KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

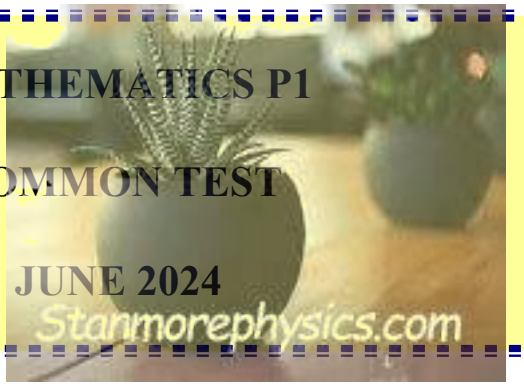
NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P1

COMMON TEST

JUNE 2024



MARKS: 100

TIME: 2 hours

This question paper consists of 6 pages including cover page

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 5 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(1-2x)=0$ (2)

1.1.2 $2x^2-5x+3=0$ (3)

1.1.3 $x(3x-5)=7$ (correct to TWO decimal places) (4)

1.1.4 $(1-x)(x+3)\leq -5$ (4)

1.1.5 $\dfrac{x^2+7}{x^2-2x-3}+\dfrac{2}{x+1}=-\dfrac{1}{x-3}$ (5)

1.2 Given that: $x-3=2\sqrt{x}$

1.2.1 Solve for x. (4)

1.2.2 Hence, if $\sqrt[3]{3t}-2\sqrt{\sqrt[3]{3t}}=3$, determine the value of t. (3)

1.3 Solve simultaneously for x and y:
$3y=x+1$ and $(x-y)(5y-3x)=0$ (5)

1.4 The roots of the equation $x^2-3x-k=0$ are $x=\dfrac{3\pm 3\sqrt{5}}{2}$. Determine the value of k. (4)

[34]

QUESTION 2

2.1 Simplify the following fully, **without using a calculator**:

2.1.1 $\dfrac{2^{4x+1}.9^{x}.6^{2x-1}}{2^{6x}.27^{x}.3^{x}}$ (4)

2.1.2 $\dfrac{3^{2021}-3^{2017}}{60.\left(\sqrt[3]{3^{6048}}\right)}$ (4)

2.2 Solve for x, **without using a calculator:**

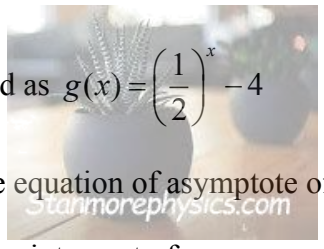
2.2.1 $m^{8x-4}=1$ (3)

2.2.2 $4.3^{x}-3^{x-2}+3^{x}=\dfrac{44}{3}$ (4)

2.2.3 $x^{x}=2^{2048}$ (3)
[18]

QUESTION 3

The function g is defined as $g(x)=\left(\frac{1}{2}\right)^x-4$



3.1 Write down the equation of asymptote of g. (1)

3.2 Calculate the y-intercept of g. (2)

3.3 Calculate the x-intercept of g. (2)

3.4 Draw a neat sketch of g. Clearly show all the intercepts with the axes and the asymptote. (3)

3.5 Calculate the average gradient of g between $x=0$ and $y=0$. (2)

3.6 Write down the equation of k if it is given that $k(x)=g(x)+4$. (1)

3.7 It is further given that $h(x)=2^{x+3}-4$. Explain, in words, how graph g must be transformed to obtain graph h. (3)

[14]

QUESTION 4

The function $f(x)=\frac{a}{x+p}+1$ has an axis of symmetry with an equation of $x=3-y$.

