GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINSIALE EKSAMEN

JUNIE 2024

GRAAD 11

WISKUNDE
(VRAESTEL 2)

TYD: **2 uur**

PUNTE: **100**

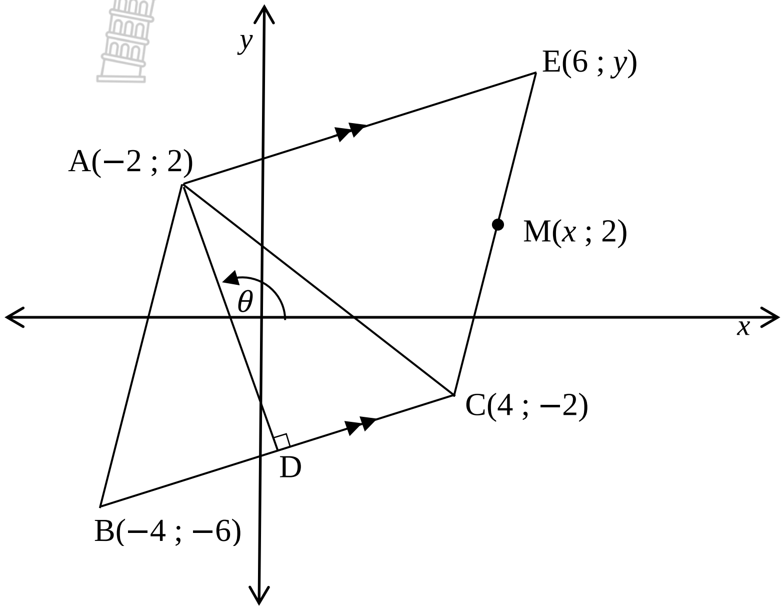
10 bladsye

INSTRUKSIES EN INLIGTING

1. Beantwoord AL die vrae.
2. Hierdie vraestel bestaan uit 8 vrae.
3. Dui ALLE berekeninge, diagramme, grafieke ensovoorts wat jy gebruik het om jou antwoorde te bepaal, duidelik aan.
4. Volpunte sal NIE noodwendig aan antwoorde alleen toegeken word NIE.
5. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders aangedui.
6. Diagramme is NIE noodwendig volgens skaal geteken NIE.
7. Indien nodig rond die antwoorde tot TWEE desimale plekke af, tensy anders aangedui.
8. Nommer die antwoorde korrek volgens die nommeringstelsel wat in hierdie vraestel gebruik word.
9. Skryf netjies en leesbaar.

VRAAG 1

In die diagram hieronder is A(−2 ; 2), B(−4 ; −6), C(4 ; −2) en E(6 ; y) die hoekpunte van ʼn vierhoek met AE || BC. D lê op BC sodanig dat AD ⊥ BC en AC is getrek. M(x ; 2) is ʼn punt op lyn EC.



1.1 Bereken die lengte van BC. (Laat jou antwoord in wortelvorm.) (2)

1.2 Indien M die middelpunt is van EC, bepaal die waarde van x en y . (2)

1.3 Bereken die gradiënt van BC. (2)

1.4 Bepaal die vergelyking van AD in die vorm $y = mx + c$. (3)

1.5 Bepaal θ, die inklinasiehoek van lyn AD. (2)

1.6 F is ʼn punt in die 4de kwadrant sodat ABFC ʼn parallelogram is.
Bepaal die koördinate van F. (2)

1.7 Toon aan dat die koördinate van D is (0,8 ; −3,6). (4)

1.8 Vervolgens of andersins, bepaal die oppervlak van ABCE, indien dit gegee is dat ABCE ʼn parallelogram is. (3)

[20]

VRAAG 2

2.1 Gegee: $\sin 20° = p$. Bepaal die volgende in terme van p:

2.1.1 $\sin(-20°)$ (2)

2.1.2 $\sin 160°$ (2)

2.1.3 $1 - \sin^2 70°$ (3)

2.2 Vereenvoudig tot 'n enkele trigonometriese funksie **sonder die gebruik van 'n sakrekenaar**:

$$\frac{\tan 225° - \sin(180° + \theta)\cos(\theta + 90°)}{\sin\theta.\sin(360° + \theta) + \cos\theta(-\theta)}$$ (6)

2.3 Gegee: $k \sin\beta = -5$ en $k\cos\beta - 1 = 0$, waar $k > 0$.

2.3.1 Verduidelik waarom $\beta \in [270°\,;360°]$. (4)

2.3.2 **Sonder die gebruik van 'n sakrekenaar**, bepaal die waarde van $\tan\beta$. (1)

