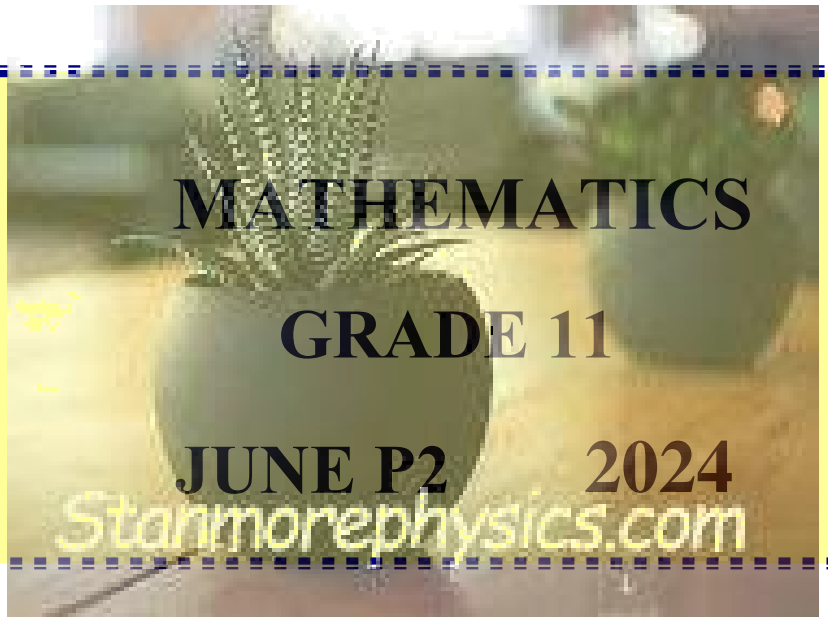
education

Department of Education
FREE STATE PROVINCE



MARKS: 100

TIME : 2 HOUR

This question paper consists of 8 pages.

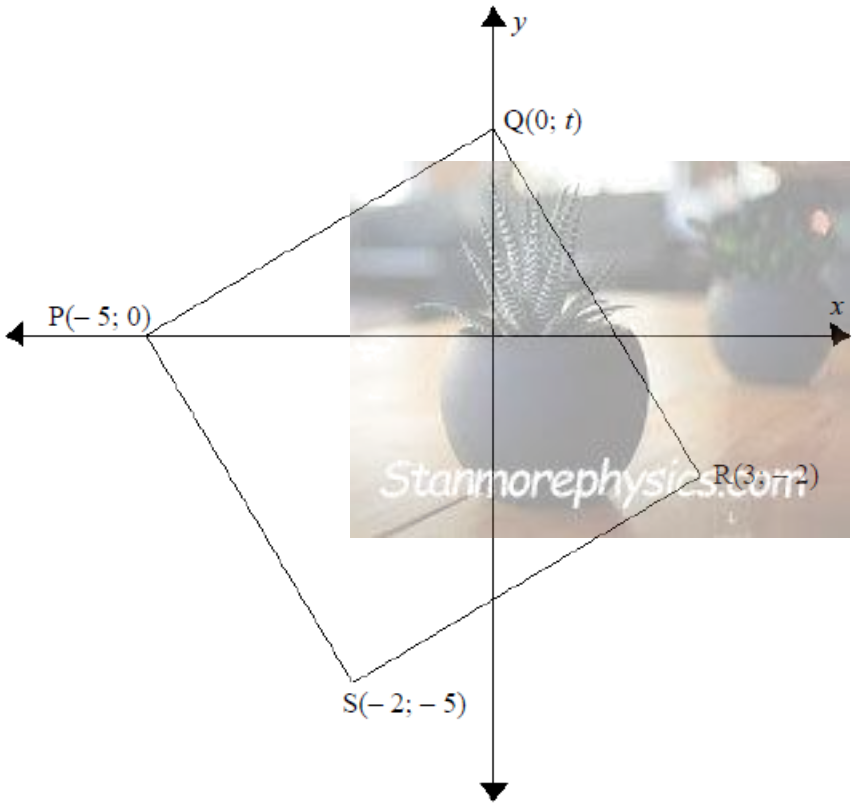
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless otherwise stated.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise
6. Information sheet with formulae is attached at the end of the question paper.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.
9. Answers only will NOT necessarily be awarded full marks.

QUESTION 1

PQRS is a quadrilateral with vertices P(– 5; 0), Q(0; *t*), R(3; – 2) and S(– 2; – 5).
P is on the *x*-axis and Q is on the *y*-axis.



