LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

SEKHUKHUNE EAST DISTRICT – DISTRICT ON THE RISE

GRADE 12

MATHEMATICS

TEST 1

DURATION: 2 HOURS

This question paper consists of 9 pages including the cover page, formula and answer sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of EIGHT questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

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QUESTION 1

1.1 Solve for x:

1.1.1 $x(5x+2)=0$ (2)

1.1.2 $x(2x-3)=4$ (correct to TWO decimal places) (4)

1.1.3 $x^2-x-6\geq 6$ (4)

1.2 Solve for x and y simultaneously if:

$x-y=3$ and $x^2+xy-2y^2=0$ (5)

[15]

QUESTION 2

2.1 Given:

$\frac{3x-1}{4};\frac{2x-1}{3};\frac{7x-5}{12}$

2.1.1 If $x=5$, determine the values of the first three terms. (1)

2.1.2 What type of sequence is this? Give a reason for your answer. (2)

2.1.3 Which term will be equal to $-44{,}5$? (3)

2.2 Given the series:

$18+6+2+\cdots$

2.2.1 What is the value of the first negative term, if any? Explain your answer. (2)

2.2.2 Determine the tenth term, T_{10}. (2)

2.2.3 Determine $S_\infty - S_{10}$. (5)

[15]

QUESTION 3

3.1 Determine the value of:

$$\sum_{k=2}^{33}(1-2k)$$ (3)

3.2 $6\,;5+x\,;-6\,;\ 6x$ form the first 4 terms of a quadratic sequence.

3.2.1 Show that $x = -3$. (4)

3.2.2 Determine an expression for the general term of the sequence. (4)

[11]

QUESTION 4

Given: $f(x) = \dfrac{3}{x-1} - 2$

4.1 Write down the equation of the:

4.1.1 Horizontal asymptote of *f*. (1)

4.1.2 Vertical asymptote of *f*. (1)

4.2 Determine the *x*- and *y*-intercepts of *f*. (3)

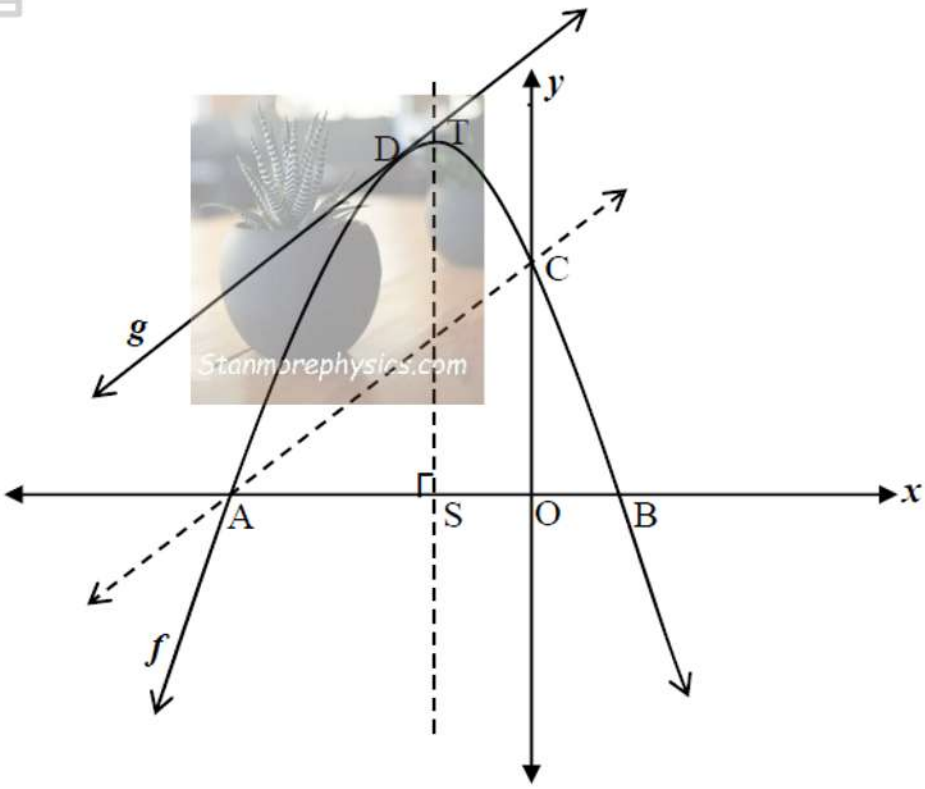
4.3 Sketch the graph of *f*, showing clearly the asymptotes and the intercepts with the axes. (3)

