KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

LAST PUSH LEARNER REVISION DOCUMENT

MATHEMATICS: PAPER 1&2

GRADE 12

2024

TABLE OF CONTENTS

	TOPIC	PAGE NUMBER
1.	ALGEBRA, EQUATIONS AND INEQUALITIES	3 – 6
2.	SEQUENCES AND SERIES	7 – 12
3.	FUNCTIONS AND INVERSES	13 – 24
4.	FINANCE, GROWTH AND DECAY	25 – 30
5.	CALCULUS	31 – 38
6.	PROBABILITY	38 – 44
7.	DATA HANDLING	45 – 54
8.	ANALYTICAL GEOMETRY	54 – 65
9.	TRIGONOMETRY	66 – 76
10.	EUCLUDEAN GEOMETRY	77 – 88
11.	ANSWERS	89 – 100

TOPIC	1. Algebra, Equations and Inequalities: [± 25]	
GUIDELINES, SUMMARY NOTES, & STRATEGIES		
CONCEPT	**HOW TO LEARN IT**	**RELEVANT FORMULAE AND KEYWORDSE**
Surd equations	Isolate the surd and square both sides. Remember to check solutions of a surd equation.	
Simultaneous equations	Solve equations with two unknowns, one of which is linear and the other quadratic, algebraically.	Involves making y or x subject of the formula in the linear equation and then substituting into the other equation.
Quadratic formula	Solve quadratic equations (by factorisation, completing the square and using quadratic formula)	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Solve exponential equations	Apply the laws of exponents to expressions involving rational exponents	
Inequalities	Remove brackets, standard form, factorise, critical values, method and solve	
Nature of roots	The nature of roots and the conditions for which the roots are real, non-real, equal, unequal, rational and irrational.	$\Delta = b^2-4ac$ $\Delta<0$ non-real/imaginary $\Delta\geq 0$ real $\Delta=0$ real and equal (1 root) $\Delta>0$ real and unequal (2 roots) $\Delta>0$ and a perfect square eg $\Delta=16$ $\Delta>0$ and not a perfect square eg $\Delta=20$

ACTIVITIES

1. Solve for x

1.1	$-5x(1-4x)=0$	(2)	**L1**
1.2	$(1-3x)(x+4)=0$	(2)	**L1**
1.3	$(x+3)(x-1)=-x+1$	(2)	**L1**
1.4	$2x^2-5x+3=0$	(3)	**L1**
1.5	$x=\frac{5}{3x-2}$	(4)	**L2**
1.6	$10x=3x^2-8$	(3)	**L1**
1.7	$2x+p=p(x+2)$ (stating any restriction)	(4)	**L2**
1.8	$3x^2=4x$	(3)	**L1**
1.9	$x^{\frac{1}{2}}-3x^{\frac{1}{4}}-10=0$	(4)	**L2**
1.10	$x(x-1)=2(x-1)$	(3)	**L2**
1.11	$x=\frac{a^2+a-2}{a-1}$ if $a=888\ 888\ 888\ 888$	(2)	**L2**

1.12 $x^{-1} - x^{-2} = 20$ (4) **L2**

1.13 $x^{x} = 2^{2048}$ (4) **L3**

MIXED PROBLEMS

1.14 Solve for x if: $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + ...}}}}$ **(NW SEPT 2019)** (4) **L4**

1.15 If $(x-3)(y+4) = 0$ determine x if: **(NW SEPT 2021)**

$a)$ $y = 4$ (1) **L1**

$b)$ $y = -4$ (1) **L1**

1.16 Given $k + 5 = \dfrac{14}{k}$

$a)$ Solve for k (3) **L2**

$b)$ Hence or otherwise, solve for x if $\sqrt{x+5} + 5 = \dfrac{14}{\sqrt{x+5}}$ (3) **L2**

1.17 If $x - 6 = 0$ is one of the solutions of the equation $x + \dfrac{40}{x} = 16$, determine ONE value of y for which $2y + 3 + \dfrac{40}{2y+3} = 16.$ (3) **L2**

1.18 Calculate the maximum value of S if $S = \dfrac{6}{x^2 + 2}$ (2) **L2**

1.19 Consider the equation: $x^2 + 5xy + 6y^2 = 0$

$a)$ calculate the values of the ratio $\dfrac{x}{y}$. **(ANSWER SERIES, A3,5)** (5) **L3**

$b)$ Hence, calculate the values of x and y if $x + y = 8$ (2) **L3**

1.20 Given: $(3x - y)^2 + (x-5)^2 = 0.$
Solve for x and y. (4) **L3**

1.21 Consider: $5x^2 - kx + 16 = (x+2).Q(x) + 10$ where k is a constant and $Q(x)$ is a polynomial in terms of x. Calculate k. **(NW SEPT 2020)** (4) **L3**

1.22 Prove that $x^2 + 2xy + 2y^2$ cannot be negative for $x,\ y \in R$. (4) **L4**

2 Solve for x (leave your answer to TWO decimal places not unless otherwise stated)

2.1 $x(3x-5) = 7$ (4) **L2**

2.2 $x^2 - 2x = 5$ (correct to THREE decimal places) (4) **L1**

2.3 $3x^3 + x^2 - x = 0$ (5) **L2**

2.4 $-4x^2 + 3x + 6 = 0$ (answer correct to TWO decimal place) (3) **L1**

2.5 $2x^2 - 1 = -7x$ (correct to ONE decimal place) (4) **L2**

3 **Inequalities**

Solve for x

3.1 $x^2 - 4 \geq 5$ (3) **L2**

3.2 $(x+1)(x-3) > 12$ (4) **L2**

3.3 $x^2 - 2x \leq 15$ (4) **L2**

3.4 $(x-1)(x-4) > x + 11$ (5) **L2**

3.5 $\frac{4}{(x-1)(x-5)}<0$ (4) **L3**

3.6 $(2-x)(1-x)^2 \leq 0$ (3) **L3**

3.7 Given: $x^2-x-20<0$

a) Solve for x if $x^2-x-20<0$. (3) **L2**

b) Hence, or otherwise, determine the sum of all the integers satisfying the inequality $x^2-x-20<0$. (2) **L2**

3.8 Solve for x if $x^2-4x\leq 21$ (4) **L3**

4 SURDS AND EXPONENTS

Solve for x

4.1 $12^{5+3x}=1$` (4) **L1**

4.2 $\sqrt[x]{32}=128$ (3) **L2**

4.3 $3^{x+1}-4+\frac{1}{3^x}=0$ (4) **L2**

4.4 $12^{2x}=8.36^x$ (4) **L3**

4.5 $2\sqrt{2-7x}=\sqrt{-36x}$ (3) **L2**

4.6 $\sqrt{2-x}-x=-2$ (4) **L2**

4.7 $26-5^{2x}=(5^x-6)^2$ (4) **L2**

4.8 Show that the equation $2^{2x+1}+7.2^x-4=0$ has only ONE solution. (4) **L2**

SIMPLIFICATION

4.9 Simplify the following, $\frac{x^2}{1+x}$ if $x=1+\sqrt{3}$ (4) **L2**

4.10 If $\frac{1}{\sqrt{m}+\sqrt{m+1}}=\sqrt{m+1}-\sqrt{m}$, determine, without the use of a calculator, the exact value of: $\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+...+\frac{1}{\sqrt{1680}+\sqrt{1681}}$ (3) **L3**

4.11 Consider the product: $1\times2\times3\times4\times......\times30$

Determine the largest value of k such that 2^k is a factor of this product. (4) **L4**

4.12 If $3^{9x}=64$ and $5^{\sqrt{p}}=64$, calculate, WITHOUT the use of a calculator, the value of: $\frac{(3^{x-1})^3}{\sqrt{5^{\sqrt{p}}}}$ **(NOV 2018)** (4) **L4**

4.13 If n is the largest integer for which $n^{200}<5^{300}$, determine the value of n. **(NOV 2020)** (3) **L4**

Simplify the following WITHOUT the use of a calculator. Show all workings.

4.14 $\sqrt{\frac{2^{x+3}-2^{x+1}}{2.2^x}+1}$ (4) **L2**

4.15 $\sqrt{\frac{15^x.3^x}{9^{x+1}.5^{x-2}}}$ (4) **L2**

4.16 $\frac{\sqrt{m^{2022}-m^{2020}}}{\sqrt{25m^{2024}-25m^{2022}}}$ (4) **L4**

4.17 $\sqrt[n]{\dfrac{10^n + 2^{2+n}}{5^{2n} + 4.(5^n)}}$ where $n \neq 0$ (4) **L3**

4.18 $\dfrac{8^{n-3}.10^{n+2}}{8^{n-1}.5^{1+n}}$ (4) **L2**

5 **NATURE OF ROOTS**

5.1 Without solving the equation, show that $x^2 + 5mx + 6m^2 - 1 = 0$ has real, unequal roots for all values of m. (4) **L3**

5.2 Given $2mx^2 = 3x - 8$ where $m \neq 0$.
Determine the value(s) of m for which the roots of the equation are non-real. (4) **L3**

5.3 Given that $f(x) = x^2 - px + 8 + 2p$ has two equal root and $p < 0$, determine the coodinates of the turning point, $h(x) = f(x) - 3$. (5) **L3**

5.4 Given: $(x+5)^2 = 1 - p^2$
Calculate the values of p for which the roots of the equation are non-real. (5) **L3**

5.5 The roots of a quadratic equation are given by $x = \dfrac{-5 \pm \sqrt{20 + 8k}}{6}$,
where $k \in \{-3; -2; -1; 0; 1; 2; 3\}$

a) Write down TWO values of k for which the roots will be rational. (2) **L1**

b) Write down ONE value of k for which the roots will be non-real. (1) **L1**

5.6 What value(s) should k represent so that the nature of the roots of the following two equations will be the same?
$x^2 - x + 3 = 0$ and $kx^2 + kx + 4 = 0$ (4) **L3**

6 **SIMULTANEOUS EQUATIONS**

6.1 If $f(2) = 0$ and $f(-6) = 0$, determine an equation for $f(x)$ in the form $f(x) = x^2 + bx + c$ (3) **L2**

Solve for the UNKNOWN in the following simultaneous equations:

6.2 $y + 7 = 2x$ and $x^2 - xy + 3y^2 = 15$ (6) **L2**

6.3 $-2y + x = -1$ and $x^2 - 7 - y^2 = -y$ (6) **L2**

6.4 $xy = 9$ and $x - 2y - 3 = 0$ (6) **L2**

6.5 $y = -2x + 7$ and $\dfrac{y+5}{x-1} = \dfrac{1}{2}$ (6) **L2**

6.6 $\dfrac{1}{x} + \dfrac{1}{y} = 3$ and $x - y = \dfrac{1}{2}$ (6) **L2**

6.7 $6r + 5rp - 5p = 8$ and $r + p = 2$ (6) **L2**

6.8 $2^{y-3x} = \dfrac{1}{16}$ and $x^2 + xy = 24$ (7) **L2**

6.9 $3^x - 9^{y+2} = 0$ and $x^2 + y^2 + 2xy + y^2 = 36$ (7) **L2**

TOPIC: SEQUENCES AND SERIES [25±3]

GUIDELINES, SUMMARY NOTES & STRATEGIES

Quadratic Pattern:

Definition: Second difference are equal where the first term forms an arithmetic sequence.

General term: $T_n = an^2 + bn + c$ **. To calculate the values of** a, b **and** c**:**

	$n=1$		$n=2$		$n=3$		$n=4$
T_n	$a+b+c$		$4a+2b+c$		$9a+3b+c$		$16a+4b+c$
1st difference		$3a+b$		$5a+b$		$7a+b$	
2nd difference			$2a$		$2a$		

NB: For a **MINIMUM** or **MAXIMUM** term: $n = \frac{-b}{2a}$ or $\frac{dT_n}{dn} = 0$ i.e. First derivative then solve for n and then substitute the value of n in the original equation.

Arithmetic number patterns:

Definition: All first differences are equal, i.e. you always add or subtract a constant difference

NB: $T_2 - T_1 = T_3 - T_2$ (same difference; d)

General term: $Tn = a + (n-1)d$

$d = T_2 - T_1$

This formula can be used to determine the value of any specific term of an arithmetic sequence.

Sum of n Terms: $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}(a+l)$

Where l is the last term or T_n

Geometric number patterns:

Definition: There exists constant ratio, i.e. you multiply by the same ratio.

NB: $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ (common ratio; r)

General term: $T_n = ar^{n-1}; r = \frac{T_2}{T_1}$

Sum of n terms: $S_n = \frac{a(r^n - 1)}{r-1}$ or

$S_n = \frac{a(1-r^n)}{1-r}$

Sum to infinity: $S_\infty = \frac{a}{1-r}$

NB: A given sum formula can be used to determine the terms of a sequence**.** $T_n = S_n - S_{n-1}$

ACTIVITIES

NSC NOV 2023

1.1 Given the arithmetic series: $7 + 12 + 17 + ...$

1.1.1 Determine the value of T_{91} (3) **L1**

1.1.2 Calculate S_{91} (2) **L1**

1.1.3 Calculate the value of n for which $T_n = 517$ (3) **L2**

1.2 The following information is given about a quadratic number pattern: $T_1 = 3$, $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$

1.2.1 Show that $T_5 = 111$ (2) **L2**

1.2.2 Show that the general term of the quadratic pattern is

$T_n = 6n^2 - 9n + 6$ (3) **L3**

1.2.3 Show that the pattern is increasing for all $n \in N$. (3) **L3**

1.3 Given the geometric series: $3 + 6 + 12 + ...$ to n terms.

1.3.1 Write down the general term of this series. (1) **L1**

1.3.2 Calculate the value of k such that:

$\sum_{p=1}^{k} \frac{3}{2}(2)^p = 98301$ (4) **L3**

1.4 A geometric sequence and an arithmetic sequence have the same first term.
The common ratio of the gemoetric sequence is $\frac{1}{3}$
The common difference of the arithmatic sequence is 3
The sum of 22 terms of the arithmetic sequance is 734 more than the sum to infinity of the geometric sequence.
Calculate the value of the first term. (5) **L4**

MPUMALANGA JUNE 2024

1.5 Write the following series in sigma notation

$\frac{1}{3}+\frac{4}{9}+\frac{7}{27}+\frac{10}{81}+\ldots+\frac{22}{6\ 561}$ (4) **L3**

1.6 Consider the geometric series: $(2x-4)+(4x^2-16)+\ldots$

For which value(s) will the series converge? (4) **L3**

KZN JUNE 2024 PRACTICE PAPER

2.1 Given a quadratic number pattern:
$-120;-99;-80;-63;\ldots$

2.1.1 Determine the n^{th} term. (4) **L2**

2.1.3 Which value must be added to T_n for the sequence to have only one value of n for which $T_n=0$ (4) **L3**

2.2 Given a finite arithmetic series: $9+14+19+\ldots+124$.

2.2.1 Determine the general term of this series. (2) **L1**

2.2.2 Write the series in sigma notation. (3) **L2**

2.3 Given: $5;10;20;\ldots$ a geometric

2.3.1 Determine the n^{th} term. (1) **L1**

2.3.2 Calculate the sum of the first 18 terms. (2) **L1**

2.4 The first and the second terms of a geometric first term of 2 and $r=\frac{1}{\sqrt{2}}$
Calculate the sum to infinity divide by the sum of two terms (3) **L1**

KZN MARCH 2024

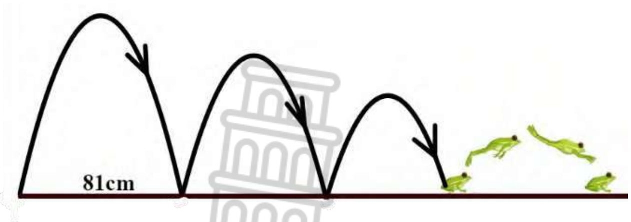
3.1 Consider the arithmatic sequence: $8;15;22;\ldots$

3.1.1 Determine the 36^{th} term. (2) **L1**

3.1.2 Calculate the sum of the first 36 terms. (2) **L1**

3.1.3 If it is given that $T_{72}+T_{72-m}=786$, determine the value of m. (4) **L3**

3.2 The diagram alongside represents a frog making a series of jumps. With every next jump, he has only enough energy left to jump $\frac{2}{3}$ the distance of his previous jump



3.2.1 If his first jump is $81cm$ long, calculate the length of his second jump. (1) **L1**

3.2.2 Determine the length of his ninth jump. (2) L1

3.2.3 If the frog continues to jump in this way, will he be able to catch a trapped insect that is $230\ cm$ away from his tarting position? Show all your calculations. (3) L2

3.3 The given number pattern is a combination of a quadratic sequence and an arithmetic sequence:

$16;32;0;28;-12;24;-20;20;...$

3.3.1 Determine the general term of the quadratic sequence. (4) **L2**

3.3.2 Determine the general term of the arithmetic sequence (2) **L2**

3.3.3 The given number patterns has two consecutive terms that are equal in value.

Determine the positions of the two terms. (4) **L3**

3.4 Calculate: $\sum_{k=3}^{9} 2(-3)^k$ (4) **L2**

KZN JUNE 2023

3.5 $T_n = -2n^2 + 40n + 103$ is the general term of a quadratic sequence.

3.5.1 Determine T_1 the first term of the quadratic sequence. (2) **L1**

3.5.2 Determine the second difference of this quadratic sequence. (2) **L1**

3.5.3 Which term of the quadratic sequence has a value of 301? (2) **L2**

3.5.4 Which term is the largest term in this quadratic sequence? (2) **L2**

KZN JUNE 2023

4.1 The following sequence is a combination of arithmetic and geometric sequence: $3;3;9;6;15;12;...$

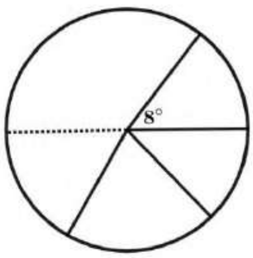
4.1.1 Write down the next TWO (2) **L1**

4.1.2 Determine $T_{20} - T_{21}$ (5) **L3**

4.2 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$

4.2.1 Prove, without the use of a calculator, that the sum of the series to infinity is $16+8\sqrt{2}$ (4) **L3**

4.3 The given circlr is completely divided into n Sectors in such a way that the angles are in Arithmetic sequence. If the smallest angle is $8°$ And the largest angle is $52°$, calculate n, the Number of sectors.



(4) **L3**

4.4 George is currently 115 kilograms and his doctor has advised him to take up a healthy eating plan so as lose some of the weight in order to maintain a healthy body. His new diet will enable him to lose 5 kilogram in the First week, thereafter he would lose $\frac{3}{4}$ of the previous week's weight loss each week.

4.4.1 If he follows the new eating plan, how many kilograms will be lose on the eighth day? Give your answer correct to 3 decimal places. (2) **L1**

4.4.2 If he follows the new eating plan indefinetely, write weight loss in sigma notation. (2) **L2**

4.4.3 If he follows the new eating plan indefinitely, what will his new weight eventually be? (3) **L3**

WBHS/SEPT 2017

5.1 The p^{th} term of the first difference of a quadratic sequence is given by $T_p = 3p - 2$

5.1.1 Determine between which two consecutive terms of the quadratic sequence the first difference is equal to 1450. (4) **L3**

5.1.2 If $T_{40} = 2290$ and $T_n = an^2 + bn + c$ n^{th} term of the quadratic sequence Calculate the value of c. (4) **L3**

DBE/MAY-JUNE 2024

6.1 Consider the geometric series:

$4+2+1+\frac{1}{2}+...$

6.1.1 Does this series converge? Justify your answer. (2) **L1**

6.1.2 Calculate S_∞ (2) **L1**

6.2 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\ 520$. Calculate the value of k (5) **L3**

6.3 Consider the quadratic number pattern:

$3;7;12;...$

6.3.1 Show that the general erm of this number is given by $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$

6.3.2 What must be added to T_{n-1} so that $T_n = 13527$? (3) **L2**

6.4 Given an arithmetic sequence with T_1=8 And $T_2 = 11$

Calculate the value of n if $T_n = 41$. (3) **L2**

6.5 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence. For the new sequence,

6.5 Calculate the value of the first term of the new arithmetic sequence (4) **L3**

KZN SEP 2023

7.1 The values below are the consecutive terms of a quadratic sequence the 4^{th} term is 49.

$-;-;-;49;77;111;151;...........$

7.1.1 Determine the third term of the quadratic sequence. (1) **L1**

7.1.2 Determine the general term, T_n of the quadratic sequence. (4) **L2**

7.1.3 Between which two consecutive terms of the quadratic sequence is the first difference 418? (3) **L2**

7.2 The first two terms of a geometric sequence are x and $x+1$.

7.2.1 Write down the common ratio. (1) **L1**

7.2.2 Write down the third term. (2) **L2**

7.2.3 If $x = 2$, will the sequence converge? Motivate your answer. (2) **L2**

7.3 The given sequence below has **four terms only**, such that the first three terms, $T_1;T_2$ and T_3 form an arithmetic

$T_2;T_3$ and T_4 form a geometric sequence: $6;a;b;16$

Calculate the values of a and b. (5) **L3**

DBE/MAY-JUNE 2023

8.1 Given the first three terms of an arithmetic sequence:

$x+5;\ 37-x\ ;\ x+13$

8.1.1 Determine the value of x (3) L1

8.1.2 Determine the general T_n of the sequence. (3) **L2**

KZN SEPT 2023

9.1 Consider the arithmetic sequence:

$3;7;11;...;399$.

9.1.1 Determine the twentieth term of the sequence. (2) **L1**

9.1.2 How many terms are in this sequence? (2) **L1**

9.2 The first term of an arithmetic sequence is a and the thirteenth term $a+24$

9.2.1 Determine the common

LIMPOPO PRE-MIDYEAR 2024

9.3 Given a geometric sequence:

36 ; -18 ; 9 ; …

9.3.1 Determine the value of r, the common ratio. (1) **L1**

9.3.2 Calculate n if $T_n = \frac{9}{4096}$ (3) L2

9.3.3 Calculate S_∞ (2) **L1**

9.3.4 Calculate the value of

$$\frac{T_1+T_3+T_5+T_7+...+T_{499}}{T_2+T_4+T_6+T_8+...+T_{500}}$$ (4) **L3**

9.4 The first three terms of an arithmetic difference of the sequence. (2) **L2**

9.2.2 Hence determine the sum of the first 200 terms in terms of a. (2) **L2**

10 Given the arithmetic sequence $85 ;82 ;79 ;76 ;\ldots$

10.1 Determine a simplified expression for T_n. (3) **L1**

10.2 Which term would be the first negative number in the sequence? (3) **L3**

11 A quadratic sequence, with general term T_n, has the following properties:

$T_{11} = 190$

$T_n - T_{n-1} = 4n - 2;\ n \geq 2$

Determine the first term of the quadratic sequence. (5) **L3**

12 The sum of the first 50 terms of an arithmetic sequence is 1 275. Calculate the sum of T_{25} and T_{26} (3) **L3**

13.1 For which values of x will $\sum_{k=1}^{\infty}(4x-1)^k$ exist? (4) **L2**

13.2 Given the quadratic sequence: $-5 ; 12 ; 27 ; \ldots$

Calculate the value of:

$$\sum_{n=3}^{10} T_n - \sum_{n=11}^{17} T_n$$ (3) **L2**

DOE NOV 2009

15 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: $5;9;13;17;21;\ldots$

Tebogo's sequence: $5;125;3\ 125;78\ 125;1\ 953\ 125;\ldots$

15.1 Write down the n^{th} term (or the rule in terms of n) of:

(a) Matthew's sequence: (3) **L1**

(b) Tebogo's sequence (2) **L1**

16 Nomsa generates a sequence which is

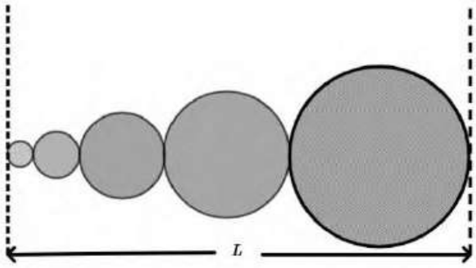
sequence are: $2p + 3$; $p + 6$ and $p - 2$.

9.4.1 Show that $p = 11$. (2) **L1**

9.4.2 Calculate the smallest value of n for which $T_n < -55$. (3) L3

GAUTENG JUNE 2024

14 The figure below shows a pattern of 5 circles, touching externally, whose centres lie on a straight line of length L units. The radii of these circles from a geometric pattern, where the radius of the smallest circle is 3 units and that of the fifth (largest) circle is 48 units.



14.1 Determine the common ratio of the geometric pattern formed by the radii of the circles. (3) **L2**

14.2 Determine the value of L. (3) **L3**

14.3 The pattern is extended by 5 more circles to circles. Calculate, in terms of π, the total area of the 10 circles of the new pattern. **L3** (4)

17 Given: $\sum_{t=0}^{99}(3t-1)$

17.1 Write down the first THREE terms of the series. (1) **L1**

17.2 Calculate the sum of the series. (4) **L2**

18 The following sequence of numbers forms a quadratic sequence: $-3;-2;-3;-6;-11$

18.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5) **L3**

PLATINUM CONTROL TEST BOOK

19 If the second term of an arithmetic sequence is 15 and the fifth term is 24,

determine the third and fourth terms of the sequence. (5) **L2**

20 The sum of the first 53 terms of an arithmetic series is 4 240, while the seventh term is equal to 20.
Find the first term and the common difference. (6) **L3**

21 If $\sum_{m=2}^{8} x.2^{1+m} = 612$

a) Find the value of x (6) **L3**

b) Hence, Write down the first 2 terms (2) **L1**

differences. (3) **L2**

18.2 Calculate the first difference between the 35th and 36th terms of the quadratic sequence. (2) **L2**

DBE/FEB. – MAR. 2011

22 The sum to n terms of a sequence of numbers is given as $S_n = \frac{n}{2}(5n+9)$

22.1 Calculate the sum to 23 terms of the sequence. (2)

22.2 Hence calculate the 23rd term of the sequence. **L1** (3) **L1**

23 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence. Determine TWO possible values for the common ratio, r, of the geometric sequence. (6) **L3**

NSC NOV 2011

24 Given the sequence:
$4; x; 32$

24.1 Determine the value(s) of x if the sequence is

(a) Arithmetic (2) **L1**

(b) Geometric (3) **L1**

25 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d, the sum to n terms can be expressed as $S_n = \frac{n}{2}[2a+(n-1)d]$ (4) **L2**

26 Consider: $3; 3; 9; 6; 15; 12; \ldots$

26.1 Write down the next TWO terms. (2) **L1**

26..2 Calculate $T_{52} - T_{51}$. (5) **L3**

27 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2) **L2**

FUNCTIONS AND INVERSES

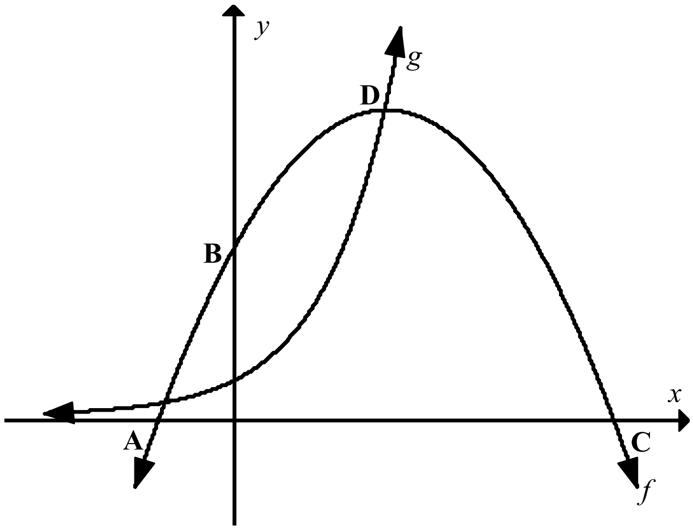
Straight Line	Parabola	Hyperbola	Exponential
$y=mx+c$ m … gradient and c … y-intercept	$y=a(x+p)^2+q$ **Axis of symmetry with equation** $x=-p$ **Maximum** or **minimum value** $(-p;q)$ Turning point	$y=\frac{a}{x+p}+q$ **Vertical asymptote with equation** $x=-p$ **Horizontal asymptote with equation** $y=q$	$y=a.b^{x+p}+q$ $b>0$ **and** $b\neq 1$ **Horizontal asymptote with equation** $y=q$
$m<0$ **… graph is decreasing** $m>0$ **… graph is increasing**	$a<0$ **… graph faces downwards(concave down) and has a minimum turning point** $a>0$ **… graph faces upwards (concave up) and has a maximum turning point**	$a<0$ **… graph is on the second and the fourth quadrant** $a>0$ **… graph is on the first and the third quadrant**	$a<0$ **… graph is below the asymptote** $a>0$ **… graph is above the asymptote**
Domain: $x\in R$ **Range:** $y\in R$	**Domain:** $x\in R$ **Range:** $y>q$ **if** $a>0$ $y<q$ **if** $a<0$	**Domain:** $x\in R$, $x\neq -p$ **Range:** $y\in R$, $y\neq q$	**Domain:** $x\in R$ **Range:** $y>q$ **if** $a>0$ $y<q$ **if** $a<0$
$y-y_1=m(x-x_1)$	$y=ax^2+bx+c$ **Axis of symmetry:** $x=\frac{-b}{2a}$ $y=a(x-x_1)(x-x_2)$ x_1 **and** x_2 **are x-intercepts**	**Axis of symmetry/lines of symmetry:** $\left\{\begin{matrix} y=x+c \\ y=-x+c \end{matrix}\right\}$ **substitute point of intersection of asymptotes** **OR** $\left\{\begin{matrix} y=(x-p)+q \\ y=-(x-p)+q \end{matrix}\right\}$	

INVERSE FUNCTION		
Straight line	**Parabola**	**Exponential**
$y=mx+c$	$y=ax^2$	$y=b^x$
Inverse is a function $x=my+c$ $y=\frac{x}{m}-\frac{c}{m}$	**Inverse is not a function** $x=ay^2$ $y=\pm\sqrt{\frac{x}{a}}$ **Restrict domain of** $y=ax^2$ **so that the inverse is a function** **Restrictions:** $\left\{\begin{matrix} x\geq 0 \\ x\leq 0 \end{matrix}\right\}$	**Inverse is a function** $x=b^y$ $y=\log_b x$
Domain: $x\in R$ **Range:** $y\in R$	**Domain:** $x\geq 0$ **or** $x\leq 0$ **Range:** $y>0$ **if** $a>0$ $y<0$ **if** $a<0$	**Domain:** $x>0$ **Range:** $y\in R$

REVISION QUESTIONS

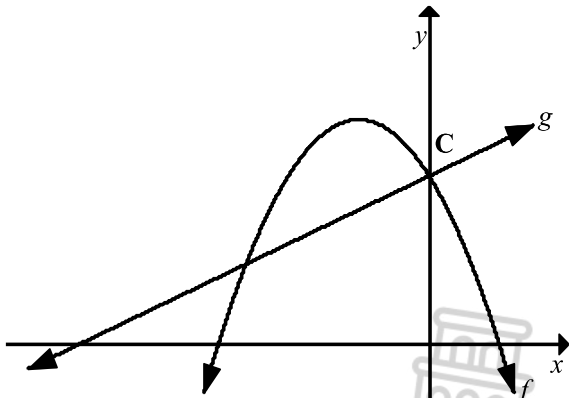
QUESTION 1 **GP/JUNE 24**

1.1 The sketch alongside, shows the graph of $f(x) = -(x-2)^2 + 9$ and $g(x) = b^x$ where b is a constant. D is a turning point of *f* and and a point of intersection of *f* and *g*. B is the *y*-intercept and A and C, the *x*-intercepts of *f*.



1.1.1	Determine the length of AC.	(4)	**L2**
1.1.2	Determine the value of b.	(2)	**L2**
1.1.3	Determine the value of *x* for which $g(x) \geq 9$.	(1)	**L1**
1.1.4	Write down the equation of h if $h(x) = f(x+2) - 9$.	(2)	**L3**
1.1.5	How can the domain of *h* be restricted so that h^{-1} will be a function?	(1)	**L1**
1.1.6	Show, algebraically, that $g\left(x+\frac{1}{2}\right) = \sqrt{3}g(x)$.	(2)	**L3**

1.2 Given: $f(x) = ax^2 + bx + c$ and $g(x) = mx + c$.

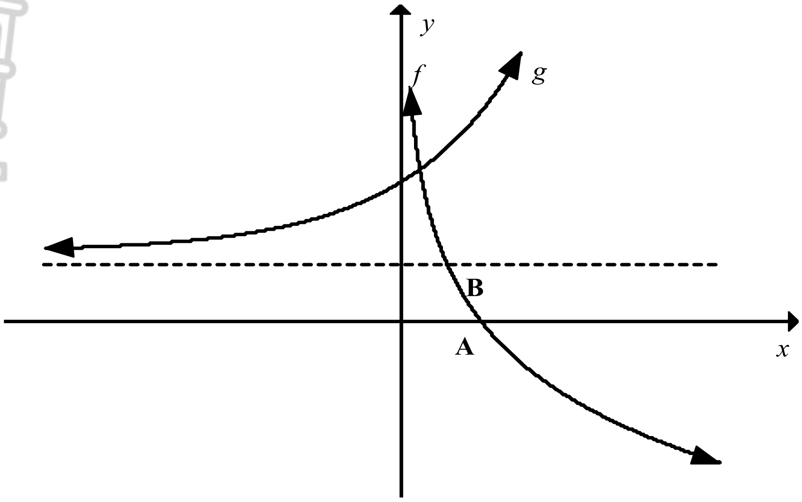


If given $f(x).g(x) < 0$ for all values of *x* when $-6 < x < -3$ or $x > 2$, determine the value of *a* in terms of m (show all workings). (5) **L4**

QUESTION 2 **GP/JUNE 24**

The sketch below shows the graphs of $g(x) = 2^x + q$ and $f(x) = \log_{\frac{1}{2}} x$.

Graph f and the asymptote of g intersect at $B\left(\frac{1}{2}; q\right)$.

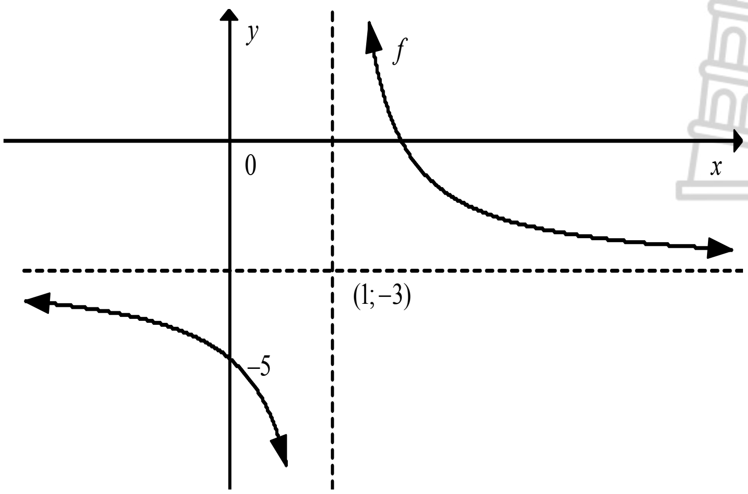


2.1	Write down the coordinates of A, the *x*-intercept of *f*.	(1)	L1
2.2	Determine the domain of *f*.	(1)	L1
2.3	Determine the equation of f^{-1} in the form of $y = ...$	(2)	L2
2.4	Sketch the graph of f^{-1}. Indicate on your graph the intercept(s) with the axis and the coordinates of one other point on the graph.	(3)	L2
2.5	Determine the equation of the asymptotes of *g*.	(1)	L2
2.6	Describe in words the transformation of g to f^{-1}.	(2)	L3

[10]

QUESTION 3 **LIMPOPO/ SEPT 23**

3.1 The graph of $f(x) = \dfrac{a}{x+p} + q$ is sketched below. The asymptotes of *f* intersect at $(1;-3)$. The y-intercept of *f* is $(0;-5)$.



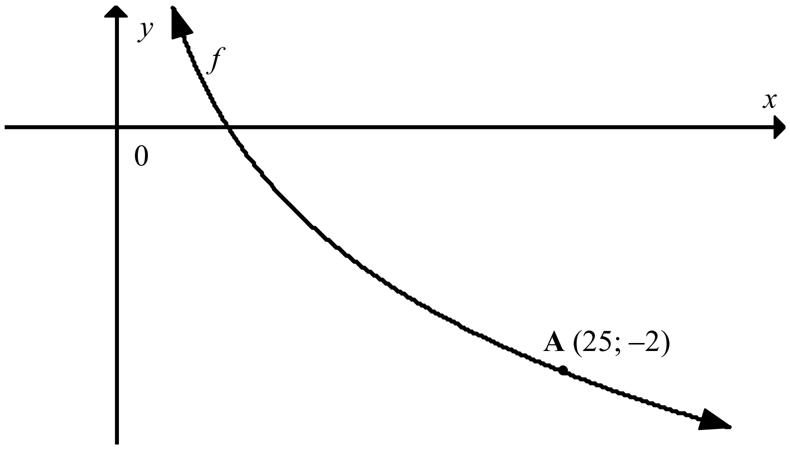
3.1.1 Determine the values of *p* and *q*. (2) L1

3.1.2 Calculate the value of *a*. (3) **L1**

3.1.3 If $f(x)=\dfrac{2}{x-1}-3$ is translated to *g* such that $g(x)=\dfrac{2}{x-3}+3$, describe the transformation from *f* to *g*. (2) **L3**

3.1.4 Write down the equation of the horizontal asymptotes of *g*. (1) **L1**

3.2 The graph of $f(x)=\log_b x$ is sketched below. $\text{A}(25;-2)$ is a point on *f*.



3.2.1 Write down the domain of *f*. (1) **L1**

3.2.2 Calculate the value of *b*. (2) **L2**

3.2.3 Determine the equation of f^{-1} in the form of $y=....$ (2) **L2**

3.2.4 For which values of *x* is $\left(\dfrac{1}{5}\right)^{x+3}-5<20$? (2) **L3**

[15]

QUESTION 4 **EC/SEPT 22**

Given $f(x)=-3^x+1$

4.1 Draw the graph of *f* . Clearly show all the intercepts with the axis as well as the asymptotes of the graph. (3) **L3**

4.2 Write down the range of *f*. (2) **L1**

4.3 Determine the equation of the asymptotes of *g*, given that $g(x)=-f(x)$. (2) **L2**

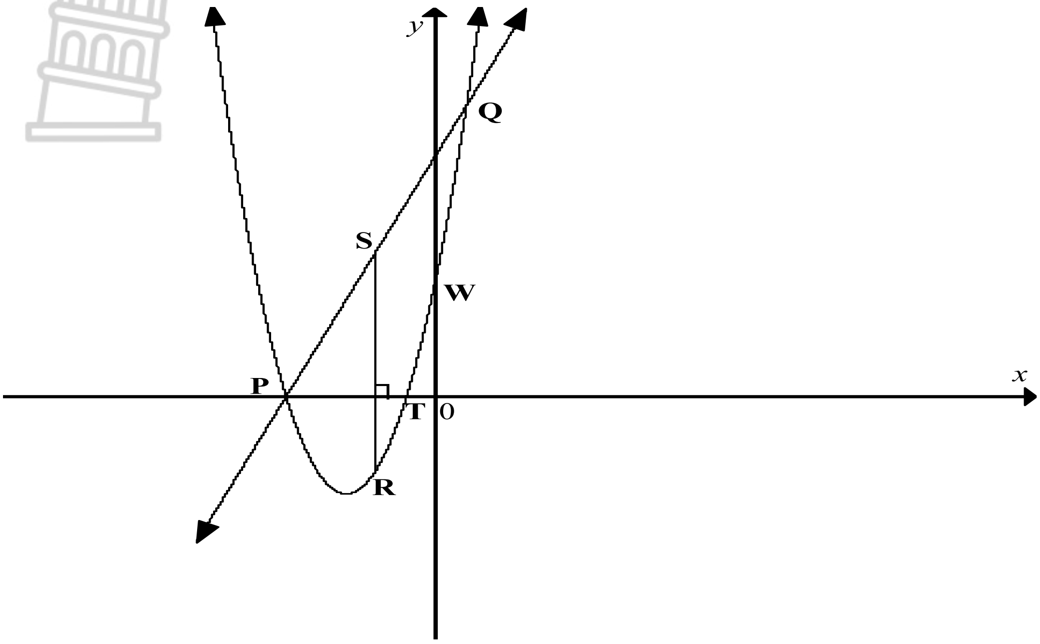
4.4 If *g* is shifted 1 unit upwards to give a new function *h*, determine the equation of h^{-1}, the inverse of *h* in the form $y=...$ (3) **L3**

[10]

QUESTION 5 **LIMPOPO/SEPT 23**

The graphs of $f(x) = ax^2 + bx + c$ and $g(x) = 2x + 10$ are sketched below.

