Downloaded from Stanmorephysics.com

education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

SEPTEMBER 2024

Stanmorephysics.com

MARKS: 150

TIME: 3 HOURS

This question paper consists of 13 pages and 1 information sheet and an answer book is provided.

Downloaded from Stanmorephysics.com

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

Downloaded from Stanmorephysics.com

QUESTION 1

A Mathematics teacher wants to create a model by which she can predict a grade 12 learner's final marks. She decided to use her 2015 results to create the model.

Preparatory exam(x)	55	35	67	85	91	48	78	72	15	75	69	37
Final exam(y)	57	50	74	80	92	50	80	81	23	80	75	42

1.1 Determine the equation of the least squares regression line in the form $y = a + bx$. (3)

1.2 Draw a scatter plot and show the regression line. (3)

1.3 Predict the final mark for a learner who attained 46% in the preparatory examination. (2)

1.4 Determine the correlation coefficient of the data. (1)

1.5 Describe the relationship between preparatory and final exam results. (1)

1.6 Could you use this equation to estimate the preparatory exam mark for a learner who attained 73% in the final exam? Give a reason for your answer. (2)

[12]

QUESTION 2

The table below shows the results from a survey of cell phone expenditure for 100 learners from a secondary school in Rustenburg.

Expenditure(in rand)	**Frequency**	**Cumulative frequency**
$50 \leq x < 100$	24	
$100 \leq x < 150$	52	
$150 \leq x < 200$	14	
$200 \leq x < 250$	6	
$250 \leq x < 300$	4	

2.1 Complete the cumulative frequency table in the ANSWER BOOK. (2)

2.2 Draw an Ogive (cumulative frequency graph) for the data. (3)

2.3 Calculate the estimated mean of cell phone expenditure. (3)

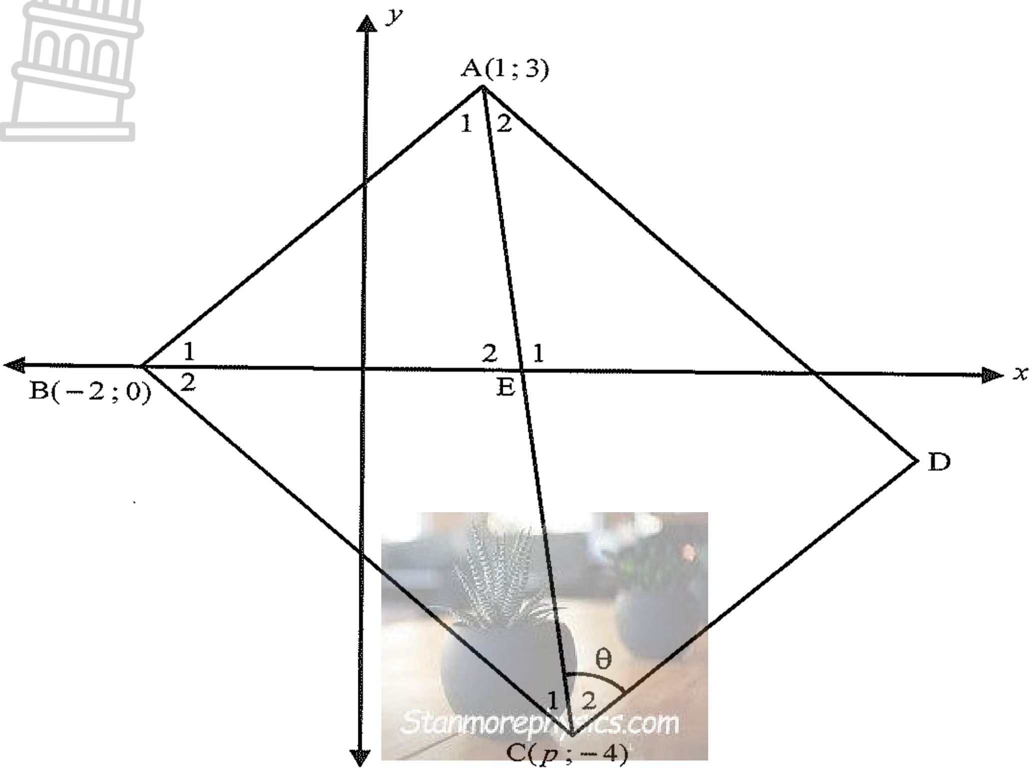
[8]

Copyright reserved

 Downloaded from Stanmorephysics.com

QUESTION 3

3.1 ABC is a triangle with vertices A(1 ; 3), B(–2 ; 0) and C(p ; –4) where $p > 0$.
The length of AC is $\sqrt{50}$ units.



3.1.1 Determine the gradient of AB. (2)

3.1.2 Show, by calculation, that $p = 2$. (4)

3.1.3. Determine the equation of the perpendicular bisector of AB. (4)

3.1.4 Write down the coordinates of D such that ABCD is a rectangle. (2)

3.1.5 Determine the equation of the circle passing through A, B and C. (4)

3.1.6 Calculate the size of θ rounded off to the nearest whole number. (5)

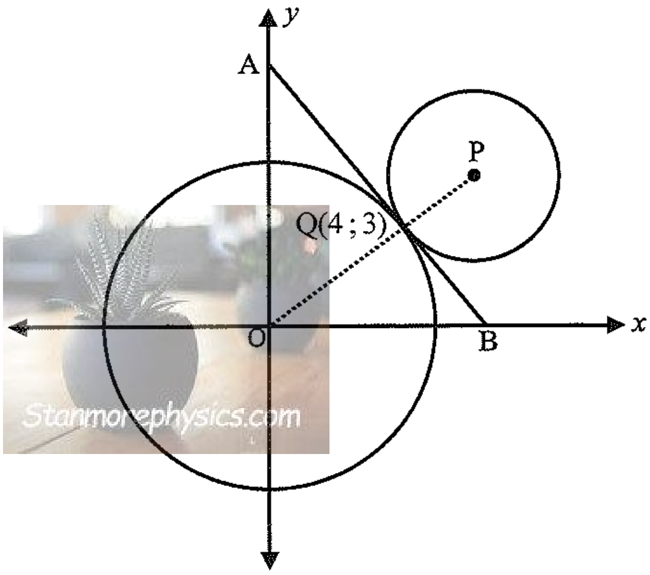
3.2 Three straight lines AB, RS and $x = -3$ intersect each other. The equation of AB is $3x + by = -2$ with $b \neq 0$, and the equation of RS is $y = -\frac{2}{3}x + 2$.
Calculate the value of b. (4)

[25]

Downloaded from Stanmorephysics.com

QUESTION 4

Two circles in the diagram below represent two interlocking gears, which touch at the point Q(4 ; 3). The circles have the following equations: $x^2 + y^2 = 25$ and $x^2 - 12x + y^2 - 9y + 50 = 0$



4.1 Show that the coordinates of P are $(6 ; 4\frac{1}{2})$. (3)

4.2 Determine the equation of the common tangent AB. (4)

4.3 If the larger gear makes one full revolution, how many times will the smaller gear turn completely? (4)

4.4 Find the area of ΔAOB. (3)

4.5 Another tangent to the circle with centre O, drawn from A, touches the circle at C. And C is the reflection of Q by the y axis.
Determine the length of CQ. (2)

