Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 11

NOVEMBER 2023

MATHEMATICS P2

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, including 1 page information sheet, and an answer book of 20 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 10 questions.

2.   Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3.   Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.

4.   Answers only will NOT necessarily be awarded full marks.

5.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

7.   Diagrams are NOT necessarily drawn to scale.

8.   An information sheet with formulae is included at the end of the question paper.

9   Write neatly and legibly.

QUESTION 1

The box-and-whisker diagram below shows the distribution of the September examination marks of a group of 24 boys in Grade 11. The median is 84 and the mean is 87.


68   78   84   105   127
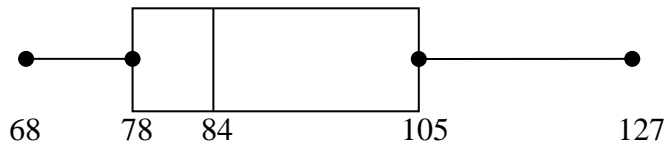
1.1 How many boys scored more than 105? (1)

1.2 In which direction are the marks skewed? (1)

1.3 Calculate the range of the data. (2)

1.4 On checking the answer book of the candidate who scored 127, an adding error is discovered and his mark is changed to 147. Determine the resulting value of the following measures:

    1.4.1 The median (1)

    1.4.2 The mean (2)

[7]

QUESTION 2

55 learners were sampled at the school tuckshop to measure their waiting time in minutes before being served. The results were published in the table below:

| Time waiting (minutes)	Number of learners	Cumulative frequency
$0 < x \leq 4$	6	
$4 < x \leq 8$	10	
$8 < x \leq 12$	19	
$12 < x \leq 16$	15	
$16 < x \leq 20$	5	
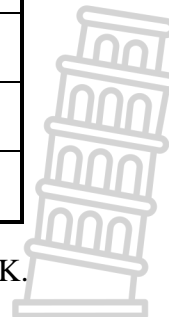
2.1 Complete the cumulative frequency table in the ANSWER BOOK. (2)

2.2 Hence, draw the cumulative frequency graph on the grid in the ANSWER BOOK. (4)

2.3 Use your graph to estimate the number of learners who waited for more than 14 minutes. (3)

2.4 Write down the modal class of the data. (1)

2.5 Use your graph to estimate the interquartile range of the data. (3)

[13]

QUESTION 3

3.1 Collinear points are points that lie on the same straight line. If A$(-8; 0)$, B$(x - 5; -8)$ and C$(x; -14)$ are collinear, then calculate the value of x. (5)

3.2 A$(2; 10)$; B$(6; -2)$ and C$(0; -6)$ are vertices of a triangle. Points D and E are x-intercepts of straight-line AC and AB respectively.
Study the diagram below and answer the questions that follow.
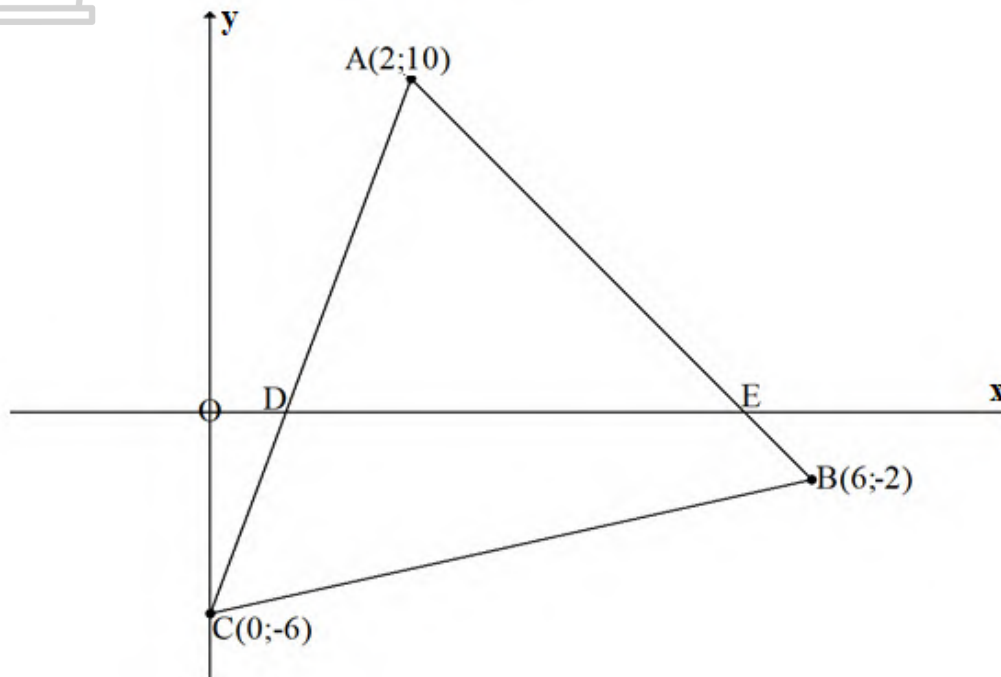
3.2.1 Calculate the coordinates of M, the midpoint of BC. (2)

3.2.2 Calculate the length of AC and leave your answer in simplified surd form. (2)

3.2.3 Write down the coordinates of F if ABCF is a parallelogram. (2)

3.2.4 Show that ABCF is not a rectangle. (3)

3.2.5 Determine the equation of straight lines AB and AC. (5)

3.2.6 Hence or otherwise, calculate the size of $\widehat{A}$. (6)

3.2.7 Calculate the area of ΔADE. (5)

[30]

QUESTION 4

4.1 If $7\tan\theta = 3$ and $\cos\theta < 0$, use a sketch in the correct quadrant to determine the value of: $\dfrac{\sin\theta + \cos\theta}{2\sin\theta}$ without using a calculator. (6)

4.2 If $\sin 32° = p$, express each of the following in terms of p:

    4.2.1 $\tan(-32°)$ (3)

    4.2.2 $\sin 418°$ (2)

4.3 Simplify the following fully and without using a calculator:

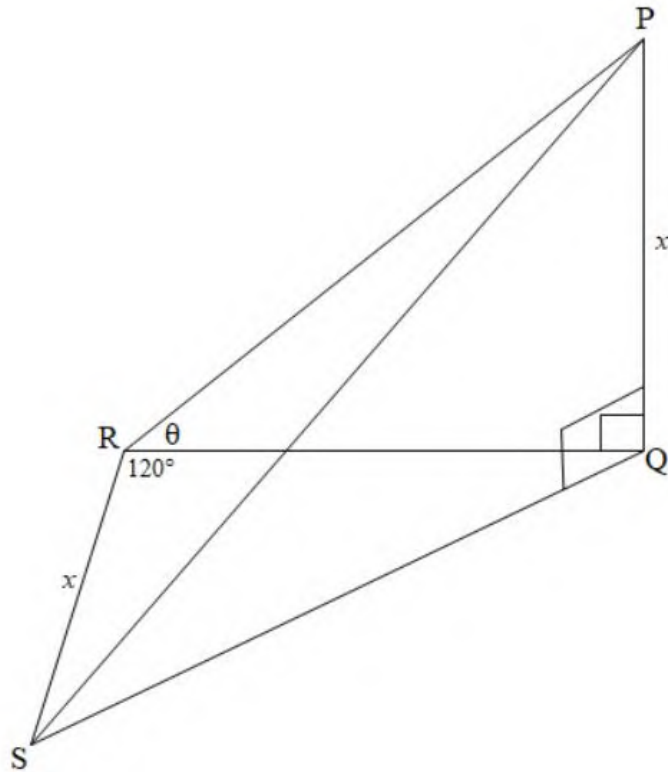$$\frac{\sin 120°.\tan 300°}{\cos(-60°).\tan 225°}$$ (7)

4.4 Prove the following identity:

$$\frac{\cos^2 x - \cos x - \sin^2 x}{2\sin x\cos x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x}$$ (5)

4.5 Determine the general solution of: $\sin(5\theta) = \cos(\theta - 40°)$ (6)

[29]

QUESTION 5

In the diagram below, PQ is a vertical tower. Q, R and S are points on the same horizontal plane. The angle of elevation from R to P is θ. $Q\hat{R}S = 120°$ and PQ = RS = x.

