



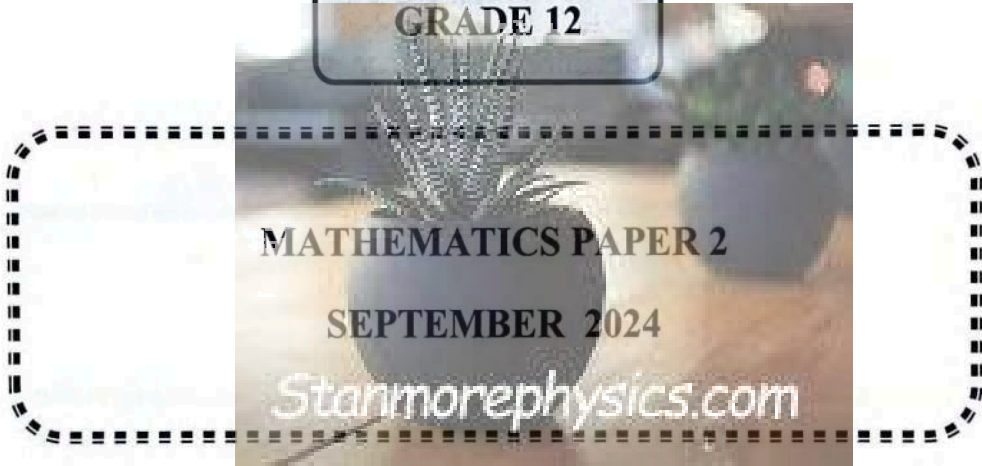
LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA



DEPARTMENT OF
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12



MATHEMATICS PAPER 2

SEPTEMBER 2024

Stanmorephysics.com

MARKS: 150

TIME: 3 HOURS



EMATHP2



This paper consists of 12 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Please read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the ANSWER BOOK.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you used to determine your answers.
4. ANSWERS ONLY will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphically) unless otherwise stated.
6. If necessary, round off answers to TWO decimal places unless otherwise noted.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this paper.
9. Write legibly and present your work neatly.



QUESTION 1

A statistician at a large corporation, with branches around the world, has been assigned to analyse the annual salaries (in \$) of 100 global customer service managers across all branches during 2023. The following data were collected:

Annual salary range (in \$)	Number of managers
$0 \leq x < 10\ 000$	3
$10\ 000 \leq x < 20\ 000$	5
$20\ 000 \leq x < 30\ 000$	12
$30\ 000 \leq x < 40\ 000$	19
$40\ 000 \leq x < 50\ 000$	20
$50\ 000 \leq x < 60\ 000$	14
$60\ 000 \leq x < 70\ 000$	12
$70\ 000 \leq x < 80\ 000$	7
$80\ 000 \leq x < 90\ 000$	5
$90\ 000 \leq x < 100\ 000$	3

- 1.1 Complete the cumulative frequency table in the ANSWER BOOK provided. (2)
 - 1.2 Draw the cumulative frequency graph (ogive) on the grid provided. (3)
 - 1.3 Use the graph to determine the interquartile range of the data. (3)
 - 1.4 The CEO of the company wants to motivate the managers whose salaries are less than \$40 000 and promise them a raise of 15% in their annual salary. The managers who earn more than \$40 000 will get a raise of 8%. What will the new mean annual salary be at the end of 2024? (4)
- [12]**

QUESTION 2

10 girls compete to see who can throw a ball the furthest. They want to see whether there is a relationship between their height and the distance which they can throw. The following table shows the height of each girl (x cm) to the nearest cm and the distance (y m) she can throw the ball.

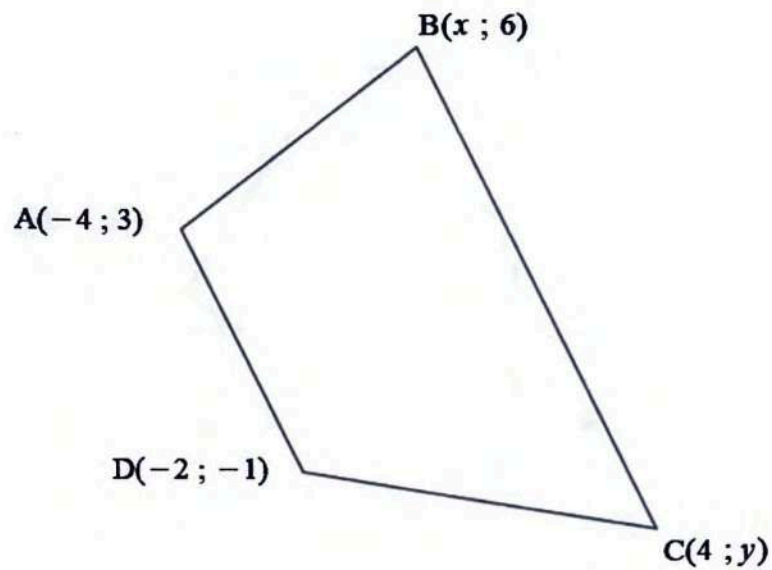
	A	B	C	D	E	F	G	H	I	J
x	122	124	133	138	144	156	158	161	164	168
y	41	38	56	56	29	54	59	61	68	67

- 2.1 Determine the equation for the least squares regression line. (3)
 - 2.2 Estimate the distance (to the nearest metre) which a ball can be thrown by a girl who is 150 cm tall. (2)
 - 2.3 Describe the trend of the data above. (1)
 - 2.4 Do you think this trend can continue indefinitely? Give a valid reason for your answer. (2)
- [8]**



QUESTION 3

The following diagram shows a trapezium ABCD with $A(-4 ; 3)$, $B(x ; 6)$, $C(4 ; y)$ and $D(-2 ; -1)$.

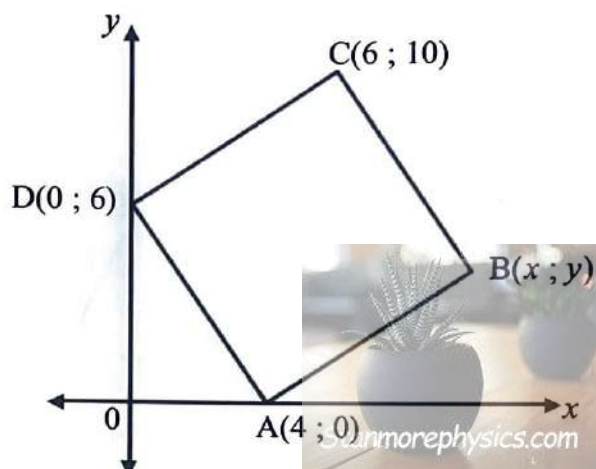


- 3.1 If $AD \parallel BC$, determine an equation in the form $y = \dots$ (4)
- 3.2 If $BC = 2AD$, determine another equation in terms of x and y . (4)
- 3.3 Determine the values of x and y . (6)
- 3.4 If $y < 0$, describe the translation of point C so that ABCD is a parallelogram. (2)

[16]

QUESTION 4

- 4.1 In the figure $A(4 ; 0)$, $B(x ; y)$, $C(6 ; 10)$ and $D(0 ; 6)$ are the vertices of a square. O is the origin.