[16]

 Downloaded from Stanmorephysics.com

QUESTION 5

5.1 If $\cos 21° = p$ determine the following in terms of p.

5.1.1 $\tan 201°$ (3)

5.1.2 $\sin 42°$ (3)

5.1.3 $\cos 51°$ (3)

5.2 Simplify : $\dfrac{\sin 210°.\cos 510°}{\cos 315°.\sin(-135°)}$ (7)

5.3 Prove the identity:

$$\frac{\cos\theta - \cos 2\theta + 2}{3\sin\theta - \sin 2\theta} = \frac{1+\cos\theta}{\sin\theta}$$ (5)

5.4 Determine the general solution of $\sin\theta \sin\frac{3\theta}{2} + \cos\frac{3\theta}{2}\cos\theta = -\frac{\sqrt{3}}{2}$. (4)

5.5 Given: $\sin\theta.\cos\beta = -1$

5.5.1 Write down the maximum and minimum value of $\cos\beta$ (1)

5.5.2 Solve for $\theta \in [0°;270°]$ and $\beta \in [-180°;90°]$. (4)

[30]

Copyright reserved

Downloaded from Stanmorephysics.com

QUESTION 6

Given that $y = f(x) = 2\cos x$ and $y = g(x) = \sin(x + 30^\circ)$:

6.1 Sketch the graphs of f and g on the ANSWER BOOK on the same set of axes for $x \in [-180^\circ; 180^\circ]$. (6)

6.2 Read the following answers from your graphs:

6.2.1 Write down the period of f. (1)

6.2.2 Determine one value of x for which $f(x) - g(x) = 1{,}5$. (1)

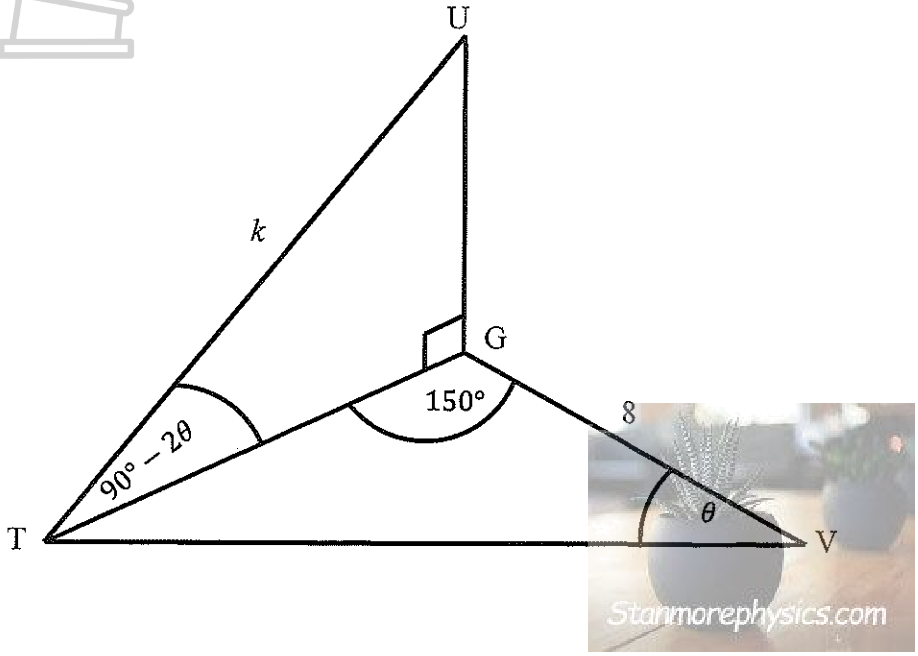
6.2.3 Determine the positive values of x for which $2\sin(x + 30^\circ).\cos x < 0$ (2)

[10]

 Downloaded from Stanmorephysics.com

QUESTION 7

A mouse on the ground at point T is looking up to an owl in a tree at point U and a cat to his right on the ground at point V. The angle of elevation from the mouse to the owl is $(90° - 2\theta)$. $TU = k \ units$, $GV = 8 \ units$, $T\hat{G}V = 150°$ and $T\hat{V}G = \theta$.



7.1 Write down the size of $T\hat{U}G$ in terms of θ (1)

7.2 Show that $TG = k \sin 2\theta$. (2)

7.3 Show that $TV = k \cos \theta$ (4)

7.4 Show that the area of $\Delta TGV = 2k \sin 2\theta$. (2)

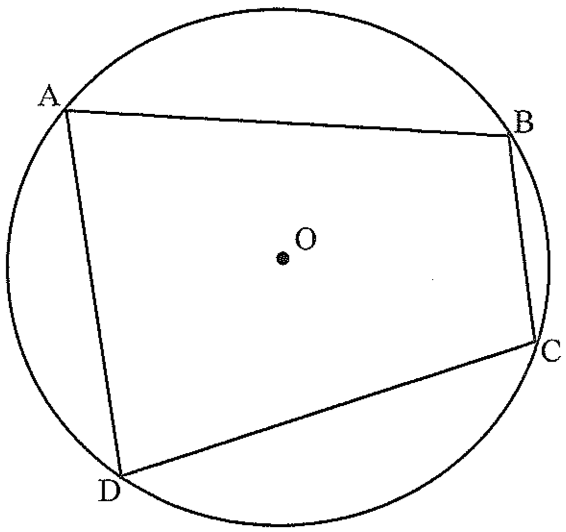
[9]

 Downloaded from Stanmorephysics.com

Give reasons for your statements and calculations in QUESTIONS 8, 9 and 10.

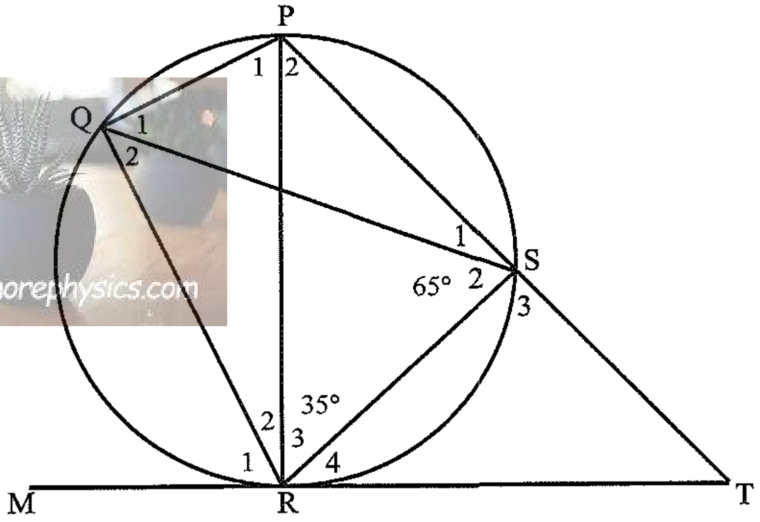
QUESTION 8

8.1 A, B, C and D are points on the circumference of the circle with centre O.
Prove the theorem which states that $\hat{A} + \hat{C} = 180°$. (5)



Downloaded from Stanmorephysics.com

8.2 In the diagram below, PQRS is a cyclic quadrilateral and PR is a diameter of the circle. The line MRT is a tangent to the circle at R. $\hat{S}_2 = 65°$ and $\hat{R}_3 = 35°$.