2.3.3 Vervolgens of andersins, bepaal die numeriese waarde van k **sonder die gebruik van 'n sakrekenaar**. (2)

2.4 Gegee die identiteit: $\sin^2\alpha\left(\tan\alpha + \frac{1}{\tan\alpha}\right) = \tan\alpha$

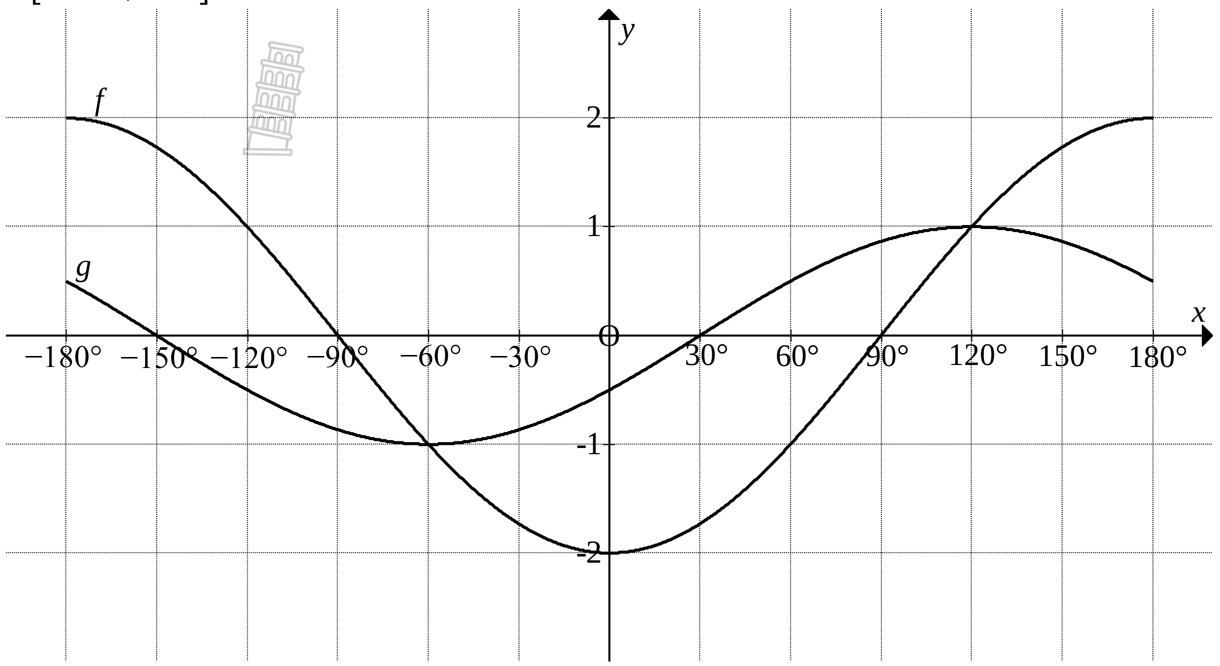
2.4.1 Bewys die identiteit. (5)

2.4.2 Vervolgens of andersins, bepaal die algemene oplossing van:
$\sin^2\alpha(\tan^2\alpha + 1) = \sqrt{3}\tan\alpha$ (4)

[29]

VRAAG 3

In die diagram hieronder, is die grafieke van $f(x) = a\cos x$ en $g(x) = \sin(x + p)$ vir $x \in [-180°; 180°]$.



3.1 Bepaal die waardes van *a* en *p*. (2)

3.2 Skryf die waardeversameling van f neer. (2)

3.3 As $h(x) = a\cos(bx)$, en die periode van *h* is 60°, skryf die waarde van *b* neer. (1)

3.4 Bepaal die waarde(s) van *x* indien:

3.4.1 $f(x) < g(x)$ (2)

3.4.2 $x.f(x) > 0$ (2)

3.5 As $k(x) = \cos(2x + t)$, bepaal die waarde van t as $k(x) = g(2x)$. (2)

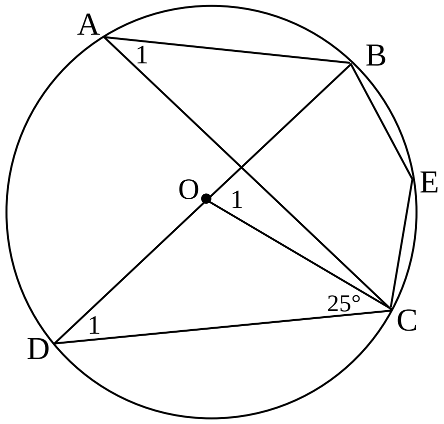
[11]

VRAAG 4

4.1 Voltooi die bewering:

Teenoorstaande hoeke van 'n koordevierhoek is … (1)

4.2 In die gegewe figuur is $D\hat{C}O = 25°$, O is die middelpunt van die sirkel. A, B, E, C en D is punte op die omtrek van die sirkel.



