GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

NOVEMBER 2023

GRADE 11

MATHEMATICS

PAPER 2

TIME: **3 hours**

MARKS: **150**

12 pages + 1 information sheet and 3 answer sheets

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.

7. Answer sheets for answering QUESTIONS 1.1, 1.2 and 9.1 are provided at the end of the question paper. Write you name in the spaces provided on the ANSWER SHEETS and submit them together with your ANSWER BOOK.

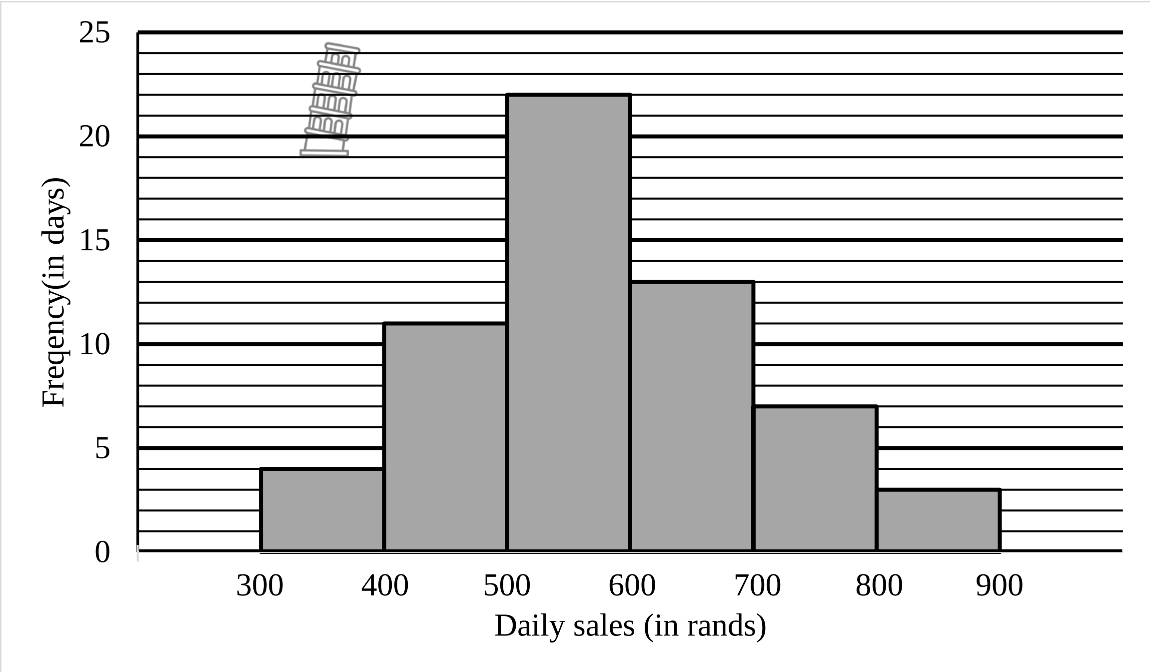
8. Diagrams are NOT necessarily drawn to scale.

9. Number the answers correctly according to the numbering system used in this question paper.

10. Write neatly and legibly.

QUESTION 1

A small tuck shop displayed a record of daily sales in rands, for the past two months (60 days) using the following histogram.



1.1 Complete the following table. Use the table provided on ANSWER SHEET A.

Class interval	Frequency	Cumulative frequency
$300 < x \leq 400$	4	4
$400 < x \leq 500$		
$500 < x \leq 600$		
$600 < x \leq 700$		
$700 < x \leq 800$		
$800 < x \leq 900$		

(2)

1.2 Draw a cumulative frequency curve for the sales over the past two months. Use the graph sheet provided on ANSWER SHEET B. (3)

1.3 Use the graph in QUESTION 1.2 and determine the estimated median value for the daily sales. (2)

1.4 The tuck shop must make R475 in sales per day to break-even. On how many days did the tuck shop make a profit? (2)

1.5 On the first day of the following month, the tuck shop made R725 in sales. Does this day lie within the top 25% of sales from the previous two months? (2)

[11]

QUESTION 2

Five data values are represented as follows: $2k$; $k + 1$; $k + 2$; $k - 3$; $2k - 2$

2.1 If the mean of the data set is 15, show that $k = 11$. (3)

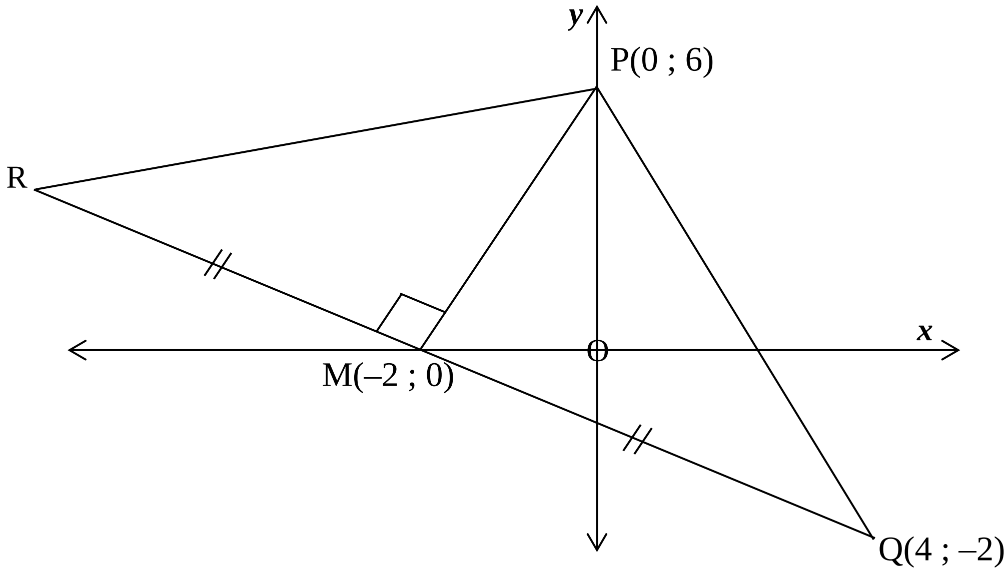
2.2 Calculate the standard deviation (σ)for this data, rounded-off to one decimal place. (2)

2.3 If t units are subtracted from each data value in the set, without further calculation, explain how the mean and standard deviation would be affected in terms of t. (2)

[7]

QUESTION 3

In the diagram below, ΔRPQ is drawn, with P(0 ; 6), Q(4 ; –2). M(–2 ; 0) is the midpoint of RQ.