1.1 Determine the lengths of PS and RS. (4)

1.2 Calculate the coordinates M, the midpoint of PR. (2)

1.3 Show that the value of $t = 3$ if it is given that PR and QS bisect each other. (1)

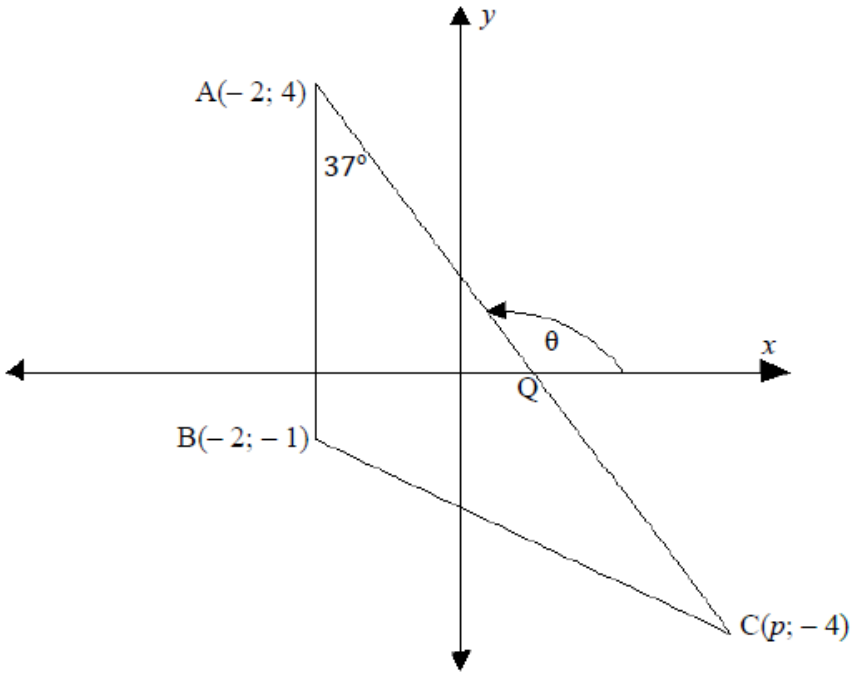
1.4 Determine the gradient of PQ. (2)

1.5 Calculate the size of $P\hat{Q}R$. (4)

1.6 What type of a quadrilateral is PQRS? Justify you answer. (3)
[16]

QUESTION 2

In the diagram below, A(– 2; 4), B(– 2; – 1) and C$(p;-4)$ are the vertices of ΔABC. Q is the *x*- intercept of the line AC. θ is the angle of inclination of AC and $B\hat{A}C=37^{o}$.



2.1 Determine the size of θ. (2)

2.2 Calculate the equation of line AC. (3)

2.3 Calculate the coordinate of Q, the *x*-intercept of AC (2)

2.4 Determine the integral value of *p* if the points A, Q and C are collinear. (3)

2.5 Determine the equation of the line parallel to BC and passing through point A. (5)

[15]

QUESTION 3

3.1 If $\theta = 89{,}4^{\circ}$ and $\alpha = 122^{\circ}$, evaluate the following (round off answers correct to three decimal places):

3.1.1 $\sin\alpha$ (1)

3.1.2 $\dfrac{\tan\alpha}{\cos^2(\theta+\alpha)}$ (2)

3.2 If $3\tan\beta + 2 = 0$ and $\sin\beta > 0$, determine without the use of a calculator and with the aid of a sketch, the value of the following:

3.2.1 $\cos\beta$ (3)

3.2.2 $\sqrt{13}\cos\beta - 13\sin^2\beta$ (2)

3.3 If $\sin 4^{\circ} = m$, determine $\tan 94^{\circ}$ in term of m. (3)

3.4 Simplify without the use of a calculator:

$$\frac{\cos\left(360^{\circ}-x\right)\tan\left(x-180^{\circ}\right)}{\sin\left(-x\right)\cos\left(90^{\circ}-x\right)}$$ (5)

3.5 Evaluate without using a calculator:

$$\frac{\tan 120^{\circ}.\sin 115^{\circ}}{\cos 240^{\circ}\cos 205^{\circ}}$$ (6)

3.6 Prove the identity:

$$\frac{1}{1-\cos x}+\frac{1}{1+\cos x}=\frac{2}{\sin^2 x}$$ (3)