5.1   Determine QR in terms of θ and x.   (2)

5.2   Show that QS = $x \sqrt{\dfrac{1}{tan^2\theta} + \dfrac{1}{tan\theta} + 1}$ (4)

5.3   If $x = 15$ cm and $\theta = 22°$, calculate

       5.3.1   QS   (2)

       5.3.2 $Q\hat{P}S$ (2)

                                                                                                    [10]

**QUESTION 6**

Given: $f(x) = \sin 2x$ and $g(x) = \tan x - 1$

6.1   Draw the sketch graphs of f and g on the same set of axes provided in the answer book for $x \in [-45°; 180°]$. Indicate all endpoints, intercepts with the axes, turning points as well as asymptotes.   (6)

6.2   Use your graphs to determine value(s) of x for which:

      6.2.1 $\sin 2x > 0$ (1)

      6.2.2 $f(x) \cdot g(x) \geq 0$ (3)

6.3   If $h(x) = f(x) + 2$, write down the range of h.   (2)

                                                         [12]

QUESTION 7

7.1   In the diagram below, O is the centre of the circle. SR = 40 cm and OT ⊥ SR. It is also given that UT = x and OU = $2x$.
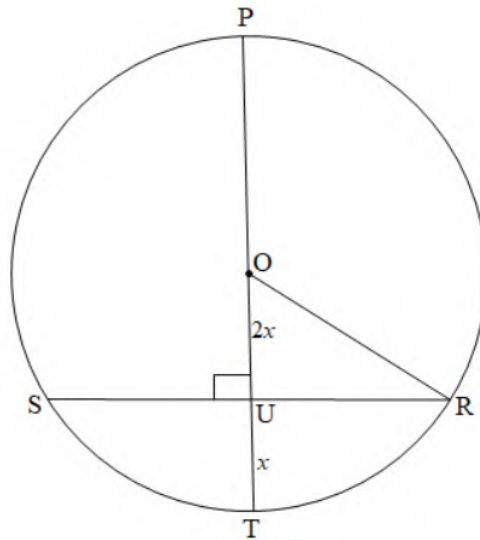
Determine:

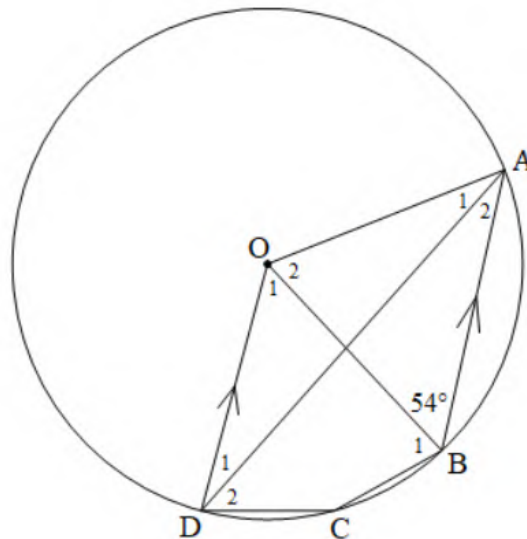7.1.1   The length of OR in terms of x (1)

7.1.2   The value of x, leaving your answer in simplest surd form   (5)

7.2   In the diagram below, O is the centre of the circle. ABCD are points on the circumference of the circle. OD ∥ AB and $O\widehat{B}A = 54°$.

Determine:

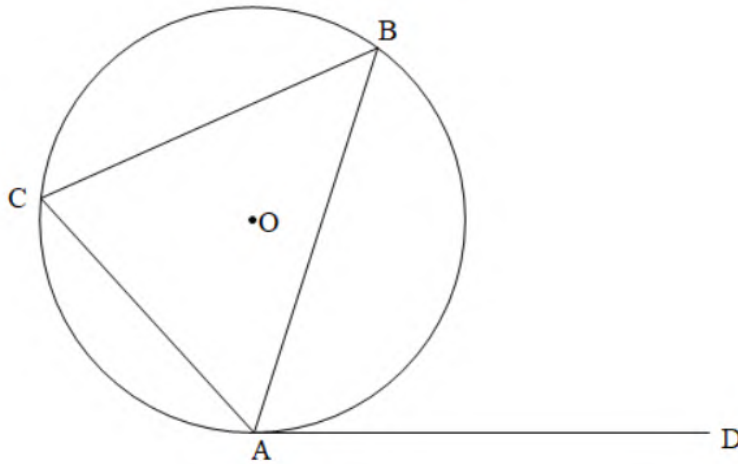7.2.1   The size of $\widehat{A}_2$ (4)

7.2.2   The size of $\widehat{C}$ (2)

7.2.3   Prove that DA bisects $O\widehat{A}B$ (3)
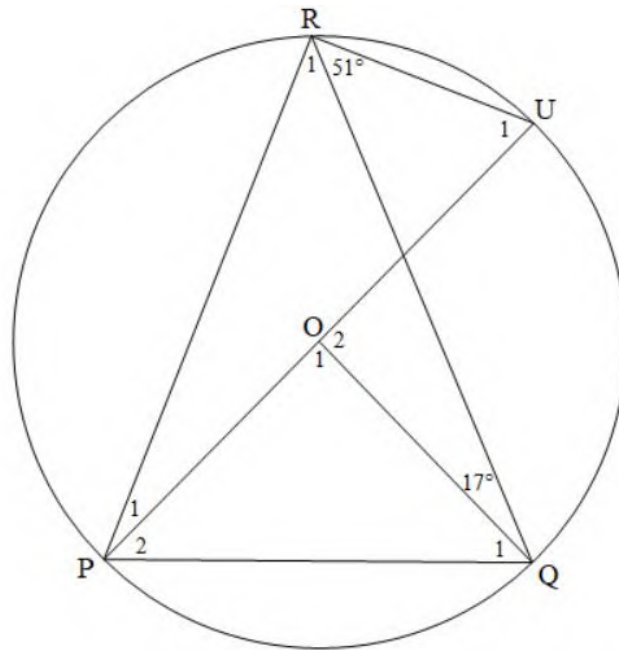
[15]

QUESTION 8

8.1 In the diagram below, DA is a tangent to the circle with centre O.

Use the diagram above to prove the theorem that states that:
$D\hat{A}B = B\hat{C}A$. (5)

8.2 In the diagram below, O is the centre of circle QPRU. $O\hat{Q}R = 17°$ and $Q\hat{R}U = 51°$.

Calculate, with reasons, the size of:

8.2.1 $\hat{R}_1$ (2)

8.2.2 $\hat{O}_1$ (2)

