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Department of Education
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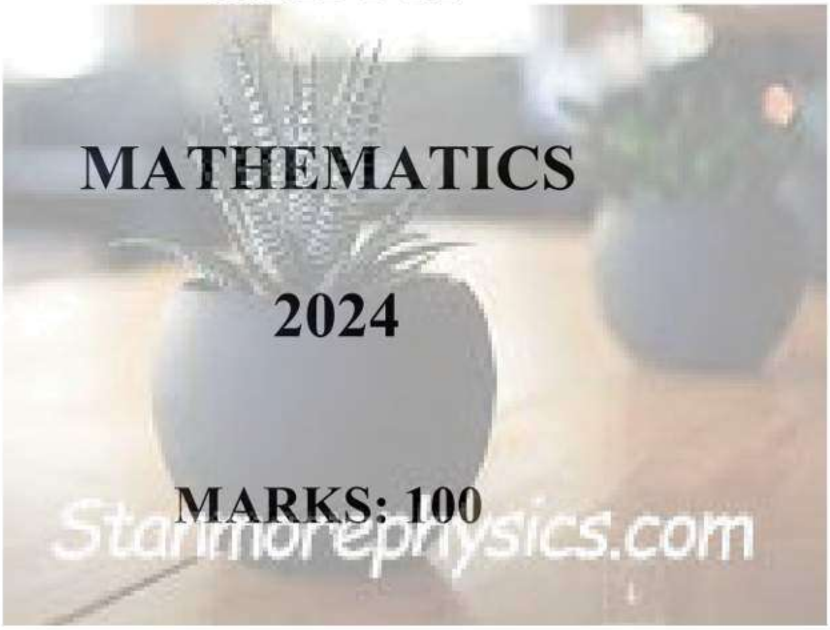
MARCH TEST

GRADE 12



MATHEMATICS

2024

MARKS: 100

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TIME: 2 hours

This paper consists of 8 pages.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used to determine the answer.
4. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
5. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
6. An INFORMATION SHEET with formulae is included at the end of the question paper.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.
9. Answers only will NOT necessarily be awarded full marks.

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QUESTION 1

Solve for *x*:

1.1 $x(2x-7)=-3$ (3)

1.2 $\sqrt{x+5}+x=3$ (correct to 2 decimal places) (5)

1.3 $-x^2+16>0$ (4)

1.4 Solve for x and y simultaneously.

$x-3y=1$ and $(2x+y-1)(x-y+1)=0$ (5)

1.5 Prove that

$\sqrt{\sqrt[3]{ab}}.\sqrt[b]{b\sqrt[3]{a}}=a^{\frac{2+b}{6b}}b^{\frac{b+6}{6b}}$ (3)

[20]

QUESTION 2

2.1 The terms: $\frac{25}{16};\frac{81}{16};\frac{169}{16};\frac{289}{16};\ldots\ldots\ldots\ldots$ forms a quadratic pattern.

2.1.1 Determine the second constant difference of the pattern. (1)

2.1.2 Show that the general term of the pattern is $T_n=n^2+\frac{1}{2}n+\frac{1}{16}$ (3)

2.1.3 Prove that all the terms of this pattern will always be positive and are all perfect squares. (3)

2.2 Consider the arithmetic sequence:

$5\,;\,3\,;\,1\,;\,-1\,;\,-3\,;\,\ldots;-375$

2.2.1 Write down T_6. (1)

2.2.2 Determine the n[th] term of the sequence. (2)

2.2.3 Determine the number of terms in this sequence. (2)

2.2.4 There are 64 terms in this arithmetic sequence that are exactly divisible by 3. Calculate their sum. (3)

2.2.5 Evaluate, **without using a calculator**: (4)

$$\sum_{y=43^\circ}^{47^\circ}\sin^2 y$$

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QUESTION 3

3.1 If $r = \frac{1}{2}$ and $a = 3$.

Which term of the sequence will have a value of $\frac{3}{128}$? (4)

3.2 The first three terms of a geometric sequence are $x\ ;\ y\ ;\ -2y - x; \ldots \ldots$ (4)
Determine the numerical value of $\frac{x}{y}$

3.3 The information below is that of the convergent geometric series:

$$12 = \sum_{n=1}^{\infty} 2P^{1-n}$$

Determine the value of:

3.3.1 The first term of the convergent geometric series. (2)

3.3.2 The common ratio, r, of the convergent geometric series. (5)

[15]

