Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

MATHEMATICS

TERM 1

CONTROLLED TEST

GRADE 12

MARKS: 100

DURATION: 2 hrs

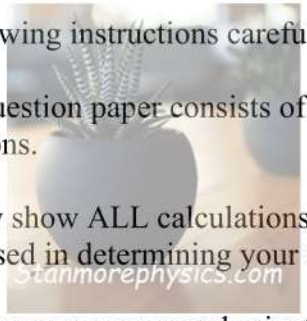
This question paper consists of 6 pages and a information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1 This question paper consists of SIX questions. Answer ALL the questions.

2 Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.



3 You may us an approved scientific calculator, unless stated otherwise.

4 Answer only will not necessarily be awarded full marks.

5 If necessary , round off answers to TWO decimal places, unless stated otherwise.

6 Diagrams are NOT necessarily drawn to scale.

7 Number the answers correctly according to the numbering system used in this question paper.

8 Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 2x - 24 = 0$ (3)

1.1.2 $2x^2 - 3x - 3 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 + 5x \leq -4$ (4)

1.1.4 $2.3^x = 81 - 3^x$ (4)

1.2 Solve simultaneously for x and y:

$x - 4y = 5$ and $x^2 - 5xy + y^2 = 7$ (6)

[20]

QUESTION 2

2.1 Given the quadratic sequence: 5 ; 18 ; 37 ; 62 ; 93 ;…. ; 557

2.1.1 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$ (4)

2.1.2 Determine the number of terms in the sequence. (4)

2.2 Given that:

$$\sum_{k=1}^{6}(x - 3k) = \sum_{k=1}^{9}(x - 3k), prove\ that \sum_{k=1}^{15}(x - 3k) = 0$$ (5)

[13]

QUESTION 3

The following geometric series is given: 10 + 5 + 2,5 + 1,25 + …

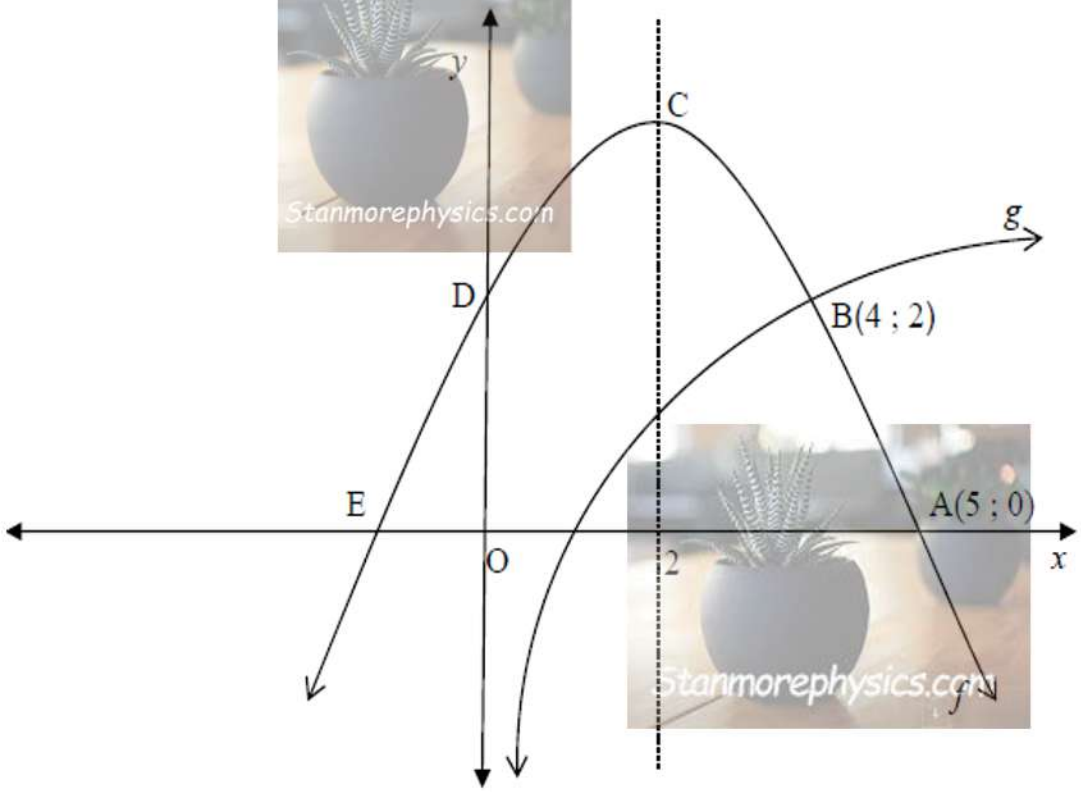
3.1 Explain why the infinite series converges? (2)