Determine the sizes of each of the following angles, with reasons:

8.2.1 $\hat{R}_1$ (1)

8.2.2 $\hat{R}_4$ (2)

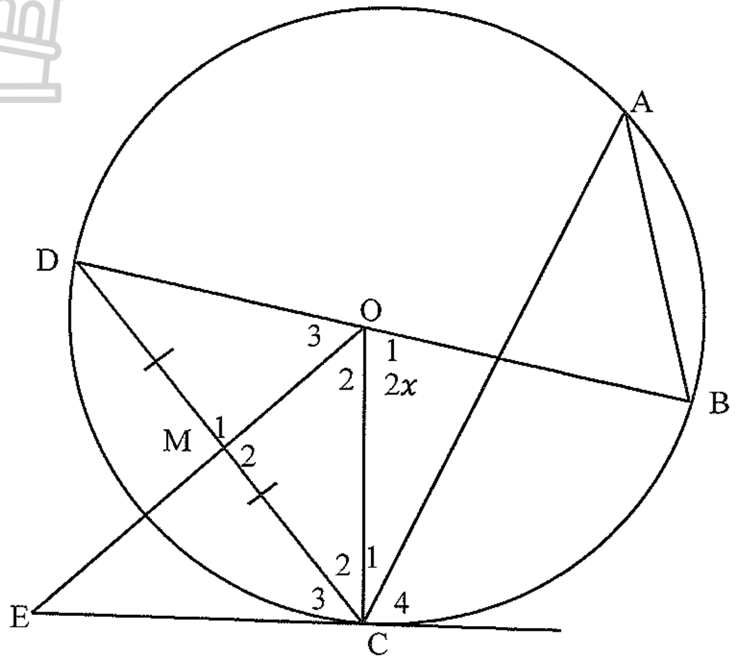
8.2.3 $\hat{T}$ (3)

[11]

 Downloaded from Stanmorephysics.com

QUESTION 9

In the diagram, O is the centre of the circle and EC is a tangent to the circle at C. DM = MC and OME is a straight line. Let $\hat{O}_1 = 2x$.

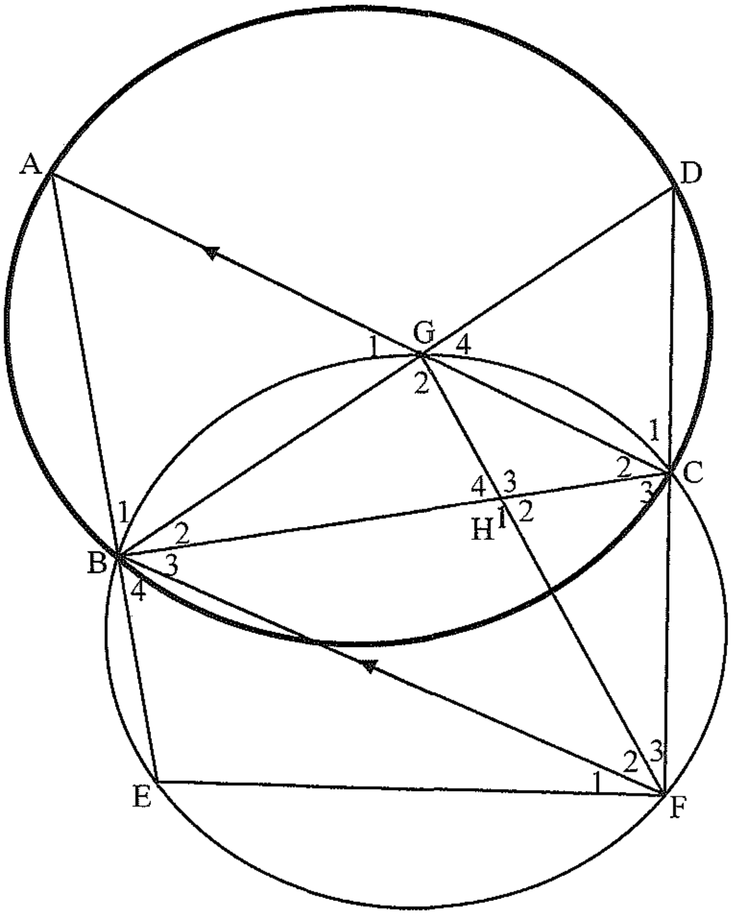


9.1	Write, with reasons, THREE angles equal to x.	(6)
9.2	Prove that $\hat{O}_2 = 90° - x$	(3)
9.3	Prove that EC is a tangent to the circle passing through points, M,C and O.	(4)
9.4	Prove that DOCE is a cyclic quadrilateral.	(3)
		[16]

Downloaded from Stanmorephysics.com

QUESTION 10

In the diagram below, two circles ABCD and BEFCG intersect at B, C and G. AC || BF. AC and BD intersect at G. BC and FG intersect at H.



10.1 Complete the following reasons:

10.1.1 $\hat{G}_1 = \hat{F}_2$ .... (1)

10.1.2 $B\hat{F}D = \hat{C}_1$ .... (1)

10.2 Prove that

10.2.1 BH = FH (4)

10.2.2 ΔBEF ||| ΔDGF (3)

10.2.3 FH.BG = BH.FC (4)

[13]

TOTAL: 150

Stanmorephysics.com

Downloaded from Stanmorephysics.com

education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2024

MARKING GUIDELINE / NASIENRIGLYNE

MARKS/PUNTE: 150

This marking guideline consist of 16 pages.

Hierdie nasienriglyne bestaan uit 16 bladsye.

Downloaded from Stanmorephysics.com

NOTE:

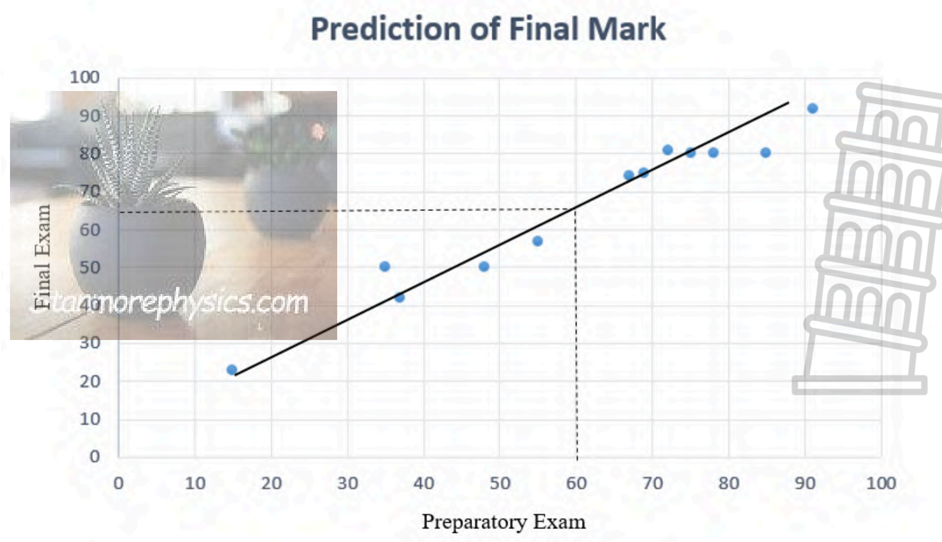
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guiselines.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyn van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

