LIMPOPO

PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

PRE TRIAL 2024

MARKS : 150

DURATION : 3 hours

This question paper consists of 11 pages and formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions and formula sheet.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1. Solve for x if:

1.1.1 $(3x-2)^2 = 5$ (4)

1.1.2 $2.3^{2x} = 9$ (3)

1.1.3 $x^3 - 3x^2 - x + 3 = 0$ (4)

1.1.4 $2x^2 + 9x - 5 \leq 0$ (4)

1.2. Solve for x and y simultaneously if:

$y - x + 3 = 0$ and $x^2 - x = 6 + y$ (6)

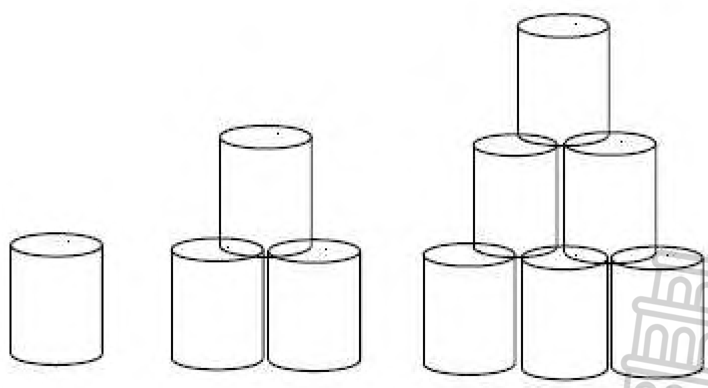
1.3. If m and n are rational numbers such that $\sqrt{m} + \sqrt{n} = \sqrt{7 + \sqrt{48}}$

Calculate a possible value of $m^2 + n^2$ (4)

[25]

QUESTION 2

A shopkeeper displays cans stacked in his window. He uses the following triangular-shaped pattern to do so:



2.1 How many cans are there in total in the next pattern? (1)

2.2 Determine the general term of the sequence. (4)

2.3 If the shopkeeper stacks the cans from the bottom-up, how many cans must he place in the bottom row of a triangular pile which has a total of 66 cans? (3)

[8]

QUESTION 3

3.1 Given: $35 + 32 + 29 + \dots\dots\dots + 5$

3.1.1 Determine the sum of the series. (5)

3.1.2 Write the series in sigma notation. (3)

3.2 Prove that the formula for the sum of a geometric series is given by:

$$S_n = \frac{a(1-r^n)}{1-r} \text{ for } r \neq 1$$ (6)