- Graph of *f* intersects the *x*-axis at $P(-5;0)$ and $T(-1;0)$, and *y*-axis at $W(0;5)$
- The two graphs intersect at points Q and P.
- R and S are points on *f* and *g* respectively such that SR is perpendicular to the *x*-axis.



5.1 Show that $f(x) = x^2 + 6x + 5$. (3) **L2**

5.2 Calculate the coordinates of Q. (4) **L3**

5.3 Show that $f(x) \neq -5$ for all values of *x*. (3) **L2**

5.4 Consider point R when SR is at maximum in the interval $x_P < x < x_Q$. Determine:

5.4.1 The gradient of the tangent to *f* at R. (4) **L3**

5.4.2 The equation of the tangent to *f* at R. (3) **L2**

5.5 Consider $x > x_P$. For which values of *x* is $g(x) - g^{-1}(x) > 15$? (3) **L4**

[20]

QUESTION 6 **DBE/JUNE 24(ADAPTED)**

Given : $g(x) = -\dfrac{1}{1-x} + 2$

6.1 Write down the equation of the asymptote of *g*. (2) **L1**

6.2 Draw a graph of *g*, indicating any intercepts with the axes and asymptotes. (4) **L2**

6.3 Determine the values of *x* where $g(x) > 0$. (2) **L2**

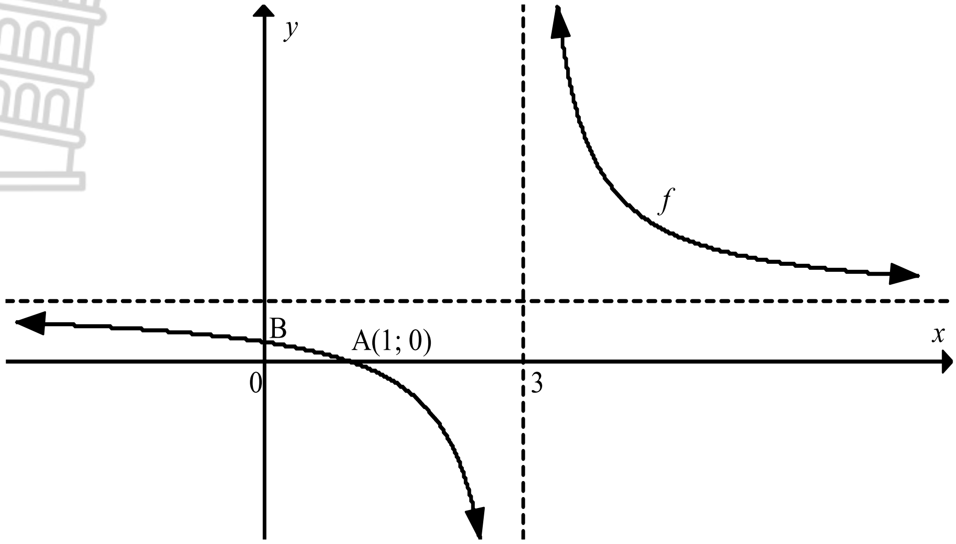
6.4 Determine the equation of the axis of symmetry of *g* which has a negative gradient. (2) **L2**

6.5 For what values of *x* will the graph of f increase (2) **L2**

[12]

QUESTION 7 **EC/SEPT 22**

In the diagram below, the graph of a hyperbolic function, $f(x)=\dfrac{x+k}{x+p}$, where k is a constant, is drawn. $A(1;0)$ and B are the x-intercept and y-intercept of f, respectively. The vertical asymptote goes through the x-axis at 3.



7.1 Write down the value of p. (1) **L1**

7.2 Determine the value of k. (2) **L1**

7.3 Calculate the coordinates of B. (2) **L2**

7.4 Determine the values of x for which $x.f(x)\leq 0$. (3) **L3**

7.5 Rewrite the equation of f in the form $f(x)=\dfrac{a}{x+p}+q$. (2) **L2**

[10]

QUESTION 8 **KZN PRACTICE/ JUNE 24**

The line $y=x+1$ and $y=-x-7$ are the axis of symmetry of the function $f(x)=\dfrac{-2}{x+p}+q$.

8.1 Show that $p=4$ and $q=-3$ (3) **L3**

8.2 Calculate the x-intercept of f. (2) **L2**

8.3 Sketch the graph of f. Clearly label ALL intercepts with the axis and the asymptotes. (4) **L3**

8.4 Write down the equation of the vertical asymptotes of the graph of h if $h(x)=f(x+5)$. (2) **L2**

8.5 Determine the values of x for which $f(x)>0$. (2) **L2**

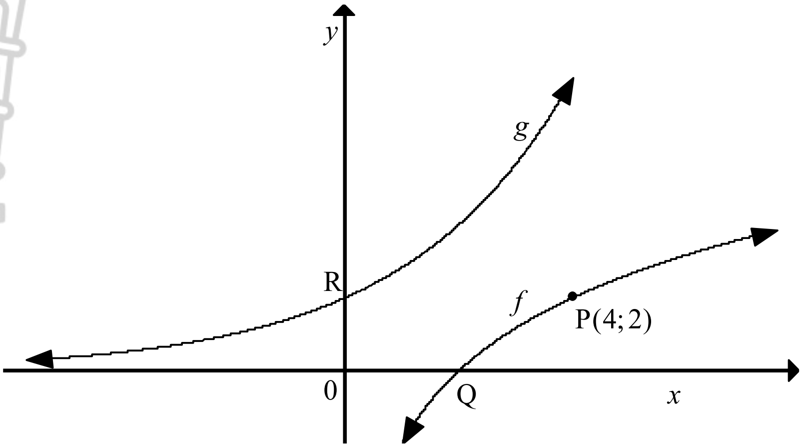
8.6 Determine the value(s) of x for which $\dfrac{-2}{x+4}\geq x+4$ (2) **L2**

8.7 Explain how would you use a graph to determine the value(s) of x for $\dfrac{-2}{x+4}=-x-4$ (3) **L3**

[18]

QUESTION 9 **DBE/JUNE 24**

In the diagram, the graphs of $f(x)=\log_a x$ and g are drawn. Graph g is the reflection of f in the line $y=x$. Graph f passes through the point $P(4;2)$. Q is the x-intercept of f and R is the y-intercept of g.

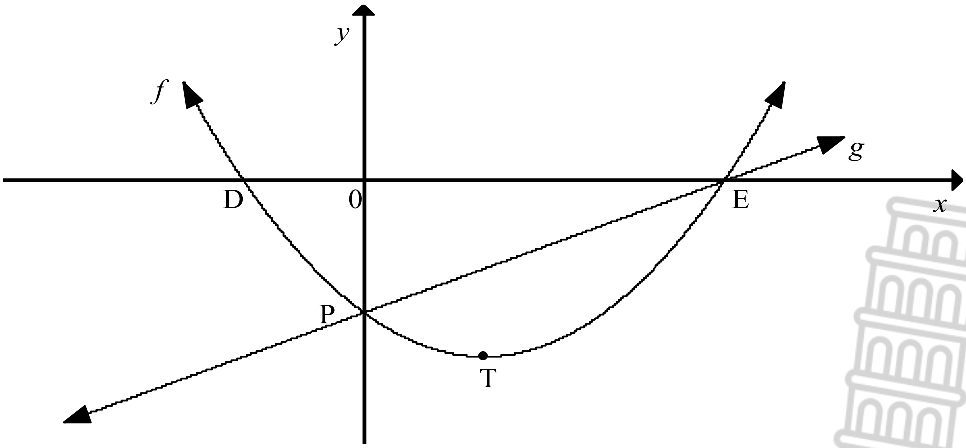


9.1	Write down the coordinates of P', the image of P on g.	(2)	**L1**
9.2	Show that $a=2$.	(2)	**L2**
9.3	Write down the equation of g in the form of $y=$....	(1)	**L2**
9.4	T is a point on f in the first quadrant where TR is parallel to the x-axis. Calculate the area of $\Delta RTP'$.	(4)	**L4**

[9]

QUESTION 10 **DBE/JUNE 24**

The graph of $f(x)=x^2-2x-3$ and $g(x)=mx+c$ are drawn below. D and E are the x-intercepts and P is the y-intercept of f. The turning point of f is $T(1;-4)$. The graph of f and g intersect at P and E.

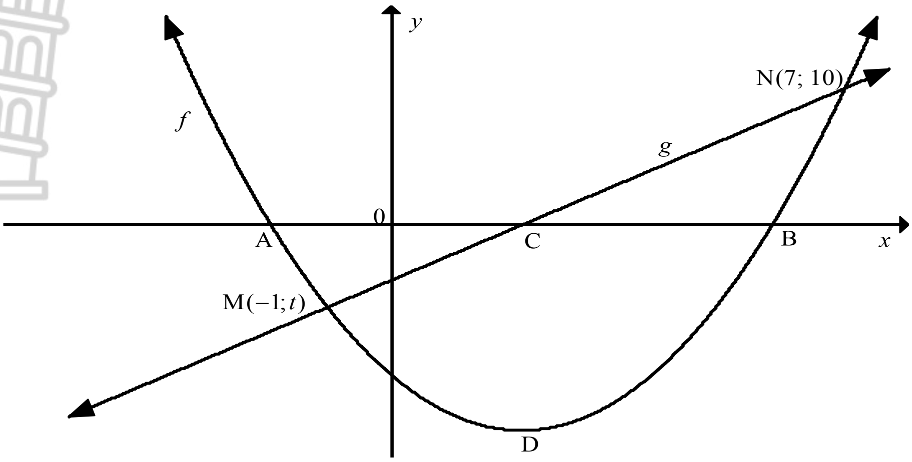


10.1	Write down the range of f.	(1)	**L1**
10.2	Calculate the coordinates of D and E.	(3)	**L2**
10.3	Determine the equation of g.	(2)	**L2**
10.4	Write down the values of x for which $f(x)-g(x)>0$.	(2)	**L2**
10.5	Determine the maximum vertical distance between h and g if $h(x)=-f(x)$ for $x\in[-2;3]$.	(5)	**L3**
10.6	Given: $k(x)=g(x)-n$. Determine n if k is a tangent to f.	(5)	**L4**

[18]

QUESTION 11 **EC/ SEPT 22**

The diagram below shows the graphs of $f(x)=x^2-4x-11$ and $g(x)=f'(x)$. A and B are the *x*-intercepts of *f* and C is the *x*-intercept of *g*. D is the turning point of *f*. *f* and *g* intersect at $M(-1;t)$ and $N(7;10)$.



11.1 Calculate the:

11.1.1 Coordinates of D. (3) **L2**

11.1.2 Distance CN. (4) **L2**

11.2 For which value(s) of x is:

11.2.1 $f(x)<g(x)$? (2) **L2**

11.2.2 $g(x)-f(x)$ a maximum? (4) **L3**

11.3 Determine the average gradient of *f* between $x=2$ and $x=7$ (2) **L2**

11.4 Determine the value(s) of k for which $f(x)+k=0$ have two roots with different signs. (2) **L2**

[18]

QUESTION 12

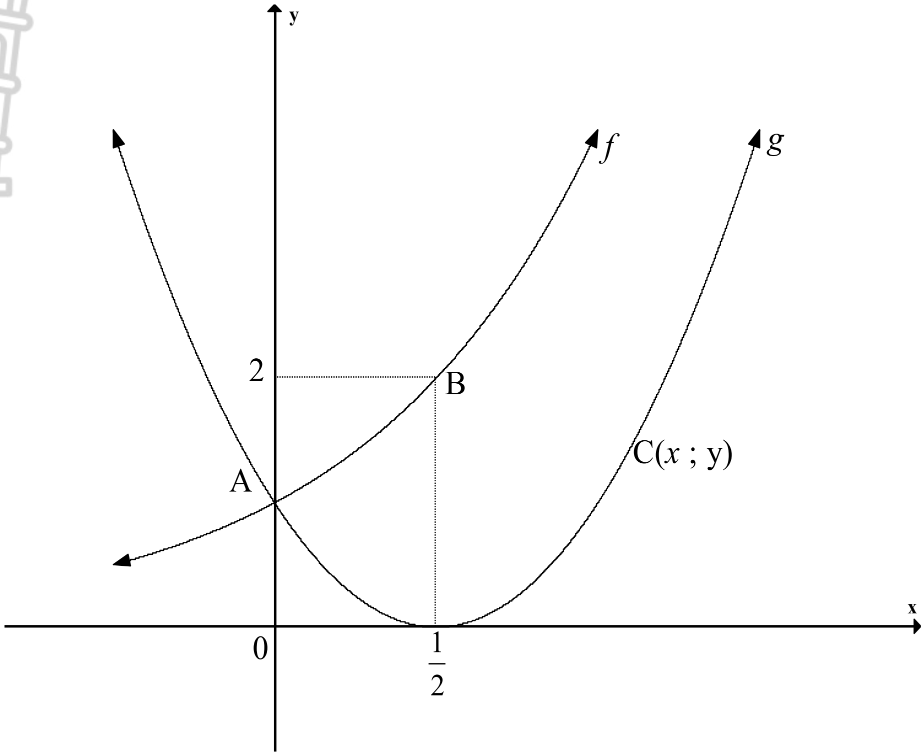
The graph of a hyperbola with equation $y=f(x)$ has the following properties:

- Domain: $x\in\mathbb{R}, x\neq 5$
- Range: $y\in\mathbb{R}, y\neq 1$
- Passes through the point $(2;0)$

Determine $f(x)$ (4) **L3**

QUESTION 13 **Ekurhuleni North/JUNE 17**

The graphs of $f(x)=k^x$ and $g(x)=ax^2+bx+c$ are sketched below. The graphs intersect at A and *g* touch the *x*-axis at $\left(\frac{1}{2};0\right)$. The coordinate of B, on the graph of *f* are indicated. AC is parallel to the *x-axis.*



13.1	Determine the coordinate of A.	(1)	**L1**
13.2	Determine the value of *k*.	(2)	**L2**
13.3	Show that $a=4$, $b=-4$ and $c=1$	(4)	**L2**
13.4	Determine the equation of f in the form $y=...$	(1)	**L1**
13.5	Determine the coordinate of *h* if *h* is reflection of *f* about the *y*-axis	(2)	**L2**
13.6	Write down the range of *g*.	(1)	**L1**
13.7	Determine the equation of the tangent to *g* at point C.	(5)	**L3**
13.8	For which values of *g* $g'(x).f(x)\geq 0$?	(2)	**L3**
13.9	Use the graphs to determine the value of *k* for which $f(x)=k$ will have non-real roots.	(2)	**L2**
13.10	Write down the equation of *f* if the graph of *f* is shifted 2 units to the left.	(1)	**L2**
		[21]	

QUESTION 14 **DBE/NOV 15**

The function defined as $y=\dfrac{a}{x+p}+q$ has the following properties:

- The domain is $x\in\mathbb{R}, x\neq -2$.
- $y=x+6$ is an axis of symmetry.
- The function is increasing for all $x\in\mathbb{R}, x\neq -2$.

Draw a neat sketch graph of this function. Your sketch must include all the asymptotes. (4) **L4**

QUESTION 15 **KZN/JUNE18**

Given $h(x)=a^x$ passes through the point $\mathrm{A}\left(-2;\frac{1}{9}\right)$.

15.1 Calculate the value of *a*. (2) **L1**

15.2 Write down the equation of h^{-1}, the inverse of *h,* in the form $y=\ldots$ (2) **L2**

15.3 Writ down the coordinates of any point on the graph of the inverse of *h*. (2) **L1**

15.4 Determine the value(s) of *x* for which $h^{-1}(x)\leq -2$ (2) **L3**

15.5 Write down the equation of *f* if $f(x)=h^{-1}\left(\frac{x}{2}\right)$ (2) **L2**

[10]

QUESTION 16

The graph of *g* is defined by the equation $g(x)=\sqrt{ax}$. The point $(8;4)$ lies on *g*.

16.1 Calculate the value of *a* (2) **L2**

16.2 If $g(x)>0$, for what values of *x* will *g* be defined (2) **L1**

16.3 Determine the range of *g*. (1) **L1**

16.4 Write down the equation of g^{-1}, the inverse of *g,* in the form $y=\ldots$ (2) **L2**

16.5 If $h(x)=x-4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of *h* and *g*. (2) **L2**

16.6 Hence, or otherwise, determine the values of *x* for which $g(x)>h(x)$ (2) **L3**

[11]

QUESTION 17

Given $f(x)=3^x$

17.1 Determine the equation for f^{-1} in the form $f^{-1}(x)=\ldots$ (1) **L1**

17.2 Sketch the graphs of *f* and f^{-1}, showing clearly ALL intercepts with the axes. (4) **L2**

17.3 Write down the domain of f^{-1}. (2) **L1**

17.4 For which values of *x* $f(x).f^{-1}(x)\leq 0$? (2) **L3**

17.5 Write down the range of $h(x)=3^{-x}-4$ (2) **L1**

17.6 Write down an equation for *g* if *g* is the image of the graph of *f* after *f* has been translated two units to the right and reflected about the *x*-axis. (2) **L3**

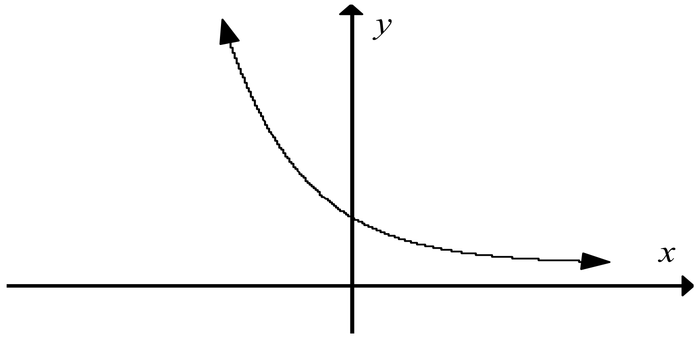
[13]

QUESTION 18 **DBE/MARCH 2016**

Determine the range of the function $y = x + \frac{1}{x}, x \neq 0$ and x is real. (6) **L4**

QUESTION 19 **FS/JUNE 24**

Sketched below is the graph of $f(x) = k^x, k > 0$. The point $\left(2; \frac{1}{9}\right)$ lies on *f*.



19.1 Determine the value of *k* (2) **L1**

19.2 Write down the range of *f* (1) **L1**

19.3 Explain the transformation of *f* to f^{-1}. (1) **L1**

19.4 Determine the equation of f^{-1} in the form $y = \ldots$ (2) **L1**

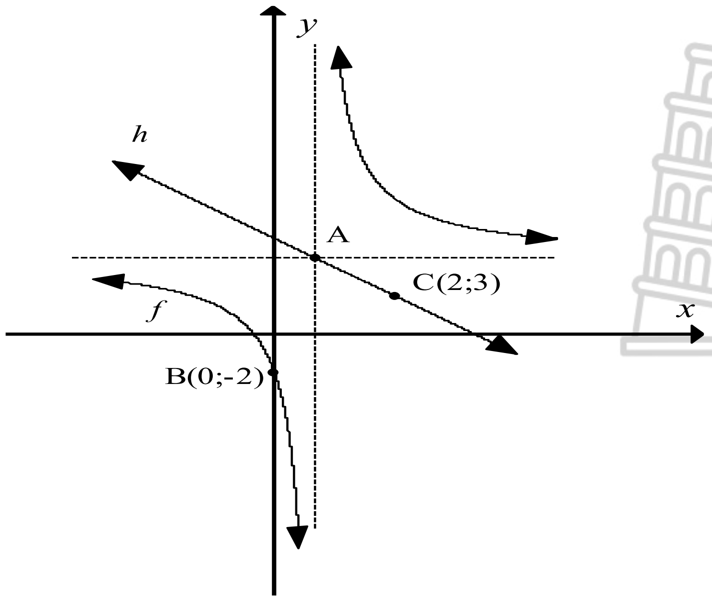
19.5 Sketch the graph of f^{-1}. Indicate on your graph the coordinate of ONE point. (3) **L2**

19.6 Prove that $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$. (3) **L4**

[12]

QUESTION 20 **FS/JUNE 24**

In the sketch below the graph of $f(x) = \frac{a}{x+p} + 4$ is given. Asymptotes of *f* intersect at point A. The graph of *f* cuts the *y*-intercept at $B(0;-2)$. The axis of symmetry of *f*, is the line *h*. Point C coordinates $C(2;3)$ is the point on *h*.



20.1 Determine the equation of *h* (2) **L1**

20.2 Determine the coordinates of point A (2) **L2**

20.3 Determine the equation *f*. (3) **L2**

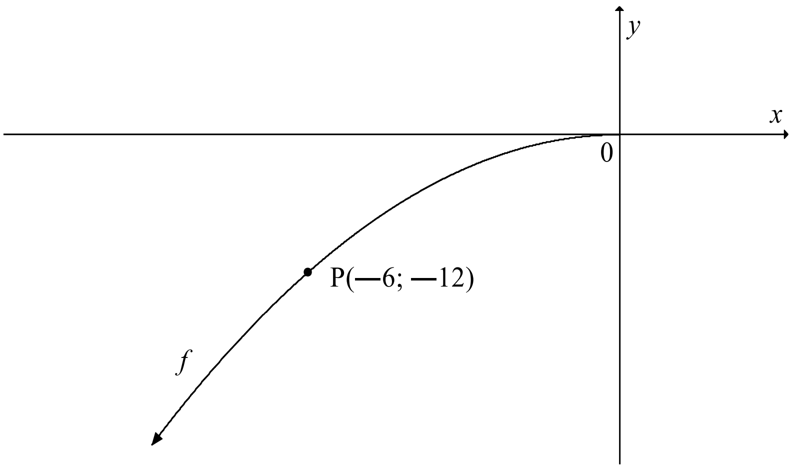
20.4 Determine the equations of the asymptotes of $f(x+1)$. (3) **L2**

20.5 Write down the coordinates of the image of $D\left(\frac{1}{2};0\right)$ if D is reflected about the axis of symmetry $y = x+3$ (2) **L3**

[12]

QUESTION 21 **DBE/NOV 18 (AMENDED)**

In the diagram below, the graph of $f(x) = ax^2, x \leq 0$. $P(-6;-12)$ is a point on *f*.



21.1 Is f^{-1} a function? Motivate your answer. (2) **L1**

21.2 If R is a reflection of P in the line $y = x$, write down the coordinates of R. (1) **L1**

21.3 Calculate the value of a. (2) **L1**

21.4 Write down the equation of f^{-1} in the form of $y = ...$ (3) **L2**

21.5 Sketch the graph of f^{-1}, showing one other point on your graph. (2) **L2**

21.6 Determine the value(s) of *x* for which $\frac{f^{-1}(x)}{f(x)} \leq 1$ (5) **L3**

[15]

QUESTION 22

The equation of parabola is given by $f(x) = ax^2 + bx + c$. The roots of *f* are $(m-5)$ and $(m+3)$ The maximum value of *f* occurs at $x = 2$.

22.1 Determine the value of *m*. (2) **L2**

22.2 Determine the equation of *f* in the form $f(x) = ax^2 + bx + c$ if it is also given that $f(1) = 15$. (4) **L2**

22.3 Determine the range of *g* if $g(x) = f(x) - 4$. (3) **L3**

[09]

TOPIC FINANCIAL MATHEMATICS (15 ± 3)

GUIDELINES, SUMMARY NOTES, & STRATEGIES

SIMPLE INTEREST AND COMPOUND INTEREST (A > P)

- On the **Simple interest**, the interest is calculated on the **original** amount invested or borrowed.
  $A = P(1+in)$
- On the **Compound interest**, the interest is calculated on the accumulated amount.
  $A = P(1+i)^n$

DEPRECIATION (A < P)

- For depreciation we use :
  $A = P(1-in)$ **Straight line depreciation**
  $A = P(1-i)^n$ **Reducing balance depreciation**

COMPOUNDING PERIOD	INTEREST (i)	Period (n)
Monthly	$\frac{i}{12}$	$n \times 12$
Quarterly	$\frac{i}{4}$	$n \times 4$
Half yearly/Semi-annually	$\frac{i}{2}$	$n \times 2$

EFFECTIVE AND NOMINAL INTEREST RATES

- For the annual effective rate, we use the formula: $1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$
- When working with different compounding periods use the formula: $(1+i_{new})^m = (1+i_{nom})^n$

ANNUITIES

- An annuity is a series of equal payments made at regular time intervals.
- The annuity formulae are used under the following conditions:
  - ✓ All payments are equal
  - ✓ The payments are made at regular intervals
  - ✓ The interest rate remains fixed and the compounding period for interest is the same as the payment intervals

THE FUTURE VALUE

- We can use the following formula to calculate the future value of an annuity:

$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$

F is the future value.
x is the payment.
i is the interest rate per interval.
n is the number of payments.

THE PRESENT VALUE

- We can use the following formula to calculate the present value of an annuity:

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

P is the present value.
x is the payment.
i is the interest rate per interval.
n is the number of payments.

SINKING FUND

- $A = P(1-i)^n$ (Scrap value of old asset)
- $A = P(1+i)^n$ (Cost of new asset)
- Sinking fund = new – old
- Calculate x
- Withdrawals (calculate x_{new}) treat it separately and add it back

THE OUTSTANDING BALANCE ON A LOAN

Outstanding Balance = Loan with interest to date – Repayments with interest to date

$$OB = P(1+i)^n - \frac{x\left[(1+i)^n - 1\right]}{i} \quad \textbf{OR} \quad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

Note:

- When using the P formula, use the remaining number of payments.
- When using OB=A-F, use n as number of payments made.

DELAYED/ DEFERRED ANNUITIES	THE LAST/FINAL PAYMENT
• When the first payment of a loan is made more than one period after the loan was received, this payment is referred to as a ***deferred annuity.*** • Apply the compound interest to the loan to move it to the same point on the timeline as the present value of the annuity	Last payment = Outstanding balance after the last full payment multiplied by $(1+i)^1$ **MISSED PAYMENTS** To calculate the new payment: • We calculate the outstanding balance immediately after the last payment made. • We then apply the compound interest to this outstanding balance, till one period before payments resume. The result is the present value of the new annuity consisting of all the remaining payments.

DBE/NOV. 2023

1.1 Patric deposited an amount of R18 500 into an account earning r% interest p.a, compounded monthly. After 6 months, his balance was R19 319,48.

1.1.1 Calculate the value of r. (3) **L1**

1.1.2 Calculate the effective interest rate. (2) **L2**

1.2 Kuda bought a laptop for R 10 000 on 21 January 2019. He will replace it with a new one in 5 years' time on the 31 January 2024.

1.2.1 The value of the old laptop depreciates annually at the rate of 20% p.a. according to the straight-line method. After how many years will the laptop have a value of R0? (2) **L2**

1.2.2 Kuda will buy a laptop that cost R 20 000. In order to cover the cost price, He made his first monthly deposit into a savings account on the 28 February 2019. He will make his 60^{th} monthly deposit on 31 January 2024. The savings account pays an interest rate of 8,7% p.a. compounded monthly. Calculate Kuda`s monthly deposits into this account. (4) **L2**

1.3 Tino wins the jackpot of R 1 600 000. He invests all his winnings in a fund that earns interest of 11,2% p.a. compounded monthly. He withdraws R 20 000 from the fund at the end of each month. His first withdrawal is exactly 1 month after his initial investment. How many withdrawals f R 20 000 will Tino be able to make from this fund? (5) **L3**

EC/SEP.2023

2.1 Lufezo deposits R 97 000 into an account that offered interest at 9,1% p.a. compounded quarterly. Calculate how many years it took for his investment to reach R 166 433. (4) **L2**

2.2 On 1 January 2018 a school bought a new bus for R 482 000. On that day they also started the sinking fund to make provisions for new bus in 5 years' time.

2.2.1 Over the next 5 years the value of the bus depreciates at 14,7% p.a. on a reducing-balance method. Calculate the trade-in value of the bus after 5 years. (2) **L2**

2.2.2 The price of these buses increases by 8,1% per year. Calculate the price of the new bus on 1 January 2023, i.e. after 5 years. (2) **L1**

2.2.3 The bank offered an interest rate of 7,3% p.a., compounded monthly, for the sinking fund. The first payment, x rands, was made in the fund on 1 January 2018 and thereafter the same amount was deposited on the 1^{st} day of every month. The last payment was made on 1 December 2011. On 31 December 2022 the school bought a new bus and used the trade-in value of the old bus as a deposit
Calculate the monthly payment into the sinking fund. (6) **L3**

NW/ SEPT.2023

3.1 Convert an effective interest rate of 11,3% p.a. to its equivalent nominal rate per annum, compounded quarterly (3) **L2**

3.2 Lisa opened a savings account and deposited R 10 000 immediately into the account. The account paid interest at 5,3% per annum, compounded monthly. She started making additional (5) **L3**

monthly deposits of R500 into the account three months after the account was opened. Her last monthly deposit of R 500 was made 5 years after the account was opened.
How much money was in the account 5 years after the account was opened?

3.3 Sam wants to buy a house and takes out a loan of R 860 000. He can only afford to pay R 7 200 per month starting 1 month after the loan is granted. The interest rate is compounded monthly at 9,5%.

3.3.1 Calculate the number of payments that Sam will make to repay the loan. (4) **L2**

3.3.2 How much will Sam pay in the last month to settle the loan? (5) **L3**

GP/SEP.2023

4.1 A survey conducted in December 2015 determined that 5,7 million South Africans were living with HIV. The researchers used a model of exponential growth $A = P(1+i)^n$ to predict that there will be 6 million people living with HIV in December 2022.
Calculate, as a percentage, the annual rate of increase that researchers used for the 7 years. (3) **L2**

4.2 Shimmy invests R 4 000 000 into an account earning interest of 6% per annum, compounded monthly. She withdraws R 30 000 per month. Her first withdrawal is exactly one month after she deposited the R 4 000 000.

4.2.1 How many withdrawals of R 30 000 will Shimmy be able to make? (5) **L3**

4.2.2 How many withdrawals will Shimmy be able to make if she changes the amount withdrawn per month to R 20 000?
Substantiate your answer. (3) **L3**

4.3 Estrid opened a saving account with a single deposit of R 1 000 on 1 April 2022. She then makes 18 monthly deposits of R 700 at the end of every month. Her first payment is made on the 30 April 2022 and her last payment on 30 Sepetember 2023.The account earns interest at 15% per annum, compounded monthly.
Determine the amount that should be in her savings account immediately after her last deposit is made (30 September 2023). (4) **L3**

MDE/SEPT. 2023

5.1 Jane deposits R x rands into an investment account. How long will it take for the value for the investment to double if the interest rate is 5,4% p.a. compounded annually? (3) **L2**

5.2 Thabo starts a printing company and needs to borrow money for start-up costs. He can make equal monthly payment of R 3 300. What amount can Thabo borrow if the interest rate on the loan is 12% per annum compounded monthly and the loan is granted over 5 years? (4) **L2**

5.3 A group of investors consider investing in a fund that promises growth at a rate of 5% p.a. compounded quarterly. Calculate the effective annual percentage rate of the growth promised. (3) **L1**

5.4 Sarah is 18 years old and wishes to accumulate R 10 000 000 by the month before her 50^{th} birthday. She will deposit equal monthly payment into an account that pays 15% p.a. compounded monthly. The first payment starts on her 18^{th} birthday and the last payment one month before her 50^{th} birthday.
Calculate the monthly instalments that Sarah will make. (3) **L2**

IEB/NOV. 2006

6.1 Ashika takes out loan of R 450 000 at an effective interest rate of 14% p.a. in order to purchase a town house. She repays a loan with equal monthly installments of R7 500, starting one month from the granting of a loan. The interest is compounded monthly.

6.1.1 Show that the nominal interest rate is approximately 13,17% p.a. (3) **L2**

6.2 Calculate:

6.2.1 The time span of the loan in months. (2) **L2**

6.2.2 The value of the last payment (less than R7 500). (3) **L2**

6.3 Calculate how much interest is paid:

6.3.1 In the first month (2) **L1**

6.3.2 In the first year payment (3) **L2**

RUSTERNBURG GIRLS` HIGH

7.1 Lynne purchases a new car for R 350 000. They take out a 6-year loan on 1 January 2019.The monthly instalments are paid at the end of every month. Interest is fixed at 18% p.a. compounded monthly.

7.1.1 Calculate the monthly repayment. (4) **L2**

7.1.2 Due to financial difficulty, Lynne misses the 40^{th} ,41^{st} and 42^{nd} payments. Determine the outstanding balance at the end of 42^{nd} month. (4) **L2**

7.1.3 If Lynne`s monthly repayment of R10 000. How many months will it take her to pay back the rest of the loan. (4) **L2**

8.1 Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum compounded monthly.

8.1.1 Determine the selling price of the house. (1) **L1**

8.1.2 The period of the loan is 20 years, and she starts repaying the loan one month after it was granted. Calculate her monthly instalments (4) **L2**

8.1.3 How much interest will she pay over the period of 20 years? Round off your answer to the nearest rands (2) **L1**

8.1.4 Calculate the balance of her loan immediately after the 85^{th} instalment (3) **L2**

8.1.5 She experiences financial difficulties after the 85^{th} instalment and did not pay any instalment for 4 months (that is 86 to 89) Calculate how much Siphokazi owes on her bond at the end of the 89^{th} month. (2) **L2**

8.1.6 She decides to increase her monthly payments to R 8 500 per month from the end of the 90^{th} month. How many months will it take to repay the bond after the new payment of R 8500? (4) **L3**

UNUSED PAPER NOV. 2009

9.1 A new cell phone was purchased for R 7 200. Determine the depreciation value after 3 years if the cell phone depreciates at 25% p.a. on a reducing-balance method. (3) **L1**

9.2 Jill negotiates a loan of R 300 000 with a bank which has to be repaid by means of monthly repayments of R 5 000 and a final payment which is less than R 5 000. The repayments start one month after the granting of the loan. Interest is fixed at 18% per annum, compounded monthly.

9.2.1 Determine the number of payments required to settle the loan (6) **L3**

9.2.2 Calculate the balance outstanding after Jill has paid the last R 5 000. (5) **L2**

9.2.3 Calculate the value of the final payment made by Jill to settle the loan. (2) **L1**

9.2.4 Calculate the total amount that Jill repaid to the bank (1) **L1**

KEVIN SMITH STUDY GUIDE

10.1 Mr Dasoo wants to take out a loan for a house over 20 years. He approached two banks and has been offered two different options. Two options are shown in the table below. Which option should Mr Dasoo choose?

Variables	**Option 1**	**Option 2**
Loan amount	R 800 000	R 800 000
Interest rate (compounded monthly)	12%	11,8%
Repayments	Monthly	Monthly
Bank charges	R0	R 200 per month
Commissions	R6 000	R 0

(6) **L3**

KELVIN SMITH STUDY GUIDE

11.1 Ted invests R 10 000 into an account that offers an interest rate of 3,25 % p.a. compounded quarterly. After 2 years he deposits an additional R 2 500 into the account and 3 years later withdraws R 5 000. How much will he have in his account after 10 years? (4) **L2**

11.2 x Rand is invested into an account offering an interest rate of 12% p.a. compounded monhtly.3 years later $2x$ Rand is deposited into the account. After 7 years there is R 27 655, 87 in the account. Determine the value of x. (5) **L3**

IEB/MAY. 2021

12.1 Simon wants to buy a car that costs R 345 000. He opens a saving account and six month after opening an account makes a deposit of R 12 895 and continues depositing R 12 895 at the end of every six-month period. Interest is paid at 13% p.a. compounded half yearly.

12.1.1 How much money will be in Simon`s account three years after opening the account? (3) **L2**

12.1.2 Ignoring the effects of inflation on the price of the car, Determie how long will take Simon to save the money needed to buy the car? (5) **L3**

12.1.3 If the effect of inflation is considered, determine the cost of the car 8 years after opening the bank account. Inflation for this period is calculated at 3,5% per annum (3) **L2**

12.2 Instead of the savings plan he considered a second plan which is getting loan for R R 345 00 under the following agreement:

- Interest is charged at 13% per annum compounded half yearly.
- The loan must me settled in 8 years

Determine the minimum monthly repayment. (3) **L3**

KELVIN SMITH STUDY GUIDE

13.1 Arshad`s birthday is on the 1st of January. On the day he turs 20 he stars to save for his 21st birthday party by placing R 200 into a savings account every month with his last payment made on the 21st birthday. How much money will Arshad have for his party, if the account promises an interest rate of 4,5% per annum compounded monthly and his party is to be held on the first of February? (4) **L2**

MIND ACTION SERIES NEW ADDITION

14.1 Jenny wishes to repay a loan of R 150 000, by mean of 16 equal quarterly payments, starting three months from now. The interest rate on the loan is 21,5% p.a quarterly.

14.1.1 Calculate what Jenny`s quarterly payments will be. (4) **L2**

14.1.2 Calculate the total interest that Jenny will pay on the loan. (3) **L2**

NW/ SEPT.2020

15.1 Patric take out an annuity that he can live from after he retires in twenty years' time. He needs R 3 000 000 in his annuity when he retires. The bank gives him an interest rate of 10% per annum compounded monthly.

15.1.1 Calculate his monthly instalments into the fund if he starts paying immediately and thereafter at the end of each month until his last payment in 20 years' time. (4) **L2**

15.1.2 After 20 years Patric retires but decides not to let the R 3 000 000 be paid out. Instead, he decides to withdraw monthly amounts of R 20 600 at the end of each month. He withdraws his first amount at the end of the fourth month. The interest that he earns over this period is 8% per year, compounded monthly. Determine how many months can he continue with his lifestyle. (7) **L3**

15.1.3 Calculate the amount of Patric`s final withdrawal. (4) **L2**

16.1 How long will it take (answer to the nearest year) for the value of an investment to depreciate with quarter of its original value? Rate of depreciation is 8,2% p.a. on the reducing balance method. (4) **L2**

16.2 Ina wants to travel overseas in 6 years' time. She will need R 58 480 to do that. Calculate her monthly payment into a saving account with an interest rate of 9% p.a. compounded monthly if she makes her first payment immediately and her last payment two months before the end of the 6 years. (5) **L2**

16.3 Jacob- took out a loan of R 1 500 000 to buy a house. He will repay the loan with monthly payments over 20 years. The interest rate is 8% p.a. compounded quarterly.