- 4.1.1 Determine the coordinates of B. (2)
- 4.1.2 Determine the length of DB. (2)
- 4.1.3 Determine the equation of the circle passing through B, C and D in the form $(x - a)^2 + (y - b)^2 = r^2$. (6)
- 4.1.4 Determine the equation of the tangent to the circle at $D(0 ; 6)$ in the form $y = \dots$. (4)
- 4.1.5 E is the point intersection of the diagonals of ABCD. Show, by using analytical methods, that OE bisects $\hat{D}O\hat{A}$. (3)
- 4.2 $(x - 1)^2 + (y + 1)^2 = 2(x - y)$ is the equation of a circle. Determine the coordinates of :
- 4.2.1 the centre of the circle. (4)
- 4.2.2 the length of the radius of the circle. (1)



[22]

QUESTION 5

5.1 If $\cos 12^\circ = m$, determine the value of the following in terms of m , without using a calculator:



5.1.1 $\cos(-12^\circ)$ (2)

5.1.2 $\cos 72^\circ$ (3)

5.1.3 $\cos 6^\circ$ (4)

5.2 Show that: $\frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x-90^\circ)\cos(90^\circ-2x)}{\sin x} = \cos 2x$ (6)

5.3 If $\sin^2 B - \cos^2 B = 1$ and $B \in [0^\circ; 90^\circ]$, determine without using a calculator:

5.3.1 $\cos 2B$. (1)

5.3.2 the size of \hat{B} . (2)

5.3.3 hence the size of \hat{C} , when $\sin(B-C) - \cos(B-C) = 0$ and $-180^\circ \leq C \leq 90^\circ$. (6)

5.4 Consider $P = 2 \cos x - \cos 2x$.

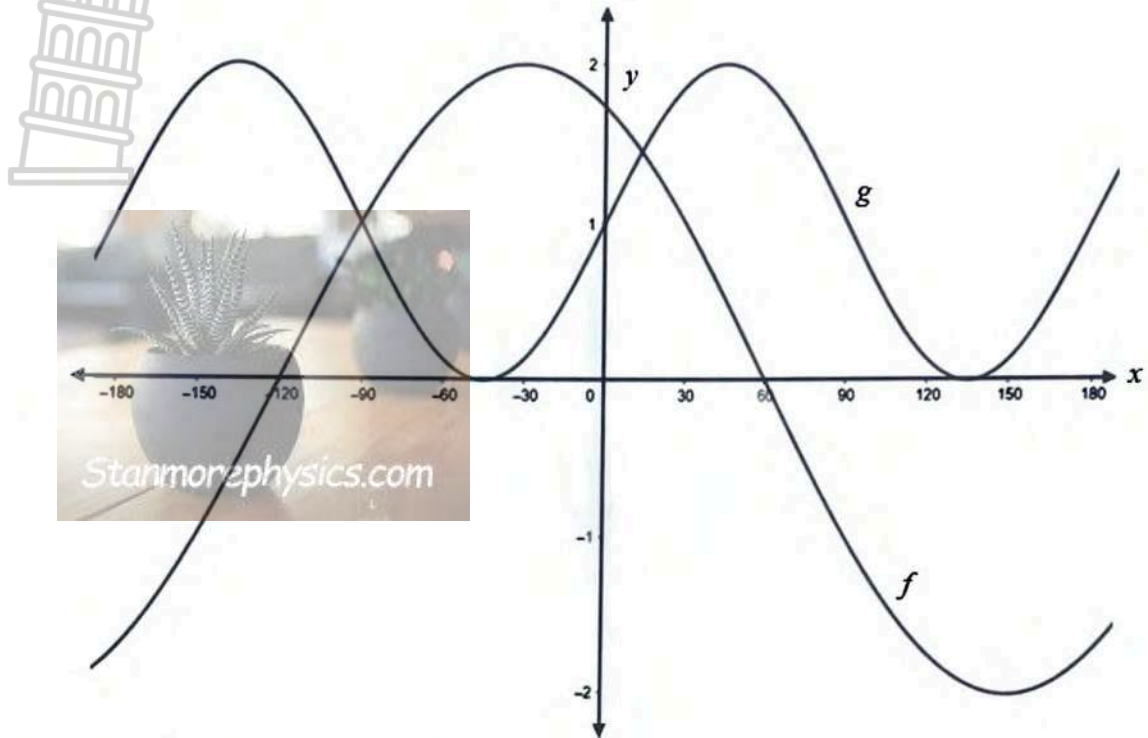
Use algebraic methods to determine the maximum value of P . (5)

[29]



QUESTION 6

In the diagram below the graphs of $f(x) = 2 \cos(x+a)$ and $g(x) = b + \sin 2x$, for $x \in [-180^\circ; 180^\circ]$ are drawn.

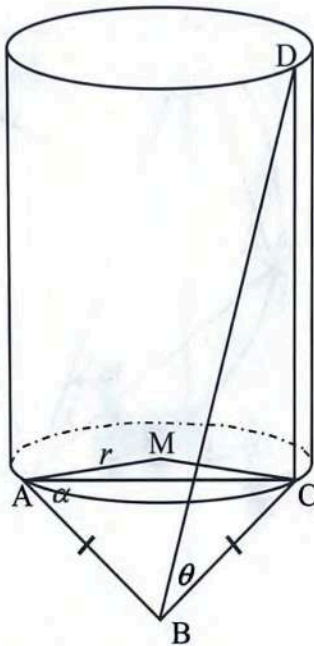
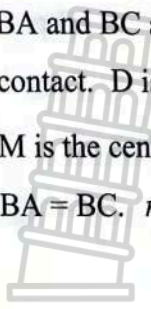


- 6.1 Determine the values of a and b . (2)
- 6.2 Determine the value of $f(x)$ if $x = 0^\circ$, without using a calculator. (3)
- 6.3 Determine the value(s) of x , by using the graphs if:
- 6.3.1 $g(x) = 2$ (2)
- 6.3.2 $f(x) \geq g(x)$; for $x \in [-180^\circ; 0^\circ]$ (2)
- 6.4 The y -axis is shifted to the left to pass through the maximum turning point of f . Determine the new equation of f in the form $y = \dots$. (2)

[11]

QUESTION 7

BA and BC are tangents drawn to the base of a right cylindrical silo with A and C the points of contact. D is a point h metres vertically above C and the angle of elevation of D from B is θ . M is the centre of the base and $\hat{BAC} = \alpha$. B, A, C and M are in the same horizontal plane with $BA = BC$. r is the radius of the circle.



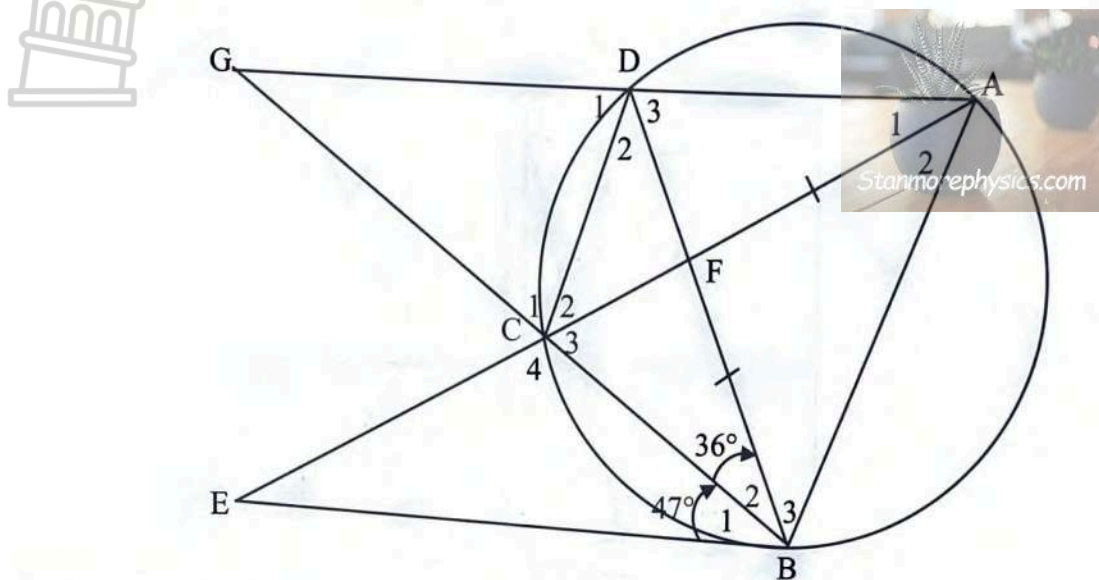
- 7.1 Write down the size of \hat{BCM} . (1)
- 7.2 Find \hat{MAC} and \hat{AMC} in terms of α . (3)
- 7.3 Hence, prove that: $AC = \frac{2h \cos \alpha}{\tan \theta}$ (5)
- 7.4 Use $\triangle AMC$ to prove that: $r = \frac{h}{\tan \alpha \tan \theta}$ (3)