Bereken met redes, die grootte van:

4.2.1 $\hat{D}_1$ (1)

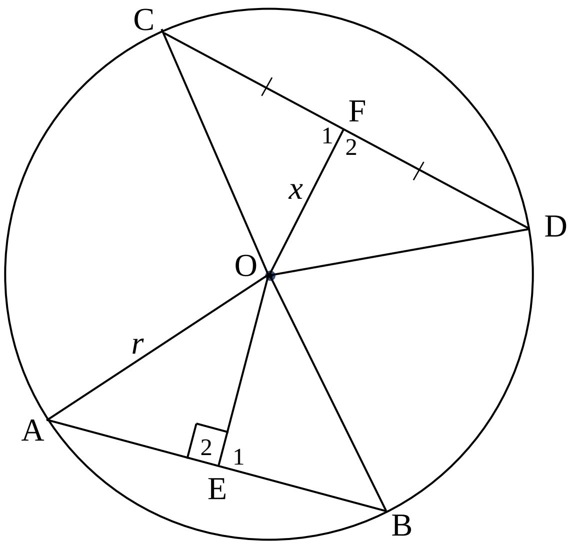
4.2.2 $\hat{O}_1$ (2)

4.2.3 $\hat{A}_1$ (2)

4.2.4 $\hat{E}$ (2)
[8]

VRAAG 5

AB en CD is koorde van ŉ sirkel met middelpunt O. AB = 12 eenhede, CD = 14 eenhede en OE = 5 eenhede. OE ⊥ AB en OF halveer CD, met OF = x.

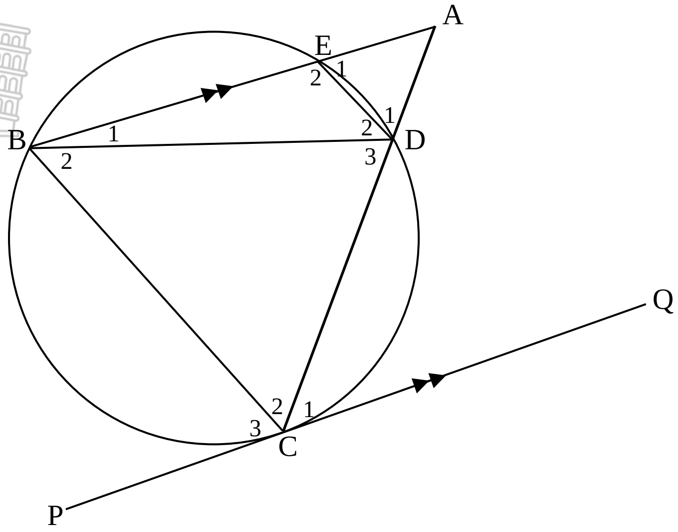


5.1 Bereken met redes, die radius van die sirkel, r. (Laat jou antwoord in wortelvorm.) (4)

5.2 Vervolgens bereken die waarde van x. (3)
[7]

VRAAG 6

PQ is ŉ raaklyn aan die sirkel by C. EBCD is 'n koordevierhoek. AEB en ADC is reguitlyne met AB || QP.



Bewys met redes dat:

6.1 $\hat{A} = \hat{B}_2$ (4)

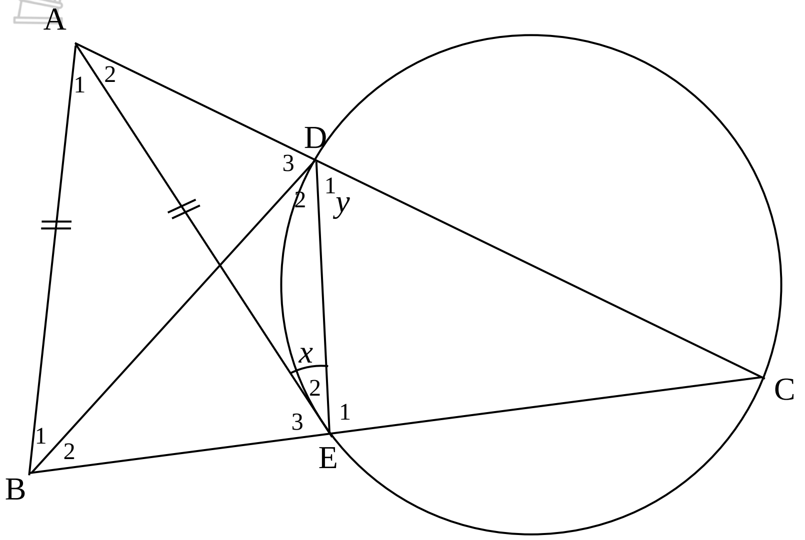
6.2 $\hat{D}_3 = \hat{D}_1$ (5)

[9]

VRAAG 7

In die diagram hieronder is koorde CD en CE verleng na A en B onderskeidelik.

