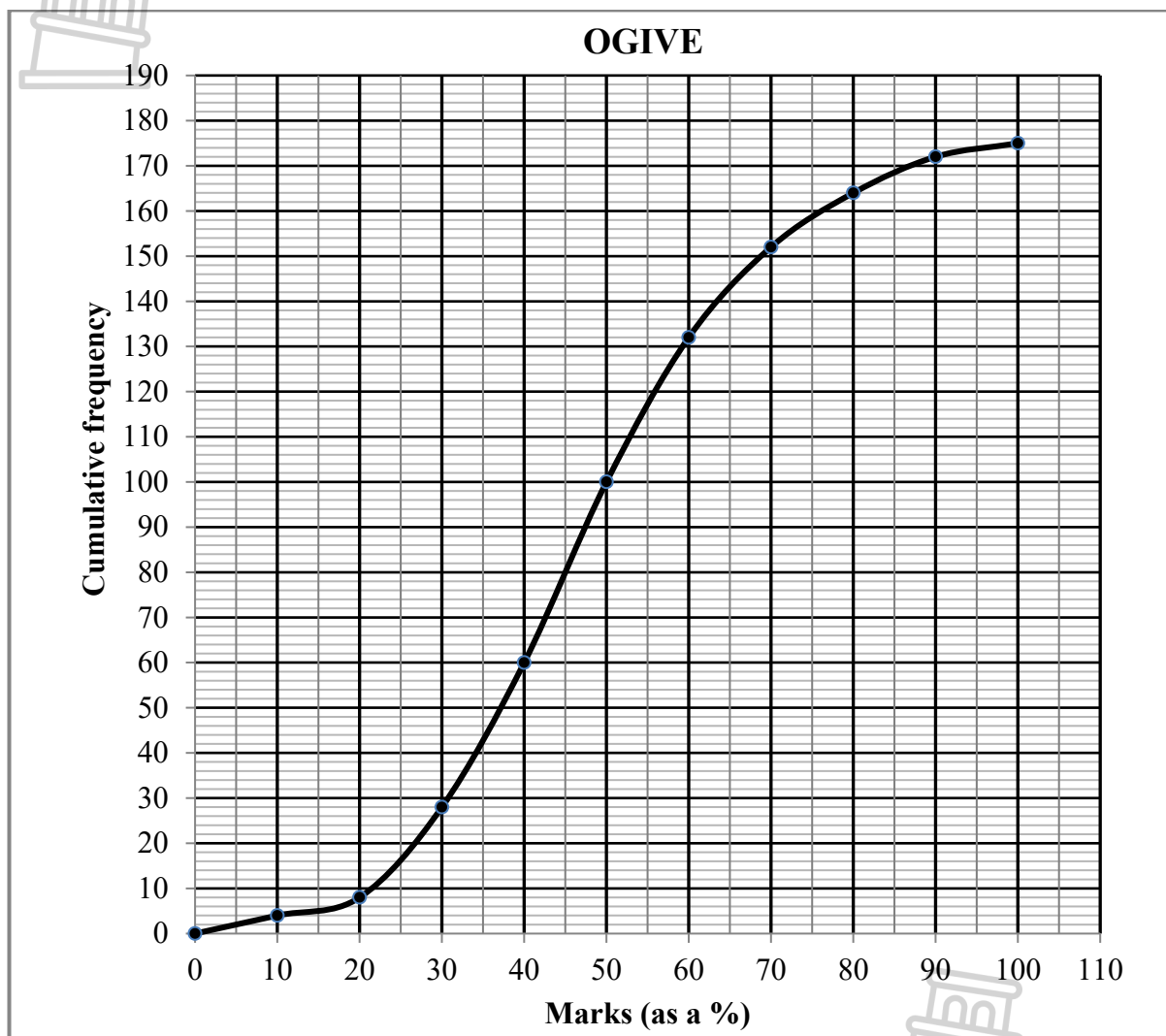
- 1.1 Describe the trend of the data. (1)
- 1.2 Determine the equation of the least squares regression line for the data. (3)
- 1.3 The owner observed that he had used one litre of milk for every 8 cups of hot drinks sold. If the temperature at 13:00 on the 11th day was expected to be 17 °C, predict the number of 1-litre boxes of milk the owner should buy for the 11th day. (3)
- 1.4 Identify an outlier in the data. (1)

[8]



QUESTION 2

2.1 Learners from various schools wrote an aptitude test in order to qualify for a bursary. Their marks (as a percentage) are represented in the ogive (cumulative frequency graph) below.



- 2.1.1 How many learners wrote the test? (1)
- 2.1.2 Write down the modal class of the data. (1)
- 2.1.3 The minimum mark to qualify for a bursary is 75%. How many learners qualified for a bursary? (2)



- 2.2 The table below shows the marks that 15 learners from one particular school obtained in the aptitude test.

Marks (as a %)	62	58	78	85	74	48	74	84	100	46	80	92	60	90	92
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Calculate the:

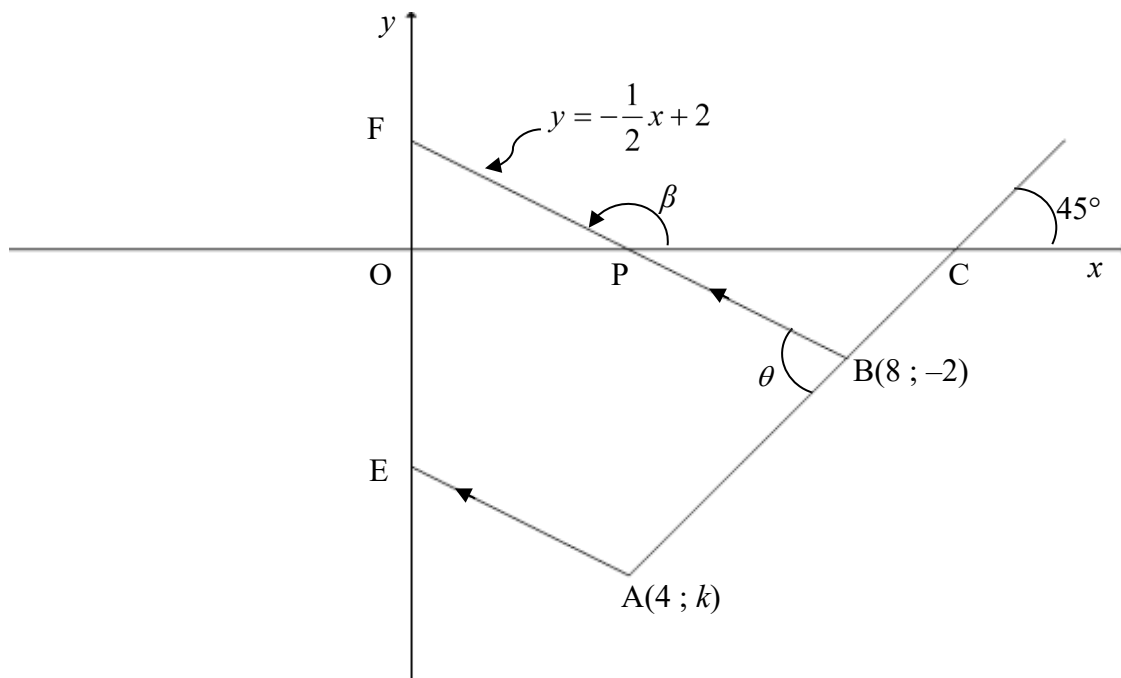
- 2.2.1 Mean mark obtained by these learners (2)
- 2.2.2 Standard deviation of these learners' marks (1)
- 2.2.3 Number of these learners whose marks lie more than one standard deviation above the mean (2)
- 2.3 The final Grade 11 marks (as a percentage) obtained by a group of learners was analysed. The one standard deviation interval about the mean was calculated as (82,7 ; 94,1).
- Calculate the standard deviation for the final Grade 11 marks. (3)

[12]



QUESTION 3

In the diagram below, the line BF is drawn from B(8 ; -2) to cut the x-axis at P and the y-axis at F. The inclination of BF is β and the equation of BF is $y = -\frac{1}{2}x + 2$. From A(4 ; k), another line is drawn parallel to BF and cuts the y-axis at E. The line passing through A and B has an inclination of 45° and cuts the x-axis at C. $\hat{ABF} = \theta$.



