KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

NOVEMBER 2024

MARKS: **150**

TIME: **3 hours**

This question paper consists of 13 pages, 1 information sheet and a SPECIAL ANSWER BOOK of 20 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

1.1 The table below shows the number of tourists who stayed at a hotel in Durban from 20 to 31 December 2023.

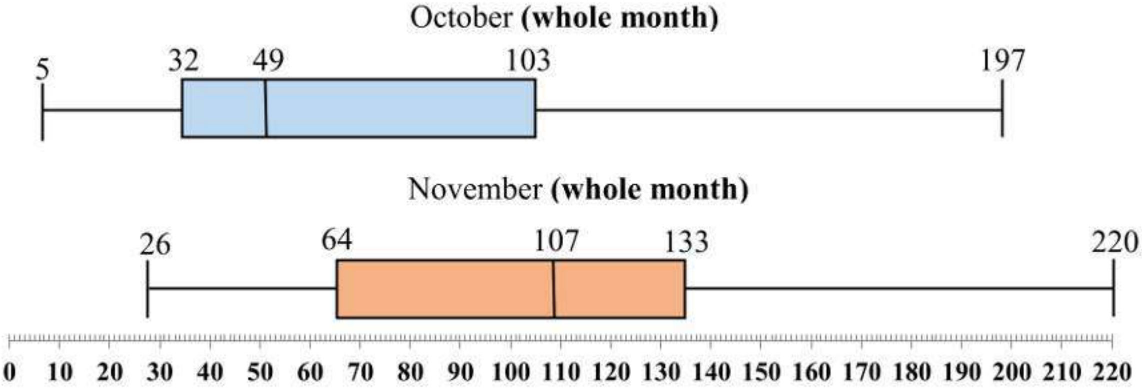
Date in December	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
Number of tourists	**286**	**68**	**150**	**147**	**176**	**255**	**132**	**174**	**172**	**197**	**172**	**39**

1.1.1 Calculate the mean of the data. (2)

1.1.2 Write down the standard deviation of the data. (1)

1.1.3 Calculate the percentage of days on which the number of tourists who stayed at this hotel was within one standard deviation of the mean. (3)

1.2 The number of tourists who stayed at this hotel during the entire month of October and the entire month of November are summarised in the box-and-whisker diagrams below.



1.2.1 Calculate the range of the number of tourists who stayed at the hotel during November. (1)

1.2.2 Comment on the skewness of the data for October. (1)

1.2.3 The maximum for October was incorrectly recorded. The correct value is higher than the recorded value. If this correction is made, what effect will it have on the:

1.2.3 (a) mean? (1)

1.2.3 (b) median? (1)