AE is 'n raaklyn aan die sirkel by E, met AB = AE. $\hat{E}_2 = x$ en $\hat{D}_1 = y$.

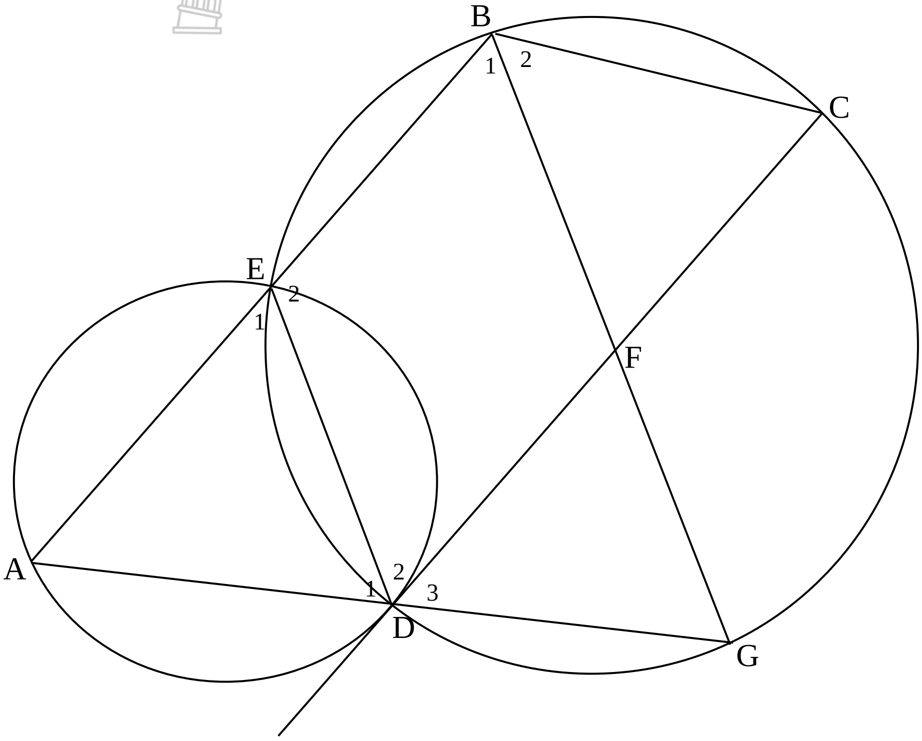


Bewys, met redes, dat ABED 'n koordevierhoek is. (6)

[6]

VRAAG 8

In die figuur sny twee sirkels mekaar by E en D. D, A, B en C is punte op die sirkel sodanig dat ABCD ’n parallelogram is. CD is ’n raaklyn aan die kleiner sirkel. Koorde DC en BG sny mekaar by F. ADG en AEB is reguitlyne.



Bewys, met redes, dat:

8.1 AD = DE (4)

8.2 DC = BG (6)

[10]

TOTAL: 100

EINDE

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

JUNE 2024

GRADE 11

MARKING GUIDELINES

MATHEMATICS (PAPER 2)

11 pages

INSTRUCTIONS AND INFORMATION

- **A** – ACCURACY
- **CA** – CONSISTENT ACCURACY
- **S** – STATEMENT
- **R** – REASON
- **S & R** – STATEMENT with REASON

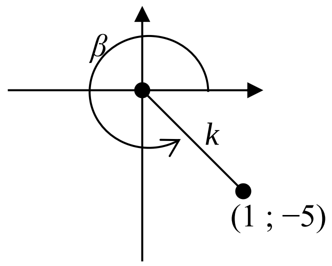
NOTES:

- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed OUT an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a question is UNACCEPTABLE.

