KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS)

TEACHER SUPPORT

DOCUMENT GRADE 11

MATHEMATICS

STEP AHEAD PROGRAMME

2022

This document has been compiled by the KZN FET Mathematics Subject Advisors.

PREFACE

This support document serves to assist Mathematics teachers on how to deal with curriculum gaps and learning losses as a result of the impact of COVID-19 since 2020. It also captures the challenging topics in the Grade 10 – 12 work. The lesson plans should be used in conjunction with the 2022 Recovery Annual Teaching Plans. Activities should serve as a guide on how to assess topics dealt with in this document. It will cover the following:

TABLE OF CONTENTS

TOPIC: ALGEBRA					
LESSON 1: EXPONENTS AND SURDS					
Term	1	**Week**		**Grade**	11
Duration	1hr	**Weighting**		**Date**	
Sub-topics	Simplify expressions using laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$				

RELATED CONCEPTS/ TERMS/VOCABULARY

Exponential laws
Rational number: definition
Square roots and other roots

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Learners are familiar with simplifying expressions using laws of exponents from the previous grades.

RESOURCES

Keeping Mathematics Simple (Clever)
Learning Channel Mathematics (Learners' Book
Platinum Mathematics
Maths Handbook and Study guide

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Learners fail to apply the exponential laws
- Inability to work with radical sign i.e., change from radical form to exponential form

METHODOLOGY

Rational exponents and roots

Concept development

Writing a number in exponential form

$9 = 3 \times 3 = 3^2$ **multiplication of powers**

$\sqrt{9} = 3$ **calculator**

$\sqrt{3^2} = 3$ **since $9 = 3^2$**

Changing radical form to exponential form

Given 3^2 , use $\sqrt{3^2}$ to investigate the value (exponent) of a radical sign

Let $m = 2$ and n to represent the value of $\sqrt{\ }$

$\sqrt{3^2} = (3^2)^m$ exponential form

$3^{mn} = 3^{2n}$ raising a power to an exponent, $(x^m)^n = x^{mn}$

$3^{2n} = 3^1$

$2n = 1$ exponential equations

$n = \frac{1}{2}$

$n = \frac{1}{m}$

Since $m = 2$

Then

$\sqrt{3^2} = \left(3^2\right)^{\frac{1}{2}}$

Law : $\left(x^{\frac{p}{q}}\right) = \sqrt[q]{x^p}$

1. Convert the following into exponential form

   1.1 $\sqrt{x}$

   1.2 $\sqrt[3]{x}$

   1.3 $\sqrt[5]{x}$

Solutions

   1.1 $\sqrt{x} = x^{\frac{1}{2}}$

   1.2 $\sqrt[3]{x} = x^{\frac{1}{3}}$

   1.3 $\sqrt[5]{x} = x^{\frac{1}{5}}$

The following rules apply $x^{\frac{p}{q}} = \left(x^p\right)^{\frac{1}{q}}$ or $x^{\frac{p}{q}} = \left(x^{\frac{1}{q}}\right)^p = \left(\sqrt[q]{x}\right)^p$

2 Write the following in exponential form, if necessary

   2.1 $\sqrt[3]{x^2} = x^{\frac{2}{3}}$

   2.2 $\sqrt[5]{x^4} = x^{\frac{4}{5}}$

   2.3 $\sqrt[8]{x^2} = x^{\frac{2}{8}} = x^{\frac{1}{4}}$

3. Evaluate the following without using a calculator

3.1 $\sqrt{100x^6}$

$=\sqrt{10^2x^6}$ **writing a coefficient in exponential form**

$=10x^3$ **exponential law**

3.2 $(0,343)^{-\frac{2}{3}}$

$=(0,7^3)^{-\frac{2}{3}}$

$=(0,7^3)^{-\frac{2}{3}}$

3.3 $16^{-\frac{1}{2}}+16^{\frac{3}{4}}-64^{-\frac{1}{3}}$

ACTIVITIES/ASSESSMENTS

Simplify the following. Leave your answer with positive exponents

1.1 $\sqrt[4]{16^3}$

1.2 $\sqrt[3]{64^2}$

1.3 $\left(\frac{81}{16}\right)^{\frac{1}{4}}$

1.4 $(0,0625)^{-\frac{1}{4}}$

1.5 $(0,25x^4y^8)^{-\frac{1}{4}}$

1.6 $\frac{9x^{-2}}{27x^{-3}}$

1.7 $\sqrt{a^3}\,.\,a^{\frac{1}{2}}$

1.8 $3^{-1}+2^{-2}$

1.9 $\frac{1}{(x+y)^{-1}}$

1.10 $\frac{a^{-2}-b^{-2}}{b^2-a^2}$

TOPIC: ALGEBRA					
LESSON 2					
Term	1	**Week**		**Grade**	11
Duration	1hr	**Weighting**		**Date**	

Sub-topics	Simplify expressions using laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Prime factorization

Exponential laws

Factorization by taking out a common factor

RESOURCES

Keeping Mathematics Simple (Clever)
Learning Channel mathematics (Learners' Book)
Platinum Mathematics
Maths Handbook and Study guide

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Removal of brackets in the exponents: If there are two terms the second term is left out during multiplication
- Confusion between problems with all operations combined

METHODOLOGY

Examples of simplification of expression using exponential laws will be done

Simplify the following

1. $\frac{4^{x+2}.8^{x-1}}{2^{x-3}.16^{x}}$

$= \frac{2^{2(x+2)}.2^{3(x-1)}}{2^{x-3}.2^{4(x)}}$ writing bases as powers of its prime bases

$= \frac{2^{2(x+2)}.2^{3(x-1)}}{2^{x-3}.2^{4(x)}}$ multiply the exponents (raising a power to an exponent)

$= \frac{2^{2x+4}.2^{3x-3}}{2^{x-3}.2^{4x}}$ remove the brackets in exponents

$= \frac{2^{2x+4+3x-3}}{2^{x-3+4x}}$ adding the bases

$= \frac{2^{5x}.2^{1}}{2^{5x}.2^{-3}}$ simplify

$= 2^4$
$= 16$

2. $\frac{15^{n+2}.45^{1-n}}{3^{3-n}}$

$=\frac{(3.5)^{n+2}.(3.3.5)^{1-n}}{3^{3-n}}$ prime factorization

$=\frac{3^{n+2}.5^{n+2}.3^{1-n}.3^{1-n}.5^{1-n}}{3^{3-n}}$ breaking down of powers

$=\frac{3^{n+2+1-n+1-n}.5^{1}.5^{-n}}{3^{3-n}}$ applying the exponential laws

$=\frac{3^{-n+4}.5.5^{-n}}{3^{3-n}}$ simplification

$=\frac{3^{-n+4-3-(-n)}.5.5^{-n}}{3^{3-n}}$

$=3.5.5^{-n}$ simplification

$=\frac{15}{5^{n}}$ negative exponent

3. $\frac{5^{x+1}.5^{2x-2}}{5^{3x}}$

$=\frac{5^{x}.5^{2x}5^{-2}}{5^{3x}}$

$=\frac{5^{x}.5^{2x}5^{-2}}{5^{3x}}$

$=\frac{5^{x}.5^{2x}5^{-2}}{5^{3x}}$

$=\frac{5^{3x}.5^{-2}}{5^{3x}}$

$=\frac{1}{5^{2}}$

$=\frac{1}{25}$

4. $\frac{2^{x}.+2^{x+1}}{2^{x}-2^{x+2}}$

$=\frac{2^{x}+2^{x}.2}{2^{x}-2^{x}.2^{2}}$ breaking down of powers

$=\frac{2^{x}(1+2)}{2^{x}(1-2^{2})}$ taking out the common factor

$=\frac{3}{-2}=-\frac{3}{2}$ simplification

5. $\frac{3^{x+1}+3^x}{3^x}$

$=\frac{3^x.3+3^x}{3^x}$ breaking down the power

$=\frac{3^x(3+1)}{3^x}$ common factor

$=4$ simplification

6. $=\frac{15^{x-1}+5^x}{5^x}$

$=\frac{3^{x-1}.5^{x-1}+5^x}{5^x}$ expand

$=\frac{3^x.3^{-1}.5^x.5^{-1}+.5^x}{5^x}$ expand

$=\frac{5^x\left(3^x.\frac{1}{3}.\frac{1}{5}+1\right)}{5^x}$ simplification

$=\frac{3^x}{15}+1$

$=\frac{3^x}{15}+\frac{15}{15}$

$=\frac{3^x+15}{15}$

ACTIVITIES/ASSESSMENTS

Simplify the following. Leave your answer with a positive exponent

2.1 $\frac{25^{x+1}.6}{10^{x-1}.15^x}$

2.2 $\frac{6^{n+3}.2^{n-1}}{12^{n+2}}$

2.3 $\frac{2^{4x+1}.9^x.6^{2x-1}}{12^{3x}.3^x}$

2.4 $\frac{8^x.6^{x-1}}{27^{\frac{x}{3}}.16^x}$

2.5 $\frac{2^{n+1}+2^{n+3}}{2^{n+1}}$

TOPIC: ALGEBRA					
LESSON 3: EXPONENTS AND SURDS					
Term	1	**Week**		**Grade**	11
Duration	1 Hr	**Weighting**		**Date**	

Sub-topics: **Solve equations using laws of exponents for rational exponents where:** $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$

RELATED CONCEPTS/ TERMS/VOCABULARY

Laws of exponents
Rational exponents – definition
Square root and other roots

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- **Laws of exponents**
- **Definition of a surd**
- **Simplification of expressions using laws of exponents for rational exponents where :**

  $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0;\ q > 0$

RESOURCES

Keeping Mathematics Simple (Clever)
Learning Channel Mathematics (Learners' Book)
Platinum Mathematics
Maths Handbook and Study guide

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- **Manipulation of rational exponents**
- **Raising an exponent to another exponent**
- **Splitting the powers**
- **Factorization of powers**
- **Checking the solution for validity**

METHODOLOGY

1. Revision of the exponential equations learnt in Gr10 (in which an exponent is the unknown)

2. Remember:

➢ The definition : $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ we will use this definition to solve equations with rational exponents

e.g. $\sqrt[3]{a^2} = a^{\frac{2}{3}}; \sqrt[3]{b} = b^{\frac{1}{3}}; \sqrt[4]{p^3} = p^{\frac{3}{4}}; \sqrt{x} = x^{\frac{1}{2}}$

➢ By definition $\sqrt{25} = 5;$ and $\pm\sqrt{25} = \pm 5$

Example 1. Solve for x

$4^x = 8$

$(2^2)^x = 2^3$ **- write both bases as a power of 2**

$2^{2x} = 2^3$ **- raise a power to a power:** $(a^m)^n = a^{mn}$

$2x = 3$ **- equate exponents**

$x = \frac{3}{2}$

Example 2. Solve for x

$3^{x+2}+3^{x+1}=12$

$3^{x}.3^{2}+3^{x}.3^{1}=12$ - **split the powers**

$3^{x}\left(3^{2}+3^{1}\right)=12$ - **factorise by taking out** 3^{x} **as a common factor**

$3^{x}\left(9+3\right)=12$

$12.3^{x}=12$

$\frac{12.3^{x}}{12}=\frac{12}{12}$ - **divide both sides of the equation by 12**

$3^{x}=1$

$3^{x}=3^{0}$ - $a^{0}=1$

$x=0$

Example 3 Determine the value of $\boldsymbol{x}$ **if** $x^{\frac{3}{4}}=8$

$\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}}=\left(2^{3}\right)^{\frac{4}{3}}$ - **raise each side of the equation to the reciprocal exponent**

$x=2^{4}=16$

Example 4. Determine the value of $\boldsymbol{x}$ **if** $x^{\frac{1}{2}}-3x^{\frac{1}{4}}-10=0$

Solution: this can be easily solved by using substitution.

Let $k=x^{\frac{1}{4}}$

$\therefore k^{2}=x^{\frac{1}{4}}.x^{\frac{1}{4}}=x^{\frac{1}{2}}$

And ,if $x^{\frac{1}{2}}-3x^{\frac{1}{4}}-10=0$

Then $k^{2}-3k-10=0$

$(k-5)(k+2)=0$ - **factorise**

$k=5$ or $k=-2$ - **solve for** $\boldsymbol{k}$

$x^{\frac{1}{4}}=5$ $x^{\frac{1}{4}}=-2$ - **substitute for** $k=x^{\frac{1}{4}}$

$\left(x^{\frac{1}{4}}\right)^{4}=5^{4}$ **No solution to** $x^{\frac{1}{4}}=-2:\sqrt[4]{x}\geq 0$

$x=625$

ACTIVITIES/ASSESSMENTS

SOLVE FOR X:

3.1 $x^{\frac{2}{3}} = 4$

3.2 $x^{\frac{1}{3}} = -1$

3.3 $x^{\frac{1}{2}} = -5$

$x^{\frac{3}{2}} = 27$

3.4 $3\sqrt[3]{x} = 24$

3.5 $x^{\frac{1}{2}} - 2x^{\frac{1}{4}} - 3 = 0$

3.6 $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} - 4 = 0$

3.7 $3x^{\frac{3}{4}} = 24x^{\frac{1}{4}}$

3.8 $6 + 4\sqrt[4]{x} = 18$

3.9 $3x^{\frac{2}{3}} - 5x^{\frac{1}{3}} - 12 = 0$

3.10 $x - 7x^{\frac{1}{2}} - 10 = 0$

TOPIC: EXPONENTS AND SURDS					
LESSON 4					
Term	1	**Week**		**Grade**	11
Duration	1hr	**Weighting**		**Date**	

Sub-topics	DEFINITION OF SURDS SURD LAWS

RELATED CONCEPTS/ TERMS/VOCABULARY

Introduction to surds
Definition of a surd
Application of surd laws
Exponential laws

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE
Prime factorization
Exponential laws
Factorization by taking out a common factor

RESOURCES

Keeping Mathematics Simple (Clever)
Learning Channel mathematics (Learners' Book
Platinum Mathematics
Maths Handbook and Study guide

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Writing down a surd as a product of any two factors without ensuring that one must be rational and the other irrational
- Removal of brackets in the exponents: If there are two terms the second term is left out during simplification
- Confusion between problems with multiplication and division and addition and subtraction.
- Inability to factorise into prime bases
- Negative exponents are still a challenge

METHODOLOGY

Definition :
Surd: A root of a number that produces an Irrational number
Irrational number: A number that cannot be written in the form of $\frac{a}{b}$ where a and b are integers, $a \neq 0$
Examples of surds: $\sqrt{2}; \sqrt{7}; \sqrt[3]{17}; \sqrt[5]{11}$

Discussion of the rules and the restrictions on surds will be done: Refer to worksheet

EXAMPLES: APPLICATION OF THE RULES

Simplify the following surd without using a calculator

1. $\sqrt{32}$

$=\sqrt{2\times 16}$ factorise (ensure one of the factors is a perfect square)

$=\sqrt{16}\times\sqrt{2}$ product rule

$=4\sqrt{2}$ simplification

2. $\sqrt[3]{96}$

$=\sqrt[3]{8\times 12}$ factorise (one factor a perfect cube)/ product rule

$=2\sqrt[3]{12}$ simplification

3. $\sqrt{50x^6y^8}$

$=\sqrt{25\times 2x^6y^8}$ factorisation

$=5\sqrt{2}x^3y^4$ simplification (exponential law)

4. $\dfrac{\sqrt{32}}{12}$

$=\dfrac{\sqrt{16}\times\sqrt{2}}{12}$ product rule

$=\dfrac{4\sqrt{2}}{12}$ simplification

$=\dfrac{\sqrt{2}}{3}$

ACTIVITIES/ASSESSMENTS

Simplify the following without using a calculator (Show all your working)

4.1 $\sqrt{50}$

4.2 $\sqrt[3]{250}$

4.3 $\dfrac{\sqrt{132}}{\sqrt{3}}$

4.4 $\dfrac{\sqrt{32x^{12}y^8}}{\sqrt{8x^2y^4}}$

1. INVESTIGATING THE PRODUCT RULE

- Use your calculator to complete the table below.
- Write your answer in decimal form
- The format of the answer has been modelled kin the first calculation

PRODUCT 1	PRODUCT 2
$\sqrt{7}\times\sqrt{3}=4,5825.........$	$\sqrt{7\times 3}=$
$\sqrt{8}\times\sqrt{3}=$	$\sqrt{8\times 3}=$
$\sqrt{5}\times\sqrt{2}=$	$\sqrt{5\times 2}=$
$\sqrt[3]{13}\times\sqrt[3]{11}=$	$\sqrt[3]{11\times 13}=$
$\sqrt[4]{8}\times\sqrt[4]{12}=$	$\sqrt[4]{8\times 12}$

What do you observe about Product 1 and product 2?

....

Conclusion:

$\sqrt[n]{a\times b}=$..........................

2. RESTRICTIONS ON THE RULE

The rule applies for $n\in$ natural numbers , $n\geq 2$, a,b >0
Use two examples to show what happens when each of these occur :

2.1 $a<0$

2.2 $b<0$

2.4 $n<2$ e.g. $n=1$

INVESTIGATING THE QUOTIENT RULE

- Use your calculator to complete the table below.
- Write your answer in decimal form
- The format of the answer has been modelled kin the first calculation

QUOTIENT 1	QUOTIENT 2
$\frac{\sqrt{7}}{\sqrt{3}}=$	$\sqrt{\frac{7}{3}}=$
$\frac{\sqrt{8}}{\sqrt{3}}=$	$\sqrt{\frac{8}{3}}=$
$\frac{\sqrt{5}}{\sqrt{2}}=$	$\sqrt{\frac{5}{2}}=$
$\frac{\sqrt[3]{11}}{\sqrt[3]{13}}=$	$\sqrt[3]{\frac{11}{13}}=$
$\frac{\sqrt[4]{8}}{\sqrt[4]{12}}=$	$\sqrt[4]{\frac{8}{12}}$

Conclusion:

$\sqrt[n]{\frac{a}{b}}=$....................

Use your own examples to show that, Discuss the restrictions for the rules:

1. $\sqrt[p]{\sqrt[q]{a}}=\sqrt[pq]{a}$
2. $\left(\sqrt[p]{a}\right)^{q}=\sqrt[p]{a^{q}}$

TOPIC: EXPONENTS AND SURDS					
LESSON 5: SIMPLIFICATION OF SURDS					
Term	1	**Week**		**Grade**	11
Duration	1hr	**Weighting**		**Date**	
Sub-topics	Simplification of surds				

RELATED CONCEPTS/ TERMS/VOCABULARY

Definition of a Surd
Irrational number
Rationalize

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Application of laws in surds

RESOURCES

Keeping Mathematics Simple (Clever)
Learning Channel Mathematics (Learners' Book
Platinum Mathematics
Maths Handbook and Study guide

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Not simplifying the surds fully
- Inability to recognize like surds and simplify them

METHODOLOGY

Examples
Simplify without using a calculator

1. $\sqrt{18}+\sqrt{50}-\sqrt{8}$
2. $\dfrac{\sqrt{132}}{\sqrt{3}}$
3. $\left(\sqrt{5}+3\right)\left(\sqrt{5}-3\right)$
4. $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$
5. $\dfrac{7}{\sqrt{5}-\sqrt{3}}$ (rationalize the denominator

ACTIVITIES/ASSESSMENTS

Simplify without using a calculator

5.1 $\sqrt{32}-\sqrt{8}$

5.2 $\sqrt{32}-\sqrt{8}$

5.3 $\sqrt{50}-\sqrt{5}+\sqrt{45}$

5.4 $\left(\sqrt{3}-\sqrt{7}\right)\left(\sqrt{3}-\sqrt{7}\right)$

5.5 $\left(\sqrt{11}-3\right)\left(\sqrt{11}+3\right)$

5.6 $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ (hint: rationalise the denominator)

5.7 Show that $\dfrac{\sqrt{48}}{\sqrt{24}-\sqrt{8}}=\dfrac{\sqrt{6}+3\sqrt{2}}{2}$, without using a calculator

TOPIC: ALGEBRA

LESSON 6:

Term	1	**Week**		**Grad**	11
Duration	1 Hr	**Weighting**		**Date**	

Sub-topics SIMPLE EQUATIONS WITH SURDS

RELATED CONCEPTS/ TERMS/VOCABULARY

Laws of exponents
Rational exponents

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Laws of exponents
- Definition of a surd
- Simplification of expressions using laws of exponents for rational exponents where:

  $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$.

- Solve equations using laws of exponents for rational exponents where: $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$

RESOURCES

Grade 11 books:
Keeping Mathematics Simple (Clever)
Maths Handbook And Study Guide
Via Africa Mathematics Grade 11
Platinum Mathematics

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Manipulation of rational exponents
- Isolating a surd
- Squaring on both sides
- Checking the solution for validity

METHODOLOGY

Explain that equations with surds can be solved by :

- using the laws of rational exponents,
- by Squaring both sides of the equation.

Example 1. Solve for x

$\sqrt[3]{x} = 2$ Restriction: $x \geq 0$

$x^{\frac{1}{3}} = 2$ - express in exponential form

$x^{\frac{1}{3}\times\frac{3}{1}} = 2^{\frac{3}{1}}$ - application of the laws of rational exponents

$x = 2^3$

$x = 8$

Checking:

$LHS = \sqrt[3]{8}$

$= 2^{\frac{3}{3}}$

$= 2 = RHS$

$\therefore$ 8 is a valid solution

Example 2. Solve for x

$\sqrt{x^3} = 64$

Restriction: $x \geq 0$

$x^{\frac{3}{2}} = 4^3$

$x^{\frac{3}{2}\times\frac{2}{3}} = 4^{3\times\frac{2}{3}}$

$x = 4^2$

$x = 16$

Checking:

$LHS = \sqrt{16^3}$

$= 16^{\frac{3}{2}}$

$= 4^{2\times\frac{3}{2}}$

$= 4^3$

$= 64 = RHS$

$\therefore$ **16 is a valid solution**

Example 3 Determine the value of *x* if $3\sqrt{x} = x - 4$

Restriction: $x \geq 4$

$\left(3\sqrt{x}\right)^2 = \left(x-4\right)^2$ **- squaring on both sides**

$9(x) = x^2 - 8x + 16$

$x^2 - 17x + 16 = 0$

$(x-16)(x-1) = 0$ **- factorization**

$x = 16$ or $x = 1$ **- find the answer**

Checking:

For $x = 16 : 3\sqrt{16} = 16 - 4$

For $x = 1 : 3\sqrt{1} \neq 1 - 4$

$= 12 = RHS$

$\therefore$ ***16 is valid solution***

$\therefore x = 1$ **is not a valid solution**

Example 4. Determine the value of *x* if $\sqrt{x+1} = 2$

$restrictio : x \geq -1$

$\left(\sqrt{x+1}\right)^2 = 2^2$ **- squaring on both sides**

$x + 1 = 4$

$x + 1 - 1 = 4 - 1$ **- add the additive inverse of 1 on both sides of the equation**

$x = 3$ **- find the answer**

Checking:

$LHS = \sqrt{3+1}$

$= \sqrt{4}$

$= 2 = RHS$

$\therefore$ **3 is a valid solution**

ACTIVITIES/ASSESSMENTS

SOLVE FOR X:

6.1 $\sqrt[4]{x^3} = 27$

6.2 $-1\sqrt{x} = -7$

6.3 $\sqrt{x-1} = 6$

6.4 $\sqrt{x+3} = 10$

6.5 $\sqrt{x+12} = x$

6.6 $\sqrt{x+2} = x-4$

6.7 $\sqrt{10-x} = -2x-1$

6.8 $\frac{\sqrt{x}}{\sqrt{2}} = 3\sqrt{2}$

6.9 $\sqrt{2-7x} + 2 = x$

6.10 $-\sqrt{-9x-17} = -3-x$

TOPIC: ALGEBRA					
LESSON 7: EXPONENTS AND SURDS					
Term	1	**Week**		**Grade**	11
Duration	1Hr	**Weighting**		**Date**	
Sub-topics	**SIMPLE EQUATIONS WITH SURDS**				

RELATED CONCEPTS/ TERMS/VOCABULARY

Exponential laws
Rational exponents

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- **Definition of a surd**
- **Simplification of expressions using laws of exponents for rational exponents where :**

  $x^{\frac{p}{q}} = \sqrt[q]{x^p}\,; x > 0;\ q > 0$.