3.7 Determine the general solution:

$$5\tan\left(\theta+10^{\circ}\right)=4$$ (4)

[29]

QUESTION 4

Given: $f(x) = \sin 2x$ and $g(x) = 3\cos x$ for $x \in [-90^o; 90^o]$

4.1 Sketch the graphs of $f(x)$ and $g(x)$ on the same system of axes in the given interval. Show all coordinates of the intercepts and the turning points. (6)

4.2 Write down the period of $f(x)$. (1)

4.3 Write down the range of $g(x)$. (2)

4.4 For which value(s) of x is $f(x).g(x) \leq 0$? (2)

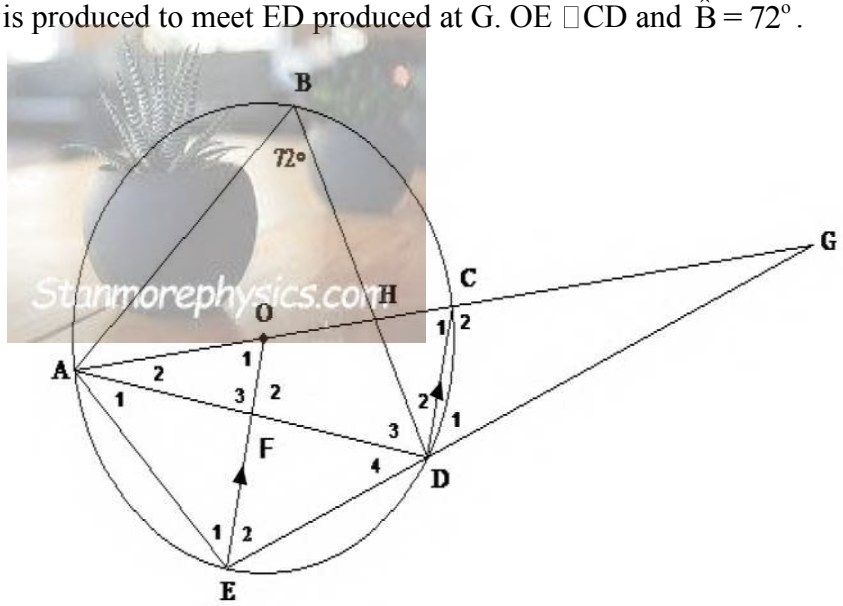
[11]

QUESTION 5

5.1 Complete the following theorem:

A line drawn from the centre of the circle, perpendicular to the chord, …… the chord. (1)

5.2 In the diagram below, O is the centre of the circle and AC is the diameter. AC is produced to meet ED produced at G. OE ⊥ CD and $\hat{B} = 72^{o}$.



5.2.1 Calculate, giving reasons, the size of:

(a) $\hat{C}_1$ (2)

(b) $\hat{O}_1$ (2)

(c) $\hat{D}_4$ (2)

(d) AED (2)

(e) $\hat{D}_1$ (2)

5.2.2 Prove that AF = FD (3)

5.2.3 Is OEDC a cyclic quadrilateral? Give a reason for your answer. (2)

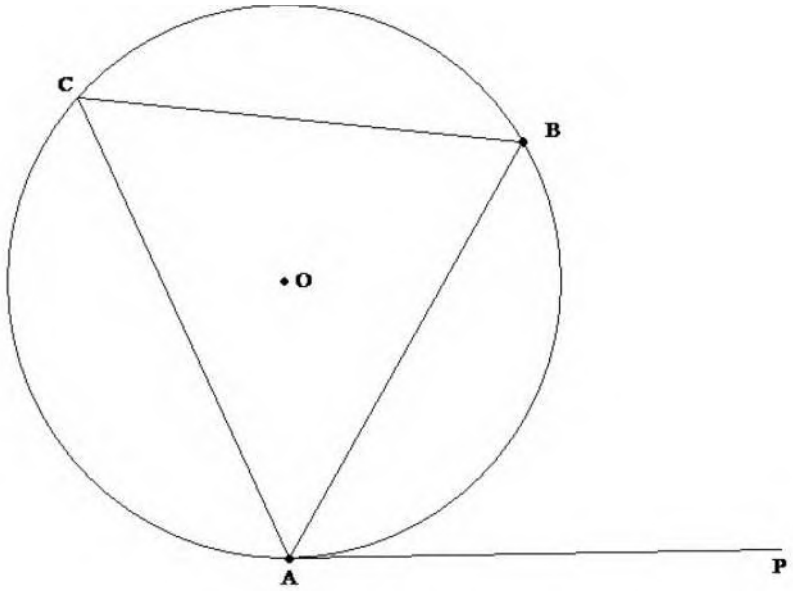
[16]