3.1 Determine the gradient of the line MQ. (2)

3.2 Determine the equation of the line MP, in the form $y = mx + c$. (3)

3.3 Determine the coordinates of R. (3)

3.4 Calculate the length of PQ, in simplified surd form. (2)

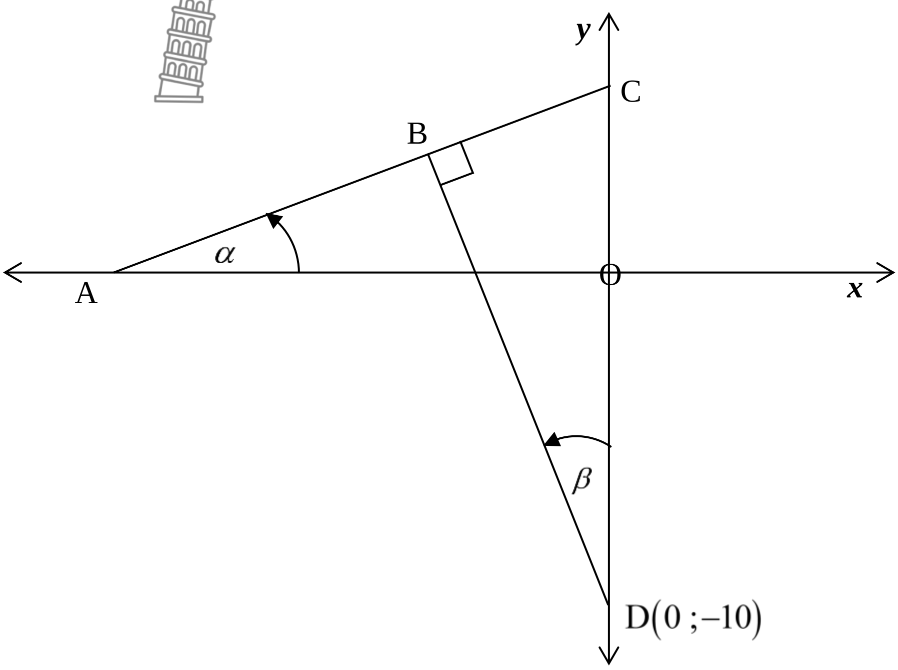
3.5 Given that RPQT is a parallelogram, determine the coodinates of T if point T is in the third quadrant. (2)

3.6 Explain why RPQT is a rhombus. (2)

[14]

QUESTION 4

In the diagram below, lines AC and BD intersect at B, where AC ⊥ BD. C and D lie on the y-axis, while A lies on the x-axis. The equation of AC is $py - x - 5p = 0$, while α is the angle of inclination for AC, with $C\hat{D}B = \beta$.



4.1 Determine the coordinates of C. (3)

4.2 If the gradient of AC is $\frac{1}{2}$, show that $p = 2$. (2)

4.3 Calculate the coordinates of B. (5)

4.4 Determine the size of α. (2)

4.5 Hence, or otherwise, prove that ABOD is a cyclic quadrilateral. (3)

4.6 Determine the coordinates of the centre of the circle which passes through D, B and C. (2)

[17]

QUESTION 5

5.1 If $3\sin\beta = 2$. and $\cos\beta < 0$, determine with the aid of a diagram and **without the use of a calculator**, the value of:

5.1.1 $3\cos^2\beta - 1$ (4)

5.1.2 $\tan(-\beta - 180°)$ (3)

5.2 Given: $t\cos 15° = 4$

Determine the following in terms of t, **without the use of a calculator**:

5.2.1 $\sin 15°$ (3)

5.2.2 $\sin 75°$ (2)

5.2.3 $1 - \tan^2 15°$ (Give the answer as a single fraction.) (3)

5.3 Simplify the following to a single trigonometric function, without the use of a calculator.

$$\frac{\cos(90° - \alpha)\sin(-\alpha - 540°)}{\tan 225° + \sin\alpha.\sin(180° + \alpha)}$$ (5)

5.4 Given: $1 - \cos\theta = 2\sin^2\theta$

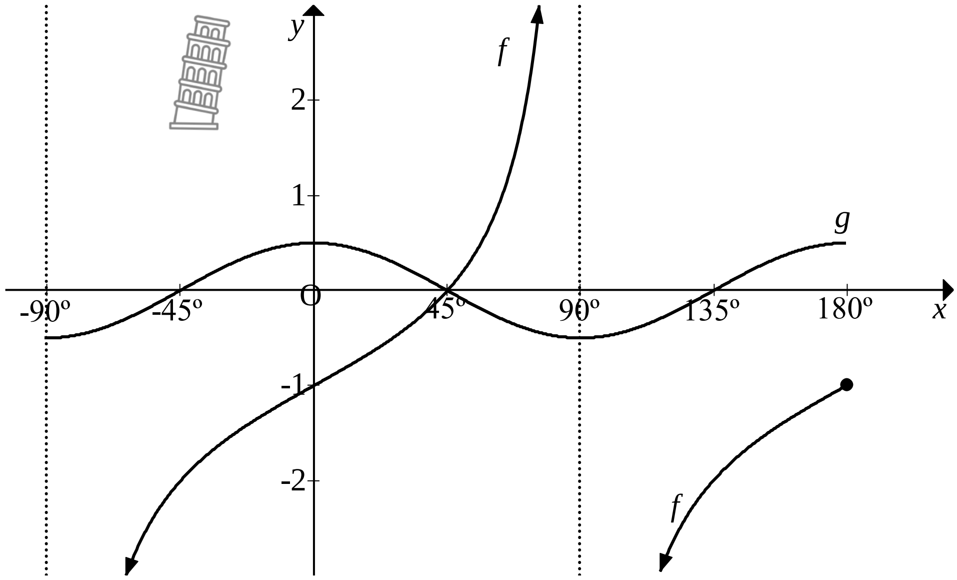
5.4.1 Show that the equation can be written as: $(2\cos\theta + 1)(\cos\theta - 1) = 0$. (2)

5.4.2 Hence, determine the general solution of $(2\cos\theta + 1)(\cos\theta - 1) = 0$. (5)

[27]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan x - 1$ and $g(x) = -\frac{1}{2}\cos 2x$ are drawn, where $x \in [-90° \,;\, 180°]$.