4.4 If another function *g* is defined as $g(x) = f(x-3) + 7$, determine the coordinates of the point of intersection of the asymptotes of *g*. (2)

[10]

QUESTION 5

The diagram shows the graphs of $f(x) = -2x^2 - 4x + 6$ and $g(x) = mx + c$. A, B and C are the intercepts of *f* with the axes. T is the turning point of the graph of *f*. The graph of g is a straight line parallel to AC, and is a tangent to the graph of *f* at D.



5.1 Determine the lengths of OC and AB. (5)

5.2 Determine the equation of the axis of symmetry of the graph of *f*. (2)

5.3 Show that the length of ST = 8 units. (2)

5.4 Calculate the gradient of AC. (2)

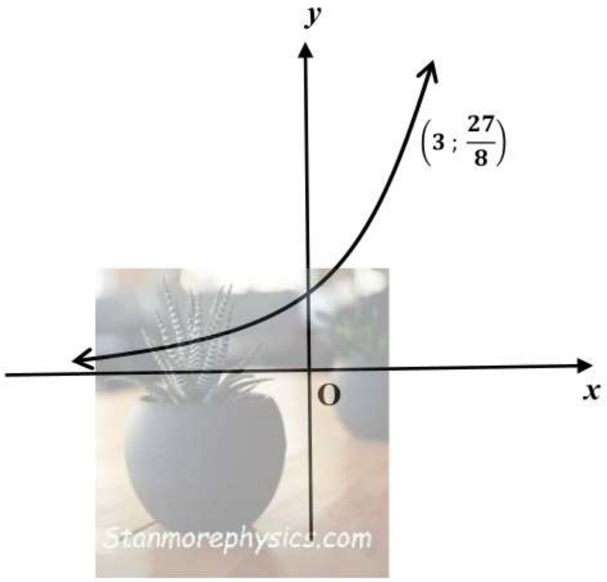
5.5 Hence, or otherwise, calculate the coordinates of D. (5)

5.6 For which value(s) of a will $f(a+t) = f(a-t)$ for all values of t? (2)

[18]

QUESTION 6

The graph of $f(x) = a^x$, where $a > 0$ and $a \neq 1$, passes through the point $\left(3\ ;\ \frac{27}{8}\right)$.



6.1 Determine the value of a. (2)

6.2 Write down the equation of f^{-1} in the form $y = \ldots$ (2)

6.3 If $h(x) = f(x-5)$, write down the domain of h. (1)

[5]

QUESTION 7

7.1 If $\sin 34^\circ = p$, determine the value of each of the following in terms of p, **WITHOUT USING A CALCULATOR.**

7.1.1 $\sin 214°$ (2)

7.1.2 $\cos 34^\circ . \cos(-22^\circ) + \cos 56^\circ . \sin 338^\circ$ (4)

7.1.3 $\cos 68°$ (2)