8.2.3 $\hat{Q}_1$ (2)
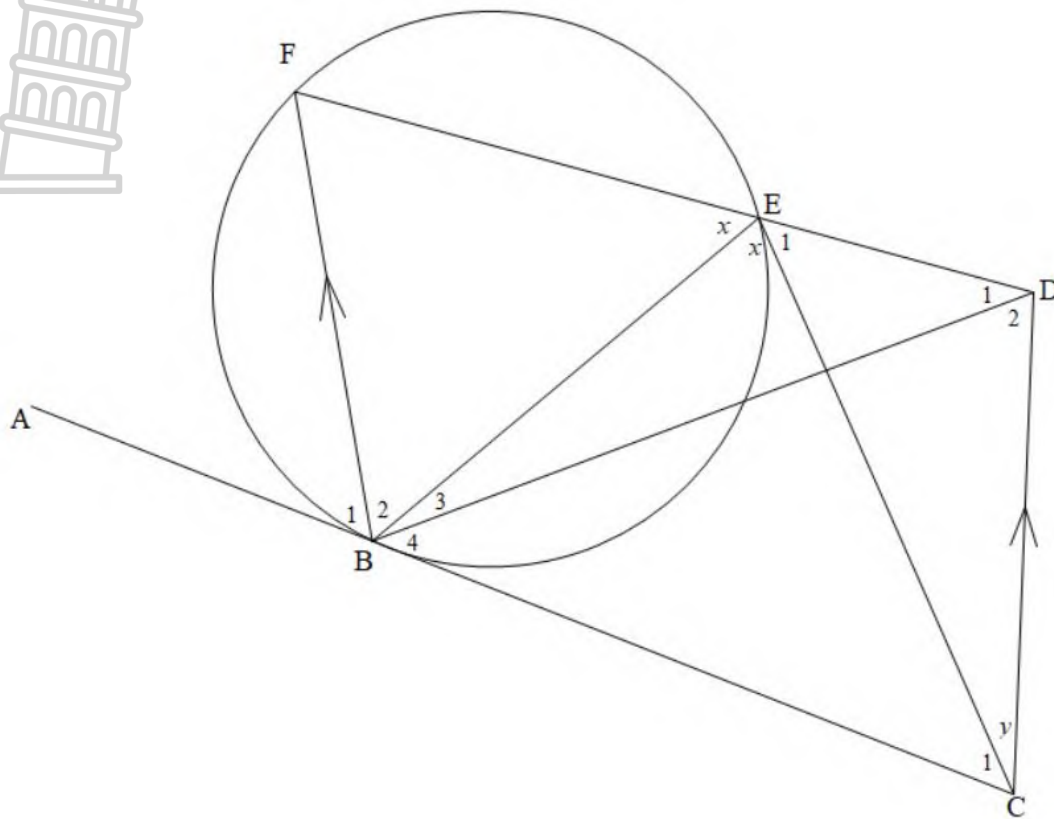
8.2.4 $\hat{U}_1$ (2)

[13]

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QUESTION 9

ABC is a tangent to circle BFE at B. From C, a straight-line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $F\hat{E}B = B\hat{E}C = x$ and $E\hat{C}D = y$.

9.1 Calculate, in terms of x and y the sizes of the following angles. Give reasons for your answers.

    9.1.1 $A\hat{B}F$ (2)
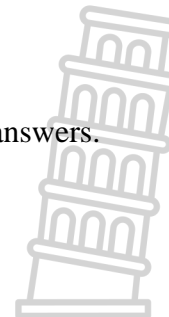
    9.1.2 $B\hat{C}D$ (2)

9.2 Why is BEDC a cyclic quadrilateral?   (1)

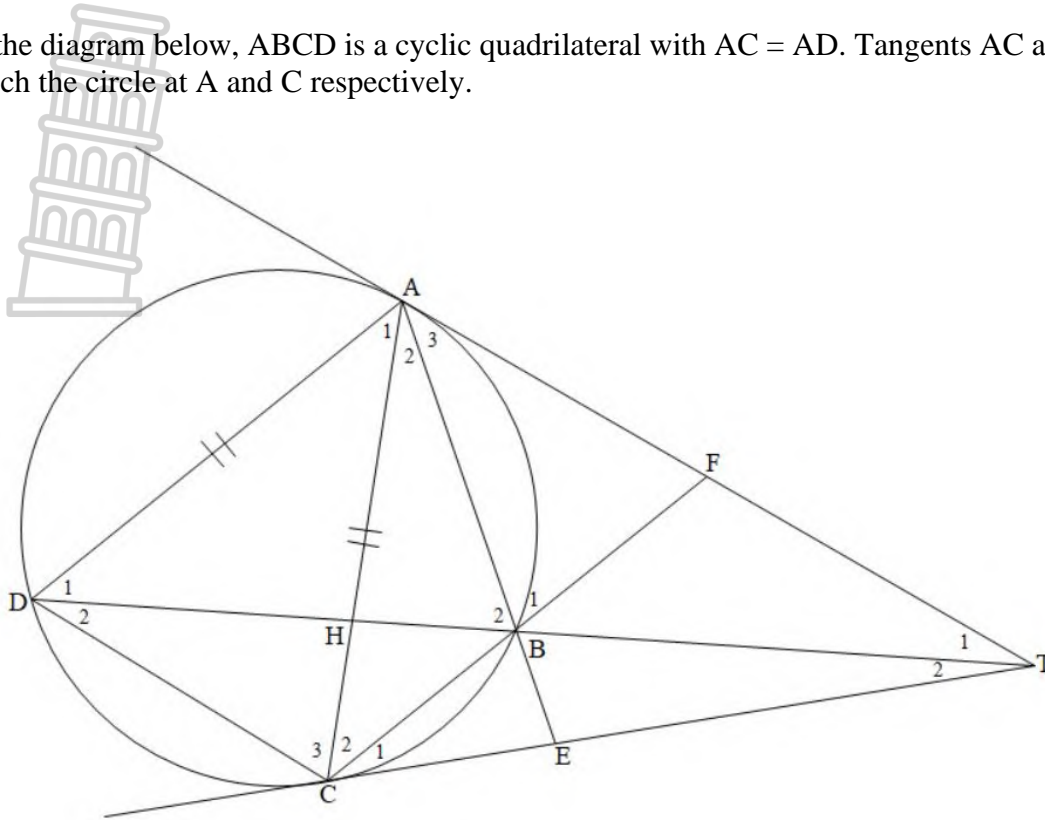9.3 Which other two angles are equal to x? Give reasons for your answers.   (4)

                                                                      [9]

                                                       Please turn over

QUESTION 10

In the diagram below, ABCD is a cyclic quadrilateral with AC = AD. Tangents AC and CT touch the circle at A and C respectively.
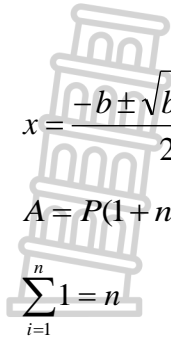

Prove that:

10.1 $\hat{B}_1 = \hat{B}_2$ (4)

10.2   BECH is a cyclic quadrilateral. (3)

10.3   CA is a tangent to the circle passing through points A, B and T. (5)

[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \;;\qquad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$In \; \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc.\cos A \quad area \; \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \; or \; B) = P(A) + P(B) - P(A \; and \; B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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CHIEF DIRECTORATE: EXAMINATIONS AND ASSESSMENT

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ERRATUM

TO: **ALL PRINCIPALS OF SCHOOLS IN THE FET BAND AND DISTRICT HEADS OF EXAMINATIONS**

FROM: **MRS P. JAPHTA**
   (a) CES: ASSESSMENTS INSTRUMENT DEVELOPMENT AND ITEM BANK MANAGEMENT SUBDIRECTORATE

SUBJECT: **MATHEMATICS P2 GR11 EXAMINATIONS ERRATUM**

DATE: **24 NOVEMBER 2023**

The Mathematics P2 Grade 11 for the November Examinations 2023 was written on Monday, the 13 November 2023. We were made aware of errors and omissions that was discovered during the marking process.