6.1 Write down the period of g. (1)

6.2 Determine the range of $g(x)$. (2)

6.3 Use the graphs to determine graphically the values of x where:

6.3.1 $f(x) \geq 0$ (2)

6.3.2 $f(x).g(x) > 0$ (2)

6.3.3 $2\tan x + \cos 2x = 2$ (3)

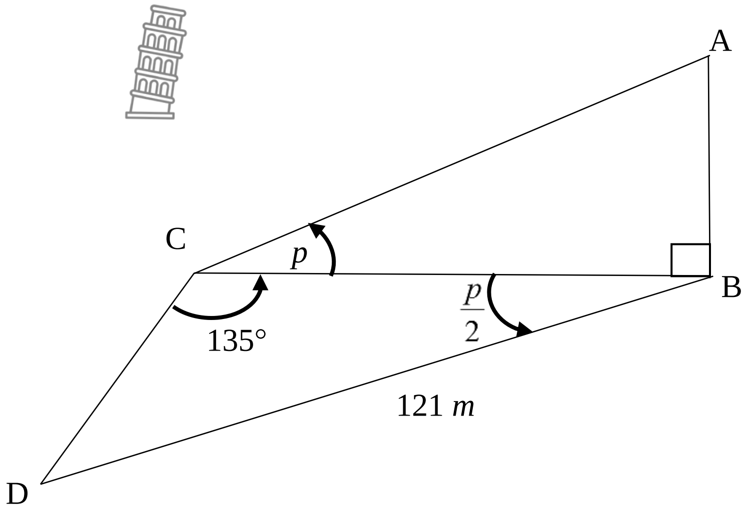
6.4 If $h(x) = \dfrac{\sin x + \cos x}{\cos x}$, describe the vertical translation of h from f. (2)

6.5 Determine the maximum value of $p(x) = 4g(x)$. (2)

[14]

QUESTION 7

In the figure below, ΔABC is drawn where $AB \perp BC$ and $A\hat{C}B = p$, with CB = 3AB.
ΔDCB is drawn such that $D\hat{C}B = 135°$, and BD = 121 *m*

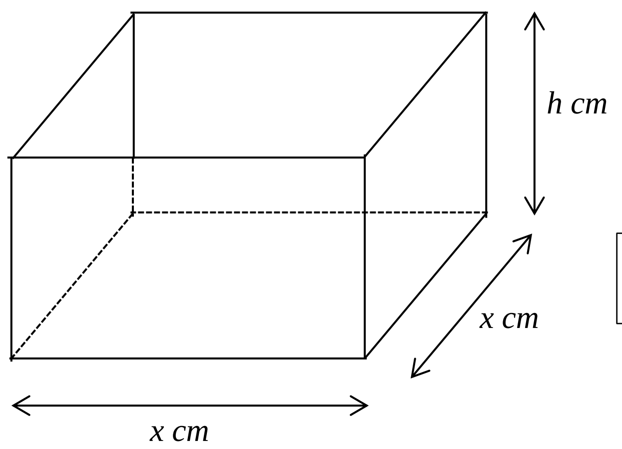


7.1 Determine the value of *p*, correct to 3 decimal places. (2)

7.2 If $p = 18{,}4°$, determine the length of CD. (3)
[5]

QUESTION 8

The diagram below represents an open tank with a square base (side dimensions of *x* cm) and a height of *h cm*. The tank has a volume of 490 cm^3.



V = Area of base x Height

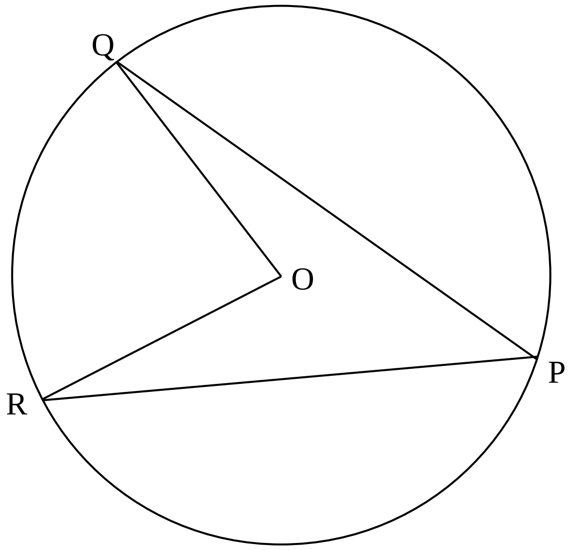
8.1 Determine the height (*h*) of the tank in terms of *x*. (2)

8.2 Show that A, the external surface area of the tank, is given by the formula:
$A = x^2 + \frac{1\ 960}{x}\ cm^2$ (2)

8.3 Given that the tank is 10 *cm* high, calculate the external surface area of the tank. (4)
[8]

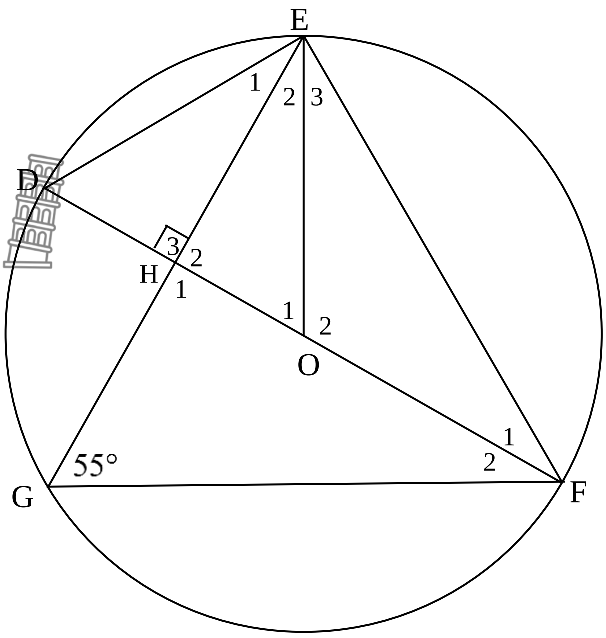
QUESTION 9

9.1 In the diagram below, O is the centre of the circle passing through P, Q and R. Chords PQ and PR are drawn, with OQ and OR joined.



Use the diagram provided in ANSWER SHEET C to prove the theorem that states that the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle, that is $R\hat{O}Q = 2\hat{P}$ (6)

9.2 In the diagram below, DF is a diameter of the circle with centre O. Chord EG intersects DF at H such that DF ⊥ EG. Chords EF and GF are drawn. $E\hat{G}F = 55°$.



