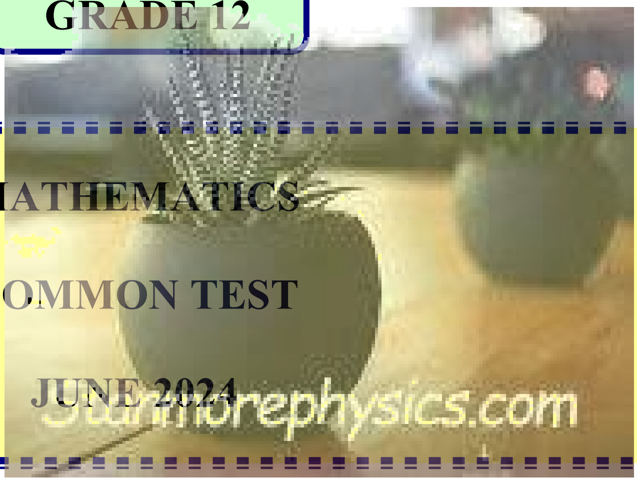
KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12



MATHEMATICS

COMMON TEST

JUNE 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages, an information sheet and an answer book of 16 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.

2. Answer **ALL** the questions in the **ANSWER BOOK** provided.

4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

8. An information sheet with formulae is included at the end of the question paper.

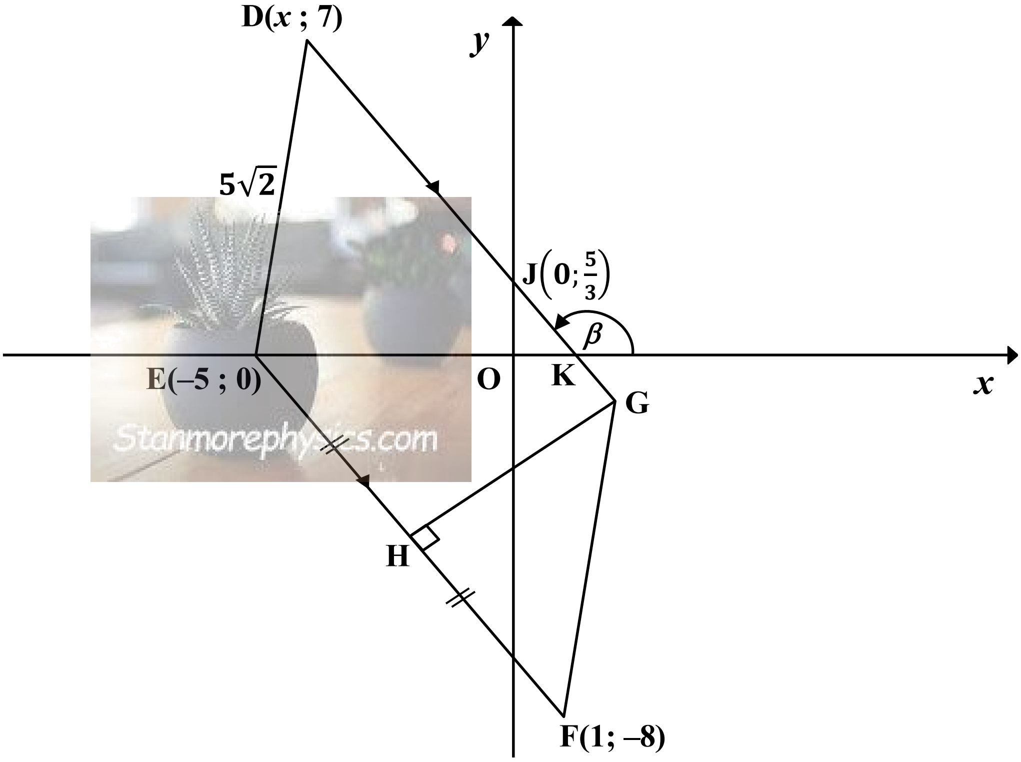
9. Write neatly and legibly.

QUESTION 1

In the diagram, DEFG is a parallelogram with vertices $D(x\,;7)$, $E(-5\,;0)$, $F(1\,;\,-8)$ and G. $GH \perp EF$, with H on EF, such that $EH = HF$. The angle of inclination of DG is β.

DE has a positive gradient. DG cuts the y-axis at $J\left(0\,;\,\frac{5}{3}\right)$ and the x-axis at K.

The length of $DE = 5\sqrt{2}$.



1.1 Calculate the gradient of EF. (2)

1.2 Calculate the coordinates of H. (2)

1.3 Determine the equation of GH in the form $y = mx + c$. (3)

1.4 Calculate the size of β. (3)

1.5 Calculate the size of $O\hat{J}K$. (2)

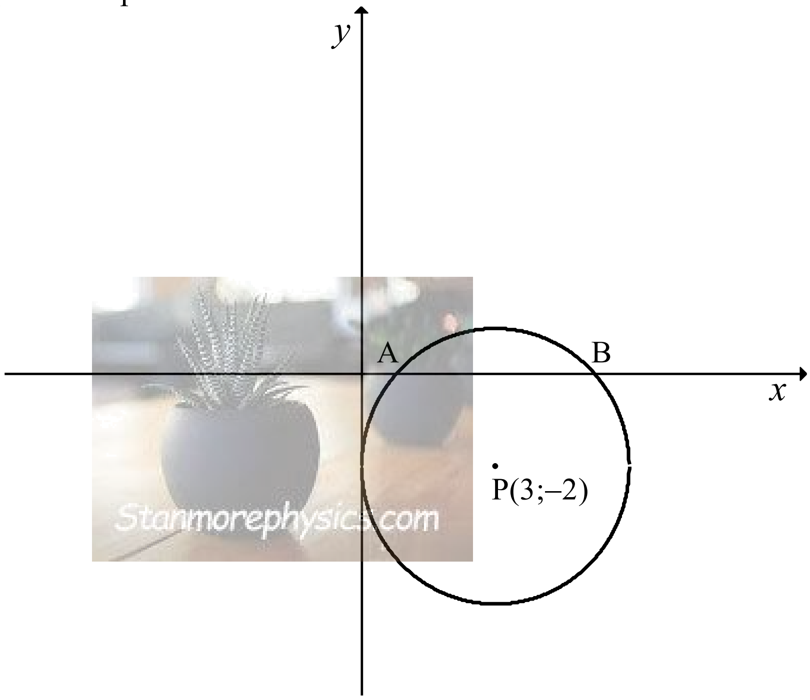
1.6 Calculate the value of x. (5)

1.7 Calculate the area of DEOJ. (6)

[23]

QUESTION 2

In the diagram below, $P(3\,;-2)$ is the centre of a circle that has the y-axis as tangent.
A and B are the x-intercepts of circle P.



