GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

JUNE 2024

GRADE 11

MATHEMATICS

(PAPER 1)

TIME: **2 hours**

MARKS: **100**

6 pages

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Present your answers according to the instructions of each question.
4. Clearly show ALL calculations, diagrams, graphs et cetera which were used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Number the answers according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Determine the value(s) of x for which the expression: $\sqrt{\frac{x^2+4}{9-x}}$

1.1.1 has real value(s). (1)

1.1.2 is not defined. (1)

1.1.3 Given, x =1, is the expression rational or irrational? Validate your answer with an appropriate calculation. (1)

1.1.4 Determine a value for x where the expression will yield a recurring decimal value? (1)

1.2 Given: $2x^2 - 3x - k = 0$. Determine x if:

1.2.1 $k = 5$ (2)

1.2.2 $k = 4$ (correct to TWO decimal places) (3)

1.3 Solve for x:

1.3.1 $x - 4 = -\sqrt{x-2}$ (5)

1.3.2 $x^2 + 4 > 3x + 2$ (4)

1.4 Solve simultaneously for x and y.

$2x = y + 2$ and $2x^2 = 2 - y^2$ (6)

1.5 For which values of p will $-2x^2 + 4x - 3 = -p$:

1.5.1 have two unequal roots? (4)

1.5.2 have roots that are BOTH positive? (2)

1.6 Show that the equation $(p^2+1)x^2 = -2pqx - q^2$ has no real roots for p and $q \in$ Real numbers and $q \neq 0$. (5)

[35]

QUESTION 2

2.1 Simplify WITHOUT using a calculator:

2.1.1 $(2^{-1}+3^{-1})^2$ (3)

2.1.2 $\dfrac{2.4^{x+2}-4^{x-3}}{2^x.2^x}$ (3)

2.1.3 $3^{-\frac{1}{2}}\left[\sqrt{12}+\sqrt[3]{3\sqrt{3}}\right]$ (4)

2.2 Solve for x WITHOUT using a calculator:

2.2.1 $2x^{\frac{3}{4}}=16$ (3)

2.2.2 $5^{x+1}+5^x=150$ (4)

2.3 WITHOUT using a calculator, show that $\dfrac{9-\sqrt{54}}{6\sqrt{2}}$ is equal to $\dfrac{3\sqrt{2}-2\sqrt{3}}{4}$. (3)

[20]

QUESTION 3

Given the number pattern:

$4\ ;\ -3\ ;\ -10\ ;\ \ldots\ ;\ -227$

3.1 Write down the value of the 4^{th} term of this pattern. (1)

3.2 Determine the general term of the number pattern in the form $T_n=an+b$. (1)

3.3 Calculate the number of terms of the pattern. (2)

3.4 The given number pattern above is also the FIRST differences of a quadratic number pattern, $T_n=an^2+bn+c$.
Determine if the quadratic pattern will have a maximum or minimum value.
Validate your answer with an appropriate calculation. (2)

[6]

QUESTION 4

Given the quadratic number pattern:

$69\ ;\ 0\ \ -63\ ;\ \ldots$

4.1 Write down the value of the next term in the pattern. (1)

4.2 Calculate an expression for the n^{th} term of the quadratic pattern. (4)

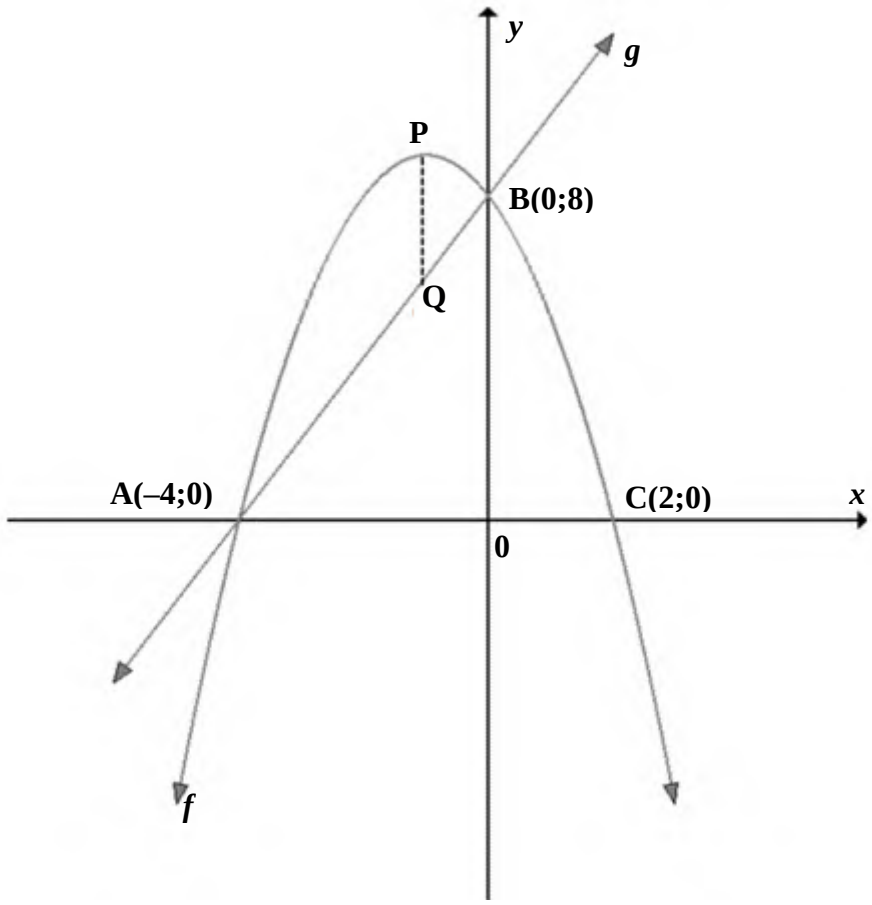
4.3 Determine the value of the SMALLEST term in this pattern. (4)

[9]

QUESTION 5

Sketched below are the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = mx + k$.

- The x-intercepts of f are at points A and C.
- The graphs of f and g intersect at points A and B respectively.
- Point P is the turning point of f.
- Q is a point on g such that line PQ is parallel to the y-axis.



5.1 Determine the equation of g. (3)

5.2 Calculate the values of a, b and c in $f(x) = ax^2 + bx + c$. (5)

5.3 Show that the coordinates of P are $(-1 ; 9)$. (2)

5.4 Determine the range of f. (1)

5.5 Determine the equation of a line p perpendicular to g passing through point C. (3)

5.6 Calculate the length of line PQ. (2)

5.7 For which value(s) of x is $\frac{f(x)}{g(x)} \geq 0$? (2)

[18]

P.T.O.

QUESTION 6

Given: $k(x)=\frac{-4}{x+3}+1$ and $h(x)=2^{-x}-4$

6.1 Write down the equations of the asymptotes of k. (2)

6.2 Determine the x and y-intercepts of k. (2)

6.3 Write down the equation of the asymptote of h. (1)

6.4 Sketch the graph of k and h on the same axes. Clearly indicate ALL intercepts with the axes as well as the asymptotes. (4)

6.5 If $p(x)=3h(x)$, write down the equation of the asymptote of p. (1)

6.6 Write down the equation of q, the reflection of h in the y-axis. (1)

6.7 Determine the x-values for which $h(x)-q(x)=0$ (1)

[12]

TOTAL: 100

END

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

JUNE 2024

GRADE 11

MARKING GUIDELINES

MATHEMATICS (PAPER 1)

11 pages

INSTRUCTIONS AND INFORMATION

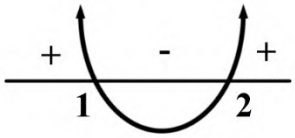
A – ACCURACY
CA. – CONTINUED ACCURACY

NOTE:

- If a candidate answerd a question TWICE, mark only the first attempt.
- If a candidate crossed OUT an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a problem is UNACCEPTABLE.

QUESTION 1

1.1	1.1.1	$x<9$	✓ answer	(1)
	1.1.2	$x=9$	✓ answer	(1)
	1.1.3	$\sqrt{\frac{(1)^2+4}{9-1}}$ $\sqrt{\frac{5}{8}}$ $\therefore$ *Irrational*	✓ answer	(1)
	1.1.4	$x=0$	✓ answer	(1)
1.2	1.2.1	$2x^2-3x-k=0$ $2x^2-3x-5=0$ $(2x-5)(x+1)=0$ $\therefore x=\frac{5}{2}\cdots of\cdots x=-1$	✓ factors ✓ answer	(2)
	1.2.2	$2x^2-3x-4=0$ $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)}$ $\therefore x=2,35\cdots of\cdots x=-0,85$ **NOTE:** Penalise 1 mark for incorrect rounding in this question ONLY.	✓ substitution ✓✓ answer	(3)
1.3	1.3.1	$x-4=-\sqrt{x-2}$ $16-8x-x^2=x-2$ $x^2-9x+18=0$ $(x-6)(x-3)=0$ $x=3$ of $x\neq 6$	✓ square both sides ✓ standard form ✓ factors ✓ critical values ✓ rejection	(5)