9.2.1 Determine, giving reasons, the size of:

(a) $\hat{D}$ (2)

(b) $\hat{O}_2$ (2)

(c) $\hat{E}_2$ (2)

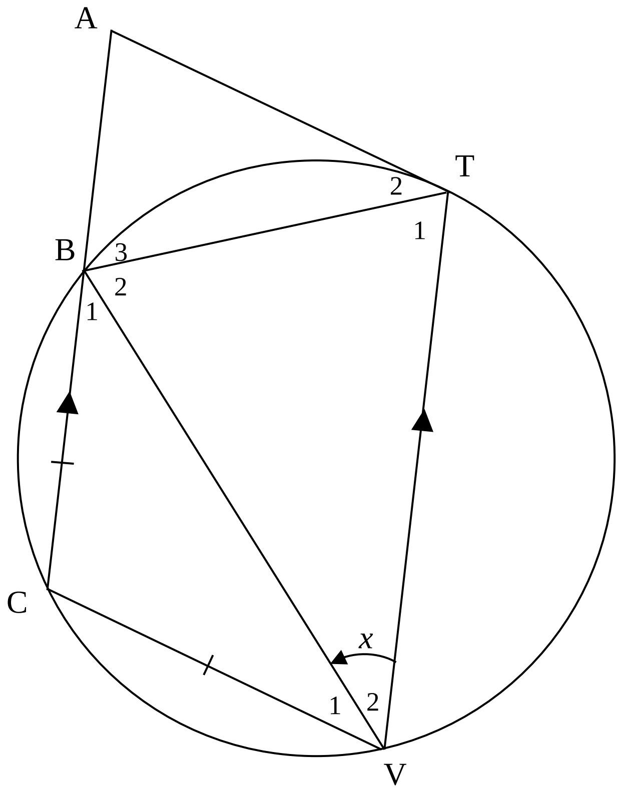
(d) $\hat{E}_3$ (3)

9.2.2 Determine the length of OH, if the diameter of the circle is 10 units and GE = 9,4 units. (4)

[19]

QUESTION 10

In the diagram below, AT is a tangent to the circle at T. Chords BT, BV and VC are drawn. CB is extended to A, such that AC || TV. BC = CV and $B\hat{V}T = x$.



10.1 Determine, with reasons, 3 angles equal to x. (6)

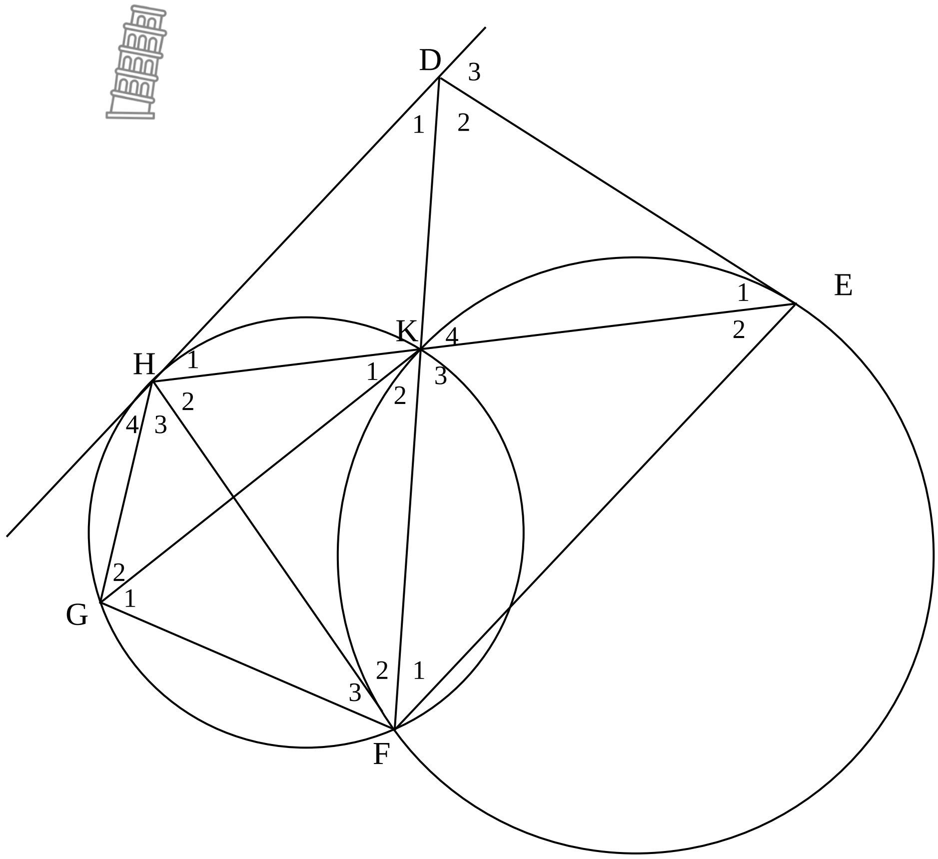
10.2 If ATVC is a parallelogram, prove that AT = BT. (5)

10.3 Determine the size of x. (2)

[13]

QUESTION 11

DE is a tangent to the larger circle at E. DH is a tangent to the smaller circle at H. Chord HK is extended to meet the larger circle at E. F and K are the points of intersections between the circles, with FK produced to D. GK is a chord of the smaller circle with FE a chord of the larger circle. HF, GH and GF are joined.



11.1 Complete the following:

$\hat{D}_3 = ... + ...$ (ext $\angle\ \Delta$) (1)

11.2 Prove that DEFH is a cyclic quadrilateral. (4)

11.3 Prove that DF bisects $H\hat{F}E$. (3)

11.4 If $\hat{K}_1 = \hat{E}_1$, prove that GK is a tangent to the larger circle at K. (7)

[15]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Name and Surname: ____________________________________ Grade: _________