2.1 Determine

2.1.1 the radius of the circle (1)

2.1.2 the equation of the circle (1)

2.2 Calculate the distance AB. (4)

2.3 Another circle has the equation $x^2+2x+y^2-8y-8=0$. Determine the radius and the coordinates of the centre of this circle. (4)

2.4 Will the two circles intersect? Clearly motivate your answer by means of calculations. (5)

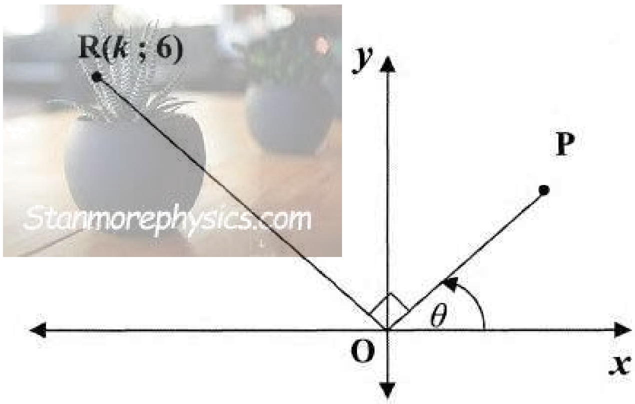
2.5 The circle with centre P is reflected about the line $y=-1$. Write down the equations of the horizontal tangents to the new circle formed through this reflection. (2)

[17]

QUESTION 3

DO NOT USE A CALCULATOR WHEN ANSWERING QUESTION 3.

3.1 In the diagram below, P is a point in the first quadrant such that $5\cos\theta = 3$. $R(k\ ;\ 6)$ is a point in the second quadrant such that $P\hat{O}R = 90°$.



Determine the value of the following:

3.1.1 $\tan\theta$ (3)

3.1.2 $\sin 2\theta$ (3)

3.1.3 k (5)

3.2 Simplify fully: $\cos(385° + \beta).\sin(35° - \beta) + \sin(25° + \beta).\sin(55° + \beta)$ (4)

3.3 Given: $\sin 3\theta = 4\sin\theta.\cos^2\theta - \sin\theta$.

3.3.1 Prove the given identity. (5)

3.3.2 Hence, or otherwise, prove the following identity: $\dfrac{\sin 3\theta + \sin\theta}{2 + 2\cos 2\theta} = \sin\theta$ (3)

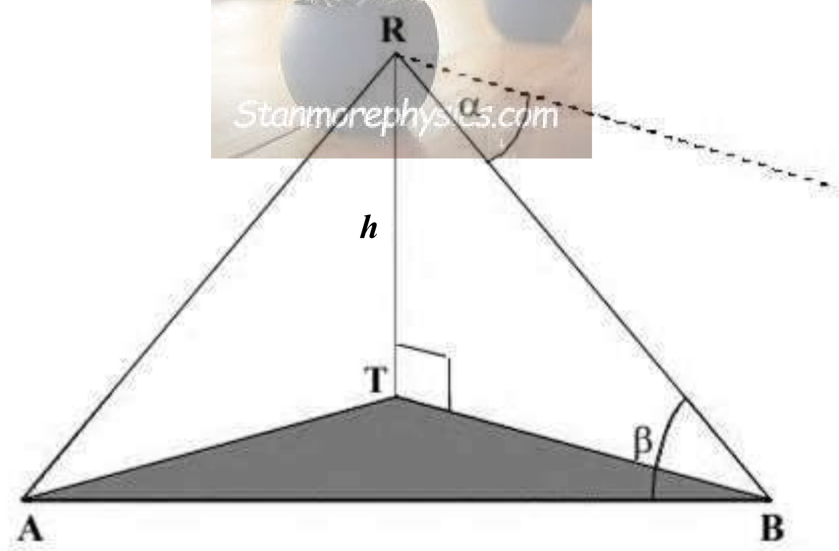
3.3.3 Determine all the values of θ for which the identity in QUESTION 3.3.2 will be undefined. (4)

3.4 Determine the minimum value of: $\cos 3x - 5$. (2)

[29]

QUESTION 4

In the diagram below, RT represents the height of a vertical tower, with T the foot of the tower. A and B represent two points equidistant from T, and which lie in the same horizontal plane as T.
The height of the tower is h. The angle of depression of B from R is α.
$\text{R}\hat{\text{B}}\text{A} = \beta$.



4.1 Determine the size of $\text{A}\hat{\text{R}}\text{B}$ in terms of β. (1)

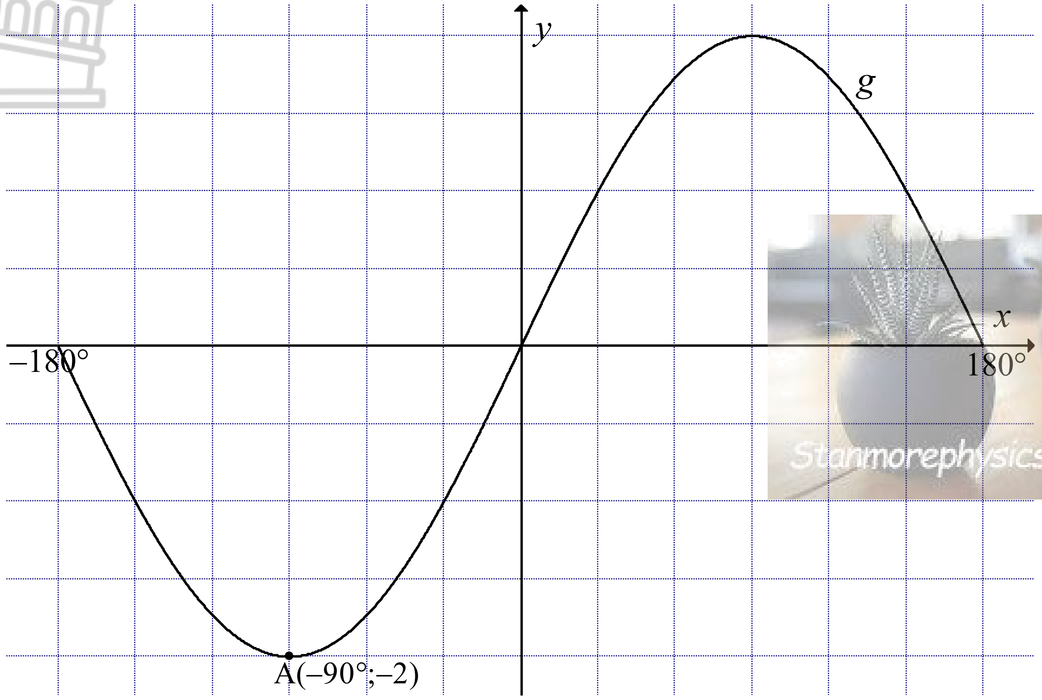
4.2 Prove that $\text{AB} = \dfrac{2h.\cos\beta}{\sin\alpha}$ (6)

4.3 Calculate the height of the tower , rounded off to the nearest unit, if AB = 5,4 units, $\alpha = 51°$ and $\beta = 65°$. (3)

[10]

QUESTION 5

In the diagram, the graph of $g(x) = a\sin x$ is drawn for the interval $x \in [-180° ; 180°]$.
$A(-90°; -2)$ are the coordinates of a turning point of the graph.