The amendment with regards to the marking was prepared in conjunction with the examiner and the moderator of the paper. This amendment addresses the errors and omissions and also ensures that learners are not disadvantaged. The following standardised approach to marking must be adopted across the Province.

QUESTION 5 – 3D DIAGRAM

- The 3D diagram is indeed a Grade 12 question and the whole question should be taken out of the question paper, resulting in the paper being marked out of 140. Although the question tests grade 11 knowledge the diagram should only be for grade 12. Teachers to use their discretion as it was not a difficult question.

QUESTION 6

6.2.2   Alternative - $-45° \leq x \leq 0°$ or $45° \leq x \leq 180°$ but $x \neq 90°$

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QUESTION 7

7.1.1   Alternative

$$OR^2 = (2x)^2 + 20^2$$

$$OR^2 = 4x^2 + 400$$

$$OR = \sqrt{4x^2 + 400}$$

QUESTION 8

(We apologise for the one line for the answers. However, learners could also use the additional space for additional answers.)

8.2.3   Since the question is only 2 marks, we must accept the memorandum reason as well (angles of a triangle) as a reason.

QUESTION 10

Statement correction: Question says tangent AC instead of AT. It then messes up the question.
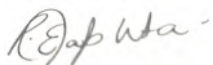
Recommendation: Mark according to what learners wrote, if they worked on AC being a tangent. QUESTION 10 has been regarded as a very difficult question by the learners and this incorrect tangent information could have given learners anxiety and stress. Happy to award an additional / free 2 ("anxiety") marks for the overall question. We do regret the typing error.

REMEDIAL ACTION

Question paper to be marked out of 140 (leaving question 5 out) and converted to 150 marks.

We request that this must be brought to the attention of all educators marking these papers and sincerely apologise for the inconvenience.

Yours in education.

24 November 2023

MRS P.E. JAPHTHA   **DATE**
(a) CES: ASSESSMENTS INSTRUMENT DEVELOPMENT
AND ITEM BANK MANAGEMENT SUBDIRECTORATE

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NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIORSERTIFIKAAT*

GRADE/*GRAAD* 11

NOVEMBER 2023

MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*
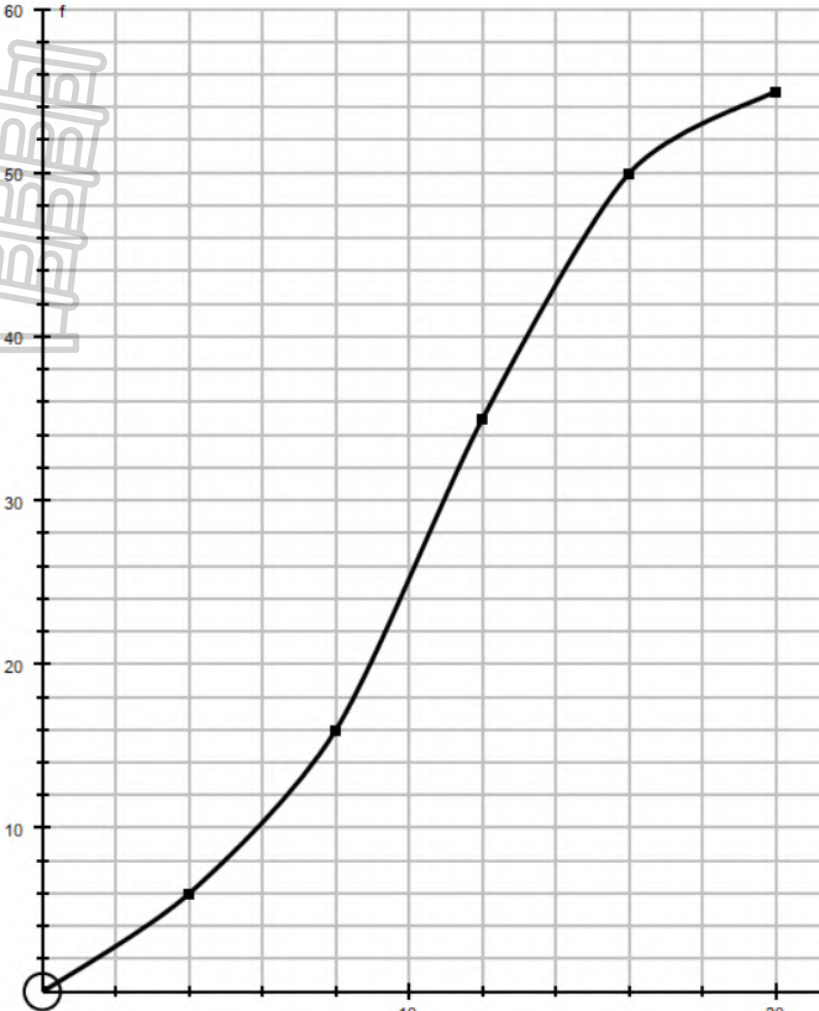
MARKS/*PUNTE*: 150

This marking guideline consists of 13 pages.
Hierdie nasienriglyn bestaan uit 13 bladsye.

QUESTION 1/*VRAAG 1*

1.1	6 boys / *seuns*	✓ answer / *antwoord*	(1)
1.2	Skewed to the right / positively skewed *Skeef na regs / Positief skeef*	✓ answer / *antwoord*	(1)
1.3	127 – 68 = 59	✓ working out / *bewerking* ✓ answer / *antwoord*	(2)
1.4.1	Median/*Mediaan* = 84 / Remains unchanged / *Bly onveranderd*	✓ answer / *antwoord*	(1)
1.4.2	$(87 \times 24 + 20) \div 24 = 87,8$ **OR/*OF*** 88	✓ working out / *bewerking* ✓ answer / *antwoord*	(2)
			[7]

QUESTION 2/*VRAAG 2*
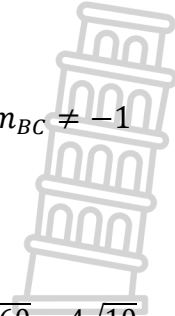
2.1	Time waiting / *Wagtyd* (minutes / *minute*)	Number of learners *Aantal leerders*	Cumulative frequency *Kumulatiewe frekwensie*	✓ for / *vir* 6; 16; 35
	$0 < x \leq 4$	6	6	✓ for / *vir* 50; 55
	$4 < x \leq 8$	10	16	
	$8 < x \leq 12$	19	35	
	$12 < x \leq 16$	15	50	
	$16 < x \leq 20$	5	55	(2)