[12]



QUESTION 8

In the diagram, ABCD is a cyclic quadrilateral. Chords AC and BD intersect at F. AD produced meets BC produced at G. EB is a tangent to the circle at B. AC produced cuts the tangent EB at E. $AF = BF$, $\hat{EBF} = 47^\circ$ and $\hat{GBD} = 36^\circ$.



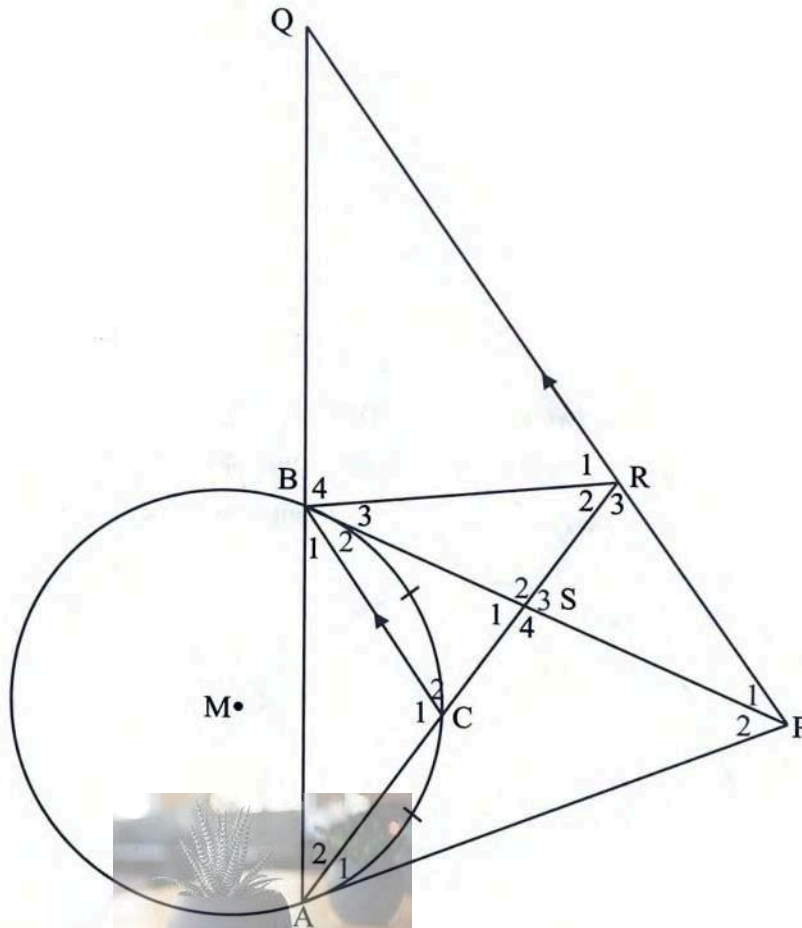
Determine with reasons:

- 8.1 \hat{A}_1 (2)
- 8.2 \hat{A}_2 (2)
- 8.3 \hat{C}_1 (2)
- 8.4 \hat{C}_4 (3)

[9]

QUESTION 9

In the diagram, PA and PB are tangents to the circle with centre M. C is the midpoint of arc ACB and $PQ \parallel CB$. It is also given that $\hat{PAC} = x$.

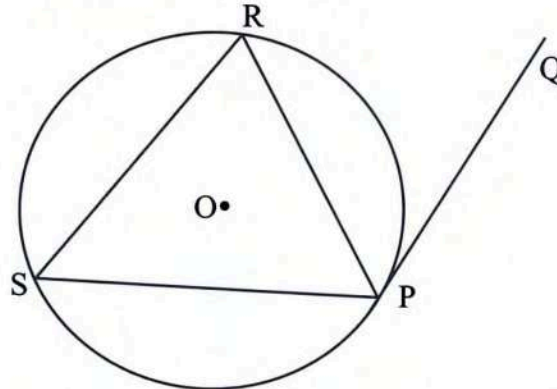


- 9.1 Write down, with reasons, 5 more angles each equal to x . (5)
- 9.2 Prove that ABRP is a cyclic quadrilateral. (2)
- 9.3 Prove that $AP = BQ$. (3)
- 9.4 Prove that PRQ is a tangent to the circle passing through BCR at R. (3)

[13]

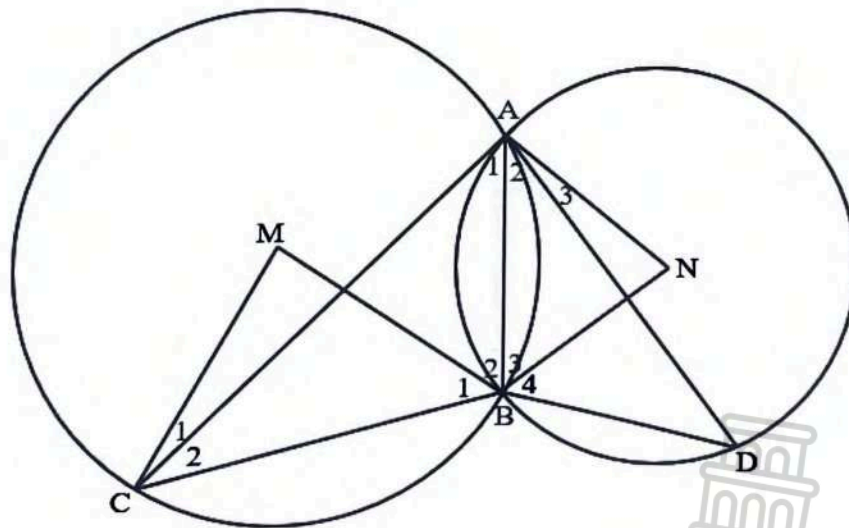
QUESTION 10

- 10.1 In the diagram PQ is a tangent to the circle with centre O. PR is a chord drawn from the point of contact and S is a point on the circumference. Proof the theorem stating that $\hat{QPR} = \hat{PSR}$.



(5)

- 10.2 In the diagram, two circles with centres M and N intersect at A and B. The radius of circle M is R, and the radius of circle N is r. AC is a tangent to circle N at A, and AD is a tangent to circle M at A.