QUESTION 1/ VRAAG 1

Preparatory exam(x)	55	35	67	85	91	48	78	72	15	75	69	37
Final exam(y)	57	50	74	80	92	50	80	81	23	80	75	42

1.1	$a = 12{,}01$ $b = 0{,}88$ $y = 12{,}01 + 0{,}88x$	✓ value of a ✓ value of b ✓ Equation (3)
1.2		✓ any correct two points ✓ straight line joining the points ✓ passing through $(\bar{x};\bar{y})$ (3)
1.3	$y = 12{,}01 + 0{,}88(46)$ $y = 52\%$ **OR** calculator $y = 52{,}50\%$	✓ Substitution ✓ Answer (2)
1.4	r = 0,98	✓ Answer (1)

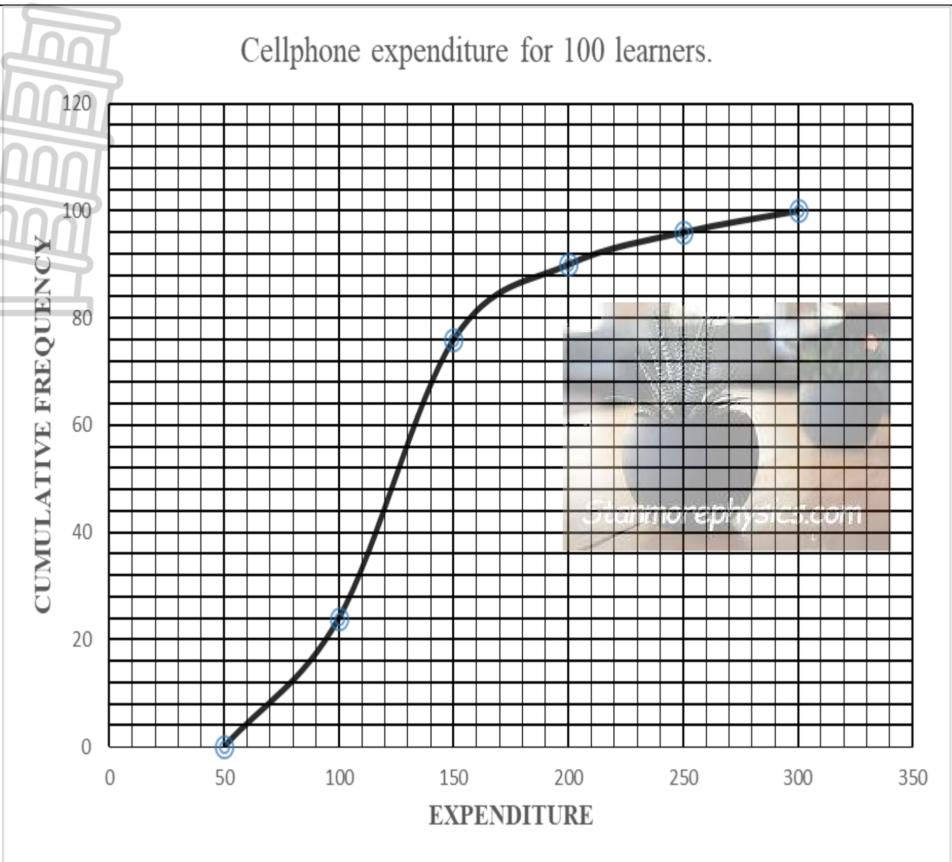
Downloaded from Stanmorephysics.com

1.5	There's a very strong positive correlation between preparatory marks and final marks. Daar is 'n baie sterk positiewe korrelasie tussen die rekordeksamen en die finale punte.	✓ Answer (1)
1.6	No, the preparatory exam mark is the independent variable. Hence we cannot determine the preparatory mark using the final mark. Nee, die rekordeksamen is die onafhanklike veranderlike. Die finale punt kan dus nie gebruik word om die rekordeksamenpunt te voorspel nie.	✓ Answer ✓ Reason (2)
		[12]

QUESTION 2 / VRAAG 2

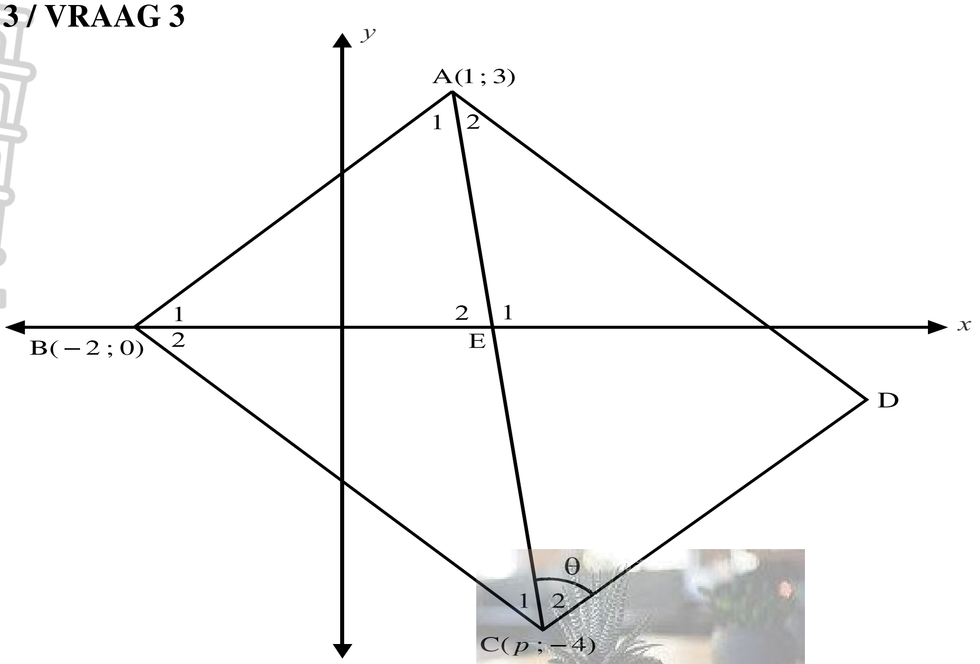
	Expenditure(in rand) Uitgawes(in rand)	Frequency Frekwensie	Cumulative frequency Kumulatiewe frekwensie	
2.1	$50 \le x < 100$	24	24	✓76 ; 90
	$100 \le x < 150$	52	76	
	$150 \le x < 200$	14	90	✓ 96 ; 100
	$200 \le x < 250$	**6**	96	
	$250 \le x < 300$	4	100	(2)

Downloaded from Stanmorephysics.com

2.2		✓ Correct points ✓ Grounding ✓ Shape (3)
2.3	$\bar{x} = \dfrac{(50\times 24)+(125\times 52)+(175\times 14)+(225\times 6)+(275\times 4)}{100}$ $= \dfrac{12600}{100}$ $= 126$	✓ Method ✓ $\div 100$ ✓ Answer (3)
		[8]