4.1 Write down the range of f. (1)

4.2 Determine the equation of the vertical asymptote of f. (2)

4.3 If $f(0)=2$, calculate the value of a. (2)

4.4 Calculate the x-intercept of f. (3)

4.5 Draw a neat sketch of f. Clearly show all intercepts with the axes and asymptotes. (3)

4.6 Write down the values of x for which $f(x)\leq 0$. (2)

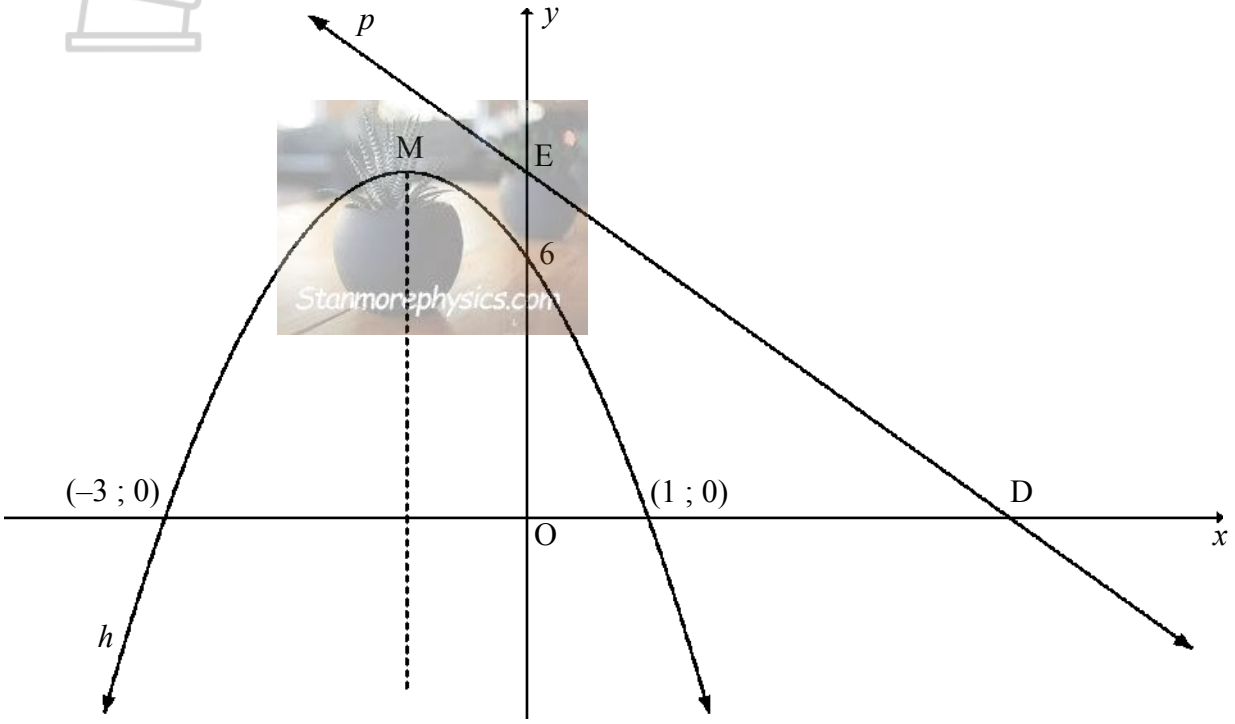
4.7 The straight line $y=-2x+5$ passes through the point of intersection of the asymptotes and intersects f at P(1 ; 3) and Q. Write down the coordinates of Q. (2)

[15]

QUESTION 5

The graphs of $h(x) = ax^2 + bx + c$ and $p(x) = 8 - 2x$ are sketched below.

The x-intercepts of h are $(-3\ ;\ 0)$ and $(1\ ;\ 0)$ and the y-intercept of h is $(0\ ;\ 6)$. M is the turning point of h. D and E are the x- and y-intercepts of p respectively.