3.3 The first two terms of a geometric sequence are: $(\tan 45^\circ)$ and $(\sin 45^\circ)$.

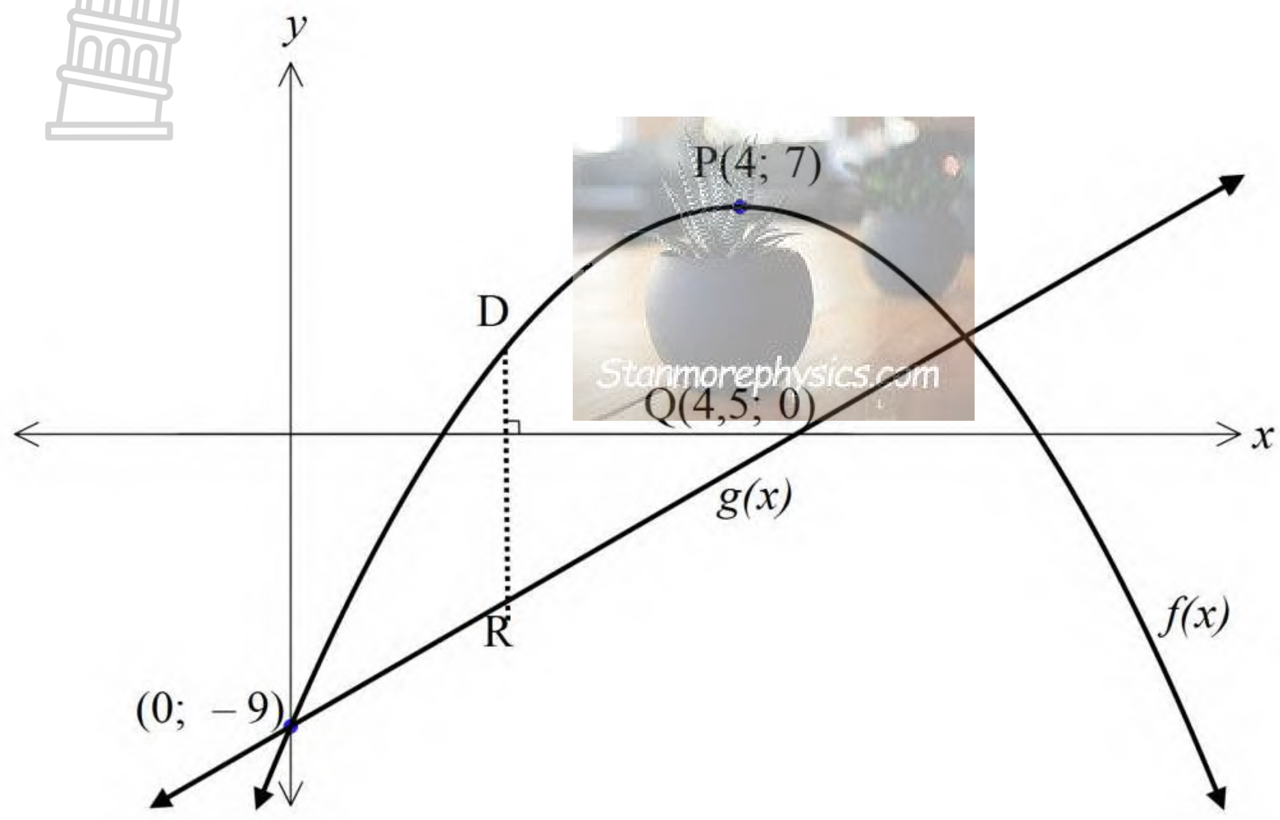
3.3.1 Determine the sum of the first eight terms of the sequence (leave your answer in simplified surd form). (5)

3.3.2 Is the sequence a converging sequence? Give a reason for your answer. (2)

[21]

QUESTION 4

The diagram shows the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = 2x - 9$. P is the turning point of the parabola. Both $f(x)$ and $g(x)$ pass through the point (0; – 9). $g(x)$ passes through Q(4,5; 0)



4.1 Write down the equation of the axis of symmetry of *f*. (1)

4.2 Write down the coordinates of the point which is a reflection of the point (0; – 9) in the axis of symmetry of *f*. (2)

4.3 Determine the values of *a*, *b* and *c*. (5)

4.4 Determine the length of DR in terms of *x* if D is on *f* and R is on *g* and DR is parallel to the line $x = 0$. (2)

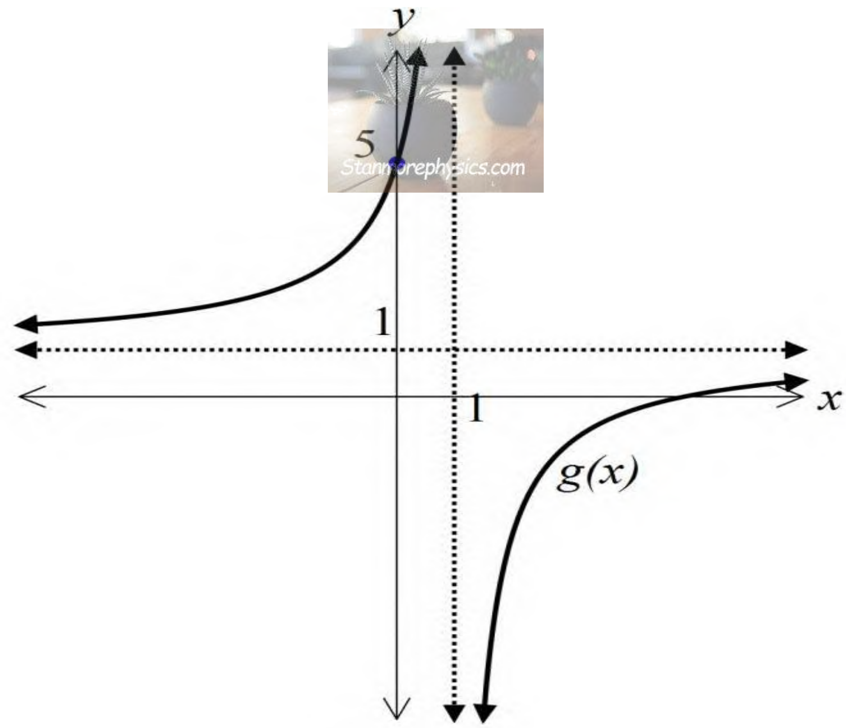
4.5 Determine the value(s) of *x* for which DR is a maximum. (2)