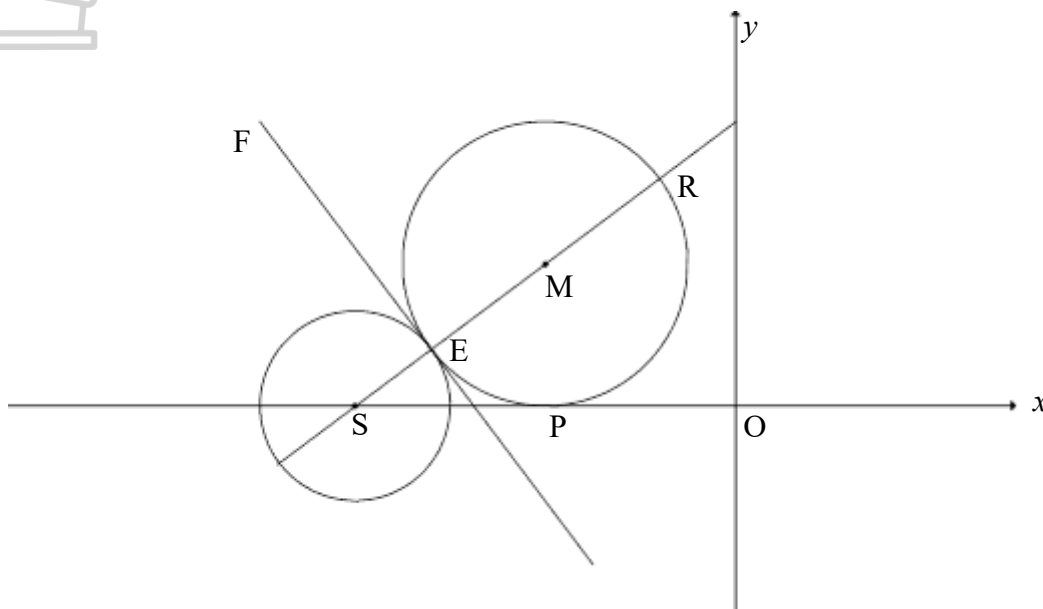
- 3.1 Calculate the gradient of AB. (1)
- 3.2 Show that the value of k is -6 . (2)
- 3.3 Determine the equation of EA in the form $y = mx + c$. (3)
- 3.4 Calculate the:
 - 3.4.1 Size of θ (3)
 - 3.4.2 Length of BF (3)
 - 3.4.3 Area of $\triangle ABF$ (4)
- 3.5 Let G be a point in the fourth quadrant such that APBG is a parallelogram. Calculate the size of \hat{PAG} . (4)

[20]



QUESTION 4

In the diagram below, S is a point on the x-axis. A circle centred at S and a circle centred at M are drawn. The two circles touch each other externally at E. FE is a common tangent to the circles at E. The circle centred at M, having ER as a diameter, touches the x-axis at P.



- 4.1 The equation of the circle centred at S is $(x + 8)^2 + y^2 = 4$.
 - 4.1.1 Write down the coordinates of S. (2)
 - 4.1.2 Show that the diameter of the circle centred at S is 4 units. (1)

- 4.2 If it is further given that $SR = 8$ units and $R\left(-\frac{8}{5}; \frac{24}{5}\right)$, calculate the:
 - 4.2.1 Length of EM (2)
 - 4.2.2 Gradient of the tangent FE (3)
 - 4.2.3 Coordinates of M (4)
 - 4.2.4 Coordinates of E (2)

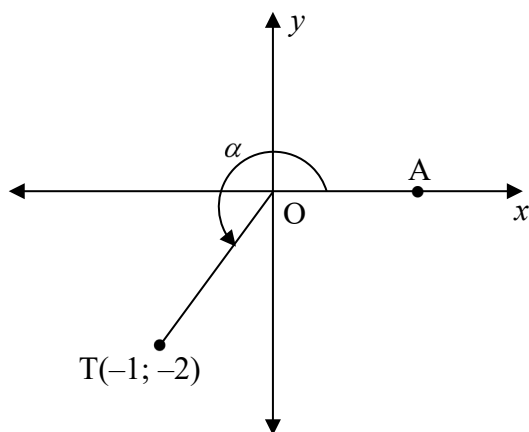
- 4.3 The circle centred at $M(-4 ; 3)$ is shifted 1 unit to the left and reflected in the x-axis to form a new circle centred at K. Determine whether the point $(-8 ; 0)$ lies inside or outside the circle centred at K. Show ALL calculations. (5)

[19]



QUESTION 5

5.1 Point $T(-1; -2)$ is given in the diagram below. A is a point on the x -axis such that reflex $\hat{AOT} = \alpha$.



Determine, **without using a calculator**, the value of each of the following:

5.1.1 $\tan \alpha$ (1)

5.1.2 $\cos \alpha$ (2)

5.1.3 $\cos(\alpha + 45^\circ)$ in simplest form (4)

5.2 Determine, **without using a calculator**, the value of the following expression:

$2 \sin(-20^\circ) \cdot \sin 160^\circ - \cos 40^\circ$ (4)

5.3 Consider: $3 \cos x \cdot \sin x + \tan x \cdot \cos^2(180^\circ - x)$

5.3.1 Simplify the expression to a single trigonometric ratio. (4)

5.3.2 Hence, write down the range of:

$f(x) = 3 \cos x \cdot \sin x + \tan x \cdot \cos^2(180^\circ - x)$ (2)