QUESTION 6

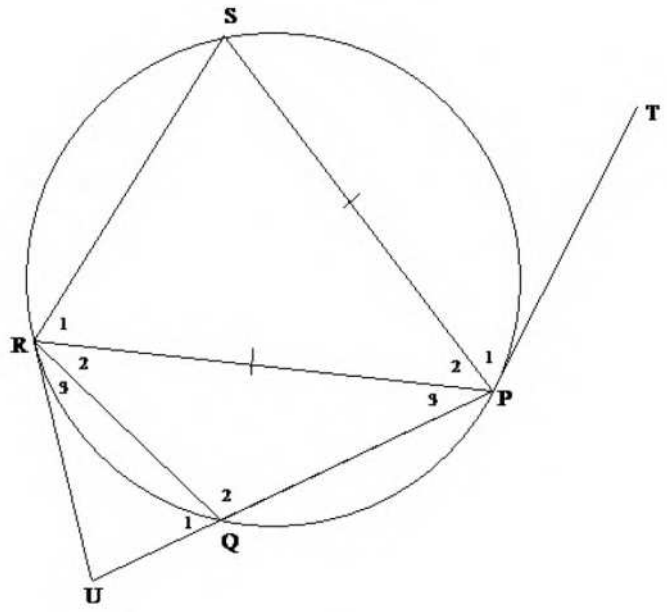
6.1 Complete the theorem:

The tangent is ……to the radius drawn from to the point of contact. (1)

6.2 Use the diagram below, A, B and C are on the circle with centre O. PA is a tangent to the circle at A. Use the diagram to prove the theorem which states that: $B\hat{A}P = \hat{C}$ (6)



6.3 In the figure below, PQRS are points on the circle. PQ is produced to meet RU at U. TP is the tangent to the circle at P. PR = SP and $\hat{U} = \hat{P}_1$



Prove that: RU = RQ. (6)

[13]

TOTAL: 100

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1};\ r \neq 1 \qquad S_\infty = \frac{a}{1-r};\ -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