5.1 Write down the value of a. (1)

5.2 On the grid provided in the ANSWER BOOK, draw the graph of $f(x) = 2\cos(x - 30°)$ for $x \in [-180° ; 180°]$. Clearly indicate all intercepts with the axes, as well as the turning points and end points of the graph. (4)

5.3 Write down:

5.3.1 the range of f (2)

5.3.2 the period of $g(3x)$ (2)

5.4 Determine algebraically the values of x if $f(x) = g(x)$, for $x \in [-180° ; 180°]$. (5)

5.5 Determine the value(s) of x, in the interval $x \in [-180° ; 180°]$, for which

5.5.1 $g'(x) = 0$? (2)

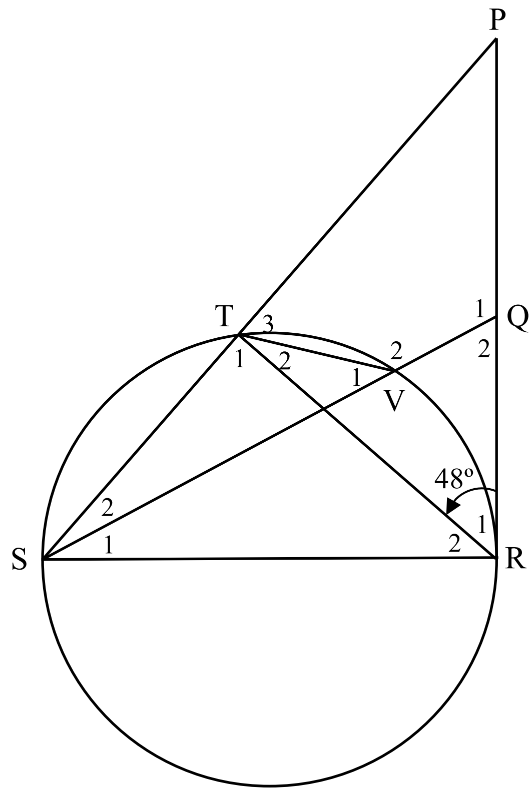
5.5.2 $f(x) > g(x)$? (2)

[18]

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 6, 7 AND 8.

QUESTION 6

6.1 SR is a diameter of the circle in the sketch below. Chord ST is produced to P. PR is a tangent to the circle at R. Chord SV produced meets PR at Q. TV and TR are drawn.



6.1.1 Write down the size of $P\hat{R}S$. Provide a reason. (2)

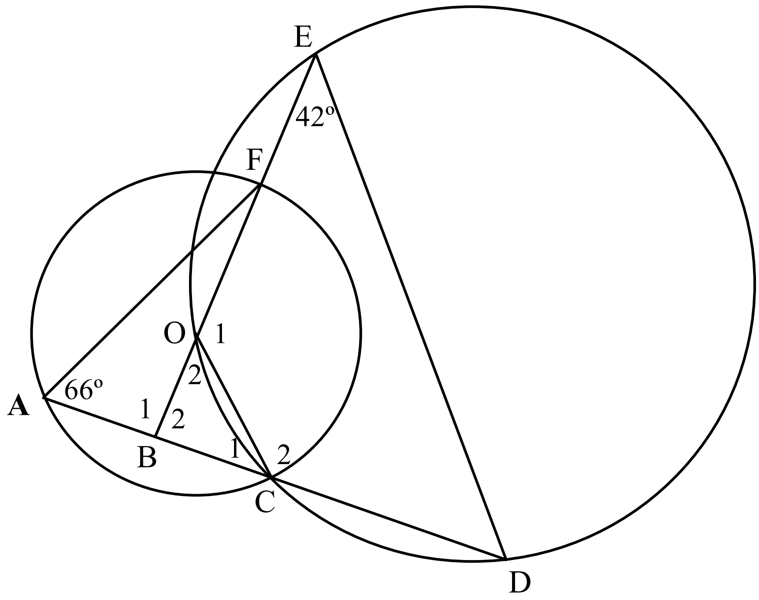
6.1.2 Calculate the size of:

(a) $\hat{P}$ (3)

(b) $\hat{V}_1$ (3)

6.1.3 Prove that $P\hat{Q}S = V\hat{T}S$. (4)

6.2 In the diagram below, the bigger circle has points E, O, C and D on its circumference. O is the centre of the smaller circle. C is a point of intersection between the two circles, and A and F are two more points on the circumference of the smaller circle.
ABCD and BOFE are straight lines.
$\hat{A} = 66°$ and $\hat{E} = 42°$.

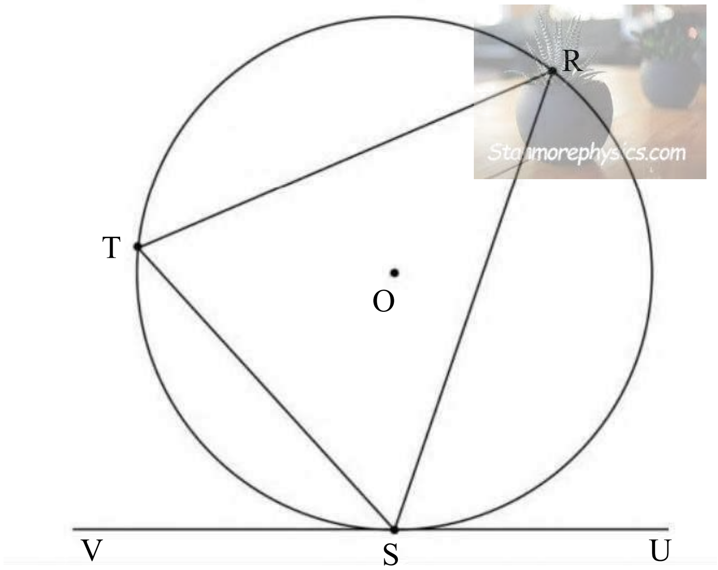


Prove that AB = BC. (6)

[18]

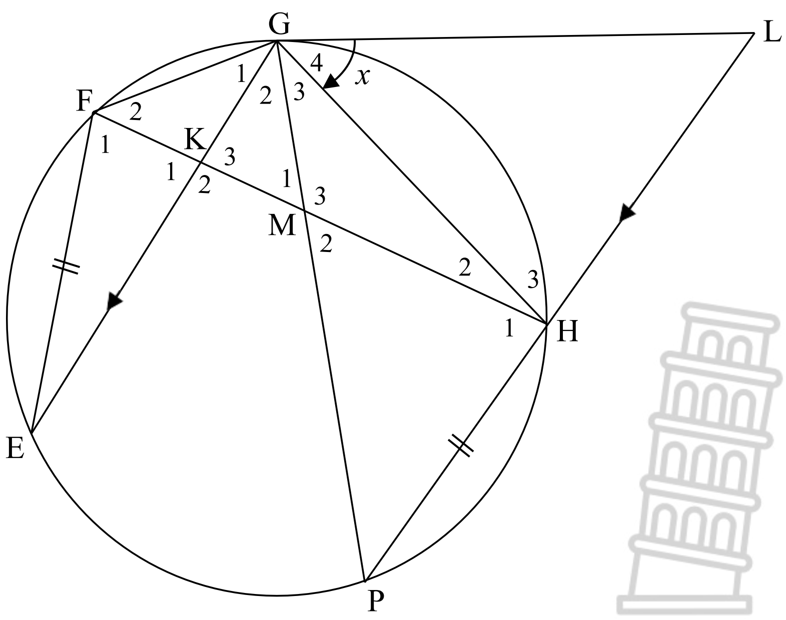
QUESTION 7

7.1 In the diagram, chords ST, SR and TR are drawn in the circle with centre O. VSU is a tangent to the circle at S.



Use the diagram to prove the theorem which states that $V\hat{S}T = \hat{R}$. (5)