[12]

QUESTION 5

The diagram shows the hyperbola defined by $g(x) = \frac{-4}{x+r} + t$

The asymptotes of g cut both the x and y-axes at 1.



5.1 Write down the values of r and t. (2)

5.2 Write down the equation of the axis of symmetry with a negative gradient. (2)

5.3 Write down the equation of the vertical asymptote of $g(x+4)$. (2)

[6]

QUESTION 6

6.1 Given: $g(x) = \left(\frac{1}{2}\right)^x$

6.1.1 Write down the equation of $g^{-1}(x)$. (1)

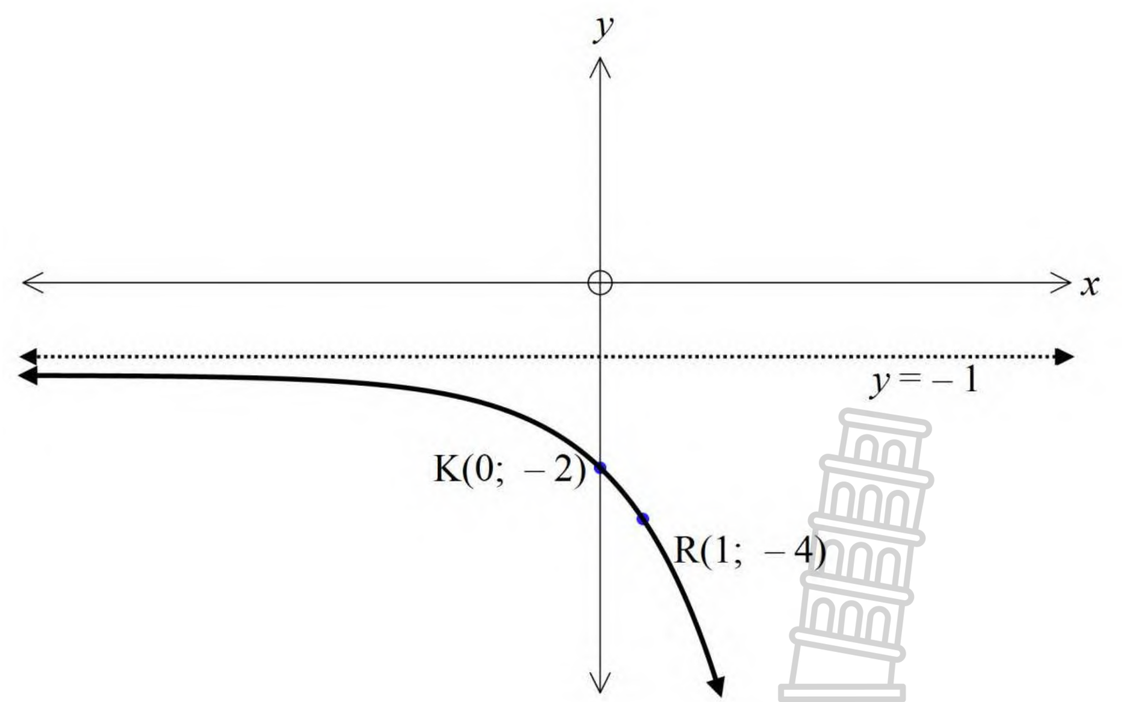
6.1.2 Using the axes on the diagram sheet, sketch the graph of $g^{-1}(x)$, showing at least two points, which must include any intercepts with the axes. Any asymptotes must also be clearly shown. (2)

6.1.3 If the point G (4; *a*) lies on $g^{-1}(x)$, determine the value of *a*. (2)

6.1.4 For which values of *x* is $g^{-1}(x) > 2$? (2)

6.1.5 Give the equation of *h*(*x*), the reflection of *g*(*x*) in the line *x* = 0. (1)

6.2 The graph of $f(x) = a.b^x + q$ is sketched below. Points $K(0;\ -2)$ and $R(1;\ -4)$ are on the curve.



Determine the value(s) of *a* ,*b* and *q*. (4)

[12]

QUESTION 7

7.1 Vladimir needed R500 urgently. A 'loan shark' agreed to give it to him for one month but he would have to repay R600.