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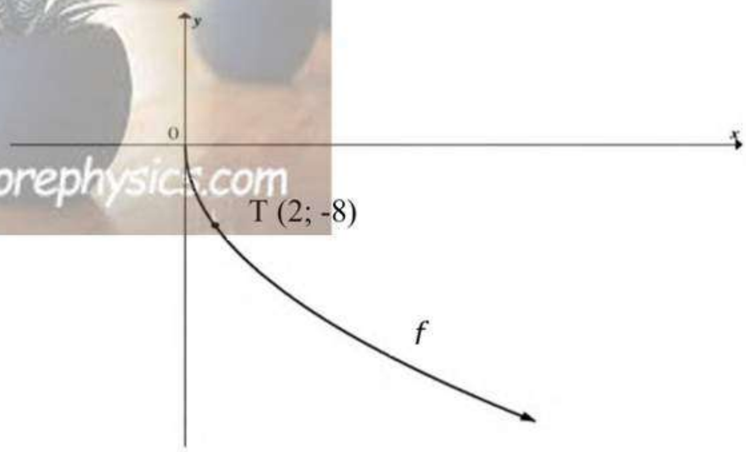
QUESTION 4

4.1 The graph of f for $x \leq 0$ is sketched below. The point T (2 ; - 8) lies on the graph of f



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4.1.1 Write down the domain of f^{-1}. (1)

4.1.2 Write down the coordinates of T^{-1}, a point which lies on f^{-1}. (1)

4.1.3 Use your graph to determine the values of x for which $f(x)^{-1} < 2$ (2)

4.2 $\text{P}(8; 1\frac{1}{2})$ is a point on the graph of $f(x) = \log_a x$.

4.2.1 Show that $a = 4$ (2)

4.2.2 Determine the equation of f^{-1} in the form $y = \cdots$ (2)

4.2.3 Determine the equation of g the reflection of f about the $x - axis$. (1)

4.2.4 Sketch the graphs of f, g and f^{-1} on the same set of axes. (5)

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QUESTION 5

5.1 Given: $sin\ 24° = k$.

Determine the following in terms of k and **without the use of a calculator.**

5.1.1 $\sin 336°$ (2)

5.1.2 $\tan(-24°)$ (2)

5.2 Given: $\cos\theta = \frac{-5}{7}$ and $0° < \theta < 180°$.

Determine $\sin(\theta + 60°)$, **without the use of a calculator**. (4)

5.3 Simplify the following to a single trigonometric ratio.

$$\frac{\sin(180° - 2x).\cos x}{2\cos(90° - x)} - \tan(180° - x).\cos(180° + x).\sin(x + 720°)$$ (7)

5.4 Given: $\frac{2\tan\beta}{1+\tan^2\beta}$

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5.4.1 Prove that:

$$\frac{2\tan\beta}{1+\tan^2\beta} = \sin 2\beta$$ (4)

5.4.2 Hence, or otherwise determine the maximum value of:

$$\frac{(1+\tan\beta)^2}{1+\tan^2\beta}$$ (3)

5.5 Determine the general solution of:

$2\cos^2 x - cosx - 1 = 0$ (5)

[27]

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QUESTION 6

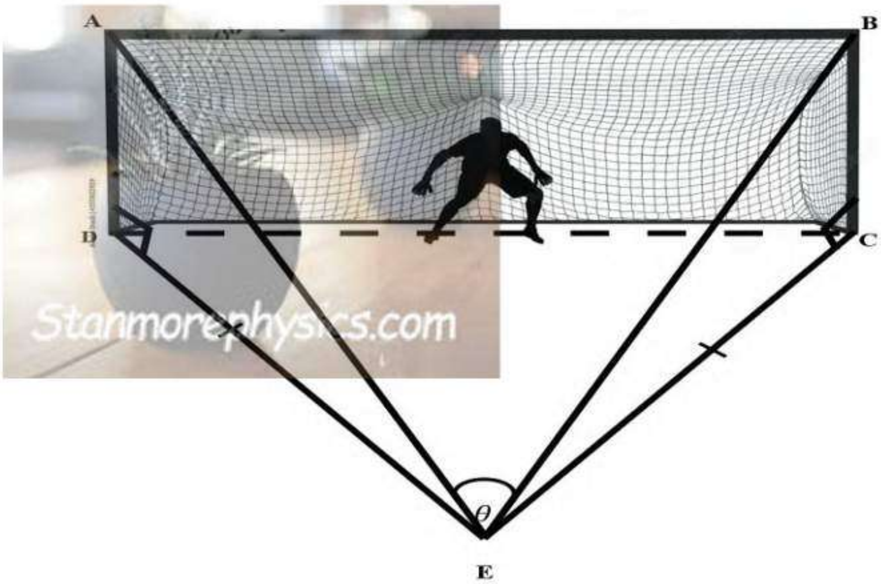
6.1 Rectangle ABCD is formed by soccer goal posts. A penalty spot (where a penalty is being kicked from) is at point E. Lines DE and CE are drawn from the bottom of the vertical poles AD and BC respectively to point E. Lines AE and BE are drawn from the top of the vertical poles AD and BC respectively to point E. C, D are E are in the same horizontal plane. It is further given that,