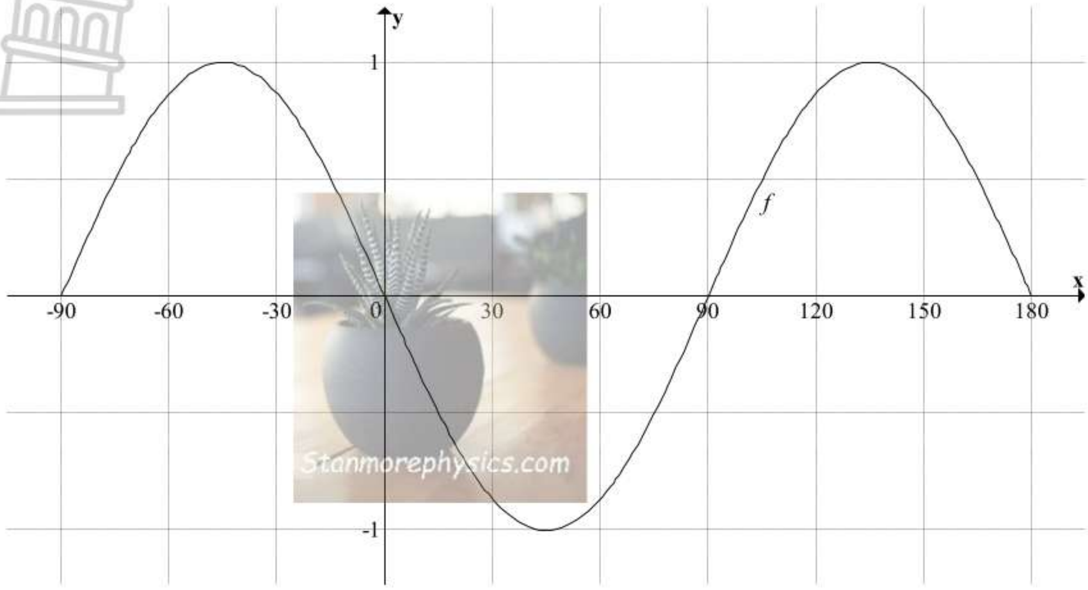
7.2 Determine the value of each of the following expressions:

$$\frac{\cos(90^\circ - 2\theta).\sin\theta}{\sin^2(180^\circ + \theta).\cos(720^\circ + \theta)}$$ (6)

[14]

QUESTION 8

In the diagram, the graph of $f(x) = -\sin 2x$ is drawn for the interval $x \in [-90°; 180°]$.



8.1 Draw the graph of g, where $g(x) = \cos(x - 60°)$, on the same system of axes for the interval $x \in [-90°; 180°]$ in the ANSWER SHEET. (3)

8.2 Determine the general solution of $f(x) = g(x)$. (5)

8.3 Use your graphs to solve x if $f(x) \leq g(x)$ for $x \in [-90°; 180°]$ (2)

8.4 Which transformation must the graph of g undergo to form the graph of h, where $h(x) = \sin x$? (2)

[12]

TOTAL: 100 MARKS

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

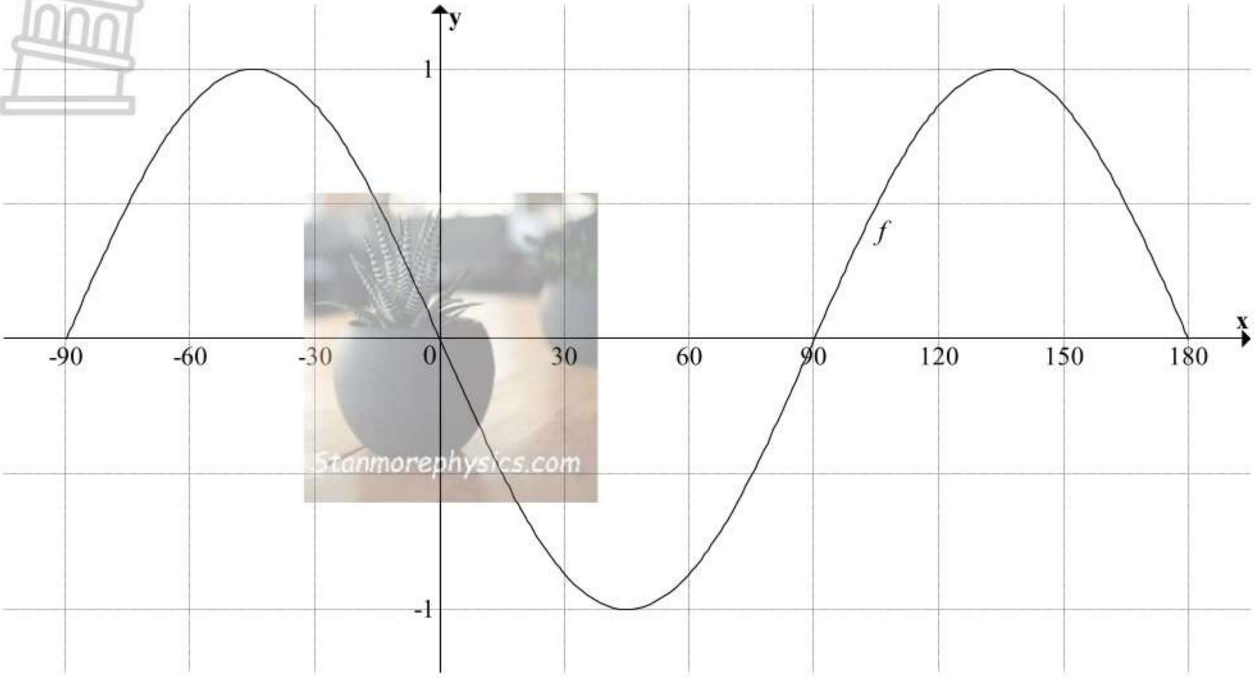
$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