7.1.1 Determine the monthly interest rate that the "Loan shark" is charging for this one month loan. (2)

7.1.2 If this monthly rate is compounded for 12 months, then determine the equivalent effective interest rate per annum. (3)

7.2 Hugo bought a property for R1 500 000. He took out a loan for the property at 9, 2% interest p.a., compounded monthly over 20 years. He begins repaying the loan in 1 months' time.

7.2.1 What will his monthly payments be? (5)

7.2.2 Hugo experiences financial difficulties and after 7 years he skips 7 consecutive payments.

(a) What is the balance outstanding on the loan after 7 years? (4)

(b) What will the balance on the loan be once he can begin making payments again? (2)

[16]

QUESTION 8

8.1 Given that $f(x) = 2x - x^2$, determine $f'(x)$ from first principles. (4)

7.2 Determine $\dfrac{dy}{dx}$ if $y = \dfrac{x^2}{2} + \dfrac{2}{x^2}$. (3)

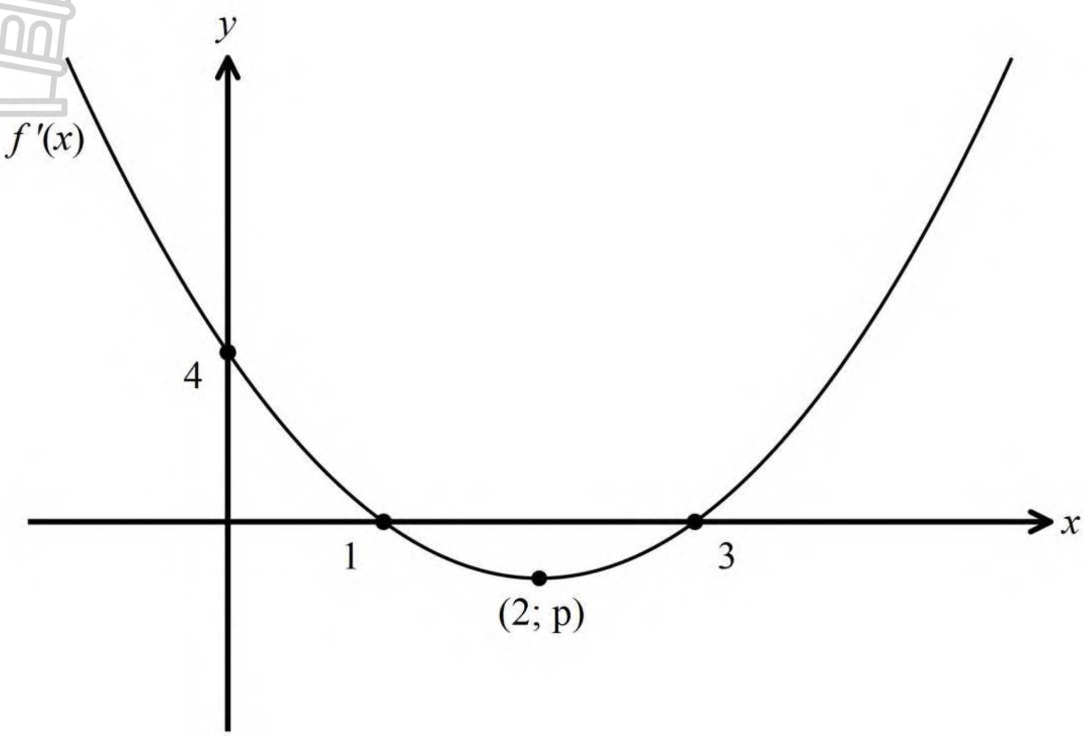
7.3 Determine $D_x\left[\dfrac{2x^3 + 4x}{\sqrt[5]{x}}\right]$ (4)

7.4 If $f(x) = ax^3 + bx^2 + cx - 5$ and the gradient at any point $(x; f(x))$ is given by $6x^2 - 24$, find the values of *a, b* and *c* (4)

[15]

QUESTION 9

The parabola in the figure below represents the curve of $f'(x)$. The parabola is the derivative of the cubic function $f(x) = ax^3 + bx^2 + cx + d$.