Downloaded from Stanmorephysics.com

QUESTION 3 / VRAAG 3



3.1.1	$m_{AB} = \dfrac{3-0}{1-(-2)} = 1$	✓ substitution ✓ answer (2)
3.1.2	$AC^2 = (p-1)^2 + (-4-3)^2$ $\therefore (\sqrt{50})^2 = p^2 - 2p + 1 + 49$ $\therefore 50 = p^2 - 2p + 50$ $\therefore 0 = p^2 - 2p$ $\therefore 0 = p(p-2)$ $p = 0$ or $p = 2$ $\therefore p = 2$	✓ dist formula used ✓ correct substitution ✓ standard form ✓ working to $p = 2$ (4)
3.1.3	Midpoint of AB / Middelpunt van AB: $M = \left(\dfrac{-2+1}{2} ; \dfrac{0+3}{2}\right) = \left(-\dfrac{1}{2} ; \dfrac{3}{2}\right)$ Gradient of $\perp$ line through M: $m = -1$ Equation of perp bisector / Vergelyking van middelloodlyn: $y - \dfrac{3}{2} = -1\left(x + \dfrac{1}{2}\right)$ $\therefore y - \dfrac{3}{2} = -x - \dfrac{1}{2}$ $\therefore y = -x + 1$	✓ midpoint M ✓ grad of $\perp$ line ✓ sub into formula ✓ $y = -x + 1$ (4)

Copyright reserved

Downloaded from Stanmorephysics.com

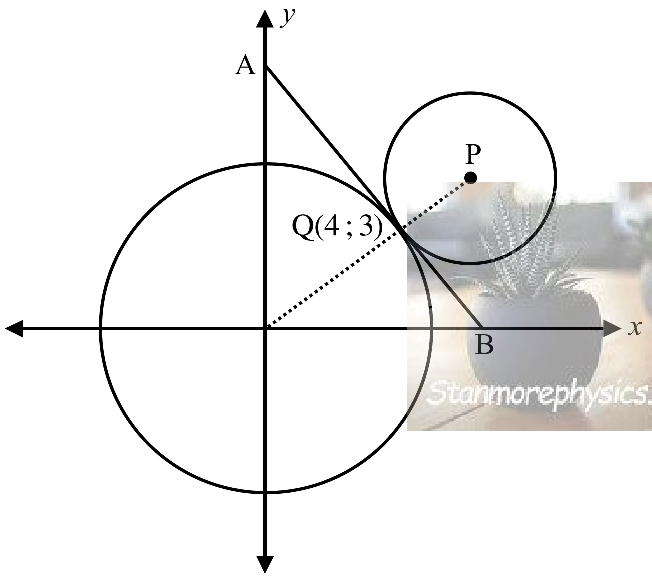
3.1.4	D(5 ; −1)	✓ $x_D = 5$ ✓ $y_D = -1$ (2)
3.1.5	$\hat{B}_1 + \hat{B}_2 = 90°$ ABCD is a rectangle / reghoek $\therefore$ AC is a diameter / middellyn Centre is the midpoint of AC/Middelpunt van sirkel is middelpunt van AC. $\left(\frac{1+2}{2} ; \frac{-4+3}{2}\right) = \left(\frac{3}{2} ; -\frac{1}{2}\right)$ Radius is half of the length of AC/Radius is helfte van AC: $r = \frac{\sqrt{50}}{2}$ Equation of circle / Vergelyking van sirkel: $\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \left(\frac{\sqrt{50}}{2}\right)^2$ $\therefore \left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{2}$	✓ $\hat{B}_1 + \hat{B}_2 = 90°$ ✓ $\left(\frac{3}{2} ; -\frac{1}{2}\right)$ ✓ $r = \frac{\sqrt{50}}{2}$ ✓ equation of circle (4)
3.1.6	$\hat{A}_1 = \theta$ alt $\angle$s = ; AB ∥ CD $\tan \hat{B}_1 = m_{AB}$ $\therefore \tan \hat{B}_1 = 1$ $\therefore \hat{B}_1 = 45°$ $\tan \hat{E}_1 = m_{AC} = \frac{-4-3}{2-1} = -7$ $\therefore \tan \hat{E}_1 = -7$ $\therefore \hat{E}_1 = 98,13010235°$ $\theta = 98,13010235° - 45°$ $\therefore \theta = 53°$	✓ $\hat{A}_1 = \theta$ ✓ $\hat{B}_1 = 45°$ ✓ $m_{AC} = -7$ ✓ $\hat{E}_1 = 98,13010235°$ ✓ $\theta = 53°$ (5)

Stanmorephysics.com

Downloaded from Stanmorephysics.com

3.2	$3x+by=-2$ $\therefore by=-3x-2$ $\therefore y=-\frac{3}{b}x-\frac{2}{b}$ $-\frac{3}{b}x-\frac{2}{b}=-\frac{2}{3}x+2$ $\therefore -9x-6=2bx+6b$ Substitute $x=-3$: $\therefore -9(-3)-6=-2b(-3)+6b$ $\therefore 27-6=6b+6b$ $\therefore 21=12b$ $\therefore b=\frac{21}{12}=\frac{7}{4}$	✓ $y=-\frac{3}{b}x-\frac{2}{b}$ ✓ equating equations Stel vergelykings gelyk ✓ substituting $x=-3$ ✓ $b=\frac{7}{4}$ (4)
		[24]

QUESTION 4 / VRAAG 4

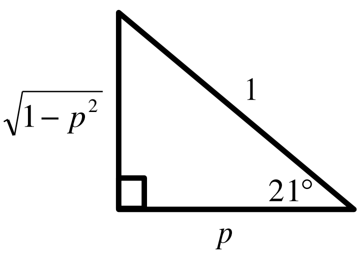


4.1	$x^2-12x+y^2-9y+50=0$ $\therefore x^2-12x+36+y^2-9y+\frac{81}{4}=-50+36+\frac{81}{4}$ $\therefore (x-6)^2+\left(y-\frac{9}{2}\right)^2=\frac{25}{4}$	✓ $(x-6)^2$ ✓ $\left(y-\frac{9}{2}\right)^2$ ✓ $\frac{25}{4}$ (3)

Downloaded from Stanmorephysics.com

4.2	$m_{OB}=\frac{3}{4}$ $\therefore m_{AB}=-\frac{4}{3}$ $y-3=-\frac{4}{3}(x-4)$ $\therefore y-3=-\frac{4}{3}x+\frac{16}{3}$ $y=-\frac{4}{3}x+\frac{25}{3}$	✓ $m_{OB}=\frac{3}{4}$ ✓ $m_{AB}=-\frac{4}{3}$ ✓ substitution of point ✓ equation (4)
4.3	Circumference of small $=2\pi\left(\frac{5}{2}\right)=5\pi$ Circumference of large $=2\pi(5)=10\pi$ Two revolutions / Twee omwentelings .	✓ $r=\frac{5}{2}$ ✓ $r=5$ ✓ $5\pi;\ 10\pi$ ✓ two revolutions (4)
4.4	$A\left(0;\frac{25}{3}\right)$ $B\left(\frac{25}{4};0\right)$ $\therefore OA=\frac{25}{3}$ and $OB=\frac{25}{4}$ Area $\Delta AOB=\frac{1}{2}\times\frac{25}{4}\times\frac{25}{3}=\frac{625}{24}$ units^2	✓ $OA=\frac{25}{3}$ ✓ $OB=\frac{25}{4}$ ✓ calculating area (3)
4.5	$Q(4;3)$ $C(-4;3)$ $\therefore CQ=8$ units	✓ $C(-4;3)$ ✓ $CQ=8$ (2)
		[16]

