education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

FURTHER EDUCATION AND TRAINING

GRADE 12

MATHEMATICS

PRE TRIAL EXAMINATION

AUGUST 2024

MARKS: 150

TIME: 3 HOURS

This paper consists of 10 pages including a formula sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 10 questions. Answer **ALL** the questions.
2. Clearly show **ALL** the calculations, reasoning, diagrams, graphs, etc., that you have used in determining your answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
5. Number the answers **EXACTLY** as the questions are numbered.
6. Diagrams are not drawn to scale.
7. It is in your own interest to write legibly and to present the work neatly.

1.1 Solve for x, correct to TWO decimal places, where necessary:

1.1.1 $(x+3)(2-x)=0$ (2)

1.1.2 $2x^2+3x-7=0$ (4)

1.1.3 $2^x-8=2.2^{\frac{x}{2}}$ (4)

1.1.4 $7x^2+18x-9>0$ (4)

1.2 Solve for x and y if

Given: $4y-x=4$ and $xy=8$

1.2.1 Solve for x and y simultaneously. (6)

1.2.2 Write down both lines of symmetry of the graph $xy=8$. (2)

1.3 The solutions of a quadratic equation are given by $x=\frac{-2\pm\sqrt{36-4k}}{2k}$ for which value(s) of k will this equation:

1.3.1 Have Non-real roots (2)

1.3.2 Be undefined (1)

[25]

QUESTION 2

2.1 The first term of a quadratic sequence, $Tn = n^2 + an + b$, is 9 and the term of the first difference is 11.

2.1.1 Determine the a and b , hence the general term. (3)

2.1.2 What is the value of the first term of the sequence that is greater than 240 (4)

2.2 Given the arithmetic sequence: 3; -1; -5;…-85; -89

2.2.1 Calculate the number of terms in the sequence. (3)

2.2.2 Calculate the sum of all negative terms in this sequence (3)

2.2.3 Consider the sequence: 3; -1; -5…-85; -89…; -389 (4)

Determine the number of terms in this sequence that will be exactly divisible by 3.

[17]

QUESTION 3

The first 3 terms of a geometric series are 3 +2 + $\frac{4}{3}$ + $\frac{8}{9}$ +…

3.1 Explain why the series converges (1)

3.2 Calculate the sum to infinity of the series (2)

3.2 Express $s_\infty - s_n$ in the form ab^n (5)

[8]

QUESTION 4

Consider the function $f(x) = \frac{-2x+5}{x-1}$

4.1 Show, using necessary calculations, that f can be written in the form (4)

$f(x) = \frac{a}{x+p} + q$

4.2 write down the equations of the asymptotes of f, If $f(x) = \frac{3}{x-1} - 2$. (2)

4.3 Calculate the intercepts of the graph of f with the axes. (3)

4.4 Sketch the graph of f. (3)

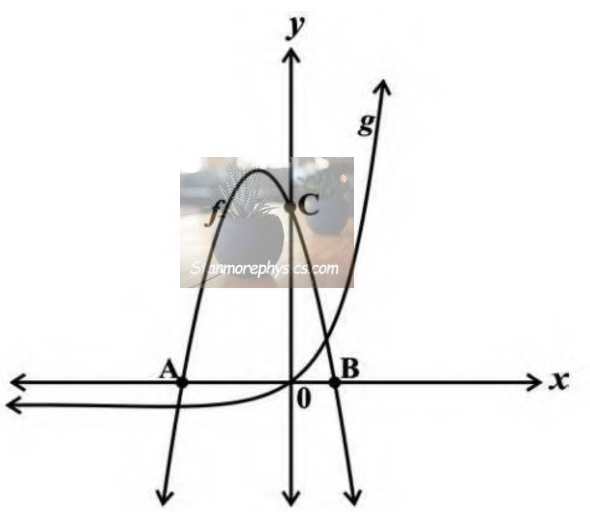
4.5 Write down the range of $y = -f(x)$. (1)

4.6 Describe, in words, the transformation of f to g if $g(x) = \frac{-3}{x+1} - 2$ (2)

[15]

QUESTION 5

Sketched below is the graph of the functions $f(x) = ax^2 + bx + c$ and $g(x) = d^x + q$. A(-3;0) and B(1;0) are the x-intercepts, and C(0;6) is the y-intercepts of f. The graph of g passes through the origin and point (1;2)



5.1 Use the graphs to determine the values of x where $f'(x).g(x)<0$ (4)

5.2 Determine d and q. (4)

5.3 Determine g^{-1}, the inverse of g, in the form $y = \ldots$ (3)

5.4 State the domain of g^{-1}. (2)

5.5 Determine a, b and c. (4)

5.6 Draw the graph of f^{-1}, the inverse of f. Show the turning point and the intercepts with the axes. (4)

5.7 Determine the values of k for which $f(x) + k = g(x)$ has two roots that are opposite in sign. (2)

[23]

QUESTION 6

6.1 Alec invests a lump sum of R5000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a, compounded quarterly.

6.1.1 What is the quarterly interest rate for Alec's investment? (1)

6.1.2 Calculate the amount in Alec's savings account at the end of the 2 years. (3)

6.2. A school issued new laptops to each of its 110 employees at the beginning of the year. The school was advised to set up a sinking fund to ensure that there would be enough money to replace them at the end of the 5th year.