7.2 LG is a tangent to circle EFGHP at G. Chord PH is produced to L. Chord HF cuts chord GP in M and chord EG in K. EG || PL and EF = PH. $\hat{G}_4 = x$.



7.2.1 Write down, with reasons, THREE other angles, each equal to x. (6)

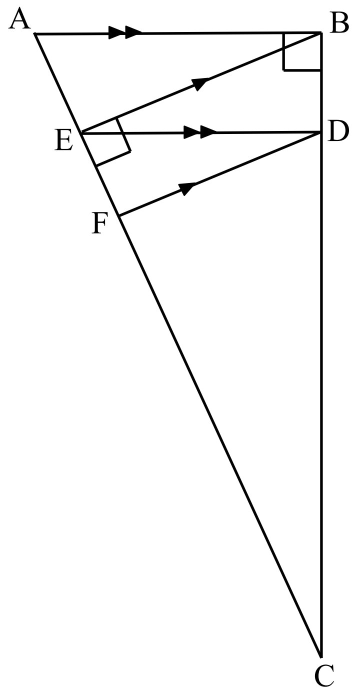
7.2.2 Prove that $\Delta HMG \ ||| \ \Delta EFG$. (5)

7.2.3 Hence, or otherwise, prove that $PH.HG = EG.HM$. (3)

[19]

QUESTION 8

In the diagram below, ΔABC is drawn with D on BC, and F and E on AC such that $AB \parallel ED$, $EB \parallel FD$, $AB \perp BC$ and $BE \perp AC$. AC = 6,5 units and BC = 6 units.



8.1 Determine the length of AB. (2)

8.2 Prove that $CB = \sqrt{CA.CE}$. (6)

8.3 Hence, determine the length of CE, correct to one decimal place. (2)

8.4 Write down the length of AE. (1)

8.5 Determine the length of EF. (5)

[16]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

COMMON TEST

JUNE 2024

MARKING GUIDELINES

MARKS: 150

TIME: 3 hours

These marking guidelines consist of 17 pages.

NOTE:

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason.)**
R	**A mark for a correct reason** **(A reason mark may only be awarded if the statement is correct.)**
S/R	**Award a mark if the statement AND reason are both correct.**

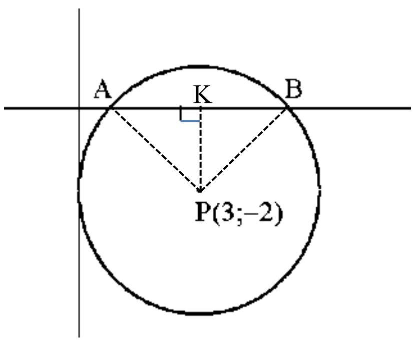
QUESTION 1

1.1	$m_{EF}=\dfrac{-8-0}{1-(-5)}$ $=-\dfrac{4}{3}$ Answer only: Full marks	A✓ substitution CA✓ answer (2)
1.2	$\left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$ $=\left(\dfrac{-5+1}{2};\dfrac{0+(-8)}{2}\right)$ $=(-2;-4)$ Answer only: Full marks	A ✓ x-coordinate A ✓ y-coordinate (2)
1.3	$m_{GH}\times\left(-\dfrac{4}{3}\right)=-1$ $m_{GH}=\dfrac{3}{4}$ Substitute $(-2;-4)$ and $m_{GH}=\dfrac{3}{4}$ into $y=mx+c$: $-4=\dfrac{3}{4}(-2)+c$ $c=-\dfrac{5}{2}$ $y=\dfrac{3}{4}x-\dfrac{5}{2}$ If $m=-\dfrac{4}{3}$ is used: maximum 1 mark	CA✓ value of m_{GH} CA✓ substitution of point and gradient CA✓ answer (3)
1.4	$m_{DG}=m_{EF}=-\dfrac{4}{3}$ $m_{DG}=\tan\beta=-\dfrac{4}{3}$ $\beta=126,87°$	CA✓ value of m_{DG} CA✓ $\tan\beta=-\dfrac{4}{3}$ CA✓ answer (3)

1.5	$\text{O}\hat{\text{J}}\text{K} = 126,87° - 90°$ [exterior ∠ of ΔOJK] $= 36,87°$ If answer is a negative angle: 0/2	CA✓ method CA✓ answer (2)
1.6	$\text{DE} = \sqrt{(x-(-5))^2 + (7-0)^2} = 5\sqrt{2}$ $(x-(-5))^2 + (7-0)^2 = 50$ $x^2 + 10x + 24 = 0$ $(x+6)(x+4) = 0$ $x = -6$ or $x = -4$ $x = -4$ only	A✓ substitution in distance formula and equating to $5\sqrt{2}$ CA✓ squaring both sides CA✓ standard form CA✓ both x-values CA✓ selecting the x-value > -5 (5)
	OR Equation of DG: $y = -\frac{4}{3}x + \frac{5}{3}$ Substitute $y = 7$: $7 = -\frac{4}{3}x + \frac{5}{3}$ $\frac{4}{3}x = -7 + \frac{5}{3}$ $\frac{4}{3}x = \frac{-16}{3}$ $\therefore x = \frac{-16}{3} \times \frac{3}{4}$ $= -4$	**OR** CA✓ equation of DG CA✓ substitute $y = 7$ CA✓ simplification CA✓✓ answer $x = -4$ (5)