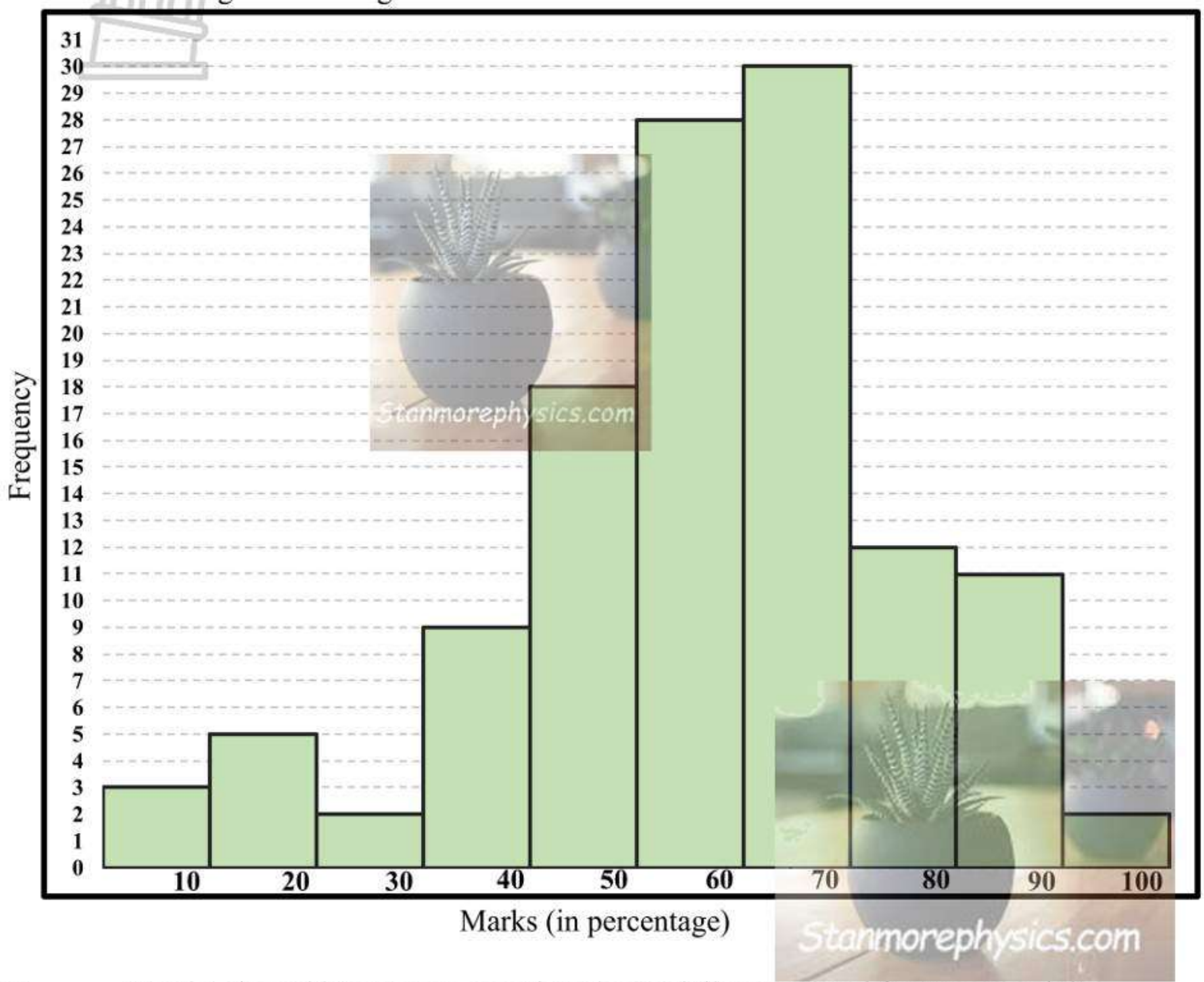
1.2.4 There was only one day during November on which 64 tourists stayed at the hotel. On how many days during November did less than 64 tourists stay at the hotel? (2)

[12]

QUESTION 2

The marks (in percentage) obtained by Grade 11 learners in their Mathematics Examination is shown in the histogram.

Histogram showing marks obtained in a Grade 11 Mathematics Examination.



2.1 Use the above histogram to complete the partially completed frequency and the cumulative frequency columns of the table provided in the ANSWER BOOK. (3)

2.2 Draw the cumulative frequency graph (ogive) of this data on the grid provided in the ANSWER BOOK. (3)

2.3 Use the ogive to estimate the:

2.3.1 number of learners who scored more than 75% in the examination. (2)

2.3.2 median mark of the results. (1)

[9]