16.3.1 Showing ALL your calculations and formulae, prove that his monthly instalment will be R 12 499,96 (5) **L2**

16.3.2 Calculate the outstanding amount after 12 years. (3) **L2**

TOPIC	DIFFERENTIAL CALCULUS
GUIDELINES, SUMMARY NOTES, & STRATEGIES	

TEACHING APPROACHES (CALCULUS)

1. FIRST PRINCIPLES:

The learners:

- ✓ Need to understand what is meant by determining the gradient from first principles and know the first principles formula.
- ✓ must be able to copy the first principle formula from the formula sheet correctly.
- ✓ Be able to simplify the first principles expression (It seems as if learners handled this question better when they determine $f(x+h)$ separately and then bring it back to the formula).
- ✓ Need to be mindful of the notation and apply it correctly when they simplify the first principle expression.
- ✓ At this stage, learners can also determine the equation of the tangent at a point.

2. RULES FOR DIFFERENTIATION

- ✓ The learners:
  - i. need to revise how to simplify surds, rational, irrational exponents.
  - ii. Must know how to simplify expressions before differentiation.
  - iii. Must know how to tell which variable they are required to differentiating with respect to.
- ✓ Must expose themselves to variety of questions having different notations including where a variable is given as constant.
- ✓ Following instructions is once more important, on how the answer should be provided whether with a + positive or - negative.
- ✓ Must always use of correct notation.

3. CUBIC FUNCTIONS $f(x) = ax^3 + bx^2 + cx + d$

The learners need to know and follow these steps when sketching a cubic function:

- ✓ Before learners can sketch a cubic function, they at least need to know the shape of their graph as guided by value of a where a could be $a > 0$ and $a < 0$.
- ✓ The learners must be able to Factories a third-degree polynomial using any other method to determine the ***x*-intercepts** (the x-intercepts are known as the: zero, roots, $f(x) = 0$. It would be an advantage if they can be able factories using a calculator.
- ✓ They must also be able to find the ***y*- intercept**, which is when $x = 0$, or given by the value of d.
- ✓ Learners must be able to use the first derivative to find the coordinates of the turning points, which are also known as the Stationery points or local minima and local maxima. In simple terms, this is finding $f'(x) = 0$, solve for x, and then find the corresponding y-values to give the coordinate of the turning point.
- ✓ Examiners often require learners to write the intercepts with the axes, stationary points and points of inflection in coordinate form $(a\ ;\ b)$ .Make sure that the learners are aware of this.

4. INTERPRETATION OF A CUBIC FUNCTION:

The learners must be able to:

- ✓ Tell what the domain is, that $x \in R$
- ✓ Understand the relationship between the graph of a function and the graph of its derivative is important in that it explains to the learners why the second derivative is zero at a point of inflection.
- ✓ Understand that the point of inflection is determined by equating the second derivative to zero and solving for x. An alternative method is to add up the x-coordinates of the turning points and divide by 2 (i.e. determining the midpoint of the two turning points).
- ✓ Tell for which values of x will $f(x)$ be concave up: $f''(x) > 0$ & Concave down: $f''(x) < 0$

- ✓ Tell where *f* is increasing or decreasing. Increasing ($f'(x) > 0$), decrease $f'(x) < 0$).
- ✓ Determine the values of *x*, for which: $x.f(x) > 0$, $f'(x) > 0$, $f'(x).f(x) < 0$
- ✓ when will f have three real roots, two real roots or one real root?

5. OPTIMIZATION

The learners need to develop the conceptual understanding on Optimization

- **Calculus of motion**
  - ✓ In this regard, the equation will be given.
  - ✓ The learners need to know that, Velocity is the derivative of displacement, and
  - ✓ Acceleration (2nd derivative) is the derivative of velocity
- **Rates of change**
  - ✓ Knowledge of formulae for the surface area and volume of right prisms is required from learners.
  - ✓ A list of relevant formulae will only be provided for the surface area and volume of cones, spheres and pyramids. Learners must select the correct one to use.

REVISION QUESTIONS

1.

KZN SEP 23

1.1 From first principles, determine the derivative of $f(x) = 2x^2 + 9$. (5) **L1**

FS SEP 23

1.2 Determine the derivative of $f(x) = 3 - x^2$ using FIRST PRINCIPLES. (5) **L2**

MP SEP 23

1.3 Given: $f(x) = -2x^2 + 1$, Determine $f'(x)$ from first principle. (5) **L2**

LP SEP 23

1.4 Determine the derivative, from first principle of $f(x) = -x^2$. (5) **L1**

KZN MAR 16

1.5 Given: $f(x) = \frac{-5}{x}$, determine $f'(x)$ from first principles. (5) **L2**

NW SEP 20

1.6 Given: $f(x) = -x^2 + 7x + 9$, determine $f'(x)$ from first principles. (3) **L2**

MIND ACTION GR12

1.7 Determine $f'(x)$ from first principles if $f(x) = 4$ (3) **L1**

MATHS HANDBOOK GR12

1.8 Determine $f'(x)$ from first principles if $f(x) = x$ (5) **L1**

FS SEP 16

1.9 Determine $f'(x)$ from first principles if $f(x) = x^3$ and hence find $f'(-2)$ (5) **L2**

KZN JUN 20

1.10 Determine $f'(x)$ from first principles given $f(x) = x^2 - bx$. (5) **L2**

2 **Rules for Differentiation:**

FS SEP 23

2.1 Determine $D_x\left(\frac{2}{x} - \sqrt{x}\right)$ (4) **L2**

FS SEP 23

2.2 Determine $\frac{dy}{dx}$ if $y = \left(x^3 - 1\right)^2$ (3) **L1**

GP SEP 23

2.3 Determine the derivative of $f(x) = \left(2\sqrt{x} - \frac{1}{x}\right)^2$ (5) **L2**

MP SEP 23

2.4 Determine $f'(x)$ if $f(x) = \frac{1}{2}x^2 - \frac{5}{x}$ (3) **L1**

LP SEP 23

2.5 Determine $\frac{dy}{dt}$ given that $y = 2t^5 + \sqrt[4]{t^7}$ (3) **L2**

KZN MAR 20

2.6 Determine $f'(x)$ if $f(x) = \frac{x^3 - 8}{2 - x}$ (4) **L2**

MP SEP 16

2.7 $\frac{dy}{dx}$ if $y=-2\sqrt{x}+x-\frac{1}{\sqrt{x}}$ (4) L2

MP SEP 17

2.8 Determine $D_x\left[\left(x^2-2\right)\left(\frac{1}{x^2}+3\right)\right]$ (4) L2

KZN SEP 23

2.9 Determine $\frac{dy}{dx}$ if $\sqrt{y+x}=x+3$ (3) L2

KZN SEP 23

2.10 Determine $\frac{d}{dx}\left[\frac{4+\sqrt{3x}}{x}\right]$ (3) L2

WC SEP 18

2.11 Differential with respect to x,

$xy=\left(x-\frac{1}{x^2}\right)\left(x+\frac{1}{x^2}\right)$ (4) L3

EC SEP 20

2.12 Determine $D_t\left[\frac{1}{2}gt^2-\frac{5}{t}+3g\right]$ (4) L3

3

EC SEP16

3.1 Given $s(t)=t^3$.Show that the gradient of any tangent to s will never be negative. (2) L3

FS SEP 17

3.2 Given $f(x)=x^3-2x^2$ Determine the equation of the tangent to f at the point where $x=2$ (6) L2

FS SEP 17

3.3 The line $g(x)=-\frac{1}{8}x+p$ is a tangent to the graph of $f(x)=5-2x^2$ at the point A. Determine the coordinates of A. (5) L3

NSC NOV 16

3.4 $g(x)=-8x+20$ is a tangent to $f(x)=x^3+ax^2+bx+18$ at $x=1$. Calculate the values of a and b. (5) L3

KZN SEP 17

3.5 Given: $f(x)=x^2-\frac{4}{x^2}$

3.5.1 Determine the gradient of the tangent to f at the point where $x=2$ (3) L2

3.5.2 Determine the equation of the tangent to f at $x=2$ (3) L2

NW SEP 17

3.6 The graph $h(x)=ax^3+px$ passes through the point $(3;-2)$.The gradient of the tangent to h at $(0;0)$ is 3.

3.6.1 Determine the value of a and p. (4) L3

3.6.2 Determine the gradient of the tangent to h at x (2) L2

PLATINUM MATHS GR12

3.7 Given: $g(x)=-x^3-2x^2+11x+12$.

3.71 Determine the equation of the tangent to g at $x=2$. (5) L2

3.7.2 Determine the coordinates of point where tangent intersects $g(x)$ a second time. (5) L3

PLATINUM MATHS GR12

3.8 Consider the graph $f(x)=-x^3-3x^2+4$ and $g(x)=\frac{23}{9}x^2-\frac{19}{3}x$

3.8.1 State the point where the graphs share a common tangent. (5) L4

3.8.2 Determine the equation of the common tangent at this point. (4) L3

4

FS SEP 23

4.1 Given: $f(x)=x^3-12x-16$

4.1.1 Calculate the coordinates of the turning points of the graph of f (5) L2

4.1.2 Calculate the x-intercepts of f (3) L2

4.1.3 $y = 15x + p$ is a tangent to the graph of f. Calculate the x-coordinates of the point(s) of contact (4) **L3**

4.1.4 For which value(s) of x will the given function be concave up? (3) **L2**

4.2 Given: $f(x) = 3x^3 - 3x^2 + 6x - 2$

For which values of x will f be concave up. (4) **L4**

NSC MARC16

4.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

4.3.1 Prove that $(x-2)$ is a factor of. (2) **L1**

4.3.2 Hence, or otherwise, factorize $f(x)$ fully (2) **L2**

4.3.3 Determine the x-coordinates of the turning points of f. (4) **L2**

4.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3) **L2**

4.3.5 Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is integer (6) **L3**

4.4 **NSC JUN 17**

Given: $f(x) = x^3 - x^2 - x + 1$

4.4.1 Write down the coordinates of the y-intercept of f. (1) **L1**

4.4.2 Calculate the coordinates of the x-intercepts of f. (5) **L2**

4.4.3 Calculate the coordinates of the turning point of f. (6) **L2**

4.4.4 Sketch the graph of f. Clearly indicate all intercepts with the axes and the turning points. (3) **L2**

4.4.5 Write down the values of x for which $f'(x) < 0$. (2) **L2**

4.5 **KZN JUN 18**

$f(x) = -x^3 + 3x^2 + 9x - 27 = -(x+3)(x-3)^2$ is the equation of a cubic function.

4.5.1 Write down the intercepts of f. (3) **L2**

4.5.2 Calculate the co-ordinates of the stationary points of f. (5) **L2**

4.5.3 Sketch the graph of f on a system of axes. (Clearly indicate the coordinates of the stationary points and the intercepts with the axes). (4) **L2**

4.5.4 Determine the value(s) of x for which the graph is concave down. (2) **L2**

4.5.5 Determine the equation of the tangent to the graph of f at $x = 0$. (3) **L2**

4.5.6 If $f(x) = k$ has 3 unequal real roots, determine the values(s) of k. (3) **L2**

4.5.7 Write down the equation of t if f is shifted 3 units horizontally to the left. (2) **L3**

NSC NOV 17

4.6 Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

4.6.1 Show that f has a point of inflection at $x = 2$. (5) **L3**

4.6.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4) **L2**

4.6.3 For which values of x will $y = -f(x)$ be concave down? (2) **L3**

4.6.4 Use your graph to answer the following question:

4.6.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2) + 3$. (2) **L2**

4.6.4.2 Claire claims that $f'(2) = 1$.
Do you agree with Claire? Justify your answer. (2) **L3**

4.7 **GP SEP 19**

A cubic function has following essential properties:

- $f(4) = f(1) = 0$
- $f(0) = 8$
- $f'(3) = f'(1) = 0$
- $f(3) = 8$

4.7.1 Sketch the graph of f, clearly indicating the turning point(s) and the points of intersection of the graph with the axes. (3) **L3**

4.7.2 Show that the defining equation of f is $f(x) = -2x^3 + 12x^2 - 18x + 8$. (4) **L3**

4.7.3 Determine the value(s) of x for which graph of f is concave down. (3) **L2**

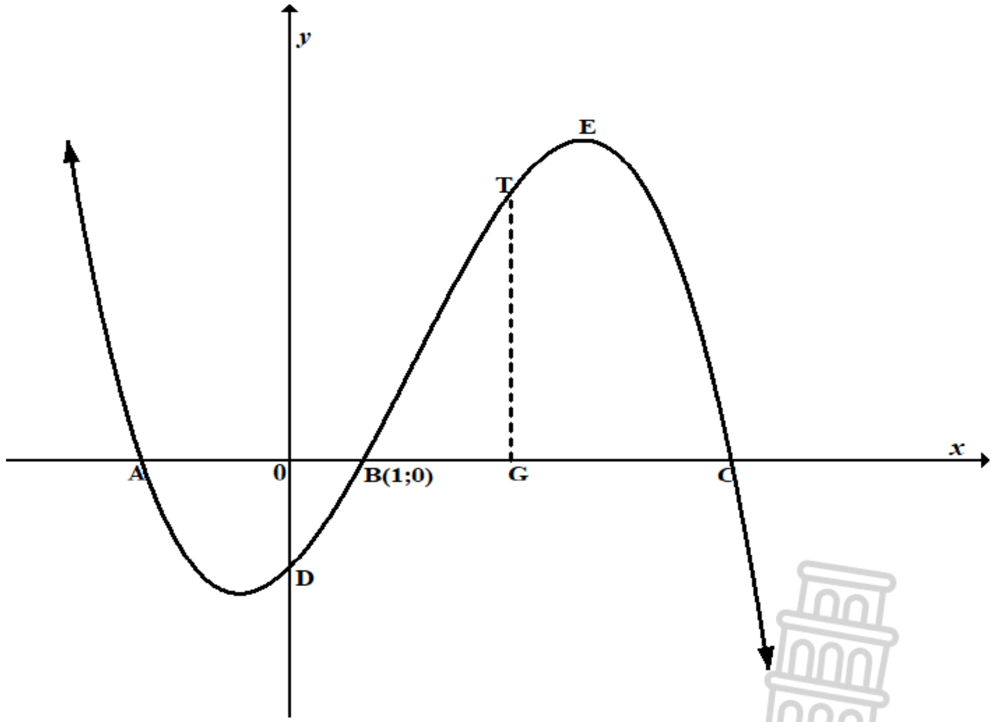
4.8 Use the information given below to sketch then graph of $f(x) = ax^3 + bx^2 + cx + d$

- $f(1) = f(4) = 0$
- $f'(1) = f'(3) = 0$
- $f'(x) > 0$ for $1 < x < 3$
- $f(0) = f(3) = 4$

(5) **L3**

4.9 **MP SEP 23**

In the diagram, the graph of $f(x) = -x^3 + 5x^2 + 8x - 12$ is drawn. A, B and C are the x-intercepts of f. E is a turning point of f. T is a point f and G is a point on the x-axis such that TG is perpendicular to the x-axis. D is the y-inercept of f.



4.9.1 Calculate the coordinates of C if B(1;0). (4) **L2**

4.9.2 Determine the coordinates of E. (5) **L2**

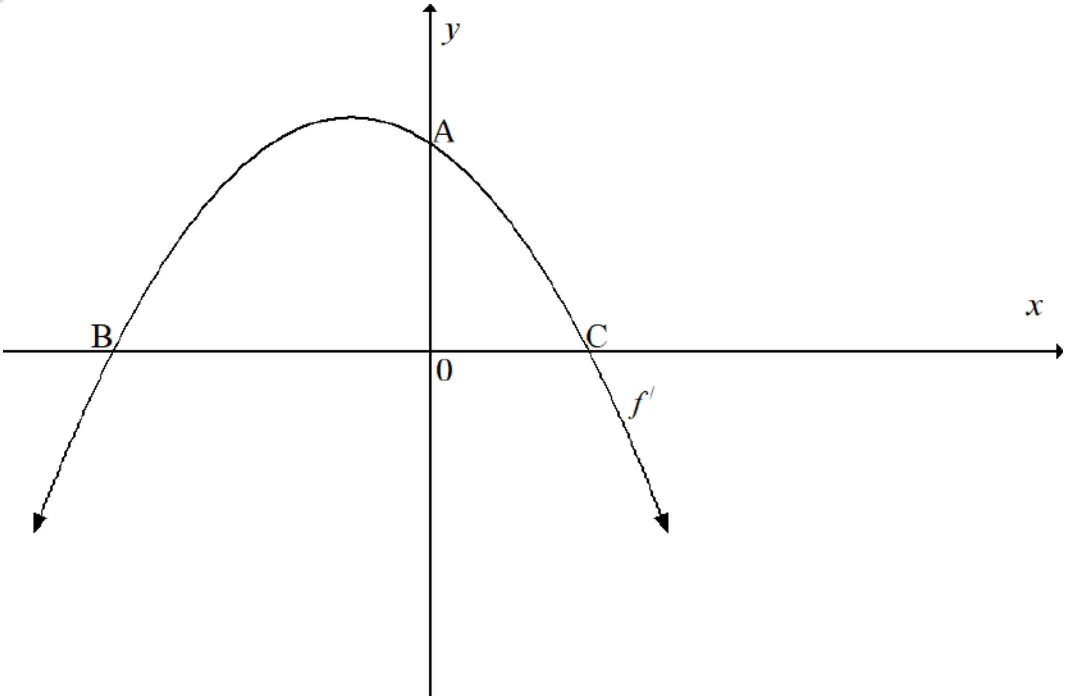
4.9.3 For which value(s) of x will f be concave up? (2) **L2**

4.9.4 Calculate the length of OG if the tangent to the curve at T is parallel to the tangent to the curve at D. (5) **L4**

4.9.5 Determine the value of m if $y = mx + c$ intersects f perpendicularly at $x = 5$. (3) **L3**

4.10 **GP SEP 23**

Sketch below is the graph of f'. The derivative of $f(x) = -2x^3 - 3x^2 + 12x + 20$. Points A, B and C are the intercepts of f' with the axes.



4.10.1 Write down the coordinates of A. (1) **L2**

4.10.2 Determine the coordinates of B and C (3) **L2**

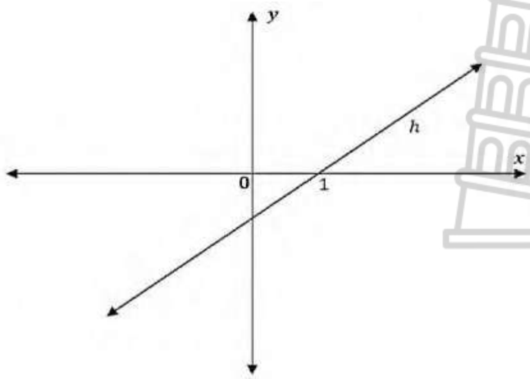
4.10.3 Which points on the graph of f will have exactly the SAME x-value(s) as B and C? (1) **L3**

4.10.4 For which values of x will f be increasing? (3) **L2**

4.10.5 Determine the y-coordinate of the point of inflection of f. (4) **L2**

4.11 **FS SEP23**

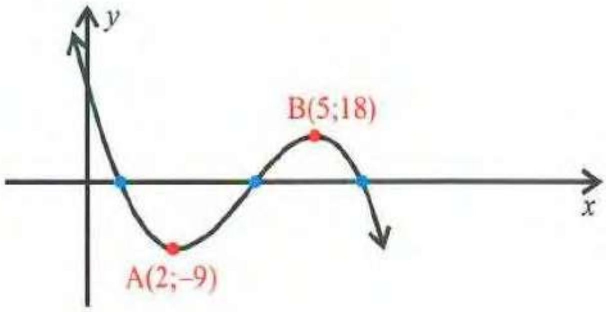
The diagram shows the straight-line h, where $h(x) = f'(x)$. The x-intercept of h is 1. The following is true for function $f: f(1) = -3$ and $f(3) = 0$.



Draw a sketch graph of the function f, clearly indicating all x-intercepts and turning point(s) (3) **L4**

4.12 **MATHS HANDBOOK GR12**

The graph shown below has the equation $f(x) = -2x^3 + bx^2 + cx + d$. The graph has stationary points at $A(2;-9)$ and $B(5;18)$. Determine the value of b, c and d.



(7) **L3**

5 5.1 **FS SEP 23**

During an experiment the temperature, T in °C Varies with time t in seconds, to the equation $T(t) = 60 + 27t - t^3$, $t \in [0;6]$.Calculate:

5.1.1 The average change of temperature between 3 and 6 seconds. (3) **L1**

5.1.2 After how many seconds the temperature will be a maximum. (3) **L2**

5.2 **STUDY AND MASTER GR12**

A particle moves according to the formula $s = 6t - 24\sqrt{t} + 30$, where s is its distance in millimetres from a fixed point P after t seconds.

5.2.1 Determine its distance from P after 9 seconds. (2) **L1**

5.2.2 The particle reaches its minimum distance from P after t seconds. Determine the values of t. (5) **L2**

5.3 **STUDY AND MASTER GR12**

A length of wire, 4 metres long, is cut into two pieces. One piece is bent into the shape of a square and the other into the shape of a circle.

5.3.1 If the length of wire that is used to make the circle is x metres, show that the sum of the areas of the circle and the square is given by:

$$f(x) = \left(\frac{1}{16} + \frac{1}{4\pi}\right)x^2 - \frac{x}{2} + 1 \quad \text{m}^2$$

(6) **L3**

5.3.2 How should the wire be cut so that the sum of the areas of the circle and the square is a minimum? (4) **L2**

5.4 **NW SEP 16**

A marathon athlete trans between towns A and C. He starts at point B which lies between towns A and C. The athlete runs from point B to town C and back to point B. The road between the towns is in a straight line. The displacement S, in kilometres, from point B after t hours, is given by: $s(t) = -t^3 + 12t^2 - 32t$

5.4.1 How many hours will it take the athlete to return to point B? (3) **L1**

5.4.1 Calculate the distance between point B and town C. (5) **L2**

5.4.3 Calculate the maximum speed that the athlete has reached while training. (4) **L3**

5.5 **STUDY AND MASTER GR12**

A mirror is set into a wooden frame which is 2 cm wide. The outside perimeter of the wooden frame is 72 cm.

5.5.1 The length of the frame is x cm. Determine the breadth of the frame in terms of x. (1) **L2**

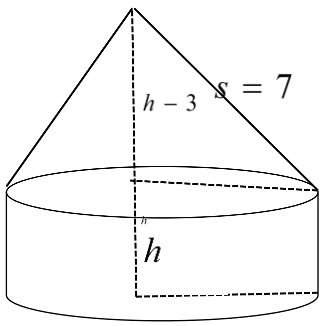
5.5.2 Determine the length and the breadth of the mirror (without the frame) in terms of x. (3) **L2**

5.5.3 Show that the area of the mirrors is given by $A(x) = -x^2 + 36x - 128$ cm^2. (2) **L3**

5.5.4 Calculate the dimensions of the mirror with largest area that can fit into the frame. (3) **L2**

5.6 **STUDY AND MASTER GR12**

Motsumi is making model of a rondavel out of clay for his design project. Motsumi's model has a radius of x cm and a height of h cm. The height of the roof itself(the cone) is $(h-3)$ cm and the volume of the conic roof is $90\,\text{cm}^3$



5.6.1 Show that $h=\dfrac{270+3\pi r^2}{\pi r^2}$. (Volume of a cone $=\dfrac{1}{3}\pi r^2 h$) (5) **L4**

5.6.2 Motsumi wants to paint the entire clay model. Determine the maximum surface area for the clay rondavel, without a floor. Leave your answer correct to the nearest cubic cm. (Total surface area without base $=\pi rs+2\pi h$) (7) **L3**

TOPIC 6. PROBABILITY [±15 MARKS]

GUIDELINES, SUMMARY NOTES, & STRATEGIES

The probability scale: $0\le P\le 1$. If P (an event) = 0, the event is impossible; If P (an event) = 1, the event is certain to happen.

The **definition of probability**: $P(E)=\dfrac{n(E)}{n(S)}$

Addition Rule for any 2 events A and B: **P (A or B) = P(A) + P(B)−P (A and B)**
Mutually exclusive events A and B: **P (A or B) = P(A) + P(B)**
NOTE: Since P (A and B) = 0
Independent events A and B: **P (A and B) = P(A) × P(B)**
The complementary rule: **P (not A) = 1−P(A)**
Venn-Diagram, Tree diagram and Contingency Table

The fundamental counting principle: If one operation can be done in *m* ways and a second operation can be done in *n* ways then the total possible number of different ways in which both operations can be done is $m\times n$.

- *Pin codes and Passwords*
- *Arrangements [(a) Different/Selection (b) Identical)]*
- *Re-arrangements*

REVISION QUESTIONS

1 **KZN SEPTEMBER 2018 QUESTION 12**

Study the table below and answer the questions that follow.

	Like Sport	Do not Like Sport	Totals
Males	80	*b*	*c*
Females	*a*	90	*d*
Totals	200	150	350

1.1 Write down the values of *a, b, c* and *d.* (4) **L1**

1.2 Is the event liking a sport independent of gender? Show all working (4) **L2**

2 **KZN SEPTEMBER 2018 QUESTION 13**

Consider the letters of the word "DEPENDENT". Determine, using all letters:

2.1 the number of unique arrangements of the letters that can be formed? (3) **L2**

2.2 the number of unique arrangements of letters that can be formed in 2.1 starting with the letter "N" ? (3) **L2**

2.3 the number of unique arrangements of letters that can be formed in 2.1 starting and ending with the letter "N" ? (3) **L2**

3 **EC SEPTEMBER 2018 QUESTION 11**

3.1 In a survey done at a local traffic department, the following information was obtained.

	Failed	Passed	Total
Male	**A**	**B**	1200
Female	**C**	**D**	400
Total	200	1400	1600

3.1.1 Calculate the probability that a person selected at random will be male (1) **L1**

3.1.2 Calculate the probability that a person selected at random failed the test (1) **L1**

3.1.3 If being male and failing the test are independent events, show that the value of **A** = 150. (3) **L3**

3.1.4 Use the value of **A** to determine the values of **B**, **C** and **D**. (3) **L1**

3.1.5 Calculate the probability of choosing a female who failed. (2) **L2**

3.2 9 cars of different makes of which 4 are black are to be parked in a straight line.

3.2.1 In how many different ways can all the cars be parked? (2) **L1**

3.2.2 If the 4 black cars must be parked next to each other, determine in how many different ways the cars can be parked. (3) **L2**

4 **GAUTENG TRIAL 2023 QUESTION 12**

4.1 When Marge turned eight, her friends Emily, Klara, Cory, Liza, Shirley and Penny were invited to her birthday party. Marge and her friends sat in a row and played a game. In how many ways can they be seated if:

4.1.1 They sit in alphabetical order (1) **L1**

4.1.2 Emily and Klara do NOT want to sit next to each other? (3) **L3**

4.2 The probability that a certain rugby team has all its players fit to play is 70%. The probability that they will win a game if all their players are fit is 90%. When they are not fit the probability of them winning becomes 45%. Calculate the probability of them winning the FIRST game. (2) **L2**

5 **KZN SEPT 2019 PREPARATORY EXAMINATIONS QUESTION 12**

A bag contains 12 blue balls, 10 red balls and 18 green balls. 2 balls are chosen at random without replacement. Determine the probability:

5.1 if the two balls chosen at random are green. (3) **L3**

5.2 if the two balls chosen at random are blue and red. (3) **L3**

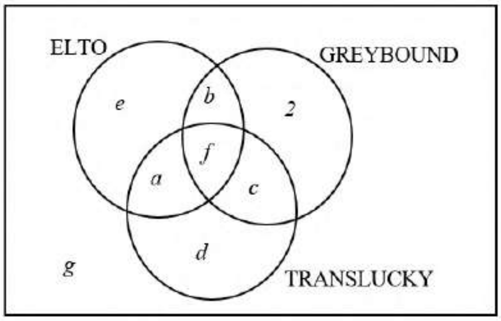
6 **KZN TRIALS 2023 QUESTION 12**

A group of 40 people was asked which bus company they liked to travel by: Elto, Greybound or Translucky bus company.

- 3 people like travelling by all 3 bus companies.
- 7 people liked to travel by Translucky and Elto.
- 3 people did not like using any of the bus companies.

- The probability of a randomly selected person liking to travel by Greybound and Elto is $\frac{2}{5}$
- The probability of a randomly selected person liking to travel by Greybound and Translucky is $\frac{1}{5}$
- The probability of a randomly selected person liking to travel by Translucky only is $\frac{1}{10}$
- The probability of a randomly selected person liking to travel by Elto is $\frac{13}{20}$

The partially completed Venn diagram drawn below represents the given information.



6.1 Use the given information to determine the values of *a, b, c, d* and *e* (5) **L3**

6.2 How many people liked to travel by Greyhound bus company? (1) **L2**

6.3 Calculate the probability of a randomly selected person liking to travel by only one bus company. (2) **L2**

7 **KZN TRIALS 2023 QUESTION 13**

Nonhle who is a Grade 12 learner has 8 textbooks from eight different subjects: Mathematics, English, Accounting, History, Tourism, Afrikaans, Geography and Drama which she wants to arrange in a line on a shelf.

7.1 In how many ways can the textbooks be arranged? (1) **L1**

7.2 In how many ways can the textbooks be arranged if the Mathematics textbook and the Accounting textbook must be on each end of the shelf? (3) **L2**

7.3 If the Mathematics textbook and the Accounting textbook must be on each end of the shelf, what is the probability that the History textbook and the Tourism textbook are not next to each other? (4) **L3**

8 **SEPT 2019 PREPARATORY EXAMINATIONS QUESTION 13**

The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are used to form 3 - digit codes, eg. 567, 218, etc. Determine the number of different codes that can be formed:

8.1 if repetition is allowed. (2) **L2**

8.2 such that the code is greater than 500 and repetition is NOT allowed (2) **L2**

8.3 such that the middle digit is 5 and repetition is allowed (2) **L2**

9 **GAUTENG JUNE 2022 QUESTION 10**

9.1 In a survey, 1 530 people were asked whether they had ever broken a limb. The results of the survey were as follows:

	Broken a limb	Not Broken a limb	Total
Male	463	b	782
Female	a	c	d
Total	913	617	1530

9.1.1 Calculate the values of a, b, c, and d. (4) **L2**

9.1.2 If a person is chosen at random, what is the probability that it will be a female who has not broken a limb? (2) **L2**

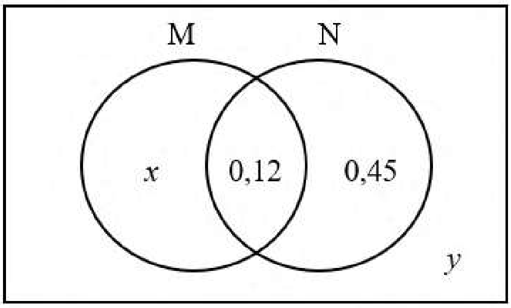
9.1.3 Is having a broken limb dependent on gender? Motivate your answer. (3) **L2**

9.2 Two learners are selected at random from a group of 10 boys and 12 girls. Determine the probability that …

9.2.1 they are both girls. (2) **L2**

9.2.2 one is a boy and one is a girl. (3) **L3**

9.3 In the Venn diagram below, M and N are independent events.



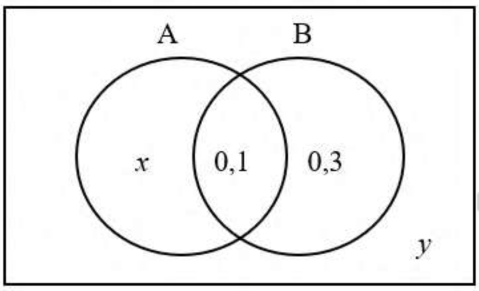
Calculate, giving answers correct to two decimal places:

9.3.1 The value of x (3) **L3**

9.3.2 The value of y (2) **L2**

10 **GAUTENG TRIAL 2023 QUESTION 11**

10.1 Machine A and machine B are two different coin-pressing machines that operate at the same time. The probability that machine A ONLY presses a R5 coin, is x and the probability that machine B ONLY presses a R5 coin, is 0,3. The probability that both the machines press R5 coins at the same time is 0,1.



10.1.1 If A and B are independent events, determine the values of x and y. (4) **L2**

10.1.2 Determine the probability that exactly one of the machines is pressing a R5 coin (1) **L2**

10.2 Wilson takes a driver's test. The probability that he will succeed on bis first attempt is $\frac{3}{7}$. For each attempt that he redoes the test, the probability of passing increases to $\frac{3}{5}$

10.2.1 What is the probability that Wilson will succeed after 2 attempts? (2) **L2**

10.2.2 Determine the probability that Wilson will succeed after 3 attempts (2) **L2**

11 **GAUTENG TRIAL 2021 QUESTION 10**

Events A, B and C occur as follows, where A and B are independent events:

- $P(A) = 0,38$
- $P(B) = 0,42$
- *P(A and B)=0,1596*
- $P(C) = 0,28$

11.1 Are A and B mutually exclusive events? Motivate your answer. (2) **L2**

11.2 By using an appropriate formula, show that the value of *P(A or B)=0,64* (2) **L2**

11.3 Calculate the number of people in the sample space. (2) **L2**

11.4 Determine *n(not C)* (2) **L3**

12 **GAUTENG TRIAL 2021 QUESTION 11**

12.1 Each of the digits: 1 ; 1 ; 2 ; 3 ; 4 ; 7 is written on a separate card.
The cards are then placed next to each other to create a 6 digit number.

12.1.1 How many numbers start and end with the same digit? (1) **L2**

12.1.2 Find the probability that the number is 112347 or 743211. (4) **L4**

12.2 n people (numbered 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; … ; n) are arranged randomly in a line.

12.2.1 Find the number of ways, in terms of n, that person 1 and person 2 are standing next to each other. (You do not need to simplify your answer.) (3) **L4**

13 **DBE NOVEMBER 2021 QUESTION 12**

13.1 A and B are independent events. It is further given that:
$P(A \text{ and } B) = 0,3$ and $P(\text{only } B) = 0,2$

13.1.1 Are A and Bmutually exclusive? Motivate your answer. (1) **L2**

13.1.2 Determine:

(a) P (only A) (4) **L2**

(b) P (not A or not B) (2) **L3**

13.2 A teacher has 5 different poetry books, 4 different dramas and 3 different novels. She must arrange these 12 books from left to right on a shelf.

13.2.1 Write down the probability that a novel will be the first book placed on the shelf. (1) **L2**

13.2.2 Calculate the number of different ways these 12 books can be placed on the shelf if any book can be placed in any position (2) **L3**

13.2.3 Calculate the probability that a poetry book is placed in the first position, the three novels are placed next to each other and a drama is placed in the last position. (4) **L4**

14 **DBE NOVEMBER 2023 QUESTION 10**

14.1 A and B are independent events. $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$

Determine

14.1.1 P(A and B) (2) **L2**

14.1.2 P (at least ONE event occurs) (2) **L3**

14.2 The probability that it will snow on the Drakensberg Mountains in June is 5%.

- When it snows on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0°C is 72%.

- If it does not snow on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0°C is 35%.

14.2.1 Represent the given information on a tree diagram. Clearly indicate the probabilities associated with EACH branch. (3) **L3**

14.2.2 Calculate the probability that the temperature in Central South Africa will NOT drop below 0°C in June 2024. (3) **L3**

14.3 Ten learners stand randomly ina line, one behind the other.

14.3.1 In how many different ways can the ten learners stand in the line? (1) **L2**

14.3.2 Calculate the probability that there will be 5 learners between the 2 youngest learners in the line. (4) **L4**

15 **IEB NOV 2023 QUESTION 11**

After some research you decide that the best option for your online security is to have two unique passwords before opening important documents.

15.1. The first ten-digit password consists of two parts.

15.1.1 The first part is made up of six numbers. The numbers that can be used are the numbers from 1 to 9. How many unique six-digit codes can be created if **repetition is allowed**? (2) **L2**

15.1.2 The second part is a four-digit code compiled from the digits 1 to 9, **repetition is not allowed**. How many even ten-digit passwords can be created by combining these two parts? (3) **L2**

15.2 The second nine-digit password has to be a number greater than 600 000 000 with the last digit being divisible by 3. How many nine-digit passwords are possible if you can use the digits from 1 to 9, **no repetition is allowed**? (4) **L3**

16 **IEB NOV 2014 QUESTION 6**

16.1 In a sample space S, the number of elements in S, $n(S) = 30$ and there are two events A and B such that $n(A) = 15$, $n(B) = 20$ with $n(A \text{ and } B) = 6$.

16.1.1 Draw a Venn diagram to represent this situation. (3) **L2**

16.1.2 Write down the value of $n(A \text{ or } B)$. (1) **L1**

16.1.3 An element is randomly selected from S.

- Write down the probability that the element is in both events A and B. That is, P(A and B). (1) **L1**
- Showing all working, determine whether the events A and B are independent (3) **L3**

16.2 Steve needs to set up a format for passwords onto his website. He has decided on having letters from the alphabet (of 26 letters), followed by digits 0 to 9. Letters and digits can be repeated.

16.2.1 Calculate the number of passwords that can be created using 2 letters followed by 2 digits. (2) **L2**

16.2.2 Steve thinks that he will need to cater for 3 million different passwords. He will stick with 2 letters but will need more digits. Determine the least number of the digits he will need. (4) **L4**

17 **IEB NOV 2019 QUESTION 11**

17.1 Ten coins are arranged in a row:

- five are R1 coins
- three are R2 coins
- two are R5 coins

How many different arrangements are possible, knowing that all the coins of the same value are identical? (3) **L2**

17.2 The trees in an orange orchard are harvested twice a year. During the first harvest, 70% of the oranges are picked while the rest are left. At the second harvest, 35% of the remaining oranges are picked while the rest are not picked. Assume no oranges were added between harvests.

17.2.1 Calculate the probability that a randomly selected orange will not be picked. (3) **L3**

17.2.2 If it is further given that all the oranges that are picked are packaged with:

- 9% from each harvest selected for export
- 31% sold to the local market and
- the rest are sent to a factory to be made into juice.