QUESTION 1			
1.1	$BC=\sqrt{(4-(-4))^2+(-2-(-6))^2}$ $BC=\sqrt{80}$ $BC=4\sqrt{5}$	✓ Substitution into correct formula ✓ Answer	(2)
1.2	$\frac{6+4}{2}=x$ $\quad\frac{y+(-2)}{2}=2$ $x=5$ $\quad y=6$	✓ $x=5$ ✓ $y=5$	(2)
1.3	$m_{BC}=\frac{-2-(-6)}{4-(-4)}$ $m_{BC}=\frac{1}{2}$	✓ Substitution into correct formula ✓ Answer	(2)
1.4	$m_{BC}=\frac{1}{2}$ $\therefore m_{AD}=-2$ $\quad AD\perp BC$ $y-(2)=-2(x-(-2))$ $y=-2x-2$	✓ $m_{AD}=-2$ ✓ Substitute A$(-2\ ;\ 2)$ ✓ Equation	(3)
1.5	$\tan\theta=-2$ $\theta=180°-63,434...°$ $\theta=116,57°$	✓ Substitution into correct formula ✓ Answer	(2)

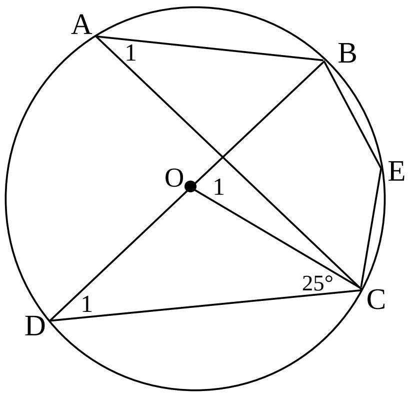
1.6	$F(2\,;-10)$	✓ $x=2$ ✓ $y=-10$	(2)
1.7	Equation of BC: $y-(-2)=\frac{1}{2}(x-4)$ $y=\frac{1}{2}x-4$ D is a point of intersection between AD and BC $y=\frac{1}{2}x-4$ (1) and $y=-2x-2$ (2) (1) = (2) $\frac{1}{2}x-4=-2x-2$ $x-8=-4x-4$ $5x=4$ $x=\frac{4}{5}=0,8$ $y=-2(0,8)-2$ $y=-\frac{18}{5}=-3,6$ $\therefore D(0,8\,;\,-3,6)$	✓ $m_{BC}=-2$ ✓ Substitute B(−4;−6) of C (4;−2) ✓ Equation of BC ✓ Simultaneous equation substitution of y	(4)
1.8	$AD=\sqrt{(-2-(0,8))^2+(2-(-3,6))^2}$ $AD=\sqrt{39,2}=\frac{14\sqrt{5}}{5}$ Area ABCE = BC × AD Area ABCE $=4\sqrt{5}\times\frac{14\sqrt{5}}{5}$ Area ABCE = 56 units^2	✓ Length of AD ✓ Substitution into area of a parallelogram formula ✓ Answer	(3)
			[20]

QUESTION 2				
2.1	2.1.1	$\sin(-20°)$ $= -\sin 20°$ $= -p$	✓ $-\sin 20°$ ✓ $-p$	(2)
	2.1.2	$\sin 160°$ $= \sin 20°$ $= p$	✓ $\sin 20°$ ✓ p	(2)
	2.1.3	$\sin^2\theta + \cos^2\theta = 1$ $1 - \sin^2 70°$ $= \cos^2 70°$ $= \sin^2 20°$ $= p^2$ **OR** $1 - \sin^2 70°$ $= 1 - \cos^2 20°$ $= \sin^2 20°$ $= p^2$ **OR** $1 - \sin^2 70°$ $= 1 - \cos^2 20°$ $= 1 - \left(\sqrt{1-p^2}\right)^2$ $= p^2$	✓ $\cos^2 70°$ ✓ $\sin^2 20°$ ✓ p^2 ✓ $1 - \cos^2 20°$ ✓ $\sin^2 20°$ ✓ p^2 ✓ $1 - \cos^2 20°$ ✓ $\left(\sqrt{1-p^2}\right)^2$ ✓ p^2	(3)
2.2	$\dfrac{\tan 225° - \sin(180° + \theta)\cos(\theta + 90°)}{\sin\theta.\sin(360° + \theta) + \cos\theta\cos(-\theta)}$ $= \dfrac{1 - (-\sin\theta)(-\sin\theta)}{\sin\theta.\sin\theta + \cos\theta.\cos\theta}$ $= \dfrac{1 - \sin^2\theta}{\sin^2\theta + \cos^2\theta}$ $= \cos^2\theta$		✓ $\tan 225° = 1$ ✓ $-\sin\theta$ ✓ $-\sin\theta$ ✓ $\sin\theta.\sin\theta$ ✓ $\cos\theta.\cos\theta$ ✓ $\cos^2\theta$	(6)