Proof the following:

10.2.1 $\triangle ABC \parallel \triangle DBA$ (3)

10.2.2 $AB^2 = DB \cdot BC$ (2)

10.2.3 $\triangle CBM \parallel \triangle BAN$ (5)

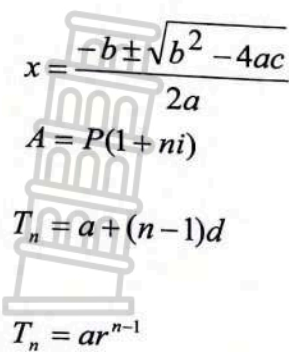
10.2.4 $R^2 : r^2 = BC : DB$ (3)

[18]

GRAND TOTAL:

150

INFORMATION SHEET: MATHEMATICS



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$T_n = a + (n-1)d$$

$$T_n = ar^{n-1}$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\hat{y} = a + bx$$

$$A = P(1 - ni)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$A = P(1 - i)^n$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$M \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$





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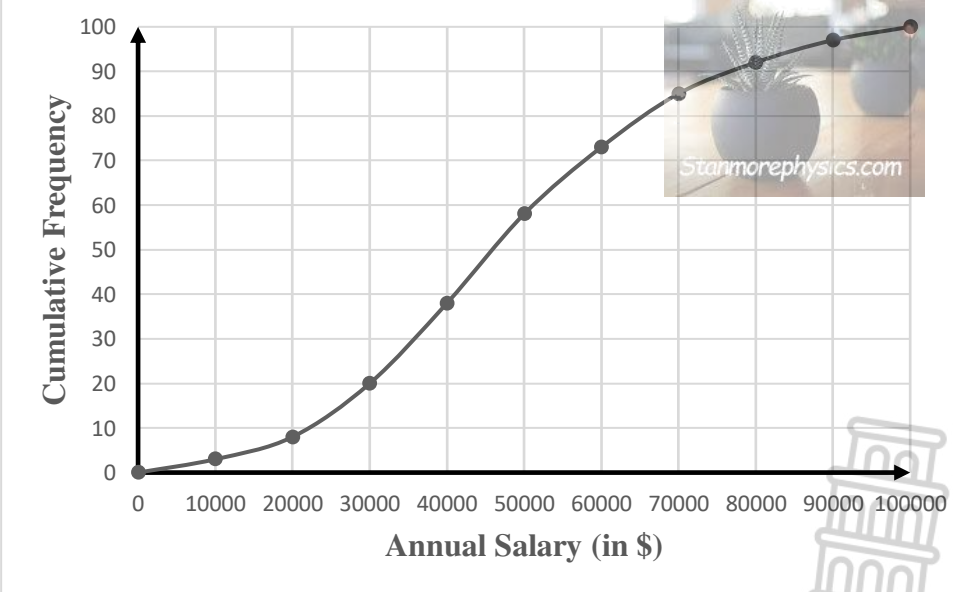
GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2
SEPTEMBER 2024
Stanmorephysics.com
MARKING GUIDELINES/NASIEENRIGLYNE

MARKS/PUNTE: 150

This marking guidelines consist of 15 pages/*Hierdie nasienriglyne bestaan uit 15 bladsye.*

QUESTION/VRAAG 1

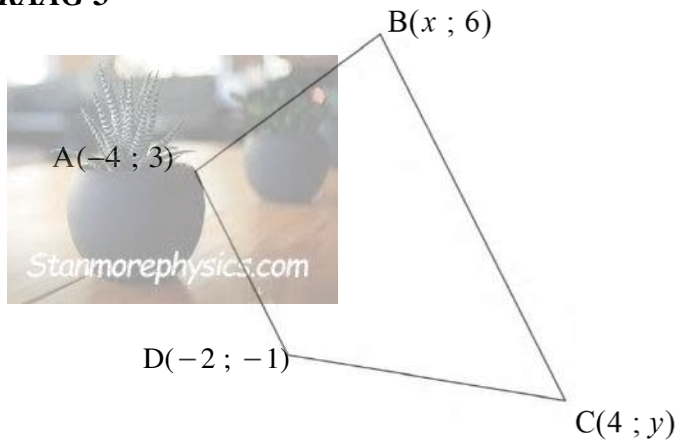
1.1	<table border="1"> <thead> <tr> <th>Annual salary range (in \$)/ <i>Jaarlikse salaris (in \$)</i></th> <th>Number of managers/ <i>Aantal bestuurders</i></th> <th>Cum Freq/ <i>Kum frekw</i></th> </tr> </thead> <tbody> <tr> <td>$0 \leq x < 10\ 000$</td> <td>3</td> <td>3</td> </tr> <tr> <td>$10\ 000 \leq x < 20\ 000$</td> <td>5</td> <td>8</td> </tr> <tr> <td>$20\ 000 \leq x < 30\ 000$</td> <td>12</td> <td>20</td> </tr> <tr> <td>$30\ 000 \leq x < 40\ 000$</td> <td>19</td> <td>39</td> </tr> <tr> <td>$40\ 000 \leq x < 50\ 000$</td> <td>20</td> <td>59</td> </tr> <tr> <td>$50\ 000 \leq x < 60\ 000$</td> <td>14</td> <td>73</td> </tr> <tr> <td>$60\ 000 \leq x < 70\ 000$</td> <td>12</td> <td>85</td> </tr> <tr> <td>$70\ 000 \leq x < 80\ 000$</td> <td>7</td> <td>92</td> </tr> <tr> <td>$80\ 000 \leq x < 90\ 000$</td> <td>5</td> <td>97</td> </tr> <tr> <td>$90\ 000 \leq x < 100\ 000$</td> <td>3</td> <td>100</td> </tr> </tbody> </table>	Annual salary range (in \$)/ <i>Jaarlikse salaris (in \$)</i>	Number of managers/ <i>Aantal bestuurders</i>	Cum Freq/ <i>Kum frekw</i>	$0 \leq x < 10\ 000$	3	3	$10\ 000 \leq x < 20\ 000$	5	8	$20\ 000 \leq x < 30\ 000$	12	20	$30\ 000 \leq x < 40\ 000$	19	39	$40\ 000 \leq x < 50\ 000$	20	59	$50\ 000 \leq x < 60\ 000$	14	73	$60\ 000 \leq x < 70\ 000$	12	85	$70\ 000 \leq x < 80\ 000$	7	92	$80\ 000 \leq x < 90\ 000$	5	97	$90\ 000 \leq x < 100\ 000$	3	100	<p>✓39</p> <p>✓100</p>	(2)
Annual salary range (in \$)/ <i>Jaarlikse salaris (in \$)</i>	Number of managers/ <i>Aantal bestuurders</i>	Cum Freq/ <i>Kum frekw</i>																																		
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$90\ 000 \leq x < 100\ 000$	3	100																																		
1.2	<p style="text-align: center;">Annual Salary 2023</p> 	<p>✓(0 ; 0) ✓shape/ vorm ✓8 other points/ ander punte</p>	(3)																																	
1.3	<p>$IQR/IKV = 61\ 000 - 33\ 000$ $= 28\ 000$</p>	<p>✓accept/ aanvaar 61000 - 63000 ✓accept/ aanvaar 32000 - 34000 ✓answ/ antw</p>	(3)																																	

1.4	Less/Minder \$40 000: $\bar{x} = \frac{1055000}{39} = 27051,28$ After 15% increase/na 15% verhoging: $\bar{x} = 31108,97$ More than/Meer as \$40 000: $\bar{x} = \frac{3685000}{61} = 60409,84$ After 8% increase/na 8% verhoging: $\bar{x} = 65242,62$ New/nuwe: $\bar{x} = \frac{31108,97 \times 39 + 65242,62 \times 61}{100} = \frac{5034799,92}{100} = 50348$	✓ 27051,28 ✓ 31108,97 ✓ 65242,62 ✓ 50348	(4)
			[12]