What percentage of oranges will be sent to the factory to be made into juice? (4) **L3**

17.2.3 There are 120 oranges in an export box. If 172 export boxes are produced, then how many oranges were there in the total crop? (4) **L3**

18 **THE ANSWER SERIES KZN 2024 MATHS WORKSHOP**

18.1 Six friends go to watch a movie. They will sit next to each other in a straight row. Themba and Linna have had an argument and refuse to sit next to each other. How many possible seating arrangements are there? (3) **L4**

18.2 Mr and Mrs Brown and their four children line up for a photograph.

18.2.1 In how many ways can they line up if anyone may sit anywhere? (1) **L1**

18.2.2 In how many ways can they line up if Mr and Mrs Brown must each sit at an end? (2) **L3**

18.2.3 In how many ways can they line up if Mr and Mrs Brown must sit next to each other in the middle? (2) **L3**

18.2.4 In how many ways can they line up if Mr and Mrs Brown must sit next to each other anywhere? (3) **L3**

18.3 Consider the letters AHMST. If all five letters are used, and all the possible arrangements that can be formed are placed in alphabetical order, in which position is the word MATHS? (3) **L4**

18.4 A four-digit code is made from the digits 0 to 6.

How many four-digit codes can be made if the code has to be greater than 2 000, less than 3 000, and must be even?

You may not repeat digits. (3) **L3**

18.5 Consider the word MILLION

18.5.1 Determine the number of seven-letter words that can be made. (2) **L2**

18.5.2 Determine the probability that the vowels will be next to each other. (3) **L3**

TOPIC: STATISTICS/DATA HANDLING

GUIDELINES, SUMMARY NOTES, & STRATEGIES

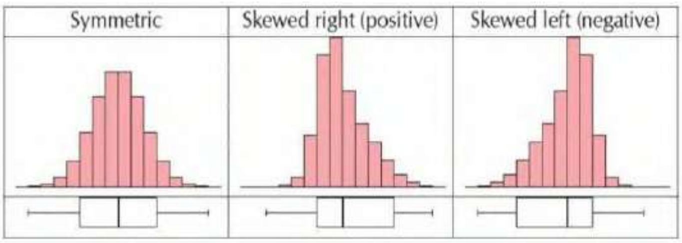
Definition:
Data Handling is a process during which data (information) is collected, recorded, and presented.

Terminology:

- ❖ **Ungrouped data** – a set of random data elements gathered for analysis.
- ❖ **Grouped data** – data elements aggregated into different classes, groups, or intervals.

- **Measures of central tendency** – single numbers around which all data items seem to be spread.
  - ❖ The **Mean**, also known as the average, is the sum of all the data values in a set, divided by number of all elements in the set.
  - ❖ The **Median,** (Q_2) it presents the middle value in a data set.
  - ❖ The **Mode** is the most frequent data item in a set. In grouped data, the modal group will have the highest frequency. Data sets may have no mode, two modes (bimodal), three modes (trimodal), etc.
- **Measures of dispersion** – numbers that describe the spread of the data.
  - ❖ The **Range** is the difference between the maximum and the minimum data values in a given data set.
  - ❖ The **Inter-Quartile-Range** (IQR) is the difference between the third and first quartiles, i.e. $IQR = Q_3 - Q_1$
  - ❖ **Standard Deviation** (σ) is a measure of how dispersed data is around the mean. The square of the standard deviation is the **variance.**

- Five Number Summary – five numbers that separate a data set into quarters.
  - ❖ Minimum value
  - ❖ Lower quartile (Q_1) position $\frac{1}{4}(n+1)$
  - ❖ Median
  - ❖ Upper quartile Q_3 position $\frac{3}{4}(n+1)$
  - ❖ Maximum value



- Box – and – Whisker Diagram (drawn using the five number summary)
  - ❖ It is important in analysing the distribution of data in a given set.
  - ❖ If mean – median = 0, then the distribution is symmetric.
  - ❖ If mean – median > 0, then the distribution is positively skewed.
  - ❖ If mean – median < 0, then the distribution is negatively skewed.

- Outliers – data items that are a lot bigger or smaller than the rest of the elements in the data set.

They are determined as follows:

  - ❖ Lower outliers are numbers $< Q_1 - 1.5 \times IQR$
  - ❖ Upper outliers are numbers $> Q_1 + 1.5 \times IQR$
- Graphical representations
  - ❖ **Histogram** – represents grouped data as condensed bars whose widths and lengths represent class intervals and frequency respectively.
  - ❖ **Ogive (Cumulative Frequency Curve)** – an *s*-shaped smooth curve drawn by plotting upper limits of class intervals of a grouped data against cumulative frequency of a set.
  - ❖ **Scatter plot** – representation of bivariate data as discrete data points.
- Bivariate data summaries
  - ❖ **Regression line (line of best fit) -** a line drawn on the scatter plot that shows a general trend that bivariate data seems to follow.
  - ❖ **Least squares regression line** – is a straight line that passes through the mean point relating bivariate data
  - ❖ **Correlation Coefficient** – indicates the strength of the relationship between the variables in bivariate data.

REVISION QUESTIONS

FS/ PREPARATORY EXAM 2023

1 To celebrate Pi Day at school, learners participate in a competition to write down the value of Pi (π), up to the most correct decimal places, Eleven learners make it to the final round of the competition where their number of correct decimal places is counted.
The judges stop counting after the first mistake. The results of the eleven learners are shown in the table below:

63	79	50	74	75	66	150	86	72	74	60

Calculate the:

1.1 Mean of the data. (2) **L1**

1.2 The standard deviation for the given data (1) **L2**

1.3 Number of results that lie outside ONE standard deviation of the mean (3) **L2**

1.4 Identify the outlier in the given results. (1) **L1**

1.5 The result with the number of the most correct decimal places is increased by k %, while the result with the number of the lowest correct decimal places is decreased by t %. The other nine results remain unchanged.
Only one of the options below correctly reflects the new range of the data in terms of k and t. Choose the answer and write only the letter.

A. $100+k-t$

B. $150k-50t$

C. $150k+50t$

D. $100+3/2k+1$ (2) **L2**

IEB/ NOVEMBER 2022

2 The test scores of 5 students are given in ascending order as:
$\{25; x+3; x+6; x+9; x+13\}$. The median is 41.

2.1 Determine the value of x. (2) **L1**

2.2 Determine the standard deviation of the scores using your answer to 2.1. (2) **L2**

2.3 How many scores will lie within one standard deviation from the mean? Show all your working. (3) **L2**

3 The minimum overnight temperatures ($T\,°C$) and the number of service calls (S) made to a company that supplies gas heaters was recorded for a period of 8 days.
The equation of the least squares regression line for the data is given as: $S=-1.8T+22{,}7$

3.1 State whether the data represents a **positive** or **negative** correlation. (1) **L1**

3.2 Use the regression line to predict the number of service calls made, for an overnight temperature of $10\,°C$. (2) **L1**

3.3 The correlation coefficient for the 8-day period is −0,95. On the 9th day, the number of service calls was 8 and the minimum overnight temperature was $3\,°C$. If the 9th day data is included, what effect will it have on the correlation coefficient? Explain. (2) **L3**

DBE NOVEMBER 2023

4 Truck drivers travel a certain distance and have a rest before travelling further. A driver kept record of the distance he travelled (in km) on 8 trips and the amount of time he rested (in minutes) before he continued his journey. The information is given in the table below.

Distance travelled (in km) (x)	180	200	400	600	170	350	270	300
Amount of rest time (in minutes) (y)	20	25	55	120	15	50	40	45

4.1 Determine the equation of the least squares regression line for the data (3) **L1**

4.2 If a truck driver travelled 550 km, predict the amount of time (in minutes) that he should rest before continuing his journey. (2) **L2**

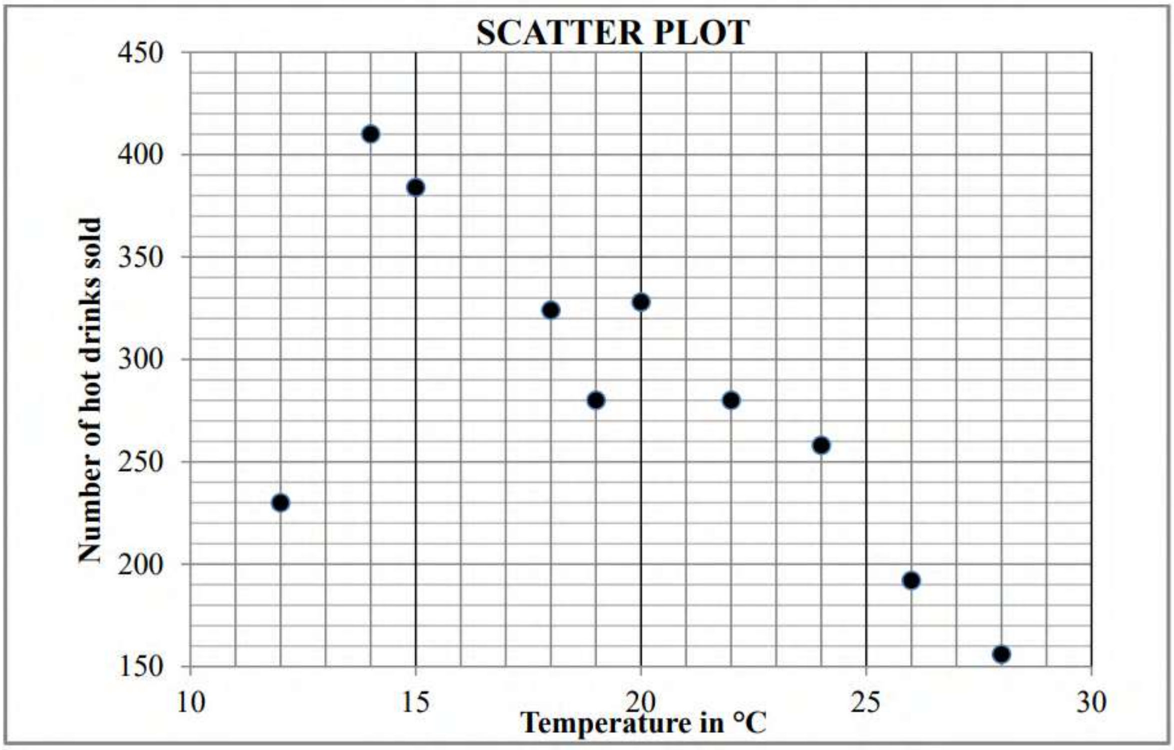
4.3 Write down the correlation coefficient for the data. (1) **L1**

4.4 Interpret your answer to QUESTION 4.3 (1) **L1**

DBE NOV 2020(2)

5. An annual sports festival is held over a period of 11 days. A tuckshop sells hot drinks at this festival. On each of the first 10 days, the owner of the tuckshop recorded the temperature at 13:00 and the number of cups of hot drinks sold. This information is presented in the table and scatter plot below.

Temperature (in °C)	14	24	26	18	20	28	22	15	12	19
Number of hot drinks sold	410	258	192	324	328	156	280	384	230	280



5.1 Describe the trend of the data. (1) **L1**

5.2 Determine the equation of the least squares regression line for the data. (3) **L1**

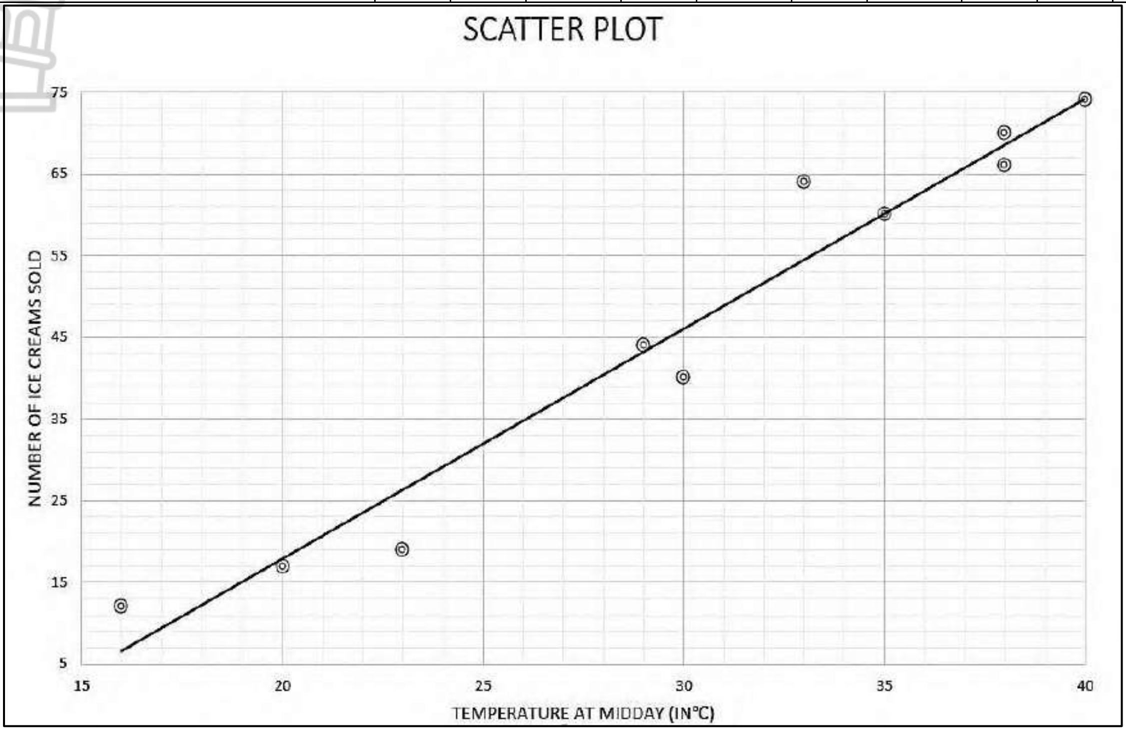
5.3 The owner observed that he had used one litre of milk for every 8 cups of hot drinks sold. If the temperature at 13:00 on the 11th day was expected to be 17 °C, predict the number of 1-litre boxes of milk the owner should buy for the 11th day. (3) **L2**

5.4 Identify an outlier in the data. (1) **L1**

FS/ PREPARATORY EXAM 2023

6 On the first Saturday of a months, information was recorded about the temperature at midday (in $°C$); and the number of ice creams sold at an ice cream stand at a particular beach. The data is shown in the table below and represented on the scatter plot. This data's least squares regression line is drawn on the scatter plot.

Temperature at midday (in $°C$)	16	20	23	29	33	38	40	38	35	30
Number of ice creams sold	12	17	19	44	64	70	74	66	60	40



6.1 Refer to the scatter plot. Would you say that the relationship between the temperature at midday and the number of ice creams sold is weak or strong? Motivate your answer (2) **L1**

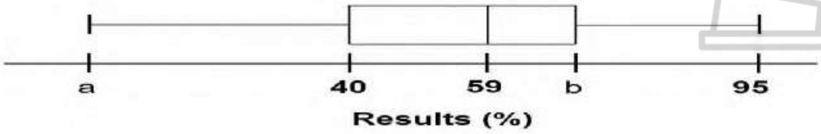
6.2 Determine the equation of the least squares regression line. (3) **L1**

6.3 Predict the number of ice creams that will be sold on a Saturday if the temperature is 2 $°C$ at midday. (2) **L2**

6.4 On another first Saturday of the month, the temperature at midday was 24 $°C$ and 40 ice creams were sold. If this data was added to the data set, how will the prediction of the number of ice creams sold within the given domain be affected? Motivate your answer. (2) **L3**

IEB/ NOVEMBER 2023

7. The percentage results obtained by 26 learners in a mathematics test is displayed in the box and whisker plot below:



If the range of the data is **80** and the interquartile range (IQR) is **30**:

7.1 Determine the value of ***a***. (1) **L1**

7.2 Determine the value of ***b***. (1) **L1**

7.3 Determine whether the minimum result obtained (using your answer to 7.1) is an outlier or not. Use the formula Q1 − 1,5 x IQR (2) **L2**

7.4 The table illustrates the approximate time spent (x) in minutes by 7 learners studying for a mathematics test and their mark obtained as a percentage.

Time spent (x) in minutes	0	90	90	80	90	120	150
Mark obtained (y) as a %	15	59	60	73	85	90	95

7.4.1 Predict, using the equation of the least squares regression line in the form $y = a + bx$ for this data, what mark a learner who studies for 180 minutes will obtain. Round your answers to 3 decimal places. (3) **L3**

7.4.2 Is this prediction in 7.4.1 a reliable one? Explain. (2) **L4**

DBE/ MAY/JUNE 2023

8. The ages of the people who attended a music concert was summarised in the table below.

AGE	NUMBER OF PEOPLE
$5 < x \leq 15$	20
$15 < x \leq 25$	25
$25 < x \leq 35$	60
$35 < x \leq 45$	90
$45 < x \leq 55$	55
$55 < x \leq 65$	40
$65 < x \leq 75$	30

8.1 Write down the modal class of the data. (1) **L1**

8.2 How many people attended the music concert? (1) **L1**

8.3 Draw a cumulative frequency graph (ogive) to represent the above data. (4) **L2**

8.4 Use the cumulative frequency graph to determine the median age of the people who attended the music concert. (2) **L2**

MAY/JUNE 2024

8.5 Fifty athletes need to access suitable training facilities. The table below shows the distances in km. that they need to travel to obtain access to suitable training facilities.

Distance (x km)	Number of athletes	Cumulative frequency
$0 < x \leq 5$	3	3
$5 < x \leq 10$	….	10
$15 < x \leq 15$	20	….
$15 < x \leq 20$	….	42
$20 < x \leq 25$	5	….
$55 < x \leq 30$	….	50

8.5.1 Complete the table above (2) **L1**

8.5.2 On the grid provided draw the cumulative frequency curve(ogive) (3) **L2**

8.5.3 Calculate the interquartile range (2) **L2**

GP/SEPTEMBER 2022

9. Formula 1 (F1) race car drivers have to endure high G-forces at extremely high temperatures. They tend to lose close to 4 kg of weight after every race.
The table below shows the total weight lost by 40 different race car drivers after the duration of one race.

INTERVAL OF WEIGHT LOST (IN GRAMS)	NUMBER OF DRIVERS
$0 < w \leq 500$	1
$500 < w \leq 1000$	2
$1000 < w \leq 1500$	3
$1500 < w \leq 2000$	8
$2000 < w \leq 2500$	6
$2500 < w \leq 3000$	15
$3000 < w \leq 3500$	5
Total	**40**

9.1 Write down the modal class of the data. (1) **L1**

9.2 Calculate the estimated mean weight-loss of the race car drivers. (3) **L2**

9.3 The recording of the weight lost in a second race was made a month later. The amount of weight lost in race 2 was k grams more than in race 1. It is given that the maximum value of the ogive, representing race 2 was (3 504 ; 40) and the graph was grounded at (4 ; 0).

9.3.1 Sketch the ogive (cumulative frequency graph) representing race 2 in the. (4) **L2**

9.3.2 How will the range of race 2 compare with the range of race 1? (1) **L1**

9.3.3 Determine the average weight lost in race 2. (4) **L2**

LIMPOPO DOE/SEPTEMBER 2023

10. The following set of data 3 ; 4 ; 4 ; 4 ; 6 ; 10 ; 12 ; 12; y has mean of 7

10.1 Determine:

10.1.1 the value of y (2) **L1**

10.1.2 the median of this set of data points (1) **L1**

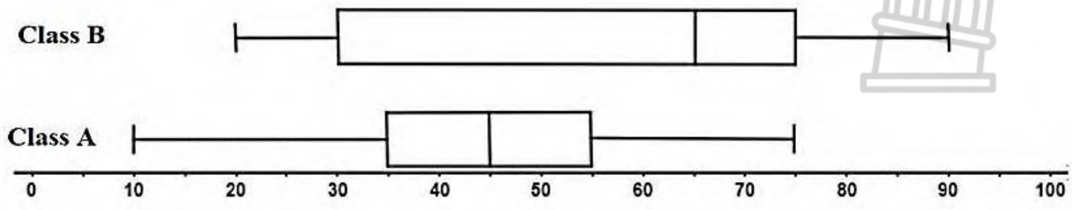
10.2 Two additional numbers, $7-n$ and $7+n$, are added to the data set.

10.2.1 Calculate the mean of these eleven numbers. (2) **L1**

10.2.2 Determine the standard deviation if the data points, that are within ONE standard deviation of the mean, lie in the interval $3 < x \leq 11$ (2) **L2**

EC/JUNE 2022

11. The box and whisker diagrams below show the Mathematics results of class A and class B in the June Examination. It is also given that class B has a Median of 65%.



11.1 Which class had the top learners? (1) **L1**

11.2 Determine which class had the greatest Inter Quartile Range (IQR). (1) **L1**

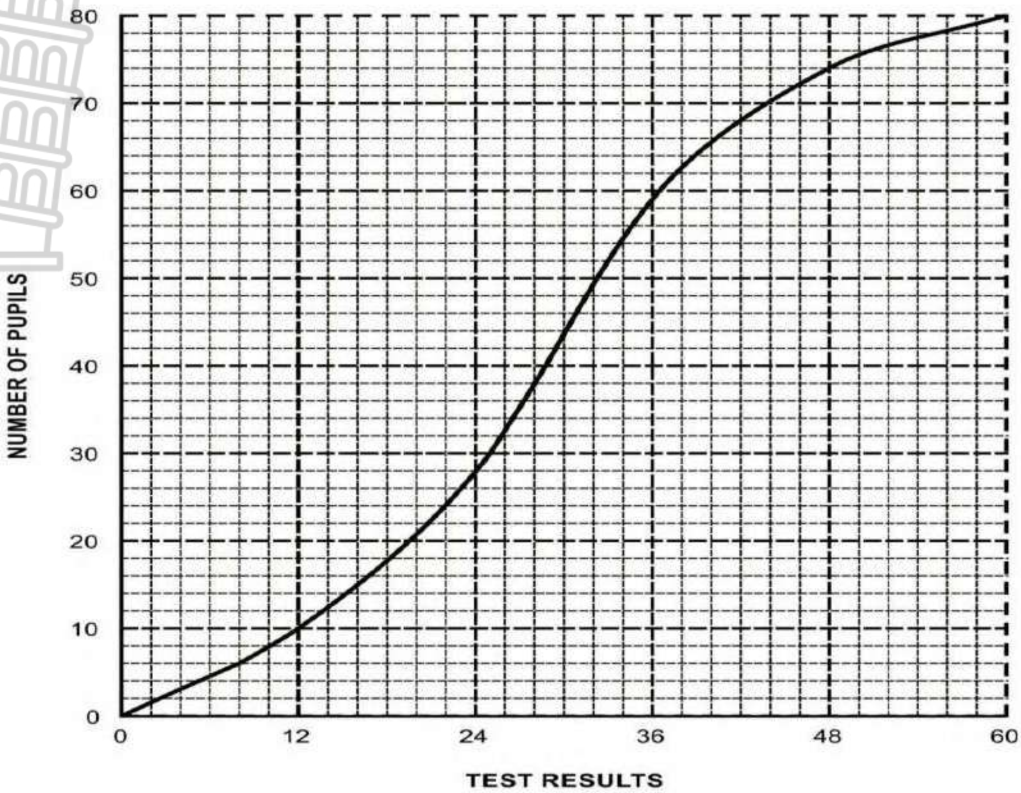
11.3 What percentage of class A scored less than 60%? (1) **L2**

11.4 If all the learners in class A were given an extra 5%, what would happen to the standard deviation of the marks in class A? (1) **L2**

11.5 Determine the semi-interquartile range of class B. (1) **L1**

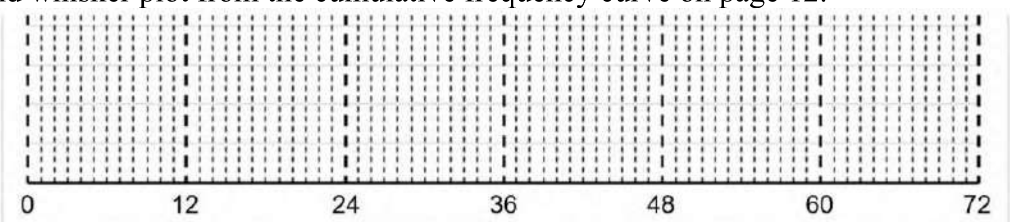
IEB/ MAY 2022

12. Refer to the cumulative frequency curve below that represents the results of 80 pupils that wrote a test out of 60 marks.



12.1 How many pupils got between 12 and 48 for the test? (1) **L2**

12.2 If it is given that the lowest mark was 8 and the highest mark was 60 then sketch a box and whisker plot from the cumulative frequency curve on page 12.



(5) **L3**

12.3 What percentage of pupils got more than 60% for the test? (2) **L2**

EC/JUNE 2022

13 A group of 30 pupils was asked to complete an obstacle course at their Grade 11 camp. The times (in seconds) taken by the pupils to complete the obstacle course are given in the table below.

Time taken	$60 < x \leq 90$	$90 < x \leq 120$	$120 < x \leq 150$	$150 < x \leq 180$	$180 < x \leq 210$
No of pupils	3	6	7	8	6

13.1 Complete the cumulative frequency table for above data. (1) **L1**

13.2 Draw a cumulative frequency curve for the above data on the grid provided. (4) **L2**

13.3 Indicate on your graph where you would read off:

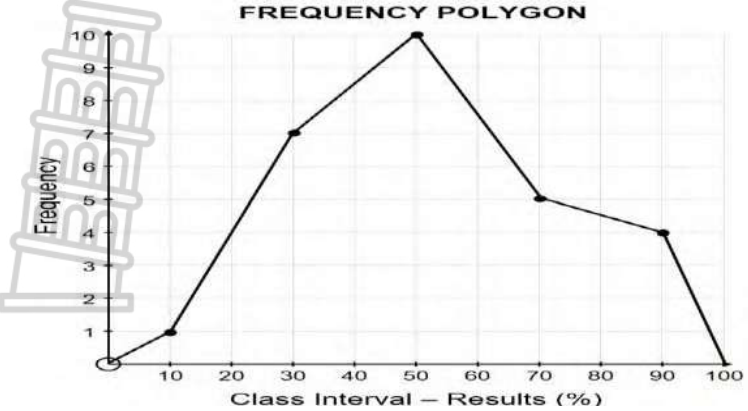
13.3.1 The number of pupils that took 135 seconds to complete the course (Use the letter A) (1) **L1**

13.3.2 The value of t if 60% of the pupils took less than t seconds to complete the obstacle course. (Use the letter B). (1) **L1**

13.3.3 The 75th percentile. (Use the letter C) (1) **L2**

IEB/NOVEMBER 2023

14. The frequency polygon represents the results obtained by 27 learners in a mathematics test. The class intervals are given as [0;20); [20;40); [40;60); [60;80); [80;100).



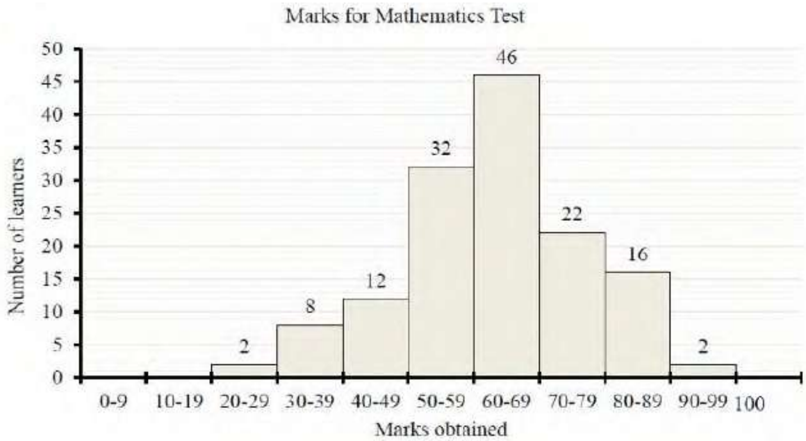
14.1 Determine the estimated mean of this set of results. (3) **L1**

14.2 What percentage of learners obtained a result in the interval $60 \leq x < 80$? (1) **L1**

14.3 Use the information given on the frequency polygon to draw a Cumulative Frequency Curve (Ogive) that represents the results of this group of learners. (3) (3) **L2**

Gauteng north strategy for learner attainment.

15 The marks obtained by learners of a certain school in mathematics is represented by a histogram below



15.1 How many learners wrote the test? 140 (1) **L1**

15.2 Write down the modal class. 60-69 (1) **L1**

Free state preparatory 2017

16. The June results of 10 learners in grade 11 class taking accounting and mathematics are recorded as follows. The table below shows the marks (out of 100) achieved by learners in both subjects

accounting(x)	72	78	90	78	46	67	80	73	63	35
mathematics (y)	56	64	85	93	32	74	86	77	72	54

16.1 CALCULATE

16.1.1 The mean mark for mathematics (1) **L1**

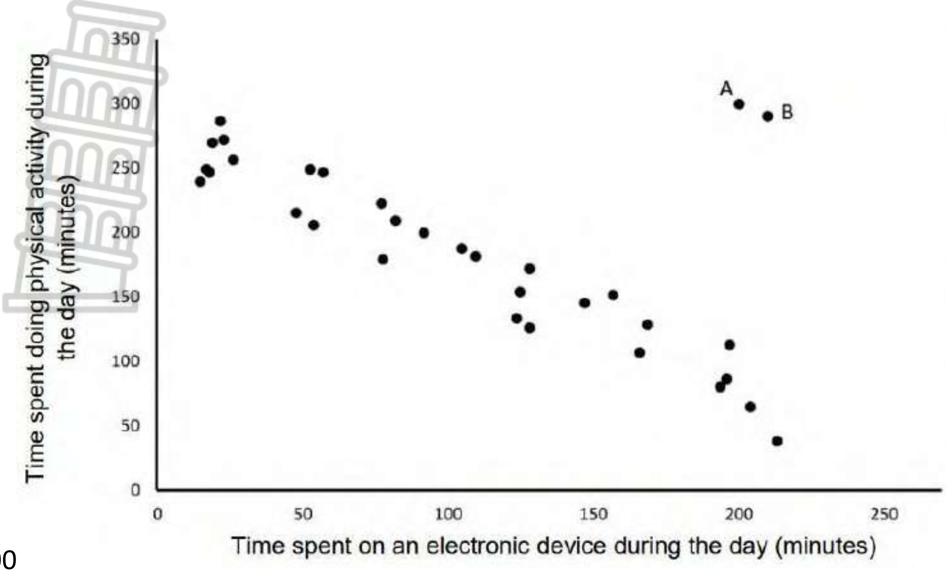
16.1.2 The standard deviation of the data for mathematics (1) **L1**

16.2 How many of these 10 learners obtained a mark for mathematics that is above ONE standard deviation from the mean. (2) **L2**

16.3 Calculate the least squares regression line of the above data. (3) **L2**

IEB/ NOVEMBER 2019

17. The diagram below shows the relationship between the time spent on an electronic device and the amount of time spent doing physical activity during the day.



00

17.1 Circle the correlation coefficient that best describes the data represented in the diagram above: $r = 1$ $\quad r = -1$ $\quad r = 0{,}8$ $\quad r = -0{,}8$ (1) **L1**

17.2 If A and B were removed from the data set above, what would happen to the:

17.2.1 Correlation coefficient? (1) **L1**

17.2.2 gradient of the line of best fit? (1) **L1**

17.2.3 Circle the line below which best describes the person represented by A.
A person who has just bought an i-pad and plays computer games.
A person who watches sport on television and likes to read books.
A person who plays professional sport and studies via the internet. (1) **L1**

17.3 Please refer to the information in the table below and answer the questions that follow:

	Coffee shop A	**Coffee shop B**
days of the week	cups of coffee sold per day	cups of coffee sold per day
Monday	low	Fairly high
Tuesday	low	Fairly high
Wednesday	low	Fairly high
Thursday	low	Fairly high
Friday	high	Fairly high
Saturday	high	Fairly high
Sunday	high	low
mean	350 cups/day	350 cups/day
STANDARD DEVIATION	m cups/day	P cups/day

17.3.1 Explain why the standard deviation at Coffee Shop B is smaller than the standard deviation at Coffee Shop A. (1) **L2**

17.3.2 If Coffee Shop A decides to sell coffee at a higher price on the weekends, then how would this affect the mean and standard deviation? (2) **L2**

17.3.3 What possible strategy could coffee shop B introduce so that the mean and standard deviation both increase? Explain your answer. (1) **L2**

LIMPOPO/SEPTEMBER 2023

18 A mathematics teacher wants to make an unbiased prediction of her Grade 12 learners' final marks. She uses their SBA mark and the final mark. The results are as follows:

SBA MARK(%)	FINAL MARK (%)		SBA MARK(%)	FINAL MARK (%)
42	51		48	59
35	43		72	85
69	76		57	63
62	73		25	35
83	85		65	59
75	72		68	75

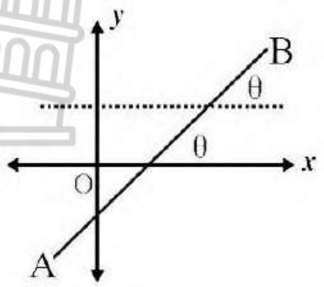
18.1 Draw the scatter plot for the data. (4) **L2**

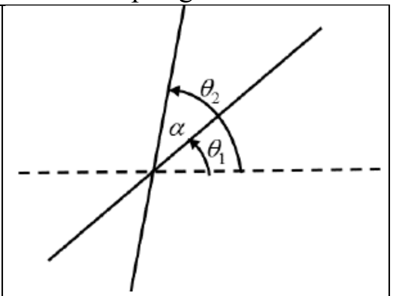
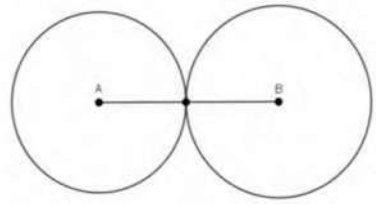
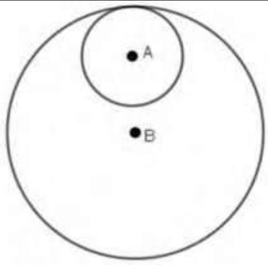
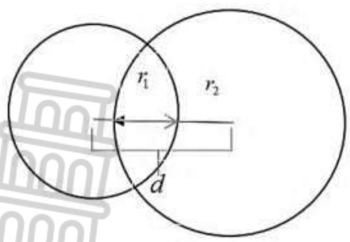
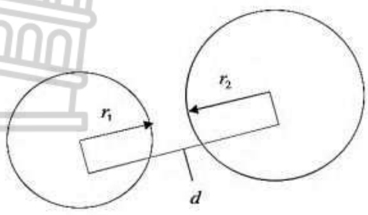
18.2 Calculate the correlation coefficient for the data. (2) **L1**

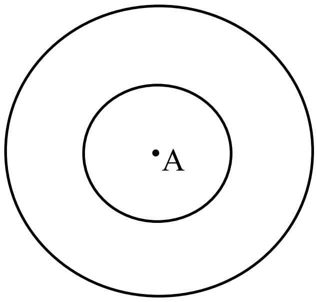
18.3 Is the SBA mark a reliable predictor of the final mark? Provide a reason for your answer. (2) **L2**

18.4 Determine the equation of the least squares regression line. (3) **L1**

18.5 Predict Toby's final if his SBA mark was 66%. (2) **L1**

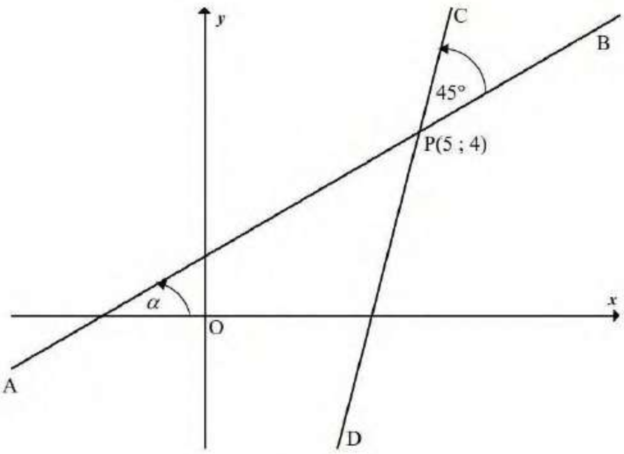
TOPIC	**Analytical Geometry** $[40 \pm 3]$		
GUIDELINES, SUMMARY NOTES, & STRATEGIES			
1. Distance formula: $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$	When given the distance between two points with both points given but one value being a variable. Use this formula through substitution.		
2. Midpoint: $\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$	Remember, midpoint might be given and be requested to calculate any end point.		
3. Gradient: $m=\frac{y_1-y_2}{x_1-x_2}$	3.1	**Collinear** points and **parallel** lines have same (equal) gradient. i.e. $m_1=m_2$	
	3.2	The product of the gradient of **perpendicular** lines is -1. i.e. $m_1 \times m_2=-1$	
4. Inclination: $\tan\theta=m$	4.1	The inclination of a line is the angle formed with the horizontal in an anti-clockwise direction. On the cartesian plane, the inclination of a line is calculated by finding the angle formed at the x-axis measured in anticlockwise direction. θ is the angle of inclination of line AB.	

	4.2	The angle between two lines: $\alpha = \theta_2 - \theta_1$	
	4.3	Sometimes you will be given the angle of inclination then requested to determine the gradient.	
5. Equation of a straight line: $y = mx + c$ or $y - y_1 = m(x - x_1)$	Take note of the following form: $ax + by + c = 0$		
6. Point of intersection	Equate the equations of those two lines or solve using simultaneous equations in a case where a line intersects a circle.		
7. Equation of the circle: $(x-a)^2 + (y-b)^2 = r^2$	$(a;b)$ is the centre and r is the radius, so, when centre is the origin then: $x^2 + y^2 = r^2$		
1. Interpretation of circles			
8.1 Touching at one point	Externally: $d_{AB} = r_A + r_B$ where A and B are centres		
	Internally: $d_{AB} = r_A - r_B$ where A and B are centres		
8.2 Intersecting at two points	$d < r_1 + r_2$		
8.3 Not touching at all	$d > r_1 + r_2$		

	Same centre (concentric circles)	
2. Equation of a tangent	9.1 Tangent is always perpendicular to the radius at the point of contact. Meaning: $m_{rad} \times m_{tan} = -1$ 9.2 Tangents from the same point outside the circle are equal in length.	
3. Calculation of Areas	10.1 Area of a triangle (known height): $A = \frac{1}{2}\text{base}\times\text{height}$ 10.2 Area of a triangle (known angle): use area rule 10.3 Area of regular Quads: use standard formulas from grade 10 10.4 Area of irregular Quads: use the difference of areas of known figures.	