2.3	2.3.1	 $\sin\beta = \frac{-5}{k}$ and $\cos\beta = \frac{1}{k}$ $\sin\beta$ is negative and $\cos\beta$ is postive in quadrant IV **OR** In quadrant IV, y is negative and x is positive.	✓ Isolate $\sin\beta$ ✓ Isolate $\cos\beta$ ✓ Explanation ✓ Diagram	(4)
	2.3.2	$\tan\beta = -\frac{5}{1}$ $\tan\beta = -5$	✓ $\tan\beta = -5$	(1)
	2.3.3	$x^2 + y^2 = r^2$ $(1)^2 + (-5)^2 = k^2$ $k = \sqrt{26} \qquad k > 0$	✓ Substitute into Pythagoras ✓ $k = \sqrt{26}$	(2)
2.4	2.4.1	$\text{LHS} = \sin^2\alpha\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$ $= \sin^2\alpha\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha.\cos\alpha}\right)$ $= \frac{\sin\alpha(1)}{\cos\alpha}$ $= \tan\alpha$ $= \text{RHS}$	✓ $\frac{\sin\alpha}{\cos\alpha}$ ✓ $\frac{\cos\alpha}{\sin\alpha}$ ✓ Add fractions ✓ $\sin^2\alpha + \cos^2\alpha = 1$ ✓ conclusion	(5)
	2.4.2	$\sin^2\alpha\left(\tan^2\alpha + 1\right) = \sqrt{3}\tan\alpha$ $\sin^2\alpha\left(\tan\alpha + \frac{1}{\tan\alpha}\right) = \sqrt{3}$ $\therefore \tan\alpha = \sqrt{3}$ $\alpha = 60° + k.180° \quad ; \quad k \in Z$	✓ $\div \tan\alpha$ ✓ Substitute $\tan\alpha$ ✓ $\alpha = 60° + k.180°$ ✓ $k \in Z$	(4)
				[29]

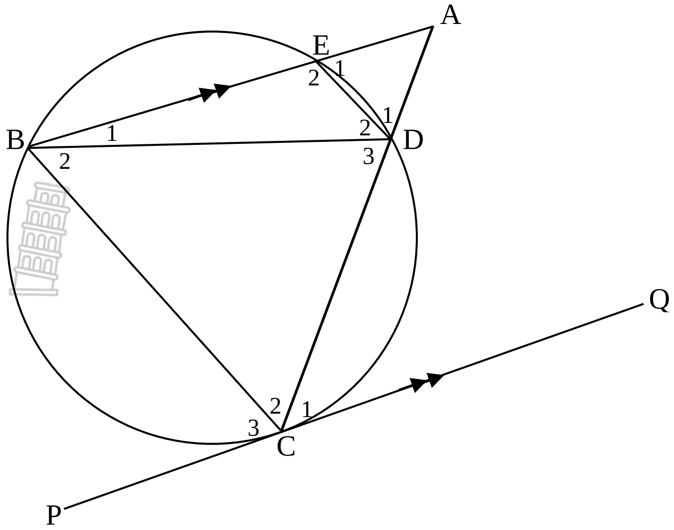
QUESTION 3

3.1	$a=-2$ $p=-30°$	✓ $a=-2$ ✓ $p=-30°$	(2)
3.2	$-2\le y\le 2$ **OR** $y\in[-2\ ;\ 2]$	✓ Interval ✓ Notation	(2)
3.3	$b=6$	✓ $b=6$	(1)
3.4	3.4.1 $-60°<x<120°$	✓ Interval ✓ Notation	(2)
	3.4.2 $-90°<x<0°$ of $90°<x\le 180°$	✓ $-90°<x<0°$ ✓ $90°<x\le 180°$	(2)
3.5	$g(2x)=\sin(2x-30°)$ $=\cos(90°-(2x-30°))$ $=\cos(120°-2x)$ $=\cos(2x-120°)$ $\therefore t=-120°$ **OR** $g(2x)=\sin(2x-30°)$ $=\sin(2(x-15°))$ $\cos(2x+t)$ must be translated 120° to the right $\therefore t=-120°$	✓ $\cos(120°-2x)$ ✓ $t=-120°$ **OR** ✓ $\sin(2(x-15°))$ ✓ $t=-120°$	(2)
			[11]

QUESTION 4				
4.1	supplementary or (add up to 180°)		✓ Answer	(1)
4.2				
4.2	4.2.1	$\hat{D}_1 = 25°$ ∠'s opp equal radii	✓ S & R	(1)
	4.2.2	$\hat{O}_1 = 50°$ ext ∠ of Δ	✓ S ✓ R	(2)
	4.2.3	$\hat{A}_1 = 25°$ ∠'s in same segments **OR** ∠ at centre = 2 × at circumference	✓ S ✓ R	(2)
	4.2.4	$\hat{E} = 155°$ opp ∠'s cyclic quad	✓ S ✓ R	(2)
				[8]