	1.3.2	$x^2+4>3x+2$ $x^2-3x+2>0$ $(x-2)(x-1)>0$ $x<1$ or $x>2$ OR $x\in(-\infty;1)\cup(2;\infty)$  	✓ standard form ✓ factors ✓✓ answers	(4)
1.4	$y+2=2x$ $\therefore y=2x-2\cdots\cdots(1)$ $2x^2=2-y^2\cdots\cdots(2)$ $\therefore 2x^2=2-(2x-2)^2$ $2x^2=2-(4x^2-8x+4)$ $2x^2=2-4x^2+8x-4$ $\therefore 6x^2-8x+2=0$ $\therefore 3x^2-4x+1=0$ $(3x-1)(x-1)=0$ $\therefore x=\frac{1}{3}\cdots of\cdots x=1$ $\therefore y=-\frac{4}{3}\cdots of\cdots y=0$ **OR** $y+2=2x$ $\therefore x=\frac{y+2}{2}\cdots\cdots(1)$ $2x^2=2-y^2\cdots\cdots(2)$ $\therefore 2(\frac{y+2}{2})^2=2-y^2$ $2(\frac{y^2+4y+4}{4})=2-y^2$ $\therefore (\frac{y^2+4y+4}{2})=2-y^2$ $\therefore y^2+4y+4=4-2y^2$ $\therefore 3y^2+4y=0$ $y(3y+4)=0$ $\therefore y=0\cdots\cdots of\cdots\cdots y=-\frac{4}{3}$ $\therefore x=1\cdots\cdots of\cdots\cdots x=\frac{1}{3}$	✓ y as subject ✓ substitution ✓ standard form ✓ factors ✓ both x- values ✓ both y- values **OR** ✓ x as subject ✓ substitution ✓ standard form ✓ factors ✓ both y-values ✓ both x-values	(6)
1.5	1.5.1	For unequal roots: $\Delta>0$	✓ condition of Δ	

		$\therefore 2x^2-4x+(3-p)=0$ $\Delta=b^2-4ac$ $\therefore \Delta=(-4)^2-4(2)(3-p)$ $\Delta=16-24+8p$ $\therefore \Delta=-8+8p$ $\therefore -8+8p>0$ $8p>8$ $\therefore p>1$	✓ substitution into Δ ✓ expression for Δ ✓ answer	(4)
	1.5.2	From 1.5.1 $3-p>0\cdots\cdots(a>0\cdots and\cdots b<0)$ $\therefore -p>-3$ $\therefore 1<p<3$ **NOTE**: Any other valid method	✓ $c>0$ ✓ answer	(2)
1.6	$(p^2+1)x^2=-2pqx-q^2$ $\therefore (p^2+1)x^2+2pqx+q^2=0$ $\Delta=(2pq)^2-4(p^2+1)q^2$ $\Delta=4p^2q^2-4p^2q^2-4q^2$ $\therefore \Delta=-4q^2$ $\therefore q^2>0,\cdots q\in\Re;\cdots q\neq 0$ $\therefore -4q^2<0$ $\therefore$ Roots non-real		✓ standard form ✓ substitute into Δ ✓ simplification ✓ expression for Δ ✓ explanation	(5)
				[35]

QUESTION 2

2.1	2.1.1	$(2^{-1}+3^{-1})^2$ $=(\frac{1}{2}+\frac{1}{3})^2$ $=(\frac{3+2}{6})$ $=(\frac{5}{6})^2$ $=\frac{25}{36}$	✓ converting exponents ✓ simplification ✓ answer	(3)
	2.1.2	$\frac{2.4^{x+2}-4^{x-3}}{2^x.2^x}$ $=\frac{2.4^x.4^2-4^x.4^{-3}}{4^x}$ $=\frac{4^x(32-\frac{1}{64})}{4^x}$ $=32-\frac{1}{64}$ $=\frac{2\,048-1}{64}$ $=\frac{2\,047}{64}$ **NOTE:** Accept $31\frac{63}{64}$	✓✓ factors ✓ answer	(3)
	2.1.3	$3^{-\frac{1}{2}}[\sqrt{12}+\sqrt[3]{(3\sqrt{3})}]$ $=3^{-\frac{1}{2}}[2\sqrt{3}+(3\sqrt{3})^{\frac{1}{3}}]$ $=3^{-\frac{1}{2}}[2.3^{\frac{1}{2}}+3^{\frac{1}{3}}.3^{\frac{1}{6}}]$ $=3^{-\frac{1}{2}}[2.3^{\frac{1}{2}}+3^{\frac{1}{2}}]$ $=3^{-\frac{1}{2}}.3^{\frac{1}{2}}(2+1)$ $=3^0(3)$ $=3$	✓ simplify brackets ✓ surds as rational exponents ✓ factors ✓ answer	(4)