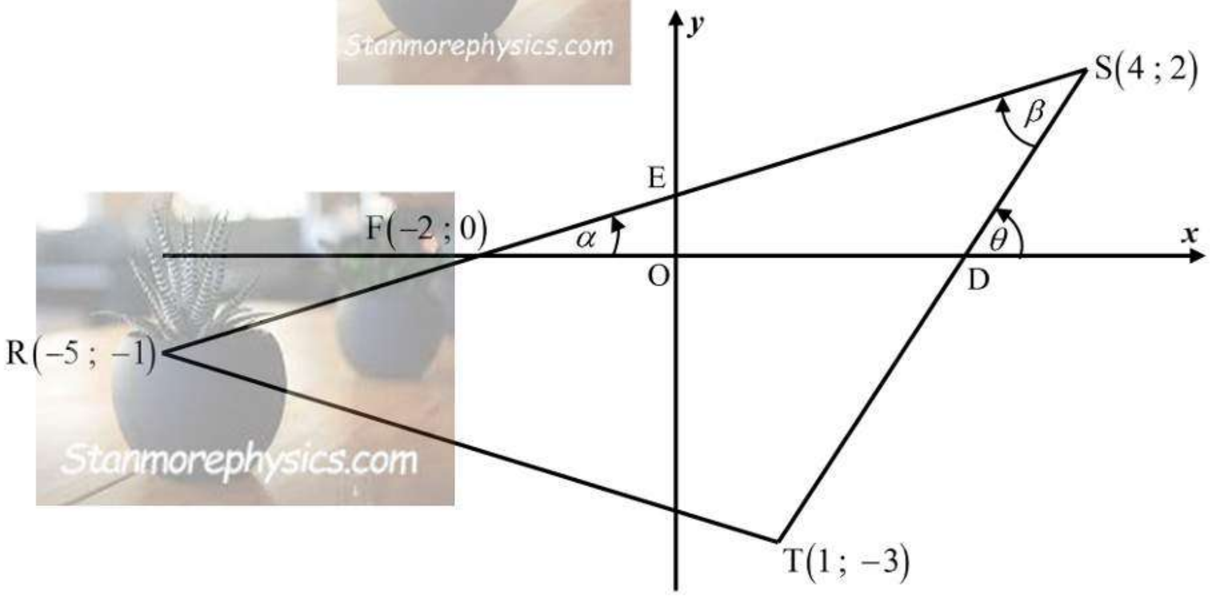
QUESTION 3

3.1 P(−1 ; 1), Q(2 ; 5) and R(5 ; y) are three points in the Cartesian plane. Calculate the value(s) of y such that:

3.1.1 P, Q and R are collinear. (4)

3.1.2 $P\hat{Q}R = 90°$. (3)

3.2 In the sketch below, R(−5 ; −1), S(4 ; 2) and T(1 ; −3) are the vertices of ΔRST. RS and ST intersect the x axis at F(−2 ; 0) and D respectively. E is the y intercept of RS. The equation of RS is $3y - x - 2 = 0$ and $R\hat{S}T = \beta$. α and θ are the angles of inclination of RS and ST respectively.



3.2.1 Calculate the length of RT. (2)

3.2.2 Calculate the gradient of ST. (2)

3.2.3 Calculate the coordinates of M, the midpoint of RS. (2)

3.2.4 Determine the equation of a line parallel to ST passing through M. (3)

3.2.5 Determine the co-ordinates of G if RGST, in that order, is a parallelogram. (2)

3.2.6 Calculate the size of β. (5)

3.2.7 Calculate the area of SEOD. (6)

[29]

QUESTION 4

4.1 If $2\cos\beta+1=0$ and $\sin\beta<0$, determine WITHOUT the use of a calculator and with the aid of a diagram, the value of:

4.1.1 $\sin\beta$ (4)

4.1.2 $2\tan^2\beta-\cos^2\beta$ (2)

4.2 Simplify $\dfrac{\cos(\theta-90°).\cos(-\theta).\tan(360°-\theta)}{\sin(180°+\theta)}$ to a single trigonometric term. (6)

4.3 WITHOUT using a calculator, calculate the value of:

$$\frac{2\sin 510°-\cos 340°.\cos 20°}{\cos^2 110°}$$ (6)

