education
Department of Education
FREE STATE PROVINCE

LEARNER NAME: ________________________________

SUBJECT	**:**	**MATHEMATICS**
CLASS	**:**	**GRADE 9**
TASK	**:**	**INFORMAL ACTIVITIES** **ALGEBRAIC EXPRESSIONS FACTORISATION**
NUMBER	**:**	**14 ACTIVITIES**

This document consists of 43 pages including cover page

ACKNOWLEDGEMENTS

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MD, Phillips., J, Basson., J, Odendaal. (2015) Mind Action Series Mathematics Grade 8 and 9 3rd edn., Sanlamhof: Allcopy Publishers.

Kevin Smith. (2017) Maths Handbook and Study Guide. Berlut Books CC.

2. PATTERNS, FUNCTIONS AND ALGEBRA

ALGEBRAIC EXPRESSIONS (FACTORISATION)

When we expand $a(b + c)$ we are multiplying by using the distributive law.

$$\therefore a(b + c) = ab + ac$$

If we reverse this process we get $ab + ac = a(b + c)$. This reverse process is called factorisation. The highest common factor (a) has been "taken out" of the expression $ab + ac$ and the expression is said to be factorised into the product of two factors, namely (a) and $(b + c)$. The two terms have been expressed as one term (or the product of factors)

TAKING OUT HIGHEST COMMON FACTOR (HCF)

Steps in factorizing by taking out the highest common factor (HCF).

Consider the expression: $6x^3y + 9x^2y^2$

Step 1: Take out highest common factor of constants (coefficients). HCF of 6 and 9 is 3.

$$\therefore 3(2x^3y + 3x^2y^2).$$

Step 2: Take out the highest common factor of variables. Look out for the lowest exponent of each variable. Take out each variable with its lowest exponent, i.e. x^2 for variable x and y for variable y.

$$\therefore \text{HCF is } x^2y.$$

Step 3: Write the complete factorisation by taking out HCF and divide each term with HCF to ensure that you are left with correct quotient inside the brackets.

$\therefore 6x^3y + 9x^2y^2$

$= 3x^2y\left(\frac{6x^3y}{3x^2y} + \frac{9x^2y^2}{3x^2y}\right)$

$= 3x^2y(2x + 3y)$

TAKING OUT BRACKET AS HIGHEST COMMON FACTOR (HCF)

Steps in factorizing by taking out a bracket as highest common factor (HCF).

Consider the expression $x(p-q)+2(p-q)$

Step 1: Recognise the expression in the brackets, is the same.

$$(p-q)$$

Step 2: Take out the expression $(p-q)$ as highest common factor and leave the quotient inside the second bracket.

$$=(p-q)(x+2)$$

Factorise the expressions:

Example 1: $2x+10$.

Solution: $2x+10$

$$=2\left(\frac{2x}{2}+\frac{10}{2}\right)$$

$$=2(x+5)$$

Example 2: $8x^2y+12xy^2$.

Solution: $8x^2y+12xy^2$

$$=4xy\left(\frac{8x^2y}{4xy}+\frac{12xy^2}{4xy}\right)$$

$$=4xy(2x+3y)$$

Example 3: $x^2y-5xy-y$.

Solution: $x^2y-5xy-y$

$$=y\left(\frac{x^2y}{y}-\frac{5xy}{y}-\frac{y}{y}\right)$$

$$=y(x^2-5x-1)$$

Example 4: $a(y-2)+b(y-2)$.

Solution: $a(y-2)+b(y-2)$

$= (y-2)(a+b)$

CLASSWORK 1	**DATE:**

Factorise the following expressions:

1. $2x+6$
2. $4t^3u^7-8t^4u^8+32t^5u^9$
3. $t(r-s)+(r-s)$
4. $p(x+15)-2(x+15)$

HOMEWORK 2	**DATE:**

Factorise the following expressions:

1. $3a-12$
2. $6q^2r-12qr^2$
3. $m(a-2b)+(a-2b)$
4. $x(y-2)+5(y-2)$

FACTORISATION INVOLVING THE SIGN – CHANGE RULE

Consider the expressions: $+(q-p)$ **and** $-(-q+p)$

Let us remove the brackets from each expression by multiplying:

$+(q-p) = q-p$ $\qquad$ $-(-q+p) = q-p$

We can conclude that: $+(q-p) = -(-q+p)$

Notice that if the sign outside the brackets changes, then the signs inside the brackets also change. This is called sign – change rule.

We can also write this as $+(q-p) = -(p-q)$

Example: $-(5-y) = +(-5+y) = (y-5)$

$+(8-3z) = -(-8+3z) = -(3z-8)$

$-(-7+a) = +(+7-a) = (7-a)$

$-(-9c-6b) = +(+9c+6b) = (6b+9c)$

Factorising by taking the expression inside the brackets as common factor.