5.4 Prove the identity: $\frac{\cos 3x}{\cos x} = 4 \cos^2 x - 3$ (5)



5.5 Determine the general solution of x in the following equation:

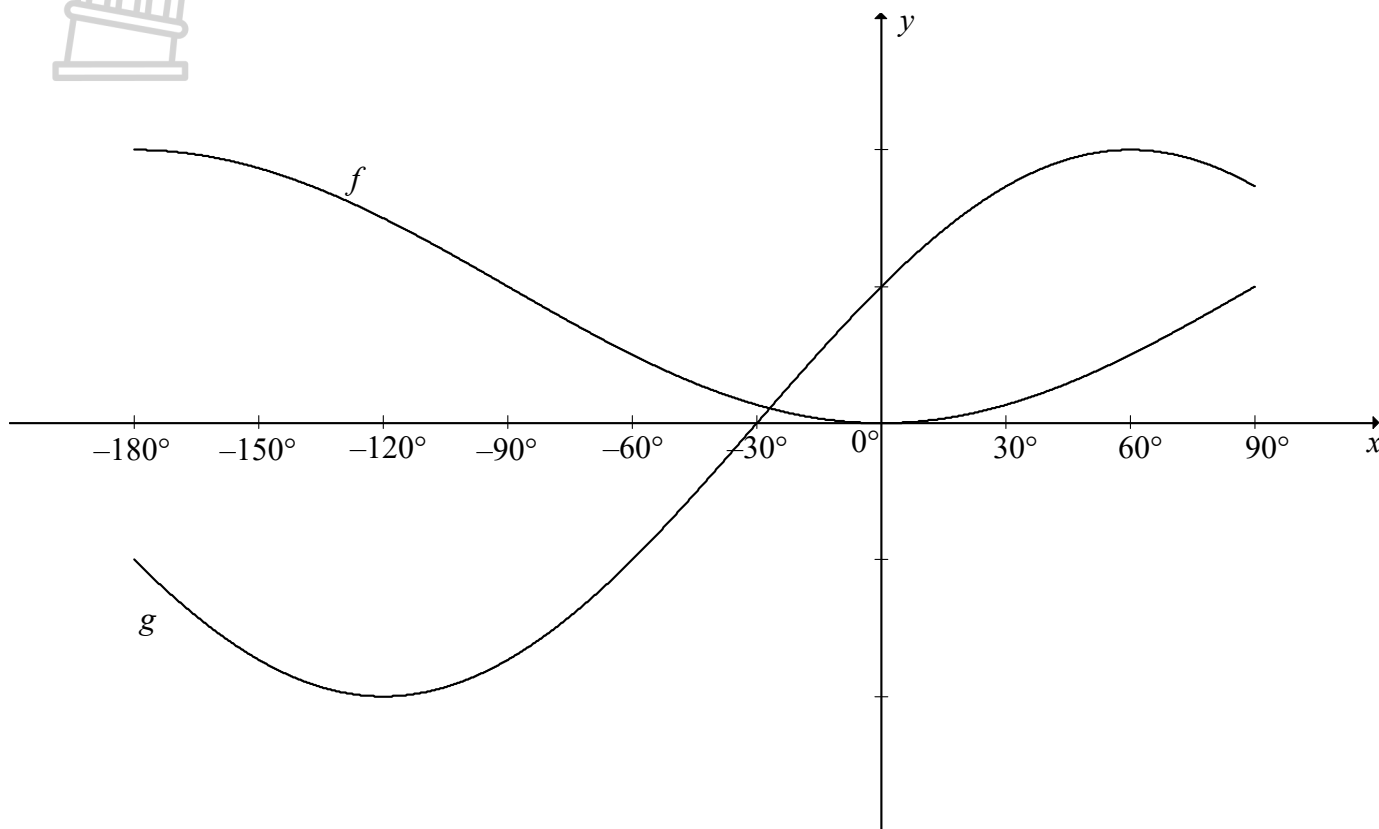
$3^{2 \tan x} - 3^{\tan x + 1} = 54$ (5)

[27]



QUESTION 6

In the diagram, the graphs of $f(x) = -\cos x + 1$ and $g(x) = 2\sin(x + 30^\circ)$ are drawn for the interval $x \in [-180^\circ; 90^\circ]$.



6.1 For which values of x , $x \in [-180^\circ; 90^\circ]$, will:

6.1.1 $f(x) \cdot g(x) \geq 0$ (2)

6.1.2 $g(x) = -1$ (2)

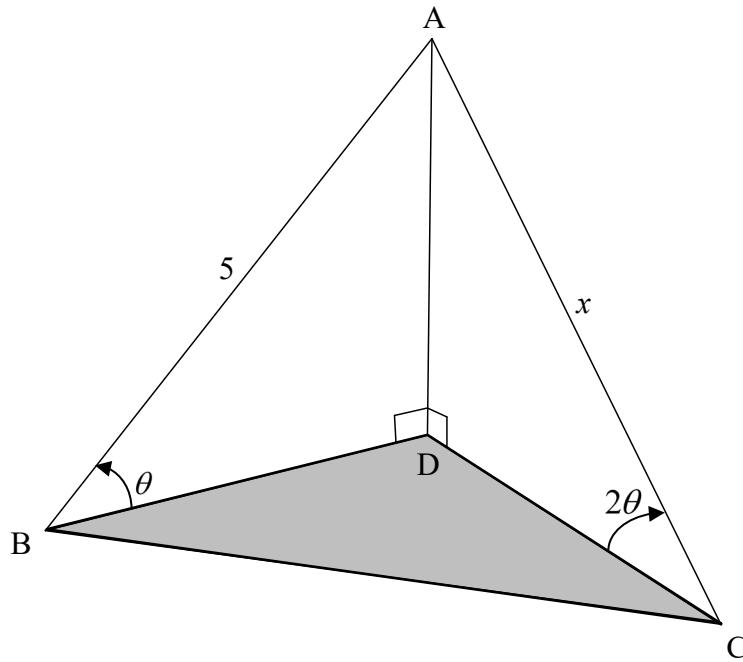
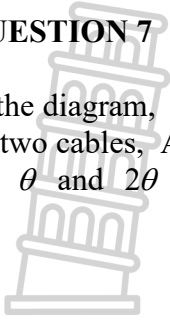
6.2 The y -axis is moved 90° to the right. Determine the new equation of the graph originally called f , in its simplest form. (2)

[6]



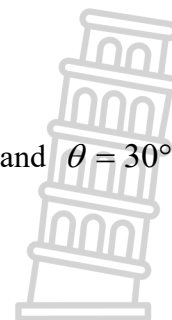
QUESTION 7

In the diagram, B, C and D are in the same horizontal plane. AD is a vertical pole anchored by two cables, AB and AC. The angles of elevation from B and C to A, the top of the pole, are θ and 2θ respectively. $AB = 5$ units and $AC = x$ units.



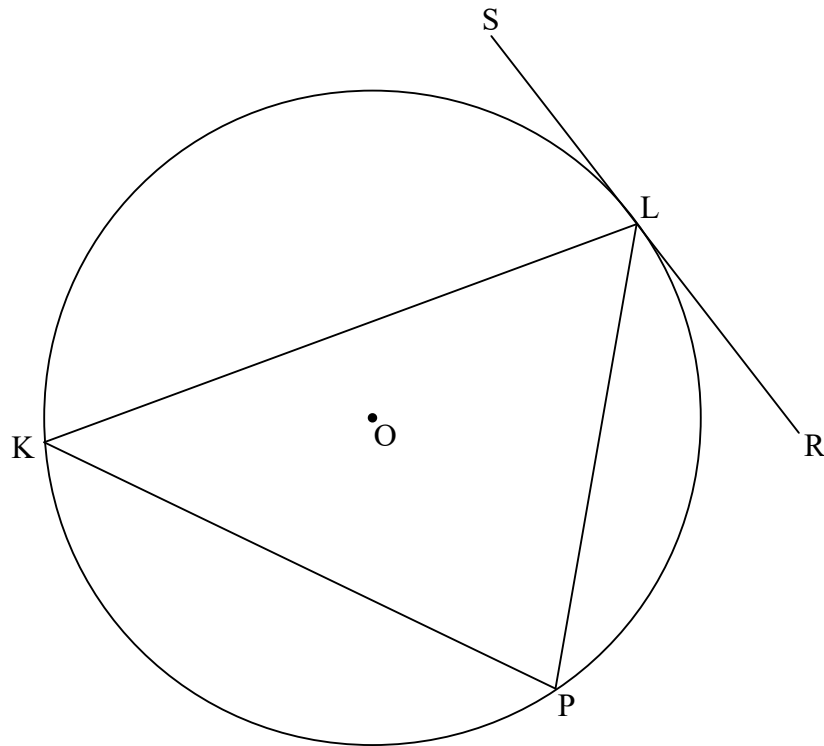
7.1 Show that $x = \frac{5}{2 \cos \theta}$ (5)