4.4 Given the following expression: $\dfrac{\sin x}{1+\cos x}+\dfrac{\cos x}{\sin x}$

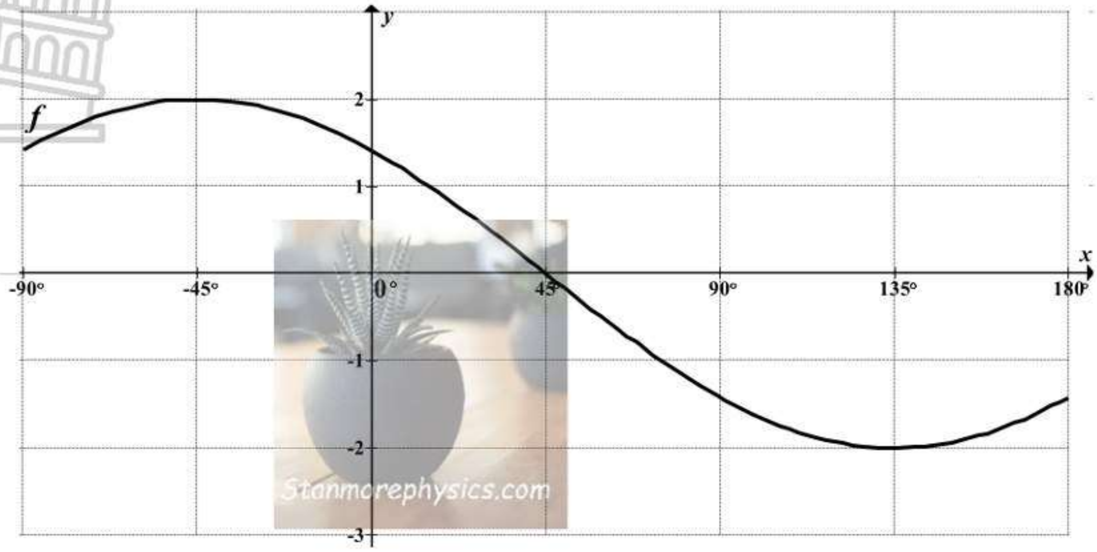
4.4.1 Prove that: $\dfrac{\sin x}{1+\cos x}+\dfrac{\cos x}{\sin x}=\dfrac{1}{\sin x}$ (4)

4.4.2 Determine the general solution for which the identity in QUESTION 4.4.1 is undefined. (5)

[27]

QUESTION 5

The graph of $f(x) = 2\cos(x + p)$ is sketched below for $x \in [-90°\ ;\ 180°]$.



5.1 Write down the amplitude of *f*. (1)

5.2 Write down the value of *p*. (1)

5.3 Given that $g(x) = \sin 2x + 1$, write down the period of *g*. (1)

5.4 On the system of axes provided in the ANSWER BOOK, sketch the graph of $g(x) = \sin 2x + 1$ for $x \in [-90°\ ;\ 180°]$. (3)

5.5 Use your graphs to write down the value(s) of *x* for which:

5.5.1 $f(x).g(x) < 0$ (3)

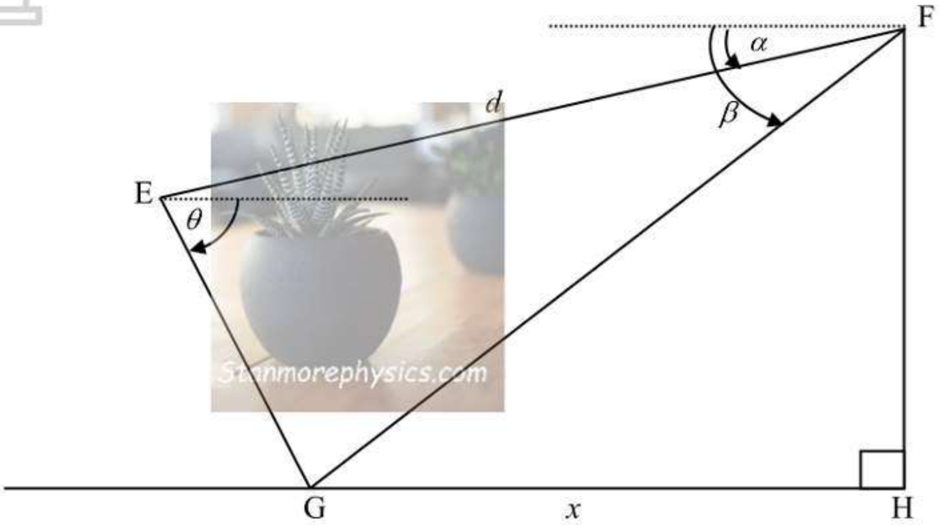
5.5.2 $f(x) = -g(x) + 2$ (3)