education

Department of Education
FREE STATE PROVINCE

GRADE 11

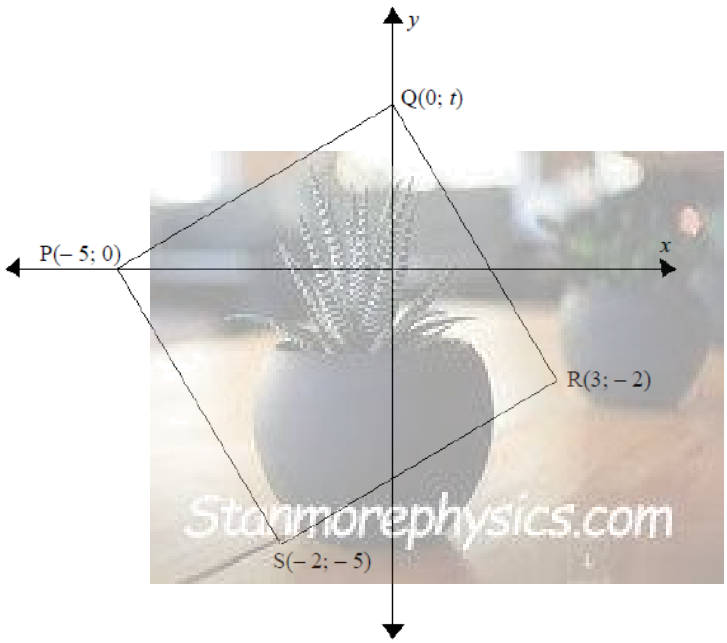
MATHEMATICS

2024 JUNE P2

MARKING GUIDELINE

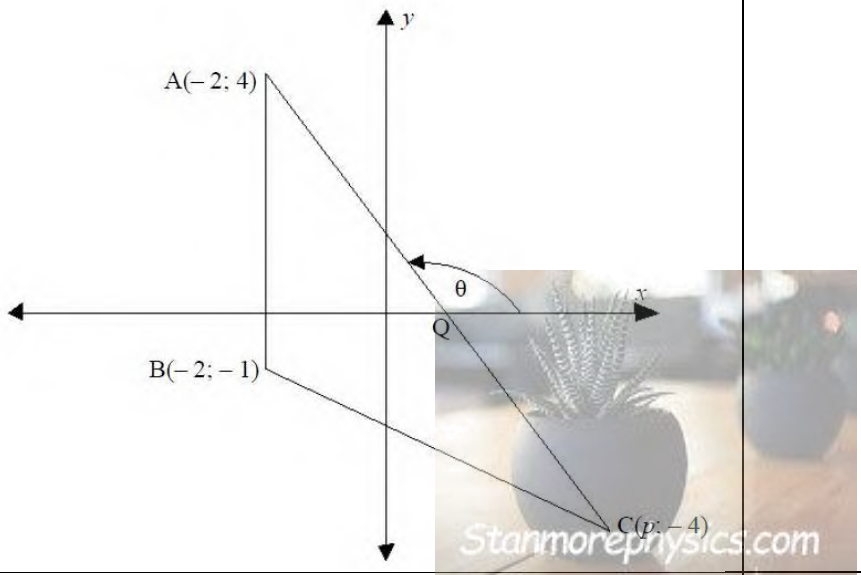
This marking guideline consists of 11 pages

QUESTION 1

		
1.1	PS$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ PS$=\sqrt{(-2+5)^2+(-5+0)^2}$ PS$=\sqrt{34}$ RS$=\sqrt{(3+2)^2+(-2+5)^2}$ RS$=\sqrt{34}$	✓subt in corr form ✓answer ✓subt in corr form ✓answer (4)
1.2	M$\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ M$\left(\frac{-5+3}{2};\frac{0-2}{2}\right)$ M(– 1; – 1)	✓x-value ✓y-value (2)
1.3	$\frac{t-5}{2}=-1$ $t-5=-2$ $t=3$	✓Method (1)
1.4	$m_{PQ}=\frac{y_2-y_1}{x_2-x_1}$ $m_{PQ}=\frac{3-0}{0+5}$ $m_{PQ}=\frac{3}{5}$	✓subt in corr form ✓answer (2)

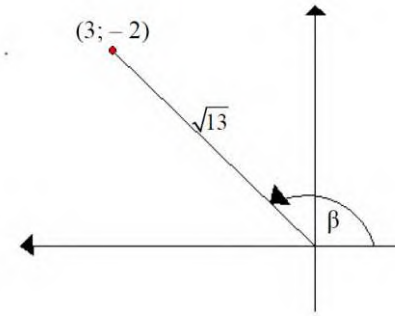
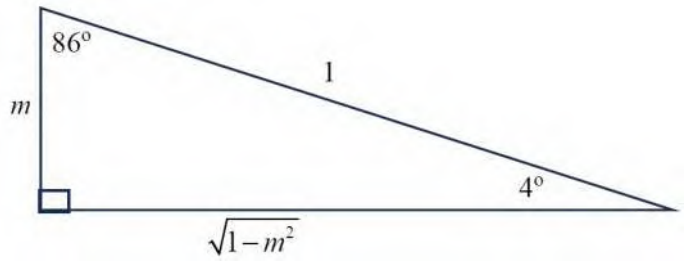
1.5	$m_{QR} = \frac{-2-3}{3-0}$ $m_{QR} = -\frac{5}{3}$ $m_{PQ} \times m_{QR} = \frac{3}{5} \times -\frac{5}{3} = -1$ $\therefore \hat{PQR} = 90^o$ OR Inclination of PQ = $\tan^{-1}\left(\frac{3}{5}\right) = 30,96^o$ $m_{QR} = \frac{-2-3}{3-0}$ $m_{QR} = -\frac{5}{3}$ Inclination of QR = $180 - \tan^{-1}\left(\frac{5}{3}\right) = 120,96^o$ $\therefore \hat{PQR} = 120,96^o - 30,96^o = 90^o$	✓subt in corr form ✓answer ✓Method ✓answer OR ✓inc PQ ✓gradient QR ✓inc QR ✓answer (4)
1.6	Square Diagonals bisect each other/diagonals have the same midpoint. All sides are equal. Interior angles = 90^o	✓square ✓diagonals ✓any of the other two reasons. (3)
		[16]