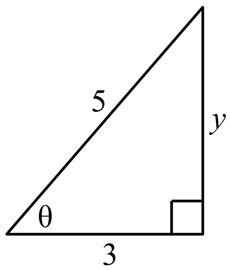
1.7	Equation of DG: $y=-\frac{4}{3}x+\frac{5}{3}$	CA✓ equation of DG
	For x-coordinate of K: $0=-\frac{4}{3}x+\frac{5}{3}$	CA ✓ substitution of $y = 0$
	$x=\frac{5}{4}=1,25$	CA✓ value of x-coordinate of K
	Area of $\Delta DEK=\frac{1}{2}\times \text{base}\times \text{height}$	
	$=\frac{1}{2}\times\left(5+\frac{5}{4}\right)\times 7$	CA✓ substitution to calculate area of ΔDEK
	$=\frac{175}{8}$	
	Area of $\Delta OJK=\frac{1}{2}\times\frac{5}{4}\times\frac{5}{3}$	CA✓ substitution to calculate area of ΔOJK
	$=\frac{25}{24}$	
	Area of DEOJ $=\frac{175}{8}-\frac{25}{24}=\frac{125}{6}=20,83$ units^2	CA✓ area of DEOJ (6)
	OR	**OR**
	Equation of DG: $y=-\frac{4}{3}x+\frac{5}{3}$	CA✓ equation of DG
	For x-coordinate of K: $0=-\frac{4}{3}x+\frac{5}{3}$	CA ✓ substitution of $y = 0$
	$x=\frac{5}{4}=1,25$	CA✓ value of x-coordinate of K
	$DK=\sqrt{\left[1,25-(-4)\right]^2+(0-7)^2}=\sqrt{\frac{1225}{16}}=8,75$	
	$EK=1,25-(-5)=6,25$	
	Area of $\Delta DEK=\frac{1}{2}\times DK\times EK\times\sin J\hat{K}O$	
	$=\frac{1}{2}\times 8,75\times 6,25\times\sin 53,13^\circ$	CA✓ substitution to calculate area of ΔDEK
	$=21,87$	
	Area of $\Delta OJK=\frac{1}{2}\times\frac{5}{4}\times\frac{5}{3}$	CA✓ substitution to calculate area of ΔOJK
	$=\frac{25}{24}$	
	Area of DEOJ $=21,87-\frac{25}{24}=20,83$ units^2	CA✓ area of DEOJ (6)
		[23]

QUESTION 2

2.1.1	radius = 3 units	A✓ answer (1)
2.1.2	$(x-3)^2+(y+2)^2=3^2$	CA✓ $(x-3)^2+(y+2)^2=3^2$ (1)
2.2	For x-intercepts, let $y=0$: $(x-3)^2+(0+2)^2=3^2$ $(x-3)^2=5$ $x-3=+\sqrt{5}$ or $x-3=-\sqrt{5}$ $x=3+\sqrt{5}=5,24$ or $x=3-\sqrt{5}=0,76$ $\text{AB}=3+\sqrt{5}-(3-\sqrt{5})$ $=2\sqrt{5}=4,47$ units **OR** For x-intercepts, let $y=0$: $(x-3)^2+(0+2)^2=3^2$ $(x-3)^2=5$ $x^2-6x+9=5$ $x=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$ $x=3+\sqrt{5}=5,24$ or $x=3-\sqrt{5}=0,76$ $\text{AB}=3+\sqrt{5}-(3-\sqrt{5})$ $=2\sqrt{5}=4,47$ units **OR**  $\text{AP}^2=\text{AK}^2+\text{PK}^2$ [Pythagoras] $3^2=\text{AK}^2+2^2$ $\text{AK}=\sqrt{5}$ $\text{AK}=\text{BK}$ [line from centre ⊥ to chord] $\therefore \text{AB}=2\sqrt{5}$	CA✓ substitute $y=0$ CA✓ values of x CA✓ subtraction CA✓ answer (4) **OR** CA✓ substitute $y=0$ CA✓ values of x CA✓ subtraction CA✓ answer **(4)** **OR** CA✓ applying Theorem of Pythagoras CA✓ substitution CA✓ length of AK CA✓ answer (4)

2.3	$x^2+2x+y^2-8y-8=0$ $x^2+2x+1+y^2-8y+16=8+1+16$ $(x+1)^2+(y-4)^2=5^2$ radius = 5 units centre: $(-1\ ;\ 4)$	A✓ completing the square A✓ $(x+1)^2+(y-4)^2=5^2$ CA✓ radius CA✓ coordinates of centre (4)
2.4	Sum of radii $=3+5=8$ Distance between centres $=\sqrt{(-1-3)^2+(4-(-2))^2}$ $=\sqrt{52}=7{,}21$ units $\therefore$ Distance between centres $<$ Sum of radii $\therefore$ The circles will intersect	CA✓ sum of radii CA✓substitution CA✓ distance between centres CA✓ distance between centres < sum of radii CA✓ conclusion (5)
2.5	$y=3$ $y=-3$	A✓ answer A✓answer (2)
		[17]

QUESTION 3

3.1.1	$\cos\theta=\frac{3}{5}$ $y^2=5^2-3^2$ [Pythagoras] $y=4$ $\tan\theta=\frac{4}{3}$  	A✓ $\cos\theta=\frac{3}{5}$ A✓ $y=4$ CA✓ answer (3)
3.1.2	$\sin 2\theta=2\sin\theta\cos\theta$ $=2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$ $=\frac{24}{25}$	A✓ expansion CA✓ substitution CA✓ answer (3)

3.1.3	$RO = \sqrt{k^2 + 6^2}$ [Pythagoras] $\sin(90° + \theta) = \cos\theta$ $\frac{6}{\sqrt{k^2+6^2}} = \frac{3}{5}$ $\sqrt{k^2+6^2} = 10$ $k^2 + 6^2 = 100$ $k^2 = 64$ $k = -8$	A✓ $RO = \sqrt{k^2 + 6^2}$ A✓ $\sin(90° + \theta) = \cos\theta$ CA✓ substitution CA✓ simplification CA✓answer (5)
	OR $RO = \sqrt{k^2 + 6^2}$ [Pythagoras] $\cos(90° + \theta) = -\sin\theta$ $\frac{k}{\sqrt{k^2+6^2}} = -\frac{4}{5}$ $-4\sqrt{k^2+6^2} = 5k$ $16(36 + k^2) = 25k^2$ $576 + 16k^2 = 25k^2$ $9k^2 = 576$ $k^2 = 64$ $k = -8$	**OR** A✓ $RO = \sqrt{k^2 + 6^2}$ A✓ $\cos(90° + \theta) = -\sin\theta$ CA✓ substitution CA✓ simplification CA✓answer (5)
	OR $RO = \sqrt{k^2 + 6^2}$ [Pythagoras] $RP^2 = OP^2 + OR^2$ [Pythagoras in ΔPOR] $(k-3)^2 + (6-4)^2 = 5^2 + (k^2 + 36)$ $k^2 - 6k + 9 + 4 = 25 + k^2 + 36$ $-6k = 48$ $k = -8$	**OR** A✓ $RO = \sqrt{k^2 + 6^2}$ A✓ Pythagoras in ΔPOR CA✓ substitution CA✓ simplification CA✓answer (5)
	OR $m_{OP} = \frac{4}{3}$ $m_{OR} = \frac{6-0}{k-0}$ $m_{OP} \times m_{OR} = -1$ [OR ⊥ OP] $\therefore \frac{4}{3} \times -\frac{6}{k} = -1$ $k = -8$	**OR** A✓ m of OP A✓ m of OR A✓ condition for gradients of ⊥ lines CA✓ substitution CA✓answer (5)