QUESTION/VRAAG 2

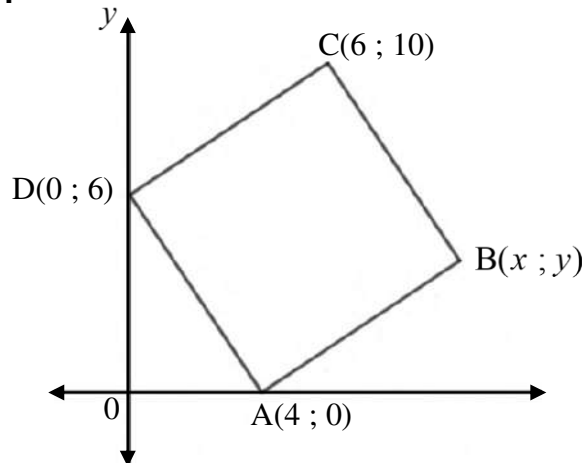
2.1	$a = -26,21740146... = -26,22$ $b = 0,5389468765... = 0,54$ $\hat{y} = -26,22 + 0,54x$	✓ -26,22 ✓ 0,53 ✓ equation/vergelyking	(3)
2.2	$\hat{y} = -26,22 + 0,54(150) = 54,78$ OR/OF $\hat{y} = 54,62$ with calculator/ met sakrekenaar gebruik word	✓ substitution/vervanging ✓ answer/Antwoord OR/OF ✓ ✓ answer/antwoord	(2) (2)
2.3	As the girls get taller, they throw the ball further/ <i>Namate die meisies langer word, gaan hulle die bal verder gooi</i>	✓ answer/antwoord	(1)
2.4	No, the length of the girls cannot get longer indefinitely/ <i>Nee, die lengte van die meisies kan nie onbepaald langer word nie</i>	✓ No/nee ✓ reason/rede	(2)
			[8]

QUESTION/VRAAG 3

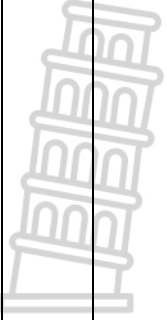


3.1	$m(AD) = m(BC)$ $\frac{3+1}{-4+2} = \frac{6-y}{x-4}$ $\frac{-2}{1} = \frac{6-y}{x-4}$ $-2x+8 = 6-y$ $y = 2x-2$	✓ $m(AD)$ ✓ $m(BC)$ ✓ equating/gelykstel ✓ equation/vergelyking	(4)
3.2	$BC = 2AD$ $\sqrt{(x-4)^2 + (6-y)^2} = 2\sqrt{(-4+2)^2 + (3+1)^2}$ $\sqrt{x^2 - 8x + 16 + 36 - 12y + y^2} = 2(2\sqrt{5})$ $x^2 - 8x + y^2 - 12y + 52 = 80$ $x^2 - 8x + y^2 - 12y = 28$	✓ subst in dist formula/ vervang in afst formule ✓ simplify/vereenvoudig ✓ 80 ✓ equation/vergelyking	(4)
3.3	$y = 2x - 2$① $x^2 - 8x + y^2 - 12y = 28$② Subst/vervang ① in ②: $x^2 - 8x + (2x - 2)^2 - 12(2x - 2) = 28$ $x^2 - 8x + 4x^2 - 8x + 4 - 24x + 24 - 28 = 0$ $5x^2 - 40x = 0$ $x^2 - 8x = 0$ $x(x - 8) = 0$ $x = 0$ or/of $x = 8$ $y = -2$ $y = 14$	✓ substitution/vervanging ✓ simplify/vereenvoudig ✓ std form/std vorm ✓ factors/faktore ✓ values/waardes x ✓ values/waardes y	(6)
3.4	x translate 2 to left/transleer 2 links y translate 4 up/transleer 4 op	✓ 2 left/links ✓ 4 up/op	(2)
			[16]

QUESTION/VRAAG 4

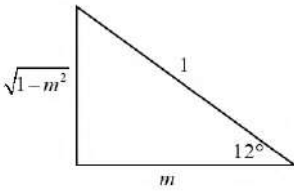


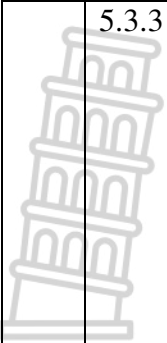
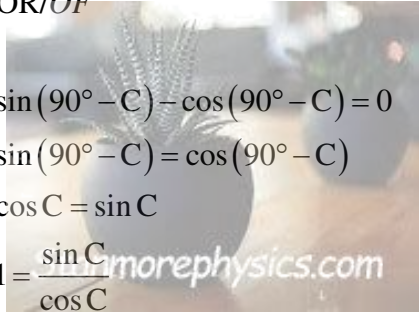
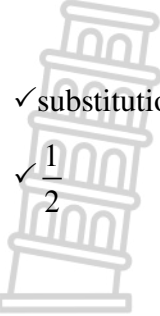
4.1	4.1.1	B(10 ; 4)	✓ x ✓ y	(2)
	4.1.2	$DB = \sqrt{(0-10)^2 + (6-4)^2} = 2\sqrt{26}$	✓ subst in dist formula/ vervang in afstand formule ✓ answer/antwoord	(2)
	4.1.3	$\hat{C} = 90^\circ \therefore DB$ is a diameter (line subtend $90^\circ \angle$) E(5 ; 5) Radius = $\frac{1}{2}(2\sqrt{26}) = \sqrt{26}$ $(x-5)^2 + (y-5)^2 = 26$	✓ state DB is diameter/ stel DB as midlyn ✓ x ✓ y (midpt DB) ✓ radius = $\sqrt{26}$ ✓ LH ✓ RH / ✓ LK ✓ RK	(6)
	4.1.4	$m(\text{radius}) = \frac{6-5}{0-5} = -\frac{1}{5}$ $m(\text{tangent}) = 5$ $y = mx + c$ $y = 5x + c$ $6 = 5(0) + c$ $c = 6$ $y = 5x + 6$	✓ m(radius) ✓ m(tangent/raaklyn) ✓ subst m and point D/ vervang m en punt D ✓ equation	(4)

		<p>OR/OF</p> $m(\text{radius}) = \frac{6-5}{0-5} = -\frac{1}{5}$ $m(\text{tangent}) = 5$ $y - y_1 = m(x - x_1)$ $y - 6 = 5(x - 0)$ $y - 6 = 5x$ $y = 5x + 6$	<p>✓ $m(\text{radius})$ ✓ $m(\text{tangent/raaklyn})$ ✓ subst m and point D/ vervang m en punt D</p> <p>✓ equation</p>	(4)
	4.1.5	$m(\text{OE}) = \frac{0-5}{0-5} = 1$ $\tan \hat{E\hat{O}A} = 1$ $\therefore \hat{E\hat{O}A} = 45^\circ \text{ and/en } \hat{D\hat{O}A} = 90^\circ$ $\therefore \text{EO bisect/halveer } \hat{D\hat{O}A}$	<p>✓ $m(\text{OE})$ ✓ tan def ✓ $\hat{E\hat{O}A} = 45^\circ$</p>	(3)
4.2	4.2.1	$(x-1)^2 + (y+1)^2 = 2(x-y)$ $x^2 - 2x + 1 + y^2 + 2y + 1 = 2x - 2y$ $x^2 - 4x + y^2 + 4y = -2$ $x^2 - 4x + 4 + y^2 + 4y + 4 = -2 + 4 + 4$ $(x-2)^2 + (y+2)^2 = 6$ $\therefore \text{Centre / midpunt } (2 ; -2)$	<p>✓ simplify/vereenvoudig</p> <p>✓ completing of square/ vierkantsvoltooiing</p> <p>✓ std form/std vorm</p> <p>✓ x and/en y</p>	(4)
	4.2.2	Radius = $\sqrt{6}$	✓ radius = $\sqrt{6}$	(1)
				[22]