7.2 Calculate the length of BC if it is given that $\hat{BAC} = 112^\circ$ and $\theta = 30^\circ$. (3)
[8]



QUESTION 8

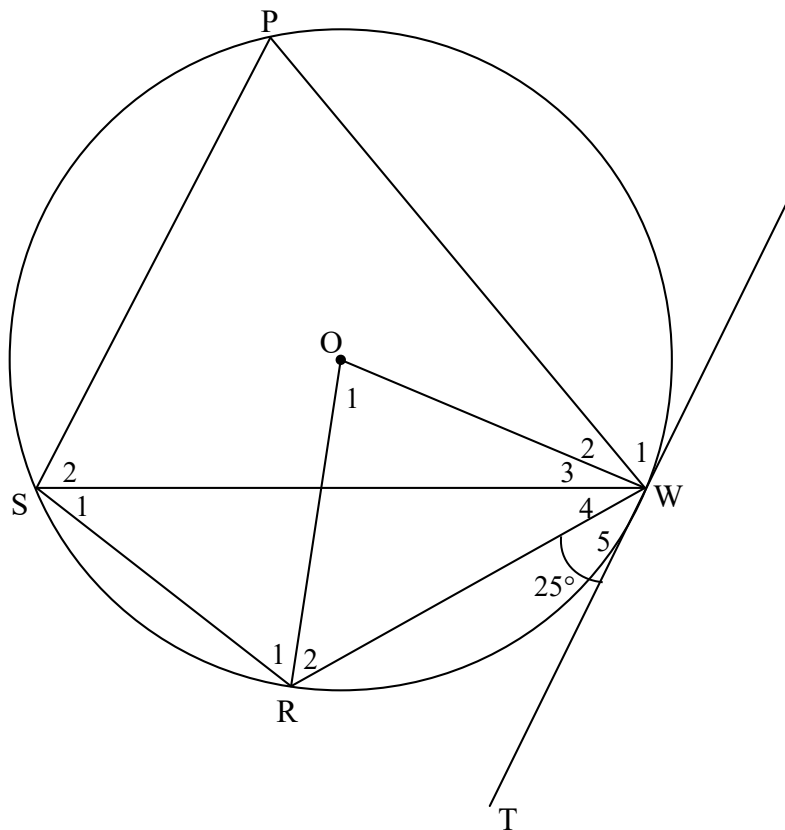
8.1 In the diagram, chords KL, LP and KP are drawn in a circle, centered at O. SLR is a tangent to the circle at L.



Prove the theorem which states that the angle between the tangent SLR and chord KL is equal to the angle in the alternate segment, that is prove that $\hat{S}LK = \hat{P}$. (6)



8.2 In the diagram below, PWRS is a cyclic quadrilateral in the circle, centered at O. $\triangle PSW$ is an equilateral triangle. TW is a tangent to the circle at W. Radii OR and OW are drawn. $\hat{W}_5 = 25^\circ$.



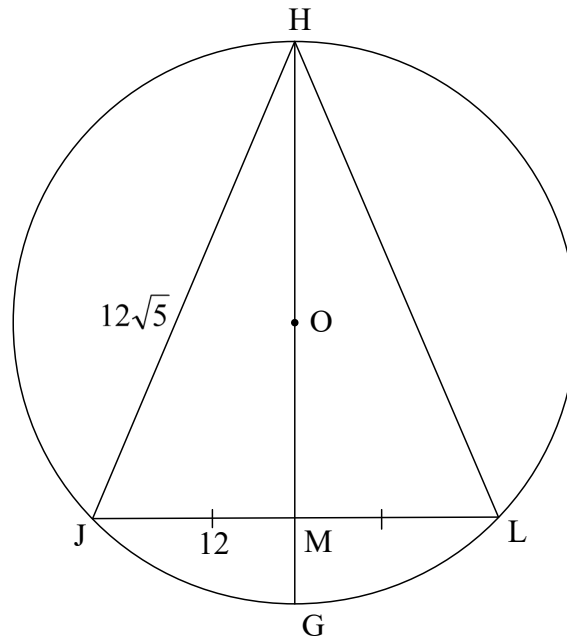
8.2.1 Determine, giving reasons, the size of:

- (a) \hat{S}_1 (2)
- (b) \hat{O}_1 (2)
- (c) \hat{R}_1 (5)

8.2.2 Prove that $SP \parallel TW$. (3)



- 8.3 In the diagram below, a circle centered at O is drawn. H, J, G and L are points on the circle. $\triangle HJL$ is drawn. HOG bisects JL at M .
 $HJ = 12\sqrt{5}$ units and $JM = 12$ units.

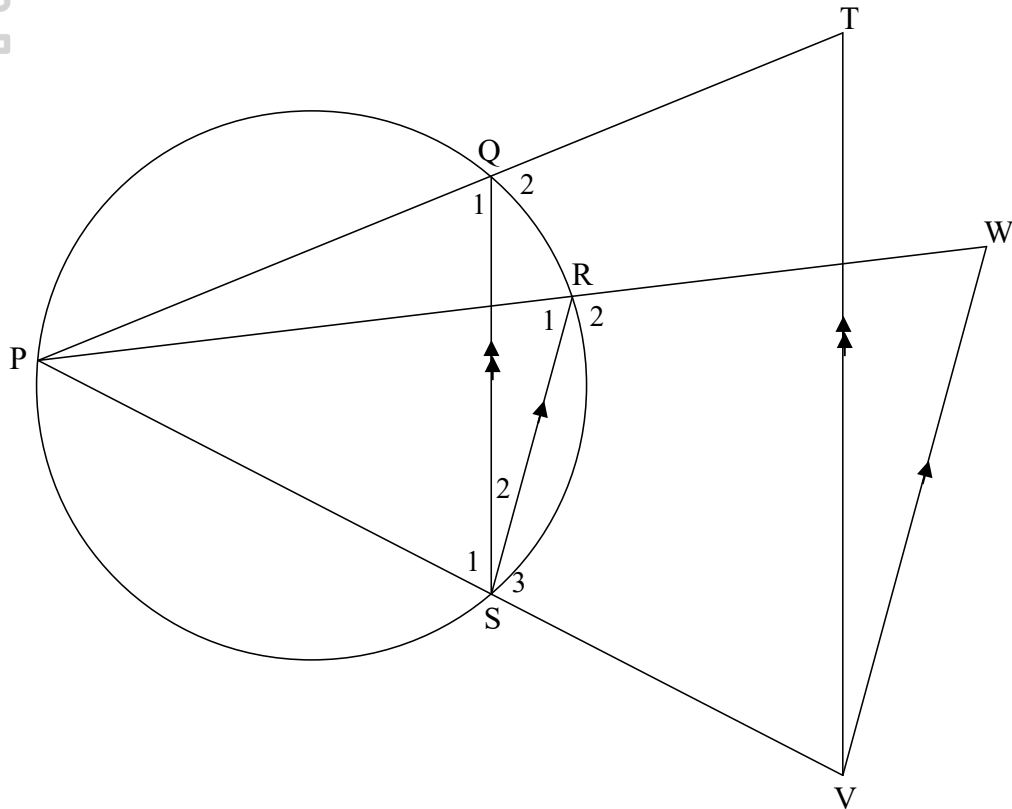
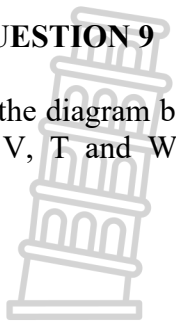


- 8.3.1 If $MG = 6$ units and $OM = x$, write HM in terms of x . (2)
- 8.3.2 Calculate, giving reasons, the length of the radius of the circle. (5)
[25]



QUESTION 9

In the diagram below, P, Q, R and S are points on a circle. PS, PQ and PR are produced to V, T and W respectively. $VT \parallel SQ$ and $SR \parallel VW$.