DBE FEB/MARCH 2010

1 The straight line AB has the equation $5y-3x-5=0$. Another straight line CD is drawn to intersect AB at $P(5;4)$ such that the acute angle between AB and CD is $45°$

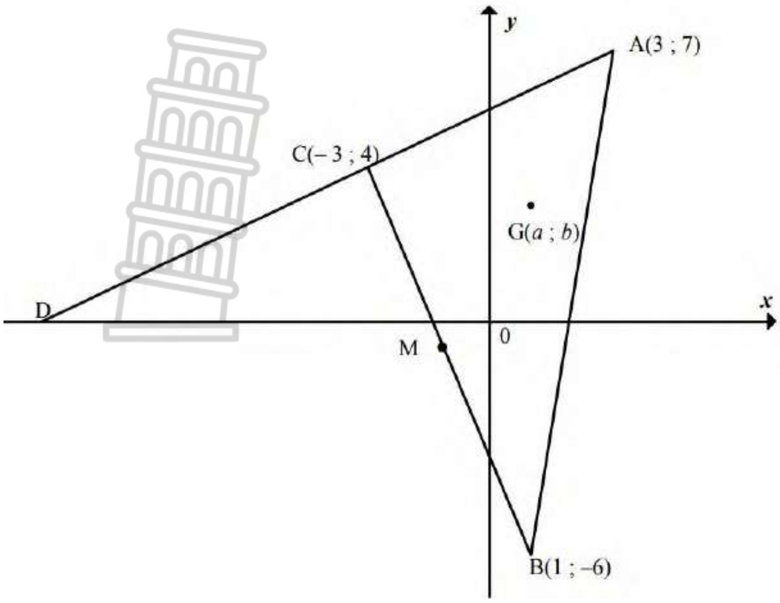


1.1 Determine the gradient of the line CD. (5) **L3**

1.2 Hence, or otherwise, determine the equation of the line CD. (2) **L1**

DBE NOV 2016

2 In the diagram below, A, B and C are vertices of a triangle. AC is extended to cut the x-axis at D



2.1 Calculate the gradient of:

a) AD (2) **L1**

b) BC (1) **L1**

2.2 Calculate the size of $D\hat{C}B$. (3) **L2**

2.3 Write down an equation of the straight-line AD. (2) **L1**

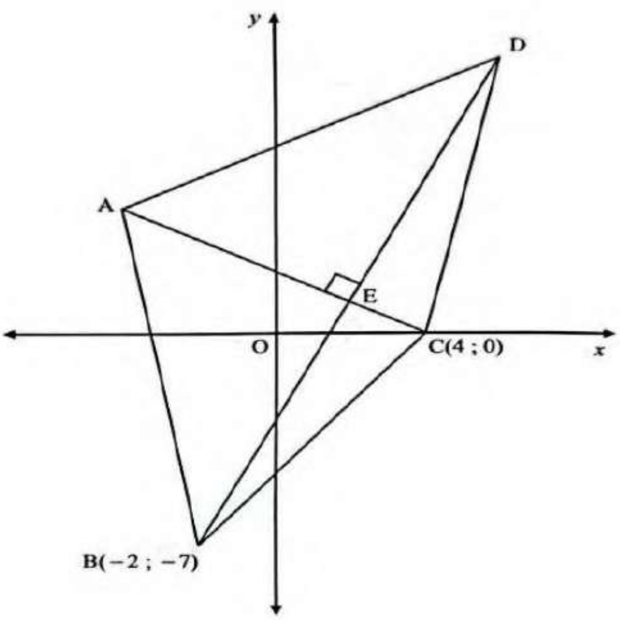
2.4 Determine the coordinates of M, the midpoint of BC. (2) **L1**

2.5 If $G(a;b)$ is a point such that A, G and M lie on the same straight line, show that $b = 2a + 1$ (4) **L4**

2.6 Hence calculate TWO possible values of b if $GC = \sqrt{7}$. (6) **L4**

SEPTEMBER 2023 LIMPOPO

3 In the diagram below A, $B(-2;-7)$, $C(4;0)$ and D are the vertices of a kite. E is the midpoint of the diagonal BD and $AC \perp BD$ at E. The equation of AC is $y = -\frac{1}{2}x + 2$.



Determine

3.1 The equation of BD (4) **L2**

3.2 The coordinates of E (3) **L1**

3.3 If the ratio $CE:EA = 1:3$, determine the coordinates of A. (2) **L1**

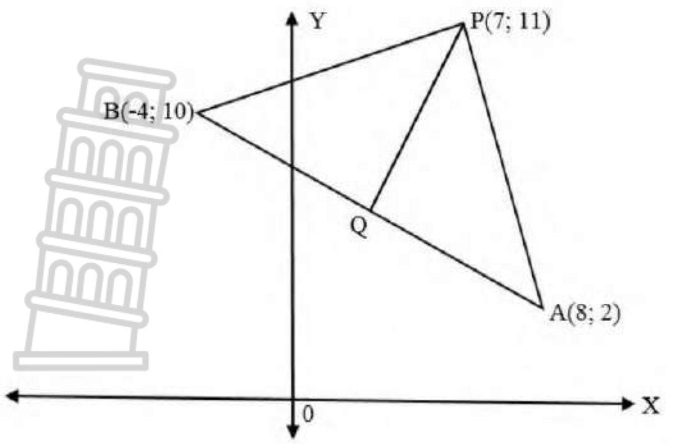
3.4 Kite PQRS is obtained after the measurements of kite ABCD is enlarged by a scale factor of 2. Calculate the area of kite PQRS. (5) **L4**

JUNE 2021 LIMPOPO

4.1 If the points $K(-2;6)$, $L(0;5)$ and $M(8;y)$ lies on the same straight line, determine the numerical value of y. (4) **L2**

4.2 Determine the equation of the line perpendicular to the line in 4.1 and passing through the point $(-2;6)$. (4) **L3**

4.3 In the figure, ΔABP has vertices $A(8;2)$, $B(-4;10)$ and $P(7;11)$



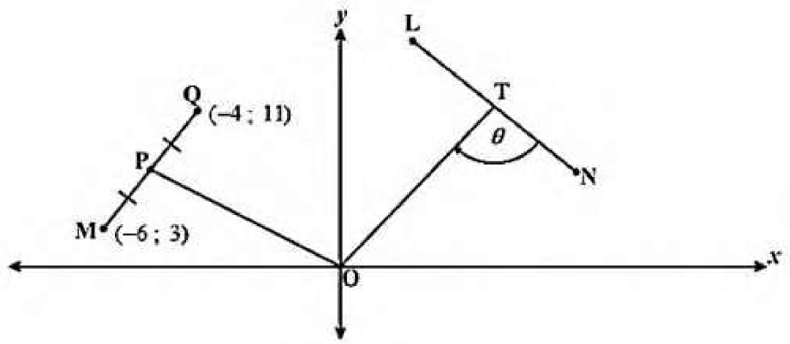
a) Calculate the length of the median PQ. Leave the answer in simplest surd form. (3) **L1**

b) If PQ is produced to point $R(x;-4)$ so that $QR = 2PQ$, determine the $x-$coordinate of point R. (6) **L4**

c) If BARP is a parallelogram, determine the coordinates of R. (2) **L1**

NORTHERN CAPE SEPT 2023

5.1 In the diagram below, P is the midpoint of the line segment joining $M(-6;3)$ and $Q(-4;11)$. The equation of OT is $y = \frac{3}{2}x$. The equation of LN is $x + y = 15$.



a) Write down the coordinates of P, the midpoint of MQ. (2) **L1**

b) Determine the coordinates of T. (3) **L2**

c) Calculate the size of θ. (4) **L3**

5.2 The distance between the origin and point $A(-2;k-1)$ is $2k$ units. Calculate the value of k. (4) **L3**

5.3 Given: $S(2;3)$, $Y(2+4a;3-5a)$ and $U(2+4b;3-5b)$ with $a \neq 0$, $b \neq 0$ and $a \neq b$.

a) Prove that the Point S, Y and U are collinear. (3) **L3**

b) Hence, determine the equation of the straight line SYU in the form $y = mx + c$. (3) **L2**

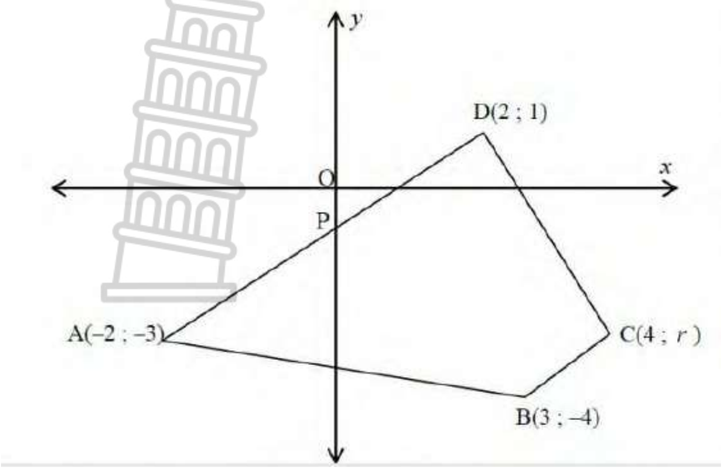
5.4 Points $T(2t-11;t+2)$, $P(-2;3)$, $Q(4;-1)$ and $R(4p;p-7)$ are given. Determine the value of:

a) P if QR is parallel to $x-$axis. (2) **L2**

b) t if points T, P and Q are collinear. (3) **L2**

GP SEPT 2018

6 In the diagram below, points $A(-2;-3)$, $B(3;-4)$, $C(4;r)$ and $D(2;1)$ are the vertices of quadrilateral ABDC. P is the midpoint of line AD.



6.1 Calculate the value of r if $AD \parallel BC$. (4) **L2**

6.2 What type of quadrilateral is ABCD? (1) **L1**

6.3 Determine the coordinates of P. (2) **L2**

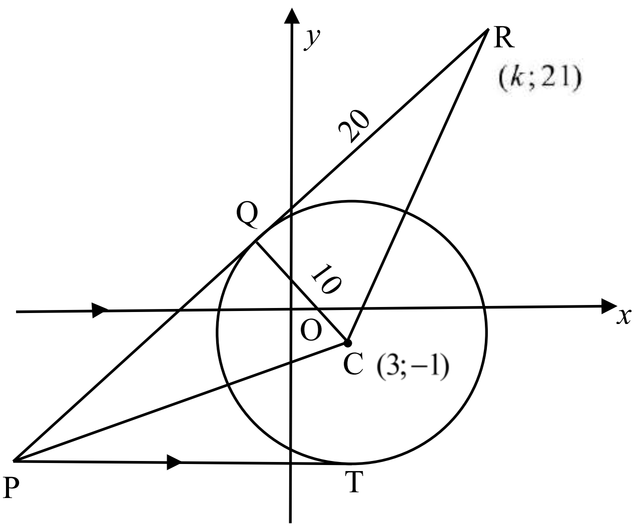
6.4 Prove that $BP \perp AD$. (2) **L2**

6.5 Determine the equation of the circle passing through PBA in the form $(x-a)^2+(y-b)^2=r^2$ (5) **L3**

6.6 Calculate the maximum radius of the circle having equation $x^2+y^2-2x\cos\theta-4y\cos\theta=-2$ (5) **L4**

EC JUNE 2024

7 In the diagram below, a circle with centre $C(3;-1)$ and a radius of 10 units is drawn. PQR and PT are tangents to the circle at Q and T respectively. PT is parallel to the x axis. $C(k;21)$, C and P are vertices of $\triangle RCP$. QR=20 units.



7.1 Write down the size of $C\hat{Q}R$. (1) **L1**

7.2 Calculate the length of RC and leave your answer in the surd form (2) **L1**

7.3 Calculate the value of k, if R lies in the first diagram. (4) **L2**

7.4 Determine the equation of the circle with centre C, passing through Q and T. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$ (2) **L1**

7.5 Determine the equation of PT (2) **L2**

7.6 The equation of the line PR is given by $3y-4x=35$

a) Calculate the coordinates of P (2) **L1**

b) Calculate the length of PQ, give a reason for your answer (2) **L1**

c) Is the area of $\triangle QRC=$ area of $\triangle QPC$? Motivate your answer by means of calculations. (3) **L2**

d) Consider the line $x=q$, for what value(s) of q will the line be not a tangent to the circle with centre C. (2) **L3**

7.7 Consider another circle with equation $(x-3)^2+(y+16)^2=16$ and having centre M.

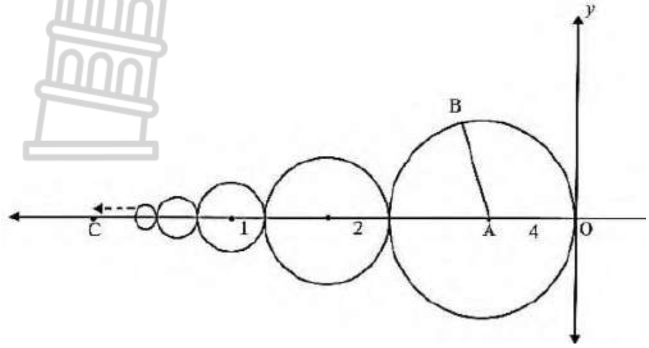
a) Write down the coordinates of the centre M. (1) **L1**

b) Write the down the length of the radius of the circle with centre M (1) **L1**

c) Prove that the circle with centre C and the circle with centre M, do not touch each other (intersect). (3) **L2**

LP SEPT 2023

8 An infinite number of circles, each touching the next, are drawn between C and O. The centres of all the centres lie on the negative x-axis. The radius of the largest circle, centred at A, is 4 units and the radius of each circle thereafter is halved. B is a point on the largest circle.



8.1 Show that OC = 16 units (2) **L2**

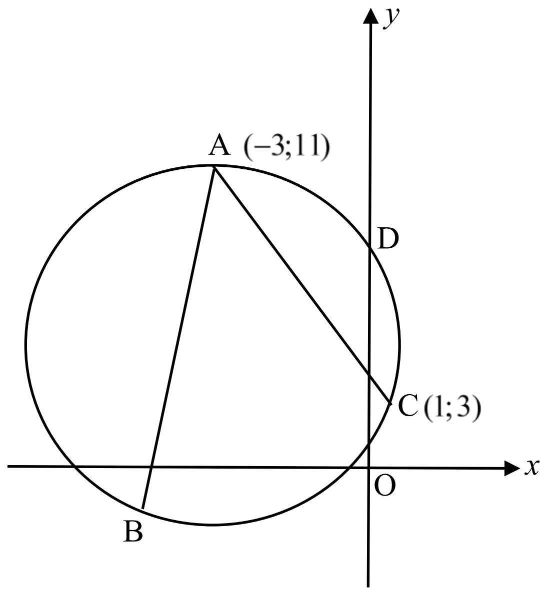
8.2 If BC is a tangent to circle A at B, write down the size of $A\hat{B}C$, providing a reason for your answer (2) **L2**

8.3 Hence, determine $\tan\hat{C}$ (4) **L2**

8.4 Determine the equation of BC. (3) **L3**

MP SEPT 2023

9 In the diagram $A(-3;11)$ and $C(1;3)$ are points on the circumference of a circle with diameter AB and centre T. The equation of AB is given $y = 3x + 20$



9.1 Determine the equation of the perpendicular bisector of AC (4) **L2**

9.2 Show that the coordinates of the centre of the circle $(-5;5)$ (3) **L3**

9.3 Calculate the length of the diameter AB (3) **L1**

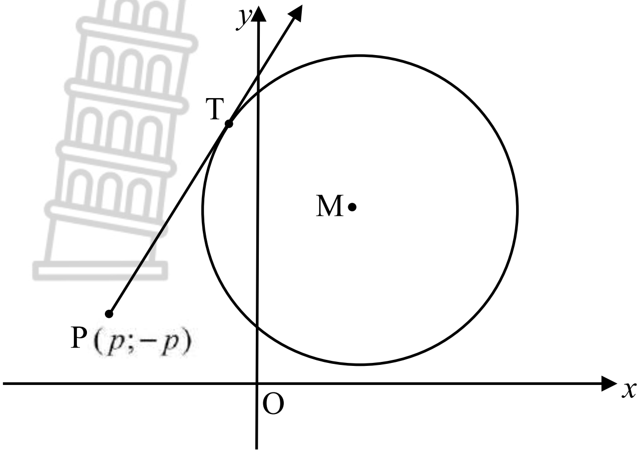
9.4 The tangent to the circle at A cuts the y-axis at $(0;p)$. Calculate the numerical value of p (4) **L2**

9.5 If the circle through A, B and C is moved 3 units to the right and 2 units upwards, and the radius is halved, write down the equation of the new circle (3) **L3**

9.6 A new circle with equation $(x-2)^2 + (y-3)^2 = 4$ and centre P is given. Will this circle intersect the original circle or nor? Motivate your answer with the necessary calculations. (4) **L2**

LIMPOPO SEPTEMBER 2016

10 M is the centre of the circle defined by $x^2+y^2-2x-4y+1=0$. P$(p;-p)$ is any point on the tangent to the circle at T.



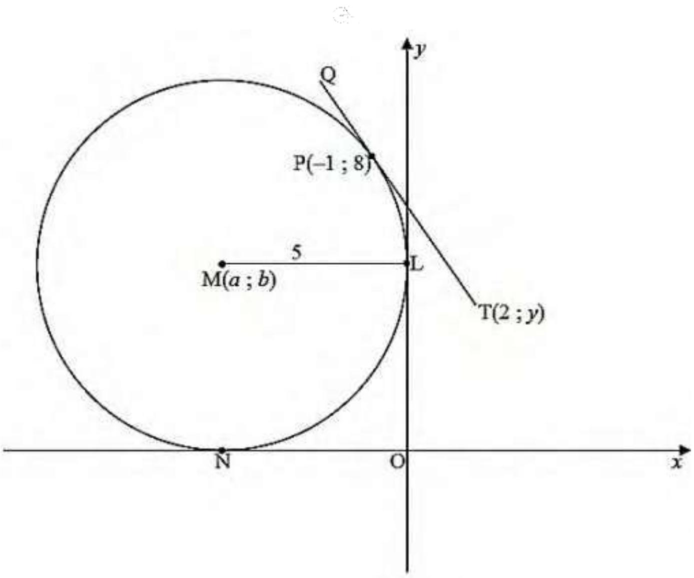
10.1 Show, by calculations that the coordinates of M are $(1;2)$. (3) **L2**

10.2 Prove that the length of PT$=\sqrt{2p^2+2p+1}$ (3) **L3**

10.3 Calculate the coordinates of P where P is a close as possible to T and hence calculate the minimum length of PT (5) **L4**

FREE STATE SEPTEMBER 2023

11 In the digram, a circle centred at M$(a;b)$ with a radius of 5 units touches the x-axis and the y-axis at points N and L, respectively. QPT is a tangent to this circle at P$(-1;8)$. The coordinates of T are $(2;y)$.



11.1 Give a reason why ML$\perp$ y-axis. (1) **L1**

11.2 Determine the:

a) Coordinates of M (2) **L1**

b) Equation of the circle having centre M. (2) **L2**

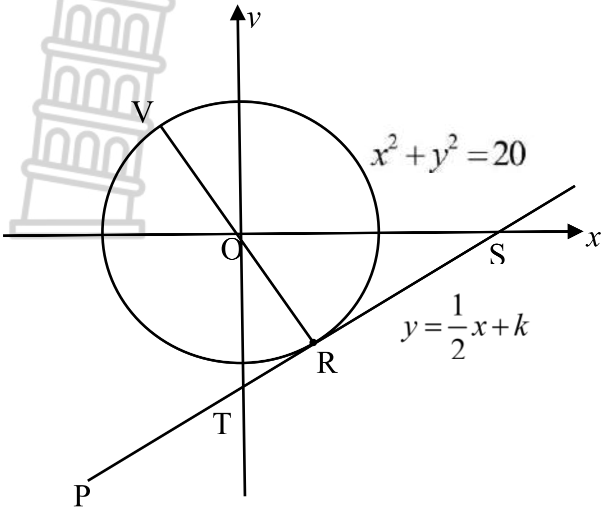
c) Equation of the tangent QPT in the from $y=mx+c$ (5) **L3**

11.3 Another circle having point T as the centre, touches the circle, having M as the centre externally. Determine the equation of the circle centred at T in the form $(x-h)^2+(y-k)^2=r^2$ (6) **L4**

11.4 The circle with centre M is translated across the Cartesian plane in such a way that both horizontal and vertical axes remain tangents to the circle simultaneously. Write down all the possible coordinates of the centres of the newly translated circles, given that $\sqrt{xy}$ must be real for ALL values of x and y (4) **L4**

NORTH WEST MARCH 2016

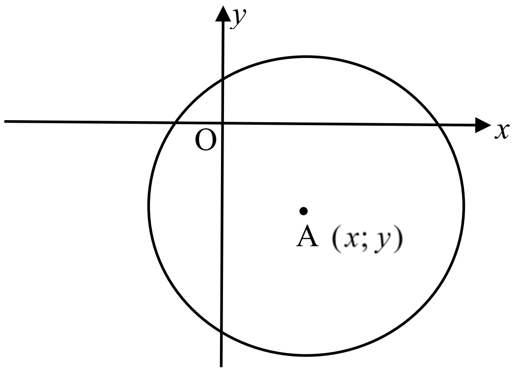
12 In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



12.1	Determine, giving reasons, the equation of OR in the form $y = mx + c$	(3)	**L2**
12.2	Determine the coordinates of R.	(3)	**L3**
12.3	Determine the $\frac{\text{area of } \triangle OTR}{\text{area of } \triangle OSR}$, given that $R(2;-4)$.	(6)	**L4**
12.4	For which value(s) of k will the line $y = \frac{1}{2}x + k$ intersect the circle at two points?	(5)	**L4**

LIMPOPO PRE JUNE 2024

13 The equation of a circle is $x^2 + y^2 - 2x + 4y - 4 = 0$.

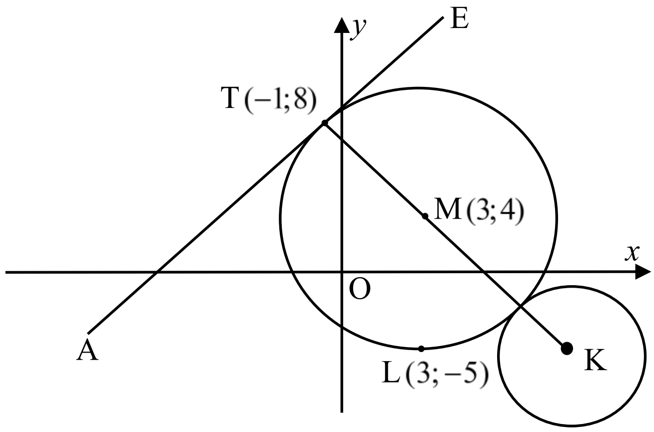


13.1	Determine the coordinates of A, the centre of the circle and the length of the radius, r	(5)	**L3**
13.2	Calculate the value of p in $N(1; p)$ with $p > 0$ is a point on the circle	(1)	**L2**
13.3	Determine the equation of the tangent to the circle at N	(2)	**L2**

13.4 A second circle, centre B, with equation $(x-4)^2 + y^2 = k^2$ cuts the circle centred A twice. Determine the values of k for which point A will be inside the circle B (6) **L4**

FREE STATE JUNE 2024

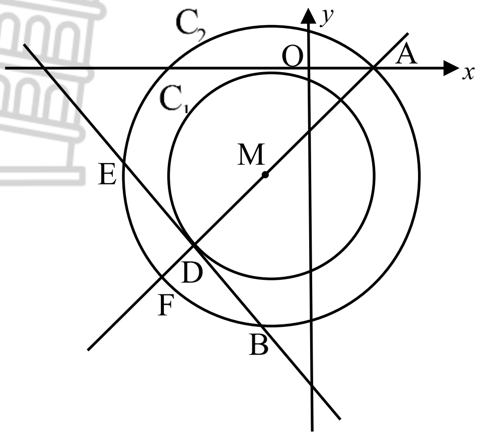
14 Drawn below is the BIGGER circle centre at M and SMALLER circle centred at K. ATE is a tangent to the bigger circle at T. TN is a diameter of the bigger circle and NK is a radius of the smaller circle. The coordinates of $T(-1;8)$, $M(3;4)$ and $L(3;-5)$.



14.1	Determine the equation of the circle in the form of	(2)	**L1**
14.2	Determine the equation of the tangent through point T	(5)	**L3**
14.3	Does point P(7;3) lie inside, outside or on the circle. Show all calculations	(4)	**L3**
14.4	If it is further given that KL is a tangent at L, to the circle centred at M. Determine the coordinates of K, the centre of the smaller circle.	(5)	**L3**

FREE STATE SEPTEMBER 2020

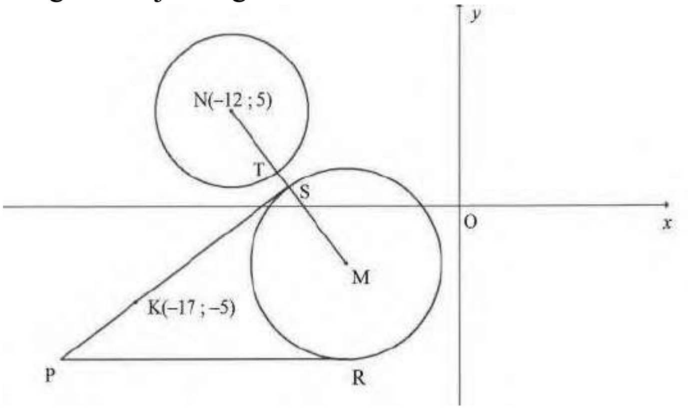
15 Circles C_1 and C_2 in the figure below have same centre M. P and A are points on C_2. PM intersects C_1 at D. The tangent BD to C_1 intersects C_2 at B and E. The equation of circle C_1 is given by $x^2+2x+y^2+6y+2=0$ and the equation of line PM is $y=x-2$.



15.1	Calculate the coordinates of centre M.	(3)	**L3**
15.2	Determine the radius of the circle C_1	(1)	**L1**
15.3	Determine the coordinates D the point where line PM and circle C_1 intersect.	(5)	**L3**
15.4	Give reason why $M\hat{D}B=90°$.	(1)	**L1**
15.5	If is given that $DB=4\sqrt{2}$, determine MB, the radius of circle C_2	(4)	**L2**
15.6	Write down the equation of circle C_2 in the form $(x-a)^2+(y-b)^2=r^2$.	(1)	**L1**

DBE MAY/JUNE 2023

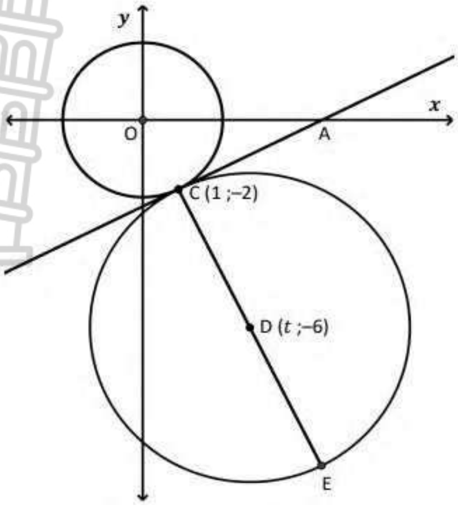
16 In the diagram, the equation of the circle centred at $N(-12;5)$ is $x^2+y^2+24x-10y+153=0$. The equation, the equation of the circle centred at M is $(x+6)^2+(y+3)^2=25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x-axis. $K(-17;-5)$ is a point on PS. The straight-line joining N and M cuts the smaller circle at T and the larger circle at S.



16.1	Write down the coordinates of M	(2)	**L2**
16.2	Calculate the:		
	a) Length of the radius of the smaller circle	(2)	**L2**
	b) Length of TS	(4)	**L3**
16.3	Determine the equation of the tangent:		
	a) PR	(2)	**L2**
	b) PS, in the form $y=mx+c$	(5)	**L2**
16.4	Quadrilateral PSMR is drawn. Calculate the:		
	a) Perimeter of PSMR	(5)	**L3**
	b) Ratio of $\dfrac{\text{area of }\triangle NPS}{\text{area of quadrilateral PSMR}}$	(2)	**L2**

WESTERN CAPE SEPTEMBER 2016

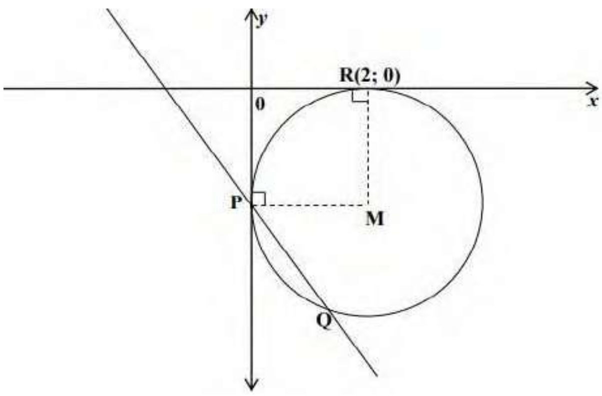
17 The diagram below consist of two circles, which touch each other externally at $C(1;-2)$. The smaller circle has its centre O at the origin. The other circle has centre $D(t;-6)$. CA is a common tangent which intersects the x-axis at A. CDE is a diameter of the larger circle.



17.1	Give a reason why the points O, C and D lie on a straight line.	(2)	**L2**
17.2	Calculate the gradient of OC	(2)	**L1**
17.3	Hence, show that the value of $t=3$	(2)	**L2**
17.4	Determine the equation of the tangent AC in the from $y=mx+c$	(3)	**L2**
17.5	Calculate the coordinates of E	(2)	**L2**
17.6	Determine the equation of a circle passing through the points $A(5;0)$, C and E in the form $(x-a)^2+(y-b)^2=r^2$	(6)	**L3**
17.7	If a circle with centre D and equation $(x-t)^2+(y+6)^2=r^2$ has to cut the circle with centre O twice, give all possible values of r.	(4)	**L4**

KZN MARCH 2016

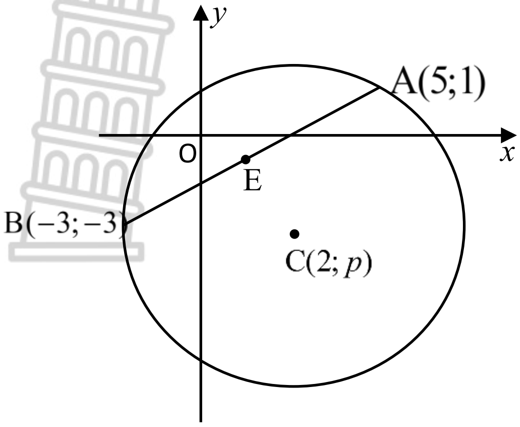
18 In the sketch below, the circle with sentre M touches, the y- axis at P and the x-axis at $R(2;0)$. The straight line defined by the equation $y=-x-2$ cut the circle at point Q and passes through point P.



18.1	Write down the coordinates of P	(1)	**L1**
18.2	Write down the coordinates of M, the centre of the circle.	(1)	**L1**
18.3	Show that the equation of the circle with centre M is : $x^2+y^2-4x+4y+4=0$	(3)	**L2**
18.4	The straight line with equation $y=-x+c$ is a tangent to the circle with centre M. Calculate the numerical values of c	(5)	**L4**

NSC MAY/JUNE 2024

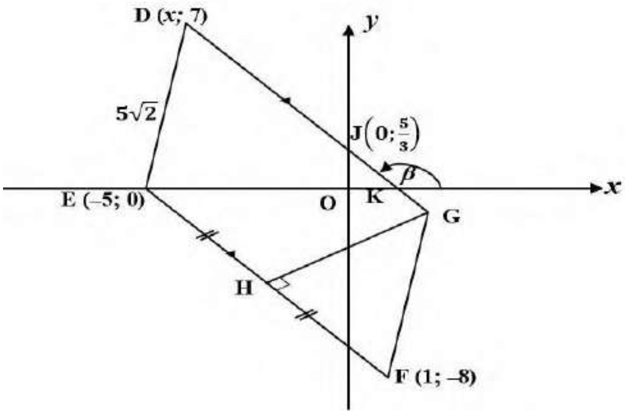
19 In the diagram, the circle centred at $C(2;p)$ is drawn. $A(5;1)$ and $B(-3;-3)$ are points on the circle. E is the midpoint of AB.



19.1	Calculate the coordinates of E, the midpoint of AB	(2)	**L1**
19.2	Calculate the length of AB	(1)	**L1**
19.3	Determine the equation of EC	(4)	**L3**
19.4	Show that $p=3$	(1)	**L1**
19.5	Show by calculations that the equation of the circle is $x^2+y^2-4x+6y-12=0$	(4)	**L3**
19.6	Calculate the values of t for which the straight line $y=tx+8$ will not intersect the circle.	(6)	**L4**

KZN JUNE 2024

20 In the diagram, DEFG is a parallelogram with vertices $D(x;7)$, $E(-5;0)$, $F(1;-8)$ and G. $GH \perp EF$, with H on EF, such that $EH = HF$. The angle of inclination of DG is β. DE has a positive gradient. DG cuts the y-axis at $J\left(0\,;\,\frac{5}{3}\right)$ and the x-axis at K. The length of $DE = 5\sqrt{2}$.



20.1	Calculate the gradient of EF.	(2)	**L1**
20.2	Calculate the coordinates of H.	(2)	**L1**
20.3	Determine the equation of GH in the form $y=mx+c$.	(3)	**L2**
20.4	Calculate the size of β.	(3)	**L2**
20.5	Calculate the size of $O\hat{J}K$.	(2)	**L2**
20.6	Calculate the value of x.	(5)	**L2**
20.7	Calculate the area of DEOJ.	(6)	**L3**

TOPIC	TRIGONOMETRY	[±50 MARKS]

GUIDELINES, SUMMARY NOTES, & STRATEGIES

1. Definitions of trig ratios:

In a right angled triangle: $\sin\theta = \frac{opposite}{hypotenuse}$; $\cos\theta = \frac{adjacent}{hypotenuse}$ and $\tan\theta = \frac{opposite}{adjacent}$

SOH CAH TOA helps you to remember these definitions.

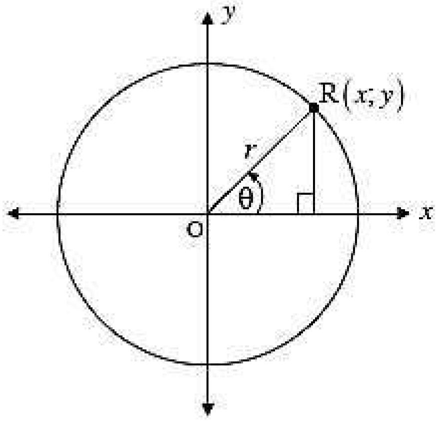
In a Cartesian plane:

$\sin\theta = \frac{y}{r}$;

$\cos\theta = \frac{x}{r}$;

$\tan\theta = \frac{y}{x}$

and $r^2 = x^2 + y^2$



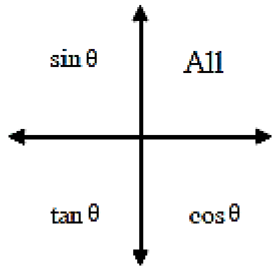
2. CAST Rule:

All trig ratios are positive in the 1st quadrant. **All**

Only $\sin\theta$ is positive in the 2nd quadrant. **Students**

Only $\tan\theta$ is positive in the 3rd quadrant. **Take**

Only $\cos\theta$ is positive in the 4th quadrant. **Care**



3. Reduction Formulae:

If θ is an acute angle, i.e. in the 1st quadrant,

$180° - \theta$ will lie in the 2nd quadrant,

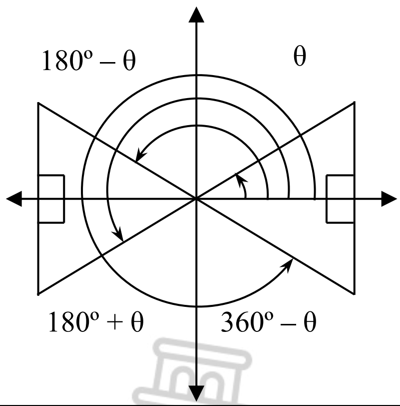
$180° + \theta$ will lie in the 3rd quadrant,

and $360° - \theta$ will lie in the4th quadrant.

$\sin\theta = \sin(180° - \theta) = -\sin(180° + \theta) = -\sin(360° - \theta)$

$\cos\theta = -\cos(180° - \theta) = -\cos(180° + \theta) = \cos(360° - \theta)$

$\tan\theta = -\tan(180° - \theta) = \tan(180° + \theta) = -\tan(360° - \theta)$



For $90° - \theta$ and $90° + \theta$ the ratio changes to its co-function.
The co-function of cos is sin and the co-function of sin is cos.

$\sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta$ $\sin(90° + \theta) = \cos\theta$; and $\cos(90° + \theta) = -\sin\theta$

Trigonometric identities:

Square identity: $\sin^2\theta + \cos^2\theta = 1$

Quotient identity: $\frac{\sin\theta}{\cos\theta} = \tan\theta$

Compound Angles:

$\sin(\theta \pm \beta) = \sin\theta\cos\beta \pm \cos\theta\sin\beta$

$\cos(\theta \pm \beta) = \cos\theta\cos\beta \mp \sin\theta\sin\beta$

Double Angles:

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

QUESTION 1 **STUDY AND MASTER GRADE 12**

If $\cos^2 12° - \sin^2 12° = m$, express, without the use of a calculator, the following in terms of m.

1.1	$\cos 24°$	(2)	**L1**
1.2	$\frac{\sqrt{3}}{2}\cos 6° + \frac{1}{2}\sin 6°$	(4)	**L2**
1.3	$\sin 426°$	(3)	**L2**
1.4	$\cos 48°$	(3)	**L2**
1.5	$\sin 132°$	(3)	**L2**
1.6	$\cos 33°$	(3)	**L3**
1.7	$2\sin^2 12°$	(4)	**L3**
1.8	$\cos 54°$	(3)	**L2**
1.9	$\sin 42°$	(3)	**L2**

QUESTION 2 **SEPTEMBER 2018 NORTH WEST**

If $\sin 32° = t$, determine in term of t, the value of the following:

2.1	$\sin 212°$	(3)	**L2**
2.2	$\cos 122°$	(3)	**L2**
2.3	$\cos 64°$	(3)	**L2**
2.4	$\sin 16°$	(4)	**L3**
2.5	$\tan 392°$	(3)	**L2**

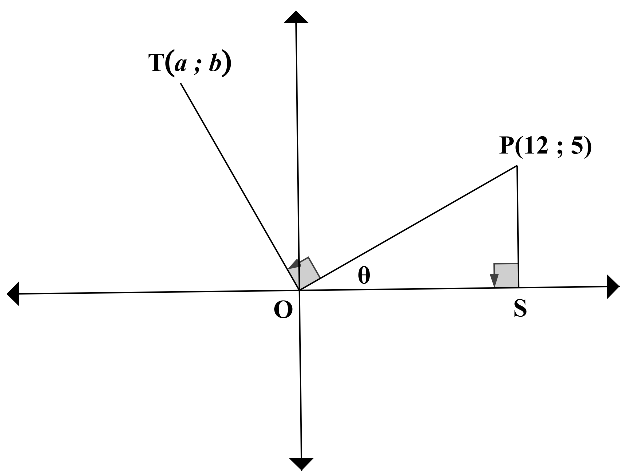
QUESTION 3

3.1 If $\cos\beta = k$, express the following in terms of k, without using a calculator:

$$\sin\left(\frac{\beta}{2} + 45°\right)\cos\left(\frac{\beta}{2} + 45°\right)$$ (5) **L3**

QUESTION 4 **GAUTENG SEPTEMBER 2023**

4.1 In the diagram below, P is the point $(12\ ;\ 5)$ and $T(a\ ;\ b)$. $OT \perp OP$; $PS \perp x$-axis and $P\hat{O}S = \theta$.