QUESTION/VRAAG 5

5.1	5.1.1	$\cos(-12^\circ)$ $= \cos 12^\circ$ $= m$		✓ reduction/reduksie ✓ answer/antwoord	(2)
	5.1.2	$\cos 72^\circ$ $= \cos(60^\circ + 12^\circ)$ $= \cos 60^\circ \cos 12^\circ - \sin 60^\circ \sin 12^\circ$ $= \left(\frac{1}{2}\right)(m) - \left(\frac{\sqrt{3}}{2}\right)(\sqrt{1-m^2})$ $= \frac{m - \sqrt{3}(1-m^2)}{2}$		✓ $60^\circ + 12^\circ$ ✓ expansion/uitbrei ✓ substitution/vervangings	(3)
	5.1.3	$\cos 12^\circ = 2 \cos^2 6^\circ - 1$ $m = 2 \cos^2 6^\circ - 1$ $m + 1 = 2 \cos^2 6^\circ$ $\frac{m+1}{2} = \cos^2 6^\circ$ $\cos 6^\circ = \sqrt{\frac{m+1}{2}}$		✓ double \angle formula/ dubbel \angle formule ✓ equating to m/gelykstel m ✓ $\frac{m+1}{2}$ ✓ answer/antwoord	(4)
5.2		$\frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x-90^\circ)\cos(90^\circ-2x)}{\sin x} = \cos 2x$ LHS = $\frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x-90^\circ)\cos(90^\circ-2x)}{\sin x}$ $= \frac{-\sin 54^\circ}{\sin 54^\circ} - \frac{(-\cos x)(\sin 2x)}{\sin x}$ $= -1 - \frac{(-\cos x)(2 \sin x \cos x)}{\sin x}$ $= -1 + 2 \cos^2 x$ $= 2 \cos^2 x - 1$ $= \cos 2x$		✓ $-\sin 54^\circ$ ✓ $\sin 54^\circ$ ✓ $-\cos x$ ✓ $\sin 2x$ ✓ $2 \sin x \cos x$ ✓ $-1 + 2 \cos^2 x$	(6)
5.3	5.3.1	$\sin^2 B - \cos^2 B = 1$ $\cos^2 B - \sin^2 B = -1$ $\cos 2B = -1$		✓ -1	(1)
	5.3.2	$2B = 180^\circ - 0^\circ + k \cdot 360^\circ$ or $2B = 180^\circ + 0^\circ + k \cdot 360^\circ$ $B = 90^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ $B = 90^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ $\therefore \hat{B} = 90^\circ$		✓ both equations/beide vergelykings ✓ value/waarde \hat{B}	(2)

<p>5.3.3</p> 	$\sin(90^\circ - C) - \cos(90^\circ - C) = 0$ $\sin(90^\circ - C) = \cos(90^\circ - C)$ $\cos C = \sin C$ $1 = \frac{\sin C}{\cos C}$ $\tan C = 1$ $\hat{C} = 45^\circ + k.180, k \in \mathbb{Z}$ $\hat{C} = \{-135^\circ ; 45^\circ\}$ <p>OR/OF</p>  $\sin(90^\circ - C) - \cos(90^\circ - C) = 0$ $\sin(90^\circ - C) = \cos(90^\circ - C)$ $\cos C = \sin C$ $1 = \frac{\sin C}{\cos C}$ $\tan C = 1$ $\hat{C} = 45^\circ + k.360, k \in \mathbb{Z} \text{ or/of } \hat{C} = 180^\circ + 45^\circ + k.360$ $\hat{C} = 225^\circ + k.360, k \in \mathbb{Z}$ $\hat{C} = \{-135^\circ ; 45^\circ\}$	<p>✓ substitution/vervanging \hat{B}</p> <p>✓ co-functions/kofunksies</p> <p>✓ $\tan C = 1$</p> <p>✓ 1 equation with $k.180^\circ$ /1 vergelyking $k.180^\circ$</p> <p>✓ -135° ✓ 45°</p> <p>OR/OF</p> <p>✓ substitution/vervanging \hat{B}</p> <p>✓ co-functions/kofunksies</p> <p>✓ $\tan C = 1$</p> <p>✓ 2 equations with $k.360^\circ$ /2 vergelykings $k.360^\circ$</p> <p>✓ -135° ✓ 45°</p>	<p>(6)</p> <p>(6)</p>
<p>5.4</p>	$P = 2 \cos x - \cos 2x$ $= 2 \cos x - (2 \cos^2 x - 1)$ $= -2 \cos^2 x + 2 \cos x + 1$ $\cos x = -\frac{b}{2a}$ $= -\frac{2}{2(-2)}$ $= \frac{1}{2}$ $P = -2 \cos^2 x + 2 \cos x + 1$ $= -2 \left(\frac{1}{2}\right)^2 + 2 \left(\frac{1}{2}\right) + 1$ $= \frac{3}{2}$	<p>✓ double/dubbel \angle</p> <p>✓ std form/std vorm</p>  <p>✓ substitution/vervanging</p> <p>✓ $\frac{1}{2}$</p> <p>✓ answer/antwoord</p>	<p>(5)</p>
			<p>[29]</p>

QUESTION/VRAAG 6

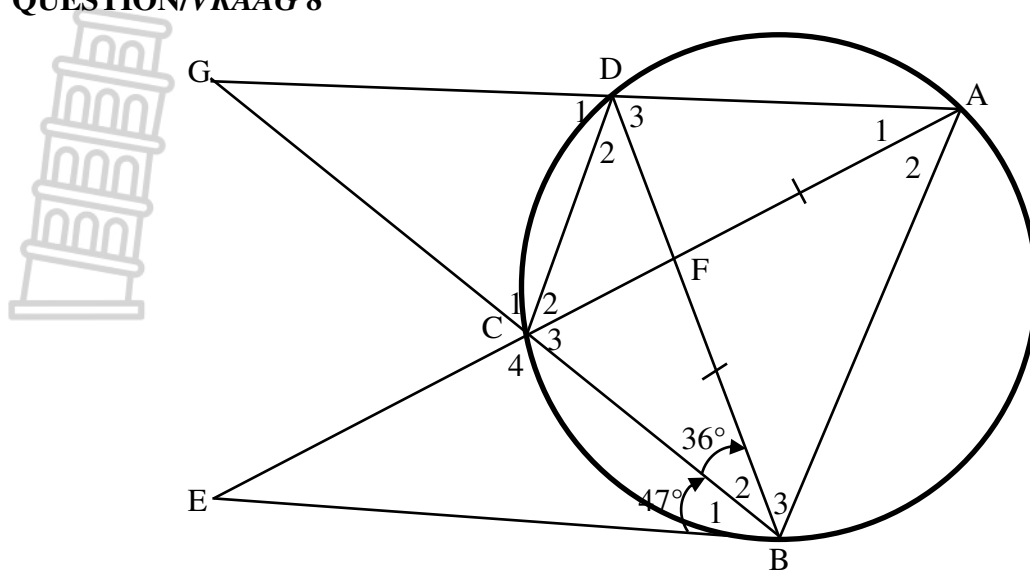
6.1	$a = 30^\circ$ $b = 1$	✓ value/waarde a ✓ value/waarde b	(2)
6.2	$f(x) = 2 \cos(0^\circ + 30^\circ)$ $f(x) = 2 \cos 30^\circ$ $f(x) = 2 \left(\frac{\sqrt{3}}{2} \right)$ $f(x) = \sqrt{3}$	✓ substitution/vervanging 0° ✓ substitution/vervanging ✓ answer/antwoord	(3)
6.3	6.3.1 $x = -135^\circ$ and/en $x = 45^\circ$	✓ -135° ✓ 45°	(2)
	6.3.2 $x \in [-90^\circ ; 0^\circ)$	✓ critical values/kritieke waardes ✓ notation/notasie	(2)
6.4	$f(x) = 2 \cos(x + 30^\circ - 30^\circ) = 2 \cos x$	✓ -30° ✓ answer/antwoord	(2)
			[11]