QUESTION 2

		
2.1	$\theta = 90^\circ + 37^\circ$ (ext $\angle$ of a triangle) $\theta = 127^\circ$	✓Method ✓answer (2)
2.2	$m = \tan 127^\circ$ $m = -1,33$ $y - y_1 = m(x - x_1)$ $y - 4 = -1.33(x + 2)$ $y = -1,33x - 2,66 + 4$ $y = -1,33x + 1,34$	✓gradient ✓Subt of A ✓answer (3)
2.3	$-1,33x + 1,34 = 0$ $-1,33x = -1.34$ $x = 1$ Q(1; 0)	✓y = 0 ✓x = 1 (2)
2.4	$\frac{y_2 - y_1}{x_2 - x_1} = m$ $\frac{4 + 4}{-2 - p} = -1.33$ $2,66 + 1.33p = 8$ $1.33p = 5.34$ $p = 4$ OR	✓Method ✓simplification ✓answer

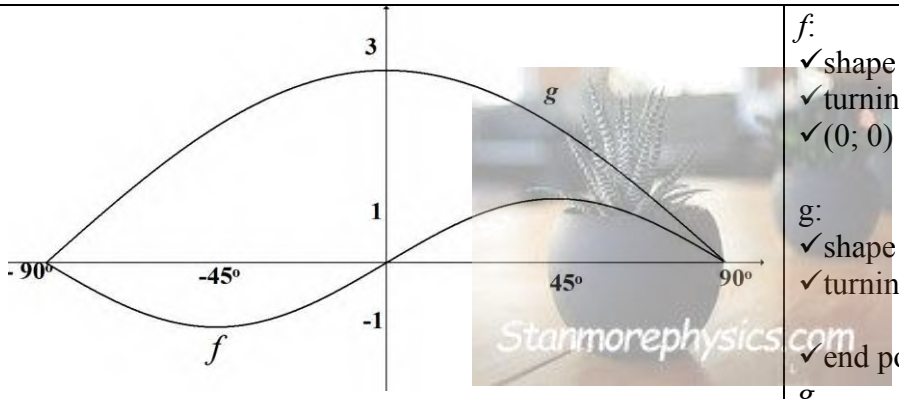
	$\frac{y_2-y_1}{x_2-x_1}=m$ $\frac{-4-0}{p-1}=-1.33$ $-1.33p+1.33=-4$ $-1.33p=-5,33$ $p=4$	✓Method ✓simplification ✓answer (3)
2.5	$m_{BC}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{-4+1}{4+2}$ $=-\frac{1}{2}$ $m_{new}=m_{BC}=-\frac{1}{2}$ $y-y_1=m(x-x_1)$ $y-4=-\frac{1}{2}(x+1)$ $y=-\frac{1}{2}x-\frac{1}{2}+4$ $y=-\frac{1}{2}x+\frac{7}{2}$	✓subt ✓m_{BC} ✓m_{new} ✓subt of point A ✓eq. (5)
		[15]

QUESTION 3

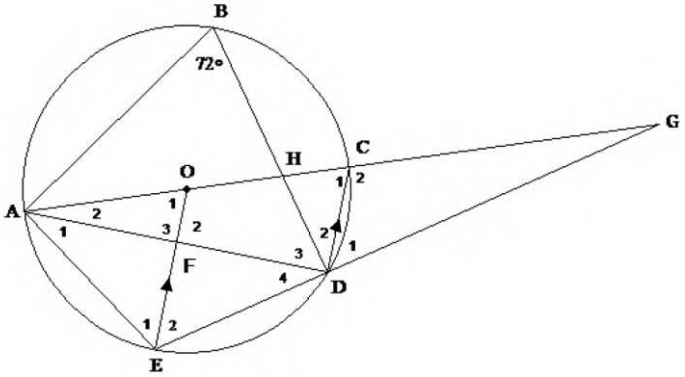
3.1.1	$\sin 122^o = 0,848$	✓answer
3.1.2	$\dfrac{\tan\alpha}{\cos^2(\theta+\alpha)}$ $= \dfrac{\tan 122^o}{\cos^2\left(89.4^o + 122^o\right)}$ $= \dfrac{\tan 122^o}{\left(\cos 211,4^o\right)^2}$ $= -2,197$	✓subt ✓answer (2)
3.2.1	$\tan\beta = -\dfrac{2}{3}$ $r^2 = x^2 + y^2$ $r^2 = (-3)^2 + (2)^2$ $r = \sqrt{13}$ $\therefore \cos\beta = -\dfrac{3}{\sqrt{13}}$ 	✓diagram ✓value of r ✓cos β (3)
3.2.2	$\sqrt{13}\cos\beta - 13\sin^2\beta$ $= \sqrt{13}\left(-\dfrac{3}{\sqrt{13}}\right) - 13\left(\dfrac{2}{\sqrt{13}}\right)^2$ $= -3 - 4$ $= -7$	✓subt ✓answer (2)
3.3	 $r^2 = x^2 + y^2$ $1 = x^2 + m^2$ $x^2 = 1 - m^2$ $x = \sqrt{1-m^2}$ $\tan 94^o = \tan\left(180^o - 86^o\right)$ $= -\tan 86^o$ $= -\dfrac{\sqrt{1-m^2}}{m}$	✓ $x = \sqrt{1-m^2}$ ✓reduction ✓answer (3)