5.6 Graph h is obtained when f is translated $45°$ to the left. Determine the equation of h. Write your answer in its simplest form. (2)

[14]

QUESTION 6

A cableway connects the top of two mountains across a valley. The distance from E to F is d. From F, the angle of depression to E is α and the angle of depression to G at the bottom of the valley is β. The angle of depression from E to G is θ. H is a point that is directly below F and in the same horizontal plane as G. The distance from G to H is x.



6.1 Write down the size of $F\hat{G}E$ in terms of θ and β. (2)

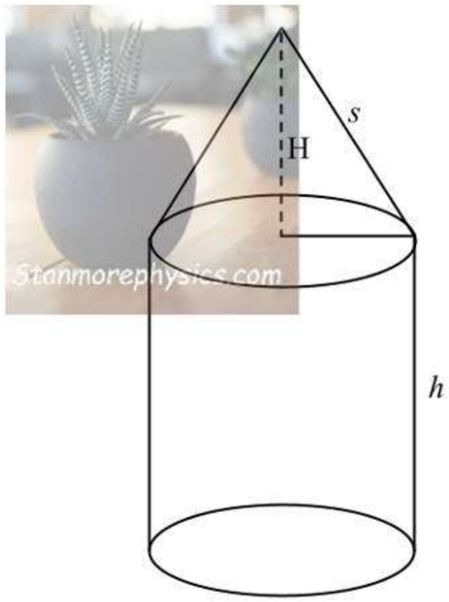
6.2 Show that $FH = \sqrt{\left[\dfrac{d\sin(\alpha+\theta)}{\sin(\theta+\beta)}\right]^2 - x^2}$ (5)

[7]

QUESTION 7

The diagram below shows a sketch of an oil container which is made up of a cylinder and a cone. The cylinder and the cone have the same radius. The height, h, of the cylinder is 15 cm and the slant height, s, of the cone is 10 cm. The volume of the cylinder is 4 000 cm^3.

$V = \frac{1}{3} \text{area of base} \times H$ $V = \pi r^2 h$ $SA = \pi r^2 + 2\pi rh$ $SA = \pi rs$