	2.2.1	$2x^{\frac{3}{4}} = 16$ $\therefore x^{\frac{3}{4}} = 8$ $\therefore x^{\frac{3}{4}\times\frac{4}{3}} = 8^{\frac{4}{3}}$ $\therefore x = (\sqrt[3]{8})^4$ $\therefore x = 16$ **NOTE**: Any other valid method.	✓ method ✓ x as subject ✓ answer	(3)
	2.2.2	$5^{x+1} + 5^x = 150$ $5^x.5^1 + 5^x = 150$ $\therefore 5^x(5^1 + 1) = 150$ $\therefore 5^x(6) = 150$ $5^x = 25$ $\therefore 5^x = 5^2$ $\therefore x = 2$ **NOTE**: Any other valid method.	✓ factors ✓ simplification ✓ 25 as base of 5 ✓ answer	(4)
2.3	$\frac{9-\sqrt{54}}{6\sqrt{2}}$ $= \frac{(9-\sqrt{9.2.3})}{6\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{9\sqrt{2} - 3\sqrt{2}.\sqrt{3}.\sqrt{2}}{6.2}$ $= \frac{9\sqrt{2} - 3.2\sqrt{3}}{12}$ $= \frac{3(3\sqrt{2} - 2\sqrt{3})}{12}$ $= \frac{3\sqrt{2} - 2\sqrt{3}}{4}$	✓ $\times \frac{\sqrt{2}}{\sqrt{2}}$ ✓ simplification ✓ factorisation	(3)
				[20]

QUESTION 3

3.1	$T_4 = -17$	✓ answer	(1)
3.2	$T_n = -7n + 11$	✓ answer	(1)
3.3	$T_n = -7n + 11$ $\therefore -227 = -7n + 11$ $-238 = -7n$ $\therefore n = 34$	✓ substitute T_n ✓ answer	(2)
3.4	$2a = -7$ $a = -\frac{7}{2}$ $\therefore$ maximum **NOTE:** Any other valid method.	✓ value for a ✓ conclusion	(2)
			[6]

QUESTION 4

4.1	69 0 −63 T_4 −69 −63 −57 6 6 next term (T_4) is: -120	✓ answer	(1)
4.2	$T_n = an^2 + bn + c$ $\therefore T_2 = a(2)^2 + b(2) + c = 4a + 2b + c$ $\therefore T_1 = a(1)^2 + b(1) + c = a + b + c$ $\therefore T_2 - T_1 = 3a + b$ $2a = 6$ $\therefore a = 3$ $3a + b = -69$ $3(3) + b = -69$ $9 + b = -69$ $\therefore b = -78$ $a + b + c = 69$ $3 + (-78) + c = 69$ $\therefore c = 144$ $\therefore T_n = 3n^2 - 78n + 144$	✓ 2^{nd} difference ✓ value for a ✓ value for b ✓ value for c	(4)

4.3	Smallest term (minimum) is at the turning point of the quadratic equation: $\therefore n = \frac{-b}{2a} = \frac{-(-78)}{2(3)} = \frac{78}{6} = 13$ $T_n = 3n^2 - 78n + 144$ $\therefore T_{13} = 3n^2 - 78n + 144$ $\therefore T_{13} = 3(13)^2 - 78(13) + 144$ $\therefore T_{13} = -363$	✓ substitute into A.O.S ✓ value for n ✓ substitution ✓ answer	(4)
			[9]