3.4	$\frac{\cos(360^\circ - x)\tan(x - 180^\circ)}{\sin(-x)\cos(90^\circ - x)}$ $= \frac{\cos x.(\tan x)}{(-\sin x).\sin x}$ $= -\frac{\cos x}{\sin^2 x}.\frac{\sin x}{\cos x}$ $= -\frac{1}{\sin x}$	✓cos x ✓tan x ✓– sin x ✓sin x ✓ $\frac{\sin x}{\cos x}$ (5)
3.5	$\frac{\tan 120^\circ.\sin 115^\circ}{\cos 240^\circ.\cos 205^\circ}$ $= \frac{-\tan 60^o.\sin 65^o}{-\cos 60.(-\cos 25^o)}$ $= -\frac{\sqrt{3}.\sin 65^o}{\frac{1}{2}\sin 65^o}$ $= -2\sqrt{3}$	✓ – tan 60^o ✓sin 65° ✓ – cos 60° ✓– cos 25° ✓sin 65° ✓answer (5)
3.6	$\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}$ $\text{LHS} = \frac{(1+\cos x)+(1-\cos x)}{(1-\cos x)(1+\cos x)}$ $= \frac{2}{1-\cos^2 x}$ $= \frac{2}{\sin^2 x}$ $= \text{RHS}$	✓numerator ✓denominator ✓simplification (3)
3.7	$5\tan(\theta + 10^\circ) = 4$ $\tan(\theta + 10^o) = \frac{4}{5}$ $\text{ref}\angle = \tan^{-1}\frac{4}{5} = 38,66^o$ First Quad: $(\theta + 10^o) = 38,66^o + k.180^o$; $\theta = 28,66 + k.180^o$ Third quad: $(\theta + 10^o) = 218^o + k.180^o$; $\theta = 208,66 + k.180$ $k \in \mathbb{Z}$	✓std form ✓ref angle ✓answer ✓answer (4)
		[29]