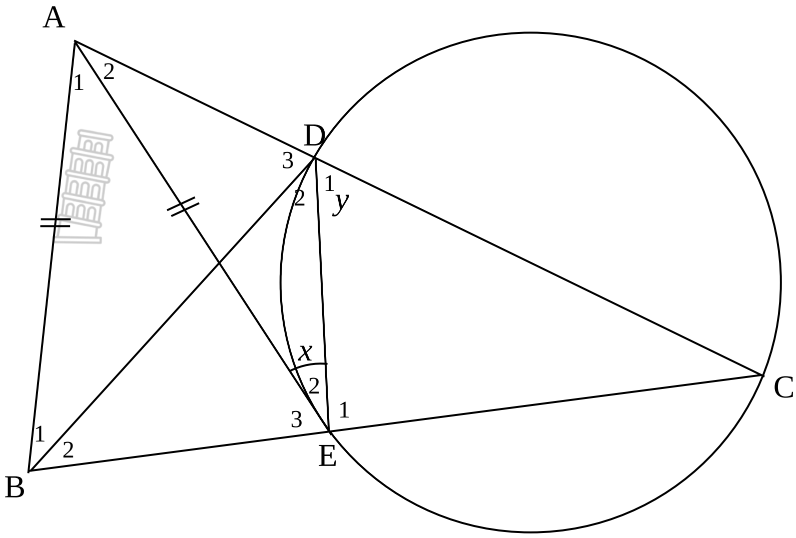
QUESTION 5			
5.1	$AE = 6$ line from centre ⊥ to chord $r^2 = 5^2 + 6^2$ Pythagoras' theorem $r = \sqrt{61}$	✓ S ✓ R ✓ S ✓ Answer in surd form	(4)
5.2	$OC = r = \sqrt{61}$ radii $F_1 = 90°$ line from centre to midpt of chord $x^2 = 61 - 7^2$ Pythagoras' Theorem $x = 2\sqrt{3} = 3,46$	✓ S ✓ R ✓ S	(3)
			[7]

QUESTION 6



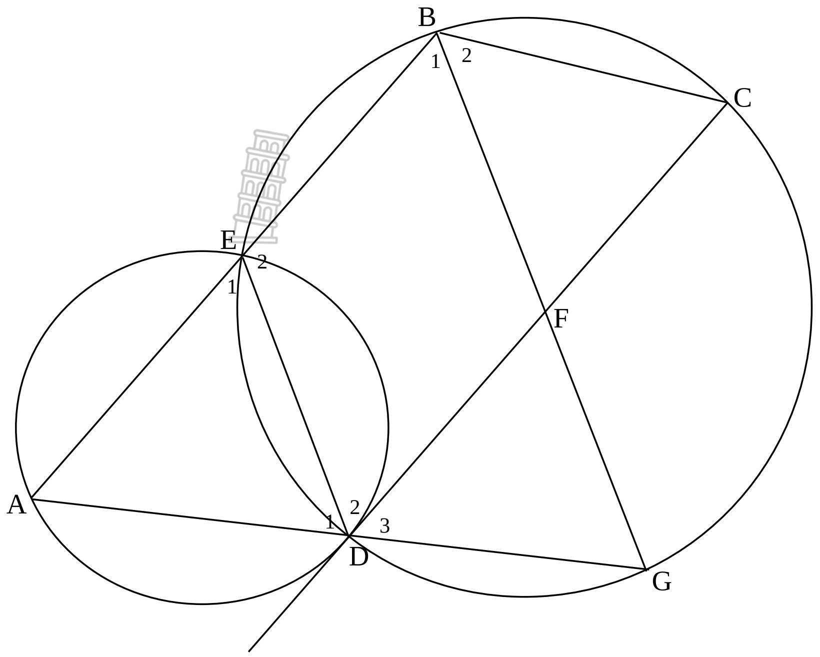
6.1	$\hat{A} = \hat{C}_1$ — alt $\angle$'s, PQ ∥ AB $\hat{B}_2 = \hat{C}_1$ — tan chord theorem $\therefore \hat{A} = \hat{B}_2$ — both equal $\hat{C}_1$	✓ S ✓ R ✓ S ✓ R	(4)
6.2	$\hat{D}_3 = \hat{B}_1 + \hat{A}$ — ext $\angle$ of Δ $\hat{D}_3 = \hat{B}_1 + \hat{B}_2$ — $\hat{B}_2 = \hat{A}$ $\hat{D}_1 = \hat{B}_1 + \hat{B}_2$ — ext $\angle$ cyclic quad $\therefore \hat{D}_3 = \hat{D}_1$ — both equal $\hat{B}_1 + \hat{B}_2$ **OR** $\hat{D}_1 = \hat{B}_1 + \hat{B}_2$ — ext $\angle$ cyclic quad $\hat{C}_3 = \hat{B}_1 + \hat{B}_2$ — alt $\angle$'s, AB ∥ PQ $\therefore \hat{D}_1 = \hat{C}_3$ — both equal $\hat{B}_1 + \hat{B}_2$ en $\hat{D}_3 = \hat{C}_3$ — tan chord theorem $\therefore \hat{D}_3 = \hat{D}_1$ — both equal $\hat{C}_3$	✓ S ✓ R ✓ S ✓ S ✓ R ✓ S ✓ R ✓ S & R ✓ S ✓ R	(5)
			[9]