5.1 Write down the coordinates of D. (1)

5.2 Show that $a = -2$, $b = -4$ and $c = 6$. (4)

5.3 Calculate the coordinates of M, the turning point of h. (2)

5.4 Write down the range of h. (1)

5.5 Determine the values of x for which $h(x).p(x) < 0$. (3)

5.6 If $h(x) = k$, determine the value(s) of k for which:

5.6.1 roots are non-real. (1)

5.6.2 roots have the same sign. (2)

5.7 Calculate how graph p must be translated so that it becomes a tangent to graph h. (5)

[19]

TOTAL: 100

FINAL

KWAZULU-NATAL PROVINCE
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MATHEMATICS P1

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MARKING GUIDELINES

NATIONAL SENIOR CERTIFICATE

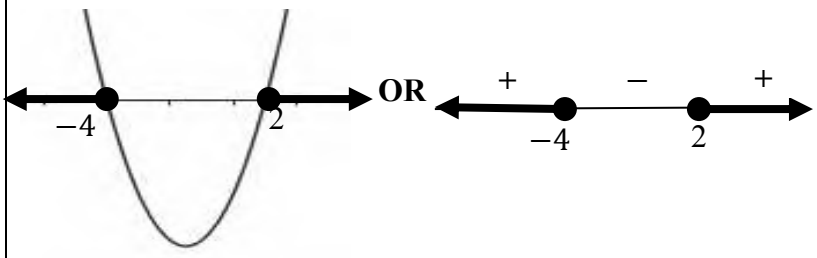
GRADE 11

MARKS: **100**

GRADE 11
Marking Guideline

These marking guidelines consist of 10 pages.

QUESTION 1

1.1.1	$x=0$ or $x=\frac{1}{2}$	✓A $x=0$ ✓A $x=\frac{1}{2}$ (2)
1.1.2	$2x^2-5x+3=0$ $(2x-3)(x-1)=0$ $x=\frac{3}{2}$ or $x=1$	✓A correct factors ✓CA answer ✓CA answer (3)
1.1.3	$x(3x-5)=7$ $3x^2-5x-7=0$ $x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-7)}}{2(3)}$ $x=-0,91$ or $x=2,57$	✓A standard form ✓CA correct substitution ✓CA answer ✓CA answer (4)
1.1.4	$(1-x)(x+3)\le -5$ $x+3-x^2-3x+5\le 0$ $-x^2-2x+8\le 0$ $x^2+2x-8\ge 0$ $(x+4)(x-2)\ge 0$  $x\le -4$ or $x\ge 2$	✓A standard form ✓CA critical values ✓CA answer ✓CA answer (4)

GRADE 11
Marking Guideline

1.1.5	$\frac{x^2+7}{x^2-2x-3}+\frac{2}{x+1}=-\frac{1}{x-3}$ $\frac{x^2+7}{(x-3)(x+1)}+\frac{2}{x+1}+\frac{1}{x-3}=0$ LCD : $(x-3)(x+1)$ $\frac{x^2+7+2(x-3)+1(x+1)}{(x-3)(x+1)}=0$ $\frac{x^2+7+2x-6+x+1}{(x-3)(x+1)}=0$ $x^2+3x+2=0$ $(x+1)(x+2)=0$ $x\neq -1$ or $x=-2$	✓A correct factorisation ✓CA correct LCD ✓CA standard form ✓CA factors ✓CA answers with rejection (5)
1.2.1	$x-3=2\sqrt{x}$ $(x-3)^2=\left(2\sqrt{x}\right)^2$ $x^2-6x+9=4x$ $x^2-10x+9=0$ $(x-9)(x-1)=0$ $x=9$ or $x\neq 1$ **OR** $x-3=2\sqrt{x}$ $x-2\sqrt{x}-3=0$ $\left(\sqrt{x}\right)^2-2\sqrt{x}-3=0$ $\left(\sqrt{x}-3\right)\left(\sqrt{x}+1\right)=0$ $\sqrt{x}=3$ *or* $\sqrt{x}\neq -1$ $\therefore x=9$	✓A squaring both sides ✓CA standard form ✓CA factors ✓CA answers with rejection (4) ✓A standard form ✓CA factors ✓CA both equations ✓CA answers with rejection (4)
1.2.2	$\sqrt[3]{3t}=9$ $\left(\sqrt[3]{3t}\right)^3=9^3$ $\frac{3t}{3}=\frac{729}{3}$ $t=243$	✓CA $\sqrt[3]{3t}=x$ from 1.2.1 ✓M cubing both sides ✓CA answer (3)