Example: Factorise $3x(p-q) - 6(q-p)$

Step 1: Ensure that expression inside brackets are the same by applying change of signs if need be,

i.e. $(p-q)$ **is not the same as** $(q-p)$.

∴ change the sign between the terms and the sign of each term inside the brackets.

$$\therefore\ -6(q-p) = +6(-q+p).$$

Then exchange terms inside the brackets as follows:

$$(-q+p) = (p-q)$$

Step 2: Then take out highest common factor in the form of brackets:

$$\text{i.e. } 3x(p-q) + 6(p-q)$$

$$= (p-q)(3x+6)$$

Step 3: Then ensure that expression left inside the bracket is factorised further if need be, i.e. $3x+6$ needs to be factorised further.

$$\therefore 3x+6=3(x+2)$$

Step 4: Write the complete factorisation:

$$\therefore 3x(p-q)-6(q-p)$$

$$=3x(p-q)+6(p-q)$$

$$=(p-q)(3x+6)$$

$$=(p-q)3(x+2)$$

$$=3(p-q)(x+2)$$

Factorise the following expressions:

Example 1: $-2x+10$.

Solution: $-2x+10$

$$=-2\left(\frac{-2x}{-2}+\frac{10}{-2}\right)$$

$$=-2(x+5)$$

Example 2: $-12a^3b^4-18a^2b^5$.

Solution: $-12a^3b^4-18a^2b^5$

$$=-6a^2b^4\left(-\frac{12a^3b^4}{-6a^2b^4}-\frac{18a^2b^5}{-6a^2b^4}\right)$$

$$=-6a^2b^4(2a-3b)$$

Example 3: $a(y-2)-b(2-y)$.

Solution: $a(y-2)-b(2-y)$

$= a(y-2)+b(-2+y)$

$= a(y-2)+b(y-2)$

$= (y-2)(a+b)$

Example 4: $2x(3m-2n)+5y(2n-3m)$

Solution: $2x(3m-2n)+5y(2n-3m)$

$= 2x(3m-2n)-5y(-2n+3m)$

$= 2x(3m-2n)-5y(3m-2n)$

$= (3m-2n)(2x-5y)$

CLASSWORK 3 DATE:

Factorise the following expressions completely:

1. $-2ab+6ac$
2. $-18x^2y-12xy^2$
3. $x(a-b)+y(b-a)$
4. $x(a-b)-y(b-a)$

HOMEWORK 4 DATE:

Factorise the following expressions:

1. $-5pq-20pr$
2. $-14m^2n^4+21m^3n^6$
3. $p(a-2)-q(2-a)$
4. $t(x-y)+(y-x)$

FACTORISING DIFFERENCE OF TWO SQUARES (DOTS)

Consider the product $(x+y)(x-y)$.

$(x+y)(x-y)$

$= x^2 - xy + xy - y^2$ → **expand using foil**

$= x^2 - y^2$ → **like terms are additive inverse ∴ cancel each other**

This only happens if the terms inside the brackets are the same but just differ with the signs of last terms.

If we reverse this process, we get: $x^2 - y^2 = (x+y)(x-y)$

Notice that x^2 and y^2 are square expressions: $x^2 = x \times x$ and $y^2 = x \times x$

When you subtract the two square expression, namely, $x^2 - y^2$, you can form two factors of the form $(x+y)(x-y)$.

Steps for factorizing difference of two squares.

Consider the expression $2x^4 - 8$

Step 1: Identify if the two terms are perfect squares if there is negative (-) sign between. For variable to be a perfect square, it must have been raised to an even exponent. If one or both of the terms are not perfect squares take out the HCF: i.e. $2x^4 - 8$

$$= 2(x^4 - 4)$$

Step 2: Check whether the expression inside the bracket is not difference of two squares. If it is, factorise as follows:

$2(x^4 - 4)$

$= 2(\sqrt{x^4} - \sqrt{4})(\sqrt{x^4} + \sqrt{4})$ or $2(\sqrt{x^4} + \sqrt{4})(\sqrt{x^4} - \sqrt{4})$

$= 2(x^2 - 2)(x^2 + 2)$ or $2(x^2 + 2)(x^2 - 2)$

NB: order of brackets does not matter hence the "or" part

Step 3: Then check again if the expression with negative between is difference of two squares or not, if it is difference of two squares, factorise further. If it was $(x^2 - 9)$, it was to be factorised further.