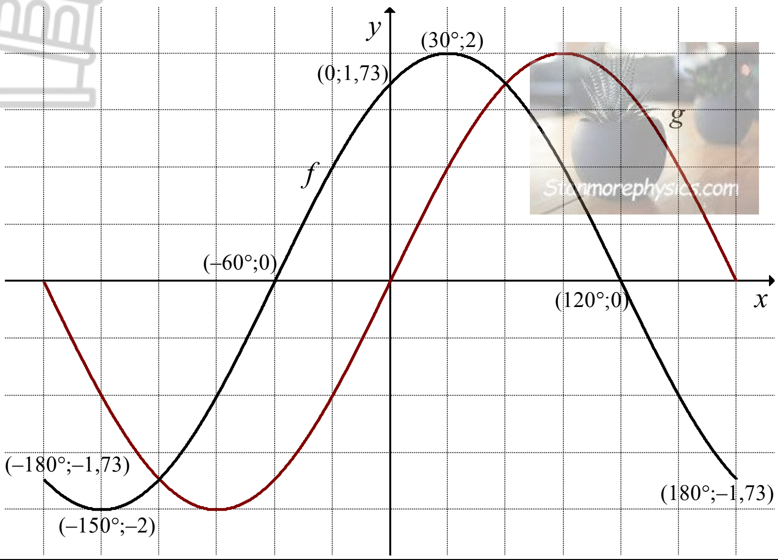
3.2	$\cos(385°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos(25°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\cos[90°-(55°+\beta)]$ $=\cos(25°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\cos(35°-\beta)$ $=\sin(25°+\beta+35°-\beta)$ $=\sin 60°$ $=\frac{\sqrt{3}}{2}$ **OR** $\cos(385°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos(25°+\beta).\cos[90°-(35°-\beta)]+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos(25°+\beta).\cos(55°+\beta)+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos[25°+\beta-(55°+\beta)]$ $=\cos(-30°)$ $=\frac{\sqrt{3}}{2}$	A✓ $\cos(25°+\beta)$ A✓ $\cos(35°-\beta)$ A✓ applying compound angle identity A✓ answer (4) **OR** A✓ $\cos(25°+\beta)$ A✓ $\sin(55°+\beta)$ A✓ applying compound angle identity A✓ answer (4)
3.3.1	$\sin 3\theta$ $=\sin(2\theta+\theta)$ $=\sin 2\theta\cos\theta+\cos 2\theta\sin\theta$ $=2\sin\theta\cos^2\theta+(2\cos^2\theta-1)\sin\theta$ $=2\sin\theta\cos^2\theta+2\sin\theta\cos^2\theta-\sin\theta$ $=4\sin\theta\cos^2\theta-\sin\theta$ **OR** $\sin 3\theta$ $=\sin(2\theta+\theta)$ $=\sin 2\theta\cos\theta+\cos 2\theta\sin\theta$ $=2\sin\theta\cos^2\theta+(1-2\sin^2\theta)\sin\theta$ $=2\sin\theta\cos^2\theta+\sin\theta-2\sin^3\theta$ $=2\sin\theta\cos^2\theta+\sin\theta-2\sin\theta(1-\cos^2\theta)$ $=2\sin\theta\cos^2\theta+\sin\theta-2\sin\theta+2\sin\theta\cos^2\theta$ $=4\sin\theta\cos^2\theta-\sin\theta$	A✓ replace 3θ by $(2\theta+\theta)$ A✓ compound angle expansion A✓sine double angle expansion A✓ cosine double angle expansion A✓ simplification (5) **OR** A✓ replace 3θ by $(2\theta+\theta)$ A✓ compound angle expansion A✓sine double angle expansion A✓ cosine double angle expansion A✓ simplification (5)

	OR $\sin 3\theta$ $= \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos^2\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$ $= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$ $= 3\sin\theta\cos^2\theta - \sin\theta(1-\cos^2\theta)$ $= 3\sin\theta\cos^2\theta - \sin\theta + \sin\theta\cos^2\theta$ $= 4\sin\theta\cos^2\theta - \sin\theta$	**OR** A✓ replace 3θ by $(2\theta+\theta)$ A✓ compound angle expansion A✓sine double angle expansion A✓ cosine double angle expansion A✓ simplification (5)
3.3.2	$\frac{\sin 3\theta + \sin\theta}{2 + 2\cos 2\theta}$ $= \frac{4\sin\theta\cos^2\theta - \sin\theta + \sin\theta}{2 + 2\cos 2\theta}$ $= \frac{4\sin\theta\cos^2\theta}{2 + 2(2\cos^2\theta - 1)}$ $= \frac{4\sin\theta\cos^2\theta}{2 + 4\cos^2\theta - 2}$ $= \frac{4\sin\theta\cos^2\theta}{4\cos^2\theta}$ $= \sin\theta$	A✓ numerator simplified A✓ cos double angle expansion A✓ denominator simplified (3)
3.3.3	$2 + 2\cos 2\theta = 0$ $\cos 2\theta = -1$ $\therefore 2\theta = 180° + k.360°, \quad k \in Z$ $\theta = 90° + k.180°, \quad k \in Z$ Penalty of 1 if $k \in Z$ is omitted	A✓ equating denominator to 0 A✓ $\cos 2\theta = -1$ CA✓ $2\theta = 180° + k.360°$ CA✓ $\theta = 90° + k.180°, \quad k \in Z$ (4)
3.4	Minimum value of $\cos 3x = -1$ $\therefore$ Minimum value of $\cos 3x - 5 = -6$ Answer only: Full marks	A✓ Minimum value of $\cos 3x = -1$ A✓ answer (2)
		[29]

QUESTION 4

4.1	$R\hat{A}B = R\hat{B}A = \beta$ [$\Delta RAT \equiv \Delta RBT$] $\therefore A\hat{R}B = 180° - (R\hat{A}B + R\hat{B}A)$ [sum of $\angle$ s of ΔRAB] $= 180° - 2\beta$	A✓ answer (1)
4.2	$R\hat{B}T = \alpha$ [alt. $\angle$ s; || lines] $\frac{h}{BR} = \sin\alpha$ $BR = \frac{h}{\sin\alpha}$ $\frac{AB}{\sin A\hat{R}B} = \frac{BR}{\sin R\hat{A}B}$ $AB = \frac{BR.\sin A\hat{R}B}{\sin R\hat{A}B}$ $= \frac{h\sin(180° - 2\beta)}{\sin\alpha\sin\beta}$ $= \frac{h\sin 2\beta}{\sin\alpha\sin\beta}$ $= \frac{h(2\sin\beta\cos\beta)}{\sin\alpha\sin\beta}$ $= \frac{2h\cos\beta}{\sin\alpha}$	A✓ $\frac{h}{BR} = \sin\alpha$ A✓ BR subject of formula A✓ applying sine rule A✓ AB subject of formula A✓ $\sin 2\beta$ A✓ $2\sin\beta\cos\beta$ (6)
	OR $R\hat{B}T = \alpha$ [alt. $\angle$ s; || lines] $\frac{h}{BR} = \sin\alpha$ $BR = \frac{h}{\sin\alpha}$ $AB^2 = BR^2 + AR^2 - 2BR.AR.\cos(180° - 2\beta)$ $= 2BR^2 - 2BR^2.\cos(180° - 2\beta)$ $= 2BR^2 + 2BR^2.\cos 2\beta$ $= 2BR^2(1 + \cos 2\beta)$ $= 2BR^2(1 + 2\cos^2\beta - 1)$ $= 4BR^2\cos^2\beta$ $\therefore AB = 2BR\cos\beta$ $= \frac{2h\cos\beta}{\sin\alpha}$	**OR** A✓ $\frac{h}{BR} = \sin\alpha$ A✓ BR subject of formula A✓ applying cosine rule A✓ $-\cos(180° - 2\beta) = \cos 2\beta$ A✓ $\cos 2\beta = 2\cos^2\beta - 1$ A✓ square root on LHS and RHS (6)
4.3	$5,4 = \frac{2h\cos 65°}{\sin 51°}$ $h = \frac{5,4 \times \sin 51°}{2\cos 65°}$ $h = 5$ units Accept 4,96 units as answer	A✓ substitution A✓ h subject of formula CA✓ answer (3)
		[10]