GRADE 11
Marking Guideline

1.3	$3y = x+1$.....(1) $(x-y)(5y-3x)=0$.....(2) $x = 3y-1$.....(3) $[((3y-1)-y)(5y-3(3y-1))]=0$ $(2y-1)(3-4y)=0$ $y=\frac{1}{2}$ or $y=\frac{3}{4}$ $x=\frac{1}{2}$ or $x=\frac{5}{4}$	✓A rewriting in terms of x ✓CA subst. of (3) into (2) ✓CA simplification ✓CA both values of y ✓CA both values of x (5)
	OR	**OR**
	$3y = x+1$.....(1) $(x-y)(5y-3x)=0$.....(2) from (2) $x=y$ or $y=\frac{3}{5}x$.....(3) sub (3) into (1) $3x = x+1$ or $3\left(\frac{3}{5}x\right)=x+1$ $2x=1$ or $\frac{9}{5}x=x+1$ $x=\frac{1}{2}$ or $x=\frac{5}{4}$ sub $x-$values into (3) $y=\frac{1}{2}$ or $y=\frac{3}{4}$	✓A solving for y ito x in (2) ✓CA subst. both into eq. (1) ✓CA simplification ✓CA both values of x ✓CA both values of y (5)

1.4	$x = \frac{3 \pm 3\sqrt{5}}{2}$ $x = \frac{3 \pm \sqrt{45}}{2}$ $\Delta = 45$ and $x^2 - 3x - k = 0$ $\Delta = b^2 - 4ac$ $\Delta = (-3)^2 - 4(1)(-k)$ $\Delta = 9 + 4k$ $\therefore 9 + 4k = 45$ $\frac{4k}{4} = \frac{45-9}{4}$ $k = 9$ **OR** $(2x - 3 + 3\sqrt{5})(2x - 3 - 3\sqrt{5}) = 0$ $(2x-3)^2 - (3\sqrt{5})^2 = 0$ $4x^2 - 12x + 9 - 9(5) = 0$ $4x^2 - 12x - 36 = 0$ $x^2 - 3x - 9 = 0$ $\therefore k = 9$	✓A rewriting Δ as $\sqrt{45}$ ✓A substitution into Δ ✓CA equating two Δs ✓CA answer (4) **OR** ✓A rewriting roots as factors of equation ✓A standard form ✓CA ÷ equation by 4 ✓CA answer (4)
		[34]

GRADE 11
Marking Guideline

QUESTION 2

2.1.1	$\frac{2^{4x+1}.9^{x}.6^{2x-1}}{2^{6x}.27^{x}.3^{x}}$ $=\frac{2^{4x+1}.\left(3^{2}\right)^{x}.\left(3\times 2\right)^{2x-1}}{2^{6x}.\left(3^{3}\right)^{x}.3^{x}}$ $=\frac{2^{4x+1}.3^{2x}.3^{2x-1}.2^{2x-1}}{2^{6x}.3^{3x}.3^{x}}$ $=2^{4x+1+2x-1-6x}.3^{2x+2x-1-3x-x}$ $=2^{0}.3^{-1}$ $=1\times\frac{1}{3}$ $=\frac{1}{3}$	✓A rewriting with prime bases ✓CA separate prime bases ✓CA simplifying: exponential laws ✓CA answer (4)
2.1.2	$\frac{3^{2021}-3^{2017}}{60.\left(\sqrt[3]{3^{6048}}\right)}$ $=\frac{3^{2017}(3^{4}-1)}{60.3^{2016}}$ $=\frac{3^{2017-2016}(80)}{60}$ $=4$	✓A common factor in numerator 3^{2017} ✓A 3^{2016} in denominator ✓A 80 in numerator ✓CA answer (4)
2.2.1	$m^{8x-4}=1$ $m^{8x-4}=m^{0}$ $8x-4=0$ $x=\frac{1}{2}$ **OR** $m^{8x-4}=1$ $\frac{m^{8x}}{m^{4}}=1$ $m^{8x}=m^{4}$ $8x=4$ $x=\frac{1}{2}$	✓A m^{0} ✓CA equating exponents ✓CA answer (3) **OR** ✓A $m^{8x}=m^{4}$ ✓CA equating exponents ✓CA answer (3)