QUESTION 5 / VRAAG 5

5.1.1	$p^2+y^2=1^2$ $y=\sqrt{1-p^2}$  $\tan 201°=\tan(180°+21°)$ $=\tan 21°$	✓ $y=\sqrt{1-p^2}$ ✓ $\tan 21°$ ✓ $\frac{\sqrt{1-p^2}}{p}$ (3)

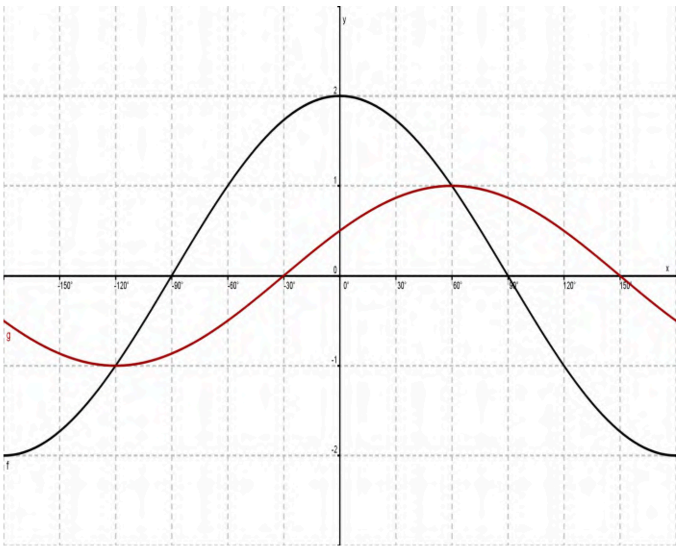
Downloaded from Stanmorephysics.com

	$= \dfrac{\sqrt{1-p^2}}{p}$	
5.1.2	$\sin 42° = \sin 2(21°)$ $= 2\sin 21° \cos 21°$ $= 2\left(\sqrt{1-p^2}\right)(p)$ $= 2p\sqrt{1-p^2}$	✓ $\sin 2(21°)$ ✓ Identity ✓ Answer. (3)
5.1.3	$\cos(30° + 21°) = \cos 30° \cos 21° - \sin 30° \sin 21°$ $= \dfrac{\sqrt{3}}{2}(p) - \dfrac{1}{2}\left(\sqrt{1-p^2}\right)$ $= \dfrac{p\sqrt{3} - \sqrt{1-p^2}}{2}$	✓ $\cos(30° + 21°)$ ✓ Compound angle ✓ Substitution ✓ Answer (3)
5.2	$\dfrac{\sin 210° \cos 510°}{\cos 315° \sin(-135°)}$ $= \dfrac{(-\sin 30°)(\cos 150°)}{(\cos 45°)(-\sin 135°)}$ $= \dfrac{(-\sin 30°)(-\cos 30°)}{(\cos 45°)(-\sin 45°)}$ $= \dfrac{\left(-\dfrac{1}{2}\right)\left(-\dfrac{\sqrt{3}}{2}\right)}{\left(\dfrac{\sqrt{2}}{2}\right)\left(-\dfrac{\sqrt{2}}{2}\right)}$ $= -\dfrac{\sqrt{3}}{2}$	✓ $-\sin 30°$ ✓ $-\cos 30°$ ✓ $\cos 45°$ ✓ $-\sin 45°$ ✓ $-\dfrac{1}{2}$ ✓ $-\dfrac{\sqrt{2}}{2}$ ✓ $-\dfrac{\sqrt{3}}{2}$ (7)
5.3	$\dfrac{\cos\theta - \cos 2\theta + 2}{3\sin\theta - \sin 2\theta}$ $= \dfrac{\cos\theta - (2\cos^2\theta - 1) + 2}{3\sin\theta - 2\sin\theta\cos\theta}$ $= \dfrac{\cos\theta - 2\cos^2\theta + 1 + 2}{\sin\theta(3 - 2\cos\theta)}$ $= \dfrac{3 + \cos\theta - 2\cos^2\theta}{\sin\theta(3 - 2\cos\theta)}$ $= \dfrac{(3 - 2\cos\theta)(1 + \cos\theta)}{\sin\theta(3 - 2\cos\theta)}$ $= \dfrac{1 + \cos\theta}{\sin\theta}$ $= RHS$	✓ $2\cos^2\theta - 1$ ✓ $2\sin\theta\cos\theta$ ✓ $3 + \cos\theta - 2\cos^2\theta$ ✓ $(3 - 2\cos\theta)(1 + \cos\theta)$ ✓ $\sin\theta(3 - 2\cos\theta)$ (5)

Downloaded from Stanmorephysics.com

5.4	$\sin\theta\sin\frac{3\theta}{2}+\cos\frac{3\theta}{2}\cos\theta=-\frac{\sqrt{3}}{2}$ $\cos\frac{3\theta}{2}\cos\theta+\sin\theta\sin\frac{3\theta}{2}=-\frac{\sqrt{3}}{2}$ $\cos\left(\frac{3\theta}{2}-\theta\right)=-\frac{\sqrt{3}}{2}$ $\cos\left(\frac{\theta}{2}\right)=-\frac{\sqrt{3}}{2}$ $\frac{\theta}{2}=150°+k.360°\ k\in\mathbb{Z}\ OR\ \frac{\theta}{2}=-150°+k.360k\in\mathbb{Z}$ $\theta=300°+k.720°k\in\mathbb{Z}\quad \theta=-300°+k.720°k\in\mathbb{Z}$ **OR** $ref\ \angle=30°$ $\frac{\theta}{2}=180°-30°+k.360°\boldsymbol{or}\frac{\theta}{2}=180°+30°+k.360°k\in\mathbb{Z}$ $\theta=300°+k.720°\ k\in\mathbb{Z}\ \boldsymbol{or}\ \theta=420°+k.720°\ \ k\in\mathbb{Z}$	✓ $\cos\left(\frac{3\theta}{2}-\theta\right)$ ✓ $\frac{\theta}{2}=\pm150°+k.360°$ ✓ $\theta=300°+k.720°$ ✓ $-300°+k.720°$ (4) **OR** ✓ $\cos\left(\frac{3\theta}{2}-\theta\right)$ ✓ $\frac{\theta}{2}=180°\pm+k.360°$ ✓ $300°+k.720°$ ✓ $420°+k.720°$ (4)
5.5.1	Maximum value is 1 and minimum value is -1	✓ Answer (1)
5.5.2	Range of both graphs is/waardevers van beide grafieke: $-1\leq y\leq 1$ $\sin\theta.\cos\beta=-1$ $\sin\theta=1\ and\ \cos\beta=-1\ \ or\ \ \sin\theta=-1\ \ and\ \cos\beta=-1$ $\theta=90°\ \ and\ \ \beta=-180°\ \ or\ \ \theta=270°\ \ and\ \beta=0°$	✓ $\theta=90°$ ✓ $\beta=-180°$ ✓ $\theta=270°$ ✓ $\beta=0°$ (4)
		[30]