Prove, giving reasons, that:

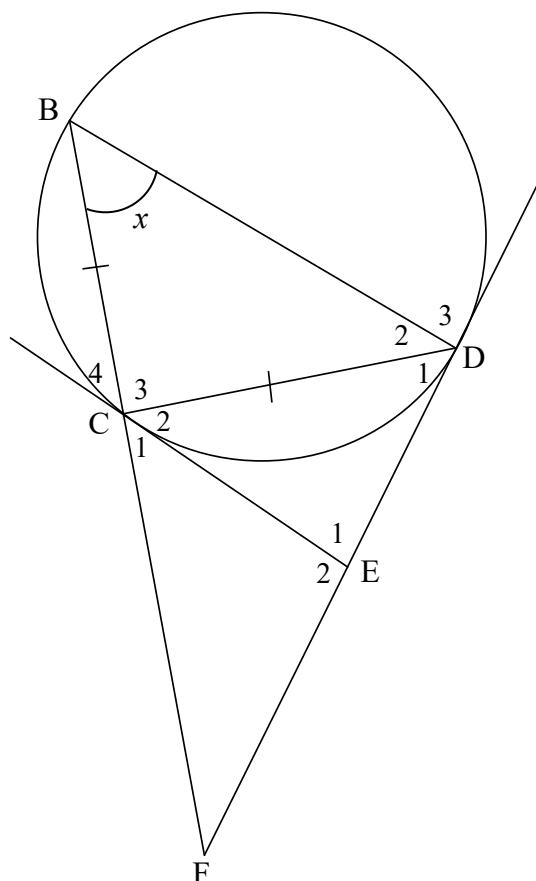
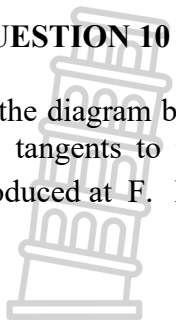
9.1 $\frac{TQ}{QP} = \frac{WR}{RP}$ (3)

9.2 TPVW is a cyclic quadrilateral (5)
[8]



QUESTION 10

In the diagram below, B, C and D are points on a circle such that $BC = CD$. EC and ED are tangents to the circle at C and D respectively. BC produced meets tangent DE produced at F. $\hat{B} = x$.



10.1 Prove, giving reasons, that:

10.1.1 $\hat{E}_1 = 180^\circ - 2x$ (5)

10.1.2 $\triangle ECD \parallel \triangle CBD$ (3)

10.2 Prove, giving reasons, that:

10.2.1 $CD^2 = CE \cdot BD$ (3)

10.2.2 $\frac{CF^2}{EF^2} = \frac{BD}{DE}$ (6)

[17]

TOTAL: 150

a



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

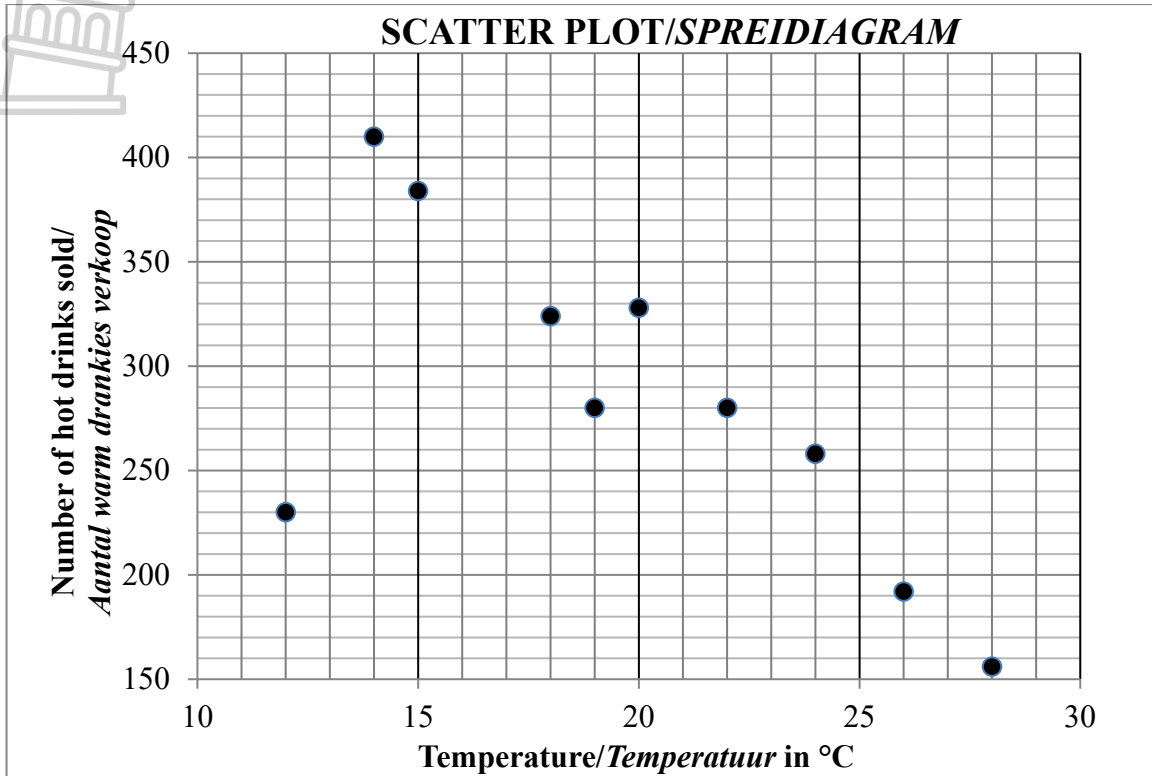


PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG
<p>1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.</p> <p>2. Remember that your own name (or the name of your school) may NOT appear anywhere on or in this answer book.</p> <p>3. Answer ALL questions in the spaces provided.</p> <p>4. NO pages may be torn from this answer book.</p> <p>5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each question paper.</p> <p>6. Candidates may NOT retain an answer book or remove it from the examination room.</p> <p>7. Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.</p> <p>8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.</p> <p>9. If you require additional space for your answers:</p> <p>9.1 Use the additional space provided at the end of the answer book.</p> <p>9.2 When answering a question in the additional space, clearly indicate the question number in the column on the LHS.</p> <p>9.3 Rule off after each answer.</p> <p>10. Draw a neat line through any work/rough work that must not be marked.</p>	<p>1. Skryf jou eksamennommer en sentrumnummer duidelik in die ruimtes verskaf en plak jou stafieskodeplakker in die ruimte verskaf.</p> <p>2. Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.</p> <p>3. Beantwoord ALLE vrae in die ruimtes wat verskaf is.</p> <p>4. GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.</p> <p>5. Lees die instruksies, wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat op elke vraestel gegee word.</p> <p>6. GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.</p> <p>7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.</p> <p>8. Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.</p> <p>9. In geval jy bykomende ruimte benodig vir jou antwoorde:</p> <p>9.1 Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word.</p> <p>9.2 As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.</p> <p>9.3 Trek 'n lyn na elke antwoord.</p> <p>10. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.</p>



QUESTION/VRAAG 1

Temperature/Temperatuur (in °C)	14	24	26	18	20	28	22	15	12	19
Number of hot drinks sold/Aantal warm drankies verkoop	410	258	192	324	328	156	280	384	230	280