QUESTION 5

5.1	$a = 2$	A✓ answer (1)
5.2		A✓ shape A✓ turning points A✓ x-intercepts A✓ y-intercept (4)
5.3.1	$y \in [-2\ ;\ 2]$ OR $-2 \leq y \leq 2$ Penalty of 1 mark if one or both end points are excluded	A✓ A✓ answer (2)
5.3.2	$\text{period} = \frac{360°}{3}$ $= 120°$ Answer only: Full marks	A✓ $\frac{360°}{3}$ A✓ answer (2)
5.4	$2\sin x = 2\cos(x - 30°)$ $\sin x = \cos(x - 30°)$ $\sin x = \sin[90° - (x - 30°)]$ $\sin x = \sin(-x + 120°)$ $x = -x + 120° + k.360°$ or $x = 180° - (-x + 120°) + k.360°$, $k \in Z$ $2x = 120° + k.360°$ $\quad$ $x = 300° + x + k.360°$ $x = 60° + k.180°$ $\quad$ no solution In the interval $x \in [-180°\ ;\ 180°]$: $x = 60°$ or $x = -120°$	A✓ equating A✓ co-function A✓ both solutions CA✓ $x = 60°$ CA✓ $x = -120°$ (5)

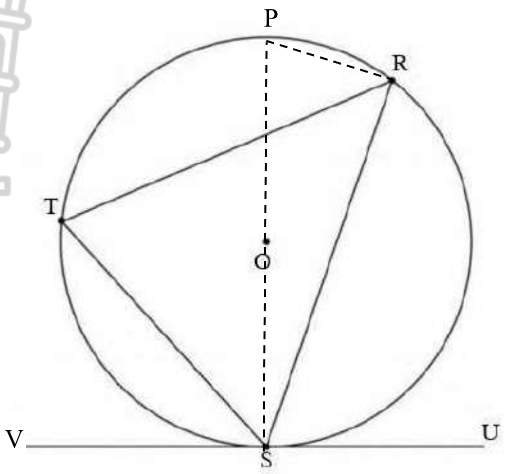
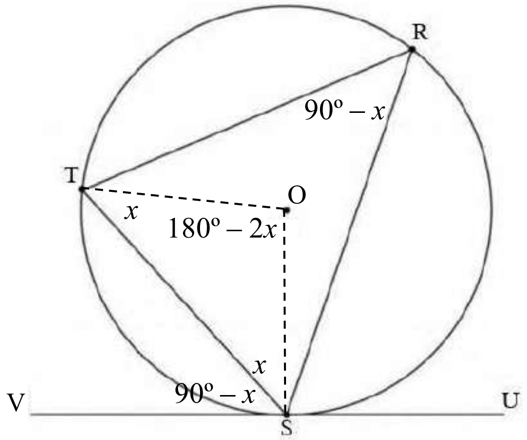
	OR $2\sin x = 2\cos(x-30°)$ $\sin x = \cos(x-30°)$ $\cos(90°-x) = \cos(x-30°)$ $90°-x = x-30°+k.360°$ or $90°-x = 360°-(x-30°)+k.360°$, $k \in Z$ $2x = 120°+k.360°$ $\quad$ $x = 300°+x+k.360°$ $x = 60°+k.180°$ $\quad$ no solution In the interval $x \in [-180° ; 180°]$: $x = 60°$ or $x = -120°$	**OR** A✓ equating A✓ co-function A✓ both solutions CA✓ $x = 60°$ CA✓ $x = -120°$ (5)
	OR $2\sin x = 2\cos(x-30°)$ $\sin x = \cos(x-30°)$ $\sin x = \cos x\cos 30° + \sin x \sin 30°$ $\sin x = \cos x.\frac{\sqrt{3}}{2} + \sin x.\frac{1}{2}$ $\frac{1}{2}\sin x = \cos x.\frac{\sqrt{3}}{2}$ $\sin x = \sqrt{3}\cos x$ $\tan x = \sqrt{3}$ $x = 60°+k.180°$ In the interval $x \in [-180° ; 180°]$: $x = 60°$ or $x = -120°$	**OR** A✓ equating A✓ compound angle expansion A✓ $\tan x = \sqrt{3}$ CA✓ $x = 60°$ CA✓ $x = -120°$ (5)
5.5.1	$x = -90°$ or $x = 90°$	A✓A✓ (2)
5.5.2	$x \in (-120° ; 60°)$ OR $-120° < x < 60°$ Penalty of 1 mark if one or both end points are included	CA✓CA✓ (2)
		[18]

QUESTION 6

6.1.1	$P\hat{R}S = 90°$ [tangent ⊥ diameter]	S✓R✓ (2)
6.1.2 (a)	$P\hat{S}R = \hat{R}_1$ [tan-chord-theorem] $= 48°$ $\therefore \hat{P} = 180° - (P\hat{S}R + P\hat{R}S)$ [sum of ∠s of ΔPSR] $= 180° - (48° + 90°)$ $= 42°$ **OR** $\hat{T}_1 = 90°$ [∠ in semi-circle] $\hat{P} = \hat{T}_1 - \hat{R}_1$ [exterior ∠ of ΔPTR] $= 90° - 48°$ $= 42°$	S/R✓ A✓ $P\hat{S}R = 48°$ CA✓ answer (3) **OR** S/R✓ S✓ A✓ answer (3)
6.1.2 (b)	$\hat{R}_2 = 90° - 48° = 42°$ $\hat{V}_1 = \hat{R}_2$ [∠s in same segment] $= 42°$	A✓ $\hat{R}_2 = 42°$ R✓ CA✓ answer (3)
6.1.3	$P\hat{Q}S = 90° + \hat{S}_1$ [exterior ∠ of ΔQSR] $\hat{T}_1 = 90°$ [∠ in semi-circle] $\therefore V\hat{T}S = 90° + \hat{T}_2$ But: $\hat{T}_2 = \hat{S}_1$ [∠s in same segment] $\therefore P\hat{Q}S = V\hat{T}S$ **OR** $\hat{V}_1 = \hat{P}$ [both $= 42°$] PTVQ is a cyclic quadrilateral [converse: ext. ∠ = opp int ∠] $\therefore P\hat{Q}S = V\hat{T}S$ [ext. ∠ = opp int ∠] **OR** $P\hat{Q}S = 180° - (\hat{P} + \hat{S}_2)$ [sum of ∠s of ΔPQS] $V\hat{T}S = 180° - (\hat{V}_1 + \hat{S}_2)$ [sum of ∠s of ΔVTS] $\therefore P\hat{Q}S = V\hat{T}S$ [$\hat{V}_1 = \hat{P}$; proved above]	S/R✓ S✓ $\hat{T}_1 = 90°$ S✓ $V\hat{T}S = 90° + \hat{T}_2$ S/R✓ (4) **OR** S✓ S✓R✓ R✓ (4) **OR** S✓ R✓ S/R✓ R✓ (4)