$CE = DE = 11{,}5\text{ m}$

$AD = 2{,}4\text{m}$

$AB = 7{,}3\text{m}$

$B\hat{C}E = A\hat{D}E = 90°$



Calculate the following, correct to one decimal place.

6.1.1 Length of AE. (2)

6.1.2 Size of θ. (3)

[5]

TOTAL = 100

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INFORMATION SHEET/INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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education

Department of Education
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MARCH TEST

GRADE12

MATHEMATICS MARKING GUIDLINE

2024

MARKS: 100

This marking guideline consists of 09 pages.

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QUESTION 1

1.1	$2x^2 - 7x + 3 = 0$ $x = \frac{1}{2}$ or $x = 3$	SF ✓ $x -$ values ✓✓ (3)
1.2	$\left(\sqrt{x+5}\right)^2 = (3-x)^2$ $x + 5 = 9 - 6x + x^2$ $x + 5 - 9 + 6x - x^2 = 0$ $-x^2 + 7x - 4 = 0$ $x^2 - 7x + 4 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(4)}}{2(1)}$ $x \neq 6.37$ or $x = 0.63$	Squaring ✓ Standard form ✓ Substitution into correct formula ✓ Solution. Panelise 1 Mark for wrong rounding and solution not rejected ✓✓ (5)
1.3	$-x^2 + 16 > 0$ $x^2 - 16 < 0$ $(x+4)(x-4) < 0$ cv: -4 and 4 $-4 < x < 4$	Factors ✓ Solution ✓✓ (3)
1.4	$x - 3y = 1$ ………① $(2x + y - 1)(x - y + 1) = 0$ ………② $x = 1 + 3y$ ………3 eqt 3 … into … eqt 2 $(2x + y - 1)(x - y + 1) = 0$ $(2(1 + 3y) + y - 1)(1 + 3y - y + 1) = 0$ $(7y + 1)(2y + 2) = 0$ $7y^2 + 8y + 1 = 0$ $(7y + 1)(y + 1) = 0$ $y = -\frac{1}{7}$ or $y = -1$ $x = \frac{4}{7}$ or $x = -2$	Eqt 3 ✓ Substitution ✓ Standard form ✓ $y -$ values ✓ $x -$ values ✓ (5)

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1.5	$\left(\sqrt[3]{ab}\right)^{\frac{1}{2}}.\left(b\sqrt[3]{a}\right)^{\frac{1}{b}}$ $\left[(ab)^{\frac{1}{3}}\right]^{\frac{1}{2}}.\left[b\left(a^{\frac{1}{3}}\right)\right]^{\frac{1}{b}}$ $a^{\frac{1}{6}}.b^{\frac{1}{6}}.b^{\frac{1}{b}}.a^{\frac{1}{3b}}$ $a^{\frac{3b+6}{6.3b}}.b^{\frac{1}{6}+\frac{1}{b}}$ $a^{\frac{b+2}{6b}}.b^{\frac{b+6}{6b}}$	$\left(\sqrt[3]{ab}\right)^{\frac{1}{2}}.\left(b\sqrt[3]{a}\right)^{\frac{1}{b}}$ ✓ $a^{\frac{1}{6}}.b^{\frac{1}{6}}$ ✓ $b^{\frac{1}{b}}.a^{\frac{1}{3b}}$ ✓ $a^{\frac{3b+6}{6.3b}}.b^{\frac{1}{6}+\frac{1}{b}}$ ✓ (4)
		[20]