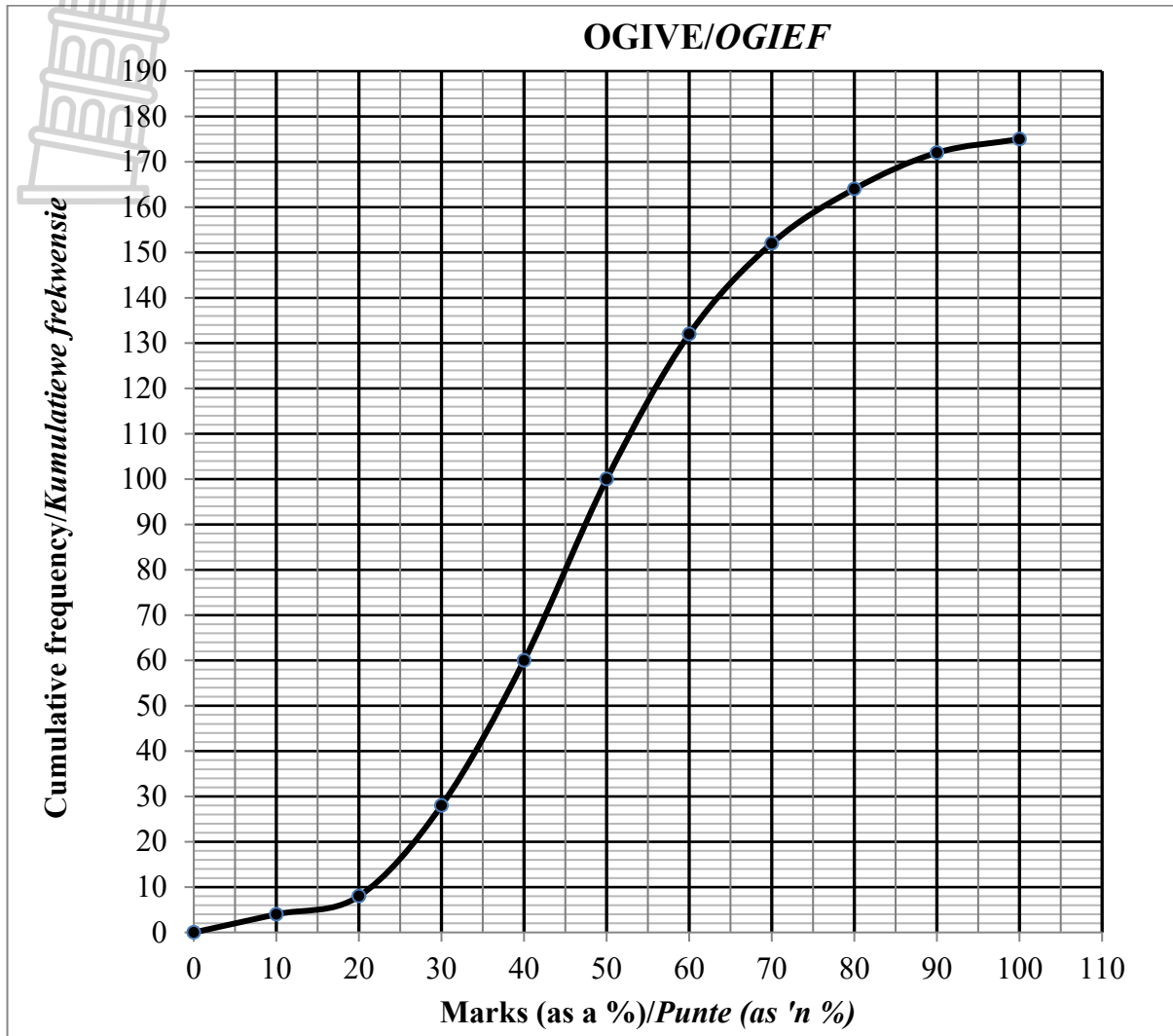


	Solution/Oplissing	Marks/Punte
1.1		(1)
1.2		(3)
1.3		(3)
1.4		(1)
		[8]



QUESTION/VRAAG 2

2.1



	Solution/Oplissing	Marks Punte
2.1.1		(1)
2.1.2		(1)
2.1.3		(2)



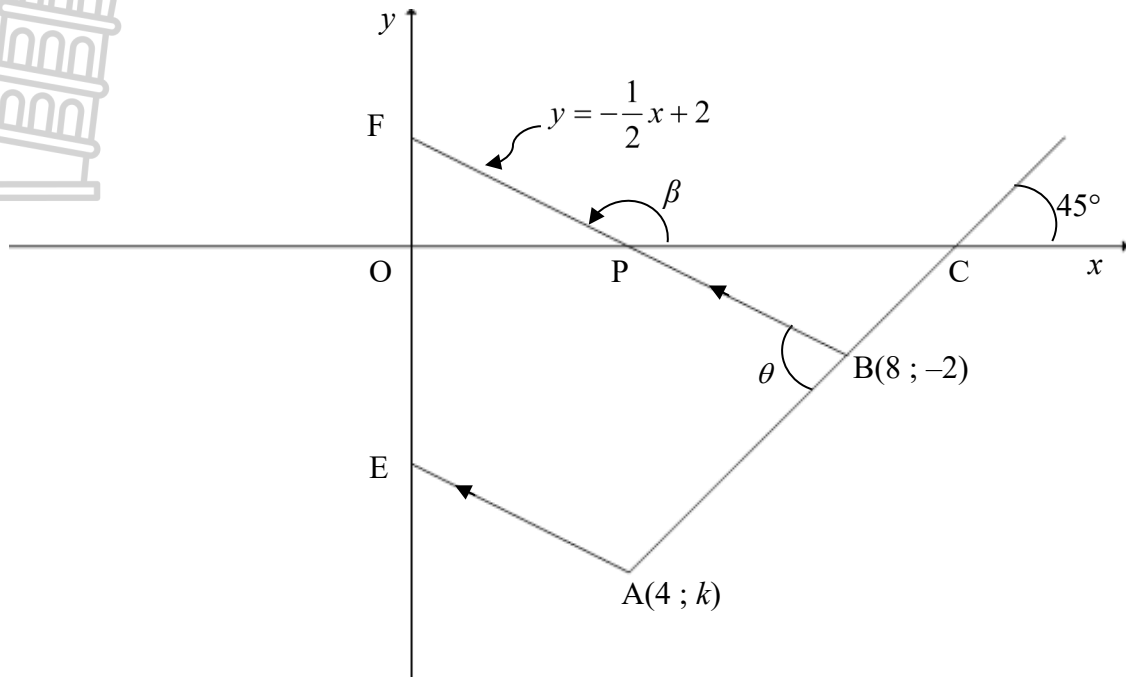
2.2

Marks (as a %) Punte (as 'n %)	62	58	78	85	74	48	74	84	100	46	80	92	60	90	92
---	----	----	----	----	----	----	----	----	-----	----	----	----	----	----	----

	Solution/Oplossing	Marks Punte
2.2.1		(2)
2.2.2		(1)
2.2.3		(2)
2.3		(3)
		[12]