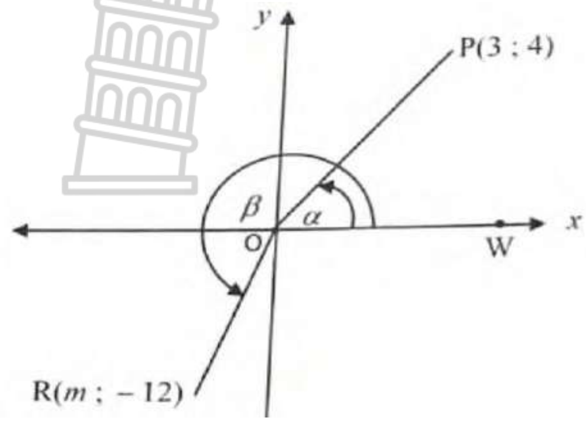
Without using a calculate, determine, the value of:

4.1.1	$\tan\theta$	(1)	**L1**
4.1.2	$\sin\theta$	(2)	**L2**
4.1.3	$\sin(-90° + \theta)$	(2)	**L2**
4.1.4	$\sin 2\theta$	(3)	**L2**
4.1.5	a, if $TO = 19{,}5$ units.	(4)	**L3**

LIMPOPO SEPTEMBER 2023

4.2 In the diagram below, $P(3\ ;4)$ and $R(m\ ;\ -12)$ are two points as indicated.

$P\hat{O}W = \alpha$ and $R\hat{O}W = \beta$.



Answer the following questions without using a calculator.

4.2.1	Write down the value of $\tan\alpha$	(1)	**L1**
4.2.2	Determine the value of $\sin(90° + \alpha)$	(3)	**L2**
4.2.3	Determine the value of m if it is given that $12 + 13\sin\beta = 0$.	(4)	**L2**
4.2.4	Determine the value of $\sin(\alpha + \beta)$	(3)	**L3**

QUESTION 5

5.1 If $4\tan\theta = 3$ and $180° < \theta < 360°$, determine with the aid of a diagram:

5.1.1	$\cos(180° + \theta)$	(4)	**L3**
5.1.2	$\tan 2\theta$	(3)	**L2**
5.1.3	$\sin(45° + \theta)$	(4)	**L2**

5.2 Given that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°\ ;\ 270°]$ and $\beta \in [90°\ ;\ 270°]$

Determine, without using a calculator, the value of the following:

5.2.1	$\cos\alpha$	(4)	**L3**
5.2.2	$\cos(450° - \beta)$	(2)	**L2**
5.2.3	$\cos(\alpha + \beta)$	(3)	**L2**

5.3 If $6\cos 2\theta + 5 = 0$, where $2\theta \in [180°\ ;\ 360°]$. Calculate without using a calculator the values in simplest form of:

5.3.1	$\sin 2\theta$	(3)	**L2**
5.3.2	$\cos\theta$	(4)	**L3**

QUESTION 6

Simplify without using a calculator:

6.1 $$\frac{\sin 260°.\cos 170°}{\sin 10°.\sin 190°.\cos 350°}$$ (6) **L2**

6.2 $$\frac{\cos 750°.\tan 315°.\cos(-\theta)}{\cos(360° - \theta).\sin 300°.\sin(180° - \theta)}$$ (7) **L2**

6.3 $$\frac{\sin 104°(2\cos^2 15° - 1)}{\tan 38°.\sin^2 412°}$$ (6) **L3**

6.4 $$\frac{\cos(180° + \theta).\tan(720° - \theta).\sin^2(90° - \theta)}{\sin(-\theta + 180°)} + \sin^2\theta$$ (6) **L2**

6.5 $\cos(90°-2x).\tan(180°+x)+\sin^2(x+360°)$ (5) **L2**

QUESTION 7

7.1 Evaluate the following without the use of a calculator: $\sin 75°-\sin 15°$ (3) **L2**

7.2 Prove without the use of a calculator: $\cos 80°+\cos 40°=\cos 20°$ (3) **L3**

7.3 Simplify the following without use of a calculator: $\sin 35°+\sin 25°-\sin 85°$ (3) **L3**

7.4 Simplify without the use of a calculator: $\sin 169°\sin 41°+\sin 79°\sin 131°$ (5) **L2**

7.5 Simplify fully: $\cos(385°+\beta).\sin(35°-\beta)+\sin(25+\beta).\sin(55°+\beta)$ (5) **L3**

7.6 Determine, without using a calculator, value of $\left(\cos 15°+\sqrt{3}\sin 15°\right)$ (5) **L4**

7.7 Given: $k=\sqrt{3}\cos x+\sin x$, Write k in the form of $t\ \sin(x+\theta)$ (4) **L3**

QUESTION 8

Determine the general solution of the following:

8.1 $\cos(2x-20°)=\sin(x+10°)$ (6) **L2**

8.2 $\cos x=3\sin(x-45°)$ (6) **L2**

8.3 $\sin x+\cos x=\sqrt{\dfrac{3}{2}}$ (6) **L3**

8.4 $\sin 2x+5\cos 2x=0$ (5) **L3**

8.5 $\cos 2x=1-3\cos x$ (6) **L3**

8.6 $\sin^2 x+\sin 2x-\sin x-2\cos x=0$ (6) **L3**

8.7 $6\sin^2 x+2\sin 2x=1$ (6) **L3**

8.8 $\tan x=2\sin 2x$ where $\cos x<0$ (6) **L3**

8.9 $3^{2\tan x}-3^{\tan x+1}=54$ (6) **L3**

QUESTION 9

9.1 Prove the following identities:

9.1.1 $\dfrac{\sin 3x}{\sin x}+\dfrac{\cos 3x}{\cos x}=4\cos 2x$ (5) **L3**

9.1.2 $2\cos 5A\cos 3A-\cos 8A+2\sin^2 A=1$ (5) **L4**

9.1.3 $\cos(45°+x)\cos(45°-x)=\dfrac{1}{2}\cos 2x$ (5) **L3**

9.1.4 $\dfrac{\cos 2x}{1+\sin 2x}=\dfrac{\cos x-\sin x}{\sin x+\cos x}$ (5) **L3**

9.1.5 $\dfrac{\cos^2 x-\cos x-\sin^2 x}{2\sin x\cos x+\sin x}=\dfrac{1}{\tan x}-\dfrac{1}{\sin x}$ (5) **L3**

9.2 For which values of x will $\dfrac{2\tan x-\sin 2x}{2\sin^2 x}$ be undefined in the interval $0°\le x\le 180°$? (3) **L2**

9.3 Calculate the value(s) of x if $\tan 2x+2\sin x$ is undefined for $x\in[0°\ ;\ 360°]$ (5) **L3**

TOPIC TRIGONOMETRIC GRAPHS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

- The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.
- Candidates must be able to use and interpret functional notation. Learners must understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$ and $a.f(x)$ where $a \in$ R.

QUESTION 10

Given : $f(x)=\sin(x-30°)$ and $g(x)=\cos 3x$ for x $\in[-90°\ ;\ 90°]$.

10.1 Write down the period of g. (2) **L1**

10.2 Draw sketch graphs of f and g for $x\in[-90°\ ;\ 90°]$. Clearly show all intercepts with the axesand the co-ordinates of all the turning points and end points of both curves. (4) **L2**

10.3 Use the graphs to determine the value(s) of x for $x\in[-90°\ ;\ 90°]$, where:

10.3.1 $f(x)>g(x)$ (2) **L2**

10.3.2 $f(x).g(x)>0$ (2) **L3**

10.3.3 $g(x)<0$ (2) **L2**

10.4 Determine the range of $h(x)=3f(x)-1$. (2) **L2**

10.5 The graph of f is shifted 60° to the right to obtain a new graph h.
Write down the equation of h. (3) **L2**

QUESTION 11 **NOVEMBER 2017 GRADE 11**

11.1 Consider the functions $f(x)=\sin(x-30°)$ and $g(x)=\cos 2x$.
On the same set of axes, draw the graphs of f and g for $x\in[-90°;180°]$.
Clearly show all intercepts with the axes, turning points and end points. (4) **L2**

11.2 Consider: $f(x)=-2\tan\frac{3}{2}x$.
Draw the graph of f for the interval $x\in[-120°:180°]$.
Clearly show all asymptotes, intercepts with the axes and endpoints of the graph. (3) **L2**

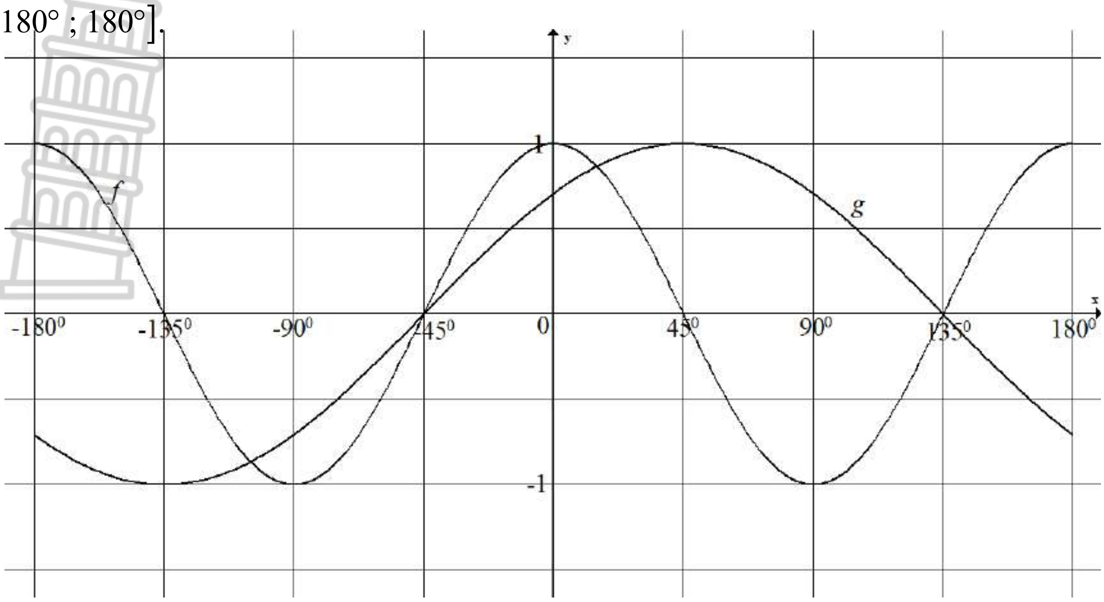
11.3 On the same system of axes, draw sketch graphs of: $f(x)=\sin(x+30°)$
$g(x)=\cos 2x$ if $-180°\leq$ x $\leq 180°$ (3) **L2**

11.3.1 What is the period of g ? (2) **L1**

11.3.2 Determine by means of calculation, the values of x if $f(x)=g(x)$ in the interval above. (5) **L3**

QUESTION 12 **NOVEMBER 2014 GRADE 11**

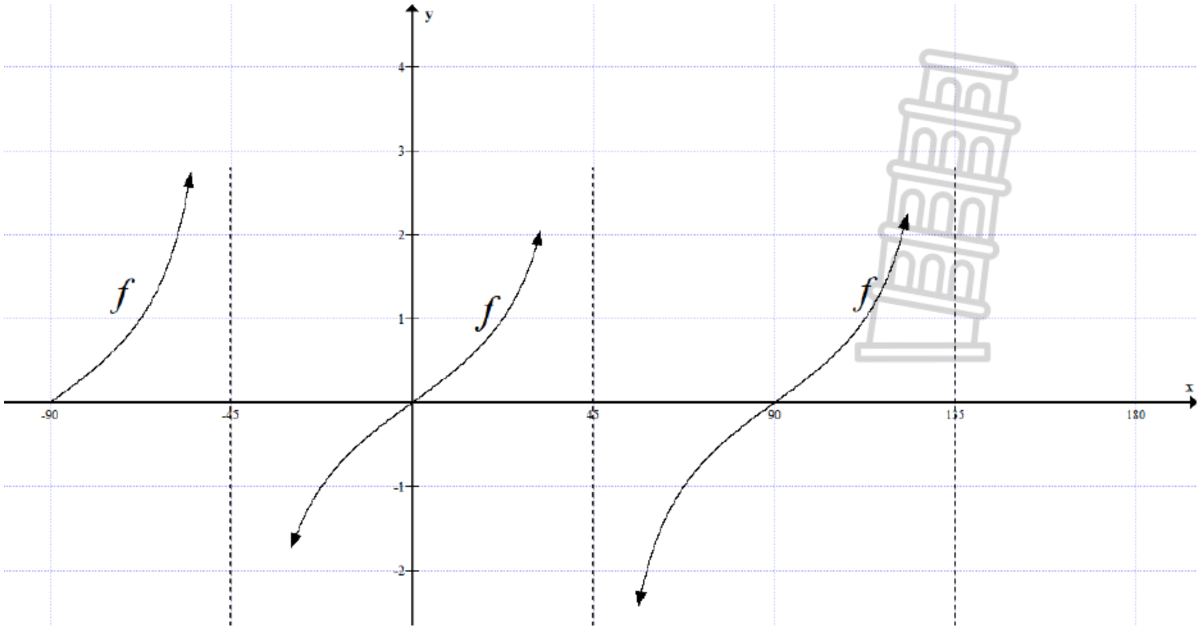
In the diagram below the graphs of $f(x)=a\cos bx$ and $g(x)=\sin(x+p)$ are drawn for $x\in[-180°\ ;\ 180°]$.



12.1	Write down the values of *a*, *b* and *p*.	(3)	**L2**
12.2	For which values of *x* in the given interval does the graph of *f* increase as the graph of *g* increases?	(2)	**L1**
12.3	Write down the period of $f(2x)$.	(2)	**L1**
12.4	Determine the maximum value of *h* if $h(x)=3f(x)-1$	(2)	**L2**
12.5	Describe how the graph *g* must be transformed to form the graph *k*, where $k(x)=-\cos x$	(2)	**L3**
12.6	Determine the value(s) of x for which $f'(x).g(x)<0$	(6)	**L3**

QUESTION 13

In the diagram, the graph of $f(x)=\tan bx$ is drawn for the interval $-90°\le x\le 135°$.

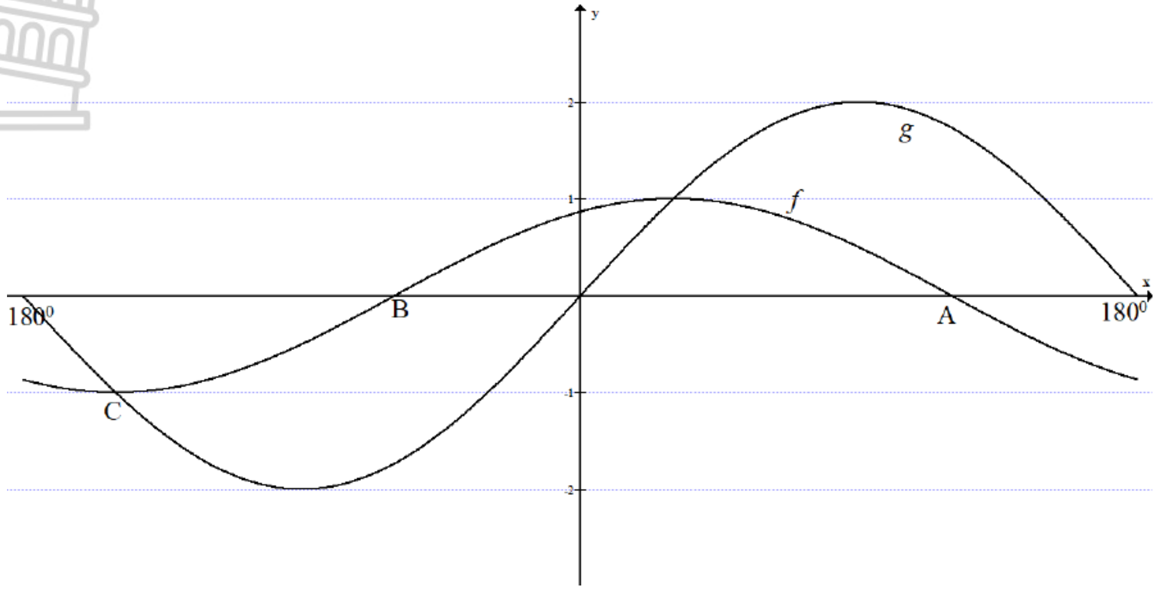


13.1	Determine the value of *b*.	(1)	**L2**
13.2	Determine the values of *x* in the interval $-90°\le x\le 135°$ for which $f(x)\le -1$.	(4)	**L3**

13.3 Graph h is defined as $h(x) = \tan b(x + 55°)$. Write down the equations of the asymptotes of h in the interval $-90° \leq x \leq 135°$. (2) **L2**

13.4 Determine the general solution of $\cos(x - 30°) = \sin x$ (6) **L3**

13.5 In the diagram, the graphs of $f(x) = \cos(x - 30°)$ and $g(x) = 2\sin x$ are drawn for the interval $x \in [-180° ; 180°]$, A and B are the x-intercepts of f. The two graphs intersect at C and D, the minimum and maximum turing points respectively of f.



13.5.1 Write down the coordinates of:

a) A (1) **L1**

b) C (2) **L1**

13.5.2 Determine the values of x in the interval $x \in [-180° ; 180°]$, for which:

a) Both graphs are increasing (2) **L2**

b) $f(x + 10°) > g(x + 10°)$ (2) **L2**

c) $g'(x) = 0$ (2) **L2**

d) $g'(x) < 0$ (2) **L2**

13.5.3 Determine the range of $y = 2^{2\sin x + 3}$ (4) **L2**

TOPIC	TRIGONOMETRY: PROBLEMS IN TWO AND THREE DIMENSIONS
GUIDELINES, SUMMARY NOTES, & STRATEGIES	

THE SINE RULE

In any ΔABC it is true that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Important: Use the Sine Rule when given **two angles and a side** in a triangle, also when **two sides and a non-included angle** are given.

It is advisable that when calculating **sides** have the **sides as numerators**: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ and when calculating **angles, have the angles as numerators:** $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.

THE COSINE RULE

In any Δ ABC it is true that: $a^2=b^2+c^2-2bc.\cos A$, $b^2=a^2+c^2-2ac.\cos B$ and $c^2=a^2+b^2-2ab.\cos C$

Important: Use the Cosine Rule when given **two sides and an included angle,** also when you are given **all the three sides.**

THE AREA RULE

In any Δ ABC it is true that:

Area of Δ ABC $=\frac{1}{2}bc.\sin A=\frac{1}{2}ac.\sin B=\frac{1}{2}ab.\sin C$

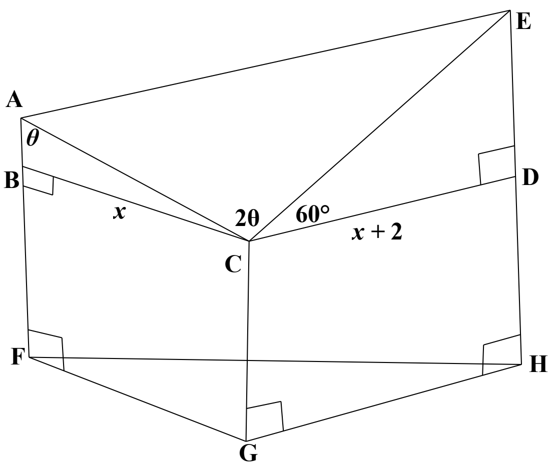
Important: To use the Area Rule, you need **two sides and an included angle** of the triangle**.**

STRATEGIES

Note: When solving 3D problems separate all the triangle so that they will be 2D and easy to solve. It is also advisable that write all your findings back to the diagrams to help you with the next sub-question.

QUESTION 14 **MAY-JUNE 2019**

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. ΔACE form the roof of an entertainment centre. BC $= x$, CD $= x+2$, B$\hat{A}$C $=\theta$, and E$\hat{C}$D $=60°$



14.1 Calculate the length of:

14.1.1 AC in terms of x and θ. (2) **L1**

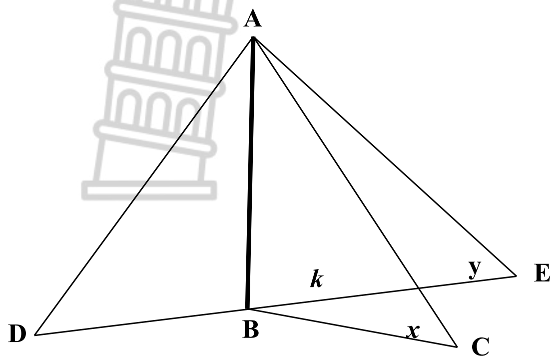
14.1.2 CE in terms of x. (2) **L1**

14.2 Show that the area of the roof ΔACE is given by $2x(x+2)\cos\theta$. (4) **L3**

14.3 If $\theta=55°$ and BC $=12$ metres, calculate the length of AE. (3) **L2**

QUESTION 15 DBE NOVEMBER 2017

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole ant points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



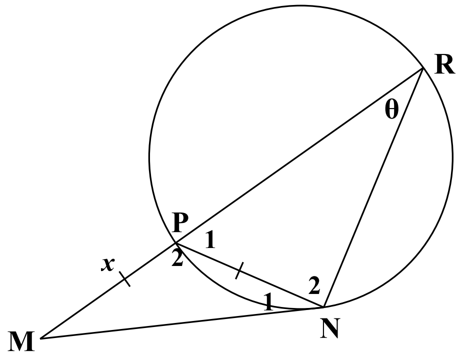
15.1 Write down the size of $\hat{ABC}$. (1) **L1**

15.2 Show that $AC = \dfrac{k.\tan y}{\sin x}$ (4) **L3**

15.3 If it is further given that $D\hat{A}C = 2x$ and $AD = AC$, show that the distance DC between the players at D and C is $2k \tan y$. (5) **L4**

QUESTION 16 KZN JUNE 2018

In the diagram below, RP is a diameter of the circle. RPM is straight line and $PM = PN = x$, $P\hat{R}N = \theta$.

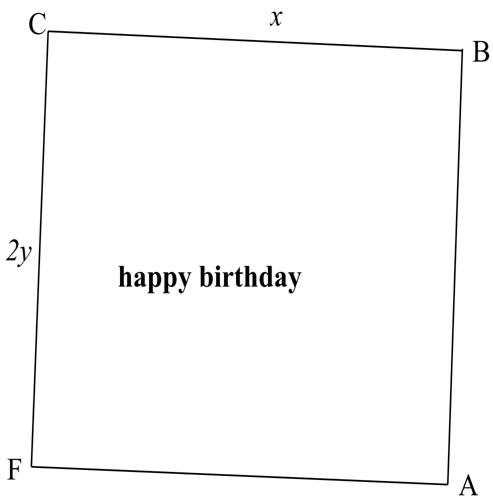


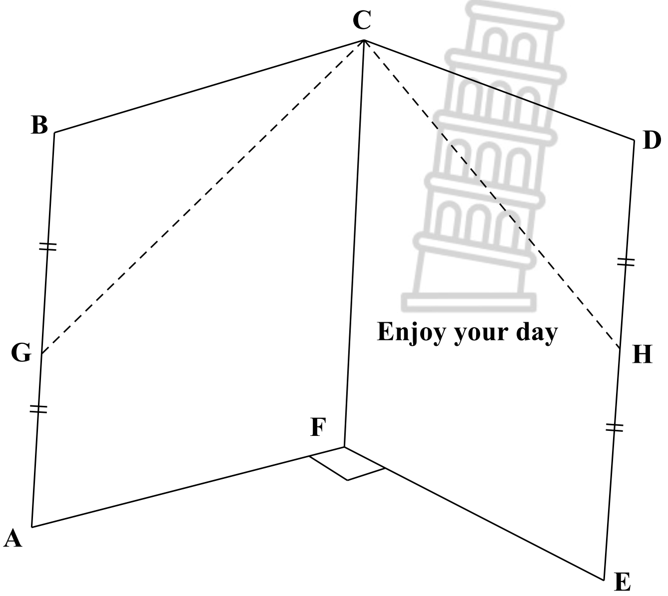
16.1 Show that: $MN^2 = 2x^2(1+\sin\theta)$ (5) **L4**

16.2 If $MN = \sqrt{12}$ units and $x = 2$ units, show, without using a calculator, that $\theta = 30°$ and $PR = 4$ units. (4) **L3**

QUESTION 17 FEB-MARCH 2012

A rectangular card is tied with ribbon at the midpoints, G and H, of the longer sides. The card is opened to read the message inside and then placed on a table in such a way that the angle $A\hat{F}E$ between the front cover and the back cover of the card is $90°$. The points G and H are joined by straight lines to the point C inside the card, as shown in the sketch. Let the shorter side of the card, $BC = x$ and the longer side, $CF=2y$.

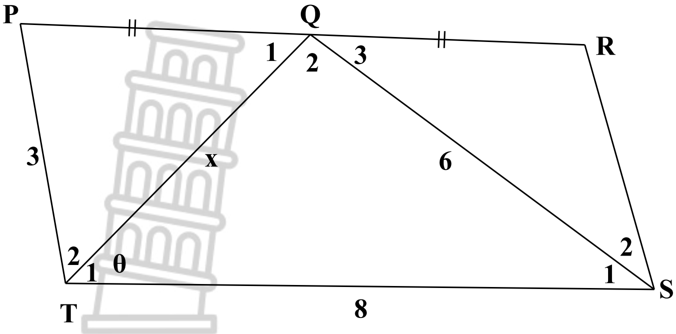




Prove that $\cos G\hat{C}H = \dfrac{y^2}{x^2 + y^2}$ (5) **L4**

NOV 2019 GRADE 11

QUESTION 18



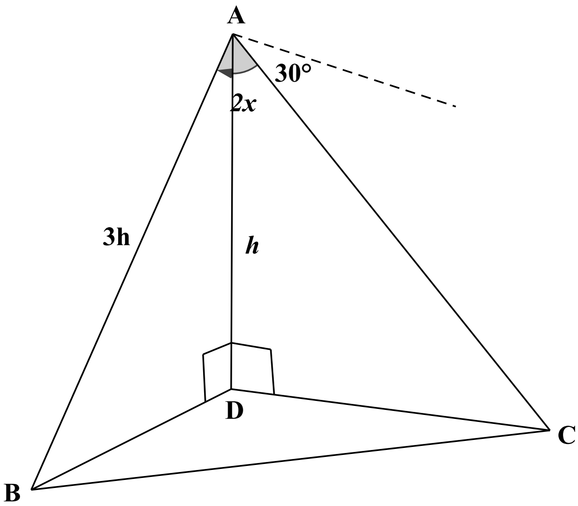
In the diagram alongside, PISR is a parallelogram. Q is the midpoint of PR. QS = 6 units, PT = 3 units, TS = 8 units, TQ = x units and $\hat{T}_1 = \theta$.

18.1 Show that $\cos\theta = \dfrac{x^2+28}{16x}$ (3) **L2**

18.2 Hence, determine the length of QT. (6) **L3**

QUESTION 19 **SEPTEMBER NORTH WEST 2022**

In the diagram below, AD is a vertical pole having height h metres. B, D and C are three points in the same horizontal plane. AB and AC are cables and the angle of depression from A to C is 30°. AB = 3h and . $B\hat{A}C = 2x$.



19.1 Write down the size of $A\hat{C}D$ (1) **L1**

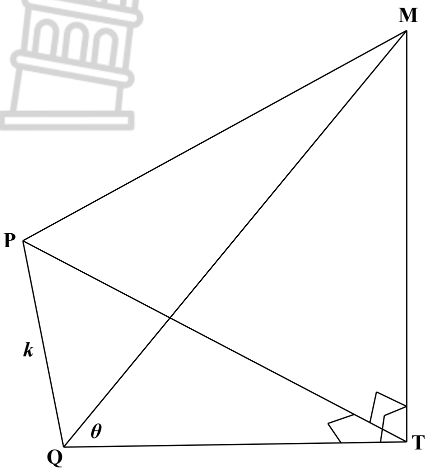
19.2 Determine the distance AC in terms of h (2) **L1**

19.3 Calculate the size of $A\hat{B}D$ (2) **L1**

19.4 Calculate the size of x if $BC = \sqrt{7}h$ (5) **L3**

QUESTION 20 **GAUTENG SEPTEMBER 2023**

In the diagram below, P, Q and Tare three points in the same horizontal plane and MT is a vertical mast. MP and MQ are two straight stay wires. The angle of elevation of M from Q is θ. PQ = k metres. PM = 2PQ. The area $\Delta MPQ = 2k^2 \sin\theta\cos\theta$



20.1 Show that $M\hat{P}Q = 2\theta$ (3) **L3**

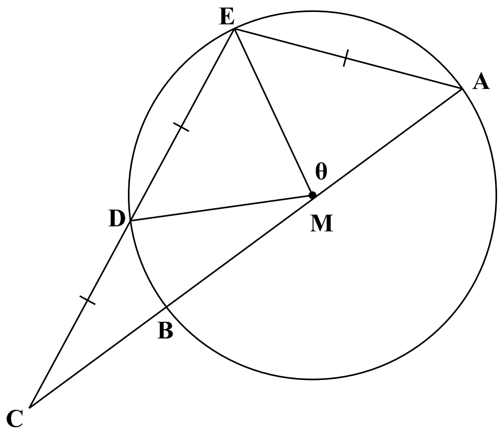
20.2 Hence, show that $MQ = k\sqrt{1+8\sin^2\theta}$ (4) **L3**

20.3 If $k = 139{,}5$ m and $\theta = 42°$, determine the length of MT. (3) **L2**

QUESTION 21

In the figure alongside, AB is diameter of the circle with centre M and radius $= r$. BC $= r$, CD = DE = AE and $A\hat{M}E = \theta$.

Prove that $\cos\theta = \dfrac{1}{4}$



(6) L4

TOPIC: EUCLIDEAN GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

DIFFERENT WAYS EUCLIDEAN GEOMETRY CAN BE TESTED

1. COMPLETING A STATEMENT OF A THEOREM IN WORDS.

- Know by heart all the theorems and be able to complete the statement.

2. DETERMINING THE VALUE OF AN ANGLE

- Know all the theorems about **lines**, **triangles** and **circles (Centre** group, **non-centre** group, **tangent** group and **cyclic quad** group**)**.
- Every statement must come with a reason and reasons must be stated according to the list of acceptable reasons from the exam guidelines

3. PROOFS IN RIDERS

Know how theorems and their converses are being formed in diagrams.

- When given 3 points on the circumference look out for a possibility of a triangle. If one side is produced then you may expect exterior angle of a triangle. If there is a tangent on the circle then there is a possibility of having a Tan Chord Theorem
- When given 4 or 5 points on the circumference then there is a possibility that 4 points may be joined and then there is a cyclic quad. In a case that one side is produced then you may expect exterior angles of a cyclic quad.
- Start with a given angle linking with what is required to prove
- Visualization: Mind picture of diagrams of theorems

DIRECT AND INDIRECT PROOFS IN RIDERS.

- In Geometry we mostly use angles to prove in questions.

1. Direct proof question: Prove $A = B$

2. Indirect proof question: Prove that a line // to another line.

Remember in Euclidean geometry- we mostly use angles to prove. This question is not asking about the angles directly. Here we need to prove sides but using angles i**ndirectly. Why indirectly?** Because we mostly use angles to prove.

∴ First, we need to change this question to be direct, and then prove. If we say it must be direct we mean that it must ask to prove angles 1^{st}, then conclude by stating the sides that are parallel

4. SIMILARITY AND PROPORTIONALITY THEOREMS

PROPORTIONALITY THEOREM

- Identify parallel lines, and use ratios for proportion.
- **Useful strategies in solving problems involving ratio in areas of triangles:**

CASE 1: If triangles share a **common angle** use area rule. Area = $\frac{1}{2}$a.bsinC

CASE 2: If triangles share a common vertex or height use Area = $\frac{1}{2}$bh

CASE 3: If none of the cases above apply then identify a common triangle and relate the two triangles in question to it, then use any of the two methods mentioned above. **OR**

Required Area = Area of big Δ − other known Area

SIMILARITY THEOREM

CASE 1: Prove that triangles are similar e.g. $\Delta ABC\ |||\ \Delta DEF$

- Angles and / or sides in proportion can be used to prove that two triangles are similar.

- Always name the triangles you are referring to when proving similar triangles

CASE 2: Prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. First prove: ΔABC|||ΔPQR and then deduce the proportion of the sides.

CASE 3: Prove that: KN. PX = NR. YP. Find two triangles in which KN, PX, NR and Y, (or sides equal to these), and thus prove that: ΔKNR|||ΔYPX, then deduce what you were asked to prove. Identify triangles. This method is used when proved similarity don't give asked ratios.

CASE 4: Prove: Proportion with square, with division, with + in between, there is a possibility that two similarities were used or Pythagoras theorem was used.

e.g. $\frac{CF^2}{EF^2} = \frac{BD}{DE}$

5. **EXAMINABLE PROOFS**

Five grade 11 proofs to be known for exam purposes:

5.1 Line from the centre ⊥ chord

5.2. **NEW**: line from centre to midpoint of chord

5.3. Angle at the centre is 2× angle at the circumference.

5.4. Opposite angles of a cyclic quad are supplementary.

5.5. Tan chord theorem.

Two grade 12 proofs:

5.6. Line drawn parallel to one side of a triangle, divides the other two sides proportionally: **Proportionality theorem**

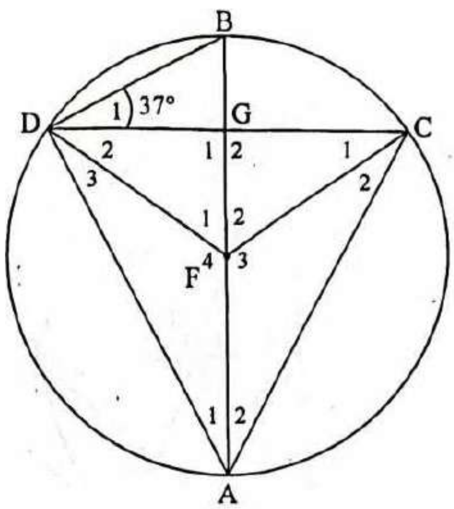
5.7 If two triangles are equiangular, then their corresponding sides are in proportion: **Similarity theorem**

NB!!!!!

- Do not make any assumption e.g. do not assume that a line is a tangent or a diameter, unless you are told that it is.
- Look for key words in the statement such as centre, || lines, tangents, cyclic quads, bisects, etc.
- Continuously update the diagram as you read the statement and as you find the angles.
- When proving theorems, no construction no marks.
- You will not always be told that you have a cyclic quadrilateral. Therefore, check lines joining four points on the circumference.
- For every statement there **must** be a reason.

EUCLIDEAN GEOMETRY

1. **DBE NSC MAY/JUNE 2024**



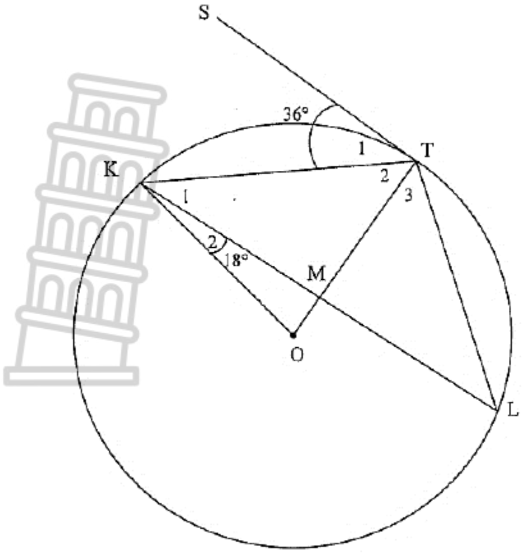
In the diagram, AB is a diameter of the circle, with centre F. AB and CD intersect at G. FD and FC are drawn. BA bisects $C\hat{A}D$ and $\hat{D}_1 = 37°$

1.1 Determine, giving reasons, any three angles equal to $\hat{D}_1$. (4) **L2**

1.2 Show that DG = GC (4) **L2**

1.3 If it is further given that the radius of the circle is 20 units, calculate the length of BG. (4) **L4**

2. **DBE NSC JUNE 2023**



In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersects KL at M. ST is a tangent to the circle at T.

$S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.

2.1 Determine, giving reasons, the size of:

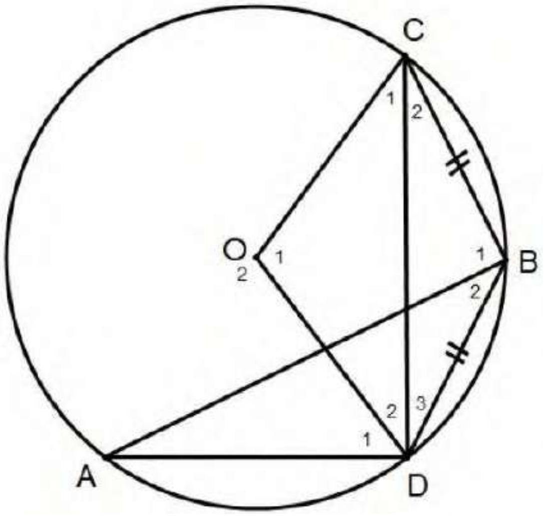
2.1.1 $\hat{T}_2$ (2) **L2**

2.1.2 $\hat{L}$ (2) **L1**

2.1.3 $K\hat{O}T$ (2) **L2**

2.2 Prove, giving reasons, that KM = ML. (3) **L3**

3. **IEB NOVEMBER 2022**



In the diagram, A, D, B and C lie on the circle with centre O. DB = BC.

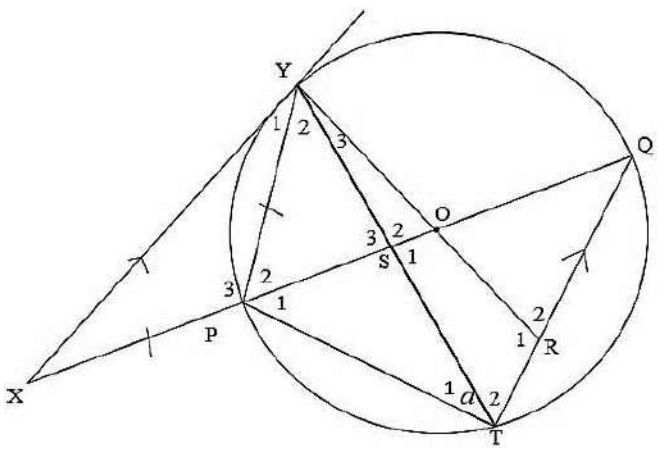
3.1 If $\hat{A} = x$, state, with reasons, two other angles equal to x. (3) **L1**

3.2 Given $\hat{O}_1 = 94°$, determine $\hat{D}_2$. (2) **L2**

3.3 Determine $\hat{B}_1 + \hat{B}_2$ (3) **L2**

3.4 Hence or otherwise determine x. (2) **L2**

4. **FS SEPTEMBER 2018**



XY is a tangent to the circle with centre O. XPQ, YOR, YST are straight lines. PX = PY, XY || TQ and $\hat{T}_1 = a$.