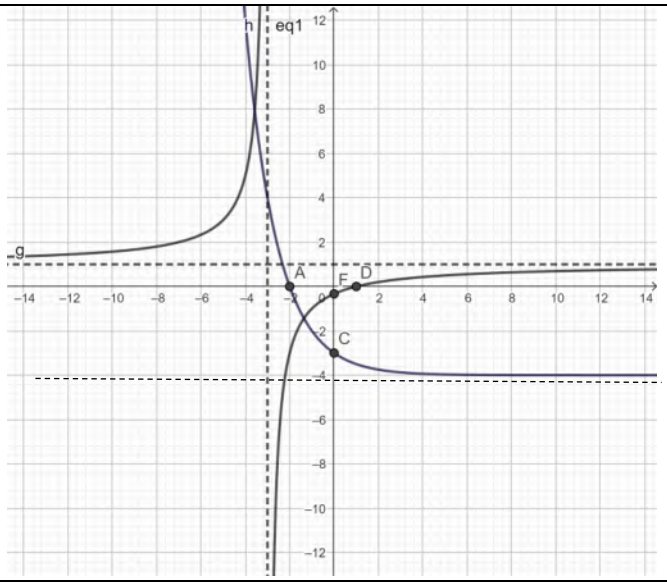
QUESTION 5

5.1	The points A$(-4\ ;\ 0)$ and B$(0\ ;\ 8)$ lie on g. $m_{AB} = \frac{8-0}{0-(-4)}$ $m_{AB} = 2$ $y = mx + c$ $\therefore 8 = 2(0) + c \cdots\cdots A(0;8)$ $\therefore c = 8$ $\therefore g(x) = 2x + 8$ **NOTE:** Any other valid method.	✓ substitute for m_{AB} ✓ value of m_{AB} ✓ value of c	(3)
5.2	$f(x) = a(x - x_1)(x - x_2)$ $f(x) = a(x + 4)(x - 2)$ subst $(0;8)$ $8 = a(0 + 4)(0 - 2)$ $8 = a(-8)$ $a = -1$ $f(x) = -(x + 4)(x - 2)$ $f(x) = -(x^2 + 2x - 8)$ $f(x) = -x^2 - 2x + 8$ $\therefore a = -1$ $b = -2$ $c = 8$	✓ substitute points A and C ✓ value of a ✓ standard form (A) ✓ value of b ✓ value of c	(5)

5.3	$f(x) = -x^2 - 2x + 8$ $\therefore x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)}$ $\therefore x = -1$ $\therefore f(-1) = -(-1)^2 - 2(-1) + 8$ $\therefore f(-1) = 9$ **NOTE:** Any other valid method.	✓ correct substitution ✓ correct substitution	(2)
5.4	$y \leq 9$	✓ answer	(1)
5.5	$m_g = 2$ $\therefore m_p = -\frac{1}{2}$ $y - y_1 = m(x - x_1)$ $\therefore y - 0 = -\frac{1}{2}(x - 2)$ $\therefore y = -\frac{1}{2}x + 1$	✓ m_p ✓ substitute point C ✓ answer	(3)
5.6	$f(-1) = 9$ $g(-1) = 6$ $\therefore f(-1) - g(-1)$ $\therefore PQ = 9 - 6$ $\therefore PQ = 3$ **OR** $DP(-1;9)$ $g(-1) = 6$ $\therefore PQ = 9 - 6$ $\therefore PQ = 3$	✓ $f(-1) = 9$ and $g(-1) = 6$ ✓ answer **OR** ✓ using TP and $g(-1)$ ✓ answer	(2)
5.7	$x \leq 2$; $x \neq -4$	✓✓ answer (A)	(2)
			[18]

QUESTION 6

6.1	$x = -3$ $y = 1$	✓ answer ✓ answer	(2)

6.2	$f(x)=\frac{-4}{x+3}+1$ $0=\frac{-4}{x+3}+1$ $-1=\frac{-4}{x+3}$ $-x-3=-4$ $-x=-1$ $\therefore x=1$ $y=\frac{-4}{0+3}+1$ $y=\frac{-4}{3}+1$ $\therefore y=-\frac{1}{3}$	✓ x-intercept ✓ y- intercept	(2)
6.3	$y=-4$	✓ answer	(1)
6.4		✓ shape of k ✓ k and h intercepts ✓ asymptotes k and h ✓ shape of h	(4)
6.5	$p(x)=3h(x)$ $\therefore p(x)=3(2^{-x}-4)$ $\therefore p(x)=3.2^{-x}-12$ asymptote: $y=-12$ **NOTE**: Answer only, full marks.	✓ answer	(1)
6.6	$q(x)=2^{x}-4$	✓ answer	(1)
6.7	$x=0$	✓ answer	(1)
			[12]

TOTAL: 100