QUESTION/VRAAG 7

7.1	$\hat{B}\hat{C}M = 90^\circ$	$\checkmark 90^\circ$	(1)
7.2	$\hat{M}\hat{A}C = 90^\circ - \alpha$ $\hat{A}M\hat{C} = 180^\circ - (90^\circ - \alpha) - (90^\circ - \alpha)$ $\hat{A}M\hat{C} = 180^\circ - 180^\circ + 2\alpha$ $\hat{A}M\hat{C} = 2\alpha$	$\checkmark 90^\circ - \alpha$ \checkmark method/metode $\checkmark 2\alpha$	(3)
7.3	In $\triangle DBC$: $\tan \theta = \frac{h}{BC}$ $BC = \frac{h}{\tan \theta}$ In $\triangle ABC$: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$ $\frac{AC}{\sin(180^\circ - 2\alpha)} = \frac{\frac{h}{\tan \theta}}{\sin \alpha}$ $AC = \frac{h}{\tan \theta} \times \frac{\sin 2\alpha}{1} \times \frac{1}{\sin \alpha}$ $AC = \frac{h \cdot 2 \sin \alpha \cos \alpha}{\tan \theta \sin \alpha}$ $AC = \frac{2h \cos \alpha}{\tan \theta}$	\checkmark tan def $\checkmark \frac{h}{\tan \theta}$ \checkmark substitute in sine rule/ vervang in sinreël $\checkmark \sin 2\alpha$ $\checkmark 2 \sin \alpha \cos \alpha$	(5)
7.4	In $\triangle AMC$: $\frac{AM}{\sin C} = \frac{AC}{\sin M}$ $\frac{r}{\sin(90^\circ - \alpha)} = \frac{\frac{2h \cos \alpha}{\tan \theta}}{\sin 2\alpha}$ $\frac{r}{\cos \alpha} = \frac{2h \cos \alpha}{\tan \theta} \times \frac{1}{2 \sin \alpha \cos \alpha}$ $r = \frac{2h \cos \alpha}{\tan \theta} \times \frac{1}{2 \sin \alpha \cos \alpha} \times \frac{\cos \alpha}{1}$ $r = \frac{h \cos \alpha}{\tan \theta \sin \alpha}$ $r = \frac{h}{\tan \theta \tan \alpha}$	\checkmark subst is sine rule/ vervang in sinreël $\checkmark \cos \alpha$ \checkmark simplify/vereenvoudig	(3)
			[12]

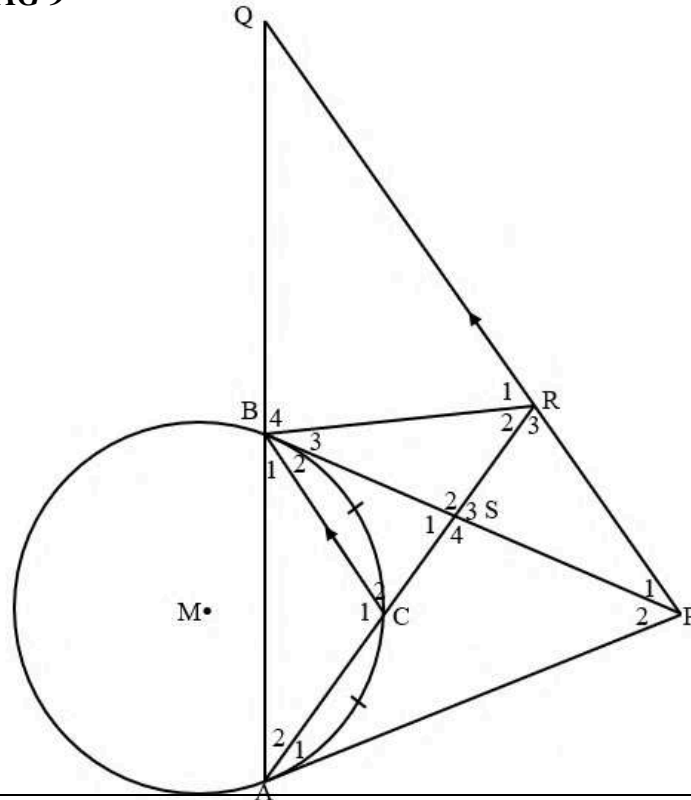
QUESTION/VRAAG 8



8.1	$\hat{B}_2 = \hat{A}_1 = 36^\circ$	\angle 's in the same segment / \angle 'e in dies segment	✓S ✓R	(2)
8.2	$\hat{B}_1 = \hat{A}_2 = 47^\circ$	tan-chord theorem/ <i>rklyn krd stelling</i>	✓S ✓R	(2)
8.3	$\hat{C}_1 = \hat{A}_1 + \hat{A}_2 = 83^\circ$	ext \angle of cyclic quad/ <i>buite \angle kvh</i>	✓S ✓R	(2)
8.4	$\hat{C}_2 = \hat{B}_3 = 47^\circ$	\angle 's in the same segment / \angle 'e in dies segment	✓S	(3)
	$\hat{C}_1 + \hat{C}_2 = \hat{C}_4$	vert. opp \angle 's / <i>regoorst \angle'e</i>	✓S/R	
	$\hat{C}_4 = 130^\circ$		✓S	
				[9]