QUESTION 2

2.1.1	$\frac{25}{16}$; $\frac{81}{16}$; $\frac{169}{16}$; $\frac{289}{16}$ $\frac{7}{2}$ $\frac{11}{2}$ $\frac{15}{2}$ 2 2	*Second constant difference* = 2 ✓ (1)
2.1.2	$2a = 2$ $a = 1$ $3a + b = \frac{7}{2}$ $3(1) + b = \frac{7}{2}$ $b = \frac{1}{2}$ $a + b + c = \frac{25}{16}$ $1 + \frac{1}{2} + c = \frac{25}{16}$ $c = \frac{1}{16}$ $\therefore T_n = n^2 + \frac{1}{2}n + \frac{1}{16}$	$a = 1$ ✓ $b = \frac{1}{2}$ ✓ $c = \frac{1}{16}$ ✓ (3)
2.1.3	$\therefore T_n = n^2 + \frac{1}{2}n + \frac{1}{16}$ $T_n = \left(n + \frac{1}{4}\right)\left(n + \frac{1}{4}\right)$ $T_n = \left(n + \frac{1}{4}\right)^2$ $\left(n + \frac{1}{4}\right)^2 > 0$ *and is a perferct square for* $n \in N$	*Factorisation* ✓ $T_n = \left(n + \frac{1}{4}\right)^2$ ✓ *Conclusion* ✓ (3)

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2.2.1	$T_6 = -5$	$T_6 = -5$ ✓ (1)
2.2.2	$T_n = a + (n-1)d$ $T_n = 5 + (n-1)(-2)$ $T_n = -2n + 7$	$Subtstitution\ of\ a = 5\ \&\ c = -2$ ✓ $T_n = -2n + 7$ ✓ (2)
2.2.3	$-375 = -2n + 7$ $-382 = -2n$ $n = 191$	$Substitution\ of\ -375\ for\ T_n$ ✓ $n = 191$ ✓ (2)
2.2.4	$S_{64} = \frac{n}{2}(2a + (n-1)d)$ $= \frac{64}{2}\big(2(3) + (64-1)(-6)\big)$ $= -11904$ **OR** $5; 3; 1; -1; +3; -5; -7; -9; -11; -13; -15; \ldots\ldots; -375$ $New\ Seq: 3; -3; -9; \ldots\ldots; -375$ $S_{64} = \frac{64}{2}\big(3 + (-375)\big)$ $= -11904$	$a = 3$ ✓ $correct\ substitution$ ✓ $answer$ ✓ **OR** $a = 3$ ✓ $correct\ substitution$ ✓ $answer$ ✓ (3)
2.2.5	$\sum_{y=43^\circ}^{47^\circ} \sin^2 y$ $= \sin^2 43^\circ + \sin^2 44^\circ + \sin^2 45^\circ + \sin^2 46^\circ + \sin^2 47^\circ$ $= \cos^2 47^\circ + \cos^2 46^\circ + \sin^2 45^\circ + \sin^2 46^\circ + \sin^2 47^\circ$ $= \cos^2 47^\circ + \sin^2 47^\circ + \cos^2 46^\circ + \sin^2 46^\circ + \sin^2 45^\circ$ $= 1 + 1 + \left(\frac{1}{\sqrt{2}}\right)^2$ $= \frac{5}{2}$	Expansion ✓ Co – ratio $sin^2 43^\circ = cos^2 47^\circ$ ✓ Or $sin^2 44^\circ = cos^2 46^\circ$ ✓ *Square Identity* Or $sin^2 46^\circ + cos^2 46^\circ$ ✓ Answer ✓ (4)
		[19]

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QUESTION 3

3.1	$ar^{n-1} = \frac{3}{128}$ $3\left(\frac{1}{2}\right)^{n-1} = \frac{3}{128}$ $\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{7}$ $n = 8$	method ✓ *substitution* ✓ *simplification* ✓ *answer*✓ (4)
3.2	$x, y, -2y - x$ $r = \frac{y}{x} = \frac{-2y-x}{y}$ $y^2 = -2xy - x^2$ $x^2 + 2xy + y^2 = 0$ $(x + y)(x + y) = 0$ $x = -y \; or \; x = -y$ $\frac{x}{y} = -1 \; or \; \frac{x}{y} = -1$	$r = \frac{y}{x} = \frac{-2y-x}{y}$ ✓ S.F ✓ Factorisation ✓ $\frac{x}{y} = -1$ ✓ (4)
3.3.1	$T_1 = 2P^{1-1}$ $T_1 = 2$	*expansion*✓ *answer*✓ (2)
3.3.2	$T_2 = 2P^{1-2}$ $2P^{-1}$ $r = \frac{T_2}{T_1} = \frac{2P^{-1}}{2} = \frac{1}{P}$ $S_\infty = \frac{a}{1-r}$ $12 = \frac{2}{1-\frac{1}{p}}$ $12 - \frac{12}{p} = 2$ $-\frac{12}{p} = -10$ $p = \frac{6}{5}$ $r = \frac{5}{6}$	*Value of r* ✓ $s\infty = 12$ ✓ *Substitution* ✓ $p = \frac{6}{5}$ ✓ $r = \frac{5}{6}$ ✓ (5)
		[15]