Factorise the following expressions completely:

Example 1: $x^2 - 9$

Solution: $x^2 - 9$

$= (\sqrt{x^2} - \sqrt{9})(\sqrt{x^2} + \sqrt{9})$

$= (x - 3)(x + 3)$

Example 2: $x^4 - 16$

Solution: $x^4 - 16$

$= (\sqrt{x^4} - \sqrt{16})(\sqrt{x^4} + \sqrt{16})$

$= (x^2 - 4)(x^2 + 4)$

$= (\sqrt{x^2} - \sqrt{4})(\sqrt{x^2} + \sqrt{4})(x^2 + 4)$

$= (x - 2)(x + 2)(x^2 + 4)$

Example 3: $(x - 2)^2 - 25$

Solution: $(x - 2)^2 - 25$

$= (\sqrt{(x-2)^2} - \sqrt{25})(\sqrt{(x-2)^2} + \sqrt{25})$

$= [(x - 2) - 5][(x - 2) + 5]$

$= (x - 7)(x + 3)$

CLASSWORK 5 DATE:

Factorise the following expressions completely:

1. $x^2 - 1$
2. $4p^2 - 1$
3. $16m^2 - 9$
4. $49r^2 - 121s^2$
5. $2a^2 - 50$

HOMEWORK 6 DATE:

Factorise the following expressions:

1. $d^2 - 36$
2. $144b^2 - 81c^2$
3. $x^2y^2 - 16z^2$
4. $3p^2 - 27$
5. $(x - y)^2 - 1$

Consider the product $(a+x)(a+y)$

By multiplying out it is clear that this product will become:

$(a+x)(a+y)$

$= a^2 + ay + ax + xy$

$= a^2 + (x+y)a + (x \times y)$

So the expression $a^2 + (x+y)a + (x \times y)$ can be factorised as $(a+x)(a+y)$.

Steps for factorizing trinomials of the form $ax^2 \pm bx \pm c$ where a is a common factor.

Consider the expression of the form: $x^2 + 9x + 20$

$ax^2 \pm bx + c$:

If the sign before the constant is '+' then both signs in the brackets will be the same as the sign before the middle term.

both signs in the brackets will be '+' or '−' as the sign before the coefficient of the middle term.

Step 1: Use two pairs of brackets multiplied together to factorise the trinomial.

$(\quad)(\quad)$

Step 2: Determine factors of first term that are similar and put them as fist term of each bracket. $x^2 = x \times x$

$(x \quad)(x \quad)$

Step 3: Both signs in the middle will be the same as the sign before the middle term which is '+'.

$(x+ \quad)(x+ \quad)$

Step 3: Determine the factors of the last term that when you add them because of the sign of the last term will give you the coefficient of the middle term, 9.

Factors of 20 in pairs: $1 \times 20 \rightarrow 1 + 20 = 21$

$2 \times 10 \rightarrow 2 + 10 = 12$

$4 \times 5 \rightarrow 4 + 5 = 9 \rightarrow$ **correct combination**

$\therefore\ (x + 4)(x + 5)$

Step 4: Write the complete factorisation:

$x^2 + 9x + 20$

$= (x + 4)(x + 5)$

Example: $x^2 - 7x + 12$

$= x^2 + (-3 - 4)x + (-3 \times -4)$

$= (x - 3)(x - 4)$

Factors of 20 in pairs:
$-1 \times -12 \rightarrow -1 - 12 = -13$
$-2 \times -6 \rightarrow -2 - 6 = -12$
$-3 \times -4 \rightarrow - - 4 = -7$
$\rightarrow$ **correct combination**

Factorise completely:

Example 1: $x^2 + 11x + 24$

Solution: $x^2 + 11x + 24$

$= (x + 8)(x + 3)$

Example 2: $a^2 - 7a + 12$

Solution: $a^2 - 7a + 12$

$= (a - 3)(a - 4)$

Example 3: $5d^2 - 45d + 100$

Solution: $5d^2 - 45d + 100$

$= 5(d^2 - 9d + 20)$

$= 5(d - 5)(d - 4)$

CLASSWORK 7	DATE:

Factorise the following expressions completely:

1. $k^2 + 8k + 15$
2. $x^2 - 10x + 9$
3. $r^2 - 7r + 12$
4. $y^2 + 13y + 40$
5. $4p^2 - 48p + 80$

HOMEWORK 8	DATE:

Factorise the following expressions completely:

1. $k^2 - 8k + 15$
2. $x^2 + 9x + 18$
3. $r^2 + 14r + 24$
4. $b^2 - 5b + 6$
5. $-2a^2 - 10a - 8$

Steps for factorizing trinomials of the form $ax^2 \pm bx \pm c$ where a is a common factor.

Consider the expression of the form: $x^2 - 4x - 12$

$$ax^2 \pm bx - c:$$

If the sign before the constant is '$-$' then there will be different signs in the brackets.

The larger factor will be given the sign before the coefficient of the middle term.

Step 1: Use two pairs of brackets multiplied together to factorise the trinomial.

$$(\quad)(\quad)$$

Step 2: Determine factors of first term that are similar and put them as fist term of each bracket. $x^2 = x \times x$

$$(x \quad)(x \quad)$$

Step 3: The signs in the middle will be opposite as the sign of the last term is negative.

$$(x- \quad)(x+ \quad)$$

Step 3: Determine the factors of the last term that when you subtract them because of the sign of the last term will give you the coefficient of the middle term, 4.