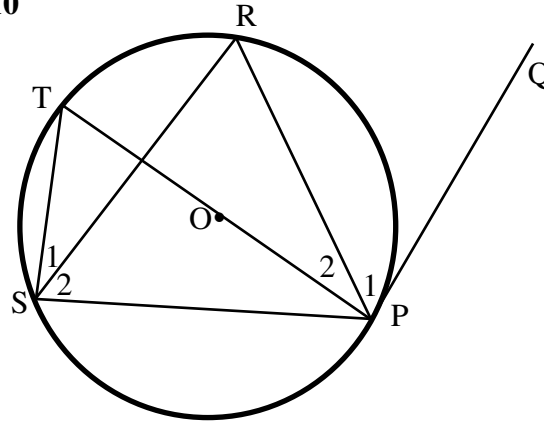


QUESTION/VRAAG 9



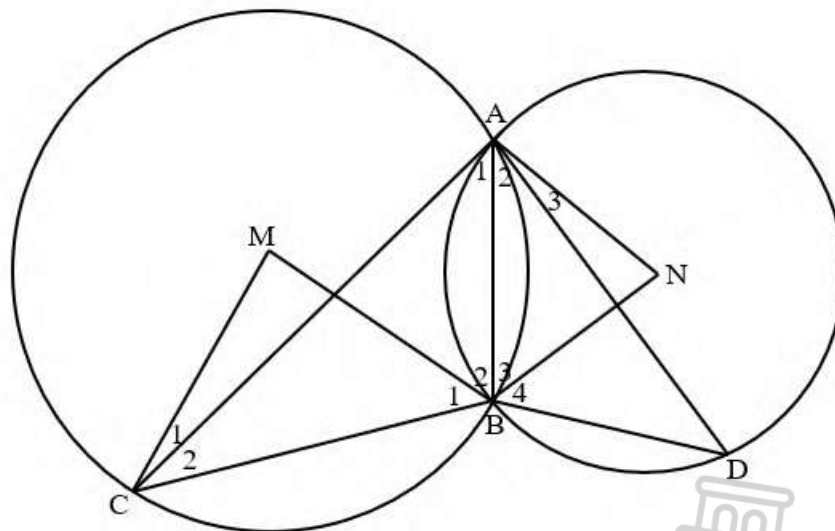
9.1	$\hat{A}_1 = \hat{B}_1 = x$ tan-chord theorem/rklyn krd stelling $\hat{B}_1 = \hat{A}_2 = x$ equal arcs/chords, equal \angle 's /gelyke boë/koorde gelyke \angle 'e $\hat{A}_2 = \hat{B}_2 = x$ tan-chord theorem/rklyn krd stelling $\hat{B}_1 = \hat{Q} = x$ corresp \angle 's /looreenk \angle 'e , BC QP $\hat{B}_2 = \hat{P}_1 = x$ alt \angle 's / verw \angle 'e , BC QP	✓S/R ✓S/R ✓S/R ✓S/R ✓S/R	(5)
9.2	$\hat{A}_2 = \hat{P}_1 = x$ ABRP is a cyclic quadrilateral/'n kvh line subtend equal \angle 's //lyn onderspan gelyke \angle 'e OR/OF converse \angle 's in same segment /omgekeerde \angle 'e in dies segment	✓S ✓R	(2)
9.3	$\hat{A}_1 + \hat{A}_2 = \hat{B}_1 + \hat{B}_2 = 2x$ BP = AP sides opp equal \angle 's / sye teenoor gelyke \angle 'e $\hat{Q} = \hat{P}_1 = x$ BQ = BP sides opp equal \angle 's /sye teenoor gelyke \angle 'e $\therefore AP = BQ$ both equal to BP/albei gelyk aan BP	✓S ✓R ✓S	(3)
9.4	$\hat{R}_1 = \hat{A}_1 + \hat{A}_2 = 2x$ ext \angle of cyclic quad/ buite \angle kvh $\hat{C}_2 = \hat{B}_1 + \hat{A}_2 = 2x$ ext \angle of Δ / buite \angle Δ $\therefore \hat{R}_1 = \hat{C}_2$ PRQ is a tangent to the circle/ converse tan-chord theorem/ raaklyn aan die sirkel omgekeerde rklyn krd stelling	✓S/R ✓S/R ✓R	(3)
			[13]

QUESTION/VRAAG 10



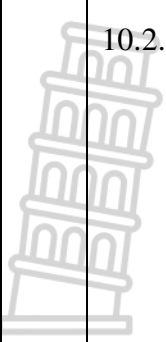
10.1	Draw diameter POT and join TS/ <i>trek midlyn POT en verbind TS</i> $\hat{P}_1 + \hat{P}_2 = 90^\circ$ radius \perp tangen/ <i>raaklyn</i> $\hat{S}_1 + \hat{S}_2 = 90^\circ$ \angle in semi circle/ <i>sirkel</i> $\hat{S}_1 = \hat{P}_2$ \angle 's in same segment/ <i>dieselde segment</i> $\therefore \hat{P}_1 = \hat{S}_2$	✓ constr/ <i>konstr</i> ✓S ✓R ✓S/R ✓S/R	(5)
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10.2



10.2.1	In $\triangle ABC$ and/en $\triangle DBA$: (i) $\hat{A}_1 = \hat{D}$ tan-chord theorem/ <i>rklyn koord stelling</i> (ii) $\hat{C}_2 = \hat{A}_2$ tan-chord theorem/ <i>rklyn koord stelling</i> $\therefore \triangle ABC \parallel \triangle DBA$ 3 \angle 's OR/OF In $\triangle ABC$ and/en $\triangle DBA$: (i) $\hat{A}_1 = \hat{D}$ tan-chord theorem/ <i>rklyn koord stelling</i> (ii) $\hat{C}_2 = \hat{A}_2$ tan-chord theorem/ <i>rklyn koord stelling</i> (iii) $\hat{CBA} = \hat{ABD}$ 3 rd \angle $\therefore \triangle ABC \parallel \triangle DBA$	✓S/R ✓S ✓R ✓S/R ✓S ✓S	(3)
			(3)

10.2.2	$\frac{AB}{DB} = \frac{BC}{AB} \quad \Delta's$ $\therefore AB^2 = DB \cdot BC$	✓S ✓R	(2)
10.2.3	<p>In $\triangle CBM$ and/en $\triangle BAN$:</p> <p>(i) $\hat{M} = 2\hat{A}_1$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\hat{A}_1 = \hat{D}$ proven/bewys $\therefore 2\hat{A}_1 = 2\hat{D}$ $\hat{N} = 2\hat{D}$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\therefore \hat{M} = \hat{N}$</p> <p>(ii) $2\hat{B}_1 = 180^\circ - \hat{M}$ int $\angle's$ of Δ / binne $\angle'e \Delta$ $\therefore \hat{B}_1 = 90^\circ - \frac{\hat{M}}{2}$ $2\hat{B}_3 = 180^\circ - \hat{N}$ int $\angle's$ of Δ / binne $\angle'e \Delta$ $\therefore \hat{B}_3 = 90^\circ - \frac{\hat{N}}{2}$</p> <p>But/maar $\hat{M} = \hat{N}$ proven/bewys $\therefore \hat{B}_1 = \hat{B}_3$ $\therefore \triangle CBM \triangle BAN$ 3 $\angle's$</p> <p>OR/OF</p> <p>In $\triangle CBM$ and/en $\triangle BAN$:</p> <p>(i) $\hat{M} = 2\hat{A}_1$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\hat{A}_1 = \hat{D}$ proven/bewys $\therefore 2\hat{A}_1 = 2\hat{D}$ $\hat{N} = 2\hat{D}$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\therefore \hat{M} = \hat{N}$</p> <p>(ii) $2\hat{B}_1 = 180^\circ - \hat{M}$ int $\angle's$ of Δ / binne $\angle'e \Delta$ $\therefore \hat{B}_1 = 90^\circ - \frac{\hat{M}}{2}$ $2\hat{B}_3 = 180^\circ - \hat{N}$ int $\angle's$ of Δ / binne $\angle'e \Delta$ $\therefore \hat{B}_3 = 90^\circ - \frac{\hat{N}}{2}$</p> <p>But/maar $\hat{M} = \hat{N}$ proven/bewys $\therefore \hat{B}_1 = \hat{B}_3$</p> <p>(iii) $\hat{M}\hat{C}\hat{B} = \hat{B}\hat{A}\hat{N}$ 3rd \angle $\therefore \triangle CBM \triangle BAN$</p>	<p>✓S/R</p> <p>✓S</p> <p>✓S/R</p> <p>✓S</p> <p>✓R</p> <p>✓S/R</p> <p>✓S</p> <p>✓S</p> <p>✓S</p>	<p>(5)</p> <p>(5)</p>

	10.2.4	$\frac{CB}{BA} = \frac{BM}{AN} \quad \Delta's$ $\frac{CB}{BA} = \frac{R}{r}$ $\frac{CB^2}{BA^2} = \frac{R^2}{r^2}$ and $AB^2 = DB \cdot BC$ <i>proven/bewys</i> $\frac{CB^2}{DB \cdot BC} = \frac{R^2}{r^2}$ $\frac{CB}{DB} = \frac{R^2}{r^2}$	✓S/R ✓squaring/vierkant ✓substitute/vervang $AB^2 = DB \cdot BC$	(3) [18]
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TOTAL/TOTAAL : 150