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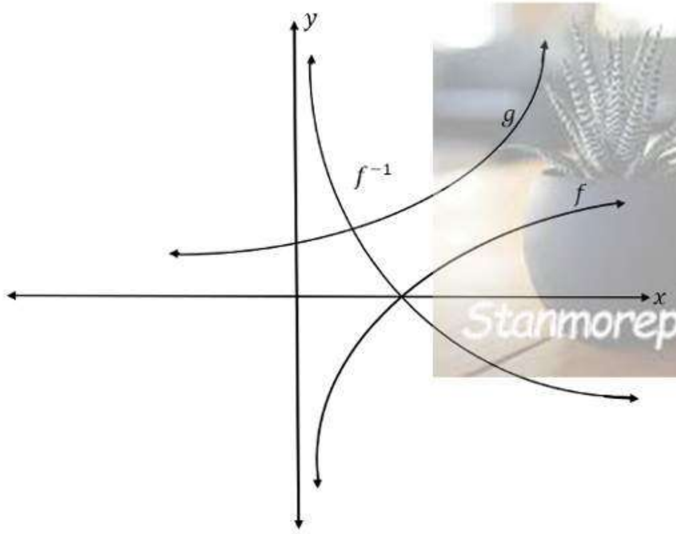
QUESTION 4

4.1.1	$x \leq 0$	*answer* ✓ (1)
4.1.2	$(-8; 2)$	*answer* ✓ (1)
4.1.3	$-8 < x \leq 0$	*Boundries* ✓ *Notation* ✓ (2)
4.2.1	$\frac{3}{2} = log_a 8$ $a^{\frac{3}{2}} = 8$ $a^{\frac{3}{2}} = 2^3$ $\left(a^{\frac{3}{2}}\right)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$ $a = 4$	$\left(a^{\frac{3}{2}}\right)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$ ✓ $a = 4$ ✓ (2)
4.2.2	$y = 4^x$	*x and y swop* ✓ *answer* ✓ (2)
4.2.3	$g(x) = -log_4 x$ or $g(x) = log_{\frac{1}{4}} x$	Answer ✓ (1)
4.2.4		*graph of f* ✓ *graph of g* ✓ *graph of* f^{-1} ✓ *Increasing shape of g and f* ✓ *Decreasing shape of* f^{-1} ✓ (5)
		[14]

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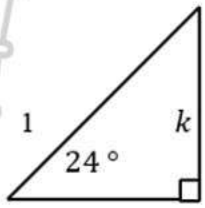
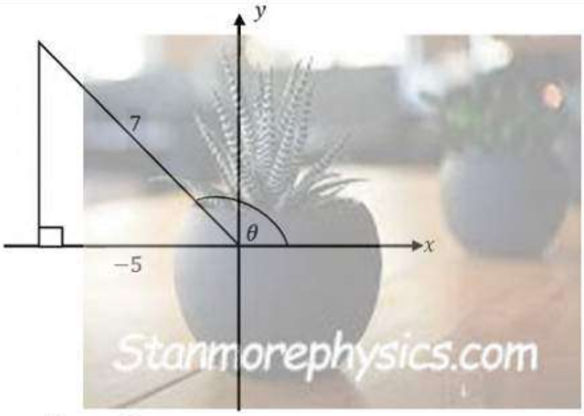
QUESTION 5