ANSWER SHEET A

QUESTION 1

1.1

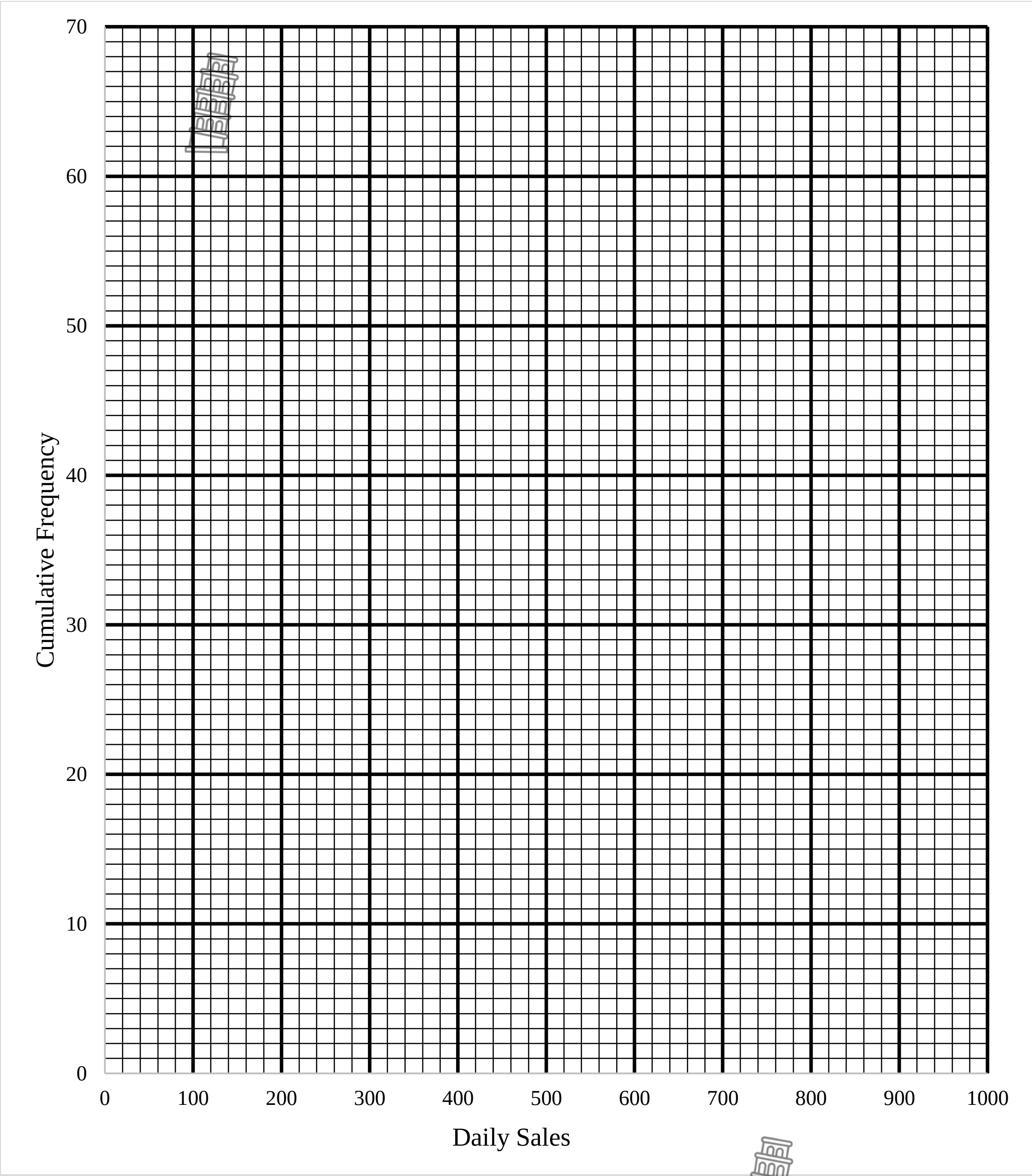
Class interval	**Frequency**	**Cumulative frequency**
$300 < x \leq 400$	4	4
$400 < x \leq 500$		
$500 < x \leq 600$		
$600 < x \leq 700$		
$700 < x \leq 800$		
$800 < x \leq 900$		

(2)

Name and Surname: ________________________________ Grade: ________

<u>ANSWER SHEET B</u>

1.2



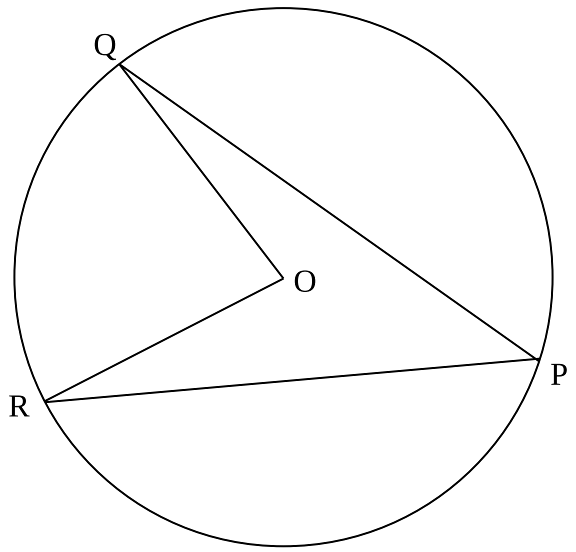
(3)

Name and Surname: ________________________________ Grade: ________

ANSWER SHEET C

QUESTION 9

9.1



(6)

GAUTENG PROVINCE
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PROVINCIAL EXAMINATION

NOVEMBER 2023

GRADE 11

MARKING GUIDELINES

MATHEMATICS (PAPER 2)

14 pages

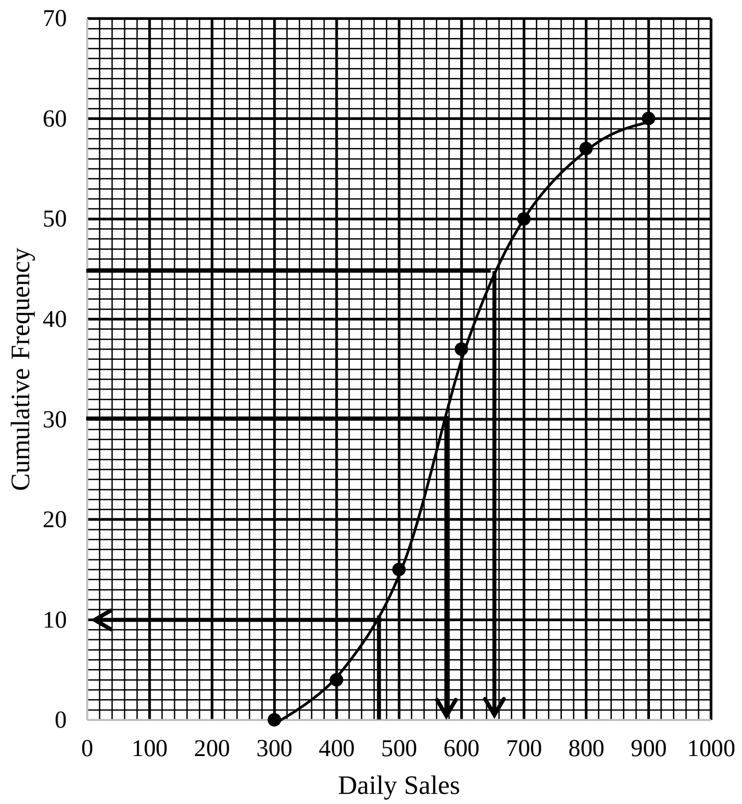
INSTRUCTIONS AND INFORMATION

- **A** – ACCURACY
- **CA** – CONSISTENT ACCURACY
- S – STATEMENT
- R – REASON
- S & R – STATEMENT with REASON

NOTES:

- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed OUT an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a question is UNACCEPTABLE.