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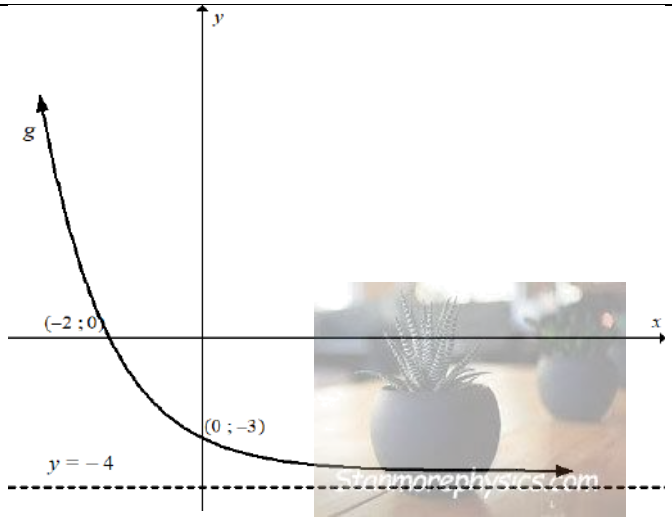
Marking Guideline

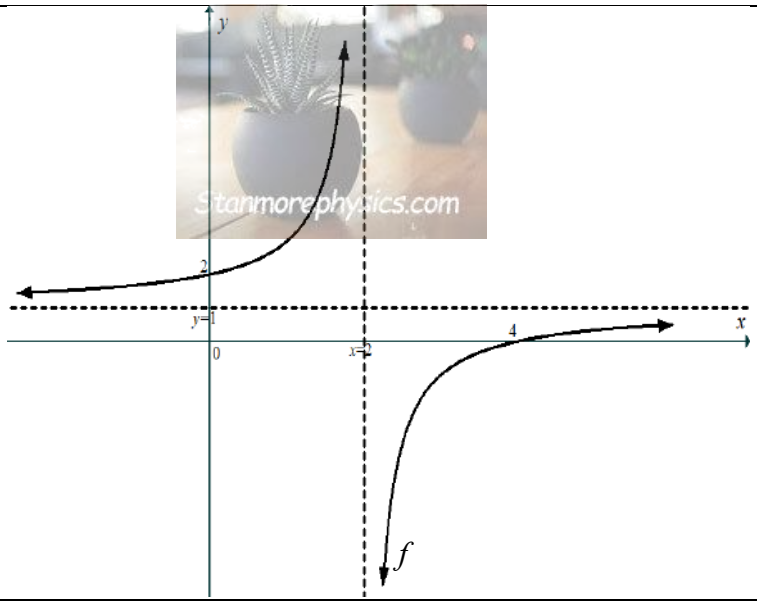
2.2.2	$4.3^x - 3^{x-2} + 3^x = \frac{44}{3}$ $4.3^x - 3^x.3^{-2} + 3^x = \frac{44}{3}$ $3^x(4 - 3^{-2} + 1) = \frac{44}{3}$ $3^x\left(\frac{44}{9}\right) = \frac{44}{3}$ $3^x = \frac{44}{3} \times \frac{9}{44}$ $3^x = 3^1$ $x = 1$	✓A common factor of 3^x ✓A factor of $\frac{44}{9}$ ✓CA simplification ✓CA answer (4)
2.2.3	$x^x = 2^{2048}$ $x^x = 2^{2(1024)}$ $x^x = 4^{2(512)}$ $x^x = 16^{(512)}$ $x^x = 16^{2(256)}$ $x^x = 256^{256}$ $\therefore x = 256$ **OR** $x^x = 2^{2048}$ $x^x = 2^{1024}.2^{1024}$ $x^x = (2.2)^{1024}$ $x^x = 4^{1024}$ $x^x = 4^{512}.4^{512}$ $x^x = 16^{256}.16^{256}$ $x^x = (16.16)^{256}$ $x^x = 256^{256}$ $\therefore x = 256$	✓A rewriting the exp. as 2(1024) ✓A rewriting as $16^{(512)}$ ✓A answer (3) ✓A rewriting the RHS as $2^{1024}.2^{1024}$ ✓A rewriting as $16^{256}.16^{256}$ ✓A answer (3)
		[18]