Factors of 12 in pairs: $1 \times 2 \rightarrow 12 - 1 = 11$

$2 \times 6 \rightarrow 6 - 2 = 4 \rightarrow$ correct combination

$3 \times 4 \rightarrow 4 - 3 = 1$

$$\therefore (x - 6)(x + 2)$$

Step 4: Write the complete factorisation:

$$x^2 - 4x - 12$$
$$= (x - 6)(x + 2)$$

Factorise the following expressions:

Example 1: $x^2 - x - 6$

Solution: $x^2 - x - 6$

$= (x - 3)(x + 2)$

Example 2: $x^2 + 3x - 28$

Solution: $x^2 + 3x - 28$

$= (x - 4)(x + 7)$

Example 3: $6x^2 - 12x - 18$

Solution: $6x^2 - 12x - 18$

$= 6(x^2 - 2x - 3)$

$= 6(x - 3)(x + 1)$

CLASSWORK 9 DATE:

Factorise the following expressions fully:

1. $n^2 - n - 20$
2. $t^2 + 4t - 60$
3. $r^2 + 5r - 36$
4. $a^2 - 7a - 18$
5. $4x^2 - 20x - 24$

HOMEWORK 10 DATE:

Factorise the following expressions fully:

1. $m^2 + m - 20$
2. $c^2 - 4c - 60$
3. $s^2 - 5s - 24$
4. $k^2 + 12k - 64$
5. $2y^2 + 4y - 16$

SIMPLIFYING ALGEBRAIC FRACTIONS

To simplify algebraic fractions, you will have to mostly factorise first.

Simplify the following expressions:

Examples 1: $\frac{8x^2-4}{4}$

Solution: $\frac{8x^2-4}{4}$ → do not cancel over a '+' or '−' sign

$= \frac{4(2x^2-1)}{4}$ → factorise or $\frac{8x^2}{4} - \frac{4}{4}$ → ÷ each term by denominator

$= 2x^2 - 1$

Examples 1: $\frac{21x^2y^4-14xy}{14xy+xyz}$

Solution: $\frac{21x^2y^4-14xy}{14xy+xyz}$

$= \frac{7xy(3xy^3-2)}{7xy(2+z)}$ → factorise both numerator and denominator

$= \frac{3x^2-2}{2+z}$

Example 4: $\frac{x^2+5x-14}{x-3} \times \frac{x^2+x-12}{2-x} \div \frac{x^2+4x}{1}$

Solution: $\frac{x^2+5x-14}{x-3} \times \frac{x^2+x-12}{2-x} \div \frac{x^2+4x}{1}$

$= \frac{(x+7)(x-2)}{x-3} \times \frac{(x+4)(x-3)}{-(x-2)} \times \frac{1}{x(x+4)}$

$= \frac{x+7}{-x}$

$= -\frac{x+7}{x}$

CLASSWORK 11 DATE:

Simplify:

1. $\frac{8a+16}{8a}$

2. $\frac{6q-7p}{14p-12q}$

3. $\frac{x^2-5x}{x^2-7x+10}$

4. $\frac{x^2-4}{x^2+5x+6} \times \frac{x-5}{x+4} \div \frac{x^2-7x+10}{x^2+7x+12}$

HOMEWORK 12 DATE:

Simplify:

1. $\frac{2p^2+p}{p^2}$

2. $\frac{8m^2+16m}{16} \times \frac{8}{m+2}$

3. $\frac{x^2-2x-15}{2x^3-4x^2-6x} \div \frac{x^2-3x-10}{2x^2-20x+42} \div \frac{x+3}{x^2+x}$

CLASSWORK 13 DATE:

REVISION OF SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS

Factorize completely:

1. $5x^3y^3 - 10x^4y^2 + 3x^2y^4$

2. $x^2(x + 11) - 16(x + 11)$

3. $2x^3 - 12x^2 + 18x$

4. $\frac{x^2-1}{3x+3}$

6. $\frac{x^2-4x}{4y} \times \frac{4xy}{xy(x-4)} \div \frac{x}{y^2-y}$

CLASSWORK 14	DATE:

CLASS TEST 1 ALGEBRAIC EXPRESSION

1. Factorise completely:

1.1 $2x^4 - 2$ **(4)**

1.2 $x^2 + 2x - 15$ **(2)**

1.3 $p^2(a - b) - (b - a)$ **(3)**

2. Simplify:

2.1 $\frac{3x+6x^2}{3x}$ **(2)**

2.2 $\frac{2a}{a+3} \div \frac{4a^2}{2a^2+6a}$ **(3)**