ANSWER SHEET

LEARNER'S NAME:____________________________________ **CLASS:**_____

QUESTION 8.1



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GRADE 12

MATHEMATICS

TEST 1

MARKING GUIDELINES

11 MARCH 2024

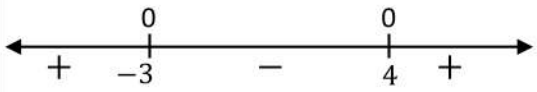
TOTAL MARKS: 50

This marking guidelines consists of 11 pages including the cover page.

NOTE:

- **If a candidate answers a question TWICE, mark only the first one.**
- **Consistent accuracy applies in ALL aspects of the marking guidelines.**

QUESTION 1

1.1.1	$x(5x+2)=0$ $x=0$ or $x=-\frac{2}{5}$	✓ $x=0$ ✓ $x=-\frac{2}{5}$	(2)
1.1.2	$x(2x-3)=4$ $2x^2-3x-4=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)}$ $x=\frac{3\pm\sqrt{41}}{4}$ $x=-0{,}85$ or $x=2{,}35$	✓ Std form ✓Substitution ✓✓Each root	(4)
1.1.3	$x^2-x-6\geq 6$ $x^2-x-12\geq 0$ $(x+3)(x-4)\geq 0$ C.V. $x=-3$ and $x=4$  $x<-3$ or $x>4$	✓ Standard form ✓ Factors ✓ Both critical values ✓ Answer	(4)

1.2	$x - y = 3$ ... ① $x^2 + xy - 2y^2 = 0$ ... ② $x = y + 3$ ... ③ Substitute ③ into ②: $\therefore (y+3)^2 + y(y+3) - 2y^2 = 0$ $\therefore y^2 + 6y + 9 + y(y+3) - 2y^2 = 0$ $\therefore y^2 + 6y + 9 + y^2 + 3y - 2y^2 = 0$ $\therefore 9y + 9 = 0$ $\therefore y = -1$ $\therefore \ x = 2$ **OR** $x - y = 3$ ... ① $x^2 + xy - 2y^2 = 0$ ... ② $y = x - 3$ ... ③ Substitute ③ into ②: $x^2 + xy - 2y^2 = 0$ .....(2) $\therefore x^2 + x(x-3) - 2(x-3)^2 = 0$ $\therefore x^2 + x^2 - 3x - 2x^2 + 12x - 18 = 0$ $\therefore 9x - 18 = 0$ $\therefore \ x = 2$ $\therefore \ y = -1$	✓ x the subject ✓ Substitution ✓ Simplifying ✓value of y ✓value of x ✓ y the subject ✓ Substitution ✓ Simplifying ✓value of x ✓value of y	(5)
			[15]