3.2 Determine $S_\infty - S_n$ in the form ab^n , where S_n is the sum of the first n terms of the series. (4)

[6]

QUESTION 4

In the sketch, the graphs of the functions given by $f(x) = ax^2 + bx + c$ and $g(x) = \log_m x$ are represented.
A(5 ; 0) is an x-intercept of f and $x = 2$ is the axis of symmetry of f.
B(4 ; 2) is the point of intersection of f and g.



4.1 Determine the value of m. (2)

4.2 Write down the domain of g. (1)

4.3 Determine the equation of g^{-1}, in the form $y = \cdots$ (2)

4.4 Write down the equation of h if h is obtained by shifting $g^{-1}(x)$ 2 units to the left. (1)

4.5 Determine the equation of the parabola f and hence show that: (4)
$a = -\frac{2}{5}$; $b = \frac{8}{5}$ and $c = 2$

4.6 Use the sketch to determine the values of x for which $f(x).g(x) > 0$ (2)
[12]

QUESTION 5

5.1 The following function is given: $f(x) = 2x^2$

5.1.1 Determine the equation of $f^{-1}(x)$, in the form $y = \cdots$ (2)

5.1.2 Is f^{-1} a function? Give a reason for your answer. (2)

5.1.3 How can the domain be restricted in order for f^{-1} to be a function? (2)

5.2 Consider: $f(x) = \frac{3}{x-2} + 1$

5.2.1 Write down the equations of the asymptotes of *f*. (2)

5.2.2 Determine the following:

(a) y −intercept of *f*. (1)

(b) x −intercept of *f*. (2)

5.2.3 Sketch the graph of *f*, showing clearly all intercepts and asymptotes. (3)

5.2.4 Determine the equation of the axis of symmetry of *f* with a positive *y*-intercept. (2)

5.2.5 If R(1 ; -3) is a point on *f*, determine the image of R if when it is reflected about the axis of symmetry in 5.2.4. (3)

[19]

QUESTION 6

6.1 If $\sin 42° = k$, determine the following in terms of *k*.

6.1.1 $\tan 42°$ (2)

6.1.2 $\sin 84°$ (3)

6.1.3 $\sin 3°$ (4)

6.2 Simplify the following expression to ONE trigonometric term: (4)

$$\frac{\sin x}{\cos x.\tan x} + \sin(180° + x)\cos(90° - x)$$

6.3 Consider the identity: $\frac{\cos 2\beta}{(\cos\beta+\sin\beta)^3} = \frac{\cos\beta-\sin\beta}{1+\sin 2\beta}$

6.3.1 Prove the identity: $\frac{\cos 2\beta}{(\cos\beta+\sin\beta)^3} = \frac{\cos\beta-\sin\beta}{1+\sin 2\beta}$ (4)

6.4 If $\cos\theta = 2p$ and $\cos 2\theta = 7p$, determine the possible value(s) of *p*. (5)

6.5 Complete the following:

$\cos(A + B) =$ (1)

6.6 Consider: $f(x) = \sin(x + 25°)\cos 15° - \cos(x + 25°)\sin 15°$ (6)

Determine the general solution of $f(x) = \tan 165°$

[30]

TOTAL: [100]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$F = \frac{x[(1+i)^n-1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a+(n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n-1)}{r-1} \; ; r \neq 1 \text{ or } S_n = \frac{a(1-r^n)}{1-r} \; ; r \neq 1$$

$$S_\infty = \frac{a}{1-r} : -1 < r < 1$$

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \qquad M\left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x-x_1) \qquad m = \frac{y_2-y_1}{x_2-x_1} \qquad m = \tan\theta$$

$$are \Delta ABC = \frac{1}{2}ab\sin C$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = cos^2\alpha - sin^2\alpha = 1 - 2sin^2\alpha = 2cos^2\alpha - 1 \qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$