- **Solve equations using laws of exponents for rational exponents where:** $x^{\frac{p}{q}} = \sqrt[q]{x^p}\,; x > 0;\ q > 0$

RESOURCES

Grade 11 books:
Keeping mathematics simple(Clever)
Maths Handbook And Study Guide
Gr11 mathematics (Via Africa)

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- **Manipulation of rational exponents**
- **Isolating a surd**
- **Squaring on both sides**
- **Checking the solution for validity**

METHODOLOGY

Equations with surds can be solved using:

- **The following Steps to solving Surd Equations**
  - **State the restrictions**
  - **Isolate the surd term**
  - **Square both sides of the equation**
  - **Solve for the unknown**
  - **Check your answer against the stated restrictions to declare a valid solution and omit the invalid solution**

Example 1. Determine the value of *x* if

$restriction: x \geq -2$ **and** $y \geq 0$

$\sqrt{x+2} - 3 = 0$

$\sqrt{x+2} = 3$ **- isolate the surd term**

$\left(\sqrt{x+2}\right)^2 = 3^2$ **- squaring on both sides**

$x + 2 = 9$

$x + 2 - 2 = 9 - 2$ **- adding the additive inverse of +2 on both sides of the equation(transposition)**

$x = 7$ **- find the answer**

Checking:

$LHS = \sqrt{7+2} - 3$

$= \sqrt{9} - 3$

$= 3 - 3$

$= 0 = RHS$

$\therefore$ **7 is a valid solution**

Example 2: Solve for x:

Restriction on $x+1 \geq 0$ $\therefore x \geq -1$ **on** $x-1 \geq 0$ $\therefore x \geq 1$

$1+\sqrt{x+1} = x$

$\sqrt{x+1} = x-1$ **- isolating a surd**

$\left(\sqrt{x+1}\right)^2 = (x-1)^2$ **- squaring on both sides**

$x+1 = x^2 - 2x + 1$

$x^2 - 3x = 0$

$x(x-3) = 0$ **- factorization**

$x = 0$ or $x = 3$

Checking:

For $x = 0$**:** $LHS = \sqrt{0+1} = 1$

$RHS = 0 - 1 = -1$

$\therefore x = 0$ **is not a valid solution**

For $x = 3$: $LHS = \sqrt{3+1} = \sqrt{4} = 2$

$RHS = 3 - 1 = 2$

$\therefore x = 3$ **is a valid solution**

ACTIVITIES/ASSESSMENTS

1. Solve for *x*

7.1 $\sqrt{x-3} - 4 = 0$

7.2 $\sqrt{12+x} = \sqrt{x} + 3$

7.3 $\sqrt{8-x} = 2$

7.4 $2\sqrt{\frac{x}{2} - 3} + 4 = 12$

7.5 $4\sqrt{x-3} - 3 = 2x - 9$

7.6 $\sqrt{2x-1} - \frac{3}{\sqrt{2x-1}} = -2$

7.7 $2\sqrt{4-x} = 2\sqrt{x} - 4$

7.8 $3\sqrt{\sqrt{x}+3} = 9$

7.9 $\sqrt{22-7x} - x = -4$

7.10 **A rectangular fence has a perimeter of 35 meters. It has a width of 5meters and a length of $\sqrt{2x+3}$ meters. Determine the value of x.**

TOPIC: EUCLIDEAN GEOMETRY

LESSON 8

Term	1	**Week**		**Grade**	11
Duration	1 hour	**Weighting**	50 ± 3	**Date**	
Sub-topics	**Revise lines and angles, Triangles and Quadrilaterals**				

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Types and properties of angles, triangles and quadrilaterals

RESOURCES

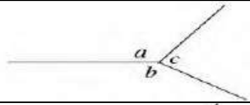
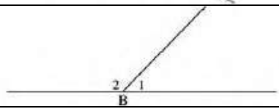
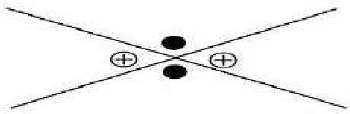
- Via Africa Study Guide Grade 11
- Mind Action Series Grade 11

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

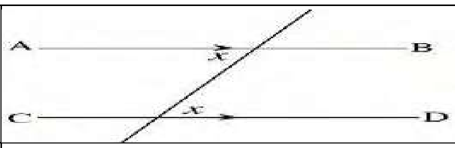
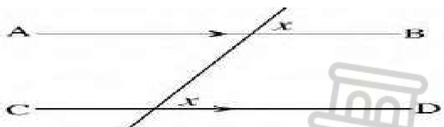
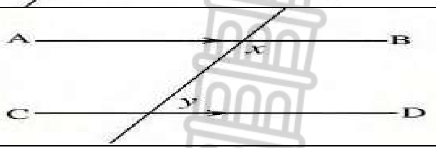
Assuming that lines are parallel. Using congruency conditions without understanding

METHODOLOGY

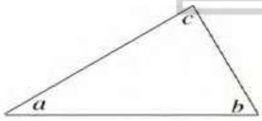
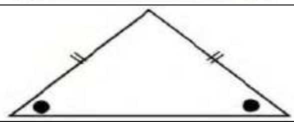
Intersecting Lines

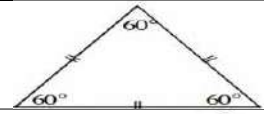
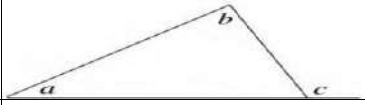
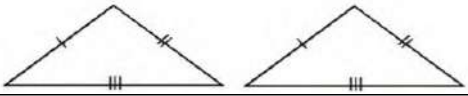
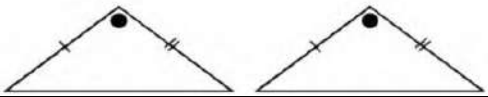
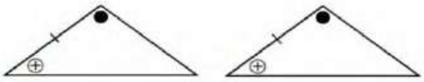
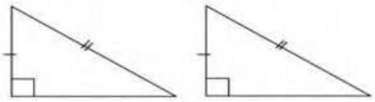
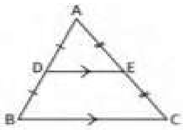
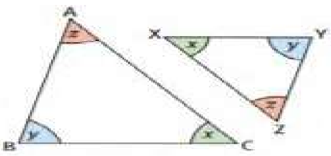
The **sum** of the angles around a point is **360°.**	 $a+b+c=360°$
Adjacent angles at a point on a line segment are **supplementary**.	 $\hat{B}_1+\hat{B}_2=180°$
When **two lines intersect**, the **vertically opposite angles** are **equal**.	

Parallel lines

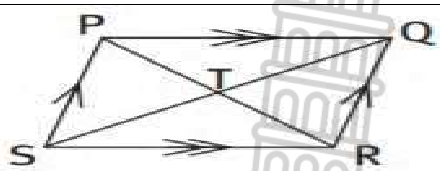
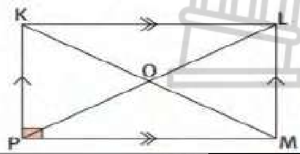
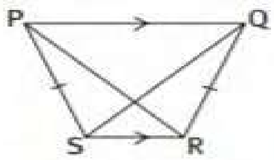
When a transversal cuts **parallel lines**, the **alternate** angles are **equal**.	
When a transversal cuts two **parallel lines**, the **corresponding angles** are **equal**.	
When a transversal cuts **parallel lines**, the **co-interior angles** are **supplementary**.	

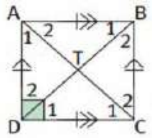
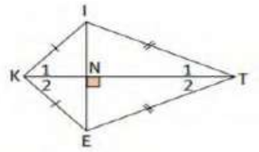
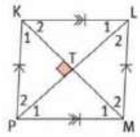
Triangles

The **interior angles** of a triangle are **supplementary**.	 $a+b+c=180°$
The **angles opposite two equal sides** of an isosceles triangle are **equal**.	

All **interior angles** of an equilateral triangle are **60°.**	
An **exterior angle** of a triangle is equal to the sum of the opposite interior angles	 $c = a + b$
When the **sides** of one triangle are equal to the to the three sides of the other triangle, the two triangles are **congruent. [SSS]**	
Two triangles are **congruent** when **two sides** and the **included angle** are equal to two sides and the included angle of the other triangle. **[SAS]**	
When **two angles** and a **side** of one triangle are equal to two angles and the corresponding side of another triangle, the two triangles are **congruent. [AAS]**	
Two triangles are **congruent** when the **hypotenuse** and **one side** of a **right-angled** triangle are equal to the hypotenuse and one side of another right-angled triangle. **[90°HS]**	
The line segment joining the **midpoints** of two sides of a triangle is **parallel to the third side** and **half the length of the third side**. [DE||DB]	
When the **corresponding sides** of two triangles are in the same ratio, the two triangles are **similar. [SSS]**	
When the **corresponding angles** of two triangles are equal, the two triangles are **similar. [AAA]**	

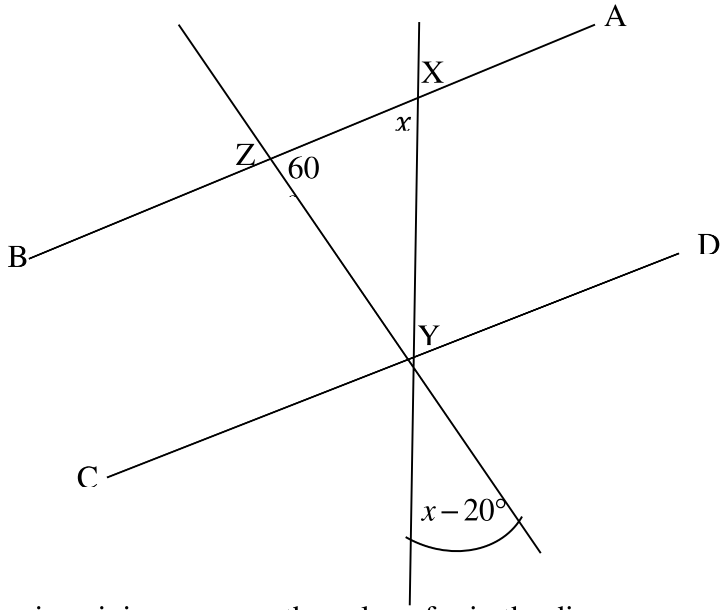
Quadrilaterals

A **parallelogram** is a quadrilateral with opposite sides parallel. [PQ||SR, PS||QR]	
A **rectangle** is a parallelogram with a 90° interior angle. [∠KPM = 90°]	
An isosceles **trapezium** is a quadrilateral with one pair opposite sides parallel and the other pair equal. [PQ||SR, PS = QR]	

A **square** is a rhombus with a 90° interior angle. [AB = BC = CD = DA]	
A **kite** is a quadrilateral with two pairs of adjacent sides equal. [KI = KE, ET = TI]	
A **rhombus** is a parallelogram with all sides equal. [KL = LM = MP = PK]	

ACTIVITIES/ ASSESSMENT

8.1 In the diagram below, AB and DC are two parallel lines cut by two transversal lines at X, Y and Z respectively.



8.1.1 Determine giving reasons, the value of x in the diagram:

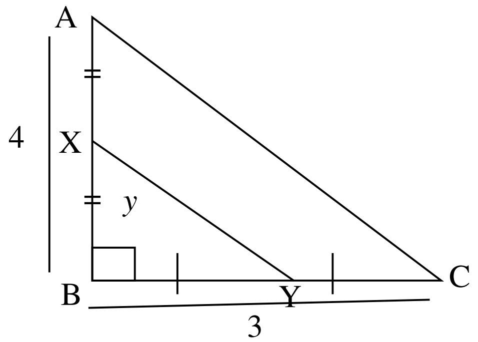
8.1.2 Name one pair of co-interior angles

8.1.3 Name one pair of alternate angles

8.1.4 Complete: If two parallel lines are cut by a transversal, then the co-interior angles are ....

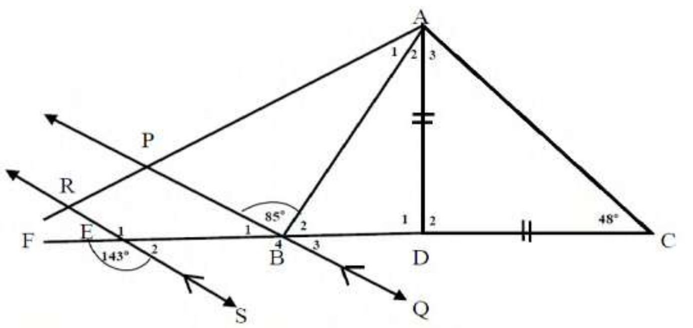
8.1.5 Complete: The size of angle XYD = Reason ....

8.2 Determine with reasons, the value of y (XY) in the diagram below. Given that AB = 4 units and BC = 3 units. X and Y are the midpoints of AB and BC respectively.



8.3 In the diagram below, AD = CD and $PQ \parallel RS$. AR and FC are straight lines. RS and FC intersect

at E also PQ intersects FC at B.



(a) Determine the sizes of the following angles, giving appropriate reasons:

1) $\hat{D}_1$

2) $\hat{B}_1$

3) $\hat{A}_2$

(b) Show that $R\hat{E}F = \hat{B}_3$

TOPIC: EUCLIDEAN GEOMETRY					
LESSON 9					
Term	1	**Week**		**Grade**	11
Duration	1 hour	**Weighting**	5 ± 3	**Date**	
Sub-topics	**Circle Geometry: Line from Centre to chord**				

RELATED CONCEPTS/ TERMS/VOCABULARY

Segment, Chord, perpendicular bisector

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Circle, diameter, radius, circumference, Congruent, Pythagoras theorem

RESOURCES

- Mind action Series
- Mind Action Series New Edition
- Siyavula

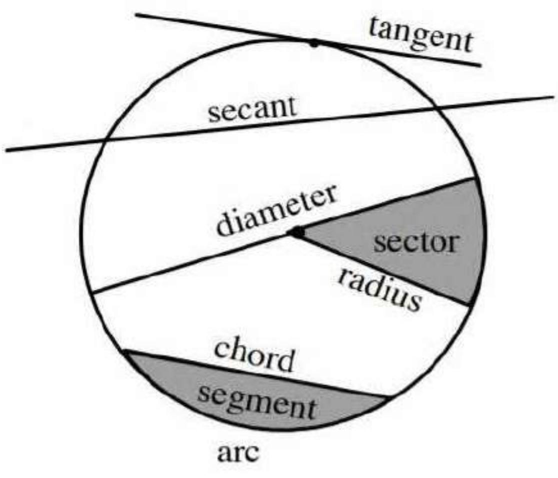
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Proving congruency and reasons for congruency
Failing to write reasons correctly/ write wrong reasons, sometimes do not understand reasons

METHODOLOGY

Terminology
The following terms are regularly used when referring to circles:

- **Arc** — a portion of the circumference of a circle.
- **Chord** — a straight line joining the ends of an arc.
- **Circumference** — the perimeter or boundary line of a circle.
- **Radius** (r) — any line from the centre of the circle to a point on the circumference.
- **Diameter** — a **special chord** that passes through the centre of the circle.
- **Segment** — part of the circle that is cut off by a chord. A chord divides a circle into two segments.
- **Tangent** — a line that touches a circle at only one point on the circumference.
- **Secant -----** a line that cuts the circle in two places.



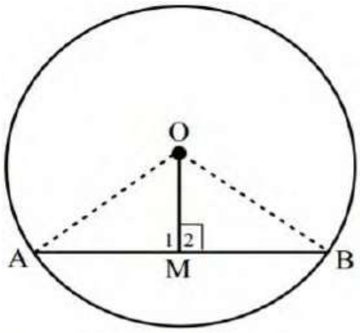
A **theorem** is a hypothesis (proposition) that can be shown to be true by accepted mathematical operations and arguments.
A **proof** is the process of showing a theorem to be correct.
The **converse** of a theorem is the reverse of the hypothesis and the conclusion.

Hint to educators: Before proving each theorem use the investigative approach to investigate the statement of the theorem

Theorem:

1. The line drawn from the centre of a circle **perpendicular** to a chord **bisects the chord.**

Given: Circle with centre O with OM ⊥ AB. AB is a chord
Required to prove (RTP): AM = MB
Proof:



Join OA and OB
In Δ OAM and Δ OBM

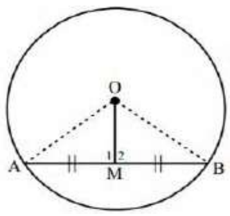
1. $OA = OB$	Radii
2. $\hat{M}_1 = \hat{M}_2 = 90°$	Given
3. $OA = OB$	Common
$\therefore \Delta OAM \equiv \Delta OBM$	HRS
$\therefore$ AM = MB	

Reason: **Perpendicular from centre to chord**

Converse:

2. The line drawn from the centre of a circle to the **midpoint** of a chord **is perpendicular** to the
chord.

Given: Circle with centre O with AM =MB. AB is a chord
Required to prove (RTP): AM ⊥ MB
Proof:



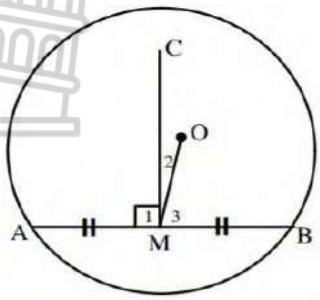
Join OA and OB
In Δ OAM and Δ OBM

1. OA = OB	Radii
2. AM = MB	Given
3. OM = OM	Common
$\therefore \Delta OAM \equiv \Delta OBM$	SSS
$\therefore O\hat{P}A = O\hat{P}B$	
$O\hat{P}A + O\hat{P}B = 180°$	∠s in a str. line
$O\hat{P}A = O\hat{P}B = 90°$	
$\therefore$ AM ⊥ MB	

Reason: **Line from centre midpoint of chord**

3. The **perpendicular bisector** of a chord **passes through the centre** of the circle.

Given: Chord AB of circle with centre O
CM ⊥ AB cutting AB at M, AM = BM
Required to prove (RTP): O lies on CM
Proof: Suppose that the centre of the circle does not lie on CM. Draw OM.



$\hat{M}_1 = 90°$ Given

But $\hat{M}_3 = 90°$ Line from centre to midpoint of chord

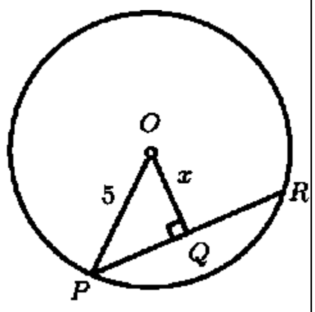
This is impossible unless $\hat{M}_2 = 0°$

$\therefore$ O lies on CM

Reason: **Perpendicular bisector of chord**

Examples:

1. Given a circle with centre O with PR = 8 units. Determine the value of x.



PQ = QR = 4 units …. **Perpendicular from centre to chord**

In ΔOPQ

$OP^2 = OQ^2 + PQ^2$ (Pythagoras Theorem)

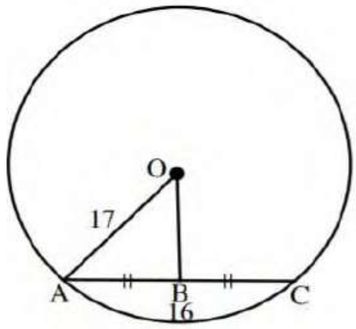
$5^2 = x^2 + 4^2$

$25 - 16 = x^2$

$9 = x^2$

$\therefore x = 3$

2. O is the centre. AC = 16, AB = BC and OA = 17. Calculate the length of OB.



AB= BC = 8 **(Line from centre midpoint of chord)**

$OB^2 = AO^2 - AB^2$ (Pythagoras Theorem)

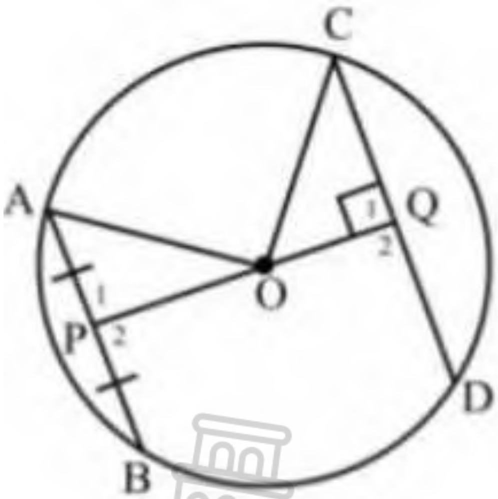
$= 17^2 - 8^2$

$= 289 - 64$

$OB^2 = 225$

$OB = 15$

3. Given the circle with centre O. AB = 6 cm, CD = 8 cm and the radius is 5 cm.



(a) Determine the length of:

1) OP

AP = 3 cm (Line from centre to midpoint of chord)

OA = 5 cm (Radius)

$OP^2 = OA^2 - AP^2$ (Pythagoras Theorem)

$OP^2 = 5^2 - 3^2 = 16$

$OP = 4cm$

2) PQ

CQ = 4 cm (Line from centre to midpoint of chord)

OC = 5 cm (Radius)

$OQ^2 = OC^2 - CQ^2$ (Pythagoras Theorem)

$OQ^2 = 5^2 - 4^2 = 9$

$OQ = 3cm$

$PQ = OP + OQ$

$PQ = 4 + 3 = 7cm$

(a) Explain why $AB \parallel CD$.

$\hat{P}_1 = 90°$ (Line from centre to midpoint of chord)

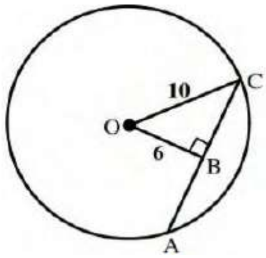
$\hat{Q}_1 = 90°$ (given)

$\hat{P}_1 + \hat{Q}_1 = 180°$ (both angles are equal to 90° each

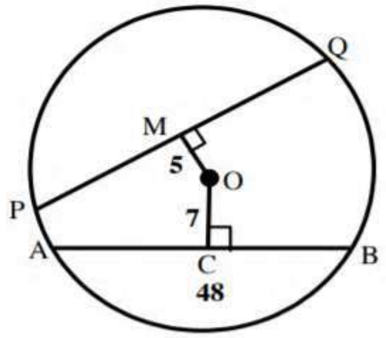
$\therefore AB \parallel CD$ (co-interior angles supplementary)

ACTIVITIES/ ASSESSMENT

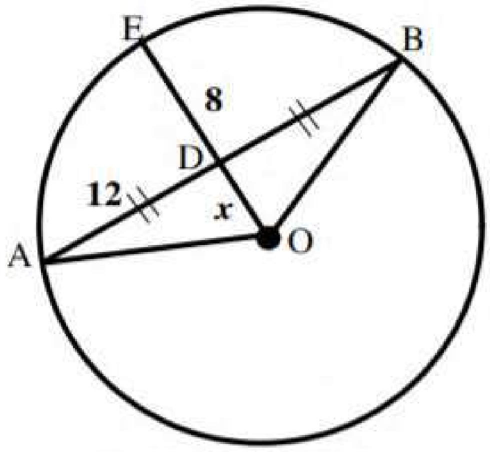
9.1 O is the centre. OB = 6, OC = 10 and $OB \perp AC$.
Calculate the length of AC.



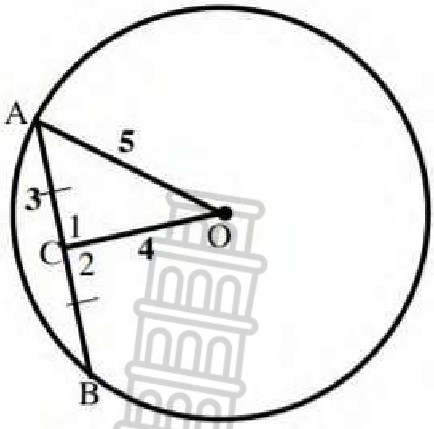
9.2 O is the centre of the circle. PQ and AB are both chords of the circle with $OM \perp PQ$ and $OC \perp AB$. OM = 5cm, OC = 7cm and AB = 48cm. Calculate the length of:
(a) the radius of the circle.
(b) PQ.



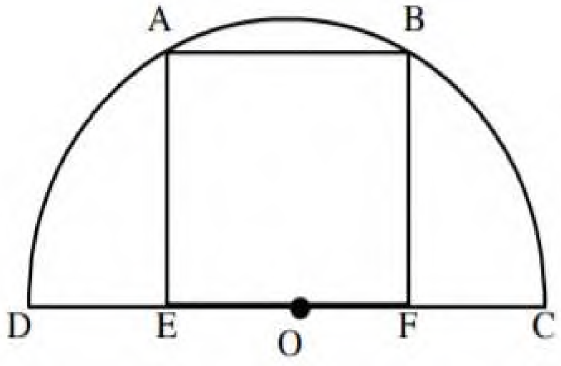
9.3 AB is a chord of circle centre O. OE bisects AB.
AD = 12cm, ED = 8cm and OD = x. $OD = x$
(a) Determine the radius OB in terms of x.
(b) Hence, calculate the length of the radius OB.



9.4 AB is a chord of the circle. AC = CB, and the length of the radius is 5 units. AC = 3 units and OC= 4 units.
(a) Show that $OC \perp AB$ and
(b) explain why OC passes through the centre O



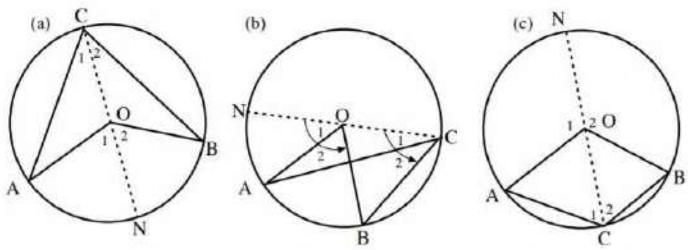
9.5 The radius of the semi-circle centre O is 5 cm.
A square is fitted into the semi -circle as shown in the diagram.
Calculate the area of the square.
(Hint: Let the length of the square equal x.)