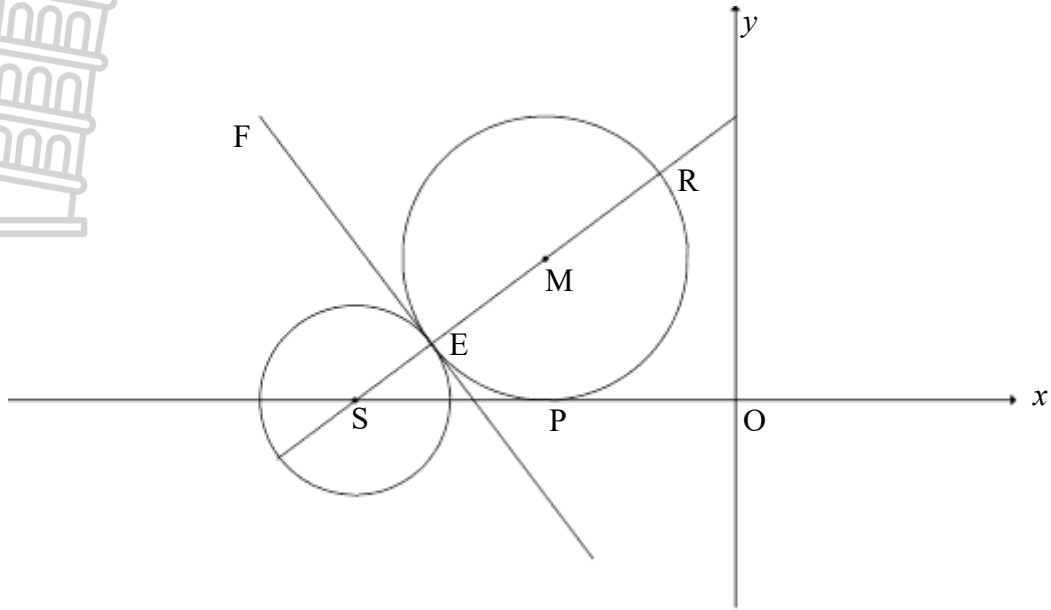
QUESTION/VRAAG 3



	Solution/Oplissing	Marks Punte
3.1		(1)
3.2		(2)
3.3		(3)



QUESTION/VRAAG 4



	Solution/Oplissing	Marks Punte
4.1.1		(2)
4.1.2		(1)
4.2.1		(2)
4.2.2		(2)

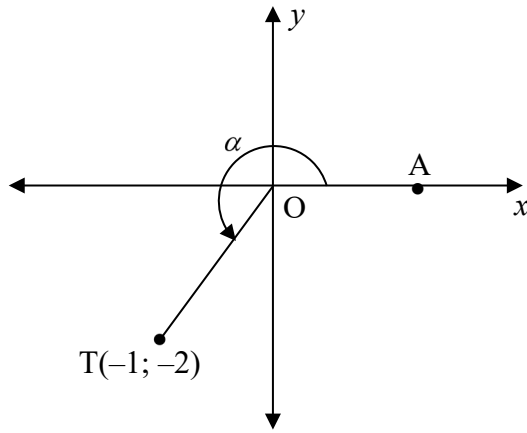


	Solution/Oplissing	Marks Punte
4.2.2 (cont./ verv.)		(3)
4.2.3		(4)
4.2.4		(2)
4.3		(5)
		[19]



QUESTION/VRAAG 5

5.1

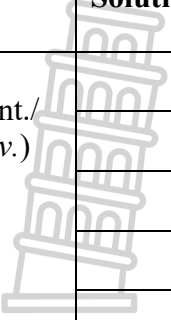



	Solution/Oplissing	Marks Punte
5.1.1		(1)
5.1.2		(2)
5.1.3		(4)



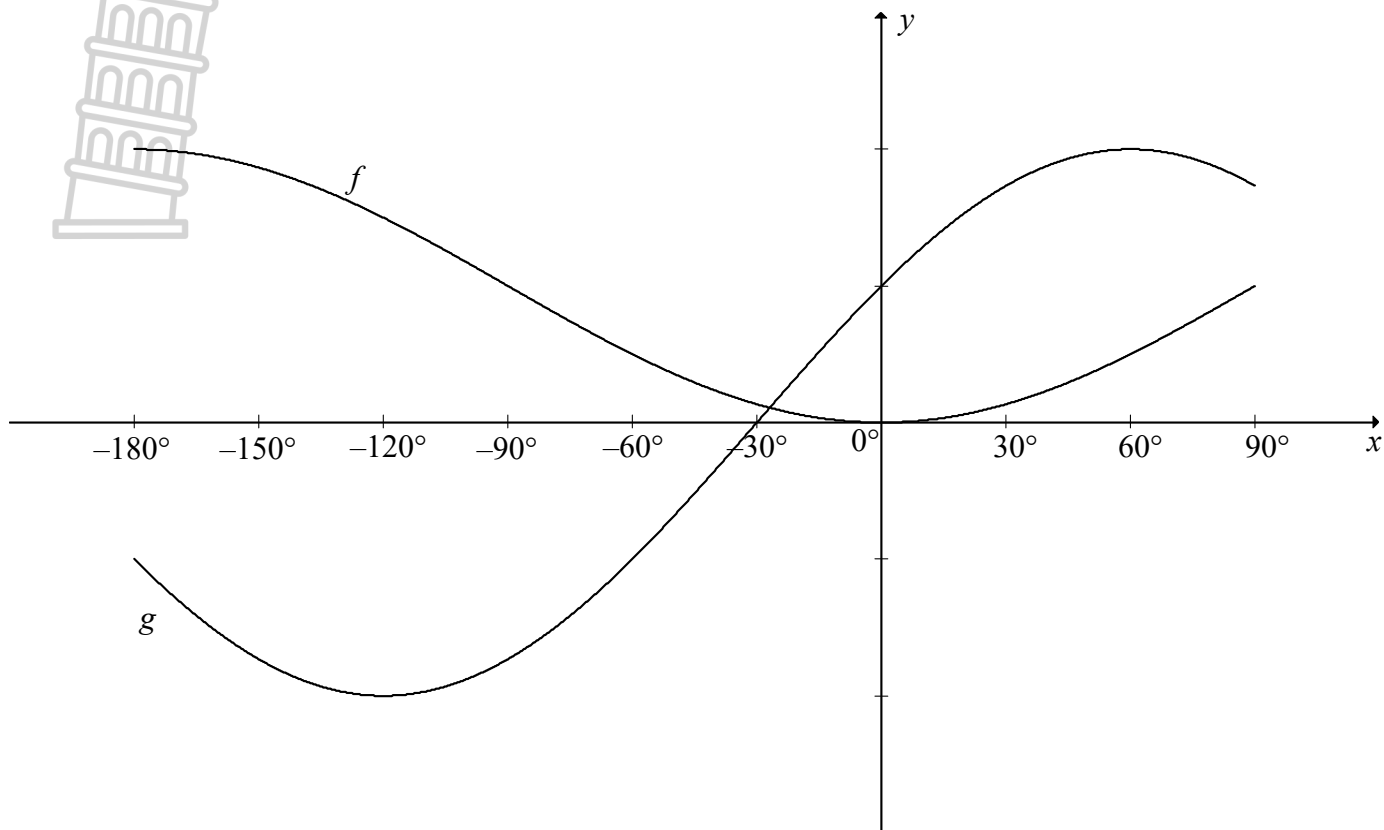
	Solution/Oplissing	Marks Punte
5.2		(4)
5.3.1		(4)
5.3.2		(2)
5.4		



	Solution/Oplissing	Marks Punte
5.4 (cont./ verv.)		
5.5		(5)
		(5)
		[27]