4.1 Write down, with reasons, FOUR other angles each equal to a. (6) **L1**

4.2 Prove that $\hat{T}_2 = 2\hat{T}_1$. (2) **L2**

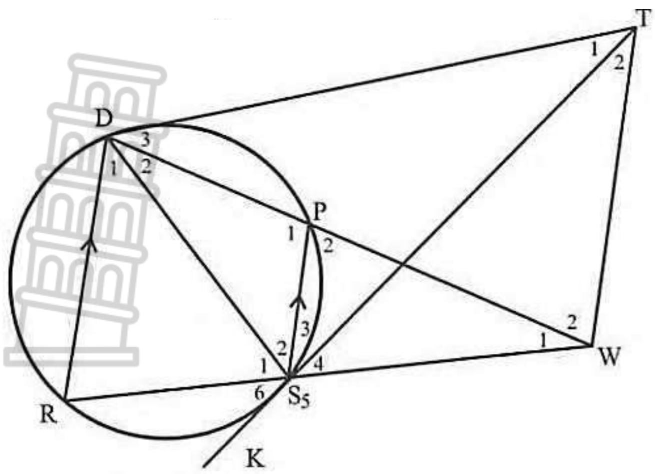
4.3 Prove that $\hat{T}_2 = 90° - a$ (2) **L2**

4.4 Prove that SORT is a cyclic quadrilateral. (2) **L3**

4.5 Determine the value of a. (2) **L2**

4.6 Show that TR = RQ (2) **L2**

5. **KEVIN SMITH GRADE 11 TEXTBOOK**



In the diagram alongside, TD is a tangent to the circle at D. RS and DP are produced to meet at W and KST is a straight line. If $\hat{S}_4 = \hat{S}_2$ and $DR \parallel PS$.

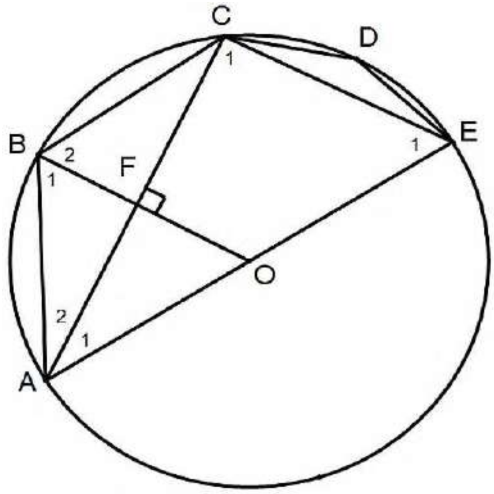
Prove that:

5.1 SWTD is a cyclic quadrilateral (4) **L2**

5.2 TK is a tangent to the circle at S (4) **L3**

5.3 $TW \parallel PS$ (3) **L2**

6. **IEB MAY/JUNE 2023**



In the diagram alongside, A, B, C, D and E lie on the circle with centre O. AC is perpendicular to OB and they intersect at F. AOE is a straight line. $\hat{A}_1 = 38°$.

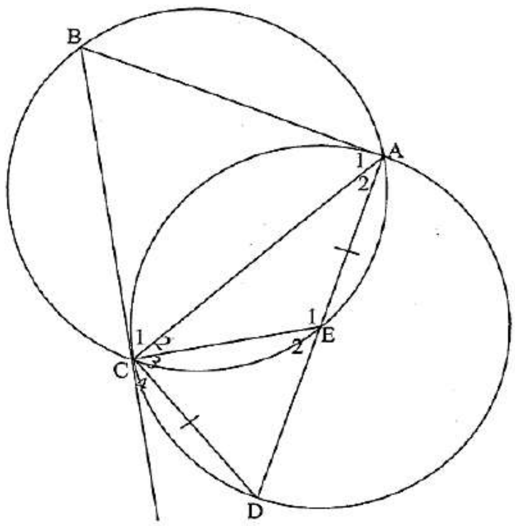
6.1 Determine $\hat{C}_1$. (2) **L1**

6.2 Determine $\hat{D}$. (2) **L1**

6.3 Determine $A\hat{B}C$. (3) **L2**

6.4 If $AC = 8cm$ and $BC = 5cm$, determine the length of BF. (3) **L2**

7. **NW SEPTEMBER 2018**



Two equal circles cut each other in A and C. BA and BC are tangents to one circle at A and C respectively and they are chords of the other circle. E is a point on the circumference of one circle and AE produced cuts the other circle in D. Chords AE and CD are equal.

Prove that:

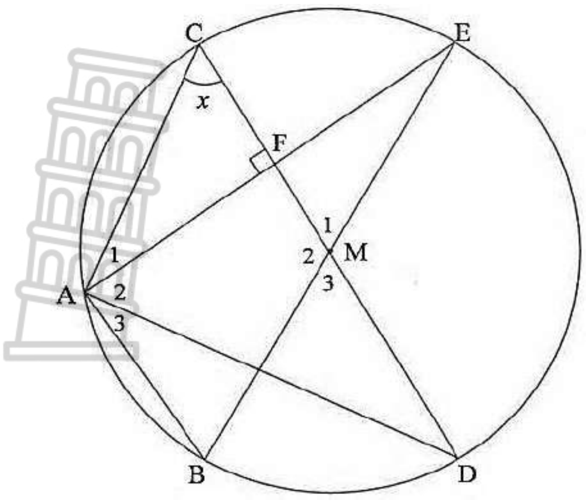
7.1 $\hat{C}_2 = \hat{C}_4$ (4) **L3**

7.2 $\hat{C}_3 = \hat{A}_1$ (3) **L2**

7.3 E is the centre of the circle that passes through A, C and D. (4) **L3**

7.4 ΔECD is equilateral (2) **L2**

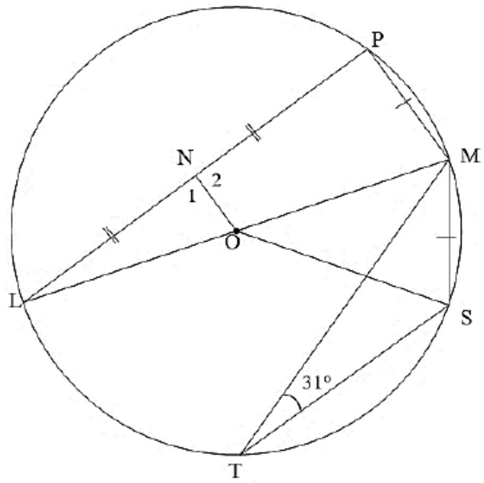
8. **DBE NSC NOVEMBER 2021**



In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. AE ⊥ CD. Let $\hat{C} = x$.

8.1	Give a reason why AF = FE.	(1)	**L1**
8.2	Determine, giving reasons, the size of $\hat{M}_1$ in terms of x.	(3)	**L2**
8.3	Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F.	(4)	**L3**
8.4	Given that CF = 6 units and AB = 24 units, calculate, giving reasons, the length of AE.	(5)	**L4**

9. **DBE NSC JUNE 2019**



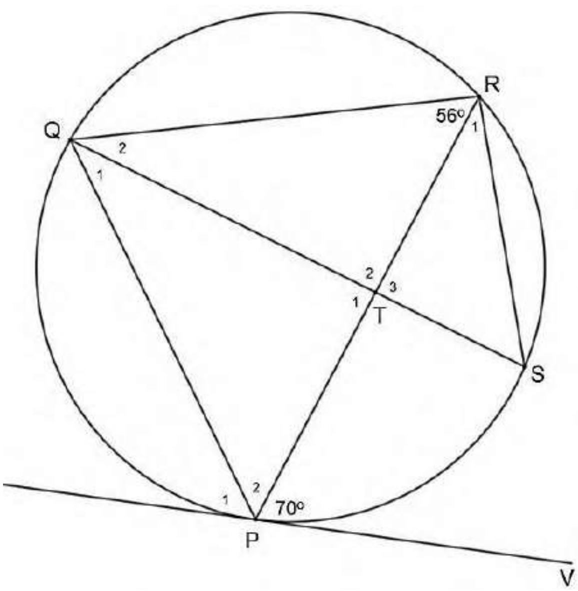
In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. PM = MS and $M\hat{T}S = 31°$

9.1 Determine, with reasons, the size of each of the following angles:

9.1.1	$M\hat{O}S$	(2)	**L1**
9.1.2	$\hat{L}$	(2)	**L2**

9.2	Prove that $ON = \frac{1}{2}MS$.	(4)	**L3**

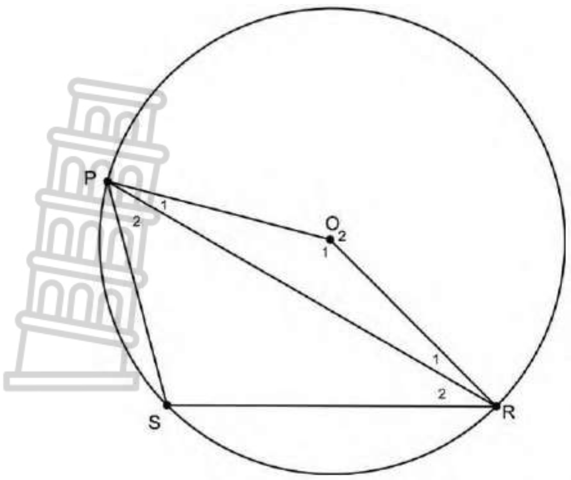
10. **IEB NOVEMBER 2018**



In the diagram alongside, P, Q, R and S are points on the circle. QS and PR intersect at point T. The line from V is a tangent at P. $Q\hat{R}P = 56°$ and $R\hat{P}V = 70°$.

10.1	Calculate the size of $R\hat{S}T$.	(3)	**L2**
10.2	If $\hat{Q}_1 = 37°$, then explain why QS is not the diameter of the circle.	(3)	**L3**

11. **IEB MARCH 2017**

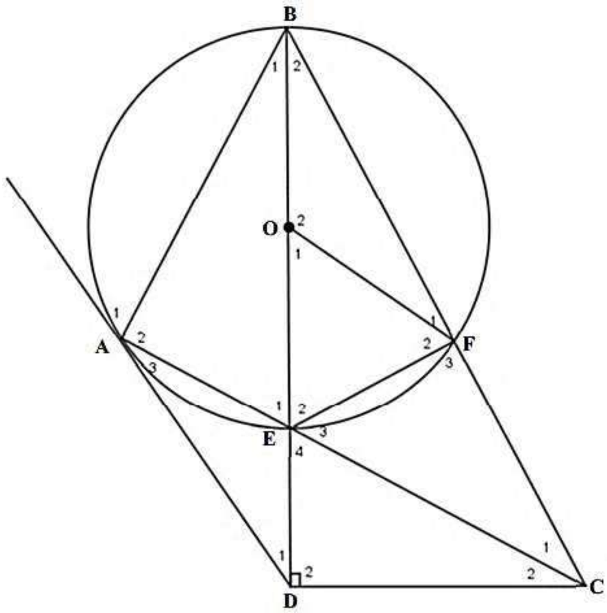


In the diagram alongside, P, R and S lie on the circle with the centre O.

11.1 If $P\hat{S}R = 102°$, determine the size of $\hat{O}_1$, giving reasons (3) **L2**

11.2 Calculate the radius, *r*, of the circle if PR = 10 *units*. (3) **L3**

12. **GP JUNE 2024**



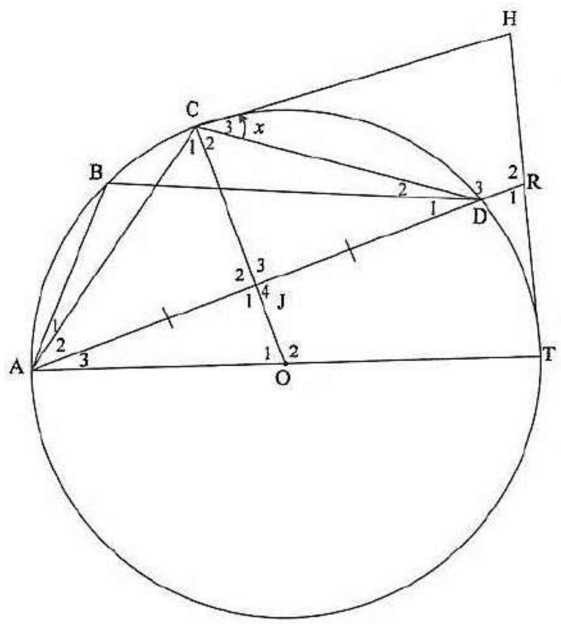
In the diagram below, the diameter BE of circle O is produced to D. DA is a tangent to the circle and CD ⊥ BD. AC and BC cut the circle at E and F respectively. OF and EF are drawn.

12.1 Prove, with reasons, that ABCD is a cyclic quadrilateral. (3) **L3**

12.2 Prove, with reasons, that BD bisects $A\hat{B}C$. (3) **L3**

12.3 Prove, with reasons, that EC is a tangent to circle OEF. (4) **L4**

13. **DBE GRADE 11 NOVEMBER 2019**



In the diagram, O is the centre of the circle through the points A, B, C, D and T. HC and HT are tangents to the circle at C and T respectively. AD is produced to meet HT at R.

OC bisects AD at J. Let $\hat{C}_3 = x$.

13.1 Write down, with a reason, another angle equal to $\hat{C}_3$. (2) **L1**

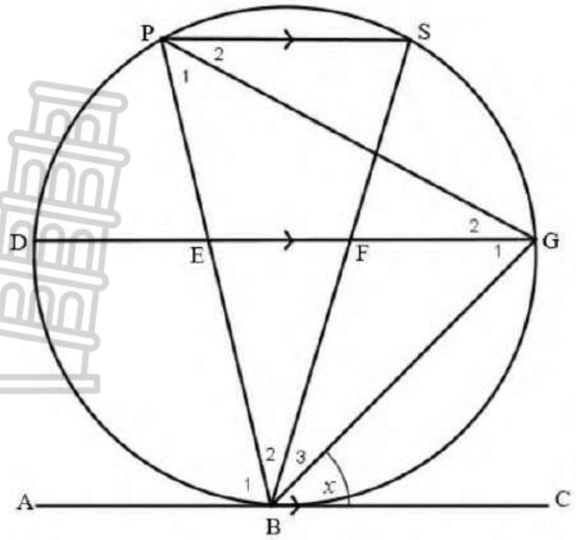
13.2 Show that CHRJ is a trapezium. (5) **L3**

13.3 Prove that OC bisects $A\hat{C}D$. (3) **L3**

13.4 Write down, with a reason, $A\hat{B}D$ in terms of x. (2) **L3**

13.5 Determine $\hat{R}_2$ in terms of x. (6) **L4**

14\. In the diagram, P, S, G, B and D are points on the circumference of the circle such that PS || DG || AC. ABC is a tangent to the circle at B. $G\hat{B}C = x$. Prove that:

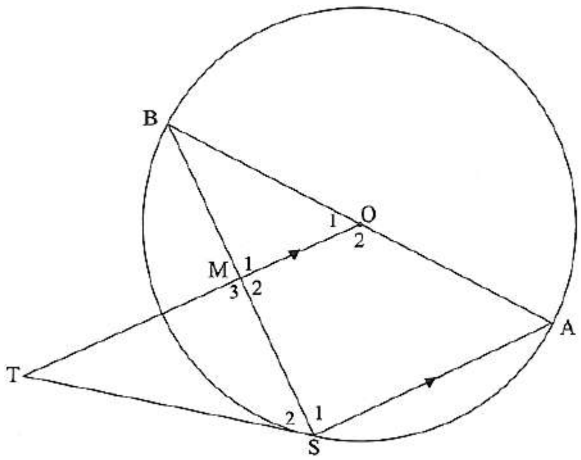


14.1 $BE = \dfrac{BP.BF}{BS}$ (2) **L2**

14.2 ΔBGP ||| ΔBEG (3) **L2**

15\. **EC SEPTEMBER 2022**

In the diagram, AB is a diameter of the circle centred at O. Δ ABS is drawn with S a point on the circle. M is a point on BS and OM is produced to T such that AS || OM. TS is drawn such that BOST is a cyclic quadrilateral.



Prove, giving reasons, that:

15.1 TS is a tangent to the circle at S. (4) **L3**

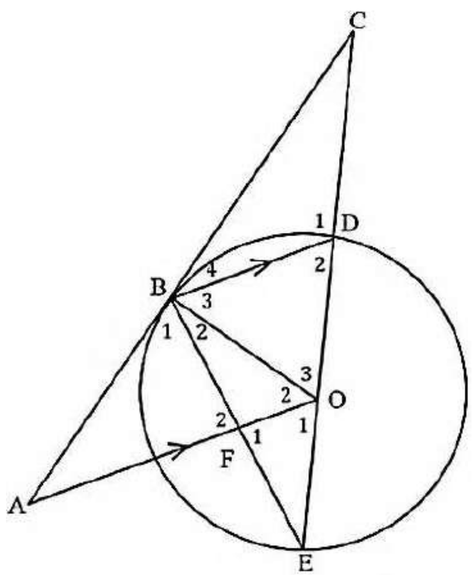
15.2 TS is a diameter of a circle passing through points T, M and S.

15.3 Δ ABS ||| Δ STM (3) **L2**

15.4 $AS.MT = 2SM^2$ (3) **L3**

16\. **WC SEPTEMBER 2023**

In the diagram alongside B, D and E are points on the circle with centre O. BD || AO and ABC is a tangent to the circle at B. AO intersects with BE at F. EODC is a straight line.



16.1 Prove, giving reasons that

16.1.1 Δ CBD ||| Δ CEB (3) **L2**

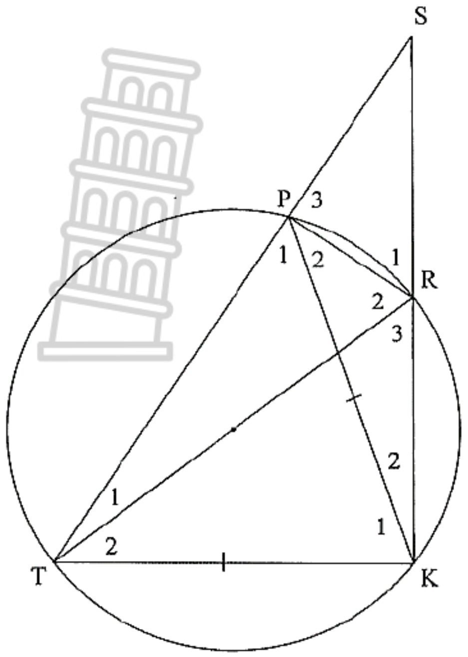
16.1.2 2EF.CB = CE.BD (3) **L3**

16.2 Determine the value of $\dfrac{CO}{OE}$ if it is further given that CB:BA = 4:3 (4) **L3**

16.3 Prove with reasons that:

$$\frac{\text{Area } \Delta FEO}{\text{Area } \Delta BED} = \frac{1}{4}$$ (4) **L3**

17. **DBE MAY JUNE 2023**



In the diagram alongside, TR is a diameter of the circle. PRKT is a cyclic quadrilateral. Chords TP and KR are produced to intersect at S. Chord PK is drawn such that PK = TK.

17.1 Prove, giving reasons, that:

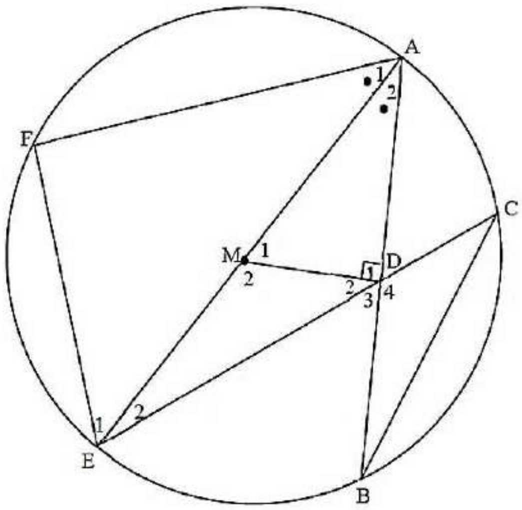
17.1.1 SR is a diameter of a circle passing through points S, P and R. (4) **L2**

17.1.2 $\hat{S} = \hat{P}_2$ (5) **L2**

17.1.3 $\Delta SPK \parallel\!| \Delta PRK$ (3) **L2**

17.2 If it is further given that SR = RK, prove that $ST = \sqrt{6}RK$ (5) **L4**

18. **GP SEPTEMBER 2023**



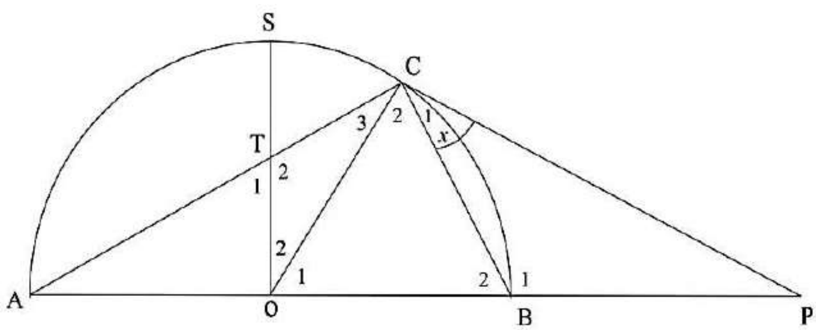
In the diagram alongside, diameter EMA of a circle with centre M bisects $F\hat{A}B$. MD is perpendicular to the chord AB. ED produced meets the circle at C. Chords CB and FE are drawn.

18.1 Prove that $\Delta AEF \parallel\!| \Delta AMD$. (4) **L2**

18.2 Determine the numerical value of $\frac{AF}{AD}$. (3) **L3**

18.3 Prove that $AD^2 = CD \times DE$. (6) **L3**

19. **EC SEPTEMBER 2019**



In the diagram alongside, O is the centre of a semi-circle ACB. S is a point on the circumference and T lies on AC such that $STO \perp AB$. Diameter AB is produced to P, such that PC is a tangent to the semi-circle at C. Let $\hat{C}1 = x$.

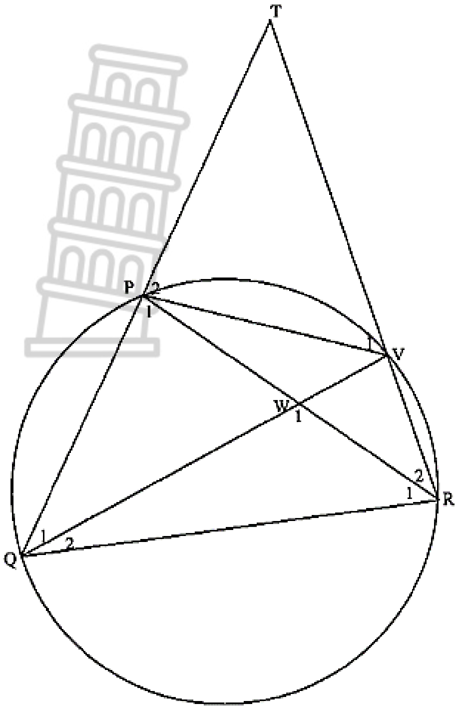
19.1 Write down, with reasons, 2 other angles equal to x. (3) **L2**

19.2 Prove that $\Delta TOC \parallel\!| \Delta BPC$. (5) **L3**

19.3 Prove that TO.PC = OB.BP. (2) **L2**

19.4 If BP = OB, show that $3OC^2 = PC^2$. (3) **L3**

20. **GP SEPTEMBER 2019**



In the diagram alongside, ΔPQR is an equilateral triangle inscribed in a circle. V is a point on the circle. QP produced meets RV produced at T. PR and QV intersect at W.

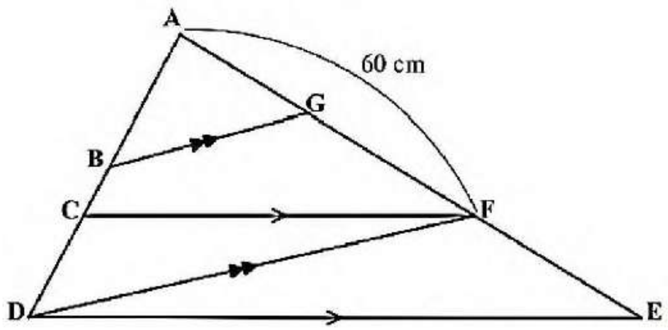
Prove, giving reasons that:

20.1 $\hat{W}_1 = T\hat{R}Q$. (3) **L3**

20.2 $\Delta TQR ||| \Delta QRW$. (3) **L2**

20.3 $\dfrac{PT}{QW} = \dfrac{PV}{WR}$. (6) **L3**

21. **NC SEPTEMBER 2023**



In ΔADE, $BG \parallel DF$ and $CF \parallel DE$.

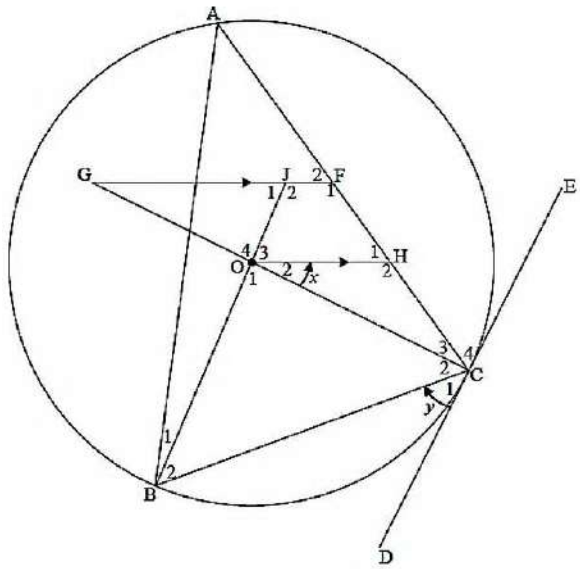
AF = 60 cm and AF : FE = 3 : 2.

Determine, with reasons:

21.1 The length of FE. (2) **L1**

21.2 The value of $\dfrac{BC}{CD}$, if it is further given that AG : GF = 7 : 8. (4) **L4**

22. **NW SEPTEMBER 2023**



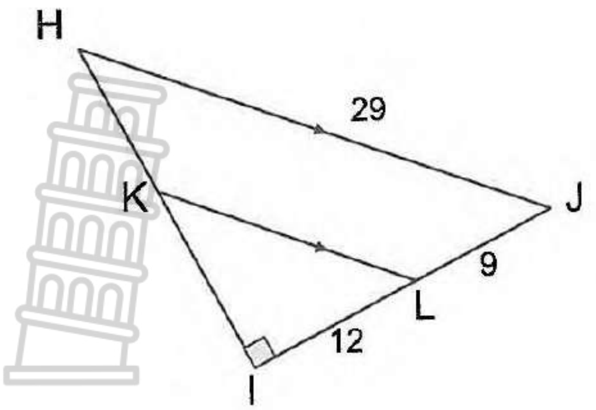
In the diagram alongside, O is the centre of the circle with points A, B and C on the circle. DCE is a tangent to the circle at C. GOC, BOJ and GJF are straight lines. F and H are points on AC such that $GF \parallel OH$.

$\hat{C}_1 = y$, $\hat{O}_2 = x$ and FH : HC = 2 : 3.

22.1 Calculate, giving reasons, $\hat{J}_1$ in terms of x and y. (6) **L3**

22.2 Determine, giving reasons, the value of $\dfrac{GO}{GC}$. (3) **L1**

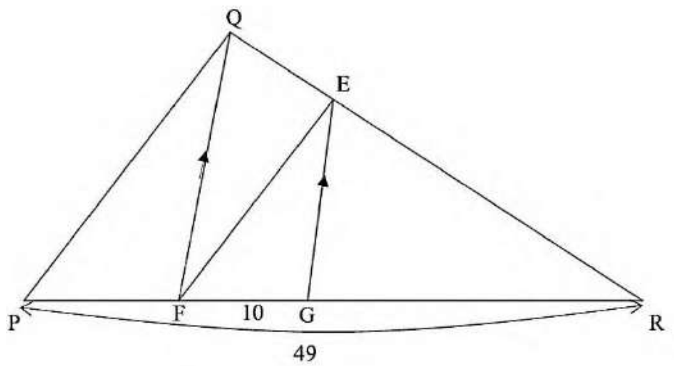
23. **IEB NOVEMBER 2023**



In the diagram:
HIJ is a right-angled triangle at I. L lies on IJ such that JL = 9 units and LI = 12 units. K lies on HI with $KL \parallel HJ$. HJ = 29 units.

23.1 Determine the length of IK. (5) **L2**

24. **EC JUNE 2024**

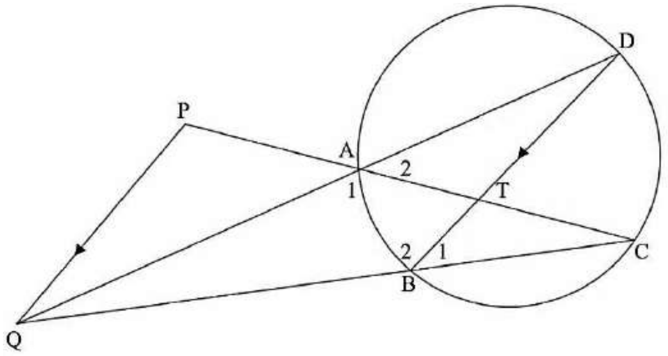


In the diagram alongside, ΔPQR is drawn. $EG \parallel QF$ and EF is a straight line. QE:ER = 2 : 5. PR = 49 units and FG = 10 units.

24.1 Calculate, giving reasons, the length of GR. (4) **L2**

24.2 Prove that $FE \parallel PQ$. (3) **L2**

25. **EC JUNE 2024**



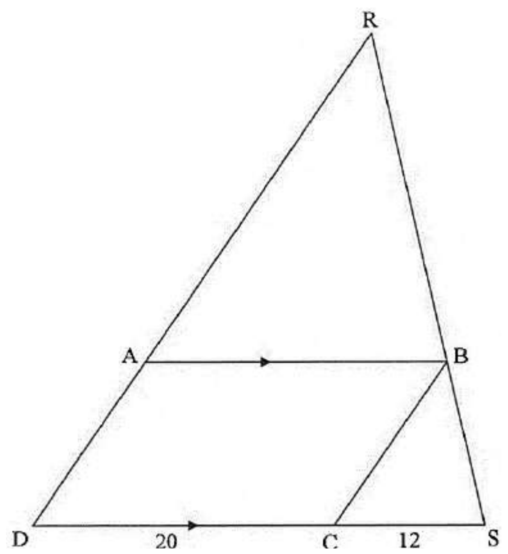
In the diagram alongside, A, B, C and D are points on the circumference of the circle. PC and QC are drawn from P and Q respectively and intersect at C. QP is joined. $DB \parallel PQ$. QB = 5BC. Prove that:

25.1 $\dfrac{CT}{PC} = \dfrac{1}{6}$ (3) **L1**

25.2 $\Delta QAC \parallel\!\mid \Delta QBD$. (4) **L2**

25.3 $QD.QA = 30BC^2$ (3) **L3**

26. **DBE MAY/JUNE 2023**

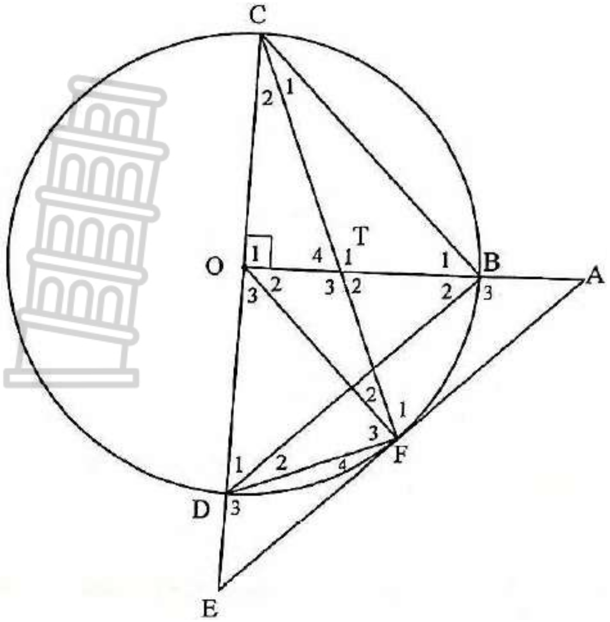


In the diagram alongside, ΔRDS is drawn. A, B, and C are points on RD, RS and DS respectively such that $AB \parallel DS$ and RB : BS = 5 : 3. DC = 20 units and CS = 12 units. Prove that:

26.1 Prove, giving reasons, that $BC \parallel AD$. (3) **L2**

26.2 If it is further given that RD = 48 units, calculate, giving reasons, the value of the ratio AD : AB. (3) **L3**

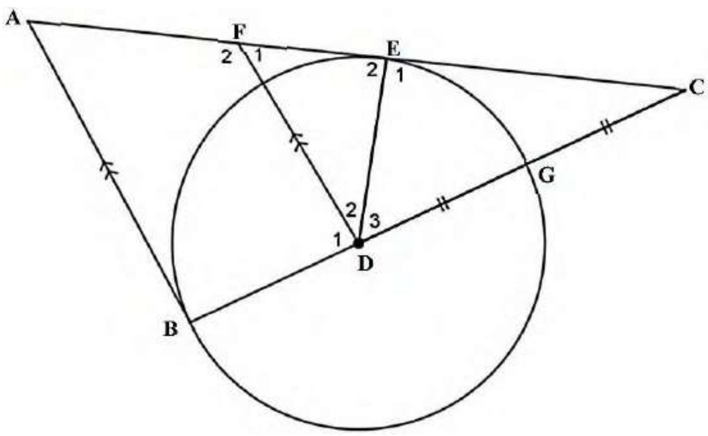
27. **DBE MAY/JUNE 2024**



In the diagram, COD is the diameter of the circle with centre O. EA is a tangent to the circle at F. $AO \perp CE$. Diameter COD produced intersects the tangent to the circle at E. OB produced intersects the tangent to the circle at A. CF intersects OB in T. CB, BD OF and FD are drawn. Prove, with reasons, that:

27.1	TODF is a cyclic quadrilateral.	(4)	**L2**
27.2	$\hat{D}_3 = \hat{T}_1$	(3)	**L1**
27.3	$\Delta TFO \mid\mid\mid \Delta DFE$	(5)	**L2**
27.4	If $\hat{B}_2 = \hat{E}$, prove that $DB \parallel EA$.	(2)	**L2**
27.5	$FO = \dfrac{TO.FE}{AB}$	(5)	**L3**

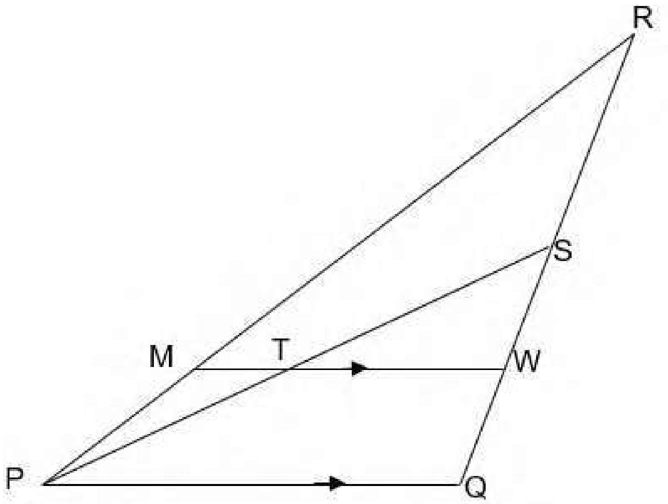
28. **GP JUNE 2024**



In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that DF ∥ AB.

28.1	Find, with reasons, the ratio of $\dfrac{AC}{FC}$.	(3)	**L3**
28.2	Prove, giving reasons, that $\Delta ABC \mid\mid\mid \Delta DEC$	(3)	**L2**
28.3	Prove that $DE^2 = \dfrac{AE.EC}{3}$.	(5)	**L3**
28.4	Find the ratio of: $\dfrac{\text{Area } \Delta FDC}{\text{Area } \Delta ABC}$.	(3)	**L3**

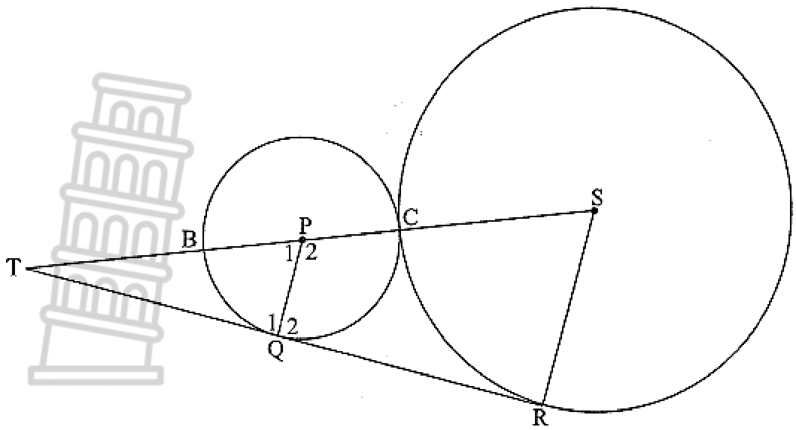
29.