TOPIC: EUCLIDEAN GEOMETRY					
LESSON 10					
Term	1	**Week**		**Grade**	11
Duration	1 hour	**Weighting**	50±3	**Date**	
Sub-topics	**Circle Geometry: Angles subtended by a chord/arc**				
RELATED CONCEPTS/ TERMS/VOCABULARY					
Subtends, arc, segment, chord, semi-circle Converse of a theorem, corollary					
PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE					
Exterior angle of a triangle					
RESOURCES					
• Mind Action Series • Mind Action Series New Edition • Siyavula • Via Afrika					
ERRORS/MISCONCEPTIONS/PROBLEM AREAS					
METHODOLOGY					

- **Arc** — a portion of the circumference of a circle.
- **Chord** — a straight line joining the ends of an arc.
- **Segment** — part of the circle that is cut off by a chord. A chord divides a circle into two segments.
- **Subtend** ---- side opposite an angle subtends that angle

➢ **Theorem:**
The angle that an **arc** of a circle **subtends at the centre** of the circle is **twice** the angle it **subtends at any point on the circumference**



Given: Circle with centre O. Arc AB subtends $A\hat{O}B$
At centre and $A\hat{C}B$ at the circumference.
Required to prove (RTP): $A\hat{O}B + 2A\hat{C}B$
Proof: Join CO and produce to N

$\hat{O}_1 = \hat{C}_1 + \hat{A}$ (Ext $\angle$ of Δ OAB)

But $\hat{C}_1 = \hat{A}$ (AO = OC, Radii)

$\hat{O}_1 = 2\hat{C}_1$

Similarly, in Δ OCB: $\hat{O}_1 = 2\hat{C}_2$

Diagram (a) and (c)

$\hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$

$\hat{O}_1 + \hat{O}_2 = 2(\hat{C}_1 + \hat{C}_2)$

$\therefore A\hat{O}B = 2A\hat{C}B$

Diagram (b)

$\hat{O}_2 - \hat{O}_1 = 2\hat{C}_2 - 2\hat{C}_1$

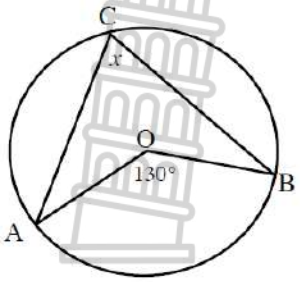
$\hat{O}_2 - \hat{O}_1 = 2(\hat{C}_2 - \hat{C}_1)$

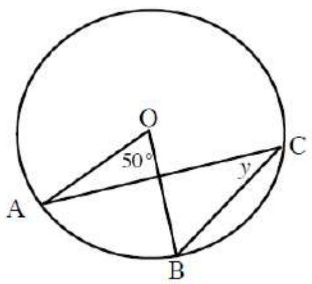
$\therefore A\hat{O}B = 2A\hat{C}B$

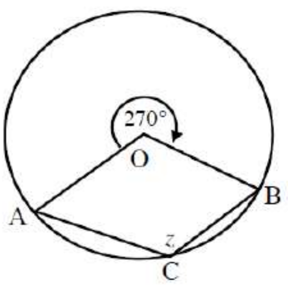
Reason: $\angle$ **at centre = 2 ×** $\angle$ **at circum**

Examples:

1. O is the centre of the circle, determine x, y and z





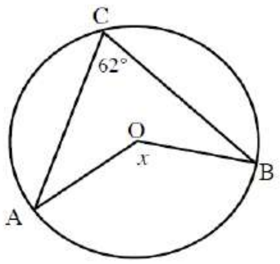


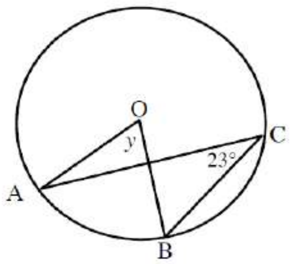
$x = 65°$ ∠ at centre = 2 x ∠ at circumf

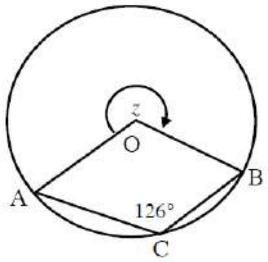
$y = 25°$ ∠ at centre = 2 x ∠ at circumf

$z = 135°$ ∠ at centre = 2 x ∠ at circumf

2. O is the centre of the circle, determine x,y and z





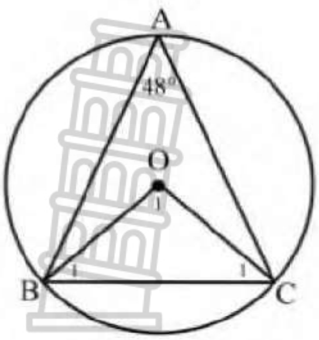


$x = 124°$ ∠ at centre = 2 x ∠ at circumf

$y = 46°$ ∠ at centre = 2 x ∠ at circumf

$z = 252°$ ∠ at centre = 2 x ∠ at circumf

3. In the given diagram, O is the centre of the circle. $B\hat{A}C = 48°$
Determine with reasons the size of:



(a) $\hat{O}_1$

$\hat{O}_1 = 2\hat{A} = 96°$ (∠ at centre = 2 x ∠ at circumf)

(b) $\hat{B}_1$

$\hat{B}_1 = \hat{C}_1$ (∠s opposite = sides)

$2\hat{B}_1 + 96° = 180°$ (∠s of Δ)

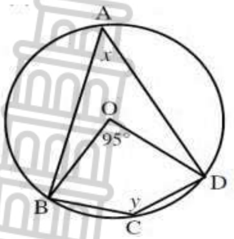
$2\hat{B}_1 = 84°$

$\hat{B}_1 = 42°$

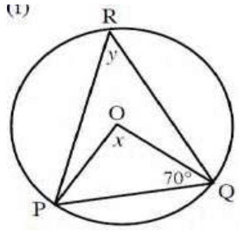
ACTIVITIES/ ASSESSMENT

10.1 Determine, with reasons, the value of the unknowns. O is the centre of the circle.

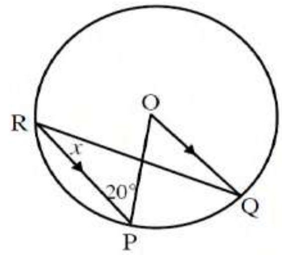
(a)



(b)



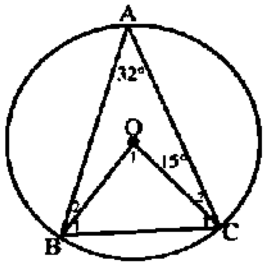
(c)



10.2 In the diagram alongside, O is the centre of the circle.

Determine, with reasons, the size of:

(a) $\hat{B}_1$ (b) $\hat{B}_2$



TOPIC: EUCLIDEAN GEOMETRY					
LESSON 11					
Term	1	**Week**		**Grade**	11
Duration	1 hour	**Weighting**	50 ± 3	**Date**	
Sub-topics	**Circle Geometry: Angles subtended by a chord/arc**				

RELATED CONCEPTS/ TERMS/VOCABULARY

Subtends, arc, segment, chord, semi-circle Converse of a theorem, corollary

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Exterior angle of a triangle

RESOURCES

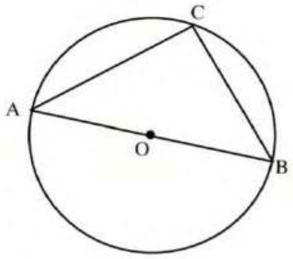
- Mind action Series
- Mind Action Series New Edition
- Siyavula
- Via Afrika

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

- **Theorem:**
  The angle **subtended by a diameter** at the circumference of a circle is a **right angle.**

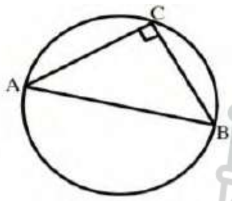
If AB is a diameter, the $\hat{C}_1 = 90°$



Reason: $\angle$ **in semi-circle**

Converse: Reverse of a theorem
If the angle subtended by a chord at a point on the circle is 90°, then the chord is a diameter

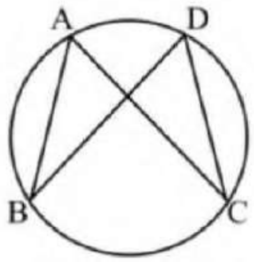
I, $\hat{C}_1 = 90°$ then AB is a diameter



Reason: **Chord subtends 90°**

- **Theorem:**
  Angles subtended by a chord/arc at the circumference of a circle on the same side of the chord are equal; or angles in the same segment of a circle are equal.

If AD subtends $\hat{B}$ *and* $\hat{C}$,
then $\hat{B} = \hat{C}$

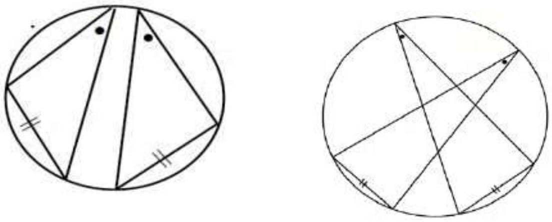


Reason: $\angle$ s **in same segment**

Corollaries

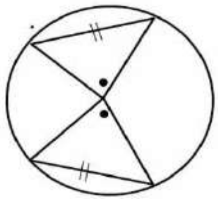
Corollary is a true statement that is a simple deduction from a theorem.

1. Equal chords (arcs) of a circle subtend equal angles at the circumference of a circle.



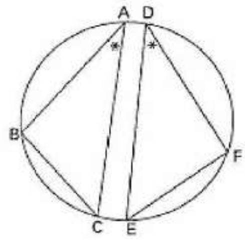
Reason:
Equal chords subt equal ∠s at circumf

2. Equal chords (arcs) of a circle subtend equal angles at the centre of the circle.



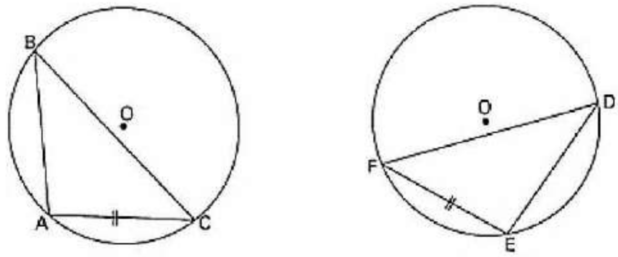
Reason:
Equal chords subt equal ∠ at centre

3. Chords (arcs) of a circle are equal when they subtend equal angles at the circumference or at the centre of the circle.



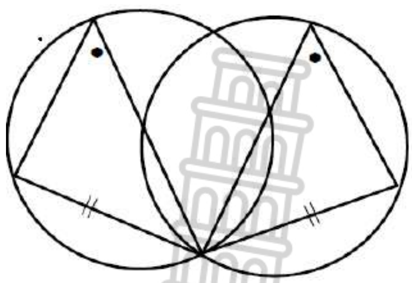
Reason:
Equal ∠s subt by equal chords

4. Equal chords (arcs) in different circles with equal radii (diameter) subtend equal angles on the circles.



Reason:
Equal chord, equal radii, equal ∠s

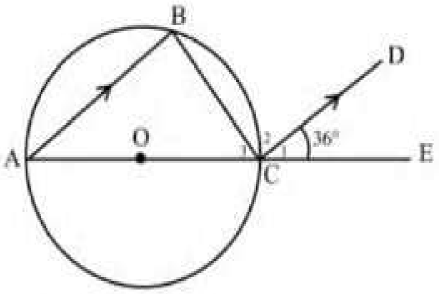
Equal chords of equal circles subtend equal equal angles at the circumference.



Reason:
Equal chords, equal circles, equal ∠s

Examples:

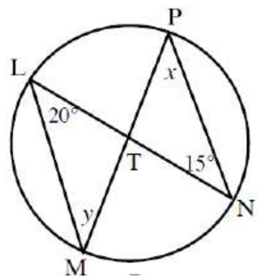
1. In the diagram alongside, O is the centre of the circle.
$AB \parallel CD$ and $\hat{C}_1 = 36°$
Determine with reasons, the size of:
(a) $\hat{C}_3$ (b) $\hat{A}$



(a) $\hat{B} = 90°$ (∠ **in semi-circle)**
$\hat{C}_2 = \hat{B} = 90°$ (alternate ∠s, $AB \parallel CD$)
$\hat{C}_3 = 180° - 90° - 36°$ (∠s on a straight line)
$\hat{C}_3 = 54°$

(b) $\hat{A} = 36°$ (∠s of Δ)

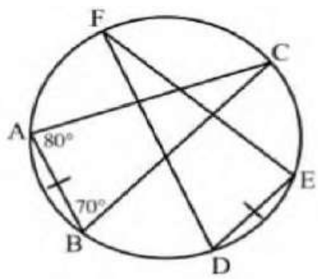
2. Calculate with reasons, the value of the unknowns:



$x = 20°$ (∠s **in same segment** -subtended by MN)
$y = 15°$ (∠s **in same segment** -subtended by LP)

3. In the diagram alongside, AB = DE
$\hat{A} = 80°$ and $\hat{B} = 70°$

Determine, with reasons, the size of $\hat{F}$

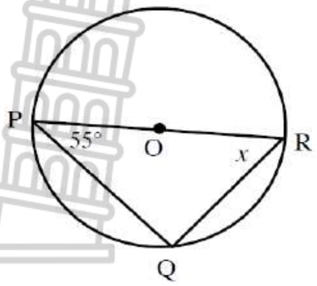


$\hat{C} = 30°$ (∠s of Δ)
$\hat{F} = \hat{C} = 30°$ (equal chords sub. Equal ∠s)

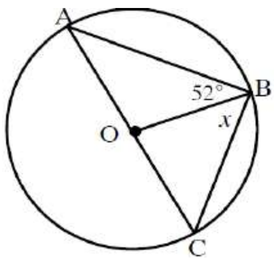
ACTIVITIES/ ASSESSMENT

11.1 Determine, with reasons, the value of the unknowns. O is the centre of the circle.

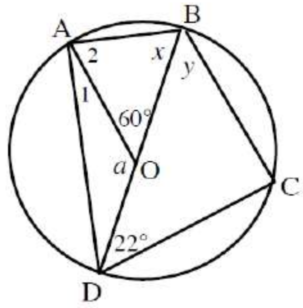
(a)



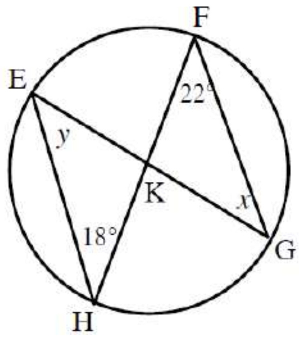
(b)



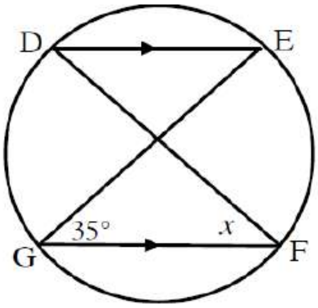
(c)



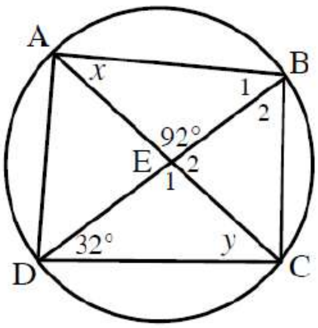
(d)



(e)

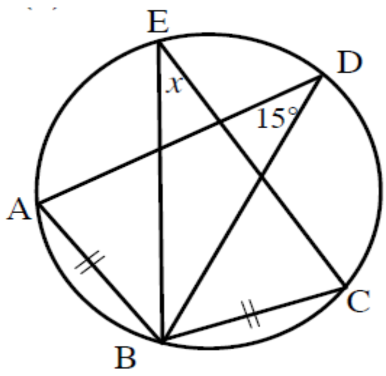


(f)

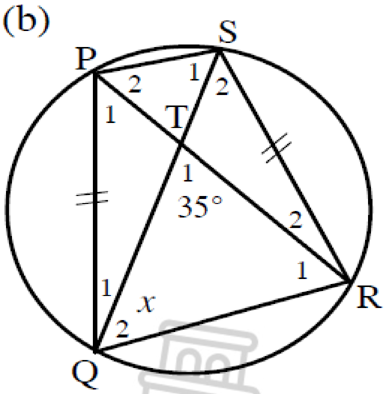


11.2 Calculate the value of the unknown.

(a)



(b)



TOPIC: EUCLIDEAN GEOMETRY					
LESSON 12					
Term	1	**Week no.**		**Grade**	11
Duration	1 hour	**Weighting**	50 ± 3	**Date**	
Sub-topics	**Cyclic Quadrilateral**				

RELATED CONCEPTS/ TERMS/VOCABULARY

Subtends, arc, segment, chord, semi-circle Converse of a theorem, corollary

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Quadrilateral, exterior angle, supplementary

RESOURCES

- Mind action Series
- Mind Action Series New Edition
- Siyavula

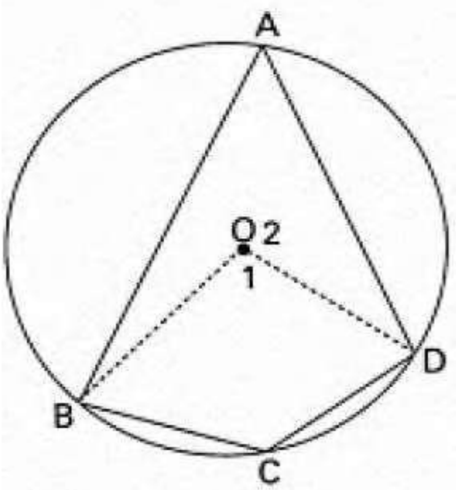
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Treating opposite angles of a cyclic quadrilateral as opposite angles of a parallelogram

METHODOLOGY

Cyclic quadrilateral is a quadrilateral with **all vertices** (corner) lying on/touching the circumference

Theorem: The opposite angles of a cyclic quadrilateral are supplementary.



Given: Circle with centre O. ABCD is a cyclic quadrilateral
Required to prove: $\hat{A} + \hat{C} = 180°$ and $A\hat{B}C + A\hat{D}C = 180°$

Proof: Join OB and OD

$\hat{O}_1 = 2\hat{A}$ (**$\angle$ at centre = 2 x $\angle$ at circumf)**

$\hat{O}_2 = 2\hat{C}$ (**$\angle$ at centre =2 x $\angle$ at circumf)**

$\hat{O}_1 + \hat{O}_2 = 360°$ ($\angle$s **round a point)**

$2\hat{A}+2\hat{C}=360°$

$2(\hat{A}+\hat{C}) = 360°$

$\therefore \hat{A}+\hat{C} = 180°$

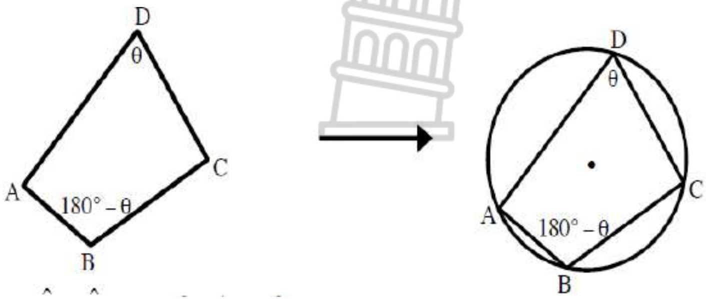
Similarly, by joining AO and OC, it can be proven that $A\hat{B}C + A\hat{D}C = 180°$

Reason: Opp. $\angle$s of cyclic quad.

Converse: IF the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

If $\hat{A}+\hat{C} = 180°$ and $\hat{B} + \hat{D} = 180°$, then ABCD is a cyclic quadrilateral.

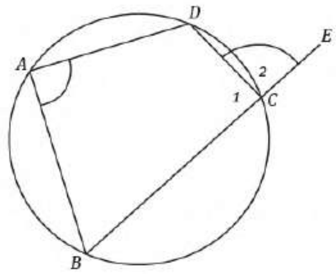
Reason: Opp. $\angle$s of cyclic quad suppl.



Theorem: An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

If BC is produced to E, then $\hat{C}_2 = \hat{A}$

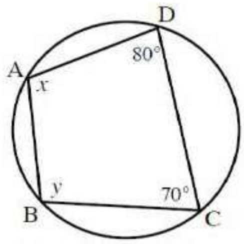
Reason: Ext. $\angle$ of cyclic = int opp. $\angle$



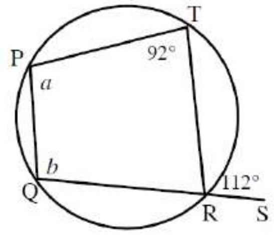
Examples:

1. Calculate, with reasons, the value of the unknowns

(a)



(b)



$x = 110°$ (Opp. $\angle$s of cyclic quad)

$y = 100°$ (Opp. $\angle$s of cyclic quad)

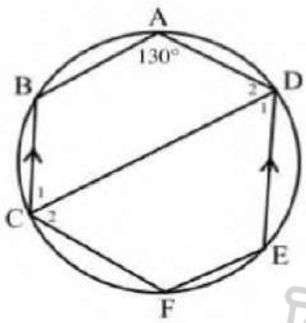
$a = 112°$ (Ext. $\angle$ of cyclic = int opp. $\angle$s)

$b = 88°$ (Opp. $\angle$s of cyclic)

2. In the following diagram, $BC \parallel ED$ and $\hat{A} = 130°$

Determine, with reasons, the size of

(a) $\hat{C}_1$ (b) $\hat{F}$



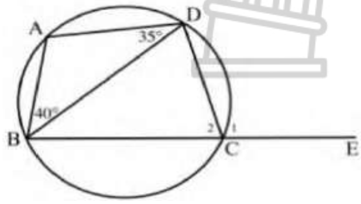
(a) $\hat{C}_1 = 50°$ (Opp. $\angle$s of cyclic quad)

(b) $\hat{D}_1 = \hat{C}_1 = 50°$ (alt. $\angle$s, $BC \parallel ED$)

$\therefore \hat{F} = 130°$ (Opp. $\angle$s of cyclic quad)

3. In the given diagram, $A\hat{B}D = 40°$ and $A\hat{D}B = 35°$

Determine, with reasons, the size of $\hat{C}_1$



$\hat{A} = 105°$ ($\angle$s of Δ)

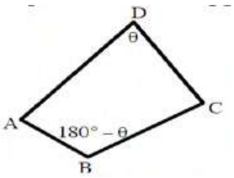
$\hat{C}_1 = \hat{A} = 105°$ (Ext. $\angle$ of cyclic = int opp. $\angle$)

PROVING THAT A QUADRILATERAL IS CYCLIC

- Prove that the opposite angles are supplementary

If $\hat{A}+\hat{C}=180°$ and $\hat{B}+\hat{D}=180°$, then

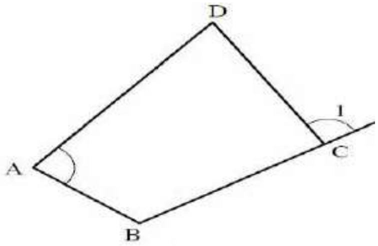
ABCD is a cyclic quadrilateral **Opp. ∠s supp**



- Prove that an exterior angle is equal to the interior opposite angle

If $\hat{A}=\hat{C}_1$ then ABCD is a cyclic quadrilateral

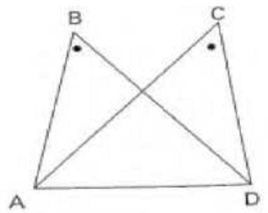
Ext. ∠ = int opp. ∠



- Prove that two points subtend equal angles at two other points on the same side.

If $\hat{B}=\hat{C}$, then A, B, C and D are concyclic
i.e., ABCD is a cyclic quadrilateral

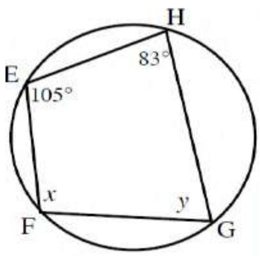
Two points subtend equal ∠s on the same side



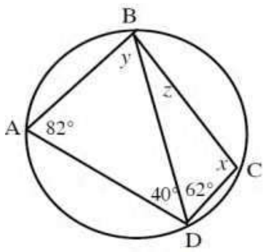
ACTIVITIES/ ASSESSMENT

12.1 Determine, with reasons, the value of the unknown

(a)



(b)



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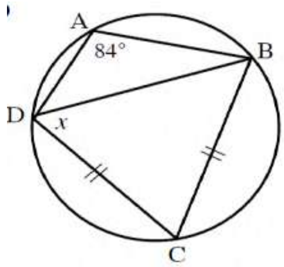

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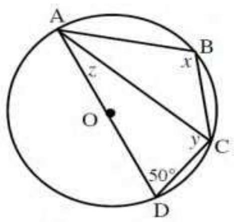

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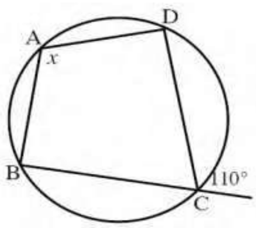

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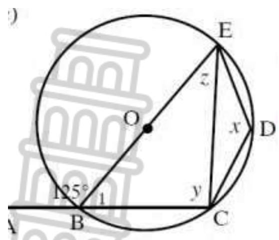
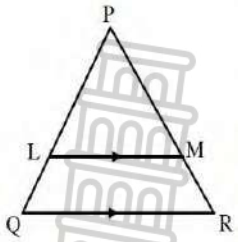


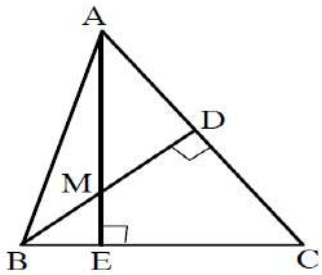
12.2 Show that LMRQ is a cyclic quadrilateral

5.3 Write down with reasons, two cyclic

if PQ = PR and LM || QR



quadrilateral in the diagram below.



TOPIC: EUCLIDEAN GEOMETRY					
LESSON 13					
Term	1	**Week no.**		**Grade**	11
Duration	1 hour	**Weighting**	50 ± 3	**Date**	
Sub-topics	**Tangents to the circle**				

RELATED CONCEPTS/ TERMS/VOCABULARY

Tangent

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Congruency, perpendicular

RESOURCES

- Mind action Series
- Mind Action Series New Edition
- Siyavula
- Mathematics up to date

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Understanding the term tangent in relation to the only point that touches the curve.

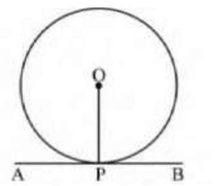
METHODOLOGY

A **tangent** is a straight line that touches the circle at only one point.

Theorem:
The **radius** of a circle is **perpendicular** to the **tangent** at the point of contact.