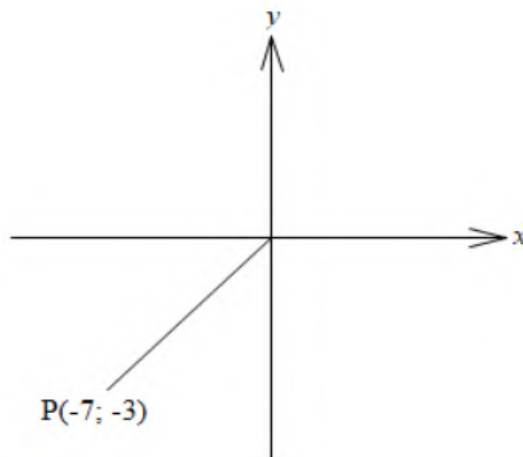| 2.2		✓ for grounding *vir anker* ✓ for endpoint *vir eindpunt* ✓✓ for any 2 correct points *vir enige 2 korrekte punte*	(4)
2.3	55 – 43 = 12 learners / *leerders* (accept/*aanvaar* 11 en 13)	✓ 55 ✓43 ✓12	(3)
2.4	$8 < x \leq 12$	✓ answer / *antwoord*	(1)
2.5	IQR / *IKW* = 14 – 7 = 7	✓ UQ ✓ LQ ✓ answer / *antwoord*	(3)
			[13]

QUESTION 3 / VRAAG 3

3.1	$m_{AB} = m_{BC}$ $\dfrac{-8-0}{x-5+8} = \dfrac{-14-0}{x+8}$ $\dfrac{-8}{x+3} = \dfrac{-14}{x+8}$ $-8x - 64 = -14x - 42$ $6x = 22$ $x = \dfrac{11}{3}$	✓ $\dfrac{-8-0}{x-5+8}$ ✓ $\dfrac{-14-0}{x+8}$ ✓ $-8x - 64$ ✓ $-14x - 42$ ✓ $x = \dfrac{11}{3}$	(5)
3.2.1	$M\left(\dfrac{6+0}{2}; \dfrac{-2-6}{2}\right)$ $= M(3; -4)$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
3.2.2	$AC = \sqrt{(2-0)^2 + \left(10-(-6)\right)^2}$ $AC = \sqrt{4 + 256}$ $AC = 2\sqrt{65}$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
3.2.3	$F(-4; 6)$	✓ x ✓ y	(2)
3.2.4	$m_{AB} = \dfrac{10-(-2)}{2-6} = \dfrac{12}{-4} = -3$ $m_{BC} = \dfrac{-2+6}{6-0} = \dfrac{4}{6} = \dfrac{2}{3}$ $m_{AB} \times m_{BC} \neq -1$ ∴ABCF is not a rectangle / *is nie 'n reghoek nie* **OR/*OF*** $AC = \sqrt{260} = 2\sqrt{65}$ $AB = \sqrt{160} = 4\sqrt{10}$ $BC = \sqrt{50} = 5\sqrt{2}$ $260 \neq 160 + 50$ $AC^2 \neq AB^2 + BC^2$	✓ m_{AB} ✓ m_{BC} ✓ $m_{AB} \times m_{BC} \neq -1$ ✓ $AB = \sqrt{160} = 4\sqrt{10}$ ✓ $BC = \sqrt{50} = 5\sqrt{2}$ ✓ $260 \neq 160 + 50$	(3)

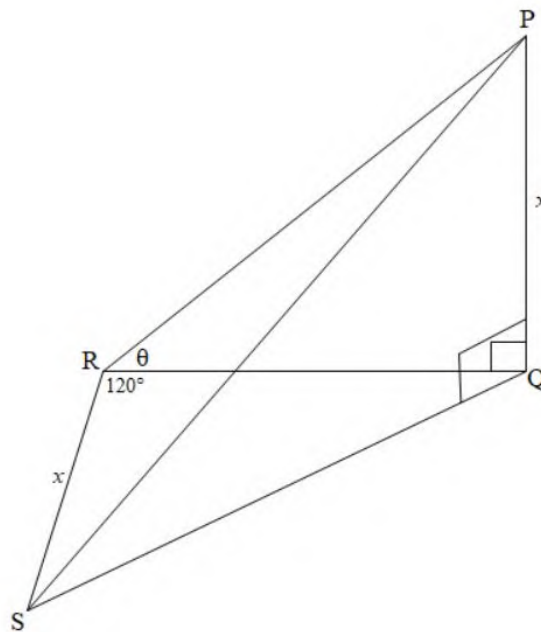| 3.2.5	$m_{AB} = -3$ $\therefore y = -3x + c$ $10 = -3(2) + c$ $c = 16$ $\therefore y = -3x + 16$ $m_{AC} = \frac{10-(-6)}{2-0} = 8$ $\therefore y = 8x - 6$	✓ m_{AB} ✓ substitution / *vervanging* ✓ equation of AB / *vergelyking van* AB ✓ gradient of AC/*gradiënt van* AC ✓ equation of /*vergelyking van* AC	(5)
3.2.6	$\tan A\hat{E}X = -3$ $\therefore A\hat{E}X = 108,43°$ $\tan A\hat{D}E = 8$ $\therefore A\hat{D}E = 82,87°$ $\hat{A} = 108,43° - 82,87° = 25,56°$	✓ $\tan A\hat{E}X = -3$ ✓ $A\hat{E}X$ ✓ $\tan A\hat{D}E = 8$ ✓ $A\hat{D}E$ ✓✓ answer / *antwoord*	(6)
3.2.7	$D\left(\frac{3}{4}; 0\right)$ $E\left(\frac{16}{3}; 0\right)$ $\therefore DE = \frac{55}{12}$ units / *eenhede* Perpendicular height / *Loodregte hoogte* $= 10$ units / *eenhede* $\therefore A = \frac{1}{2} \times \frac{55}{12} \times 10 = 22,92$ units^2/ *eenhede*^{2} **OR/*OF*** $AD = 10,08$ $AE = 10,54$ $Area \ of \ \Delta \ ADE = \frac{1}{2} \ AD.AE \sin A$ $Area \ of \ \Delta ADE = \frac{1}{2}(10,08)(10,54)\sin 25,56$ $Area \ of \ \Delta ADE = 22,92 \ units/eenhede \ ^2$	✓ coordinates of D / *koördinate van* D ✓ coordinates of E / *koördinate van* E ✓ length of DE / *lengte van* DE ✓ height / *hoogte* ✓ answer / *antwoord* ✓ $AD = 10,08$ ✓ $AE = 10,54$ ✓ Formula/formule ✓ substitution/vervanging ✓ answer/antwoord	(5)
			[30]

QUESTION 4 / VRAAG 4

4.1	$\tan \theta = \frac{3}{7}$ $r^2 = (-7)^2 + (-3)^2$ $r^2 = 58$ $r = \sqrt{58}$ $\dfrac{\sin \theta + \cos \theta}{2 \sin \theta}$ $= \dfrac{\frac{-3}{\sqrt{58}} + \left(\frac{-7}{\sqrt{58}}\right)}{2\left(\frac{-3}{\sqrt{58}}\right)}$ $= \dfrac{-10}{\sqrt{58}} \times \dfrac{\sqrt{58}}{-6}$ $= \dfrac{5}{3}$	✓ correct quadrant / *korrekte kwadrant* ✓ $\tan \theta = \frac{3}{7}$ ✓ for/*vir* ✓ substitution / *vervanging* ✓ simplification / *vereenvoudiging* ✓ answer / *antwoord*	(6)