In the diagram alongside,
S is the midpoint of RQ in Δ PRQ.
T is the midpoint of PS
and $MTW \parallel PQ$.
Calculate the numerical value of the following:

29.1	$\dfrac{RM}{RP}$	(6)	**L3**
29.2	$\dfrac{\text{area } \Delta \text{ RPS}}{\text{area } \Delta \text{ RMW}}$	(4)	**L3**

30. **NW SEPTEMBER 2019**



Two circles with centres P and S touch each other externally at C. SP produced intersects the circle P at B. A common tangent at R and Q meets SB produced at T. Prove that:

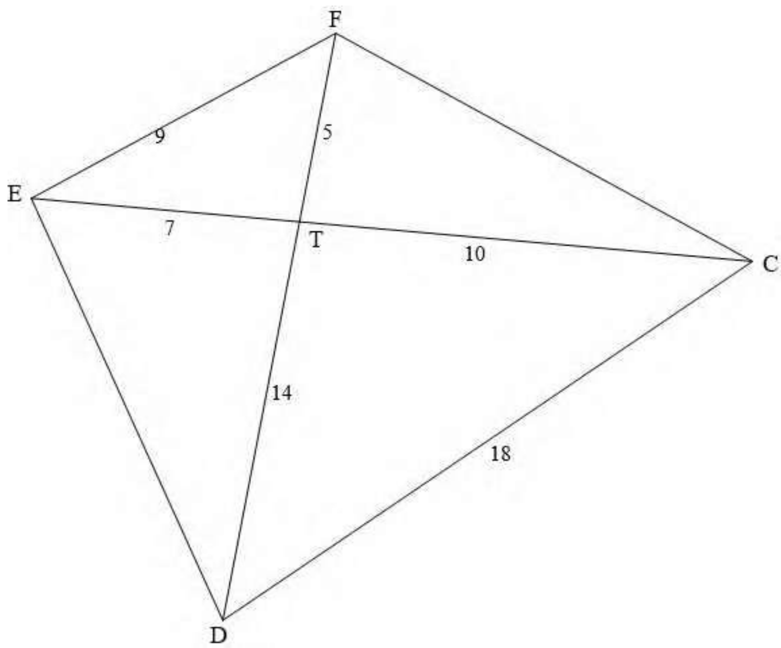
30.1 $PQ \parallel SR$ (4) **L2**

30.2 $TP = \frac{TQ(BP+SR)}{QR}$. (4) **L3**

30.3 $\Delta TQP \,|||\, \Delta TRS$ (3) **L2**

30.4 $\sqrt{TS^2 - CS^2} = \frac{\sqrt{(TP^2 + BP^2 - 2TP.BP\cos S)}.CS}{BP}$ (6) **L4**

31. **DBE NOVEMBER 2019**



In the diagram, the diagonals of quadrilateral CDEF intersect at T. EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.
Prove, with reasons, that:

31.1 $E\hat{F}D = E\hat{C}D$ (4) **L3**

31.2 $D\hat{F}C = D\hat{E}C$ (3) **L2**

ANSWERS
ALGEBRA, EQUATIONS AND INEQUALITIES

1.1	$x=0$ or $x=\frac{1}{4}$
1.2	$x=\frac{1}{3}$ or $x=-4$
1.3	$x=-4$ or $x=1$
1.4	$x=\frac{3}{2}$ or $x=1$
1.5	$x=\frac{5}{3}$ or $x=-1$
1.6	$x=\frac{-2}{3}$ or $x=4$
1.7	$x=\frac{p}{2-p}$, $p\neq 2$
1.8	$x=0$ or $x=\frac{4}{3}$
1.9	$x=625$, $x\neq 16$
1.10	$x=2$ or $x=1$
1.11	888 888 888 890
1.12	$x=\frac{1}{25}$, $x\neq\frac{1}{16}$
1.13	$x=256$
1.14	$x=4$ *or* $x\neq -3$
1.15	a) $x=3$ b) $x\in R$
1.16	a) $k=-7$ or $k=2$ b) $x=-1\,625$, $x\neq 44$
1.17	$y=\frac{3}{2}$
1.18	3
1.19	a) $\frac{x}{y}=-3$ or -2 b) $x=16$ or $x=12$ $y=-8$ or $x=-4$
1.20	$x=5$ or $y=\frac{3}{2}$
1.21	$k=-13$
1.22	$(x+y)^2+y^2$ is always positive
2.1	$x=2,57$ or $x=-0,91$
2.2	$x=3,450$ or $x=-1,450$
2.3	$x=0$ or $x=0,43$, $x=-0,77$
2.4	$x=1,66$ or $x=-0,91$
2.5	$x=0,1$ or $x=-3,6$

3.1	$x\leq -3$ or $x\geq 3$
3.2	$x<-3$ or $x>5$
3.3	$5<x<2$
3.4	$x<-1$ or $x>7$
3.5	$x<1$ or $x>5$
3.6	$x\geq 2$
3.7	a) $-4<x<5$ b) 4
3.8	$-3<x<7$
4.1	$x=-\frac{5}{3}$
4.2	$x=\frac{5}{7}$
4.3	$x=0$
4.4	$x=\frac{3}{2}$
4.5	$x=-1$
4.6	$x=2$ or $x\neq 1$
4.7	No solution
4.8	proof
4.9	2
4.10	$m=39$
4.11	$k=14$
4.12	$\frac{1}{54}$
4.13	$n=11$
4.14	2
4.15	$\frac{3}{5}$
4.16	m^{-1}
4.17	$\frac{2}{5}$
4.18	5.2^{n-4}
5.1	$m^2+4>0$
5.2	$m>\frac{9}{64}$
5.3	$TP(-2,-3)$
5.4	$p<-1$ or $p>1$
5.5	a) $k=-2$ or $k=2$ b) $k=-3$
5.6	$0<k<16$

6.1	$f(x)=x^2+4x-12$
6.2	$x=3$ or $x=4$ $y=-1$ or $y=-1$
6.3	$x=3$ or $x=-3$ $y=2$ or $y=-1$
6.4	$x=6$ or $x=-3$ $y=\frac{3}{2}$ or $y=-3$
6.5	$x=5$ $y=-3$
6.6	$x=1$ or $x=\frac{1}{6}$ $y=\frac{1}{2}$ or $y=\frac{-1}{3}$
6.7	$p=\frac{4}{5}$ or $p=-1$ $p=\frac{6}{5}$ $y=$ or $p=3$
6.8	$x=3$ or $x=5$ $y=-2$ or $y=-10$

SEQUENCES AND SERIES

1.1.1	$T_{91}=457$
1.1.2	$S_{91}=21\,112$
1.1.3	$n=103$
1.2.1	Proof
1.2.2	Proof
1.2.3	Proof
1.3.1	$T_n=3(2)^{n-1}$
1.3.2	$k=15$
1.4	$a=2$
1.5	$\sum_{k=1}^{8}\frac{3k-2}{3^k}$
1.6	$-\frac{-5}{2}<x<-\frac{-3}{2}$
2.1.1	$T_n=-2n^2+2n+143$
2.1.2	Add -1 to T_n
2.2.1	$T_n=5n+4$
2.2.2	$\sum_{n=1}^{24}5n+4$
2.3.1	$T_n=(5)(2)^{n-1}$
2.3.2	$S_{18}=1\,310\,715$
2.3.3	$\frac{S_\infty}{S_2}=2$
3.1.1	$T_{36}=253$
3.1.2	$S_{36}=4698$
3.1.3	$m=32$
3.2.1	$54\ cm$
3.2.2	$T_9=\frac{256}{81}=3.16\ cm$
3.2.3	$243\ cm\ >230\ cm$
3.3.1	$T_n=2n^2-22n+36$
3.3.2	$T_n=-4n+36$
3.3.3	T_{17} and T_{18}
3.4	$S_7=-29538$
3.5.1	$T_1=141$
3.5.2	The second difference is -4
3.5.3	$n=9$ or $n=11$
3.5.4	T_{10}
4.1.1	$21;24$
4.1.2	$T_{20}-T_{21}=449$
4.2.1	Proof
4.3	$n=12$
4.4.1	$T_8=0{,}0067$
4.4.2	$\sum_{n=1}^{\infty}5\left(\frac{3}{4}\right)^{n-1}$
4.4.3	95 kg
5.1.1	T_{484} and T_{485}
5.1.2	$c=-30$
6.1.1	Yes, because $-1<\frac{1}{2}<1$
6.1.2	$S_\infty=8$
6.2	$k=3$
6.3.1	Proof
6.3.2	164 must be added
6.4	$n=12$
6.5	$a=-\frac{4}{3}$
7.1.1	$T_3=27$
7.1.2	$T_n=3n^2+n-3$

7.1.3	Between T_{69} and T_{70}
7.2.1	$r=\frac{x+1}{x}$
7.2.2	$T_3=\frac{(x+1)^2}{x}$
7.2.3	No, $r>1$
7.3	$a=1$ or $a=9$ $b=12$ or $b=-4$
8.1.1	$x=14$
8.1.2	$T_n=4n+15$
9.1.1	$T_{20}=79$
9.1.2	$n=100$
9.2.1	$d=2$
9.2.2	$S_{200}=200a+39\ 800$
9.3.1	$r=-\frac{1}{2}$
9.3.2	$n=15$
9.3.3	$S_\infty=24$
9.3.4	$\frac{S_{odd}}{S_{even}}=-2$
9.4.1	$p=11$
9.4.2	Term 12 will be the first term smaller that -55
10.1	$T_n=88-3n$
10.2	T_{30}

11		$T_1=-40$
12		$T_{25}+T_{26}=51$
13.1		$0<x<\frac{1}{2}\quad x\neq\frac{1}{4}$
13.2		76
14.1		$r=2$
14.2		$L=186$ units
14.3		$S_\infty=3\ 145\ 725\pi$
15		
	(a)	$T_n=4n+1$
	(b)	$T_n=5(25)^{n-1}$
16		Nomsa is correct
17.1		$-1+2+5$
17.2		$S_{100}=14\ 750$
18.1		$T_n=-2n+3$

18.2		35th first difference $=-67$
19		$T_3=19$ $T_4=21$
20		$a=2$ and $d=3$
21		
	(a)	$a=\frac{127}{153}$
	(b)	$T_1=\frac{1016}{153}$ $T_2=\frac{2032}{153}$
22.1		$S_{23}=1426$
22.2		$T_{23}=117$
23		$r=\frac{3}{2}$ or $r=\frac{1}{2}$
24.1		
	(a)	$x=18$
	(b)	$x=\pm 8\sqrt{2}$
25		Proof
26.1		21; 24
26.2		$T_{52}-T_{51}=100663143$
27		Proof

FUNCTIONS

1.1.1	$x=-1/x=5$
1.1.2	$b=3$
1.1.3	$x\geq 2$
1.1.4	$h(x)=-x^2$
1.1.5	$x\geq 0$ or $x\leq 0$
1.1.6	Proof
1.2	$a=-m$
2.1	A(1; 0)
2.2	$x>0$
2.3	$y=\left(\frac{1}{2}\right)^x$
2.4	Graph
2.5	$y=1$
2.6	Reflect about y-axis and shift 1 unit down.
3.1.1	$p=-1$ and $q=-3$
3.1.2	$a=2$
3.1.3	Shift 2 units to the right and 6 units up.

3.1.4	$y=3$
3.2.1	$x>0$
3.2.2	$b=\frac{1}{5}$
3.2.3	$y=\left(\frac{1}{5}\right)^x$
3.2.4	$x>-5$
4.1	Graph
4.2	$y<1$
4.3	$y=-1$
4.4	$y=\log_3 x$
5.1	Proof
5.2	Q(1;12)
5.3	Proof
5.4.1	$m=2$
5.4.2	$y=2x+1$
5.5	$x>0$
6.1	$x=1$ and $y=2$
6.2	Graph
6.3	$x<\frac{1}{2}$ or $x>1$
6.4	$y=-x+3$
6.5	$x\in\mathbb{R}, x\neq 1$
7.1	$p=-3$
7.2	$k=-1$
7.3	$B(0;\frac{1}{3})$
7.4	$x\leq 0$ or $1\leq x<3$
7.5	$f(x)=\frac{2}{x-3}+1$
8.1	Proof
8.2	$x=-\frac{14}{3}$
8.3	Graph
8.4	$x=-9$
8.5	$-\frac{14}{3}<x<-4$
8.6	$x<-4$
8.7	x-values at the points of intersection of f and axis of symmetry with negative gradient.
9.1	$P'(2;4)$
9.2	Proof

9.3	$y=2^x$
9.4	Area $=1$ unit^2
10.1	$y>-4$
10.2	$D(-1;0)$ and $E(3;0)$
10.3	$y=x-3$
10.4	$x<0$ or $x>3$
10.5	Distance $=\frac{25}{4}=6,25$ units
11.1.1	$D(2;-15)$
11.1.2	$CN=5\sqrt{5}$
11.2.1	$-1<x<7$
11.2.2	$x=3$
12	$f(x)=\frac{3}{x-5}+1$
13.1	$A(0;1)$
13.2	$k=4$
13.3	Proof
13.4	$f(x)=4^x$
13.5	$h(x)=4^{-x}$
13.6	$y\geq 0$
13.7	$y=4x-3$
13.8	$x\geq\frac{1}{2}$
15.1	$a=3$
15.2	$y=\log_3 x$
15.3	$\left(\frac{1}{9};-2\right)$
15.4	$0<x\leq\frac{1}{9}$
15.5	$f(x)=\log_3\frac{x}{2}$
16.1	$a=2$
16.2	$x\geq 0$
16.3	$y\geq 0$
16.4	$y=\frac{x^2}{2},\ x\geq 0$
16.5	$(2;2)$ and $(8;4)$
17.1	$f^{-1}(x)=\log_3 x$
17.2	Graph

17.3	$x > 0$
17.4	$0 < x < 1$
17.5	$y > 0$
17.6	$g(x) = -3^{x-2}$
18	$y \leq -2$ or $y \geq 2$
19.1	$k = \frac{1}{3}$
19.2	$y > 0$
19.3	Reflect about the line $y = x$
19.4	$y = \log_{\frac{1}{3}} x$
19.5	Graph
19.6	Proof
20.1	$l(x) = -x + 5$
20.2	$\text{A}(1;4)$
20.3	$f(x) = \frac{6}{x-1} + 4$
20.4	$x = 0$
21.1	Yes, Inverse of one -to – one relation is a function
21.2	$\text{R}(-12;-6)$
21.3	$a = -\frac{1}{3}$
21.4	$y = -\sqrt{-3x},\ x \leq 0$
21.5	Graph
22.1	$m = 3$
22.2	$y = -x^2 + 4x + 12$
22.3	$y \in (-\infty;12]$

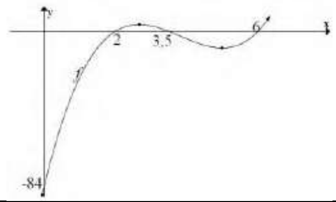
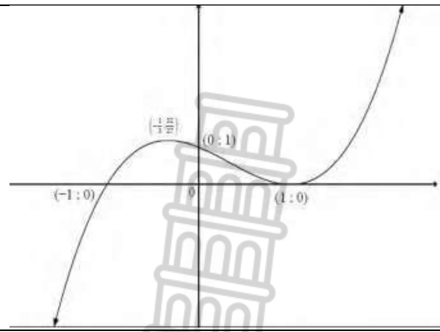
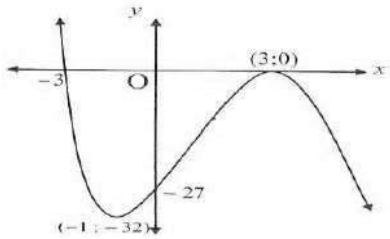
FINANCIAL MATHEMATICS

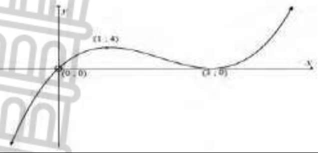
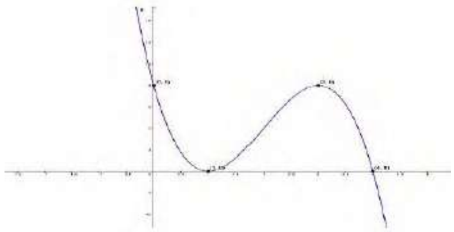
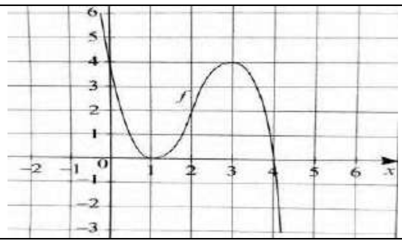
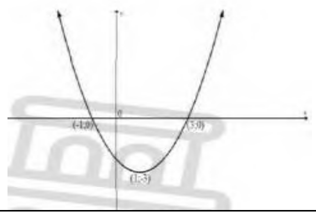
1.1	$r = 8{,}7\%$
1.1.2	$r = 9{,}06\%$
1.2.1	$n = 5$
1.2.2	$R\,267{,}26$
1.3	Tino will make 147 withdrawals of $R\,20000$
2.1	$n = 6$
2.2.1	R 217 666,80
2.2.2	R 711 500, 99
2.2.3	R 6 803,01
3.1	$r = 10{,}85\%$
3.2	R 45 997,22
3.3.1	Sam will have 370 instalments
3.3.2	R 1 536,69
4.1	$r = 0{,}74\%$
4.2.1	$n = 220$
4.2.2	She can make any number (an infinite numbers) of withdrawals.
4.3	R15 282,91
5.1	R 13,18%
5.2	R148 351,63
5.3	5,09%
5.4	$x = R$1 068, 85
6.1.1	proof
6.2.1	$n = 155{,}51$
6.2.2	R 1,574.70
6.3.1	Capital Repayment $= R$7 500 - R 4 935,50 $\approx$ R2 564,50
6.3.2	Capital Repayment = R 90,000 - R 59,268.40 $\approx$ R30,731.60
7.1.1	$x = R$7 982,73
7.1.2	R216 021,16
7.1.3	$n = 27$ therefore 27 months
8.1.1	R 850000
8.1.2	R 6 729,95
8.1.3	R 867 188
8.1.4	R 615 509,74
8.1.5	R634 183,84
8.1.6	110 *months*
9.1	R 3 037,50
9.2.1	$n = 155$
9.2.2	Outstanding balance R 3 230,50
9.2.3	Last payment R3 278,96
9.2.4	Total repaid $= R$773 278,96
10.1	**Option 1** R 2 120 088 **Option 2** R 2 135 376 Mr Dasoo will choose option 1 as it has lowest total repayment
11.1	R 11 182,68
11.2	$x = 5000$
12.1	R103 119,19
12.1.1	$n =$ 13,5 years
12.1.2	R451 919,19
12.1.3	R4 733,39
13.1	R2 669,28
14.1	R 14 212.35
14.1.1	R 14 212.35

14.1.2	$R77\ 597{,}60$
15.1.1	$R3\ 912{,}89$
15.1.2	699, 548824061 He will survive 702 months after his retirement on his current lifestyle
15.1.3	$R11\ 322{,}72$
16.1	4 *years*
16.2	$R617{,}45$
16.3.1	$R12\ 499{,}96$
16.3.2	$R885\ 814{,}82$

CALCULUS

1.1	$f'(x)=4x$
1.2	$f'(x)=-2x$
1.3	$f'(x)=-4x$
1.4	$f'(x)=-2x$
1.5	$f'(x)=\frac{5}{x^2}$
1.6	$f'(x)=-2x+7$
1.7	$f'(x)=0$
1.8	$f'(x)=1$
1.9	$f'(x)=3x^2$ $f'(2)=12$
1.10	$f'(x)=2x-b$
2.1	$-2x^{-2}-\frac{1}{2}x^{-\frac{1}{2}}$
2.2	$x^6-2x^{-3}+1$
2.3	$4+2x^{-\frac{3}{2}}-2x^{-3}$
2.4	$x+5x^{-2}$
2.5	$10t^4+\frac{7}{4}t^{\frac{3}{4}}$
2.6	$-2x-2$
2.7	$-x^{-\frac{1}{2}}+1-2x^{-\frac{3}{2}}$
2.8	$6x+4x^{-3}$
2.9	x^2+5x+9
2.10	$-4x^{-2}-\frac{\sqrt{3}}{2}x^{-\frac{3}{2}}$
2.11	$1+3x^{-2}$
2.12	$gt+5t^{-2}$
3.1	$3t^2\geq 0$
3.2	$y=4x-8$
3.3	$\mathrm{A}\left(\frac{1}{32};5\right)$
3.4	$a=-4, b=-3$

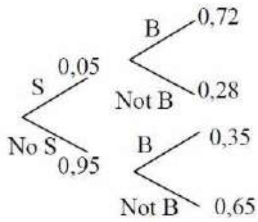
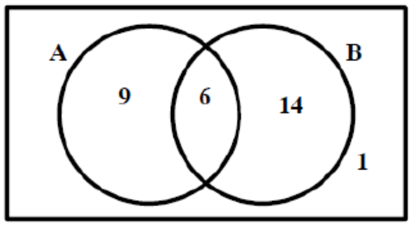
3.5	3.5.1	$f'(2)=5$
	3.5.2	$y=5x-7$
3.6	3.6.1	$p=3,\ a=\frac{-11}{27}$
	3.6.2	$h'(3)=-8$
3.7	3.7.1	$g'(2)=-9$
	3.7.2	$(-6;90)$
3.8.1	$(3;-4)$	
3.8.2	$y=9x-23$	
4.1.1	$(-2;0)$	
4.1.2	$x=4$	
4.1.3	$x=\pm 3$	
4.1.4	$x>0$	
4.2	$x>\frac{1}{3}$	
4.3.1	$f(2)=2(2)^3-23(2)^2+80(2)-84=0$	
4.3.2	$(x-2)(2x-7)(x-6)$	
4.3.3	$x=\frac{8}{3}$ or $x=5$	
4.3.4		
4.3.5	$(0;-65)$	
4.4.1	$(0;1)$	
4.4.2	$(-1;0)$, $(1;0)$	
4.4.3	$\left(\frac{1}{3};\frac{32}{27}\right)$, $(1;0)$	
4.4.4		
4.4.5	$\left(-\frac{1}{3};1\right)$ or $-\frac{1}{3}<x<1$	
4.5.1	$x=-3$ or $x=3$ or $y=-27$	
4.5.2	$(-1;-32)$, $(3;0)$	
4.5.3		

4.5.4	$x > 1$
4.5.5	$y = 9x - 27$
4.5.6	$-32 < k < 0$
4.5.7	$f(x+3) = -(x+3)^3 + 3(x+3)^2 + 9(x+3) - 27$
4.6.1	$f''(x) = 6x - 12$
4.6.2	
4.6.3	$x > 2$
4.6.4.1	$(3;7)$
4.6.4.2	$= -3 \quad \neq 1$
4.7.1	
4.7.2	$8 = a(3-1)^2(3-4)$
4.7.3	$x > 2$
4.8	
4.9.1	C(6;0)
4.9.2	E(4;36)
4.9.3	$x < \frac{5}{3}$
4.9.4	$\text{OG} = \frac{10}{3}$
4.9.5	$m = \frac{1}{17}$
4.10.1	A(0;12)
4.10.2	B(−2;0) , C(1;0)
4.10.3	Turning points
4.10.4	$-2 < x < 1$ or $x \in (-2;1)$
4.10.5	13,5
4.11	
4.12	$f(x) = -2x^3 + 12x^2 - 60x + 43$
5.1.1	-36
5.1.2	$t = 3$
5.2.1	12 mm
5.2.2	$t = 4$ seconds
5.3.1	$1 - \frac{1}{2}x + \frac{1}{16}x^2 - \frac{1}{4\pi}x^2$
5.3.2	$x = 1,76$ m
5.4.1	$t = 8$
5.4.2	24,63 *km*
5.4.3	16 *km/h*
5.5.1	$36 - x$
5.5.2	$l = x - 4, b = 36 - x - 4 = 32 - x$
5.5.3	$A(x) = (x-4)(32-x)$
5.5.4	$x = 18$ cm
5.6.1	$h - 3 = \frac{270}{\pi r^2}$
5.6.2	297,01 cm^2

PROBABILITY

1.1	$a = 120; b = 60; c = 140; d = 210$
1.2	The events are independent
2.1	15120
2.2	3360
2.3	420
3.1.1	$\frac{3}{4}$
3.1.2	$\frac{1}{8}$
3.1.3	150
3.1.4	$B = 1050; C = 50; D = 350$
3.1.5	$\frac{1}{32}$
3.2.1	362880
3.2.2	17280
4.1.1	1 or 2
4.1.2	3600
4.2	76,5%
5.1	$\frac{51}{260}$ or 0,1962 or 19,62%
5.2	$\frac{2}{13}$ or 0,1538 or 15,38%
6.1	$a = 4; b = 13; c = 5; d = 4; e = 6$
6.2	23 people
6.3	$\frac{3}{10}$
7.1	40320
7.2	1440

7.3	$\frac{2}{3}$
8.1	729
8.2	280
8.3	81
9.1.1	$a = 450; b = 319; c = 298; d = 748$
9.1.2	$\frac{298}{1530}$
9.1.3	The events are independent. $P(Male\text{ and }Broken\ Limb)$ $= P(Male) \times P(BrokenLimb)$
9.2.1	$\frac{2}{7} \approx 0,29$
9.2.2	$\frac{40}{77} \approx 0,52$
9.3.1	$x = 0,09$
9.3.2	$y = 0,34$
10.1.1	$x = 0,15; y = 0,45$
10.1.2	$0,45$
10.2.1	$\frac{12}{35}$
10.2.2	$\frac{24}{75}$
11.1	No, $(P \text{ and } B) \neq 0$
11.2	$P(A \text{ or } B) = P(A) + P(B) - (P \text{ and } B)$
11.3	1200
11.4	864
12.1.1	24 ways
12.1.2	$\frac{1}{180}$ *or* 0,01
12.2	$= (n-1)!$ $\therefore 2! \times (n-1)!$
13.1.1	No, because $P(A \text{ and } B) \neq 0$
13.1.2	(a) 0,3
	(b) 0,7
13.2.1	$\frac{1}{4}$
13.2.2	479001600
13.2.3	$\frac{1}{99}$
14.1.1	$\frac{1}{4}$
14.1.2	$\frac{5}{6}$

14.2.1	
14.2.2	0,6315
14.3.1	10!
14.3.2	$\frac{4}{45}$
15.1.1	531 441
15.1.2	531 441
15.2	50 400
16.1.1	
16.1.2	n(A or B) = 29
16.1.3	0,2
16.2.1	67 600
16.2.2	≥ 3,64717...
17.1	2520
17.2.1	0,195
17.2.2	48,3%
17.2.3	284886
18.1	480
18.2.1	720
18.2.2	48
18.2.3	48
18.2.4	240
18.3	53rd position
18.4	60
18.5.1	1260
18.5.2	$\frac{1}{7}$

TRIGONOMETRY

1.1	$\cos 24° = m$
1.2	$\cos 24° = m$
1.3	$\sin 66° = m$
1.4	$\cos 48° = 2m^2 - 1$
1.5	$\sin 132° = 2m\sqrt{1-m^2}$
1.6	$\cos 66° = \sqrt{\frac{\sqrt{1-m^2}+1}{2}}$

1.7	$2\sin^2 12° = 1 - m$
1.8	$\cos 54° = \frac{\sqrt{3}}{2}m - \frac{1}{2}\sqrt{1-m^2}$
1.9	$\sin 42° = 2m^2 - 1$
2.1	$\sin 212° = -t$
2.2	$\cos 122° = -t$
2.3	$\cos 64° = 2\sqrt{1-t^2} - 1$
2.4	$\cos 32° = \sqrt{\frac{1-\sqrt{1-t^2}}{2}}$
2.5	$\tan 392° = \frac{t}{\sqrt{1-t^2}}$
QUESTION 3	
$\sin\left(\frac{\beta}{2}+45°\right)\cos\left(\frac{\beta}{2}+45°\right) = \frac{\left(\frac{1}{\sqrt{2}}\right)\left(\sqrt{1-m^2}\right)-\left(\frac{1}{2}\right)(k)}{2}$	
4.1.1	$\tan\theta = \frac{5}{12}$
4.1.2	$\sin\theta = \frac{5}{13}$
4.1.3	$\sin(-90°+\theta) = -\frac{12}{13}$
4.1.4	$\sin 2\theta = \frac{120}{169}$
4.1.5	$a = -\frac{15}{2}$
4.2.1	$\tan\alpha = \frac{4}{3}$
4.2.2	$\cos = \frac{3}{5}$
4.2.3	$m = -5$
4.2.4	$\sin(\alpha+\beta) = -\frac{56}{65}$
5.1	$\cos(180°+\theta) = \frac{4}{5}$
5.2	$\tan 2\theta = \frac{24}{7}$
5.3	$\sin(45°+\theta) = \frac{-7\sqrt{2}}{10}$
5.2.1	$\cos\alpha = -\frac{4}{3}$
5.2.2	$\cos(90°-\beta) = \frac{5}{3}$
5.2.3	$\cos(\alpha+\beta) = \frac{1}{15}$

5.3.1	$\sin 2\theta = -\frac{\sqrt{11}}{6}$
5.3.2	$\cos\theta = -\sqrt{\frac{1}{12}}$
6.1	
6.2	$-\frac{1}{\sin\theta}$
6.3	$\sqrt{3}$
6.4	1
6.5	$3\sin^2 x$
7.1	$\frac{\sqrt{2}}{2}$
7.2	$\cos 20°$
7.3	$\sin 25°$
7.4	$\frac{\sqrt{3}}{2}$
7.5	$\frac{\sqrt{3}}{2}$
7.6	$\sqrt{2}$
7.7	$k = 2\sin(60°+x)$

8.1	$x = 33{,}33° + k120\ k \in \mathbb{Z}$ or $x = 300° + k360k \in \mathbb{Z}$
8.2	$x = 62{,}74° + k180\ k \in \mathbb{Z}$
8.3	$x = 15° + k180\ k \in \mathbb{Z}$ or $x = 75° + k180\ k \in \mathbb{Z}$
8.4	$x = 50{,}66° + 180\ k \in \mathbb{Z}$ or $x = 140{,}66° + k180\ k \in \mathbb{Z}$
8.5	$x = 60° + k360\ k \in \mathbb{Z}$ or $300° + k360\ k \in \mathbb{Z}$
8.6	$x = 116{,}57° + k180\ k \in \mathbb{Z}$
8.7	$x = 11{,}3° + k180\ k \in \mathbb{Z}$ or $x = 135° + k180\ \mathrm{k} \in \mathbb{Z}$
8.8	$x = 180° + k360\ k \in \mathbb{Z}$ or $x = 120° + k360\ \mathrm{k} \in \mathbb{Z}$ $x = 240° + k360\ k \in \mathbb{Z}$

8.9	$x = 63{,}43° + k180\ k \in \mathbb{Z}$
10.1	period $= 120°$
10.3.1	$x \in (30°\ ;\ 90°)$
10.3.2	$x \in (-90°\ ;\ -30°)$
10.3.3	$x \in (-90°\ ;\ -30°) \cup (30°\ ;\ 90°)$
10.4	$y \in [-4;\ 2]$
10.5	$h(x) = -\cos x$
11.1	graph sketching
11.2	graph sketching
11.3.1	period 360°
11.3.2	$x = -80°;\ x = 40°;\ x = 160°$

12.1	$a = 1;\ p = -4;\ b = 2$
12.2	$x \in (-90°\ ;\ 0°)$
12.3	period $= 45°$
12.4	maximum $= 2$
12.5	the graph of has been shifted $45°$ to the right
12.6	$x \in (-90°\ ;\ -45°) \cup (0°\ ;\ 90°) \cup (135°\ ;\ 180°)$
13.1	$b = 2$
13.2	$x \in (45°\ ;\ 67.5°)$
13.3	$x = 10°;\ x = 145°$
13.4	$x = 60° + k.180°\ k \in \mathbb{Z}$
13.5.1(a)	$A(120°\ ;\ 0°)$
13.5.1(b)	$C(-150°\ ;\ -1)$
13.5.2(a)	$x \in (-90°\ ;\ 30°)$
13.5.2(b)	$x \in (-160°\ ;\ 20°)$
13.5.2(c)	$x = \pm 90°$
13.5.2(d)	$x \in (-180\ ;\ -90°) \cup (90°\ ;\ 180°)$
13.5.3	range $y \in [2;32]$
14.1	$AC = \dfrac{x}{\sin\theta}$
14.2	$CE = 2(x+2)$
14.3	proof
14.4	$AE = 35.77m$
15.1	$A\hat{B}C = 90°$
15.2	proof
15.3	proof
19.1	$A\hat{C}D = 30°$ Alt $\angle s\ \parallel$
19.2	$AC = 2h$
19.3	$A\hat{B}D = 19{,}47°$
19.4	$x = 30°$

ANALYTICAL GEOMETRY

1.1	$m = 4$	
1.2	$y = 4x - 16$	
2.1	a)	$m = \dfrac{1}{2}$
	b)	$m = \dfrac{-5}{2}$
2.2	$D\hat{C}B = 85{,}2^0$	
2.3	$x - 2y + 11 = 0$	
2.4	Proof	

2.5	$b = \dfrac{7}{5}$ or $b = 3$	
3	3.1	BD: $y = 2x - 3$
	3.2	E(2;1)
	3.3	A(−4;4)
	3.4	329 square units
4	4.1	$y = 1$
	4.2	$y = 2x + 10$
	4.3 (a)	$PQ = 5\sqrt{2}$
	4.3 (b)	$x \neq 8$
	4.3 (c)	$R(-3;1)$
5	5.1 (a)	$P(-5;7)$
	5.1 (b)	$T(6;9)$
	5.1 (c)	$\theta = 78{,}69$
	5.2	$k = 1$
	5.3 (a)	Proof
	5.3 (b)	$y = -\dfrac{5}{4}x + \dfrac{11}{2}$
	5.4 (a)	$p = 6$
	5.4 (b)	$t = 3$
6	6.1	$r = -3$
	6.2	Trapezium
	6.3	$P(0;-1)$
	6.4	Proof
	6.5	$\left(x - \dfrac{1}{2}\right)^2 + \left(y + \dfrac{7}{2}\right)^2 = \dfrac{13}{2}$
	6.6	$r = \sqrt{3}$
7	7.1	90 tan ⊥ rad
	7.2	$10\sqrt{5}$
	7.3	$k = 7$
	7.4	$(x-3)^2 + (y+1)^2 = 100$
	7.5	$y = -11$
7.6	a)	$P(-17;11)$
	b)	$PQ = 20units$
	c)	$Yes, \Delta QRC \equiv \Delta QPC (SAS)$
	d)	$q \neq -7$ or $q \neq 17$
7.7	a)	$M(3;-16)$

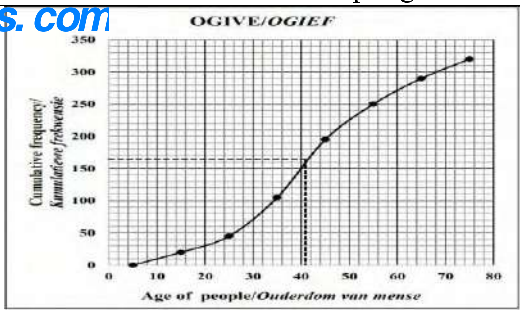
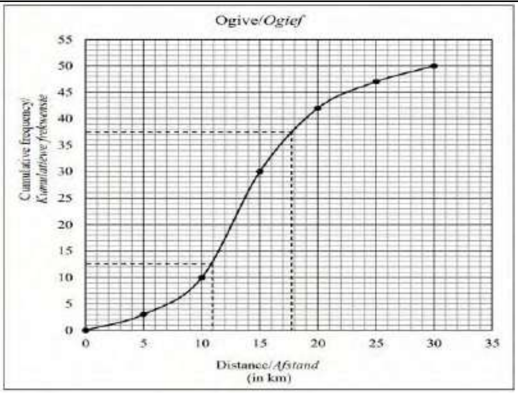
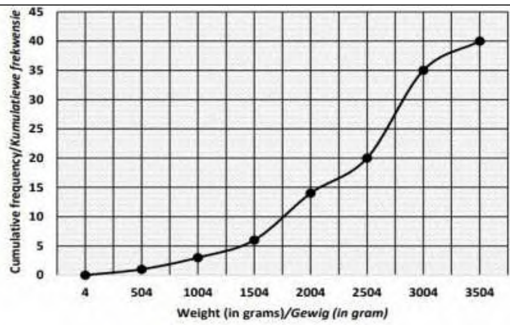
	b)	$r = 4$
	c)	Proof
8	8.1	Proof
	8.2	90^0, Radius $\perp$ tangent
	8.3	$\tan \hat{C} = \frac{\sqrt{2}}{4}$
	8.4	$y = \frac{\sqrt{2}}{4}x + 4\sqrt{2}$
9	9.1	$y = \frac{1}{2}x + \frac{15}{2}$
	9.2	Centre $(-5\ ;\ 5)$
	9.3	Diameter = 12,65
	9.4	$(x+5)^2 + (y-5)^2 = 40$
	9.5	$P = 10$
	9.6	$(x+2)^2 + (y-7)^2 = 10$
10	10.1	Proof
	10.2	Proof
	10.3	$\frac{\sqrt{2}}{2}$
11	11.1	$\tan \perp rad$
	11.2 (a)	$M(-5;5)$
	11.2 (b)	$(x+5)^2 + (y-5)^2 = 25$
	11.2 (c)	$y = \frac{-4}{3}x + \frac{20}{3}$
	11.3	$(x-2)^2 + (y-4)^2 = 75 - 50\sqrt{2} \approx 4,29$
	11.4	$M'(5;5)$ and $M''(-5;-5)$
12	12.1	$y = -2x$
	12.2	$R(2;-4)$
	12.3	$\frac{1}{4}$
	12.4	$-5 < k < 5$
13	13.1	$A(1;-2)$ and $r = 3$
	13.2	$N(1;1)$
	13.3	$y = 1$
	13.4	$5,39 < r < 8,39$
14	14.1	$(x-3)^2 + (y-4)^2 = 81$
	14.2	$y = x + 7$
	14.3	inside
	14.4	$K(12;-5)$
15	**15.1**	$M(-1;-3)$
	15.2	$r = \sqrt{8}$
	15.3	$D(-3;-5)$

	15.4	Angle between a radius and a tangent
	15.5	$MB = \sqrt{40}$
	15.6	$(x+1)^2 + (y+3)^2 = 40$
16	16.1	$M(-6;-3)$
	16.2 a)	$r = 4$
	b)	$TS = 1$ unit
	16.3 a)	$y = -8$
	b)	$y = \frac{3}{4}x + \frac{31}{4}$
	16.4 a)	Perimeter = 40units
	b)	$\frac{1}{2}$
17	17.1	$rad \perp \tan$
	17.2	$m = -2$
	17.3	Proof
	17.4	$y = \frac{1}{2}x - \frac{5}{2}$
	17.5	$E(5;-10)$
	17.6	$(x-5)^2 + (y+5)^2 = 25$
	17.7	$2\sqrt{5} < r < 4\sqrt{5}$
18	18.1	$P(0;-2)$
	18.2	$M(2;-2)$
	18.3	$x^2 + y^2 - 4x + 4y + 4 = 0$
	18.4	$c = \pm 2,82$
19.	19.1	$E(1;-1)$
	19.2	$AB = 4\sqrt{5}$
	19.3	$y = -2x + 1$
	19.4	Proof
	19.5	Proof
	19.6	$t \neq -\frac{3}{4}$ or $t \neq \frac{24}{7}$
20.	20.1	$m = -\frac{4}{3}$
	20.2	$(-2;-4)$
	20.3	$y = \frac{3}{4}x - \frac{5}{2}$
	20.4	$\beta = 126,87$
	20.5	$O\hat{J}K = 36,87$
	20.6	$x = -4$ only
	20.7	$Area$ DEOJ $= 20,83\ units^2$
	20.3	$y = \frac{3}{4}x - \frac{5}{2}$
	20.4	$\beta = 126,87$

	20.5	$O\hat{J}K = 36,87$
	20.6	$x = -4$ only
	20.7	*Area* DEOJ = 20,83 $units^2$

DATA HANDLING

1.1	77,18
1.2	24,86
1.3	2 results
1.4	150
1.5	D
2.1	x = 35
2.2	7,8
2.3	3 scores
3.1	negative
3.2	Either 4 or 5 calls
3.3	The correlation will increase slightly less (less
4.1	y=-23,85 + 0,23x
4.2	y=102,65
4.3	r = 0,98
4.4	Very strong positive correlation
5.1	The higher the temperature, the lesser hot drink are sold
5.2	A=489,46 B=-10,36
5.3	39 litres
5.4	(12,230)
6.1	*Strong, The majority of the points lie lose to the regression line*
6.2	$\hat{y} = -38,48 + 2,82x$
6.3	*34 ice creams*
6.4	*Regression line will be pulled slightly upwards*
7.1	15
7.2	70
7.3	40-1,5x30=-5 not an outlier
7.4	
7.4.1	118,603
7.4.2	No, 180 min is outside of the data set, therefore extrapolation. Implies that anyone who studies for over 180 min will obtain 100%.
8.1	$35 \langle x \leq 45$
8.2	320 people

8.3	
8.4	Median = 41
8.5.1	A=7,b=30,c=12,d=47,e=3
8.5.2	
8.5.3	IQR=6,8
9.1	$2500 < x \leq 3000$
9.2	2262,5 grams
9.3	
9.3.1	
9.3.2	It will not deviate./it will remain the same.
9.3.3	2266,5 grams
10.1.1	Y=8
10.1.2	Median=6
10.2.1	Mean=7
10.2.2	Standard deviation=4
11.1	B
11.2	B
11.3	75%
11.4	Nothing. It remains the same. No change in standard deviation.
11.5	22,5
12.1	64 People