APB is a tangent to the circle with centre O.
OP is a radius drawn to P.
Then OP $\perp$ APB

Reason: **rad $\perp$ tan**



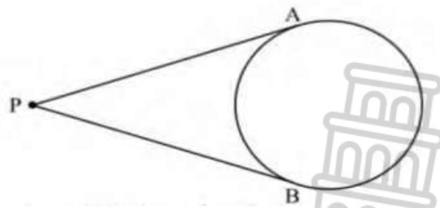
Theorem:

Two tangents drawn to a circle from the **same point** outside the circle are **equal** in length

If tangents PA and PB are from P,

then PA = PB

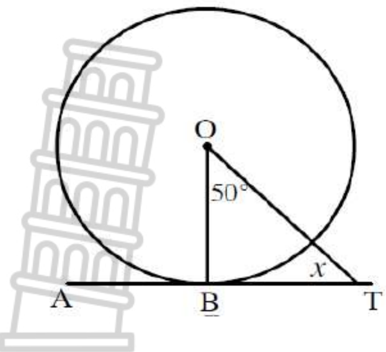
Reason: **tangents from same point**



Examples:

O is the centre of the circle. Determine, with reasons. the value of the unknown.

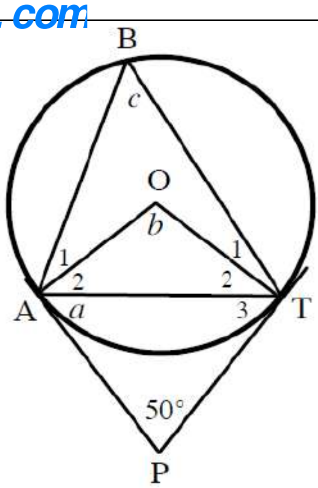
(a)



$O\hat{B}T = 90°$ (rad ⊥ tan)

$\hat{T} = 40°$ (∠s of Δ)

(b)



$a = \hat{T}_3$ (tans from same point)

$a + \hat{T}_3 = 130°$ (∠s of Δ)

$a = 65°$

$\hat{A} = 25°$ (rad ⊥ tan)

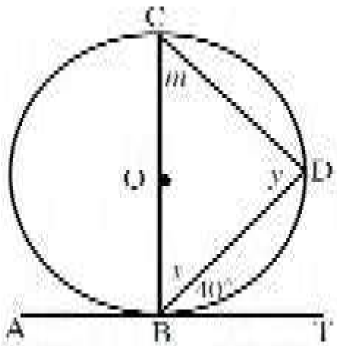
$b = 130°$ (∠s of Δ)

$c = 65°$ (∠ at centre = 2 x ∠ at circum)

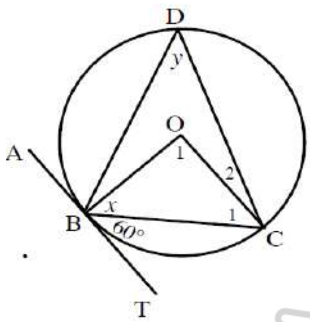
ACTIVITIES/ ASSESSMENT

13.1 O is the centre of the circle and ABT is a tangent of the circle . Determine, with reasons, the value of the unknowns.

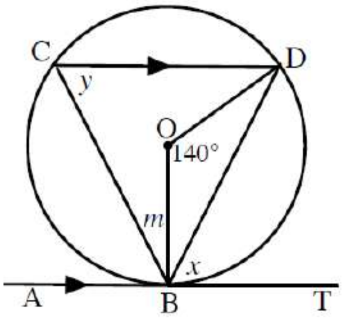
(a)



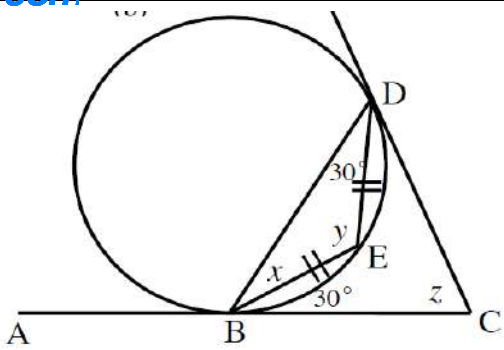
(b)



(c)



13.2 In the diagram below, DE = EB, $B\hat{D}E = C\hat{B}E = 30°$.
Determine, with reasons, the values of *x, y and z*



TOPIC: EUCLIDEAN GEOMETRY					
LESSON 14					
Term	1	**Week no.**		**Grade**	11
Duration	1 hour	**Weighting**	50 ± 3	**Date**	
Sub-topics	**Tangents to the circle**				

RELATED CONCEPTS/ TERMS/VOCABULARY

Tangent

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Congruency, perpendicular

RESOURCES

- Mind action Series
- Siyavula

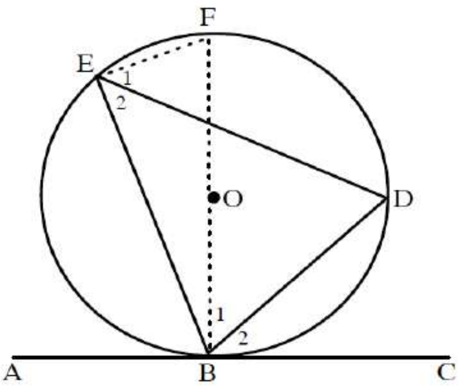
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Failing to identify the relevant angles i.e the one between the chord and the tangent and the one that is subtended by the same chord in the circumference.

METHODOLOGY

Theorem:

The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.



Given: Circle with centre O. Tangent ABC, chord BD

Required to prove: $C\hat{B}D = B\hat{E}D$ or $A\hat{B}E = B\hat{D}E$

Proof: Draw diameter BOF and join EF

$\hat{E}_1 + \hat{E}_2 = 90°$ ($\angle$ **in semi-circle)**

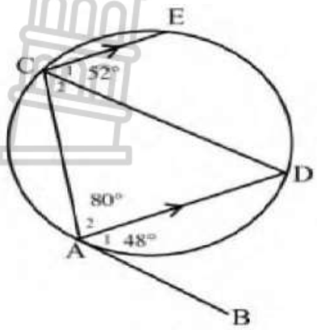
$\hat{B}_1 + \hat{B}_2 = 90°$ **(rad $\perp$ tan)**

$\hat{E}_1 = \hat{B}_1$ ($\angle$s in the same segment)

$\therefore \hat{E}_2 = \hat{B}_2$

Examples:

1. In the diagram



(a) Determine with reasons, the size of $\hat{D}$

(b) Prove that AB is a tangent to circle ACD

Solution:

(a) $\hat{D} = \hat{C}_1$ (Alt. $\angle$s, CE $\parallel$ AD)

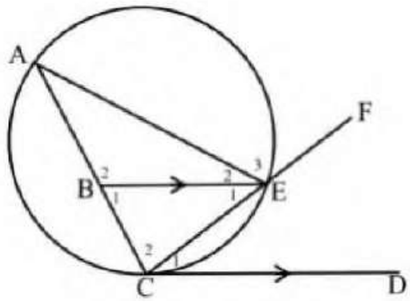
$\therefore \hat{D} = 52°$

(b) $\hat{C}_2 + 52° + 80° = 180°$ (sum of $\angle$s in Δ)

$\hat{C}_2 = 48°$

$\therefore$ AB is a tangent to circle ACD (converse tan-chord thm)

2. In the diagram $BE \parallel CD$ and CD is a tangent to the circle.



If $\hat{C}_1 = x$, name with reasons two other angles that are equal to x.

Solution:

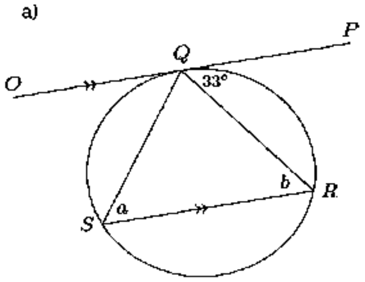
$\hat{E}_1 = x$ (Alt. $\angle$s, $CD \parallel BE$)

$\hat{A} = x$ (tan-chord thm)

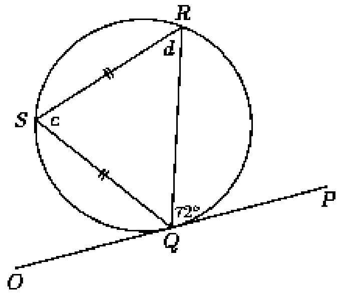
ACTIVITIES/ ASSESSMENT

14.1 Determine the values of the unknown letters, stating reasons.

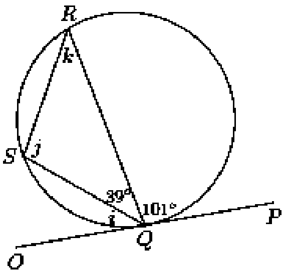
a)



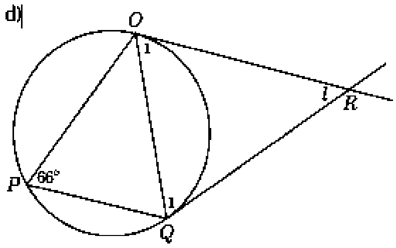
b)



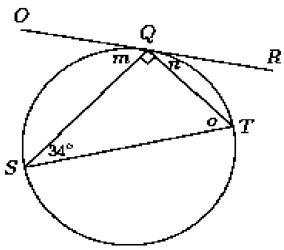
(c)



d)

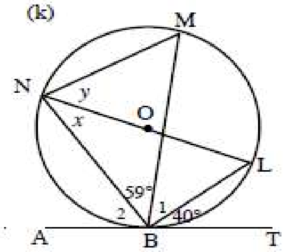


(e)

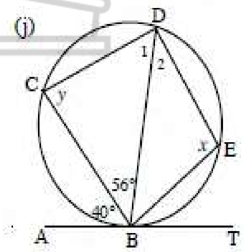


14.2 ABT is a tangent. Calculate the value of x and y.

(k)



(j)



O is the centre of the circle

TOPIC: EUCLIDEAN GEOMETRY					
LESSON 15					
Term	1	**Week no.**		**Grade**	11
Duration	1 hour	**Weighting**	50 ± 3	**Date**	
Sub-topics	**Circle Geometry problems and proofs of riders**				

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

All theorems, converses and corollaries

RESOURCES

- Mind action Series
- Siyavula Mathematics

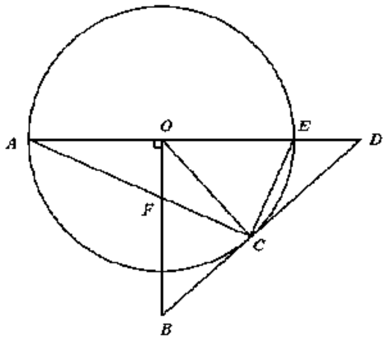
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Learners finding it difficult to apply different theorems in one rider.

METHODOLOGY

Examples:

1. BD is a tangent to the circle with centre O, with $BO \perp AD$.



Prove that:

(a) $CFDE$ is a cyclic quadrilateral

(b) $FB = BC$

(c) $A\hat{O}C = 2B\hat{F}C$

Solution:

(a) $BO \perp OD$ (given)

$\therefore F\hat{O}E = 90°$

$F\hat{C}E = 90°$ ($\angle$ in semi circle)

$\therefore CFOE$ is a cyclic quad (opp. $\angle$s suppl.)

(b) $B\hat{C}F = C\hat{E}O$ (tan-chord theo)

$B\hat{F}C = C\hat{E}O$ (ext. $\angle$ cyclic quad)

$\therefore B\hat{F}C = B\hat{C}F$

$\therefore FB = BC$ (lines opp. equal $\angle$ s)

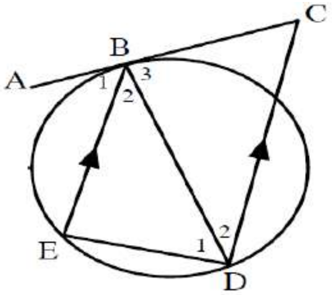
(c) $A\hat{O}C = 2A\hat{E}C$ ($\angle$ at centre = 2 $\angle$ at circum.)

and $A\hat{E}C = B\hat{F}C$ (ext. $\angle$ cyclic quad.)

$\therefore A\hat{O}C = 2B\hat{F}C$

2. ABC is a tangent to the circle BED. $BE \parallel CD$.

Prove that: $\hat{D}_1 = \hat{C}$



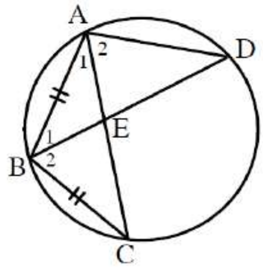
Solution:

$\hat{D}_1 = \hat{B}_1$ (tan-chord theo)

$\hat{C} = \hat{B}_1$ (corresp $\angle$s , $BE \parallel CD$)

$\therefore \hat{D}_1 = \hat{C}$

3. In the circle ABCD, AB=BC.
Prove that AB is a tangent to the circle AED in A
Solution:



$\hat{D} = \hat{C}$ (subt. By same chord)

$\hat{A}_1 = \hat{C}$ ($\angle$s opp. equal sides)

$\hat{D}_1 = \hat{A}_1$

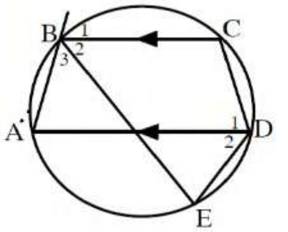
$\therefore$ AB is a tangent to circle AED (converse tan-chord thm)

ACTIVITIES/ ASSESSMENT

15.1 In circle EBCDE, BC and AD are parallel chords.
(a) Name two cyclic quadrilaterals
(b) Prove that:

(1) $\hat{B}_1 = \hat{E}$

(2) $\hat{D}_1 = \hat{A}$

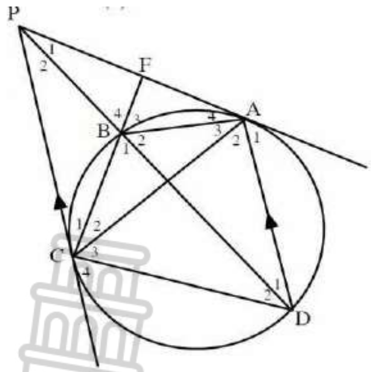


15.2 PA and Pc are tangents to the circle at A and C. $AD \parallel PC$, and PD cuts the circle B. CB is Produced to meet AP at. AB, AC and Dc are drawn.
Prove that:

(a) AC bisects $P\hat{A}D$

(b) $\hat{B}_1 = \hat{B}_3$

(c) $A\hat{P}C = A\hat{B}D$

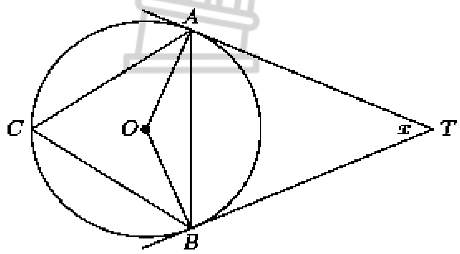


15.3 TA and TB are tangents to the circle with centre O.
C is a point on the circumference and $A\hat{T}B = x$.
Express the following in terms of x, giving reasons:

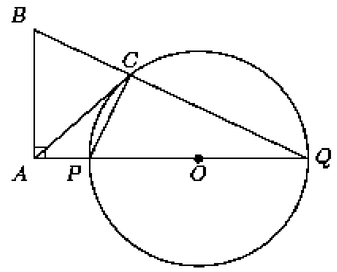
(a) $A\hat{B}T$

(b) $O\hat{B}A$

(c) $\hat{C}$



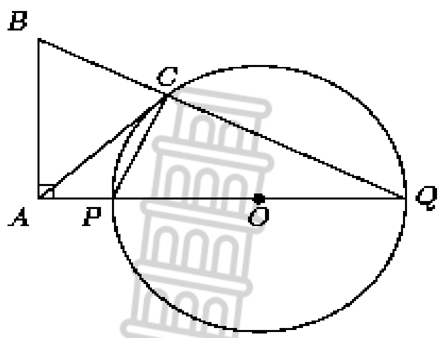
15.4 POQ is a diameter of the circle with centre O. QP is produced to A and AC is a tangent to the circle.
$BA \perp AQ$ and BCQ is a straight line.
Prove that:
(a) $P\hat{C}Q = B\hat{A}P$
(b) BAPC is a cyclic quadrilateral
(c) AB=AC



TOPIC: EUCLIDEAN GEOMETRY					
LESSON 16					
Term	1	**Week**		**Grade**	
Duration	1 hour	**Weighting**	50 ± 3	**Date**	
Sub-topics	**Circle Geometry problems and proofs of riders**				
RELATED CONCEPTS/ TERMS/VOCABULARY					
PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE					
All theorems, converses and corollaries					
RESOURCES					
• Mind action Series • Siyavula Mathematics					
ERRORS/MISCONCEPTIONS/PROBLEM AREAS					
Learners finding it difficult to apply different theorems in one rider.					
METHODOLOGY					

Example:

1. POQ is a diameter of the circle with centre O.
QP is produced to A and AC is a tangent to the circle.
$BA \perp AQ$ and BCQ is a straight line.
Prove that:
(a) $P\hat{C}Q = B\hat{A}P$
(b) BAPC is a cyclic quadrilateral
(c) AB = AC



Solution:

(a) $P\hat{C}Q = 90°$ ($\angle$ in a semi-circle)
$B\hat{A}Q = 90°$ (given)
$\therefore P\hat{C}Q = B\hat{A}P$

(b) $B\hat{A}Q = 90°$ (given)
$B\hat{C}P = 90°$ (sum of $\angle$ s in a straight line)
$\therefore$ BAPC is a cyclic quad. (converse Opp. $\angle$ s of a cyclic quad.)

(c) $\hat{B}+\hat{Q}+90° = 180°$ (sum of ∠s of Δ)

$\hat{B}+\hat{Q} = 90°$

$A\hat{C}P + B\hat{C}A + 90° = 180°$ (sum of ∠s in a straight line)

$A\hat{C}P + B\hat{C}A = 90°$

$A\hat{C}P = \hat{Q}$ (tan-chord thm)

$B\hat{C}A = \hat{Q}$

$\therefore B\hat{C}A = \hat{B}$

AB = AC (Equal ∠s subt by equal sides)

ACTIVITIES/ ASSESSMENT

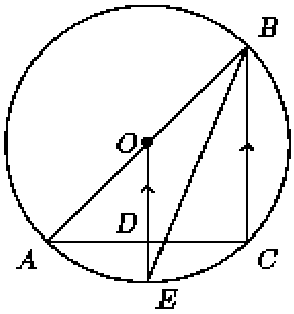
16.1 AOB is a diameter of the circle AECB with centre O.
$OE \parallel BC$ and cuts AC at D.

(a) Prove AD=DC

(b) Show that $A\hat{B}C$ is bisected by EB

(c) If $O\hat{E}B = x$ express $B\hat{A}C$ in terms of x

(d) Calculate the radius of the circle if AC=10cm and DE =1cm



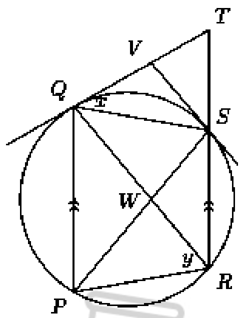
16.2 PQ and RS are chords of the circle and $PQ \parallel RS$.
The tangent to the circle at Q meets PS produced at T.
The tangent at S meets QT at V. QS and PR are drawn.
Let $T\hat{Q}S = x$ and $Q\hat{R}P = y$
Prove that:

(a) $\hat{C}_2 = \hat{C}_3$

(b) QVSW is a cyclic quadrilateral

(c) $Q\hat{P}S + \hat{T} = P\hat{R}T$

(d) W is the centre of the circle



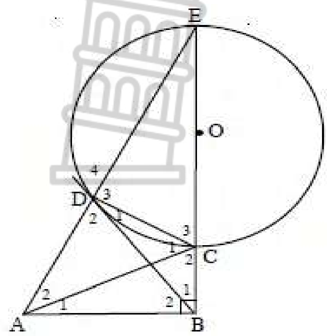
16.3 EC is a diameter of circle DEC. EC is produced to B.
BD is a tangent at D. ED is produced to A and
$AB \perp BE$.
Prove that:

(a) ABCD is a cyclic quadrilateral

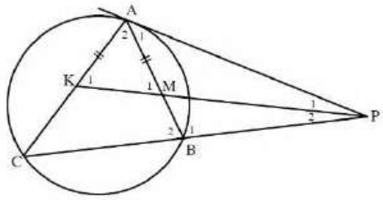
(b) $\hat{A}_1 = \hat{E}$

(c) ΔBDA is isosceles

(d) $\hat{C}_2 = \hat{C}_3$

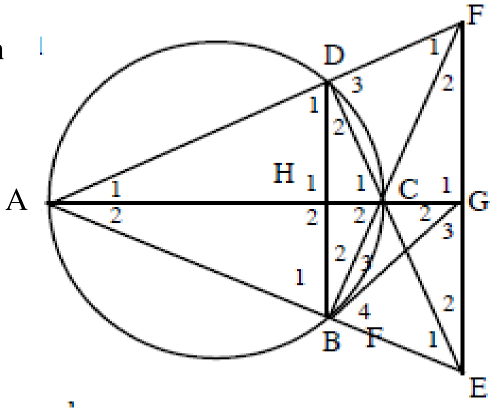


16.4 In the diagram, PA is a tangent to the circle at A. AC and AB are chords and PM is produces to K such that AK = AM. K and M lie on AC and AB respectively. Chord CB is Produced to P.



Prove that KP bisects $A\hat{P}C$

16.5 In the figure, ABCD is a cyclic quadrilateral with AB=AD and DC=BC. DC and BC, both produced meet AB and AD, both produced, at E and F respectively. AC produced meets FE at G with $\hat{G}_1 = 90°$



Prove that:

(a) AC is a diameter of the circle.

(b) DBEF is a cyclic quadrilateral

(c) BC bisects $D\hat{B}G$

TOPIC: TRIGONOMETRY					
LESSON 17					
Term	1	**Week**		**Grade**	11
Duration	2 hours	**Weighting**	±50 marks	**Date**	
Sub-topics	• Introduction to Trigonometric ratios and Trig Identities				

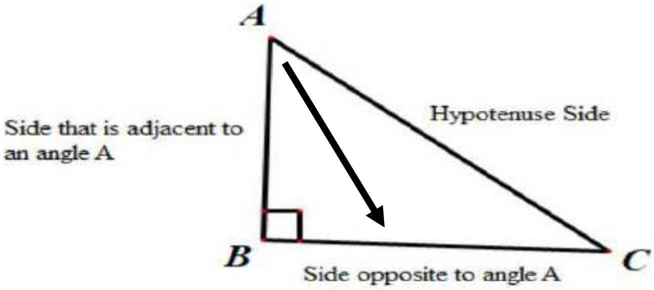
RELATED CONCEPTS/ TERMS/VOCABULARY

- Ratios(Grades 8 & 9)
- Lengths(Distances)
- Angles
- Triangles
- Trig Ratios such as: sin, cos & tan (Grade 10)

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

The trigonometric functions in relation to a right triangle are displayed in the figure below. For example, the triangle contains an angle *A*, and the ratio of the side opposite to *A* and the side opposite to the right angle (the hypotenuse) is called the sine of *A*, or sin *A*; the other trigonometry functions are defined similarly.

- $\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse Side}} = \frac{BC}{AC}$
- $\cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse Side}} = \frac{AB}{AC}$
- $\tan A = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{BC}{AB}$



Trigonometric functions are used in obtaining unknown angles and distances from known or measured angles in geometric figures.

Trigonometry developed from a need to compute angles and distances in such fields as astronomy, mapmaking, surveying, and artillery range finding.

RESOURCES

- Maths Set (Instruments)
- Scientific Calculators
- Pen / Pencils

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Language barrier
- Failure to relate technical world with trigonometry
- Failure to apply fractions correctly, viz.… sin, cos & tan

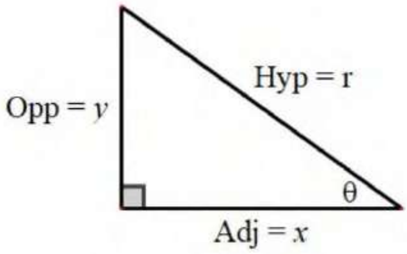
METHODOLOGY

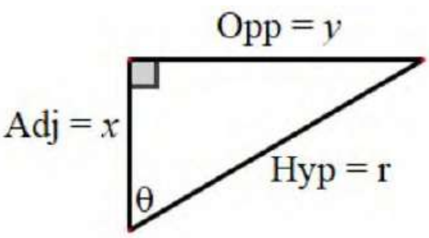
INTRODUCTION TO TRIG RATIOS(in terms of x, y and r)

Naming of sides in a right-angled triangle with respect to given angles.