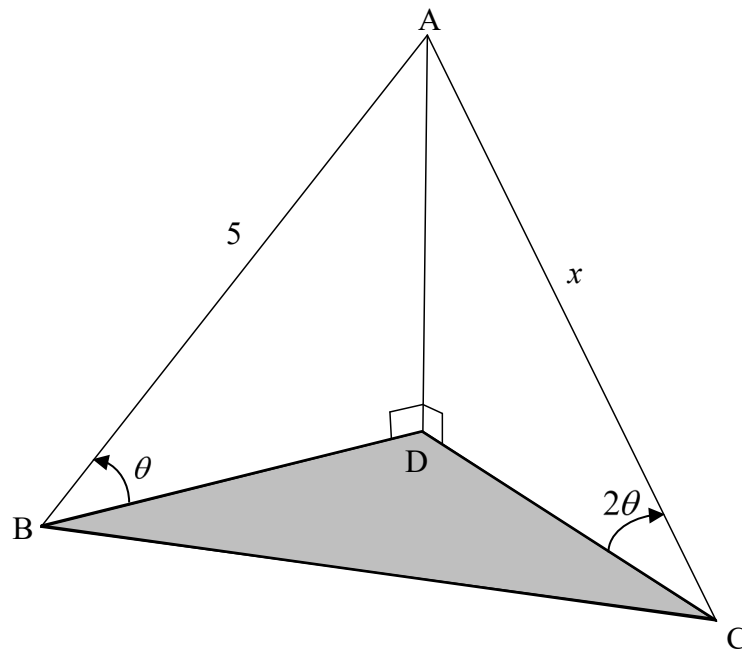
QUESTION/VRAAG 6



	Solution/Oplissing	Marks Punte
6.1.1		(2)
6.1.2		(2)
6.2		(2)
		[6]



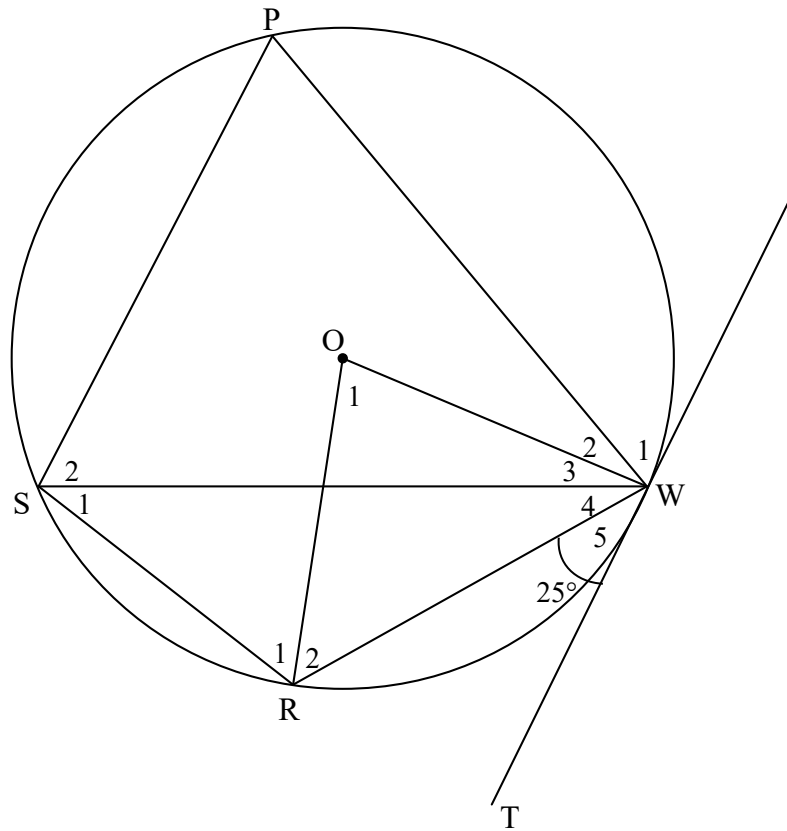
QUESTION/VRAAG 7



	Solution/Oplissing	Marks Punte
7.1		(5)
7.2		(3)
		[8]



8.2



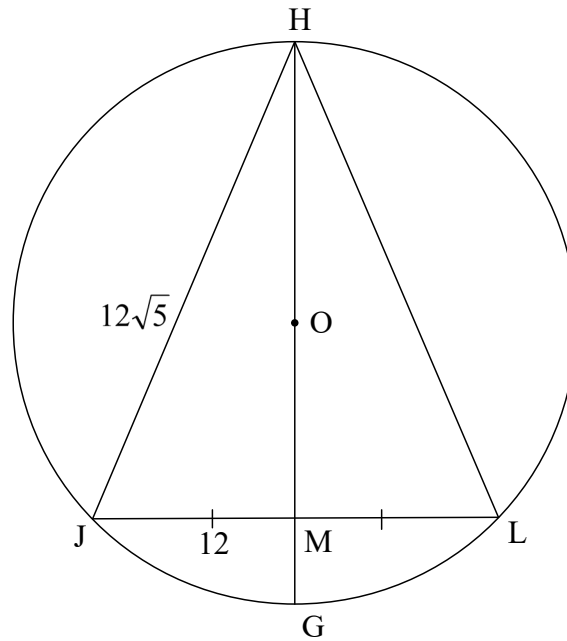
	Solution/Oplissing	Marks Punte
8.2.1(a)		(2)
8.2.1(b)		(2)



	Solution/Oplissing	Marks Punte
8.2.1(c)		(5)
8.2.2		(3)



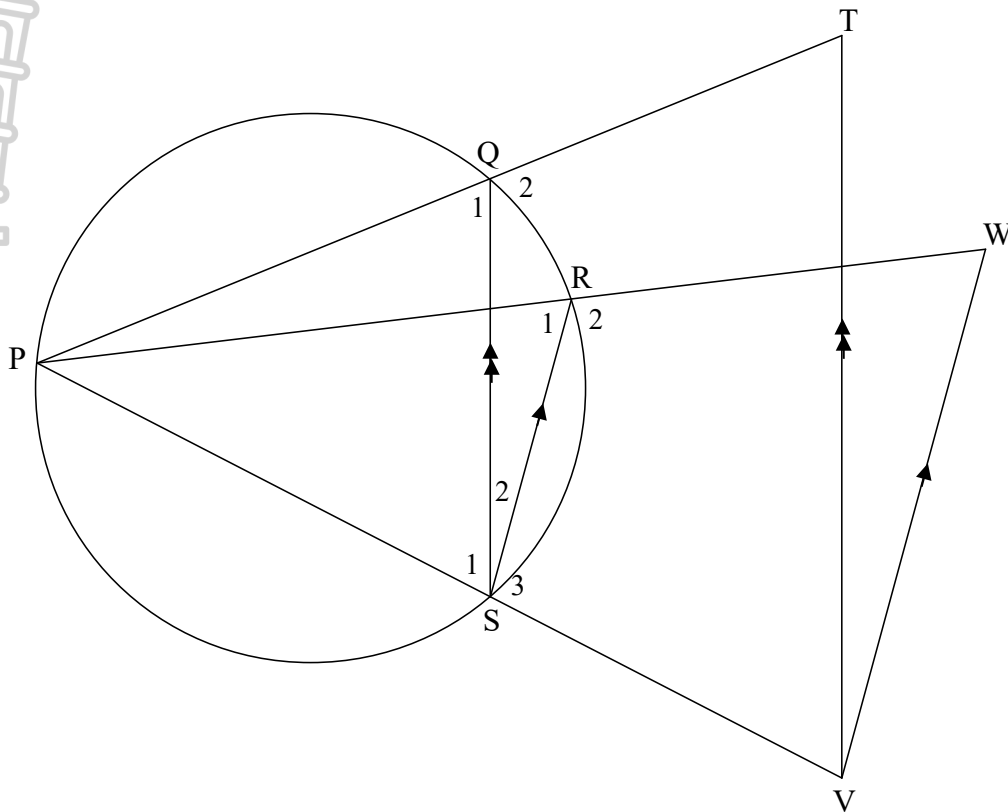
8.3



	Solution/Oplissing	Marks Punte
8.3.1		(2)
8.3.2		(5)
	[25]	



QUESTION/VRAAG 9



	Solution/Oplissing	Marks Punte
9.1		(3)
9.2		(5)
		[8]



	Solution/Oplissing	Marks Punte
10.2.1		(3)
10.2.2		(6)
		[17]