QUESTION 2

2.1.1	$\frac{7}{2}; 3; \frac{5}{2}$ **OR** $3\frac{1}{2}\ ;\ 3\ ;\ 2\frac{1}{2}$	✓ Answer	(1)
2.1.2	Arithmetic sequence; $d = -\frac{1}{2}$ or $\frac{1}{2}$ is subtracted each time	✓ Arithmetic sequence ✓ $d = -\frac{1}{2}$	(2)
2.1.3	$a = 3\frac{1}{2}\ ;\ d = -\frac{1}{2}\ ;\ T_n = -44{,}5$ $\therefore\ -44\frac{1}{2} = 3\frac{1}{2} + (n-1)\left(-\frac{1}{2}\right)$ $\therefore 4 - \frac{1}{2}n = -44\frac{1}{2}$ $\therefore -\frac{1}{2}n = -\frac{97}{2}$ $\therefore n = 97$	✓ Substitution ✓ $T_n = -44{,}5$ ✓Answer	(3)
2.2.1	$18 + 6 + 2 + \ldots$ Geometric sequence with $r = \frac{1}{3}$ Thus no negative term.	✓ GS with $r = \frac{1}{3}$ ✓ no negative term	(2)
2.2.2	$T_{10} = ar^{n-1}\ ;\ a = 18;\ r = \frac{1}{3}\ ;\ n = 10$ $= 18\left(\frac{1}{3}\right)^9$ $= \frac{2}{2187}$	✓ Substitution ✓Answer	(2)
2.2.3	$S_\infty - S_{10}\ ;\ a = 18;\ r = \frac{1}{3}\ ;\ n = 10$ $= \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$ $= \frac{18}{1-\frac{1}{3}} - \frac{18\left[1-\left(\frac{1}{3}\right)^{10}\right]}{1-\frac{1}{3}}$ $= 27 - 27\left[1-\left(\frac{1}{3}\right)^{10}\right]$ $= 4{,}57 \times 10^{-4}$	✓ Correct sum formulae ✓✓ Substitution into each formular ✓ Simplifying ✓ Answer	(5)
			[15]

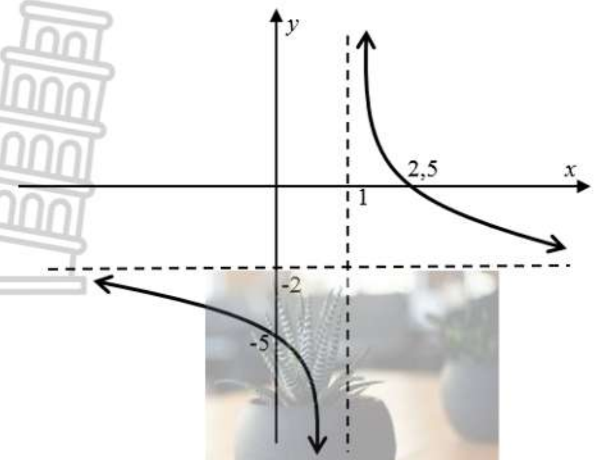
QUESTION 3

3.1	$\sum_{k=2}^{33}(1-2k)$ $-3-5-7-\cdots-65$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{32}=\frac{32}{2}[2(-3)+(32-1)(-2)]$ $=\frac{32}{2}[-6-62]$ $=-1088$ **OR** $S_n=\frac{n}{2}[a+l]$ $S_{32}=\frac{32}{2}[(-3)+(-65)]$ $S_{32}=-1088$	✓ $n=32$. ✓ Substitution ✓ Answer **OR** ✓ $n=32$. ✓ Substitution ✓ Answer	(3)
3.2.1	$6\,;5+x\,;-6\,;\ 6x$ $x+5-6;-6-(5+x);6x\ -(-6)$ $x-1;-x-11;\ 6x+6$; 1st diff $-x-11-(x-1)=6x+6-(-x-11)$; 2nd diff $\therefore -2x-10=7x+17$ $\therefore x=-3$	✓ 1st difference ✓ 2nd difference ✓ 2nd difference equal ✓ Simplifying	(4)

3.2.2	Sequence $: \mathbf{6}\ ;\ \mathbf{2}\ ;\ \mathbf{-6}\ ;\ \mathbf{-18}; \ldots$ $-4\ ;\ -8\ ;\ -12$ 1st diff $-4\ ;\ -4$ 2nd diff $2a = -4$ $\therefore a = -2$ $3a + b = -4$ $3(-2) + b = -4$ $\therefore b = 2$ $a + b + c = 6$ $-2 + 2 + c = 6$ $\therefore c = 6$ $\therefore T_n = -2n^2 + 2n + 6$	✓ $a = -2$ ✓ $b = 2$ ✓ $c = 6$ ✓ Equation	(4)
			[11]

QUESTION 4

4.1.1	$y = -2$	✓ Answer	(1)
4.1.2	$x = 1$	✓ Answer	(1)
4.2	$x -$ intercept $\frac{3}{x-1} - 2 = 0$ $\frac{3}{x-1} = 2$ $3 = 2x - 2$ $5 = 2x$ $x = \frac{5}{2}$ $y -$ intercept $y = \frac{3}{0-1} - 2$ $y = -5$	✓$y = 0$ ✓x – intercept ✓y– intercept	(3)