The following applies:

- They paid R6000.00 for each laptop.
- The laptops depreciate at 15% p.a on reducing balance basis.
- Inflation is estimated to be 6% p.a over the 5-year period.
- A sinking fund is set up such that all payments will receive 12% interest p.a compounded monthly.

6.2.1 Determine the amount of money required at the end of 5 years to replace the laptops (5)

6.2.2 Determine the monthly payments that should be made to into the sinking fund to ensure that all 110 laptops can be replaced at the end of 5-years. (4)

[13]

QUESTION 7

7.1 Differentiate f from first principle if $f(x) = 3 - 2x^2$ (5)

7.2 Evaluate:

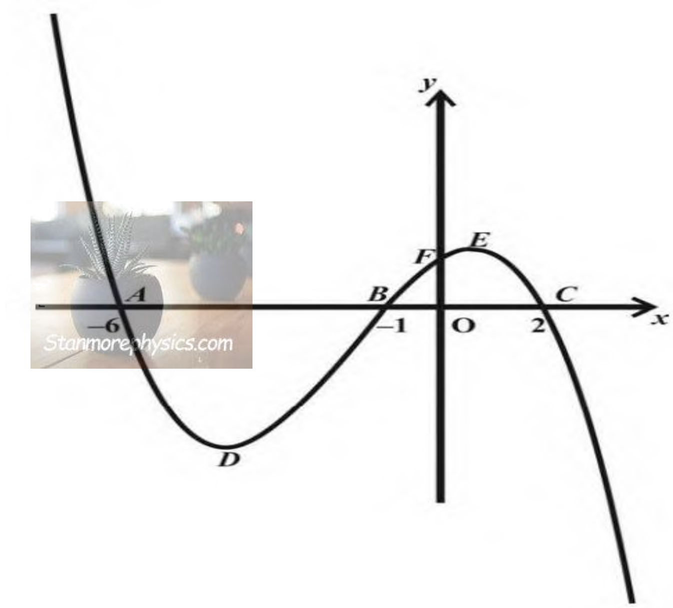
7.2.1 $\frac{dy}{dx}$ if $y = -\frac{1}{x} + \sqrt{x}$ (3)

7.2.2 $D_x\left[\frac{8 - 3x^6}{8x^5}\right]$ (3)

[11]

QUESTION 8

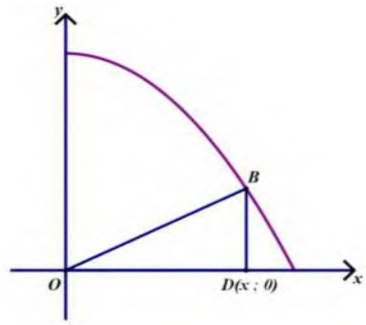
Sketched below is a graph of a cubic function $f(x) = ax^3 + bx^2 + cx + d$. A(-6; 0), B(-1; 0), C(2; 0) and F(0; 24) are intercepts with the axes, with D and E as turning points.



8.1	Show that $a = -2$, $b = -10$, $c = 16$ and $c = 24$	(5)
8.2	Determine the coordinates of D	(4)
8.3	Write down the value of p and q if the graph is translated in such a way that the point D is moved to the origin. That is, the new graph has equation $y = f(x - p) + q$, where p and q are constants.	(2)
8.4	For which values which values of x will $f''(x).f(x) > 0$	(3)
8.5	For which values k will $f(x) + k = 0$ have two negative and one positive roots.	(3)
		[17]

QUESTION 9

Refer to the figure showing the parabola given by $f(x) = 4 - \frac{x^2}{4}$ with $0 \leq x \leq 4$. D is the point $(x; 0)$ and DB is parallel to the $y - axis$, with B on the graph of f.



9.1 Write down the coordinates of B in terms of x. (2)

9.2 Show that the area, A, of ΔOBD is given by: A=$2x - \frac{x^2}{8}$. (3)

9.3 Determine how far D should be from O in order that the area of ΔOBD is as large possible. (4)

[9]

QUESTION 10

In a survey 1530 skydivers were asked if they had broken a limb. The results of the survey were as follows:

	Broken a limb	Not broken a limb	Total
Male	463	b	782
Female	a	c	d
TOTAL	913	617	1 530

10.1.1 Calculate the values of a, b, c and d. (4)

10.1.2 Calculate the probability of choosing at random in the survey, a female skydiver who has not broken a limb. (2)

10.1.3. Determine whether the events being female and having broken a limb are independent. (3)

10.2 Given the word HAMMERHEAD.

10.2.1 How many unique ten-letter words can be formed using the letters in the word above. (3)

10.2.2 Find the probability of ten-letter words formed such that the letters R and D are next to each other. (3)

[15]

TOTAL:150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad A = P(1 + in) \qquad A = P(1 - in)$$

$$A = P(1+i)^n \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1};\ r \neq 1 \qquad S_\infty = \frac{a}{1 - r};\ -1 < r < 1 \qquad F = \frac{x\left[(1+i)^n - 1\right]}{i}$$

$$P = \frac{x\left[1 - (1+i)^{-n}\right]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2};\ \frac{y_1 + y_2}{2}\right) \qquad y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \vartheta \qquad (x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2ab.\cos A$$

$$\text{Area of } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2\sin^2 A \\ 2\cos^2 A - 1 \end{cases}$$

$$\sin 2A = 2\sin A.\cos A \qquad P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\bar{x} = \frac{\sum x}{n} \qquad \hat{y} = a + bx$$