QUESTION 1

1.1	Class interval	Frequency	Cumulative frequency	✓ Frequency values ✓ Cumulative frequency values	
	$300 < x \leq 400$	4	4		
	$400 < x \leq 500$	11	15		
	$500 < x \leq 600$	22	37		
	$600 < x \leq 700$	13	50		
	$700 < x \leq 800$	7	57		
	$800 < x \leq 900$	3	60		(2)
1.2				✓ (300 ; 0) ✓ End points ✓ Shape	(3)
1.3	Q_2 = R580			✓✓ Answer Accept R560 to R600	(2)
1.4	On 10 days they made a loss. ∴ 50 days they made a profit.			✓✓ Answer	(2)
1.5	To be in the top 25% of the previous months, the tuck shop would need to have sold more than R650. Therefore, R725 would have been in the top 25%.			✓ R650 ✓ Conclusion	(2)
					[11]

QUESTION 2

2.1	$\frac{2k+k+1+k+2+k-3+2k-2}{5}=15$ $\frac{7k-2}{5}=15$ $7k-2=75$ $7k=77$ $k=11$	✓ $7k-2$ ✓ Divide by 5 ✓ Manipulation	(3)
2.2	22 ; 12 ; 13 ; 8 ; 20 $\sigma = 5{,}22$	✓✓ Answer	(2)
2.3	Mean would decrease by *t* units but the standard deviation would be unaffected.	✓ Mean decreases by *t* units ✓ Standard deviation unaffected	(2)
			[7]

QUESTION 3

3.1	$m_{MQ}=\frac{0-(-2)}{-2-(-4)}$ $m_{MQ}=-\frac{1}{3}$	✓ Substitution into correct formula ✓ Answer	(2)
3.2	$m_{MQ}\times -\frac{1}{3}=-1$ MP ⊥ MQ $m_{MP}=3$ $y=3x+6$	✓ Perpendicular gradients ✓ Gradient of MP ✓ Answer	(3)
3.3	$-2=\frac{x_R+4}{2}$ $0=\frac{y_R+(-2)}{2}$ $x_R=-8$ $y_R=2$ R (–8 ; 2)	✓ Substitution into midpoint formula ✓ x_R ✓ y_R	(3)

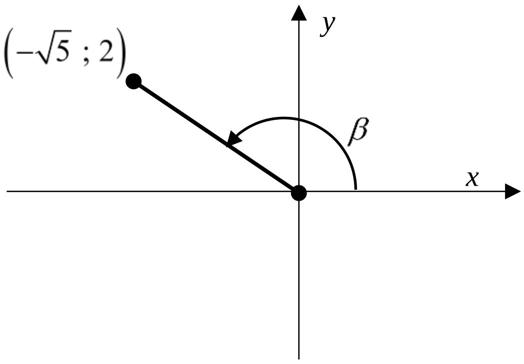
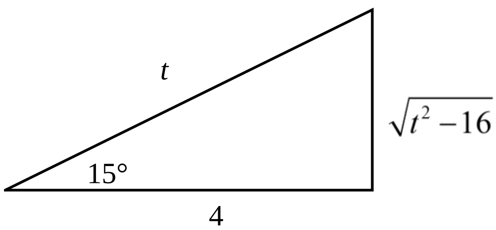
3.4	$PQ = \sqrt{(0-4)^2 + (6-(-2))^2}$ $PQ = \sqrt{80}$ $PQ = 4\sqrt{5}$	✓ Substitute into distance formula ✓ Simplified surd	(2)
3.5	E(–4 ; –6)	✓ $x_E = -4$ ✓ $y_E = -6$	(2)
3.5	The diagonals of the parallelogram are perpendicular.	✓ Diagonals perpendicular ✓ Given parallelogram	(2)
			[14]

QUESTION 4

4.1	subst. $x = 0$ $py - x - 5p = 0$ $py - (0) = 5p$ $y = \frac{5p}{p}$ $y = 5$ C(0 ; 5)	✓ $x = 0$ ✓ Manipulation ✓ $x = 5$	(3)
4.2	$py - x - 5p = 0$ $py = x + 5p$ $y = \frac{x}{p} + 5$ $m_{AC} = \frac{1}{2}$ $\therefore \frac{1}{p} = \frac{1}{2}$ $\therefore p = 2$ **OR** A(–10 ; 0) $m_{AC} = \frac{1}{2}$ $p(0) - (-10) - 5p = 0$ $10 = 5p$ $p = 2$	✓ Standard form ✓ Equate gradients ✓ A (–10 ; 0) ✓ Substitute A	(2)

4.3	Equation of AC: $2y - x - 10 = 0$ $y = \frac{1}{2}x + 5$ Equation of BD: $y = -2x - 10$ $\frac{1}{2}x + 5 = -2x - 10$ $x + 10 = -4x - 20$ $5x = -30$ $x = -6$ $y = 2$ $\therefore B(-6\,;2)$	✓ Substitute $p = 2$ ✓ Equation of BD ✓ Substitute y into other equation ✓ $x = -6$ ✓ $y = 2$	(5)
4.4	$\tan\alpha = \frac{1}{2}$ $\alpha = 26{,}57°$	✓ Substitution into correct formula ✓ Answer	(2)
4.5	$A\hat{C}D = 63{,}43°$ int $\angle$'s of Δ $\beta = 26{,}56°$ int $\angle$'s of Δ $\therefore \alpha = \beta$ $\therefore$ ABOD is a cyclic quadrilateral (converse $\angle$'s in same segment)	✓ $A\hat{C}D$ ✓ β ✓ Rede	(3)
4.6	DB is a diameter (line subtends 90°) $M_{CD} = \left(\frac{0+0}{2};\frac{5+(-10)}{2}\right)$ $= M_{CD}\left(0\,;-\frac{5}{2}\right)$	✓ $x = 0$ ✓ $y = -\frac{5}{2}$	(2)
			[17]