TRIG RATIOS:

- $\sin\theta = \frac{y}{r}$
- $\cos\theta = \frac{x}{r}$
- $\tan\theta = \frac{y}{x}$





SQUARE IDENTITY:

- $\sin^2 x + \cos^2 x = 1$

From the above, it can be deduced that:

- $\sin^2 x = 1 - \cos^2 x$
- $\cos^2 x = 1 - \sin^2 x$

DERIVING IDENTITIES (Using *x*, *y* and r)

1. $\sin^2 x + \cos^2 x$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{x^2 + y^2}{r^2}$ NB... $(x^2 + y^2 = r^2)$

$= \frac{r^2}{r^2}$

$= 1$

$\therefore \sin^2 x + \cos^2 x = 1$

2. $\frac{\sin\theta}{\cos\theta}$

$= \frac{y}{r} \div \frac{x}{r}$

$= \frac{y}{x}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$

ACTIVITIES / ASSESSMENTS			
Simplify as far as possible:			
17.1	$\frac{1-\sin^2 x}{\cos x}$	17.2	$\frac{\cos^2\beta-1}{1-\sin^2\beta}$
17.3	$\frac{1}{\sin^2 x}-\frac{\cos^2 x}{\sin^2 x}$	17.4	$\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}$
17.5	$\frac{\sin^3\theta+\sin\theta.\cos^2\theta}{\cos\theta}$		

TOPIC: TRIGONOMETRY					
LESSON 18					
Term	1	**Week**		**Grade**	11
Duration	1 Hour	**Weighting**	±50 marks	**Date**	
Sub-topics	• APPLICATION OF TRIGONOMETRIC RATIOS				

RELATED CONCEPTS/ TERMS/VOCABULARY

- Ratios(Grades 8 & 9)
- Angles
- Triangles
- Trig Ratios such as: sin, cos & tan (Grade 10)

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Trig. Ratios
- Pythagoras theorem

RESOURCES

- Maths Set (Instruments)
- Scientific Calculators
- Pen / Pencils

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Language barrier
- Failure to relate technical world with trigonometry
- Failure to apply fractions correctly, viz.… sin, cos & tan

METHODOLOGY

SIGNS OF TRIGONOMETRIC RARIOS IN ALL 4 QUADRANTS

2nd QUADRANT
Only SIN is +ve

$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

$\tan\theta = \frac{y}{x} = \frac{+}{-} = -$

1st QUADRANT
ALL RATIOS are +ve

$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$

$\tan\theta = \frac{y}{x} = \frac{+}{+} = +$

$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

$\tan\theta = \frac{y}{x} = \frac{+}{+} = +$

3rd QUADRANT
Only TAN is +ve

$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$

$\tan\theta = \frac{y}{x} = \frac{-}{+} = -$

4th QUADRANT
ONLY COS is +ve

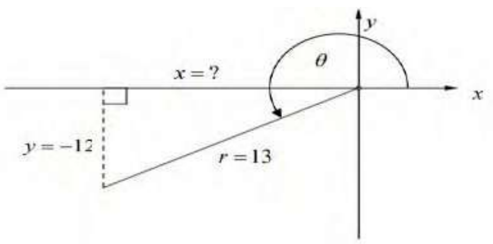
WORKED EXAMPLE

If $\sin\theta = -\frac{12}{13}$ and $90^\circ \leq \theta \leq 270^\circ$, Determine the values of the following:

1.	$\frac{\sin\theta}{\cos\theta}$	2.	$\cos^2\theta$	3.	$1-\sin^2\theta$

When **answering this question**, you need to define your trig ratio. Like $\sin\theta = -\frac{12}{13} = \frac{y}{r}$. Then you will know $y = -12$ and $r = 13$, r will never be negative, then the negative sign will be taken by y.

Sine is negative in the 3rd and 4th quadrants. $90^\circ \leq \theta \leq 270^\circ$ is an angle in between 2nd and 3rd quadrants. To know which quadrant from the two conditions, we must choose the quadrant that satisfies both conditions. Hence the 3rd quadrant.



$x^2 + y^2 = r^2$

$x^2 = r^2 - y^2$

$x = -5$
(in the 3rd quadrant)

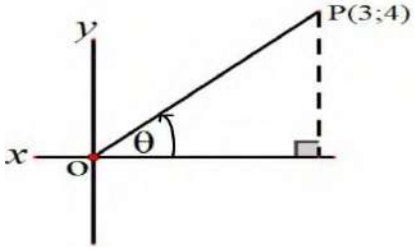
1. $\frac{\sin\theta}{\cos\theta} = \frac{-12}{13} \div \frac{-5}{13} = \frac{12}{5}$

2. $\cos^2\theta = \left(\frac{-5}{13}\right)^2 = \frac{25}{169}$

3. $1 - \sin^2\theta = 1 - \left(\frac{-12}{13}\right)^2 = \frac{25}{169}$

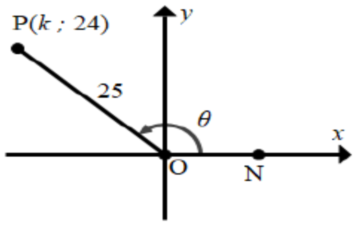
ACTIVITIES/ASSESSMENTS

18.1 **Complete this exercise without using a calculator:**
P(3;4) is a point in the Cartesian plane.
OP makes an angle θ with the positive x-axis.
Determine:



a) OP
b) $\sin\theta$

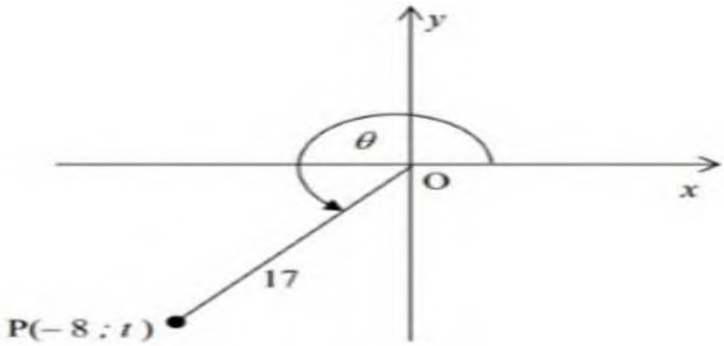
18.2 **In the diagram alongside, P(k ; 24) is a point in the second quadrant such that:**
OP = 25 units. N is a point on the positive x-axis and $P\hat{O}N = \theta$.



WITHOUT calculating the size of θ, determine the value of the following:

a) k
b) $\tan\theta$
c) $\sin\alpha$ if $\theta + \alpha = 360°$
d) $\cos^2\theta - \sin^2\alpha$

18.3 **In the diagram below, P(-8 ; t) is a point in the Cartesian plane such that** $OP = 17$ **units and reflex** $X\hat{O}P = \theta$.



18.3.1 Calculate the value of t.
18.3.2 Determine the value of each of the following WITHOUT using a calculator:

a) $\cos(-\theta)$
b) $1 - \sin\theta$

18.4 **If** $\sin\alpha = \frac{3}{5}$ **with** $\alpha \in [90°;270°]$ **and** $\cos\beta = -\frac{12}{13}$ **with** $\beta \in [0°;180°]$, **calculate the aid of the diagram the value of** $\cos\alpha + \tan\beta$.

18.5 **If** $\sin A = \frac{3}{4}$ **and** $A + B = 90°$ **where A and B are acute angles,**
Evaluate:
$$\frac{\cos A + \sin B}{\tan A}$$

18.6 **If** $5\sin\theta = -3\cos\theta$ **and** $\theta \in [90°;270°]$ **calculate with an aid of the diagram:**

a) $\sin(-1440° + \theta)$

b) $\cos^2\theta - \sin^2\theta$

18.7 **If** $sin16° = k$ **, WITHOUT using a alculator, express the following in terms of k:**

a) $tan16°$

b) $sin106°$

c) $sin^2 196 + cos^2 376°$

18.8 **If** $q\sin 61° = p$**, express the following in terms of p and q.**

a) $\cos 151°$

b) $\dfrac{\sin 331°}{\tan 29°}$

c) **Calculate without using a calculator the value of:**

$\dfrac{\tan 209°.\sin 119°}{\cos 61°}$

18.9 **If** $\tan 202° = t$ **write the following in terms of t.**

a) $\tan(-202°)$

b) $\cos 518°$

c) $\sin 338°$

d) $\dfrac{\cos 68°}{\cos 22°}$

e) $\dfrac{\cos(-202°)}{\tan 22°}$

18.10 **If** $\sin 40° = p$ **write the following in terms of p.**

a) $\sin 50°$

b) $\sin 140°$

c) $\cos 50°$

d) $\sin(-40°)$

e) $\tan 320°$

18.11 **Given:** $\sin 28° = p,$ **determine the following in terms of** ***p*****.**

a) $\tan^2(-28°)$

b) $\sin^2(-28°) - \cos^2 62°$

c) $\dfrac{\cos(-28°)}{\sin(-62°)} - \tan 62°$

d) $\dfrac{\cos(-28°)}{\sin(-62°)} - \tan 62°$

e) $\dfrac{\cos(152°)}{\sin(-28°)} - \tan 298°$

TOPIC: TRIGONOMETRY					
LESSON 19					
Term	1	**Week**		**Grade**	11
Duration	3 Hours	**Weighting**	±50 marks	**Date**	
Sub-topics	• Special Angles and Reduction Formulae				

RELATED CONCEPTS/ TERMS/VOCABULARY

- Ratios(Grades 8 & 9)
- Lengths(Distances)
- Angles
- Triangles
- Trig Ratios such as: sin, cos & tan (Grade 10)

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Special angles
Identities
CAST Diagram
Factorization

RESOURCES

- Maths Set (Instruments)
- Scientific Calculators
- Pen / Pencils

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

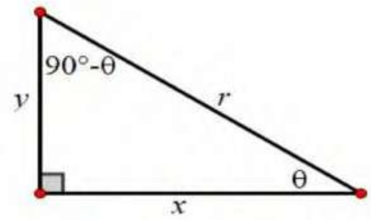
- Language barrier
- Failure to relate technical world with trigonometry
- Failure to apply fractions correctly, viz…. sin, cos & tan

METHODOLOGY

CO-RATIOS / FUNCTIONS

1. $\sin(90° - \theta) = \cos\theta$

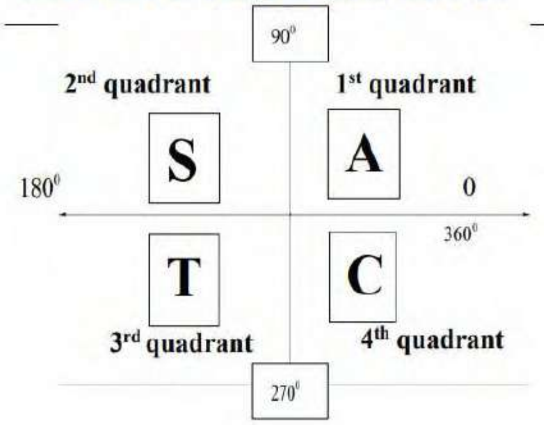
2. $\cos(90° - \theta) = \sin\theta$

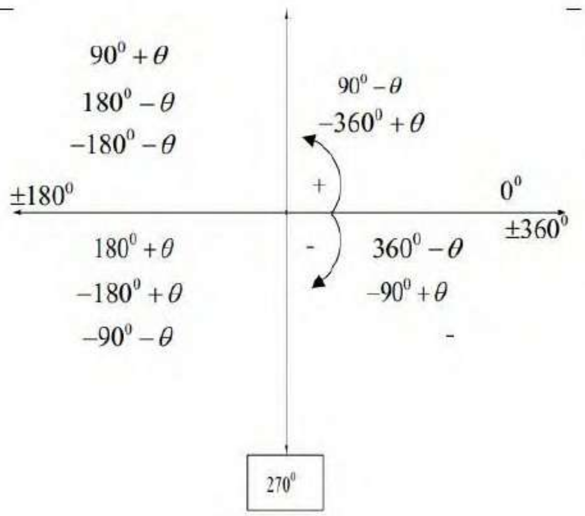


INTRODUCTION OF REDUCTION FORMULAE

CAST DIAGRAM

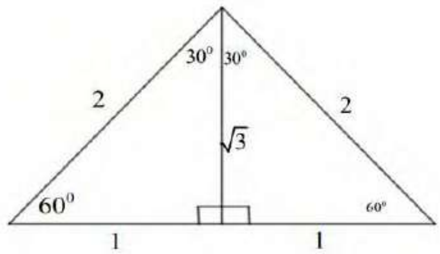
CAST: ALL STUDENTS TAKE COFFEE

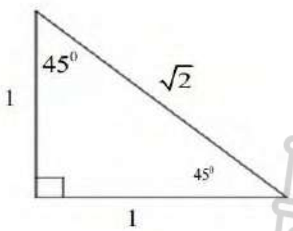




We can do reduction for angles rotating clockwise by adding $360°$ up until the angle is in the range of $0°$ to $360°$.

SPECIAL ANGLES





$\sin 0^0 = 0$	$\sin 30^0 = \frac{1}{2}$	$\sin 45^0 = \frac{1}{\sqrt{2}}$	$\sin 60^0 = \frac{\sqrt{3}}{2}$
$\cos 0^0 = 1$	$\cos 30^0 = \frac{\sqrt{3}}{2}$	$\cos 45^0 = \frac{1}{\sqrt{2}}$	$\cos 60^0 = \frac{1}{2}$
$\tan 0^0 = 0$	$\tan 30^0 = \frac{1}{\sqrt{3}}$	$\tan 45^0 = 1$	$\tan 60^0 = \frac{\sqrt{3}}{1}$

$\sin 90^0 = 1$	$\sin 180^0 = 0$	$\sin 270^0 = -1$	$\sin 360^0 = 0$
$\cos 90^0 = 0$	$\cos 180^0 = -1$	$\cos 270^0 = 0$	$\cos 360^0 = 1$
$\tan 90^0 = undefined$	$\tan 180^0 = 0$	$\tan 270^0 = undefined$	$\tan 360^0 = 0$

REDUCTION FORMULAE

Identify in which quadrant the angle(s) lie first, then you will be able to know the sign of each trigonometric ratio(s) referring to CAST diagram, then change the trig ratio to its co-function if you are reducing by 90

WORKED EXAMPLE

$90° - \theta$ **(1st quadrant)**
- $\sin(90° - \theta) = \cos\theta$
- $\cos(90° - \theta) = \sin\theta$

$90° + \theta$ **(2nd quadrant)**
- $\sin(90° + \theta) = \cos\theta$
- $\cos(90° + \theta) = -\sin\theta$

$180° - \theta$ **(2nd quadrant)**
- $\sin(180° - \theta) = \sin\theta$
- $\cos(180° - \theta) = -\cos\theta$
- $\tan(180° - \theta) = -\tan\theta$

$180° + \theta$ **(3rd quadrant)**
- $\sin(180° + \theta) = -\sin\theta$
- $\cos(180° + \theta) = -\cos\theta$
- $\tan(180° + \theta) = \tan\theta$

$360° - \theta$ **(4th quadrant)**
- $\sin(360° - \theta) = -\sin\theta$
- $\cos(360° - \theta) = \cos\theta$
- $\tan(360° - \theta) = -\tan\theta$

$-\theta$ **(4th quadrant)**
- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$

MIXED QUADRANTS

- $\sin(\theta - 90) = -\cos\theta$
- $\sin(-\theta - 90) = -\cos\theta$
- $\cos(\theta - 90) = \sin\theta$
- $\sin(-\theta - 180) = \sin\theta$
- $\cos(-\theta - 180) = -\cos\theta$
- $\tan(-\theta - 180) = -\tan\theta$
- $\sin(360° + \theta) = \sin\theta$
- $\cos(-\theta - 90) = -\sin\theta$
- $\sin(\theta - 360) = \sin\theta$
- $\cos(\theta - 360) = \cos\theta$
- $\tan(-\theta - 360) = -\tan\theta$
- $\cos(360° + \theta) = \cos\theta$
- $\sin(\theta - 180) = -\sin\theta$
- $\cos(\theta - 180) = -\cos\theta$
- $\tan(\theta - 180) = \tan\theta$
- $\sin(-\theta - 360) = -\sin\theta$
- $\cos(\theta - 360) = \cos\theta$
- $\tan(-\theta - 360) = -\tan\theta$
- $\tan(360° + \theta) = \tan\theta$

REDUCTION FORMULAE WITH SPECIAL ANGLES

WORKED MIXED EXAMPLES

- $\sin 150° = \sin(180° - 30°) = \sin 30^0 = \frac{1}{2}$
- $\cos 135° = \cos(180° - 45^{\circ}) = -\cos 45° = \frac{1}{\sqrt{2}}$
- $\sin 420° = \sin(360° + 60°) = \sin 60° = \frac{\sqrt{3}}{2}$
- $\sin 330° = \sin(360° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$
- $\tan 225° = \tan(180° + 45°) = \tan 45° = 1$
- $\tan(-405°) = \tan(-405° + 45°) = \tan 45° = 1$

ACTIVITIES/ASSESSMENTS

19.1 **Simplify the following**: **(Day 1)**

a) $\dfrac{\cos(180^\circ+\theta).\cos(90^\circ-\theta)}{\sin(90+\theta).\sin(180-\theta)}$

b) $\tan(180^\circ+\theta).\cos(90^\circ+\theta)+\sin(360-\theta).\tan(180^\circ-\theta)$

c) $\sin^2(180^\circ+\theta)-\cos^2(90^\circ-\theta)$

d) $\dfrac{\sin^2(360^\circ-\theta)}{\cos(90^\circ+\theta).\sin(540^\circ-\theta)}$

e) $\dfrac{\sin(-\theta-900^\circ).\tan(180^\circ+\theta).\cos(\theta-360^\circ)}{\sin(-\theta).\cos\theta.\tan 1485^\circ}$

19.2 **Calculate** :

a) $\cos 30^\circ \times \sin 60^\circ$

b) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

c) $\dfrac{\sin 45^\circ.\sin 30^\circ.\tan 60^\circ}{\cos 60^\circ.\tan 30^\circ.\cos 45^\circ}$

19.3 **Evaluate without the use of a calculator.** **(Day 2)**

a) $\tan^2 135^\circ$

b) $\tan(-300^\circ).\sin 600^\circ$

c) $\tan 315^\circ - 2\cos 60^\circ + \sin 210^\circ$

d) $\tan 120^\circ.\cos 210^\circ - \sin^2 315^\circ$

e) $\dfrac{\tan 150^\circ}{\tan 240^\circ} - \dfrac{\sin 300^\circ}{\sin 120^\circ}$

f) $\dfrac{\sin 315^\circ.\cos(-315^\circ).\sin 210^\circ}{\tan 225^\circ}$

g) $\sqrt{4^{\sin 150^\circ}.2^{3\tan 225^\circ}}$

h) $\dfrac{\tan 330^\circ \sin 120^\circ \sin 260^\circ}{\cos 225^\circ \sin^2 315^\circ \cos 350^\circ}$

19.4 **Prove without using a calculator that**:

a) $\dfrac{\cos 180^\circ.\sin 225^\circ.\cos 80^\circ}{\sin 170^\circ.\tan 135^\circ} = -\dfrac{\sqrt{2}}{2}$

b) $\dfrac{\cos 315^\circ + 1}{\sin 315^\circ - 1} = -1$

c) $\theta = 30°$ is a solution to $(\sin\theta)^{\sin\theta} = \frac{1}{\sqrt{2}}$

d) **Show that** $\tan 89° \times \tan 88° \times \tan 87° \times ...... \times \tan 1° = 1$ without the use of a calculator. Show all steps.

e) **Calculate the value of** $\sin^2 89° + \sin^2 88° + \sin^2 87° + ... + \sin^2 2° + \sin^2 1°$ without the use of a calculator. Show all steps.

19.5 **Prove that: (Day 3)**

a) $\sin x.\tan x + \cos x = \frac{1}{\cos x}$

b) $\frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} = 2\tan x$

c) $\tan x + \frac{1}{\tan x} = \frac{1}{\sin x.\cos x}$

d) $\frac{\tan x}{\cos x(1 + \tan^2 x)} = \sin x$

e) $\frac{(1 - \sin^2 x)^2}{\sin^2 x} = \frac{\cos^2 x}{\tan^2 x}$

f) $\frac{1 - \sin^2 x}{\sin^2 x + 2\sin x + 1} = \frac{1 - \sin x}{1 + \sin x}$

g) $\sin^2 x + \sin^2 x.\tan^2 x = \tan^2 x$

TOPIC: TRIGONOMETRY					
LESSON 20					
Term	**2**	**Week**		**Grade**	11
Duration	**2 HOURS**	**Weighting**		**Date**	

Sub-topics	**Trig. Equations**: General solutions & solutions in specified domains.

RELATED CONCEPTS/ TERMS/VOCABULARY

- Integers
- Domain
- Factorisation

$a^2 - b^2 = (a+b)(a-b)$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Solving algebraic equations.
- Rounding off.
- Special Angles.
- Solving trigonometric angles less than 90
- CAST diagram if using the quadrant approach.

RESOURCES

- Platinum Mathematics, Scientific calculator.

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

$\sin ax = a \sin x \quad i.e \quad \sin 2x = 2\sin x$

$\cos ax = a\cos x \quad i.e \quad \cos 2x = 2\cos x$

$\tan ax = a \tan x \quad i.e \quad \tan 2x = 2\tan x$

- Working with calculators not on DEGREES mode.
- Early rounding off.
- Forgetting to include $k \in \mathbb{Z}$ in the general solution
- Learners not linking identities and expressions with algebra.

METHODOLOGY

- Recap Activity from grade 10 Trig Equation. Domain $x \in [0^0; 90^0]$
- How to solve trigonometric equations with one ratio.
- We use the Shift button on the calculator to solve for an angle.

Example 1: Solve for x : Here we are solving for an angle

$\sin 2x = \frac{1}{2}$

$2x = \sin^{-1}(\frac{1}{2})$

$2x = 30$

$x = 15$

Example 2:

$4\cos(2x + 30^\circ) + 1 = 1$

$4\cos(2x + 30^\circ) = 0$

$\cos(2x + 30^\circ) = 0$

$2x + 30 = \cos^{-1}(0)$

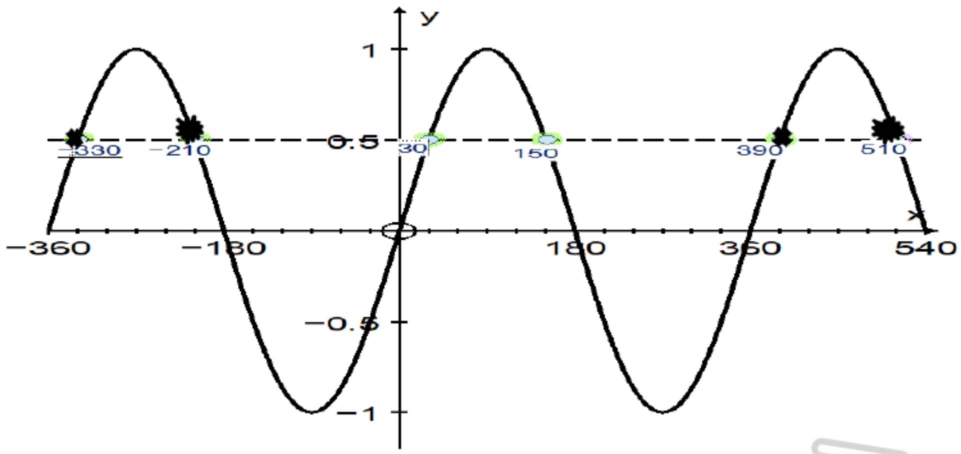
$2x + 30^\circ = 90^\circ$

$2x = 60^\circ$

$x = 30^\circ$

Illustrative Example for general solution:

Using the diagram below, solve for **x** if $\sin x = 0,5$ and $x \in [-360°; 540°]$.



- In the graph above there are 6 solutions to the given equation:

$x = 330°$ **or** $x = -210°$ **or** $x = 30°$ **or** $x = 150°$ **or** $x = 390°$ **or** $x = 510°$

- If we use a calculator to solve the problem, only one solution is obtained: $x = 30°$
- This is known as the reference angle (**RA**).
- By adding and subtracting $360°$ from the reference angle we obtain the solutions: $x = 330°$ **or** $x = 390°$
- To obtain the other solutions, take $180° - RA$ to obtain $150°$.
- By adding and subtracting $360°$ from $150°$, we obtain the solutions: $x = -210°$ **or** $x = 510°$
- This is because the sine graph has a period of $360°$ and therefore repeats itself every $360°$.

1. **General Solutions:** Method to follow when calculating the general solution for sin, cos, and tan.

If $\sin x = p$ and $-1 \leq p \leq 1$, then:

$x = \sin^{-1}(p) + 360°k$ or $x = 180° - \sin^{-1}(p) + 360°k; k \in Z$

If $\cos x = p$ and $-1 \leq p \leq 1$, then:

$x = \pm\cos^{-1}(p) + 360°k; k \in Z$

If $\tan x = p$ and $p \in \mathbb{R}$, then:

$x = RA + 180°k; k \in Z$

Types of equations and approaches to solve them:
TYPE 1: A ratio = a value

Example 1:
For the following equations, determine the general solution, correct to 2 decimal places, where necessary, then determine the specific solution for $x \in [-360°; 360°]$.