QUESTION 7



$\hat{C} = x$	tan chord theorem	✓ S & R	
$\hat{E}_2 + \hat{E}_3 = x + y$	ext $\angle$ of Δ	✓ R	
$\hat{E}_2 = x$	given	✓ S	
$\therefore \hat{E}_3 = y$		✓ S & R	
$\hat{B}_1 + \hat{B}_2 = y$	$\angle$'s opp equal sides	✓ S	
$\therefore \hat{B}_1 + \hat{B}_2 = \hat{D}1$	both equal y	✓ S & R	
$\therefore$ ABED is a cyclic quad	converse ext $\angle$ cyclic quad		
OR	ext $\angle$ = to opp int $\angle$		(6)
			[6]

QUESTION 8



8.1	$\hat{A} = \hat{C}$	opp $\angle$'s of parm	✓ S & R	
	$\hat{E}_1 = \hat{C}$	ext $\angle$ cyclic quad	✓ S ✓ R	
	$\therefore \hat{A} = \hat{E}_1$	both equal $\hat{C}$	✓ R	
	$\therefore AD = DE$	sides opp equal $\angle$'s		
	OR			
	$\hat{E}_1 = \hat{C}$	ext $\angle$ cyclic quad	✓ S ✓ R	
	$\hat{D}_3 = \hat{C}$	alt $\angle$'s, $BC \parallel DG$		
	$\therefore \hat{E}_1 = \hat{D}_3$	both equal $\hat{C}$		
	$\hat{A} = \hat{D}_3$	corr $\angle$'s, $AB \parallel DC$	✓ S & R	
	$\therefore \hat{A} = \hat{E}_1$	both equal $\hat{D}_3$	✓ R	
	$ED = AD$	sides opp equal $\angle$'s		(4)

8.2	$\hat{G} = \hat{E}_1$	ext $\angle$ cyclic quad	✓ S ✓ R	
	$\hat{A} = \hat{E}_1$	proven in 8.1	✓ S	
	$\therefore \hat{A} = \hat{G}$	both equal $\hat{E}_1$	✓ S ✓ R	
	$\therefore AB = BG$	sides opp equal $\angle$'s	✓ S & R	
	but AB = CD	opp sides of parm		
	$\therefore BG = CD$	both equal AB		
	OR			
	$\hat{G} = \hat{C}$	$\angle$'s in same seg	✓ S ✓ R	
	$\hat{A} = \hat{C}$	opp $\angle$'s parm	✓ S & R	
	$\therefore \hat{A} = \hat{G}$	both equal $\hat{C}$	✓ S ✓ R	
	$\therefore AB = BG$	sides opp equal $\angle$'s	✓ S & R	
	but AB = CD	opp sides of parm		
	$\therefore BG = CD$	both equal AB		
	OR			
	$\hat{B}_2 = \hat{G}$	alt $\angle$'s, BC $\parallel$ DG	✓ S & R	
	$\hat{B}_2 = \hat{D}_3$	$\angle$'s in the same seg	✓ S ✓ R	
	$\therefore \hat{G} = \hat{D}_3$	both equal $\hat{B}_2$		
	and $\hat{D}_3 = \hat{A}$	corr $\angle$'s, AB $\parallel$ CD	✓ S	
	$\therefore \hat{G} = \hat{A}$	both equal $\hat{D}_3$	✓ S & R	
	$\therefore BA = BG$	sides opp equal $\angle$'s	✓ S & R	
	but BA = DC	opp sides of parm		
	$\therefore BG = CD$			(6)
				[10]
			TOTAL:	**100**