GRADE 11
Marking Guideline

QUESTION 3

3.1	$y=-4$	✓A answer (1)
3.2	$g(x)=\left(\frac{1}{2}\right)^x-4$ $g(0)=\left(\frac{1}{2}\right)^0-4$ $g(0)=1-4$ $y=-3$	✓A $g(0)$ **OR** substitute $x=0$ ✓CA value of y (2)
3.3	$g(x)=\left(\frac{1}{2}\right)^x-4$ $0=\left(\frac{1}{2}\right)^x-4$ $4=2^{-x}$ $2^2=2^{-x}$ $x=-2$	✓A $g(x)=0$ ✓CA value of x (2)
3.4.		✓A shape ✓CA intercepts ✓A asymptote (3)
3.5	$m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{(0)-(-3)}{(-2)-(0)}$ $m=-\frac{3}{2}$	✓CA substitution of x- and y- intercepts from 3.2 and 3.3 ✓CA answer (2)
3.6	$k(x)=\left(\frac{1}{2}\right)^x$	✓A answer (1)
3.7	reflection about y-axis and then translation/shift of 3 units to the left **OR** translation/shift of 3 units to the right and then reflection about y-axis	✓A reflection about $y-$axis ✓A translation of 3 units ✓A left (3) **OR** ✓A translation of 3 units ✓A right ✓A reflection about $y-$axis (3)
		[14]

QUESTION 4

4.1	$y \in \mathbb{R}, y \neq 1$ or $y \in (-\infty\,;1) \cup (1\,;\infty)$	✓A answer (1)
4.2	$x = 3 - y$ $x = 3 - 1$ $x = 2$	✓A substitution ✓A answer (2)
4.3	$f(x) = \frac{a}{x+p} + 1$ $f(x) = \frac{a}{x-2} + 1$ $2 = \frac{a}{0-2} + 1$ $2 - 1 = \frac{a}{-2}$ $a = -2$	✓A substitution of a point $(0;2)$ ✓CA answer (2)
4.4	$f(x) = \frac{-2}{x-2} + 1$ $0 = \frac{-2}{x-2} + 1$ $-1 = \frac{-2}{x-2}$ $-1(x-2) = -2$ $-x + 2 = -2$ $-x = -4$ $x = 4$	✓A $f(x) = 0$ ✓CA simplification ✓CA answer (3)
4.5		✓CA shape based on a in 4.3 ✓CA vertical asymptote from 4.2 but horizontal asymptote of $y = 1$ must be accurate ✓CA x-intercept from 4.4 but y-intercept must be accurate (given in 4.3) (3)
4.6	$2 < x \leq 4$ **OR** $x \in (2;4]$	✓✓CA answer (2)
4.7	$Q(3\,;-1)$	✓A $x-$value ✓A $y-$value (2)
		[15]