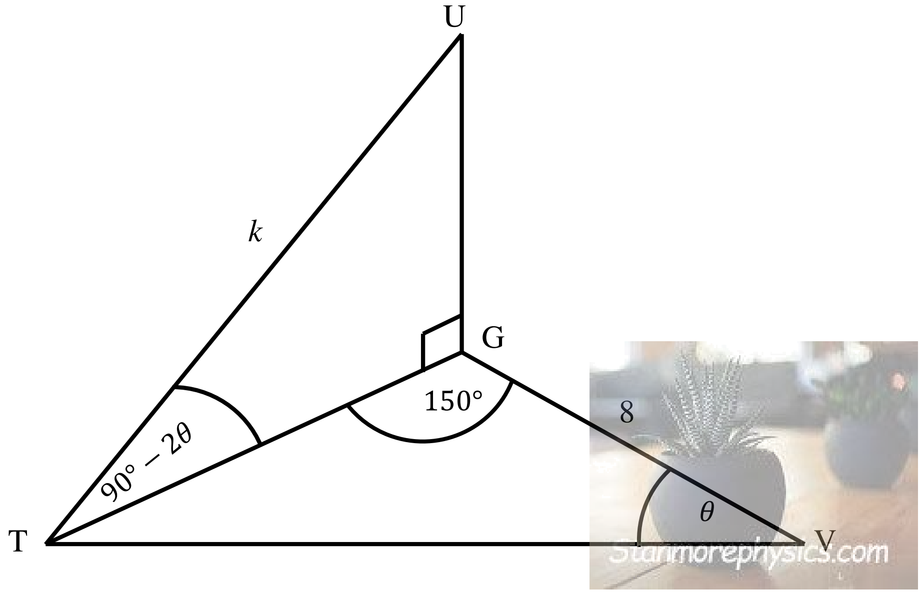
QUESTION 6 / VRAAG 6

6.1		$f(x)$ ✓ $x-intercepts$ ✓ $y-intercept$ ✓ Shape. $g(x)$ ✓ $x-intercepts$ ✓ $y-intercept$ ✓ Shape. (6)

Downloaded from Stanmorephysics.com

6.2.1	$360°$	✓ Answer (1)
6.2.2	$0°$	✓ Answer (1)
6.3.3	$90° < x < 150°$ OR $x \in (90°; 150°)$	✓ Notation ✓ Endpoints. (2)
		[10]

QUESTION 7 / VRAAG 7

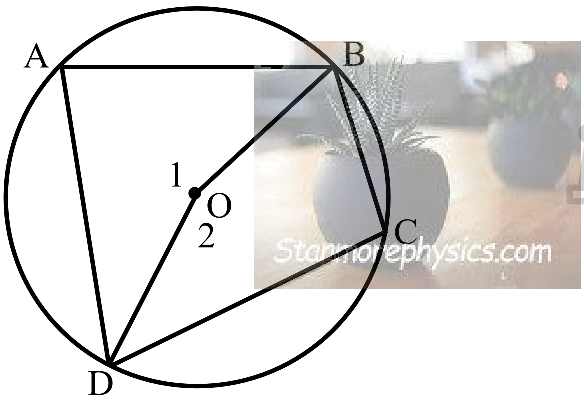


7.1	$T\hat{U}G = 2\theta$	✓Answer. (1)
7.2	$\frac{TG}{\sin 2\theta} = \frac{k}{\sin 90^\circ}$ $k\sin 2\theta = TG \times 1$ $\therefore TG = k\sin 2\theta$ OR $\cos(90^\circ - 2\theta) = \frac{TG}{k}$ $\sin 2\theta = \frac{TG}{k}$ $\therefore TG = k\sin 2\theta$	✓ Sine rule. ✓ $\sin 90^\circ = 1$ **OR** ✓ Trig ratio. ✓ $\sin 2\theta$ (2)

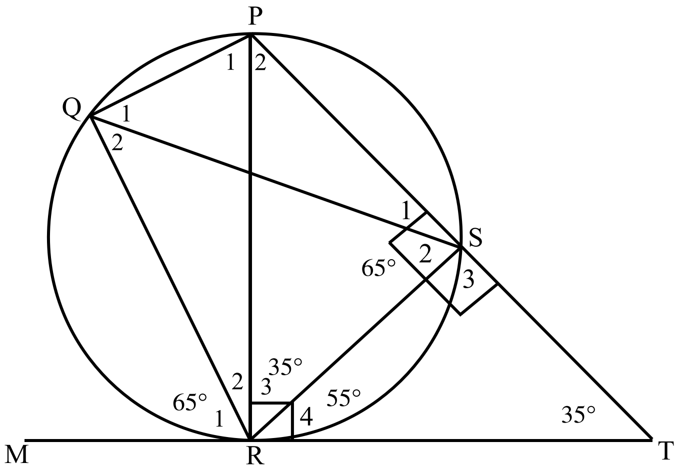
Downloaded from Stanmorephysics.com

7.3	$\frac{TV}{\sin 150^\circ} = \frac{TG}{\sin\theta}$ $\frac{TV}{\frac{1}{2}} = \frac{TG}{\sin\theta}$ $TV = \frac{\frac{1}{2}TG}{\sin\theta}$ $TV = \frac{\frac{1}{2}(k\sin 2\theta)}{\sin\theta}$ $TV = \frac{\frac{1}{2}(2k\sin\theta\cos\theta)}{\sin\theta}$ $\therefore TV = k\cos\theta$	✓ $\frac{TV}{\sin 150^\circ} = \frac{TG}{\sin\theta}$ ✓ $\frac{1}{2}$ ✓ $TV = \frac{\frac{1}{2}TG}{\sin\theta}$ ✓ $2\sin\theta\cos\theta$ (4)
7.4	$\triangle TGV = \frac{1}{2}TG.GV\sin 150°$ $= \frac{1}{2}k\sin 2\theta.8.\frac{1}{2}$ $= 2k\sin 2\theta$ **OR** $\triangle TGV = \frac{1}{2}TV.GV.\sin\theta$ $= \frac{1}{2}k\cos\theta.8.\sin\theta$ $= 4\cos\theta.\sin\theta$ $= 2k(2\cos\theta\sin\theta)$ $= 2k\sin 2\theta$	✓ Substitution. ✓ Method. (2) **OR** ✓ Substitution. ✓ Method. (2)
		[09]

QUESTION 8 / VRAAG 8

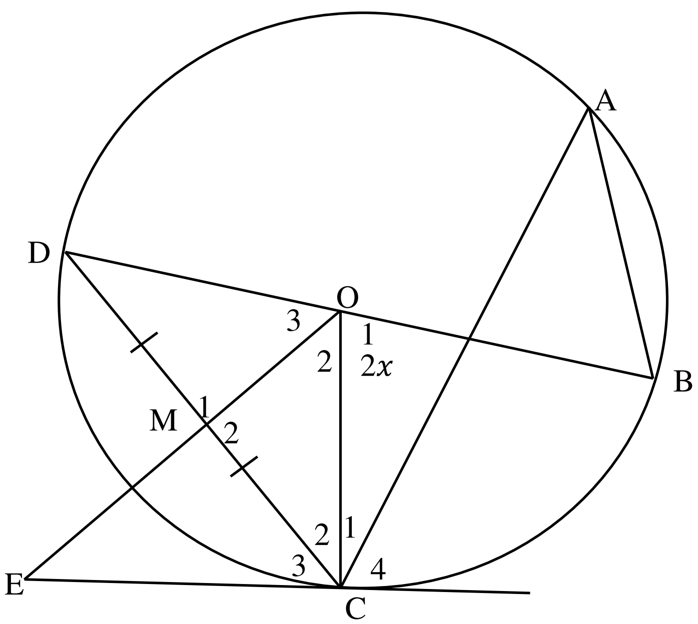
8.1	 $\hat{O}_1 = 2\hat{C}$ $\angle$ at centre $= 2\times\angle$at circ $\hat{O}_2 = 2\hat{A}$ $\angle$ at centre $= 2\times\angle$at circ	✓ construction ✓ $\hat{O}_1 = 2\hat{C}$ ✓ $\hat{O}_2 = 2\hat{A}$