5.1.1	 $r^2 = x^2 + y^2$ $1^2 = x^2 + k^2$ $x = \sqrt{1-k^2}$ $sin336° = \sin(360° - 24°)$ $= \ sin24°$ $= -k$	$sin24°$ ✓ $-k$ ✓ (2)
5.1.2	$tan(-24°) = -tan24\ °$ $= -\frac{k}{\sqrt{1-k^2}}$	$\sqrt{1-k^2}$✓ $-\frac{k}{\sqrt{1-k^2}}$ ✓ (2)
5.2	 $7^2 = (-5)^2 + y^2$ $49 - 25 = y^2$ $24 = y^2$ $\therefore y = \sqrt{24}$ $sin(\theta + 60°) = sin\theta cos60° + cos\theta sin60°$ $\left(\frac{\sqrt{24}}{7}\right)\left(\frac{1}{2}\right) + \left(\frac{-5}{7}\right)\left(\frac{\sqrt{3}}{2}\right)$ $\frac{2\sqrt{6} - 5\sqrt{3}}{14}$	$y = \sqrt{24}$ ✓ *Expansion compound identity of sine* ✓ *substitution* ✓ *Answer* ✓ (4)

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5.3	$\frac{\sin(180^\circ - 2x)\cos x}{2\cos(90^\circ - x)} - \tan(180^\circ - x)\cos(180^\circ + x)\sin(x - 720^\circ)$ $\frac{\sin(2x)\cos x}{2sinx} - (-tanx)(-cosx)sinx$ $\frac{2sinxcosxcosx}{2sinx} + tanx(-cosx)sinx$ $cosx.cosx + \frac{sinx}{cosx}(-cosx)sinx$ $\cos^2 x - \sin^2 x$ $cos2x$	$(-tanx)$ ✓ $(-cosx)$ ✓ $2sinx$ ✓ $sinx$ ✓ *Expansion double identity of sine :* $sin2x = 2\sin x\cos x \frac{sinx}{cosx}$ ✓ $cos^2x - sin^2x$ ✓ $cos2x$ ✓ (7)
5.4.1	$LHS = \frac{\frac{2sin\beta}{cos\beta}}{\frac{cos^2\beta}{cos^2\beta} + \frac{sin^2\beta}{cos^2\beta}}$ $= \frac{\frac{2sin\beta}{cos\beta}}{\frac{cos^2\beta + sin^2\beta}{cos^2\beta}}$ $= \frac{\frac{2sin\beta}{cos\beta}}{\frac{1}{cos^2\beta}}$ $= \frac{2sin\beta}{cos\beta} \times \frac{cos^2\beta}{1}$ $= \frac{2sin\beta cos\beta}{1}$ $= sin2\beta$	$sin^2\beta$ $cos^2\beta$ ✓ $cos^2\beta + sin^2\beta$ ✓ $\frac{1}{cos^2\beta}$ ✓ $2sin\beta cos\beta$ ✓ (4)
5.4.2	$= \frac{1 + 2tan\beta + tan^2\beta}{1 + tan^2\beta}$ $= \frac{1 + tan^2\beta + 2tan\beta}{1 + tan^2\beta}$ $= 1 + \frac{2tan\beta}{1 + tan^2\beta}$ $= 1 + sin2\beta$ $\therefore Max\ Value = 2$	Quadratic trinomial $1 + 2tan\beta + tan^2$ ✓ $1 + sin2\beta$ ✓ Max value = 2 ✓ (3)

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5.5	$(2cosx+1)(cosx-1)=0$ $cosx=-\frac{1}{2}$ or $\cos x=1$ $x=120°+360°.k$ or $x=240°+360°.k \quad k \epsilon Z$ $x=360°.k \quad k \epsilon Z$	Factorization ✓ $cosx=-\frac{1}{2}$ or $\cos x=1$ ✓ $x=120°+360°.k$; $k \epsilon Z$ ✓ $x=240°+360°.k$; $k \epsilon Z$ ✓ $x=360°.k \quad k \epsilon Z$ ✓ (5)
		[27]

QUESTION 6

7.4.1	$r^2=x^2+y^2$ $(AE)^2=(11{,}5)^2+(2{,}4)^2$ $=11.7$	*Substitution* ✓ *Answer* ✓ (2)
7.4.2	$\triangle ADE \equiv \triangle BCE \quad RHS\ or\ SAS$ $AE=BE \quad \equiv \triangle' s$ $a^2=b^2+c^2-2bc.cosA$ $(7.3)^2=(11.7)^2+(11.7)^2-2(11.7)(11.7).cos\theta$ $-220.49=-273.78.cos\theta$ $0.8053546643=cos\theta$ $\theta=36.4°$	*Substitution into cosine rule* ✓ *simplification* ✓ *Answer* ✓ (3)
		[5]
	TOTAL	**[100]**