4.2.1	$\tan(-32°)$ $= -\tan 32°$ $= -\dfrac{p}{\sqrt{1-p^2}}$		✓ method used / triangle *metode gebruik / driehoek* ✓ $-\tan 32°$ ✓ answer / *antwoord*		(3)
4.2.2	$\sin 418°$ $= \sin 58°$ $= \sqrt{1-p^2}$	$= \sin(90° - 32)$ $= \cos 32°$ $= \sqrt{1-p^2}$	✓ $\sin 58°$ ✓ answer / *antwoord*	✓ $\cos 32°$ ✓ answer *antwoord*	(2)
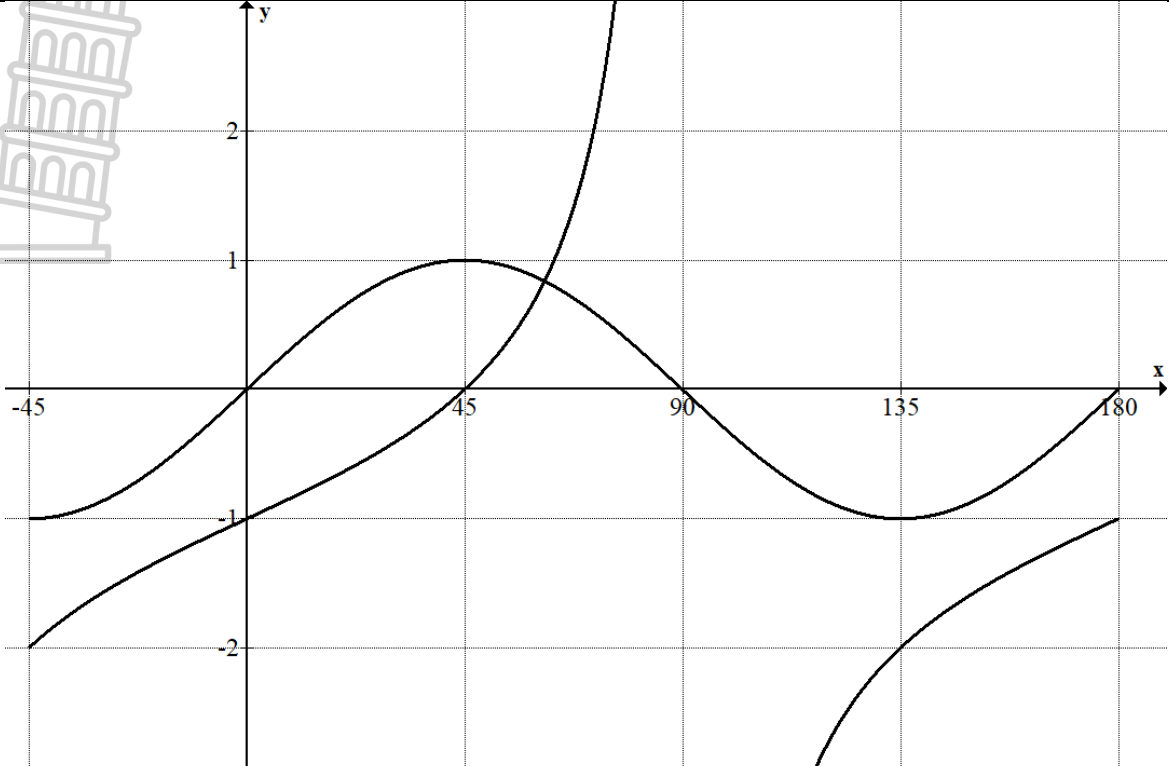
4.3	$\dfrac{\sin 120°.\tan 300°}{\cos(-60°).\tan 225°}$	$\checkmark$ $\sin 60°$	
	$= \dfrac{\sin 60° . -\tan 60°}{\cos 60°.\tan 45°}$	$\checkmark$ $-\tan 60°$ $\checkmark$ $\cos 60°$ $\checkmark$ $\tan 45°$ $\checkmark$ for numerator S/A / *vir teller* $\checkmark$ for denominator S/A / *vir noemer*	
	$= \dfrac{\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{1}}{\frac{1}{2}.\,1}$	$\checkmark$ answer / *antwoord*	
	$= -\dfrac{3}{2} \times \dfrac{2}{1} = -3$		(7)
4.4	$\dfrac{\cos^2 x - \cos x - \sin^2 x}{2\sin x \cos x + \sin x} = \dfrac{1}{\tan x} - \dfrac{1}{\sin x}$ $\text{LHS/LK} = \dfrac{\cos^2 x - \cos x - (1 - \cos^2 x)}{2\sin x \cos x + \sin x}$ $\text{LHS} = \dfrac{2\cos^2 x - \cos x - 1}{2\sin x \cos x + \sin x}$ $\text{LHS} = \dfrac{(2\cos x + 1)(\cos x - 1)}{\sin x(2\cos x + 1)}$ $\text{LHS} = \dfrac{\cos x - 1}{\sin x}$ $\text{LHS} = \dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x}$ $\text{LHS} = \dfrac{1}{\tan x} - \dfrac{1}{\sin x} = \text{RHS/RK}$	$\checkmark$ $1 - \cos^2 x$ $\checkmark$ simplifying / *vereenvoudiging* $\checkmark$ factoring the top / *faktorisering teller* $\checkmark$ factoring the bottom / *faktorisering noemer* $\checkmark$ $\dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x}$	(5)
4.5	$\sin(5\theta) = \cos(\theta - 40°)$ $\sin(5\theta) = \sin[90° - (\theta - 40°)]$ $\sin(5\theta) = \sin(130° - \theta)$ $5\theta = 130° - \theta + 360°.k$ $6\theta = 130° + 360.k$ $\theta = 21{,}67° + 60°.k$ **OR / OF** $5\theta = 180° - (130° - \theta) + 360°.k$ $4\theta = 50° + 360.k$ $\theta = 12{,}5° + 90°.k$ where/*waar* $k\epsilon Z$ **OR / OF** $0 - 5\theta = 360 - (\theta - 40) + 360k$ $-4\theta = 310 + 360k$ $\theta = -77{,}5 - 90k$	$\checkmark$ co – function / *ko-funksie* $\checkmark$ $5\theta = 130° - \theta + 360°.k$ $\checkmark$ $\theta = 21{,}67° + 60°.k$ $\checkmark$ $5\theta = 180° - (130° - \theta) + 360°.k$ $\checkmark$ $4\theta = 50° + 360.k$ $\checkmark$ $\theta = 12{,}5° + 90°.k$ $\checkmark$ co-function /*ko-funksie* $\checkmark$ $-6\theta = -130 + 360k$ $\checkmark$ $\theta = 21{,}67 - 60k$ $\checkmark$ $90 - 5\theta = 360 - (\theta - 40) + 360k$ $\checkmark$ $-4\theta = 310 + 360k$ $\checkmark$ $\theta = -77{,}5 - 90k$	(6)
			[29]