9.1 Write down the gradient of the tangent to $f(x)$ at the point where $x = 0$ (1)

9.2 Write down the x - coordinates of the turning points of the curve of $f(x)$ (2)

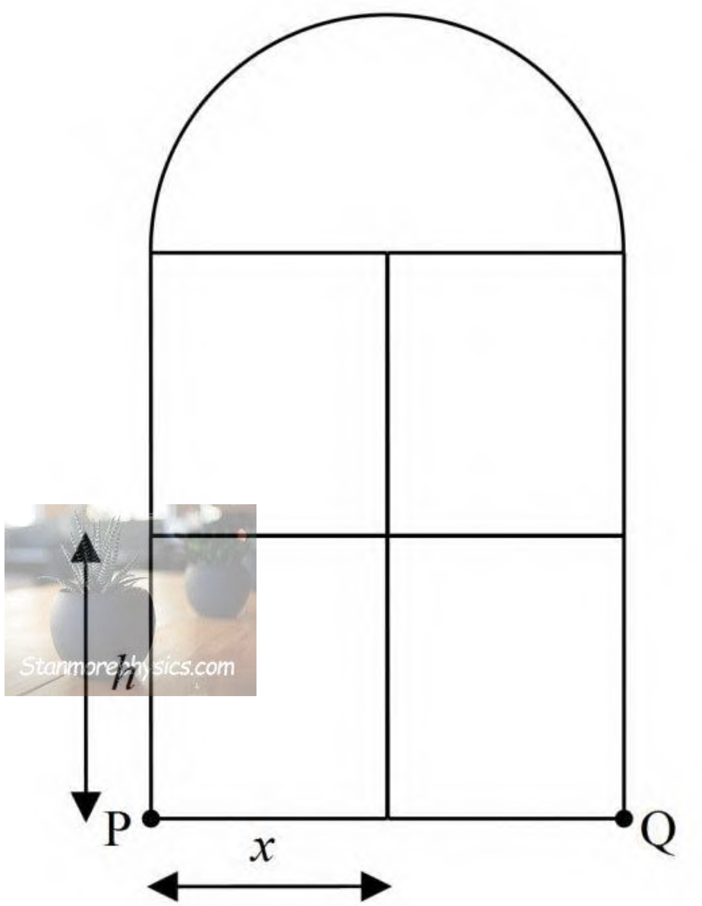
9.3 For what values of x is $f(x)$ strictly decreasing? (2)

9.4 Show that $x = \frac{-b}{3a}$ is the x - coordinate of the point of inflection of $f(x)$ (3)

[8]

QUESTION 10

A chapel window consists of four equal rectangles and a semi-circle. The length of the metal that is being used for the frame is 36 metres.



10.1 Prove that the area for the frame is given by:

$$A = 24x - 4x^2 - \frac{\pi x^2}{6}$$ (6)

10.2 Determine the length of the base PQ for a maximum area of the window. (5)

[11]

QUESTION 11

11.1 In a small town, 70% of the population received an anti-Ebola injection and 77 % of the town did not contract Ebola later that year 54 % of the people did get the injection and also did not develop Ebola.

	INJECTION	NO INJECTION	
NO EBOLA	54	**b**	77
EBOLA	**a**	7	d
	70	**c**	100

11.1 Complete the contingency table by writing down the values of **a to d**. (4)

11.2 Show calculations to determine whether receiving an anti-Ebola injection and not contracting Ebola are independent events. (4)

11.2 The letters that form the word **MATHEMATICS** are arranged as shown below on separate cards.

M	A	T	H	E	M	A	T	I	C	S

11.2.1 How many other "words" can be arranged using all these cards? (4)

11.2.2 What is the probability that a "word" made, has all the vowels above next to each other? (4)

[16]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\qquad A = P(1-ni)$ $\qquad A = P(1-i)^n$ $\qquad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\qquad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\qquad S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\qquad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\qquad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\qquad \partial^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$ $\qquad P(A) = \frac{n(A)}{n(S)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$