4.3		✓ Asymptotes ✓ x-and y-intercepts ✓ Shape	(3)
4.4	(4 ; 5)	✓ x ✓ y	(2)
			[10]

QUESTION 5

5.1	OC = 6 units A; B: x-intercepts let $y = 0$ $\therefore -2x^2 - 4x + 6 = 0$ $\therefore x^2 + 2x - 3 = 0$ $\therefore (x+3)(x-1) = 0$ $\therefore x = -3$ or $x = 1$ A$(-3;\ 0)$ and B$(1;\ 0) \Rightarrow$ $\therefore$ AB = 4 units	✓ OC = 6 ✓ $y = 0$ ✓ Factors ✓ Both x-values ✓ Answer	(5)
5.2	$x = -\dfrac{b}{2a}$ $= -\left[\dfrac{-4}{2(-2)}\right]$ $= -1$	✓ Substitution ✓ $x = -1$	(2)

5.3	Substitute $x = -1$ in $f(x)$ $\therefore ST = -2(-1)^2 - 4(-1) + 6$ $= 8$ units	✓ Substitute $x = -1$ ✓ Answer	(2)
5.4	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0-6}{-3-0}$ $= 2$	✓ Substitution ✓ Answer	(2)

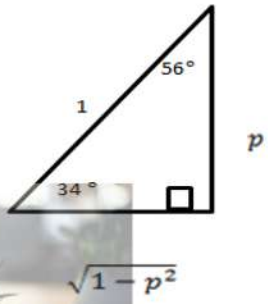
5.5	Let eq. of tangent be: $y = mx + k$
	$m_g = 2$; // lines
	$y = 2x + k$
	at D: $f(x) = g(x)$ $\therefore -2x^2 - 4x + 6 = 2x + k$
	$\therefore -2x^2 - 6x + 6 - k = 0$.....①
	For tangent: $\Delta = 0$
	$\therefore (-6)^2 - 4(-2)(6-k) = 0$
	$\therefore 36 + 48 - 8k = 0$
	$k = 10{,}5$
	$\therefore -2x^2 - 6x + 6 - 10{,}5 = 0$ .....Subst k in ①
	$\therefore 2x^2 + 6x + 4{,}5 = 0$ or $4x^2 + 12x + 9 = 0$
	$\therefore (2x+3)^2 = 0$
	$x = -\frac{3}{2}$
	$\therefore y = 2\left(-\frac{3}{2}\right) + 10{,}5 = 7\frac{1}{2}$
	$\Rightarrow D\left(-\frac{3}{2}; 7\frac{1}{2}\right)$
		✓ $m_g = 2$
		✓ $-2x^2 - 6x + 6 - k = 0$
		✓ $k = 10{,}5$
		✓ $x = -\frac{3}{2}$
		✓ $y = 7\frac{1}{2}$
			(5)

5.6	$a = -1$; the axis of symmetry **OR** $f(a+t) = f(a-t)$ $\therefore -2(a+t)^2 - 4(a+t) + 6 = -2(a-t)^2 - 4(a-t) + 6$ $\therefore -2a^2 - 4at - 2t^2 - 4a - 4t + 6$ $= -2a^2 + 4at - 2t^2 - 4a + 4t + 6$ $\therefore 8at + 8t = 0$ $\therefore 8t(a+1) = 0$ $\therefore t = o$ or $a = -1$	✓✓ $a = -1$ ✓ Subst. ✓ $a = -1$	(2)
			[18]