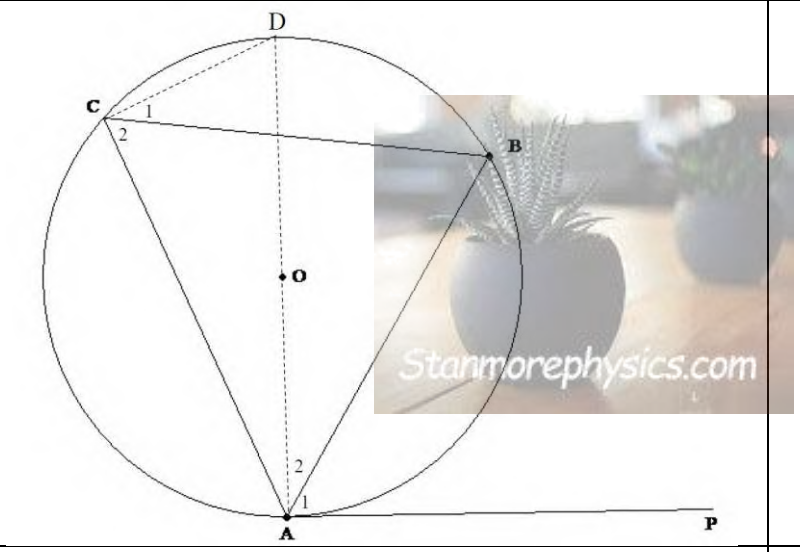
QUESTION 4

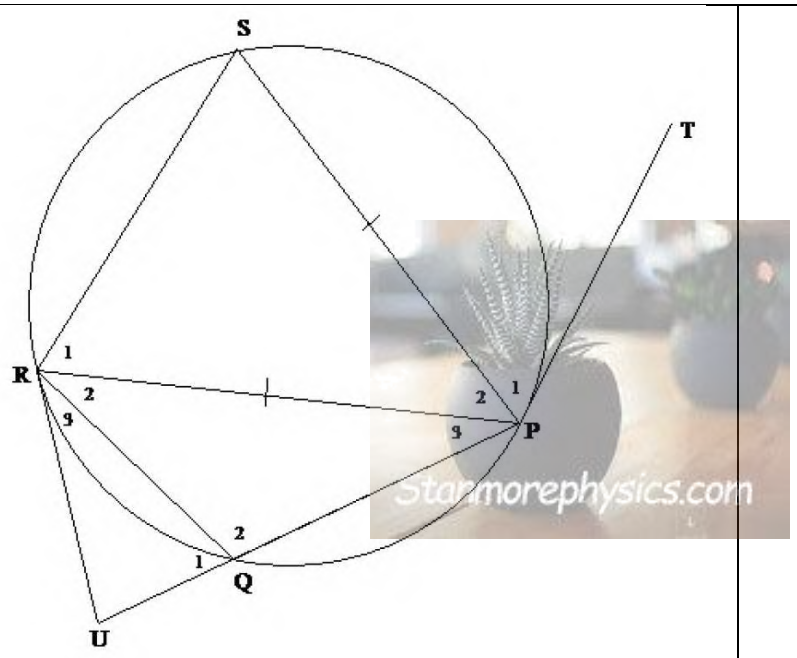
4.1		*f*: ✓shape ✓turning points ✓(0; 0) g: ✓shape ✓turning point ✓end points of *f* and *g*. (6)
4.2	Period = 180^o	✓answer (1)
4.3	$y \in [0;3]$	✓✓answer (2)
4.4	$[-90^o;0^o]$ or $-90^o \leq x \leq 0^o$	✓notation ✓critical values (2)
		[11]

QUESTION 5

5.1	bisect	✓answer (1)
		
5.2.1(a)	$\hat{C}_1 = 72^o$ (angles in the same segment)	✓S ✓R (2)
5.2.1(b)	$\hat{O}_1 = \hat{C}_1 = 72^o$ (corr angles, OE//CD)	✓S ✓R (2)
5.2.1(c)	$\hat{O}_1 = 2 \times \hat{D}_4$ (angle at centre = 2 × angle at the cirm.) $\hat{D}_4 = 36^o$	✓S ✓R (2)
5.2.1(d)	$\hat{N}_1 + \hat{N}_2 = 108^o$ (Opp angles of a cyclic quad)	✓S ✓R (2)
5.2.1(e)	$\hat{D}_1 + (\hat{D}_2 + \hat{D}_3) + \hat{D}_4 = 180^o$ (sum of angles of a triangle) $\hat{D}_1 + 90^o + 36^o = 180^o$ (angle in the semi-circle) $\therefore \hat{D}_1 = 54^o$	✓S/R ✓S/R ✓S (3)
5.2.2	$\hat{E}_2 = \hat{D}_1 = 54^o$ (corr angles OE//CD) $\hat{F}_2 = \hat{E}_2 + \hat{D}_4$ (ext. angle of a triangle) $\hat{F}_2 = 90^o$ $\therefore$ AF = FD (perpendicular from centre to chord)	✓S/R ✓S/R ✓R (3)
5.2.3	No $\hat{E}_2 + \hat{C}_1^{\,o} \neq 180^o$ (opp angles are not suppl)	✓No ✓S/R (2)
		[16]

QUESTION 6

6.1	bisect	✓answer (1)
6.2		
	$\hat{C}_1 + \hat{C}_2 = 90^o$ (angle in the semi-circle) $\therefore \hat{C}_2 = 90^o$ And $\hat{A}_1 + \hat{A}_2 = 90^o$ (tan $\perp$ radius) $\hat{A}_1 = 90^o - \hat{A}_2$ But $\hat{C}_2 = \hat{A}_2$ (angles in the same segment) $\hat{A}_1 = \hat{C}_2$	✓construction ✓S/R ✓S ✓R ✓S/R ✓S/R (6)

		
6.3	$\hat{P}_1 = \hat{S}$ (tan chord thrm) and $\hat{S} = \hat{Q}_1$ (ext angle of cyclic quad) but $\hat{P}_1 = \hat{U}$ (given) $\therefore \hat{U} = \hat{Q}_1$ $\therefore$ RU = RQ (angles opp sides are equal)	✓S ✓R ✓S ✓R ✓S ✓R (6)
		[13]

TOTAL: 100