GRADE 11
Marking Guideline

QUESTION 5

5.1	D(4 ; 0)	✓A answer (1)
5.2	$h(x)=a(x-x_1)(x-x_2)$ $h(x)=a(x-(-3))(x-(1))$ $6=a(0+3)(0-1)$ $6=a(-3)$ $\frac{6}{-3}=a$ $-2=a$ $h(x)=-2(x+3)(x-1)$ $h(x)=-2(x^2+2x-3)$ $h(x)=-2x^2-4x+6$ $a=-2\ ,\ b=-4\ ,\ c=6$	✓A defining equation ✓A substituting x-intercepts ✓A substituting $(0\ ;\ 6)$ ✓A simplification (4)
5.3	$h(x)=-2x^2-4x+6$ $x=-\frac{b}{2a}$ $x=\frac{-4}{2(-2)}$ $x=-1$ $h(-1)=-2(-1)^2-4(-1)+6$ $y=8$ M(−1 ; 8) **OR** $x_M=\frac{-3+1}{2}$ $x_M=-1$ $h(-1)=-2(-1)^2-4(-1)+6$ $y=8$ M(−1 ; 8)	✓A $x-$value ✓CA $y-$value (2) **OR** ✓A $x-$value ✓CA $y-$value (2)
5.4	$y\in\mathbb{R}\ ,\ y\leq 8$ **OR** $y\in(-\infty\ ;\ 8]$	✓CA answer (using x_M in 5.3) **OR** ✓CA answer (using x_M in 5.3) (1)
5.5	$x<-3$ or $1<x<4$ **OR** $x\in(-\infty\ ;-3)\cup(1\ ;\ 4)$ **NOTE:** CA is only on 4 i.e. x_D in 5.1	✓A $x<-3$ ✓A✓CA $1<x<4$ **OR** ✓ $x\in(-\infty\ ;-3)$ ✓A✓CA $x\in(1\ ;\ 4)$ (3)

5.6.1	$k > 8$ **OR** $k \in (8\,;\infty)$	✓CA answer (using x_M in 5.3) ✓CA answer (using x_M in 5.3) (1)
5.6.2	$6 < k \leq 8$ **OR** $k \in (6;\,8]$ **NOTE:** CA is only on 8 i.e. x_M in 5.3	✓A✓CA answer ✓A✓CA answer (2)
5.7	If graph p is to now become a tangent, then this implies that the NEW y −intercept is unknown $h(x) = p(x)$ $-2x^2 - 4x + 6 = -2x + k$ $-2x^2 - 2x + 6 - k = 0$ $2x^2 + 2x - 6 + k = 0$ $\Delta = 0$ $b^2 - 4ac = 0$ $(2)^2 - 4(2)(-6 + k) = 0$ $4 - 8(-6 + k) = 0$ $4 + 48 - 8k = 0$ $52 = 8k$ $k = \frac{52}{8} = \frac{13}{2} = 6\frac{1}{2} = 6,5$ Graph p must shift/be translated **1,5 units downwards** **NOTE:** Correct answer only: award only 1 mark **OR** $h(x) = p(x) + k$ $-2x^2 - 4x + 6 = -2x + 8 + k$ $-2x^2 - 2x - 2 - k = 0$ $2x^2 + 2x + 2 + k = 0$ $\Delta = 0$ $b^2 - 4ac = 0$ $(2)^2 - 4(2)(2 + k) = 0$ $4 - 8(k + 2) = 0$ $4 = 8(k + 2)$ $\frac{1}{2} = k + 2$ $k = -\frac{3}{2} = -1\frac{1}{2} = -1,5$ Graph p must shift/be translated **1,5 units downwards** **NOTE:** Correct answer only: award only 1 mark	✓A equating equations ✓CA standard form ✓CA equating $\Delta = 0$ ✓CA value of k ✓CA answer (5) ✓A equating equations ✓CA standard form ✓CA equating $\Delta = 0$ ✓CA value of k ✓CA answer (5)
		[19]
		TOTAL:100