6.2	$\hat{O}_1 = 2 \times \hat{A}$ [$\angle$ at centre = 2× $\angle$ at circumf.] $= 2 \times 66° = 132°$ $\hat{C}_1 = \hat{E}$ [ext. $\angle$ of cyclic quad.] $= 42°$ $\hat{B}_2 = \hat{O}_1 - \hat{C}_1$ [ext. $\angle$ of ΔOBC] $= 132° - 42° = 90°$ $\therefore$ AB = BC [line from centre $\perp$ to chord]	S/R✓ A✓ answer S/R✓ ✓A answer ✓A $132° - 42° = 90°$ R✓ (6)
		[18]

QUESTION 7

7.1	 	**NOTE:** If there is no construction: 0/5 marks
	Draw diameter SP Join P to R	✓ construction
	$P\hat{S}V = 90°$ [tangent ⊥ diameter]	S/R✓
	$V\hat{S}T = 90° - T\hat{S}P$	
	$S\hat{R}P = 90°$ [∠ in semi-circle]	S/R✓
	$T\hat{R}S = 90° - P\hat{R}T$	
	But: $T\hat{S}P = P\hat{R}T$ [∠s in same segment]	S✓R✓
	$\therefore V\hat{S}T = T\hat{R}S$ OR $V\hat{S}T = \hat{R}$	(5)
	OR	**OR**
	 	
	Draw radii OT and OS.	✓ construction
	Let $O\hat{S}T = x$.	
	$O\hat{T}S = x$ [∠s opposite equal radii]	S/R✓
	$T\hat{O}S = 180° - 2x$ [sum of ∠s of a Δ]	
	$T\hat{R}S = 90° - x$ [∠ at centre = 2× ∠ at circumf.]	S✓R✓
	$O\hat{S}V = 90°$ [tangent ⊥ radius]	S/R✓
	$V\hat{S}T = 90° - x$	
	$V\hat{S}T = T\hat{R}S$ OR $V\hat{S}T = \hat{R}$ [both $= 90° - x$]	(5)

7.2.1	$\hat{F}_2 = \hat{G}_4 = x$ [tan-chord-theorem] $\hat{P} = \hat{G}_4 = x$ [tan-chord-theorem] $\hat{G}_2 = \hat{P} = x$ [alt. ∠s; GE ‖ HP]	S✓R✓ S✓R✓ S✓R✓ (6)
7.2.2	In ΔHMG and ΔEFG : 1. $\hat{H}_2 = \hat{E}$ [∠s in the same segment] 2. $\hat{G}_3 = \hat{G}_1$ [∠s subtended by = chords] 3. $\hat{M}_3 = E\hat{F}G$ [sum of ∠s of a Δ] ∴ ΔHMG ‖‖ ΔEFG [∠∠∠]	S✓R✓ S✓ R✓ R✓ (5)
7.2.3	$\frac{HM}{EF} = \frac{HG}{EG}$ [‖‖ Δs] But: EF = PH [given] $\therefore \frac{HM}{PH} = \frac{HG}{EG}$ And PH.HG = EG.HM	S✓R✓ S✓ $\frac{HM}{PH} = \frac{HG}{EG}$ (3)
		[19]

QUESTION 8

8.1	$AB^2 = AC^2 - BC^2$ [Pythagoras] $= 6{,}5^2 - 6^2$ $\therefore AB = 2{,}5$ units	S/R✓ using Theorem of Pythagoras A✓ (2)
8.2	In ΔCBA and ΔCEB: 1. $\hat{C} = \hat{C}$ [common] 2. $A\hat{B}C = C\hat{E}B$ [both $= 90°$; given] 3. $\hat{A} = C\hat{B}E$ [sum of $\angle$s of a Δ] $\therefore \Delta CBA \mathbin{\vert\vert\vert} \Delta CEB$ [$\angle\angle\angle$] $\therefore \frac{CB}{CA} = \frac{CE}{CB}$ [$\vert\vert\vert$ Δs] $\therefore CB^2 = CA.CE$ and $CB = \sqrt{CA.CE}$ **OR** $\Delta CBA \mathbin{\vert\vert\vert} \Delta CEB$ [perpendicular from right $\angle$ vertex to hypotenuse] $\therefore \frac{CB}{CA} = \frac{CE}{CB}$ [$\vert\vert\vert$ Δs] $\therefore CB^2 = CA.CE$ and $CB = \sqrt{CA.CE}$	S✓ identifying triangles S ✓ S/R ✓ R ✓ S✓ S✓ $CB^2 = CA.CE$ (6) **OR** S✓ $\Delta CBA \mathbin{\vert\vert\vert} \Delta CEB$ R ✓✓✓ S✓ S✓ $CB^2 = CA.CE$ (6)
8.3	$6 = \sqrt{6{,}5.CE}$ $36 = 6{,}5.CE$ $CE = 5{,}5$ units Penalty of 1 mark for incorrect rounding off (5,54 units)	A✓ substitution A✓ answer (2)
8.4	$AE = 6{,}5 - 5{,}5 = 1$ unit Accept 0,96 units	CA✓ answer (1)
8.5	$\frac{BD}{BC} = \frac{AE}{AC}$ [prop. theorem; AB ∥ ED] or [line ∥ to side of Δ] $= \frac{1}{6{,}5}$ Also: $\frac{BD}{BC} = \frac{EF}{EC}$ [prop. theorem; EB ∥ FD] or [line ∥ to side of Δ] $\therefore \frac{1}{6{,}5} = \frac{EF}{5{,}5}$ Penalise only once if parallel lines are left out in reason $\therefore EF = \frac{1 \times 5{,}5}{6{,}5}$ $\therefore EF = \frac{11}{13}$ units $= 0{,}85$ units Accept 0,82 units	S✓R✓ S/R✓ CA✓ substitution CA✓ answer (5)
		[16]

TOTAL: 150