Downloaded from Stanmorephysics.com

	$\hat{O}_1 + \hat{O}_2 = 2\hat{C} + 2\hat{A}$ $360° = 2\hat{C} + 2\hat{A}$ $\angle$s round a point $\therefore 180° = \hat{C} + \hat{A}$ $\therefore \hat{A} + \hat{C} = 180°$	✓ Writing the statement $360° = 2\hat{C} + 2\hat{A}$ and hence $\hat{A} + \hat{C} = 180°$ ✓ two correct reasons (5)
8.2		
8.2.1	$\hat{R}_1 = 65°$ tan chord theorem	✓ S /R (1)
8.2.2	$\hat{R}_3 + \hat{R}_4 = 90°$ tan $\perp$ radius $\therefore \hat{R}_4 = 55°$	✓ S ✓ R (2)
8.2.3	$\hat{S}_1 + \hat{S}_2 = 90°$ $\angle$ in semi-circle $\hat{T} = 35°$ ext $\angle$ of Δ	✓ S ✓ R ✓ S (3)
		[11]

Downloaded from Stanmorephysics.com

QUESTION 9 / VRAAG 9

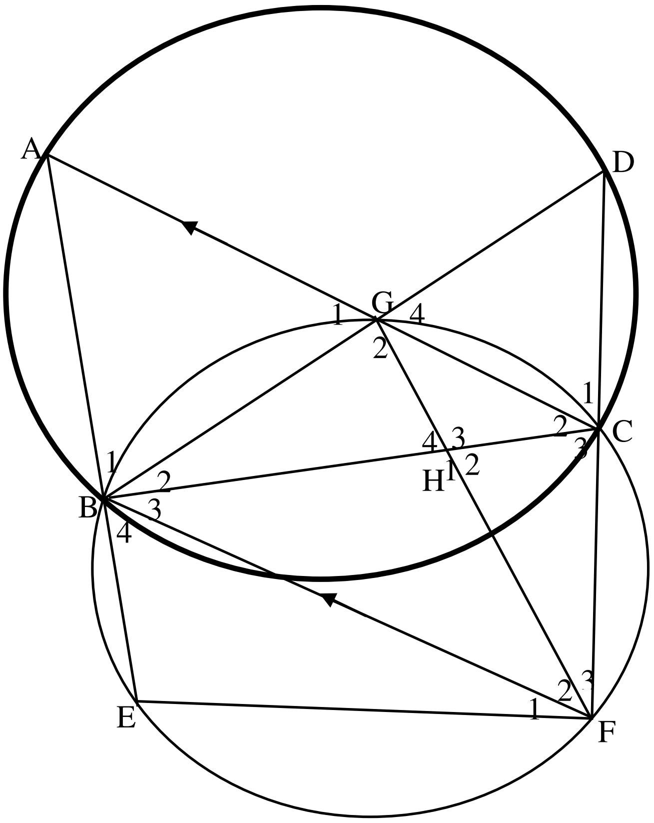


9.1	$\hat{O}_1 = \hat{A}$ [$\angle$ at the centre is = 2x $\angle$ at the circumference] $2x = \hat{A}$ $\hat{A} = x$ But $\hat{A} = \hat{D} = x$ [$\angle's$ in the same segment] $\hat{D} = \hat{C}_2 = x$ [$\angle's$ opp = sides]	✓ S ✓ R ✓ S ✓R ✓S ✓ R (6)
9.2	$\hat{M}_2 = 90°$ [$\angle$ from centre to midpoint of a chord] $\therefore \hat{O}_2 = 90° - x$ [sum of $\angle's$ in a $\triangle$]	✓ S ✓ R ✓ R (3)
9.3	$\hat{O}_2 = 90° - x$ [proved in 9.2] $\therefore \hat{C}_3 = 90° - x$ [tan $\perp$ rad] $\therefore \hat{O}_2 = \hat{C}_3$ [both $= 90° - x$] $\therefore$ EC is a tangent to circle passing through M, C & O. Converse tan-chord theorem.	 ✓S ✓ R ✓ S ✓R (4)
9.4	$\hat{C}_3 = 90° - x$ [proven in 9.3] $\hat{O}_3 = 180° - (90° - x) - 2x$ [$\angle's$ on a straight line] $\therefore \hat{O}_3 = 90° - x$ $\therefore \hat{O}_3 = \hat{C}_3$ [both $= 90° - x$] $\therefore$ DOCE is a cyclic quadrilateral. Converse $\angle's$ in the same segment.	 ✓S ✓ R ✓ R (3)
		[16]

Copyright reserved

Downloaded from Stanmorephysics.com

QUESTION 10/ VRAAG10



10.1.1	Tan-chord theorem	✓ Answer (1)
10.1.2	Corresp. $\angle s$; BF ∥ AC	✓ Answer (1)
10.2.1	$\hat{C}_2 = \hat{B}_3$ [alt. $\angle$s ; AC ∥ BF] $\hat{B}_3 = \hat{G}_3$ [$\angle s$ in the same segment] $\hat{G}_3 = \hat{F}_2$ [Alt. $\angle$s; AC ∥ BF] $\therefore \hat{B}_3 = \hat{F}_2$ [both $= \hat{G}_3$] $\therefore$ BH = FH [sides opp. = $\angle$s]	✓ S/R ✓ S/R ✓ S ✓ R (4)
10.2.2	In ΔBEF and ΔDGF $\hat{E} = \hat{G}_4$ [ext. $\angle$ of cyclic quad.] $\hat{B}_4 = \hat{A}$ [alt. $\angle s$; AC ∥ BF] $\hat{A} = \hat{D}$ [$\angle s$ in the same segment] $\therefore \hat{B}_4 = \hat{A}$ $\hat{F}_1 = \hat{F}_3$ [sum of $\angle s$ in a Δ] ΔBEF ||| ΔDGF [$\angle\angle\angle$]	✓ S/R ✓ S/R ✓ R (3)

Downloaded from Stanmorephysics.com

10.2.3	$\hat{B}_2 = \hat{F}_3$ [$\angle$s in the same segment] $\hat{H}_2 = \hat{H}_4$ [vert. opp $\angle s$] $\therefore \hat{G}_2 = \hat{C}_3$ [sum of $\angle s$ in a Δ] $\Delta BGH \parallel\!\mid \Delta FCH$ [$\angle\angle\angle$] $\frac{BG}{FC} = \frac{BH}{FH}$ $\therefore$ FH.BG = BH.FC	✓ S/R ✓ S/R ✓ R ✓ S (4)
		[13]

TOTAL/ TOTAAL:		**150**