$3\sin x + 1 = -1$ **(The intention is to isolate the trig ratio, in this case '$\sin x$') using algebraic methods.**

$3\sin x = -2$ **(Subtract 1 both sides of the equation).**

$\sin x = -\frac{2}{3}$ **(Divide by 3 both sides of the equation).**

$RA = \sin^{-1}\left(\frac{2}{3}\right)$ **(Use a calculator to find the reference angle)**

$RA = 41,81°$

General solution: $x = 41,81° + 360°k$ or $x = 138,19° + 360°k; k \in \mathbb{Z}$

Specific solutions: $x = -318,19°$ or $x = -221,81°$ or $x = 41,81°$ or $x = 138,19°$

Example 2:

$\frac{\tan^2 x}{3} - 1 = 0$ **(The intention is to isolate the trig ratio, in this case 'tan *x*').**

$\frac{\tan^2 x}{3} = 1$ **(Add 1 both sides of the equation)**

$\tan^2 x = 3$ **(Divide by 3 both sides of the equation).**

$\tan x = \pm 1{,}73$ **(Square root both sides of the equation)**

$x = \tan^{-1}(1{,}73)$ or $x = \tan^{-1}(-1{,}73)$ **(Use a calculator)**

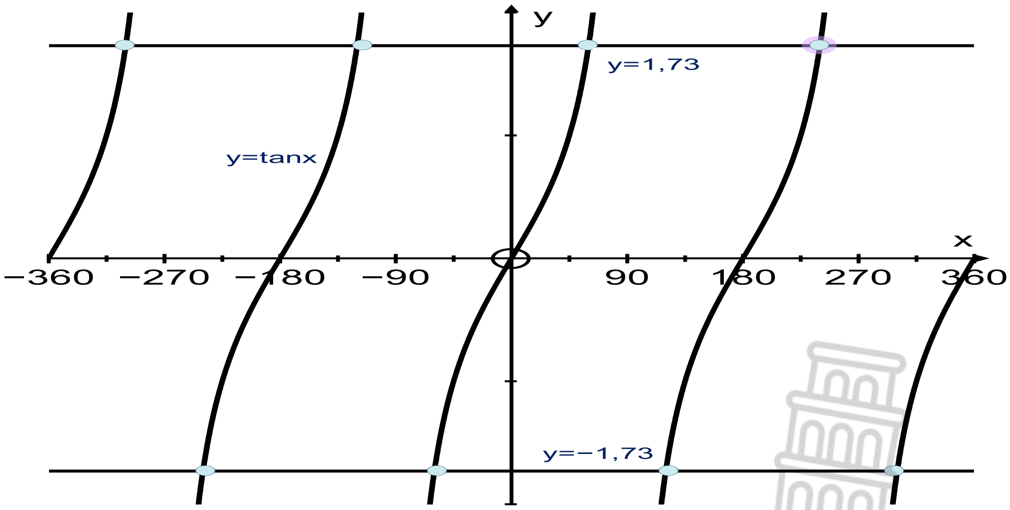
$= 59{,}97°$ or $= -59{,}97°$

General solution: $59{,}97° + 180°k$ or $-59{,}97° + 180°k; k \in \mathbb{Z}$

Specific solutions:

k	**-2**	**-1**	**0**	**1**	**2**
x	$-300{,}03^0$	$-239{,}97^0$ or $120{,}03^0$	$-59{,}97^0$ or $59{,}97^0$	$120{,}03^0$ or $239{,}97^0$	$300{,}03°$

You may use a graph to further illustrate or make the 8 specific solutions visual:



Example 3:

$2\cos(2\beta - 26°) = -1.5$ **The intention is to isolate the trig ratio, in this case '$\cos\beta$'.**

$\cos(2\beta - 26°) = -0{,}75$ **Divide by 2 both sides of the equation.**

$(2\beta - 26°) = \pm\cos^{-1}(-0{,}75) + 360°k ; k \in \mathbb{Z}$ **Use a calculator.**

$2\beta - 26° = \pm 138{,}59° + 360°k$ **The intention is to isolate the variable 'β'.**

$2\beta = 164{,}59° + 360°k$ **or** $2\beta = -112{,}59° + 360°k$ **Add** $26°$ **both sides of the equation.**

$\beta = 82{,}3° + 180°k$ **or** $\beta = -56{,}3° + 180°k$ **Divide by 2 both sides of the equation.**

General solution

$82{,}3° + 180°k$ or $= -56{,}3° + 180°k\ ;\ k \in \mathbb{Z}$

Specific solutions

$x = -277{,}7°$ or $x = -236{,}3°$ or $x = -97{,}7°$ or $x = -56{,}3°$ or $x = 82{,}3°$ or $x = 123{,}7°$ or $x = 262{,}3°$ or $x = 303{,}7°$.

ACTIVITIES/ASSESSMENTS

For the following equations, determine the general solution, correct to 2 decimal places, where necessary, then determine the specific solution for the specified domain.

Classwork

1. $\sin x = 0{,}42 \quad x \in [-180°; 180°]$
2. $\tan(x - 10°) = 1{,}015$
3. $\cos(x + 20°) = -0{,}242\ x \in [0°; 360°]$
4. $\cos 2x = \tan 40°$

Homework

1. $2{,}7 + \cos x = 3$
2. $2\sin(x + 18°) + 1 = 0\ x \in [-90°; 270°]$
3. $2\tan\left(\frac{x}{2} - 15°\right) + 3 = 0$
4. $\cos 2x = \cos 40°\ x \in [-180°; 360°]$

TOPIC: TRIGONOMETRY					
LESSON 21:					
Term	**2**	**Week**		**Grade**	11
Duration	**2 HOURS**	**Weighting**		**Date**	
Sub-topics	**Trig Equations**: General & Specified domain solution				

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Solving algebraic equations.
- Solving trigonometric equations.
- Rounding off.
- Special Angles.
- Period of Trig ratios. $\sin\theta$ *and* $\cos\theta = 360^\circ$ whereas $\tan\theta = 180^\circ$

RESOURCES

- Maths Handbook and Study Master, Platinum Mathematics, Scientific calculator.

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- $\sin ax = a\sin x$ *i.e* $\sin 2x = 2\sin x$
- $\cos ax = a\cos x$ *i.e* $\cos 2x = 2\cos x$
- $\tan ax = a\tan x$ *i.e* $\tan 2x = 2\tan x$

- Working with calculators not on DEGREES mode.
- Early rounding off.
- Dividing by a trig ratio instead of factorising.

METHODOLOGY

Factorization, as in the solution of algebraic equations, is often used. Look out for:

- Common factor
- Difference of squares
- Quadratic trinomials
- Grouping of terms

TYPE 2: A ratio = a co-ratio

For the following equation, determine the general solution, correct to 2 decimal places, where necessary, then determine the specific solution for $x \in [-360°;360°]$.

Example 1: co-ratio on either side of equal sign. Different angles.

Approach: Use reduction to co-ratio.

$\sin(2x-5°)=\cos(x-35°)$ **Co-ratios with different angles.**

$\cos[90°-(2x-5°)]=\cos(x-35°)$ **(Choose any side) and change to co-ratio.**

$95°-2x=\pm(x-35^0)+360^{\circ}k;k\in\mathbb{Z}$ **Drop/cancel trig ratio on both sides.**

Apply the rule for general solution of cos.

$-3x=-130°+360°k$ or $-x=-60°+360°k$ **Remove brackets and add like terms.**

$x=43,3°-120°k$ or $4x=60°-360^0k$ **Divide by the respective coefficients both sides**

General solution: $x=43,3°-120°k$ or $4x=60°-360°k;k\in\mathbb{Z}$

Specific solutions: $x=-316,7°;-300°;-196,7°;-77°;43,3°;60°;163,3°;283,3°$.

Example 2: co-ratio on either side of equal sign. Same angles.

For the following equation, determine the general solution, correct to 2 decimal places, where necessary, then determine the specific solution for $x\in[-360°;360°]$.

Approach: Divide both sides by cos ratio. (NB. This is the only case where we divide by a trig ratio, because the angles are the same and therefore there is no chance of division by zero)

$\sin x=\cos x$ **Co-ratios with the same angles.**

$\tan x=1$ **Divide both side by $\cos x$ (Note why we do this only in this case.)**

$\tan^{-1}(1)=45°$ **Special angle or Calculator.**

General solution: $45^0+180°k;k\in\mathbb{Z}$ **Apply the rule for general solution of tan.**

Specific solutions: $x=-315°;-135°;-45°;225°$

TYPE 3: A Ratio = ratio [Involving identities and/or factorisation]

For the following equation, determine the general solution, correct to 2 decimal places, where necessary, then determine the specific solution for $x\in[-360°;360°]$.

Example 1: same trig ratio on either side of equal sign.

Approach: drop trig ratios on both sides.

$\sin(3x-20°)=\sin(x+10°)$ **Drop/cancel trig ratio on both sides**

$3x-20^\circ = x+10^\circ+360^\circ k$; or $3x-20^\circ = 180-(x+10^\circ)+360^\circ k; k \in \mathbb{Z}$	**Apply the rule for general solution of sin.**
$2x = 30^\circ+360^\circ k$ or $3x = 190^\circ - x + 360^\circ k$	**Remove brackets and add like terms**
$2x = 30^\circ+360^\circ k$ or $4x = 190^\circ + 360^\circ k$	**Divide by the respective coefficients both sides**

General solution: $x = 15^\circ + 180^\circ k$ or $= 47,5^\circ + 90^\circ k; k \in \mathbb{Z}$

Specific solutions $-345^\circ; -312,5^\circ; -222,5^\circ; -165^\circ; -132,5^\circ; -42,5^\circ; 15^\circ; 47,5^\circ; 137,5^\circ; 195^\circ; 227,5^\circ; 317,5^\circ$

NB: Only workout the specific solution when asked to do so. Read the question and note the interval you are instructed to work out the specific solution for.

ACTIVITIES/ASSESSMENTS

For the following equations, determine the general solution and the specific solution where the domain is specified.

Classwork	Homework
1. $\cos(2x+25^\circ) = \cos(38^\circ - x)$	1. $\sin(x-30^\circ) = \cos 2x \quad x \in [-90^\circ; 180^\circ]$
2. $\cos(2x-10^\circ) = \sin(x-40^\circ) \quad x \in [0^\circ; 360^\circ]$	2. $\cos\theta - \frac{1}{\cos\theta} = \frac{5}{6} \quad \cos\theta \neq 0$
3. $\sin x\,(2\cos x - 1) = 0$	3. $\sin(x-30^\circ) = \cos(x-30^\circ) \quad x \in [-30^\circ; 360^\circ]$
4. $2\cos^2 x + 5\sin x = 4 \quad x \in [-180^\circ; 90^\circ]$	4. $2\sin x \cos x = \cos x$

TOPIC: TRIGONOMETRY					
LESSON 22:					
Term	**2**	**Week**		**Grade**	11
Duration	**1 HOUR**	**Weighting**		**Date**	
Sub-topics	**Trig. Equations**: (Restrictions).				

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Solving algebraic equations.
- Solving trigonometric equations.
- Rounding off.
- Special Angles.
- Period of Trig ratios. $\sin\theta$ *and* $\cos\theta = 360^\circ$ *whereas* $\tan\theta = 180^\circ$

RESOURCES

- Maths Handbook and Study Master, Platinum Mathematics, Scientific calculator.

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

An expression or equation will be undefined if division by 0 occurs. For this reason, it is important to note the following properties of the trig ratio tan:

$\tan\theta \neq 90^\circ$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\tan\theta$ is undefined when $\cos\theta = 0$

$\therefore$ when $\theta = \pm 90^0 + 360^0 k; k \in \mathbb{Z}$

Example 1: Solve for θ if $\tan\theta \, sin\,\theta = \tan\theta$

$\tan\theta \, sin\,\theta = \tan\theta$

$\therefore \tan\theta \, sin\,\theta - \tan\theta = 0$

$\tan\theta\,(sin\,\theta - 1) = 0$ **Never divide by** $\tan\theta = 0$.

$\tan\theta = 0$ or $sin\,\theta = 1$

$\theta = 180^\circ k$ *or* $\theta = 90^\circ + 360^\circ k$ $\theta = 90^\circ + 360^\circ k$ makes $\cos\theta = 0$

The equation is undefined when $\cos\theta = 0$

$\therefore \cos\theta = 0$ when $\theta = 90^\circ + 360^\circ k; k \in \mathbb{Z}$

Example 2: For which values of x is the equation $\frac{sin^2 x}{\sin 2x} = \frac{\tan x}{2}$ undefined?

For the equation to be undefined $\sin 2x = 0$ or $\cos x = 0$ (the equation contains $\tan x$)

$\sin^{-1}(0) = 0^0$ or $\cos^{-1}(0) = 90^0$

$2x = 0^0 + 360^0 k$ or $2x = 180^0 - 0^0 k$ or $x = \pm 90^0 + 360^0 k$

$\therefore x = 180^0 k$ or $x = 90^0 + 180^0 k$ or $x = \pm 90^0 + 360^0 k; k \in \mathbb{Z}$

ACTIVITIES/ASSESSMENTS

For which values of x are the following identities undefined?

Classwork

22.1

$$\frac{1 - 2sin^2 x}{cosx - sinx} = cosx + sinx$$

22.2

$$\frac{\cos\beta}{1 + sin\beta} = \frac{1 - sin\beta}{cos\beta}$$

Homework

22.2a

$$\frac{1}{cos\beta} + tan\beta = \frac{cos\beta}{1 - sin\beta}$$

22.2.b

$$\frac{1 - cos^2 x + sinx}{cosx \cdot sinx + cosx} = tanx$$

TOPIC: TRIGONOMETRY

LESSON 23

Term	2	**Week**		**Grade**	11
Duration	2 HOURS	**Weighting**		**Date**	
Sub-topics	Trig. Graphs: Sketching				

RELATED CONCEPTS/ TERMS/VOCABULARY

- Intercepts
- Turning points
- Amplitude
- Period
- Domain
- Range
- Asymptote
- Vertical shift
- Stretch

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Effects of parameters ***a,*** and ***q***.
- $\text{Amplitude} = \dfrac{\text{Maximum value} - \text{Minimum value}}{2}$
- Basic shapes of the sine, cosine and tan graphs

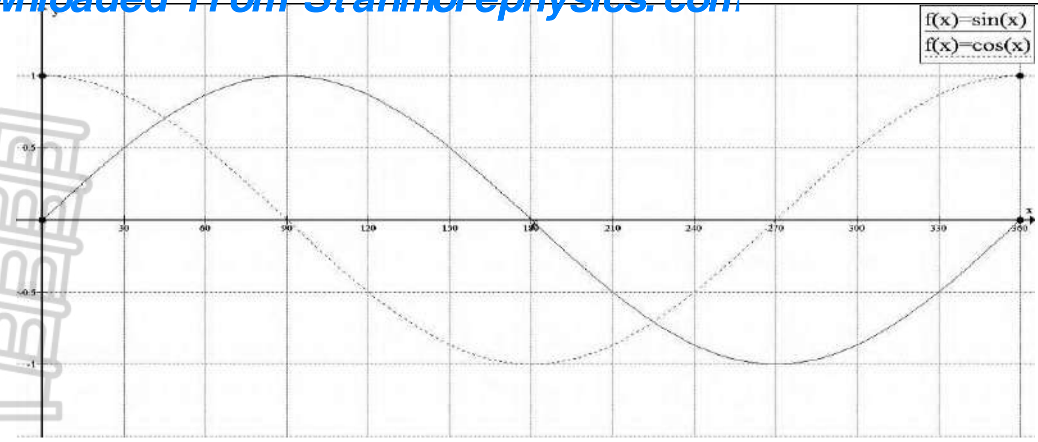
Grade 10 re-cap. Make use of a calculator or otherwise to complete the table and to plot points on a graph paper.

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$y = \sin\theta$	0	0,5	0,7	0,9	1	0,9	0,7	0,5	0

θ	210°	225°	240°	270°	300°	315°	330°	360°
$y = \sin\theta$	–0,5	–0,7	–0,9	–1	–0,9	–0,7	–0,5	0

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$y = \cos\theta$	1	0,9	0,7	0,5	0	–0,5	–0,7	–0,9	–1

θ	210°	225°	240°	270°	300°	315°	330°	360°
$y = \cos\theta$	–0,9	–0,7	–0,5	0	0,5	0,7	0,9	1



- Sketching of basic trig functions ($y = \sin\theta$; $y = \cos\theta$ and $y = \tan\theta$).
- Effect of the value of the parameters '***a***' and '***q***'.

RESOURCES

Mind action series grade 11 textbook, Pervious question papers, & Maths handbook.

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Working with calculators not in DEGREES mode.
- Relying only on the calculator for sketching without checking the values of a, p and q from the given equation. These two aspects must be linked.

METHODOLOGY

- Use the table method and calculator to sketch graphs.
- Investigate the effect of parameter *p*.
- Investigations and consolidation afterward.

ACTIVITIES/ASSESSMENTS

The influence of the value of *p* on the graphs of the different trigonometric functions

Outcomes:

At the end of this activity, you should be able to:

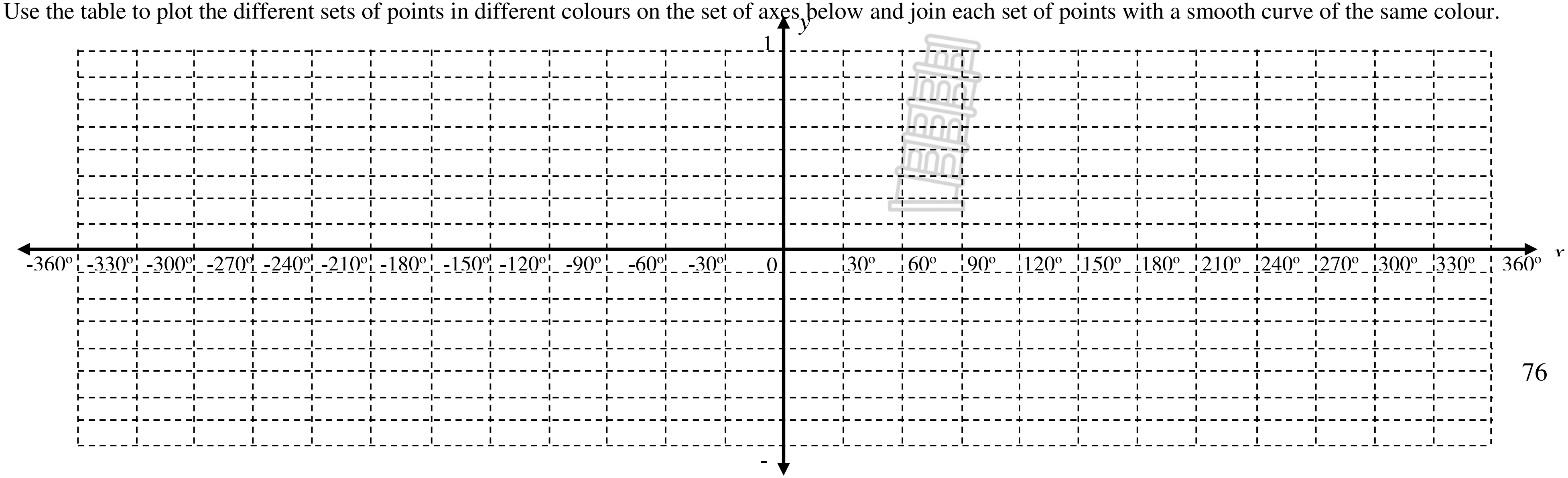
1. tell the effect that the value of p has on the graphs of the different trigonometric functions.
2. draw the graphs of the different trigonometric functions for different values of p.

1. The sine function: $y = \sin(x + p)$

23.1a Complete the following table with assistance of your calculator:

x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin x$																									
$\sin(x + 30°)$																									
$\sin(x - 60°)$																									

23.1b Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.



23.1c Use your graphs to fill out the table below.

	p	x-intersects					Turning points				Amplitude	Period
$\sin x$												
$\sin(x + 30^\circ)$												
$\sin(x - 60^\circ)$												

23.1d How did a change in the value of p affect the x-intercepts of the sine function?

____

23.1d How did a change in the value of p affect the turning points of the sine function?

____

23.1e How did a change in the value of p affect the amplitude of the sine function?

____

23.1f How did a change in the value of p affect the period of the sine function?

____

23.1g Make a general conclusion on the effect of the value of p on the graph of the sine function.

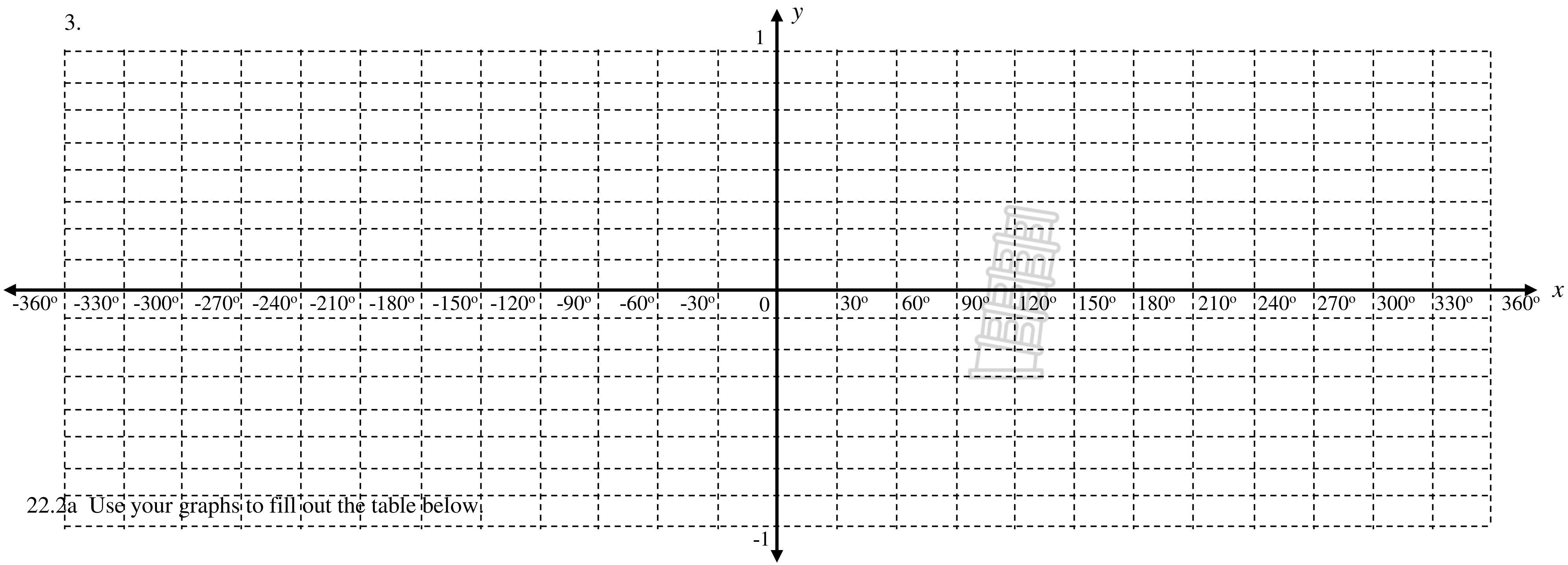
____

22. The cosine function: $y = \cos(x + p)$

22.1a Complete the following table with assistance of your calculator:

x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
sin x																									
cos(x + 60°)																									
cos(x – 30°)																									

22.1b Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.



22.2a Use your graphs to fill out the table below.

	p	x-intersects					Turning points				Amplitude	Period
$\cos x$												
$\cos(x + 60^{o})$												
$\sin(x - 30^{o})$												

22.2b How did a change in the value of p affect the x-intercepts of the cosine function?

____

22.2c How did a change in the value of p affect the turning points of the cosine function?

____

22.2c How did a change in the value of p affect the amplitude of the cosine function?

____

22.2d How did a change in the value of p affect the period of the cosine function?

____

22.2e Make a general conclusion on the effect of the value of p on the graph of the cosine function.

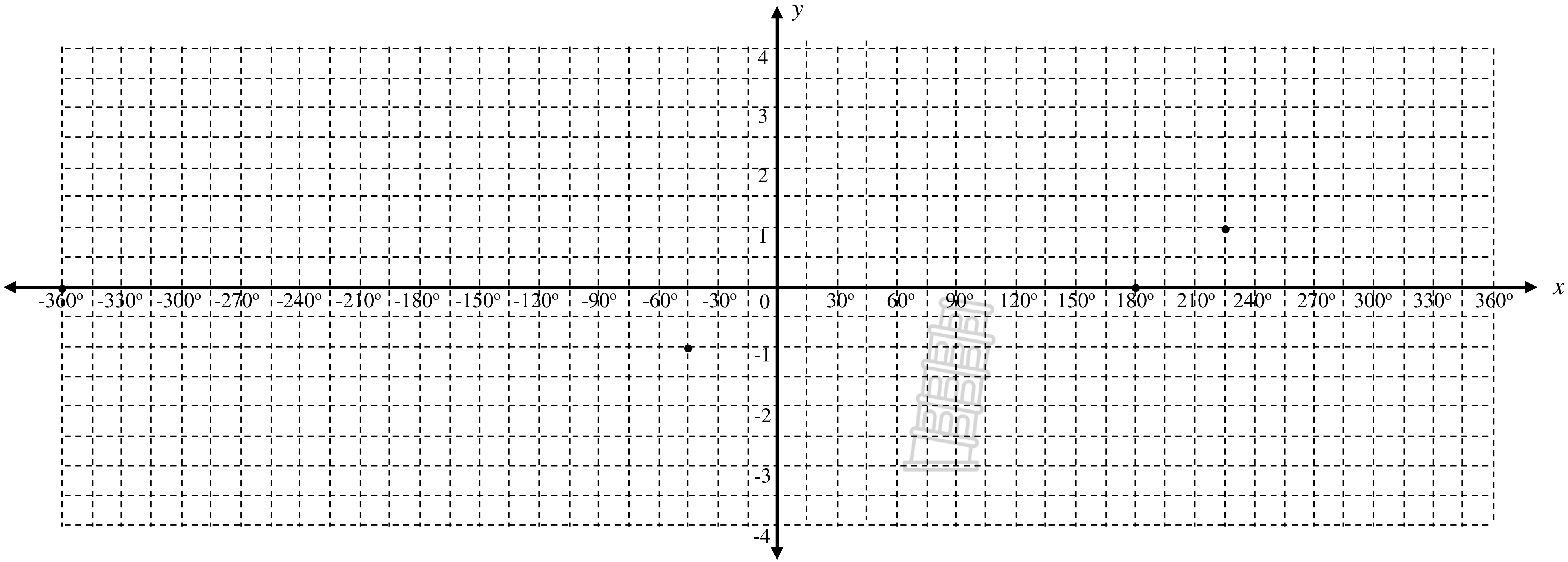
____

22.3. The tangent function: $y = \tan(x + p)$

22.3a Complete the following table with assistance of your calculator:

x	-360°	-315°	-300°	-284°	-270°	-256°	-240°	-225°	-180°	-135°	-120°	-104°	-90°	-76°	-60°	-45°	0°	45°	60°	76°	90°	104°	120°	135°	180°	225°	240°	256°	270	284	300	315°	360°
tan x																																	
tan(x +15°)																																	
tan(x –30°)																																	

22.3b Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.



22.3c Use your graphs to fill out the table below.

	p	x-intersects					Turning points				Asymptotes				Amplitude	Period
$\tan x$																
$\tan(x + 15°)$																
$\tan(x - 30°)$																

22.3d How did a change in the value of p affect the x-intercepts of the tangent function?