8 MATHEMATICS P2/WISKUNDE V2 (EC/NOVEMBER 2023)

QUESTION 5/VRAAG 5
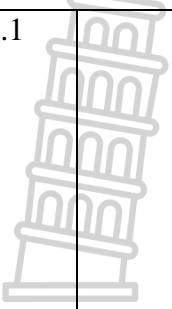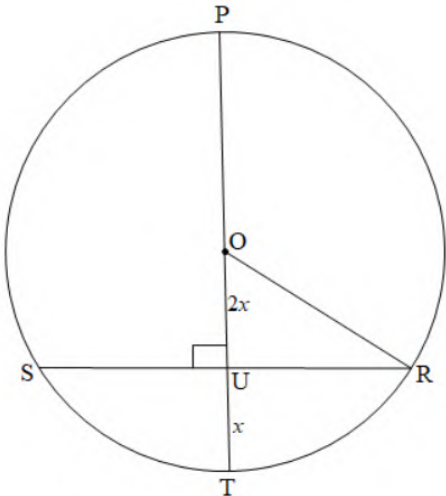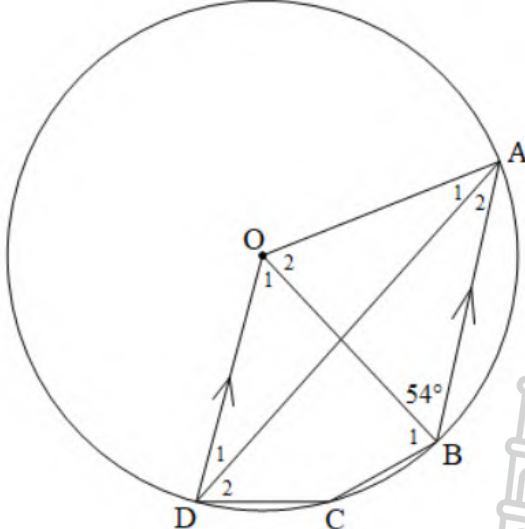
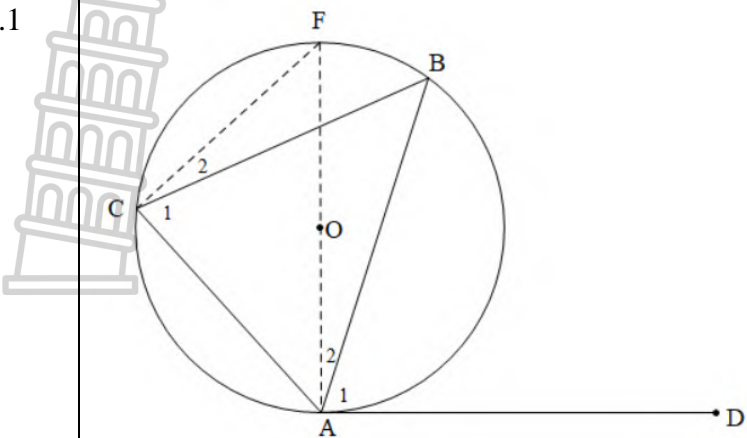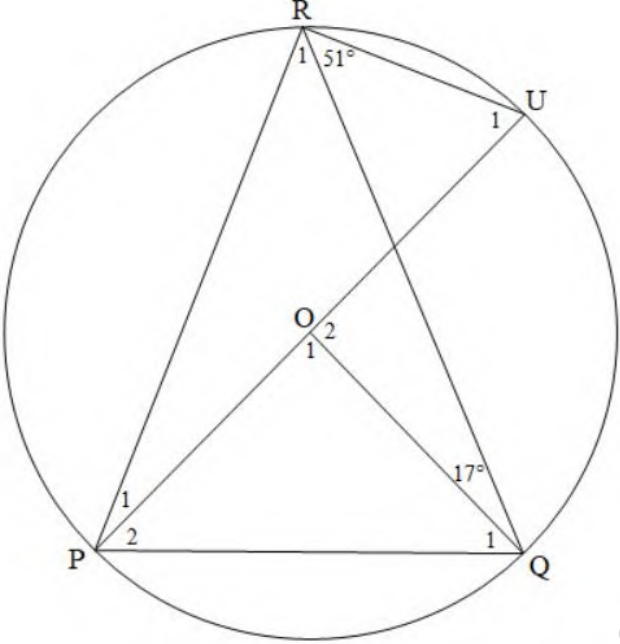| 5.1	$\tan\theta = \dfrac{x}{QR}$ $QR = \dfrac{x}{\tan\theta}$	✓ $\tan\theta = \dfrac{x}{QR}$ ✓ answer / *antwoord*	(2)
5.2	$QS^2 = x^2 + \left(\dfrac{x}{\tan\theta}\right)^2 - 2x\left(\dfrac{x}{\tan\theta}\right)\cos 120°$ $QS^2 = x^2 + \dfrac{x^2}{\tan^2\theta} - \dfrac{2x^2}{\tan\theta}\left(-\dfrac{1}{2}\right)$ $QS^2 = x^2 + \dfrac{x^2}{\tan^2\theta} + \dfrac{x^2}{\tan\theta}$ $QS = \sqrt{x^2\left(1 + \dfrac{1}{\tan^2\theta} + \dfrac{1}{\tan\theta}\right)}$ $QS = x.\sqrt{\dfrac{1}{\tan^2\theta} + \dfrac{1}{\tan\theta} + 1}$	✓ for the cosine rule *vir die cosinusreël* ✓ $-\dfrac{1}{2}$ ✓ simplification *vereenvoudiging* ✓ for square rooting *vir vierkantswortel*	(4)
5.3.1	$QS = 15\sqrt{\dfrac{1}{\tan^2 22°} + \dfrac{1}{\tan 22°} + 1}$ $QS = 46,48 \text{ m}$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
5.3.2	$\tan Q\hat{P}S = \dfrac{46,48}{15}$ $Q\hat{P}S = 72,11°$	✓ for using tan / *vir gebruik van* tan ✓ answer / *antwoord*	(2)
			[10]

QUESTION 6 / VRAAG 6

6.1		
	For f: ✓ shape ✓ endpoints ✓ TP and/*en* x-intercepts *Vir f*: ✓ *vorm* ✓ *eindpunte* ✓ DP and/*en* x-afsnitte For g: ✓ asymptote ✓ shape ✓ y-intercept *Vir g*: ✓ *asimptoot* ✓ *vorm* ✓ y-afsnit	(6)
6.2.1	$0° < x < 90°$	✓ answer / *antwoord* (1)

6.2.2	$-45° \leq x \leq 0°$ **or**/*of* $45° \leq x < 90°$
	                or/*of* $90° < x \leq 180°$
		✓✓✓ for each answer
		    vir elke antwoord (3)

6.3	$1 \leq y \leq 3$ **or**/*of* $y \epsilon [1; 3]$	✓✓ for the answer *vir die antwoord* (2)
		[12]

QUESTION 7 / VRAAG 7

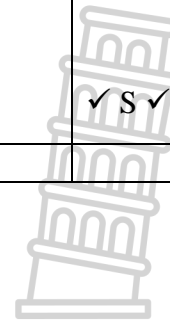|			
7.1			
7.1.1	$OR = 3x$	✓ answer / *antwoord*	(1)

7.1.2	UR = 20 cm (line from centre perpendicular…)
	    (*lyn vanaf middelpunt loodreg* …)
	 $(3x)^2 = (2x)^2 + 20^2$ (Pythagoras)
	 $9x^2 = 4x^2 + 400$
	 $5x^2 = 400$
	 $x^2 = 80$
	 $x = \sqrt{80} = 4\sqrt{5}$
		✓ S ✓ R
		 ✓ S and/*en* R
		
		 ✓ simplification
		    vereenvoudiging
		
		 ✓ answer / *antwoord*
			(5)

7.2			
7.2.1	$\hat{O}_1 = 54°$ (alt ∠s / *verw.* ∠*e*, OD ∥ AB) $\hat{A}_2 = 27°$ (∠ at centre) / (*middelpunts* ∠)	✓ S ✓ R ✓ S ✓ R	(4)
7.2.2	$\hat{C} = 153°$ (opp. ∠s of a cq) / (*teenoorst.* ∠*e van k.v*)	✓ S ✓ R	(2)
7.2.3	$\hat{A} = 54°$ (∠s opp = sides) / (∠*e teenoor = sye*) $\hat{A}_2 = 27°$ (proven) / (*reeds bewys*) ∴ $\hat{A}_1 = 27°$ ∴DA bisects $O\hat{A}B$ / DA *halveer* $O\hat{A}B$	✓ S and/*en* R ✓ S ✓ S	(3)
			[15]