QUESTION 5

5.1	5.1.1	$\sin\beta = \frac{2}{3}$ $x^2 + (2)^2 = 3^2$ $x = -\sqrt{5}$ $3\cos^2\beta - 1$ $=3\left(\frac{\sqrt{5}}{3}\right)^2 - 1$ $= \frac{5}{3} - 1$ $= \frac{2}{3}$  	✓ Quadrant II ✓ $x = -\sqrt{5}$ ✓ Substitute $\cos\beta$ ✓ Answer	(4)
	5.1.2	$\tan(-\beta - 180°)$ $= -\tan\beta$ $= -\left(\frac{2}{-\sqrt{5}}\right)$ $= \frac{2}{\sqrt{5}}$	✓ $-\tan\beta$ ✓ Substitution ✓ Answer	(3)
5.2	5.2.1	$t\cos 15° = 4$ $\cos 15° = \frac{4}{t}$ $\sin 15° = \frac{\sqrt{t^2 - 16}}{t}$  	✓ $\cos 15° = \frac{t}{4}$ ✓ $\sqrt{t^2 - 16}$ ✓ t	(3)
	5.2.2	$\sin 75°$ $= \cos 15°$ $= \frac{4}{t}$	✓ Complement ✓ Answer	(2)

	5.2.3	$1 - \tan^2 15°$ $= 1 - \left(\dfrac{\sqrt{t^2-16}}{4}\right)^2$ $= 1 - \dfrac{t^2-16}{16}$ $= \dfrac{16 - t^2 + 16}{16}$ $= \dfrac{32 - t^2}{16}$	✓ $\left(\dfrac{\sqrt{t^2-16}}{4}\right)^2$ ✓ $\dfrac{t^2-16}{16}$ ✓ Correct multiplication by LCD	(3)
5.3		$\dfrac{\cos(90° - \alpha)\sin(-\alpha - 540°)}{\tan 225° + \sin\alpha.\sin(180° + \alpha)}$ $= \dfrac{\sin\alpha.\sin\alpha}{\tan 45° + (\sin\alpha.)(-\sin\alpha)}$ $= \dfrac{\sin^2\alpha}{1 - \sin^2\alpha}$ $= \dfrac{\sin^2\alpha}{\cos^2\alpha}$ $= \tan^2\alpha$	✓ tan 45° ✓ $\sin\alpha.-\sin\alpha$ ✓ $\sin\alpha.\sin\alpha$ ✓ $1 - \sin^2\alpha$ ✓ $\dfrac{1}{\tan^2\alpha}$	(5)
5.4	5.4.1	$1 - \cos\theta = 2\sin^2\theta$ $1 - \cos\theta = 2(1 - \cos^2\theta)$ $1 - \cos\theta = 2 - 2\cos^2\theta$ $2\cos^2\theta - \cos\theta - 1 = 0$ $(2\cos\theta + 1)(\cos\theta - 1) = 0$	✓ $2(1 - \cos^2\theta)$ ✓ standard form	(2)
	5.4.2	$2\cos\theta + 1 = 0$ or $\cos\theta - 1 = 0$ $\cos\theta = -\dfrac{1}{2}$ or $\cos\theta = 1$ RA = 60° or $\theta = 0° + K.360°$; $K \varepsilon \mathbb{Z}$ QII : $\theta = 120° + K.360°$; $K \varepsilon \mathbb{Z}$ QIII : $\theta = 240° + K.360°$; $K \varepsilon \mathbb{Z}$	✓ $\cos\theta = -\dfrac{1}{2}$ ✓ $\cos\theta = 1$ ✓ Both solutions for $\cos\theta = -\dfrac{1}{2}$ ✓ $\theta = 0° + K.360°$ ✓ $+ K \varepsilon \mathbb{Z}$	(5)
				[27]

QUESTION 6

6.1		$180°$	✓ Answer	(1)
6.2		$-\frac{1}{2} \leq y \leq \frac{1}{2}$	✓ Endpoints ✓ Notation	(2)
	6.3.1	$45° \leq x \leq 90°$	✓ $x \geq 45°$ ✓ $x < 90°$	(2)
	6.3.2	$-90° < x < -45°$ or $90° < x < 135°$	✓ $-90° < x < -45°$ ✓ $90° < x < 135°$	(2)
	6.3.3	$2 \tan x + \cos 2x = 2$ $2 \tan x - 2 = -\cos 2x$ $\tan x - 1 = -\frac{1}{2} \cos 2x$ $\therefore f(x) = g(x)$ where $x = 45°$	✓ $2 \tan x - 2 = -\cos 2x$ ✓ $\tan x - 1 = -\frac{1}{2} \cos 2x$ ✓ Value of x	(3)
6.4		$h(x) = \frac{\sin x + \cos x}{\cos x}$ $h(x) = \frac{\sin x}{\cos x} + 1$ $h(x) = \tan x + 1$ $h(x) = f(x) + 2$ $\therefore$ Vertical translation up by 2 units	✓ $\tan x + 1$ ✓ Up by 2 units	(2)
6.5		$p(x) = 4\left(-\frac{1}{2}\cos 2x\right)$ $p(x) = -2 \cos 2x$ $\therefore$ Maximum value of 2	✓ $-2 \cos 2x$ ✓ Max of 2	(2)
				[14]

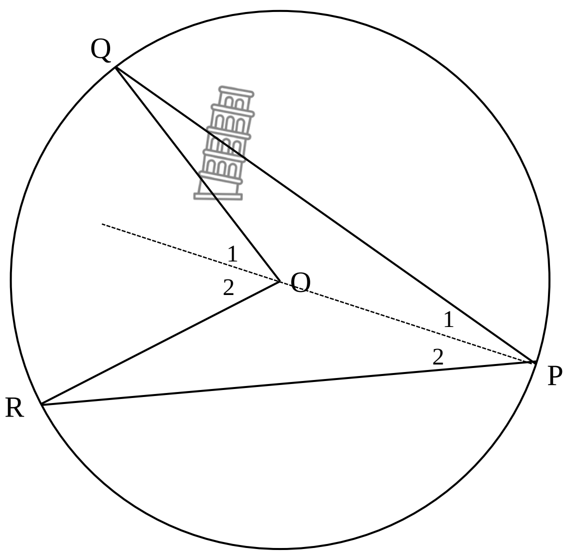
QUESTION 7

7.1	$\tan p = \frac{1}{3}$ $p = 18{,}435°$	✓ $\tan p = \frac{1}{3}$ ✓ Value of p to 3 decimal places	(2)
7.2	In ΔBCD $\frac{CD}{\sin\left(\frac{18{,}4°}{2}\right)} = \frac{121}{\sin 135°}$ $CD = \frac{121 \text{ x } \sin 9{,}2°}{\sin 135}$ CD = 27,36 *m*	✓ Correct substitution into sine rule ✓ Manipulation ✓ Length of CD	(3)
			[5]