22.3e How did a change in the value of p affect the turning points of the tangent function?

22.3f How did a change in the value of p affect the amplitude of the tangent function?

22.3g How did a change in the value of p affect the period of the tangent function?

22.3h How did a change in the value of p affect the asymptotes of the tangent function?

____

22.3i Make a general conclusion on the effect of the value of p on the graph of the tangent function.

____

TOPIC: TRIGONOMETRY					
LESSON 24					
Term	**2**	**Week**		**Grade**	11
Duration	**2 HOURS**	**Weighting**		**Date**	
Sub-topics	Trig. Graphs: Sketching				

RELATED CONCEPTS/ TERMS/VOCABULARY

- Intercepts
- Period
- Amplitude
- Asymptotes
- Turning point(s)

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Effects of parameters $\boldsymbol{a}$ and $\boldsymbol{q}$ and $\boldsymbol{p}$.

RESOURCES

- Mind action series grade 11 textbook, Pervious question papers, & Maths- handbook.

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Not knowing which angle to use as a “step” angle when using a calculator.
- Ignoring the given domain and sketching on own domain.

METHODOLOGY

- Use the table method and calculator to sketch graphs.
- Investigate the effect of parameter *k*.
- Investigations and consolidation afterwards.

ACTIVITIES/ASSESSMENTS

The influence of the value of *k* on the graphs of the different trigonometric functions

Outcomes:

At the end of this activity, you should be able to:

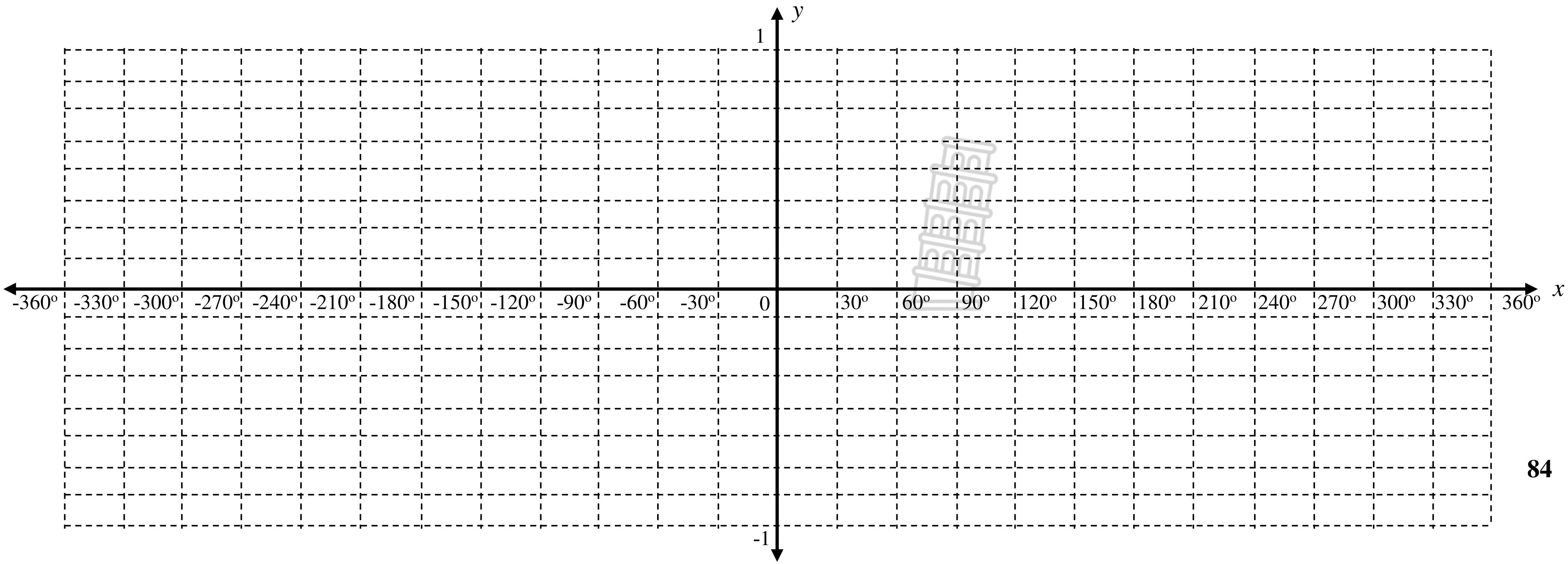
1. tell the effect that the value of *k* has on the graphs of the different trigonometric functions.
2. draw the graphs of the different trigonometric functions for different values of *k*.

23. The sine function: $y = \sin kx$

23.1 Complete the following table with assistance of your calculator:

x	-180º	-150º	-135º	-120º	-90º	-60º	-45º	-30º	0º	30º	45º	60º	90º	120º	135º	150º	180º
$\sin x$																	
$\sin 2x$																	
$\sin \frac{1}{2} x$																	

23.2 Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.



23.3 Use your graphs to fill out the table below.

	k	x-intersects					Turning points				Amplitude	Period
$\sin x$												
$\sin 2x$												
$\sin \frac{1}{2} x$												

23.4 How did a change in the value of k affect the x-intercepts of the sine function?

____

23.5 How did a change in the value of k affect the turning points of the sine function?

____

23.6 How did a change in the value of k affect the amplitude of the sine function?

____

23.7 How did a change in the value of k affect the period of the sine function?

____

23.8 Make a general conclusion on the effect of the value of k on the graph of the sine function.

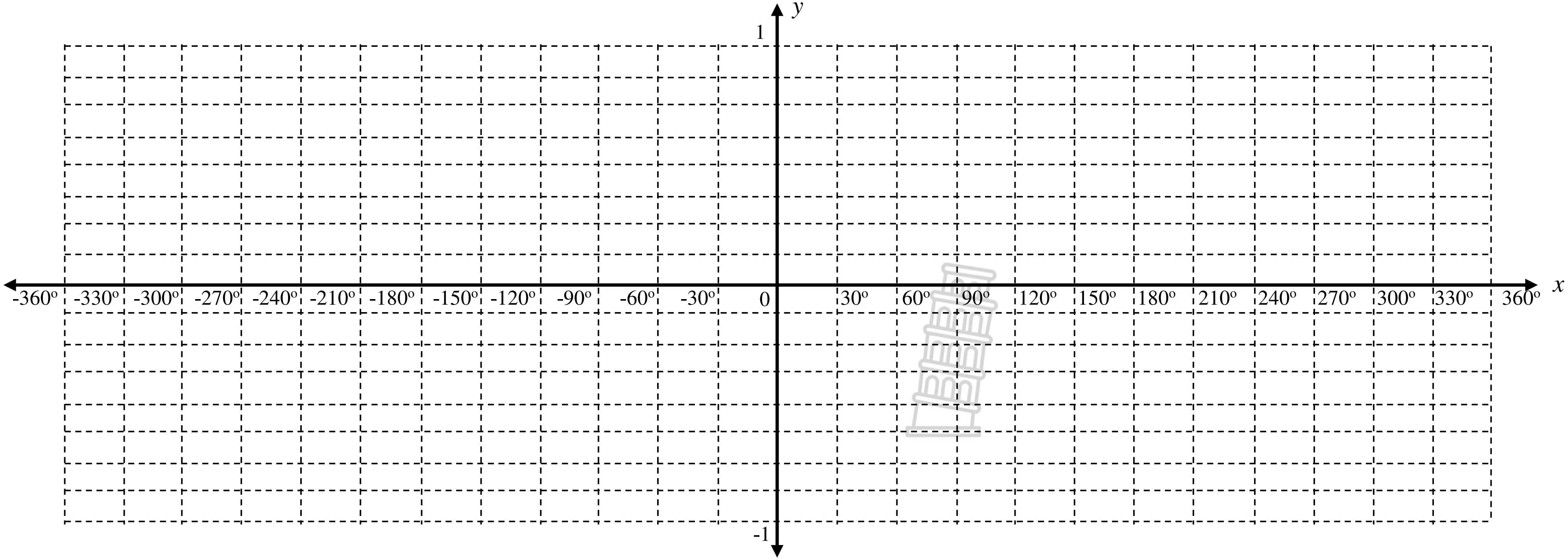
____

24. The cosine function: $y = \cos kx$

24.1 Complete the following table with assistance of your calculator:

x	-240°	-210°	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°	210°	240°
$\cos x$																	
$\cos 3x$																	
$\cos \frac{3}{2}x$																	

24.2 Use the table to plot the different sets of points in different colours on the set of axis below and join each set of points with a smooth curve of the same colour.



24.3 Use your graphs to fill out the table below.

	k	x-intersects					Turning points				Amplitude	Period
$\cos x$												
$\cos 3x$												
$\cos \frac{3}{2} x$												

24.4 How did a change in the value of k affect the x-intercepts of the cosine function?

____

24.5 How did a change in the value of k affect the turning points of the cosine function?

____

24.6 How did a change in the value of k affect the amplitude of the cosine function?

____

24.7 How did a change in the value of k affect the period of the cosine function?

____

24.8 Make a general conclusion on the effect of the value of k on the graph of the cosine function.

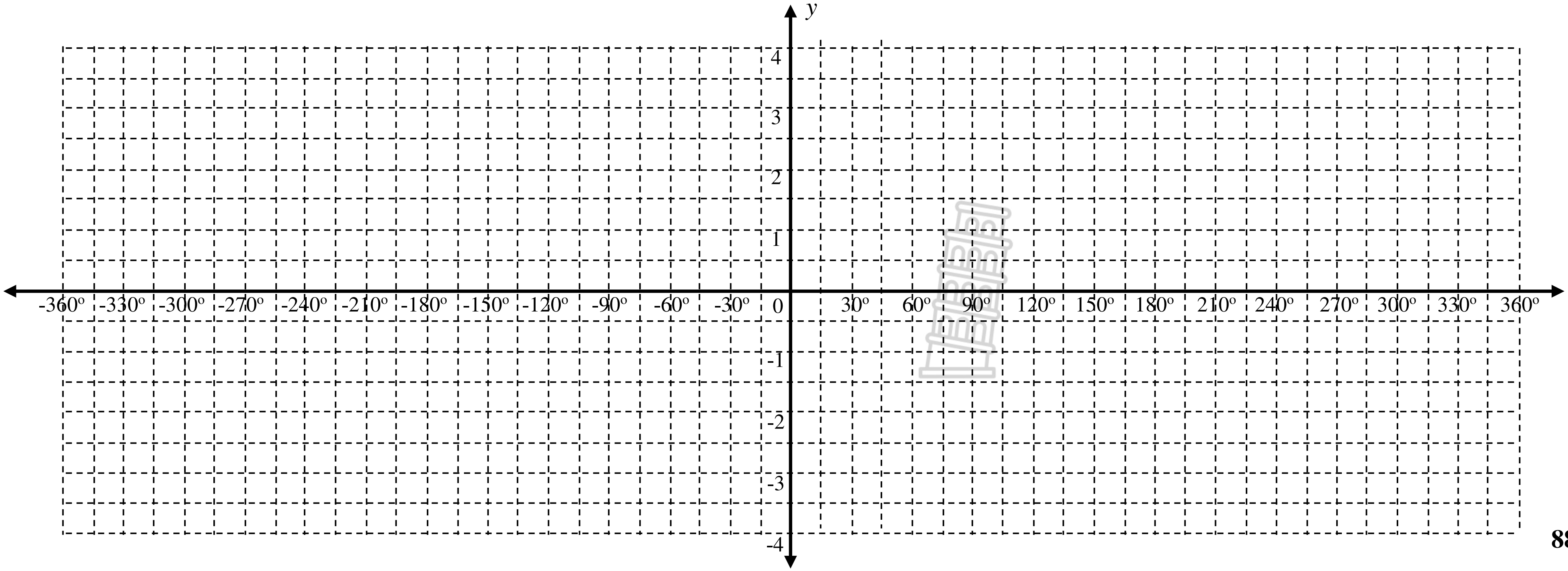
____

24.9. The tangent function: $y = \tan kx$

24.9.1 Complete the following table with assistance of your calculator:

x	-360°	-315°	-300°	-284°	-270°	-256°	-240°	-225°	-180°	-135°	-120°	-104°	-90°	-76°	-60°	-45°	0°	45°	60°	76°	90°	104°	120°	135°	180°	225°	240°	256°	270	284	300	315°	360°
tan x																																	
tan $2x$																																	
tan ½ x																																	

24.9.2 Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.



24.9.3 Use your graphs to fill out the table below.

	k	x-intersects					Turning points				Asymptotes				Amplitude	Period
tan x																
tan $2x$																
tan ½ x																

24.9.4 How did a change in the value of k affect the x-intercepts of the tangent function?

____

24.9.5 How did a change in the value of k affect the turning points of the tangent function?

____

24.9.6 How did a change in the value of k affect the amplitude of the tangent function?

____

24.9.7 How did a change in the value of k affect the period of the tangent function?

____

24.9.8 How did a change in the value of k affect the asymptotes of the tangent function?

____

24.9.9 Make a general conclusion on the effect of the value of k on the graph of the tangent function.

____

INTERPRETATION OF TRIG GRAPHS					
REVISION WORK					
Term	**2**	**Week**		**Grade**	11
Duration	**2 HOURS**	**Weighting**		**Date**	
Sub-topics	**Trig. Graphs**: Interpretation				

RELATED CONCEPTS/ TERMS/VOCABULARY

- Period
- Domain
- Range
- Asymptotes
- Determine equations
- Amplitude
- Intersection between TWO graphs
- Increasing and decreasing graphs
- Inequalities
- Distance between curves
- Transformation of functions

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

- Sketching trig graphs.
- Transformations
- The effects of parameters.

Standard form of trig graphs.

- $y = a\sin(kx + p) + q$
  $y = a\cos(kx + p) + q$
  $y = a\tan(kx + p) + q$

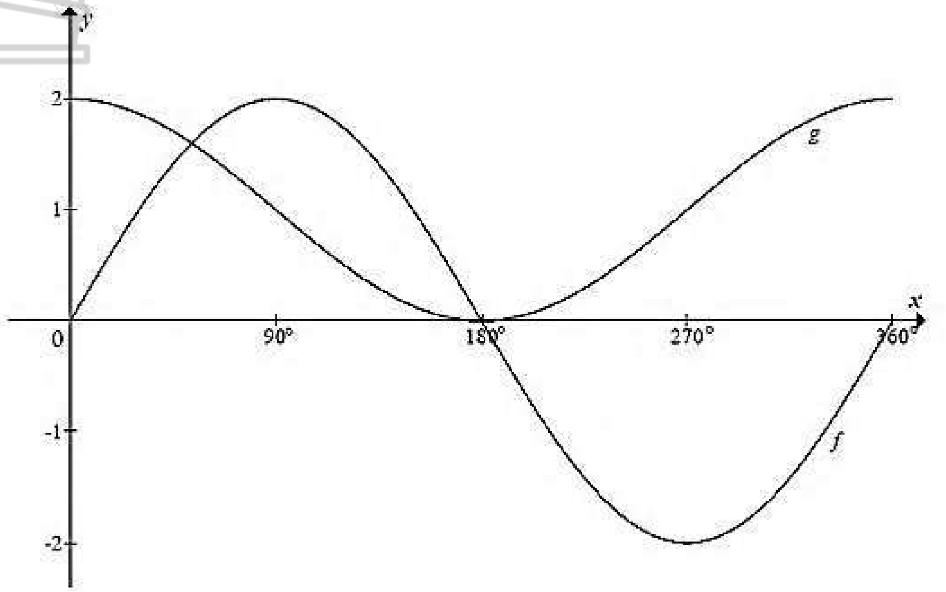
a	***q***	***p***	***k***
Change in amplitude	Vertical shift	Horizontal shift	Change in period

RESOURCES

Maths Handbook and Study Master, Platinum Mathematics, Scientific calculator.

ERRORS/MISCONCEPTIONS/PROBLEM AREAS
Not knowing which angle to use as a "step" angle when using a calculator. Ignoring the given domain and sketching on own domain.
METHODOLOGY
• Illustration through an example.

Example:



The graphs of $f(x) = a\sin x$ and $g(x) = \cos x + 1$ **for** $x \in [0^0 ; 360]$ are sketched above

Answers

1. Write down the value of a — $\boldsymbol{a = 2}$
2. Write down the period of f *and* g — **Period is 360°**
3. Write down the range of f *and* g — **Range of f:** $-2 \le y \le 2$

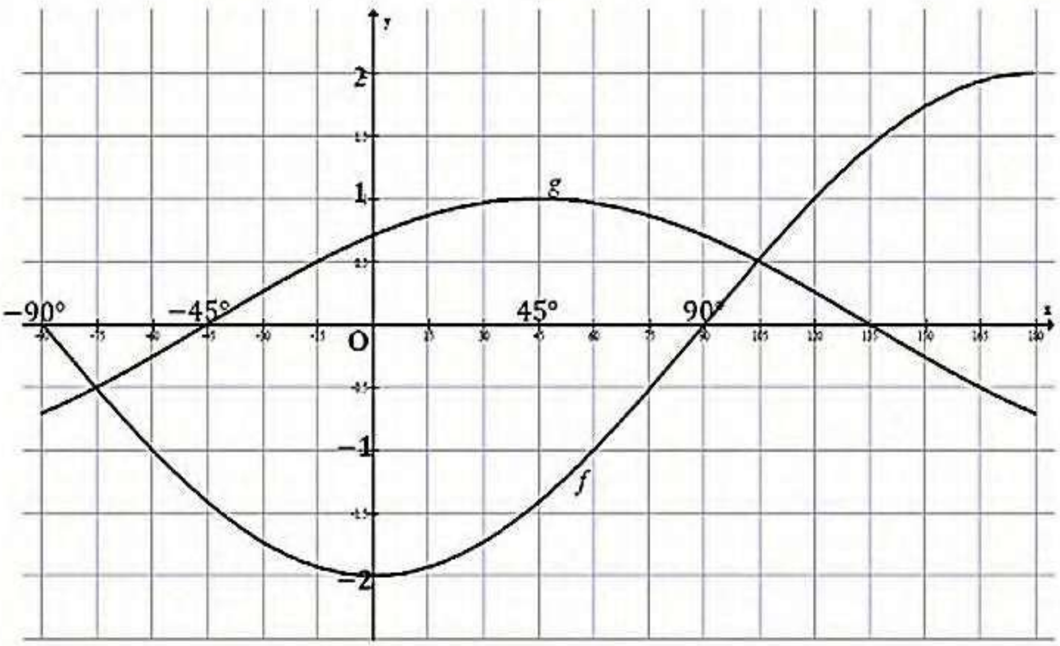
   Range of g: $0 \le y \le 2$
4. For which value(s) of x for $x \in [0°;360°]$ will:
   a. $f(x) - g(x) = 0$ — $x = 53{,}13°$ or $x = 180°$
   b. $f(x) \langle g(x)$ — $0° \le x \langle 53{,}13°$ or $180° \langle x \le 360°$
   c. $f(x).g(x) \le 0$ — $180° \le x \le 360°$
   d. $f(x).g(x) \ge 0$ — $0° \le x \le 180°$

5. The graph of g is reflected about the $x-axis$ and shifted two units upwards to form the graph of h. Determine the equation of h.	$h(x) = -\cos x + 1$

ACTIVITIES/ASSESSMENTS

Question 1

The diagram below shows the sketch graphs of $f(x) = a \cos bx$ and $g(x) = p \sin (x + r)$ for x [-90°; 180°]



1.1. Write down the values of a, b, p and r.

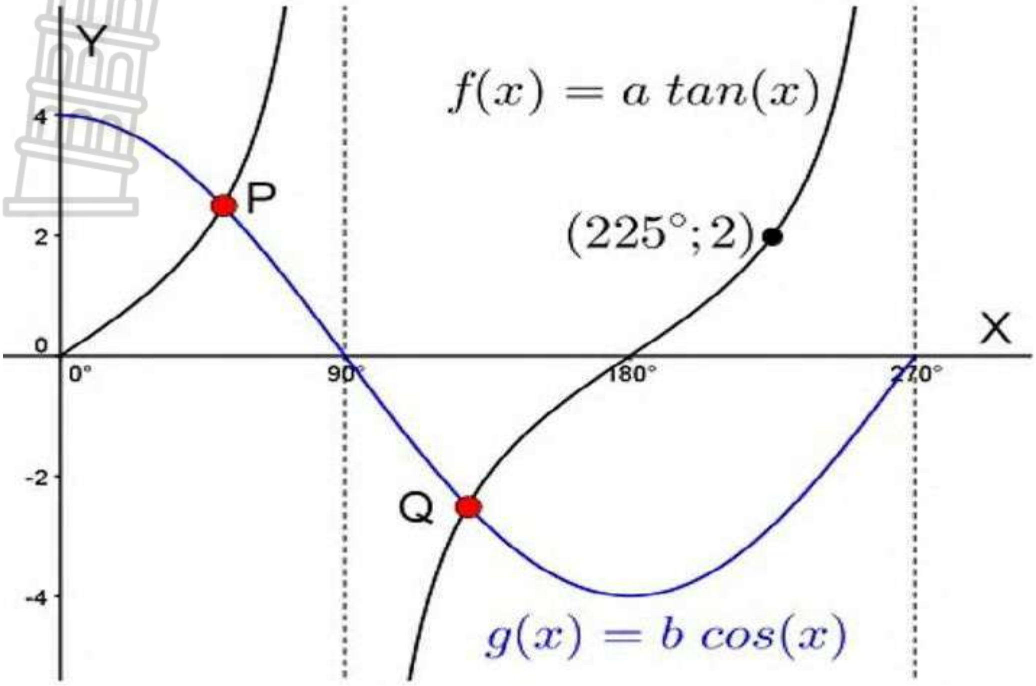
1.2. Use the graph to determine the values of x for which $f(x) - g(x) = 0$

1.3. Write down the period of f.

1.4. Write down the equation of h if h is obtained by first moving the graph of g, 45° to the right and then doubling its period.

Question 2

The graphs of the functions defined by $f(x) = a \tan x$ and $g(x) = b \cos x$ for **$0° \leq x \leq 270°$** are shown in the diagram. The point (225°; 2) lies on f. The graphs intersect at points P (51, 3° ; 2,5) and Q(128,7° ; – 2,5)



Determine:

2.1. The values of a and b.
2.2. The minimum value of $g(x) + 2$
2.3. The period of $f(x)$
2.4. The values of x for which:
    2.4.1. $f(x) > g(x)$
    2.4.2. $f(x).g(x) \leq 0$

Homework
Question 1

Given $f(x) = 1 + sin\,x$ and $g(x) = tan\,x$ for $0° \leq x \leq 180°$

1.1. Give the range of f
1.2. Determine the period of f
1.3. Draw sketch graphs of the functions f and g on the same system of axes. Clearly show all intercepts with the axes, asymptotes, and turning points where applicable.

Question 2

Given $f(x) = sin\ x - 1$ and $g(x) = 2\ cos\ x$ for $0° \leq x \leq 270°$

2.1. Sketch, on the same system of axes the graphs of f and g for $0° \leq x \leq 270°$.
2.2. Write down the following:
 2.2.1. Amplitude of g.
 2.2.2. Range of g.
2.3. Use your graph to determine the following:
 2.3.1. Number of solutions to $f(x) = g(x)$ in the interval $0° \leq x \leq 270°$
 2.3.2. Value(s) of x in the interval $0° \leq x \leq 180°$ for which $\sin x = 1 + 2\cos x$
2.4. Describe a transformation that the graph of

Question 3

Consider: $g(x) = -4\cos(x + 30°)$
3.1. Write down the maximum value of $g(x)$.

3.2. Write down the period of $g(2x)$.

3.3. Determine the range of $\frac{1}{2}g(x) + 1$.

3.4. The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form.

Question 4

Consider the function $f(x) = -2\tan\frac{3}{2}x$. Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$.