QUESTION 6

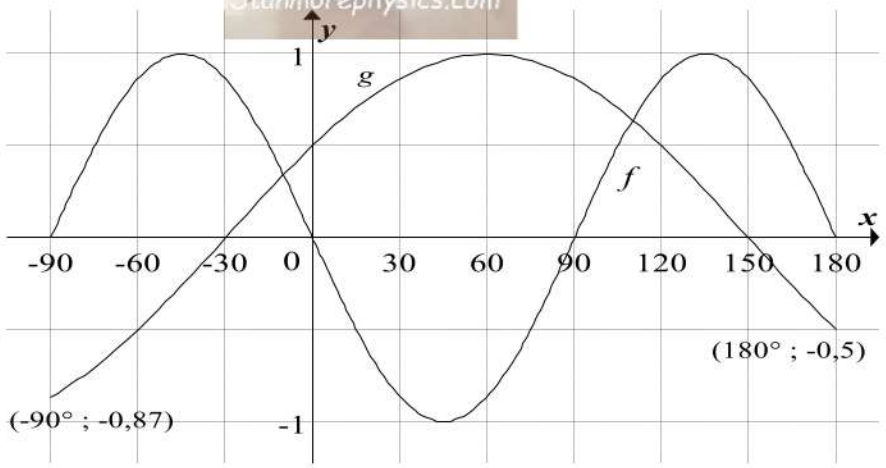
6.1	$f(x) = a^x$ $\frac{27}{8} = a^3$ $\left(\frac{3}{2}\right)^3 = a^3$ $\therefore a = \frac{3}{2}$	✓ Substitution ✓ Answer	(2)
6.2	$y = \left(\frac{3}{2}\right)^x$ $x = \left(\frac{3}{2}\right)^y$ $y = log_{\frac{3}{2}}x$ **ANSWER ONLY: FULL MARKS**	✓ Swop x and y ✓ Answer	(2)
6.3	$x \in \mathbb{R}$	✓ Answer	(1)
			[5]

QUESTION 7

	$(1)^2 = p^2 + (opp)^2$ $(1)^2 - (p)^2 = x^2$ $\therefore x = \sqrt{1-p^2}$  		
7.1.1	$\sin 214°$ $= \sin(180° + 34°)$ $= -\sin 34°$ $= -p$	✓$-\sin 34°$ ✓Answer	(2)
7.1.2	$\cos 34°.\cos(-22°) + \sin 34°.\sin 338°$ $= \cos 34° . \cos 22° + \sin 34°(-\sin 22°)$ $= \cos 34° . \cos 22° - \sin 34° . \sin 22°$ $= \cos(34° + 22°)$ $= \cos 56°$ $\therefore = p$	✓ $\cos 22°$ ✓ $-\sin 22°$ ✓ Compound angles ✓ p	(4)
7.1.3	$\cos 68°$ $= \cos 2(34°)$ $= 2\cos^2 34° - 1$ $= 2\left(\sqrt{1-p^2}\right)^2 - 1$ $= 2(1-p^2) - 1$ $= 1 - 2p^2$	✓ Double angle ✓ Answer	(2)

7.2	$\dfrac{\cos(90°-2\theta).\sin\theta}{\sin^2(180°+\theta).\cos(720°+\theta)}$ $=\dfrac{\sin 2\theta\,.\,(\sin\theta)}{(-\sin\theta)^2.(\cos\theta)}$ $=\dfrac{2\sin\theta\cos\theta\,.\,\sin\theta}{\sin^2\theta.\cos\theta\,.}$ $=2$	✓ $\sin 2\theta$ ✓ $(-\sin\theta)^2$ ✓ $\cos\theta$ ✓$\sin^2\theta$ ✓$2\sin\theta\cos\theta$ ✓ 2	(6)
			[14]

QUESTION 8

8.1		✓ Shape ✓ Endpoints ✓ Intercepts with axes	(3)
8.2	$-\sin 2x = \cos(x-60°)$ $-\sin 2x = \sin(90°-(x-60°))$ $-\sin 2x = \sin(150°-x)$ $\sin(180°+2x) = \sin(150°-x)$ $180°+2x = 150°-x+n(360°), n\epsilon Z$ $3x = -30+n(360), n\in\mathbb{Z}$ $x = -10°+n(120°)$	✓ Equate the equations ✓ Co-functions ✓ $\sin(180°+2x)$ ✓ $n(360), n\in\mathbb{Z}$ ✓ $-10°+n(120°)$	(5)
8.3	$-10°\le x\le 110°$ **OR** $x\in[-10°;110°]$	✓ Inequality/ Notation ✓ Interval	(2)
8.4	g must shift 30° right	✓ 30° ✓ right	(2)
			[12]

TOTAL: 100 MARKS