QUESTION 8 / VRAAG 8

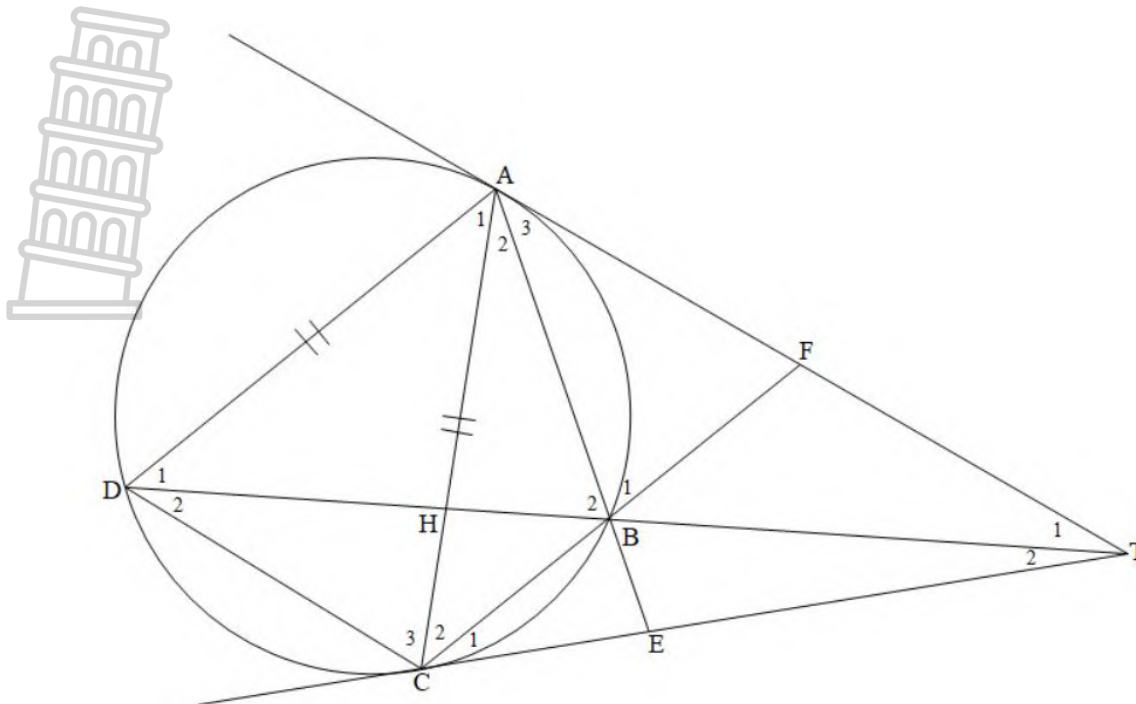| 8.1	 Let /*Laat* $\hat{A}_1 = x$ $\therefore \hat{A}_2 = 90° - x$ (tan $\perp$ rad) / (*raaklyn $\perp$ radius*) $F\hat{C}A = 90°$ ($\angle$ in a semi-circle) / ($\angle$ *in semi-sirkel*) $\hat{C}_2 = 90° - x$ ($\angle$s in the same segment) / ($\angle$e *in dies. segment*) $\therefore \hat{C}_1 = x$ $\therefore D\hat{A}B = B\hat{C}A$	✓ S ✓ R ✓ S and/*en* R ✓ S and/*en* R ✓ S	(5)
8.2			
8.2.1	$\hat{R}_1 = 39°$ ($\angle$ subtended by diameter)/ ($\angle$ *onderspan deur middellyn*)	✓ S ✓ R	(2)
8.2.2	$\hat{O}_1 = 78°$ ($\angle$ at centre) / (*Middelpunts $\angle$*)	✓ S ✓ R	(2)
8.2.3	$\hat{Q}_1 = 51°$ ($\angle$s opp = sides) / ($\angle$e *teenoor gelyke sye*)	✓ S ✓ R	(2)
8.2.4	$\hat{U}_1 = 68°$ ($\angle$s in the same segment) / ($\angle$e *in dieselfde segment*)	✓ S ✓ R	(2)
			[13]

QUESTION 9 / VRAAG 9

9.1.1	A$\hat{B}$F = x (tan-chord theorem) / (*raaklyn-koord stelling*)	✓ S ✓ R	(2)
9.1.2	B$\hat{C}$D = x (corresponding angles, BF‖CD) (*ooreenkomstige hoeke* ; BF ‖ CD)	✓ S ✓ R	(2)
9.2	Exterior angle = interior opp. angle *Buitehoek = teenoorstaande binnehoek.*	✓ R	(1)
9.3	B$\hat{D}$C = x (angles in the same segment) / (*hoeke in dieselfde segment*) F$\hat{B}$D = x (alternate angles BF‖CD) (*verwisellende hoeke* BF ‖ CD)	✓ S ✓ R ✓ S ✓ R	(4)
			[9]

QUESTION 10 / *VRAAG 10*

10.1	$\widehat{D} = \hat{C}_3$ (∠s opp. = sides) / (∠e *teenoor = sye*) $\widehat{D} = \hat{B}_1$ (ext. ∠ of a cq) / (*buite* ∠ *van k.v*) $\hat{C}_3 = \hat{B}_2$ (∠s in the same segment) / (∠e *in dieselfde segment*) $\hat{B}_1 = \hat{B}_2$	✓ S and/*en* R ✓ S ✓ R ✓ S and/*en* R (4)

10.2	$\hat{C}_1 + \hat{C}_2 = \widehat{D}$ (tan-chord theorem) / (*raaklyn-koord stelling*)
	But/*Maar* $\widehat{D} = \hat{B}_2$ (proven) / (*reeds bewys*)
	∴ $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$ (ext ∠ = int opp.angle) /
	   (*buite* ∠ = *teenoorst. binnehoek*)
	BECH is a c.q. / BECH *is 'n kv* (converse. ext. ∠ of a cq) /
	   (*omgekeerde buite* ∠ *van kv*)
		✓ S and/*en* R
		
		✓ S and/*en* R
		✓ R
		
		(3)

10.3	$\hat{B}_2 = \hat{A}_3 + \hat{T}_1$ (ext ∠ of a triangle) / (*buite* ∠ *van driehoek*)
	But/*Maar* $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$ (proven) / (*reeeds bewys*)
	And/ *En* AT = CT (tangents from the same point)
	   (*raaklyne vanaf dieselfde punt*)
	∴ $\hat{C}_1 + \hat{C}_2 = \hat{A}_3 + \hat{A}_2$ (∠s opp. = sides) / (∠e *teenoor = sye*)
	$\hat{A}_3 + \hat{A}_2 = \hat{A}_3 + \hat{T}_1$ (conv. tan-chord) /
	   (*omgekeerde raaklyn-koord*)
	∴ $\hat{A}_2 = \hat{T}_1$
	CA is a tangent to circle ABT / CA *is 'n raaklyn aan sirkel* ABT)
		✓ S and/*en* R
		✓ S and/*en* R
		✓ S and/*en* R
		✓ S and/*en* R
		✓ S and/*en* R
		
		(5)

		[12]

TOTAL/*TOTAAL*: 150