SOLUTIONS TO LESSONS

TOPIC 1: ALGEBRA AND EQUATIONS

LESSON 1	
1.1	8
1.2	16
1.3	$\frac{3}{2}$
1.4	2
1.5	$\frac{2}{xy^2}$
1.6	$\frac{x}{3}$
1.7	$a^{\frac{5}{6}}$
1.8	$\frac{7}{12}$
1.9	$x+y$
1.10	$\frac{1}{a^2b^2}$
LESSON 3	
3.1	$x=8$
3.2	$x=1$
3.3	$x=25$
3.4	$x=9$
3.5	$x=81$
3.6	$x=64$
3.7	$x=0$ or $x=64$
3.8	$x=81$
3.9	$x=-\frac{64}{27}$ or $x=27$
3.10	$x=512$
LESSON 6	
6.1	$x=81$
6.2	$x=49$
6.3	$x=37$
6.4	$x=97$
6.5	$x=4$
6.6	$x=7$
6.7	$x=-\frac{9}{4}$
6.8	$x=36$
6.9	No solution
6.10	$x=-2$
LESSON 7	
7.6	$x=3$ or $x=7$
7.7	$x=1$
7.8	$x=4$
7.9	$x=36$
LESSON 2	
2.1	$\frac{1500}{6^x}$
2.2	$\frac{3}{4}$
2.3	$\frac{1}{3}$
2.4	$\frac{1}{6}$
2.5	2^{n+3}
LESSON 4	
4.1	$5\sqrt{2}$
4.2	$5\sqrt[3]{2}$
4.3	$2\sqrt{11}$

4.4	$4\sqrt{2}x^6y^4$
5.1	$2\sqrt{2}$
5.2	$5\sqrt{2}+2\sqrt{5}$
5.3	$10-2\sqrt{21}$
5.4	2
5.5	$2+\sqrt{3}$
5.6	LHS $=\dfrac{\sqrt{48}}{\sqrt{24}-\sqrt{8}}\times\dfrac{\sqrt{24}+\sqrt{8}}{\sqrt{24}+\sqrt{8}}$ $=\dfrac{\sqrt{48\times24}+\sqrt{48\times8}}{24\text{-}8}$ $=\dfrac{\sqrt{1152}+\sqrt{384}}{16}$ $=\dfrac{24\sqrt{2}+8\sqrt{6}}{16}$ $=\dfrac{8\left(3\sqrt{2}+\sqrt{6}\right)}{16}$ $=\dfrac{3\sqrt{2}+\sqrt{6}}{2}$
5.7	Proof
7.1	$x=19$
7.2	$x=\dfrac{1}{4}$
7.3	$x=4$
7.4	$x=38$
7.5	No solution
7.10	$x=\dfrac{19}{4}$

TOPIC 2: EUCLIDEAN GEOMETRY

LESSON 8: GR. 11 EUCLIDEAN GEOMETRY		
8.1.	8.1.1	$x=70°$ (sum of $\angle$s in Δ)
	8.1.2	$B\hat{Z}Y$ and $Z\hat{Y}C$ $A\hat{Z}Y$ and $Z\hat{Y}D$ $A\hat{X}Y$ and $X\hat{Y}D$ $B\hat{X}Y$ and $X\hat{Y}C$
	8.1.3	$B\hat{Z}Y$ and $Z\hat{Y}D$ $A\hat{Z}Y$ and $Z\hat{Y}C$
	8.1.4	Supplementary
	8.1.5	$X\hat{Y}D=70°$ (Alt. $\angle$s equal, AB || DC
8.2		$y^2=2^2+\left(\dfrac{3}{2}\right)^2$ (Pythagoras theorem) $y=\dfrac{5}{2}$

8.3	(a) 1	$\hat{A}_3 = 48°$ (∠s opp. Equal sides) $\hat{D}_1 = 96°$ (Ext. ∠ of Δ)
	(a) 2	$143° + \hat{E}_2 = 180°$ (∠s in a straight line) $\hat{E}_2 = 37°$ $\hat{E}_2 = \hat{B}_3 = 37°$ (Corr. ∠s equal $PQ \parallel RS$) $\hat{B}_1 = \hat{B}_3 = 37°$ (Vert. opp. ∠s)
	(a) 3	$\hat{B}_1 + 85° = \hat{A}_2 + \hat{D}_1$ (Ext. ∠ of Δ) $37° + 85° = \hat{A}_2 + 96°$ $\hat{A}_2 = 26°$
	(b)	$\hat{B}_3 = \hat{E}_2$ (Corr. ∠s equal $PQ \parallel RS$) $R\hat{E}F = \hat{E}_2$ (Vert. opp. ∠s) $R\hat{E}F = \hat{B}_3$

LESSON 9: GR. 11 EUCLIDEAN GEOMETRY		
9.1		$BC^2 + 6^2 = 10^2$ (Pythagoras theorem) $BC = 8$ $AC = BC + AB$ (⊥ from centre to chord) $AC = 8 + 8 = 16$
9.2	(a)	$AC = CB = 24$ (⊥ from centre to chord) $OB^2 = 24^2 + 7^2$ (Pythagoras theorem) $OB = 25cm$
	(b)	$MQ^2 = 25^2 - 5^2$ (Pythagoras theorem) $PM = MQ = 10\sqrt{6}cm$ (⊥ from centre to chord) $PQ = 20\sqrt{6}cm$
9.3	(a)	$OB = OE = x + 8$ (Radii)
	(b)	$DB = 12$ (Line from centre to midpoint of chord) $OB^2 = x^2 + 12^2$ (Pythagoras theorem) $(x+8)^2 = x^2 + 12^2$

		$16x = 144 - 64$ $x = 5$ $\therefore OB = 13cm$
9.4	(a)	$AO^2 = 25$ $AC^2 + OC^2 = 9 + 16 = 25$ $\therefore AO^2 = AC^2 + OC^2$ (converse Pythagoras Theorem) $OC \perp AB$
	(b)	The **perpendicular bisector** of a chord **passes through the centre** of the circle
9.5		$AB = BF = EF = EA = x$ Area of square $= EF^2 = x^2$ In ΔOBF $OF^2 = 25 - x^2$ (Pythagoras theorem) In ΔOAE $OE^2 = 25 - x^2$ (Pythagoras theorem) $OF = OE$ EF= 2OF $x = 2\sqrt{25 - x^2}$ $x^2 = 20$ $\therefore$ Area of square = 20 square units

LESSON 10: GR. 11 EUCLIDEAN GEOMETRY		
10.1	(a)	$x = 47,5°$ ($\angle$ at centre $= 2$ x $\angle$ at circumference) $B\hat{O}D = 265°$ (Revolution) $y = 132,5°$ ($\angle$ at centre $= 2$ x $\angle$ at circumference)
	(b)	$x = 40°$ (sum $\angle$s of Δ) $y = 20°$ ($\angle$ at centre $= 2$ x $\angle$ at circumference)
	(c)	$P\hat{O}Q = 20°$ (Alt. $\angle$s equal $OQ \parallel RP$) $x = 10°$ ($\angle$ at centre $= 2$ x $\angle$ at circumference)
10.2	(a)	$\hat{O}_1 = 2(32°) = 64°$ ($\angle$ at centre $= 2$ x $\angle$ at circumference)

		$2\hat{B}_1 + 64° = 180°$ (sum of $\angle$s in Δ) $\hat{B}_1 = 58°$
	(b)	$\hat{B}_2 + 32° + 15° + 58° + 58° = 180°$ (sum $\angle$s of Δ) $\hat{B}_2 = 17°$

LESSON 11: GR. 11 EUCLIDEAN GEOMETRY		
11.1	(a)	$\hat{Q} = 90°$ ($\angle$ in semicircle) $x = 180° - 90° - 55°$ (sum of $\angle$s in Δ) $x = 35°$
	(b)	$x = 90° - 52°$ ($\angle$ in semicircle) $x = 38°$
	(c)	$a = 120°$ ($\angle$s on a straight line) $x = \hat{A}_2$ (OA=OB, Radii) $2x = 180° - 60°$ ($\angle$ in semicircle) $x = 60°$ $\hat{C} = 90°$ ($\angle$ in semicircle) $y = 68°$ (sum of $\angle$s in Δ)
	(d)	$y = 22°$ ($\angle$s in the same segment) $x = 18°$ ($\angle$s in the same segment)
	(e)	$\hat{E} = 35°$ (Alt. $\angle$s, DE || GF $x = 35°$ ($\angle$s in the same segment)
	(f)	$x = 32°$ ($\angle$s in same the segment) $\hat{B}_1 = 180° - 92° - 32° = 56°$ (sum of $\angle$s in Δ) $x = 56°$ ($\angle$s in same the segment)
11.2	(a)	$x = 15°$ (equal chords sub. Equal $\angle$s)
	(b)	$\hat{R}_1 = x$ (equal chords sub. Equal $\angle$s) $2x = 180° - 35°$ (sum of $\angle$s in Δ) $x = 145°$

LESSON 12: GR. 11 EUCLIDEAN GEOMETRY

12.1	(a)	$x = 97°$ (Opp. $\angle$s of cyclic quad) $x = 75°$ (Opp. $\angle$s of cyclic quad)
	(b)	$y = 180° - 40° - 82°$ ($\angle$s of Δ) $y = 58°$ $x = 98°$ (Opp. $\angle$s of cyclic quad) $z = 180° - 62° - 98°$ (sum of $\angle$s in Δ) $z = 20°$
	(c)	$\hat{C} = 96°$ (Opp. $\angle$s of cyclic quad) $2x = 180° - 96°$ (sum of $\angle$s in Δ) $x = 42°$
	(d)	$x = 130°$ (Opp. $\angle$s of cyclic quad) $y = 90°$ ($\angle$ in semicircle) $z = 180° - 90° - 50° = 40°$ (sum of $\angle$s in Δ)
	(e)	$x = 110°$ (Ext. $\angle$ of cyclic)
	(f)	$x = 125°$ (Ext. $\angle$ of cyclic = int opp. $\angle$) $y = 90°$ ($\angle$ in semicircle) $z = 125° - 90° = 35$ (Ext. $\angle$ Δ)
12.2		$\hat{Q} = \hat{R}$ (equal chords sub. Equal $\angle$s) $P\hat{M}L = \hat{R}$ (equal chords sub. Equal $\angle$s) $\therefore \hat{Q} = P\hat{M}L$ LMRQ is a cyclic quad. (converse Ext. $\angle$ of cyclic
12.3		ADEB is a cyclic quad (Converse Ext. $\angle$ of cyclic) EMDC is a cyclic quad (converse Opp. $\angle$s of cyclic quad)

LESSON 13: GR. 11 EUCLIDEAN GEOMETRY		
13.1	(a)	$x + 40° = 90°$ (rad $\perp$ tan) $x = 50°$ $y = 90°$ ($\angle$ in semicircle) $m + 50° + 90° = 180°$ (sum of $\angle$s in Δ) $m = 40°$
	(b)	$x + 60° = 90°$ (rad $\perp$ tan)

		$x = 30°$ $\hat{O}_1 + 30° + 30° = 180°$ (sum of $\angle$s in Δ) $\hat{O}_1 = 120°$ $\hat{O}_1 = 2y$ $\angle$ at centre = 2 x $\angle$ at circumference) $y = 60°$
	(c)	$2y = 140°$ $\angle$ at centre = 2 x $\angle$ at circumference) $y = 70°$ $y = A\hat{B}C$ (Alt. $\angle$s, DE || GF) $A\hat{B}C = 70°$ $A\hat{B}C + m = 90°$ (rad $\perp$ tan) $m = 90° - 70°$ $m = 20°$ $O\hat{D}B = O\hat{B}D$ (OB=OD, Radii) $2O\hat{B}D = 180° - 140°$ $O\hat{B}D = 20°$ $x = 90° - 20°$ (rad $\perp$ tan) $x = 70°$
13.2		$x = 30°$ (equal chords sub. Equal $\angle$s) $y + 30° + 30° = 180°$ (sum of $\angle$s in Δ) $y = 120°$ $D\hat{B}C = B\hat{D}C = 60°$ (tans from same point) $z + 60° + 60° = 180°$ (sum of $\angle$s in Δ) $z = 60°$

LESSON 14: GR. 11 EUCLIDEAN GEOMETRY		
14.1	(a)	$a = 33°$ (tan-chord theorem) $b = 33°$ (Alt. $\angle$s, OP || SR)
	(b)	$c = 72°$ (tan-chord theorem) $2d + 72° = 180°$ (sum of $\angle$s in Δ) $d = 54°$

	(c)	$i+39°+101°=180°$ (sum of $\angle$ s in Δ) $i=40°$ $j=101°$ (tan-chord theorem) $k=40°$ (tan-chord theorem)
	(d)	$\hat{O}_1=\hat{Q}_1=66°$ (tan-chord theorem) $l=180°-66°-66°$ (sum of $\angle$ s in Δ) $l=48°$
	(e)	$n=34°$ (tan-chord theorem) $o+34°+90°=180°$ (sum of $\angle$ s in Δ) $o=56°$ $m=o=56°$ (tan-chord theorem)
14.2	(k)	$x=40°$ (tan-chord theorem) $\hat{B}_1=90°-59°$ ($\angle$ in semi-circle) $\hat{B}_1=31°$ $y=\hat{B}_1=31°$ ($\angle$ s in same segment)
	(j)	$x=40°+56°$ (tan chord theorem) $x=96°$ $x+y=180°$ (Opp. $\angle$ s of cyclic quad) $y=180°-96°$ $y=84°$

LESSON 15: GR. 11 EUCLIDEAN GEOMETRY		
15.1	(a)	ABCD and BCDE
	(b)1	$\hat{B}_1=\hat{A}$ (Corr. $\angle$ s, BC || AD) $\hat{E}=\hat{A}$ ($\angle$ s in same segment) $\therefore \hat{B}_1=\hat{E}$
	(b)2	$\hat{B}_1=\hat{A}$ (Corr. $\angle$ s, BC || AD) $\hat{B}_1=\hat{D}_1$ (Ext. $\angle$ of cyclic quad) $\hat{D}_1=\hat{A}$

15.2	(a)	$P\hat{C}A = P\hat{A}C$ (2 tans from a common point) $P\hat{C}A = \hat{A}_2$ (Alt. $\angle$s, AD || PC) $\hat{A}_2 = P\hat{A}C$ $\therefore$ AC bisects $P\hat{A}D$
	(b)	$\hat{C}_4 = \hat{B}_1$ (tan-chord thm) $\hat{C}_4 = \hat{D}$ (Alt. $\angle$s, AD || PC) $\hat{D} = \hat{B}_1$ $\hat{D} = \hat{B}_3$ (Ext. $\angle$ of cyclic quad) $\hat{B}_1 = \hat{B}_3$
	(c)	$\hat{A}_1 = A\hat{B}D$ (tan chord theorem) $\hat{A}_1 = A\hat{P}C$ (Corr. $\angle$s, AD || PC) $\therefore A\hat{P}C = A\hat{B}D$
15.3	(a)	$B\hat{A}T = A\hat{B}T$ (2 tans from a common point) $2A\hat{B}T + x = 180°$ (sum of $\angle$s in Δ) $A\hat{B}T = 90° - \frac{x}{2}$
	(b)	$O\hat{B}A + A\hat{B}T = 90°$ (rad $\perp$ tan) $O\hat{B}A = \frac{x}{2}$
	(c)	$A\hat{B}T = \hat{C}$ (tan chord theorem) $\hat{C} = 90° - \frac{x}{2}$
15.4	(a)	$B\hat{A}P = 90°$ (given, BA $\perp$ AQ) $P\hat{C}Q = 90°$ ($\angle$ in semicircle) $\therefore P\hat{C}Q = B\hat{A}P$
	(b)	$\therefore P\hat{C}Q = B\hat{A}P$ (proven) BAPC is a cyclic quad (Converse Ext. $\angle$ of cyclic quad)
	(c)	$B\hat{C}A = 90° - A\hat{C}P$ ($\angle$s in a str. Line) $C\hat{P}Q = 90° - C\hat{Q}P$ (sum of $\angle$s in Δ)

		but $C\hat{Q}P = A\hat{C}P$ (tan chord theorem) $B\hat{C}A = C\hat{P}Q$ $\therefore B\hat{C}A = A\hat{B}C$ AB=AC (side opp. $= \angle$ s)
LESSON 16: GR. 11 EUCLIDEAN GEOMETRY		
16.1	(a)	$\hat{C} = 90°$ ($\angle$ in semicircle) $O\hat{D}C + \hat{C} = 180°$ (Co-int. $\angle$ s suppl, OE || BC) $O\hat{D}C = 90°$ $\therefore$ AD = DC ($\perp$ from centre to chord)
	(b)	$A\hat{B}E = O\hat{E}B$ (OB=OE, Radii) $C\hat{B}E = O\hat{E}B$ (Alt. $\angle$ s, OE || BC) $\therefore A\hat{B}E = C\hat{B}E$ EB bisect $A\hat{B}C$
	(c)	$A\hat{B}E = C\hat{B}E = x$ $B\hat{A}C + \hat{C} + \hat{B} = 180°$ ($\angle$ s of Δ) $B\hat{A}C = 90° - 2x$
	(d)	$OA^2 = OD^2 + AD^2$ (Pythagoras theorem) $OA^2 = (OA-1)^2 + 5^2$ $OA = 13°$
16.2	(a)	$S\hat{Q}V = V\hat{S}Q = x$ (2 tans from a common point) $T\hat{V}S = S\hat{Q}V + V\hat{S}Q = x + x$ $T\hat{V}S = 2x$ $S\hat{Q}V = Q\hat{R}S = x$ (tan chord theorem) $T\hat{V}S = 2Q\hat{R}S$
	(b)	$S\hat{Q}V = Q\hat{R}S = Q\hat{P}S = x$ (tan chord theorem) $P\hat{S}R = Q\hat{P}S = x$ (Alt. $\angle$ s, RS || QP) $Q\hat{W}S = P\hat{S}R + Q\hat{R}S$ (Ext. $\angle$ of Δ) $Q\hat{W}S = 2x$

		$T\hat{V}S = 2x$ (proven) $T\hat{V}S = Q\hat{W}S$ QVSW is a cyclic quad. (Converse Ext. ∠ of cyclic quad)
	(c)	$Q\hat{R}S = Q\hat{P}S = x$ (proven) $P\hat{S}R = Q\hat{P}S = x$ (Alt. ∠s, RS || QP) $Q\hat{R}P = Q\hat{S}P = y$ (∠s in the same segment) $P\hat{R}T = x + y$ $Q\hat{S}R = T\hat{Q}S + \hat{T}$ (Ext. ∠ of Δ) $x + y = x + \hat{T}$ $\hat{T} = y$ $\therefore Q\hat{P}S + \hat{T} = P\hat{R}T$
	(d)	$Q\hat{P}S = x$ (tan chord theorem) $Q\hat{W}S = 2x$ (proven) $Q\hat{W}S = 2Q\hat{P}S$ (converse ∠ at centre = 2 x ∠ at circumference)
16.3	(a)	$\hat{D}_3 = 90°$ (∠ in semicircle) $\hat{B} = 90°$ (given, AB ⊥ BE) $\hat{D}_3 = \hat{B}$ ABCD is cyclic quad. (Converse Ext. ∠ of cyclic quad)
	(b)	$\hat{D}_1 = \hat{E}$ (tan chord theorem) $\hat{A}_1 = \hat{D}_1$ (∠s in the same segment) $\therefore \hat{A}_1 = \hat{E}$
	(c)	$\hat{C}_3 = \hat{A}$ (Ext. ∠ of cyclic quad) $\hat{C}_3 = \hat{D}_4$ (tan chord theorem) $\hat{D}_4 = \hat{A}$ $\hat{D}_4 = \hat{D}_2$ (vert. opp. ∠s)

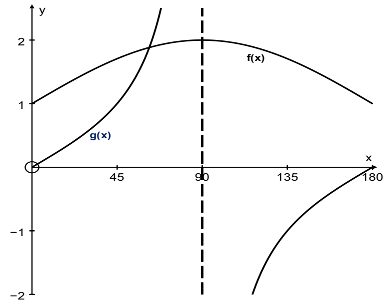
		$\therefore \hat{D}_2 = \hat{A}$ ΔBDA is isosceles
	(d)	$\hat{D}_4 = \hat{D}_2$ (vert. opp. $\angle$s) $\hat{C}_3 = \hat{D}_4$ (tan chord theorem) $\hat{C}_3 = \hat{D}_2$ $\hat{C}_2 = \hat{D}_2$ ($\angle$s in the same segment) $\hat{C}_2 = \hat{C}_3$
16.4		$\hat{A}_1 = \hat{C}$ (tan chord theorem) $\hat{K}_1 = \hat{M}_1$ ($\angle$s opp.= sides) $\hat{K}_1 = \hat{P}_2 + \hat{C}$ (Ext. $\angle$ of Δ) $\hat{M}_1 = \hat{P}_1 + \hat{A}_1$ (Ext. $\angle$ of Δ) $\hat{P}_1 = \hat{P}_2$ KP bisects $A\hat{P}C$
16.5	(a)	$\hat{D}_1 = \hat{B}_1$ (given, AB=AD) $\hat{D}_2 = \hat{B}_2$ (given, DC=BC) $(\hat{D}_1 + \hat{D}_2) + (\hat{B}_1 + \hat{B}_2) = 180°$ (Opp. $\angle$s of cyclic quad) But $\hat{D}_1 + \hat{D}_2 = \hat{B}_1 + \hat{B}_2 = 90°$ AC is a diameter of the circle
	(b)	$\hat{D}_3 = \hat{A}$ (Ext. $\angle$ of cyclic quad) $F\hat{B}E = \hat{A}$ (Ext. $\angle$ of cyclic quad) $\hat{D}_3 = F\hat{B}E$ DBEF is acyclic quad (Converse $\angle$s in the same segment)
	(c)	$\hat{G}_1$=90° $A\hat{B}F$=90° $\hat{G}_1 = A\hat{B}F$=90° $\therefore$ ABGF is a cyclic quad. (converse $\angle$s in the same segment) $\hat{B}_3 = \hat{A}_1$

		but $B_2 = A_1$ $\therefore \hat{B}_3 = \hat{B}_2$ BC bisects $D\hat{B}G$

LESSON 17: TRIGONOMETRY		
ACTIVITY 17		
17.1	$\cos x$	
17.2	$-\tan^2\beta$	
17.3	1	
17.4	$\frac{2}{\sin x}$	
17.5	$\tan\theta$	
LESSON 18: TRIGONOMETRY		
ACTIVITY 18		
18.1	a)	$OP = 5\,units$
	b)	$\sin\theta = \frac{4}{5}$
18.2	a)	$k = -7$
	b)	$\tan\theta = -\frac{24}{7}$
	c)	$-\frac{24}{25}$
	d)	$-\frac{527}{625}$
18.3		
	18.3.1a	$t = -15$
	18.3.2a	$\frac{-8}{17}$
	18.3.2b	$-\frac{32}{17}$
18.4	$-\frac{73}{60}$	
18.5	$\frac{7}{6}$	
18.6	a	$\frac{3\sqrt{34}}{34}$
	b	$\frac{8}{17}$
18.7		
	a	$\frac{k}{\sqrt{1-k^2}}$
	b	$\sqrt{1-k^2}$
	c	1
18.8		
	a	$-\frac{p}{q}$
	b	$-\frac{p}{q}$
	c	-1
18.9	a)	$-t$
	b)	$-\frac{1}{\sqrt{1+t^2}}$
	c)	$-\frac{1}{\sqrt{1+t^2}}$
	d)	t
	e)	$-\frac{1}{t\sqrt{1+t^2}}$
18.10		
	a)	$\sqrt{1+p^2}$
	b)	p
	c)	p
	d)	$-p$
	e)	$-\frac{p}{\sqrt{1-p^2}}$
LESSON 19: TRIGONOMETRY		
ACTIVITY 19		
19.1	a)	-1
	b)	0
	c)	0
	d)	-1
	e)	$-\tan\theta$
19.2		

	a)	$\frac{3}{4}$
	b)	$\frac{1}{2}$
	c)	3
19.3		
	a)	1
	b)	$-\frac{3}{2}$
	c)	$-\frac{5}{2}$
	d)	1
	e)	$\frac{2}{3}$
	f)	$\frac{1}{4}$
	g)	4
	h)	$-\sqrt{2}$
19.4	a)	Proof
	b)	Proof
	c)	Proof
	d)	Proof
	e)	6
19.5	a)	Proof
	b)	Proof
	c)	Proof
	d)	Proof
	d)	Proof
	e)	Proof
	f)	Proof
	g)	Proof

LESSON 20: TRIGONOMETRY	
Classwork	
1	$24,83^\circ + 360^\circ k$ or $155,17^\circ + 360^\circ k; k \in Z$
	$24,83^\circ; 155,17^\circ$
2	$55,43^\circ + 180^\circ k; k \in Z$
3	$84,83^\circ + 360^\circ k$ or $124^\circ + 360^\circ k; k \in Z$
	$84^\circ; 236^\circ$
4	$16,48^\circ + 180^\circ k$ or $16,48^\circ + 180^\circ k; k \in Z$
	$24,83^\circ; 155,17^\circ$
	Homework
1	$\pm 72,54^\circ + 360^\circ k; k \in Z$
2	$-48^\circ; 192^\circ$
3	$-82,62^\circ + 360^\circ k; k \in Z$
4	$\pm 20^\circ + 180^\circ k; k \in Z$
	$-160^\circ; -20^\circ; 20^\circ; 160^\circ; 200^\circ; 340^\circ$
INTERPRETATION OF TRIG GRAPHS	
REVISION WORK	
	Question 1
1.1	$a = -2; b = 2; p = 1; r = 45^\circ$
1.2	$-75^\circ; 105^\circ$
1.3	180°
1.4	$h(x) = \sin \frac{1}{2} x$
	Question 2
2.1	$a = 2; b = 4$
2.2	-2
2.3	180°
2.4.1	$45^\circ \langle x \langle 90^\circ$ or $135^\circ \langle x \langle 270^\circ$
2.4.2	$90^\circ \langle x \leq 180^\circ$

	Homework
	Question 1
1.1	$1 \leq y \leq 2$
1.2	360°
1.3	

2.2.1	2
2.2.2	$-2 \leq y \leq 2$
2.3.1	2
2.3.2	$x = 90^\circ$
LESSON 21:	
	Classwork
1	$4,33^\circ + 120^\circ k$ or $-63^\circ + 360^\circ k; k \in Z$
2	$35^\circ + 90^\circ k$ or $-120^\circ + 360^\circ k; k \in Z$
	$35^\circ; 125^\circ; 215^\circ; 240^\circ$
3	$360^\circ k$ or $180^\circ + 360^\circ k$ or
	$\pm 60^\circ + 360^\circ k; k \in Z$
4	$30^\circ + 360^\circ k$ or $150^\circ + 360^\circ k; k \in Z$
	30°
	Homework
1	$40^\circ + 120^\circ k$ or $-120^\circ - 360^\circ k; k \in Z$
	$-80^\circ; 40^\circ; 160^\circ$
2	$\pm 131,81^\circ + 360^\circ k; k \in Z$

3	$75° + 180°k; k \in Z$
	$75°; 255°$
4	$\pm 90° + 360°k$ or $30° + 360°k$ or
	$150° + 360°k; k \in Z$
LESSON 22:	
	Classwork
22.1	$45° + 180°k; k \in Z$
22.2	$\pm 90° + 360°k$ or $270° + 360°k; k \in Z$
	Homework
22.2a	$\pm 90° + 360°k; k \in Z$
22.2b	$\pm 90° + 360°k$ or $270° + 360°k; k \in Z$
LESSON 23:	
	Classwork: Investigation
	Teacher gives feedback and own answers.
LESSON 24:	
	Classwork: Investigation
	Teacher gives feedback and own answers.