QUESTION 8

8.1	$V = x^2h$ $490 = x^2h$ $h = \frac{490}{x^2}$	✓ Substitute into volume formula ✓ h in terms of x	(2)
8.2	$A = x^2 + 4xh$ $A = x^2 + 4x\left(\frac{490}{x^2}\right)$ $A = x^2 + \frac{1960}{x}\ cm^3$	✓ Base of x^2 ✓ $4x\left(\frac{490}{x^2}\right)$	(2)
8.3	$10 = \frac{490}{x^2}$ $x^2 = 49$ $x = 7 \qquad (x > 0)$ $A = (7)^2 + \frac{1\ 960}{7}$ $A = 329\ cm^2$	✓ Substitute $h = 10$ ✓ $x^2 = 49$ ✓ $x = 7$ ✓ Surface area	(4)
			[8]

QUESTION 9

9.1					
	Join PO and extend In ΔPOQ $\hat{O}_1 = \hat{Q} + \hat{P}_1$ but $\hat{Q} = \hat{P}_1$ $\therefore \hat{O}_1 = 2\hat{P}_1$ Similarly in ΔROQ $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{P}_1 + 2\hat{P}_2$ $\hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ $\therefore R\hat{O}Q = 2\hat{P}$		 ext $\angle$ of Δ $\angle$'s opp equal radii	✓ Construction ✓ SR ✓ SR ✓ S ✓ S ✓ S	(6)
9.2	9.2.1 (a)	$\hat{D} = 55°$	$\angle$'s in the same segment	✓ S ✓ R	(2)
	(b)	$\hat{O}_2 = 110°$	$\angle$ at centre $= 2\times$ $\angle$ at circumference	✓ S ✓ R	(2)
	(c)	$\hat{E}_2 = 110° - 90°$ $\hat{E}_2 = 20°$	ext $\angle$ of Δ	✓ R ✓ S	(2)

	(d)	$\hat{E}_1 = 35°$ $\hat{E}_3 = 35°$ **OR** $\hat{E}_3 = \hat{F}_1$ $\hat{E}_3 = 35°$	int $\angle$'s of Δ $\angle$'s in a semi-circle $\angle$'s opp equal radii int $\angle$'s of Δ	✓ R ✓ S ✓ R ✓ S ✓ R ✓ SR	(3)
	9.2.2	OE = 5 HE = 4,7 $OH^2 = 5^2 - (4,7)^2$ OH = 1,71 units **OR** OE = 5 $\sin 20° = \frac{OH}{5}$ OH = 1,71 units **OR** HE = 4,7 $\sin 20° = \frac{OH}{4,7}$ OH = 1,71 units	equal radii line from centre perpendicular to chord Pythagoras' Theorem radii line from centre perpendicular to chord	✓ SR ✓ S ✓ R ✓ S ✓ SR ✓✓ sin 20° ✓ OH ✓ S ✓ R ✓ tan 20° ✓ OH	(4)
					[19]

QUESTION 10

10.1	$\hat{B}_1 = x$ $\hat{V}_1 = x$ $\hat{T}_2 = x$	alt. $\angle$'s, AC ∥ TV $\angle$'s opp equal sides tan chord theorem	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R	(6)
10.2	$\hat{A} = 2x$ $\hat{B}_3 = 2x$ $\therefore \hat{A} = B_3$ AT = BT	opp $\angle$'s parm ext $\angle$ cyclic quad both equal $2x$ sides opp equal $\angle$'s	✓ S ✓ R ✓ S ✓ R ✓ R	

	OR $\hat{B}_3 = 2x$ ext $\angle$ cyclic quad $\hat{T}_1 = 2x$ alt $\angle$'s, AC ∥ TV $\therefore \hat{T}_1 = \hat{V}_1 + \hat{V}_2$ both equal $2x$ $\therefore$ BTVC is an isosceles trapezium (one pair parallel sides and one pair of base angles equal) $\therefore$ CV = BT sides of isos trap en CV = AT opp sides parm $\therefore$ AT = BT	✓ S ✓ R ✓ SR ✓ S ✓ R ✓ Logic	(5)
10.3	In ΔATB, $5x = 180°$ int $\angle$'s of Δ $x = 36°$	✓ R ✓ S	(2)
			[13]

QUESTION 11

11.1	$\hat{D}_3 = \hat{H}_1 + \hat{E}_1$ ext $\angle$ of Δ	✓ S	(1)
11.2	$\hat{D}_3 = \hat{H}_1 + \hat{E}_1$ ext $\angle$ of Δ $\hat{H}_1 = \hat{F}_2$ tan chord theorem and $\hat{E}_1 = \hat{F}_1$ tan chord theorem $\therefore \hat{D}_3 = \hat{F}_1 + \hat{F}_2$ $\therefore$ DEFH is a cyclic quadrilateral (ext $\angle$ = opp int $\angle$)	✓ S ✓ R ✓ SR ✓ R (with $\left(\hat{D}_3 = \hat{F}_1 + \hat{F}_2\right)$	(4)
11.3	$\hat{H}_1 = \hat{F}_2$ *tan chord theorem* $\hat{H}_1 = \hat{F}_1$ *$\angle$'s in the same segment* $\therefore \hat{F}_1 = \hat{F}_2$	✓ SR ✓ S ✓ R	(3)

11.4	$\hat{H}_3 + \hat{H}_4 = \hat{E}_1 + \hat{E}_2$ ext $\angle$ cyclic quad $\hat{H}_4 = \hat{K}_1$ tan chord theorem $\hat{H}_3 = \hat{K}_2$ $\angle$'s in the same circle segment $\therefore \hat{K}_1 + \hat{K}_2 = \hat{E}_1 + \hat{E}_2$ $\hat{K}_1 = \hat{E}_1$ given $\therefore \hat{K}_2 = \hat{E}_2$ $\therefore$KF is a tangent converse tan chord theorem	✓ S ✓ R ✓ SR ✓ S ✓ R ✓ S ✓ R (if $\hat{K}_2 = \hat{E}_2$ proven)	(7)
			[15]

TOTAL: 150