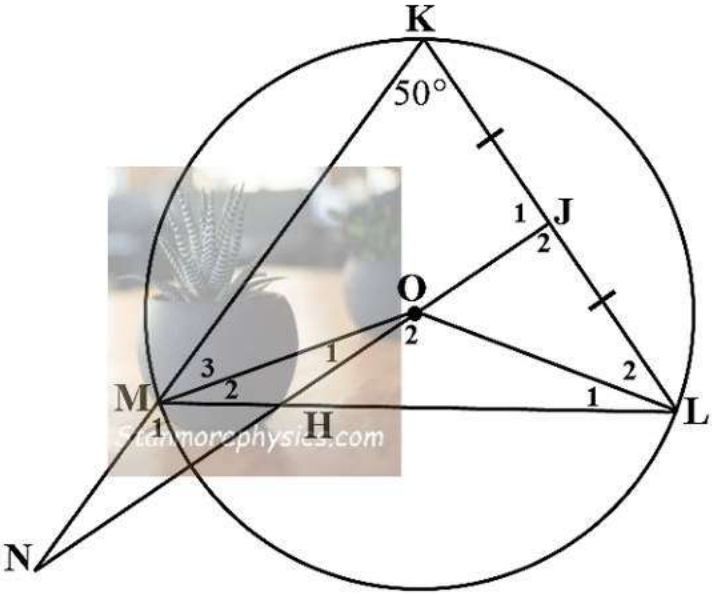


7.1 Calculate the total volume of the oil container. (7)

7.2 Calculate the total surface area of the oil container. (3)
[10]

QUESTION 8

In the diagram below, O is the centre of the circle passing through K, L and M. J is the midpoint of chord KL. Chord KM produced and JO produced meet in N. JN intersects chord ML in H. MO and LO are drawn. $\hat{K} = 50°$.



Calculate, giving reasons, the size of:

8.1 $\quad M\hat{O}L$ (2)

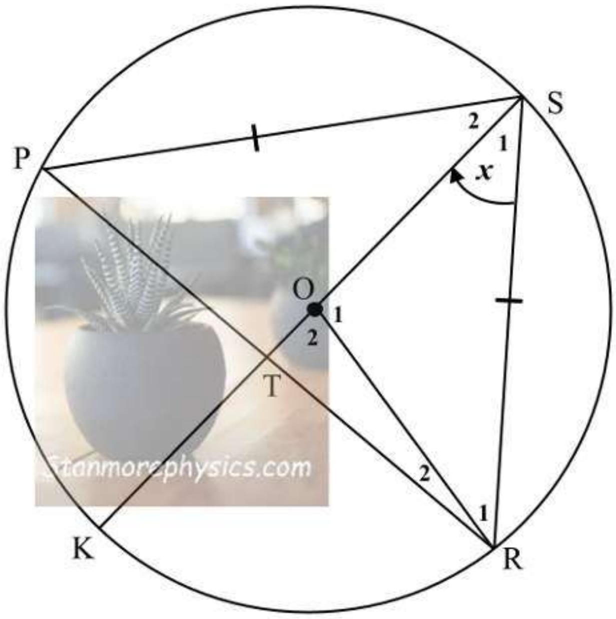
8.2 $\quad \hat{L}_1$ (2)

8.3 $\quad \hat{N}$ (3)

[7]

QUESTION 9

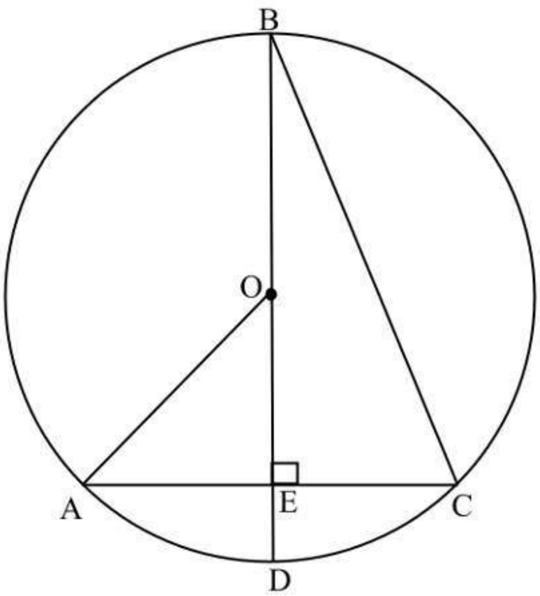
9.1 In the diagram below, O is the centre of the circle. P, S, R and K are points on the circle. SOK is a straight line intersecting PR at T. PS = SR and OR is joined. $\hat{S}_1 = x$.



9.1.1 Calculate $\hat{P}$ in terms of x. (4)

9.1.2 Prove that $\hat{S}_2 = \hat{S}_1$. (3)

9.2 In the diagram below, O is the centre of the circle. A, B, C and D lie on the circle. BOD is perpendicular to AC at E. AO and BC are drawn.

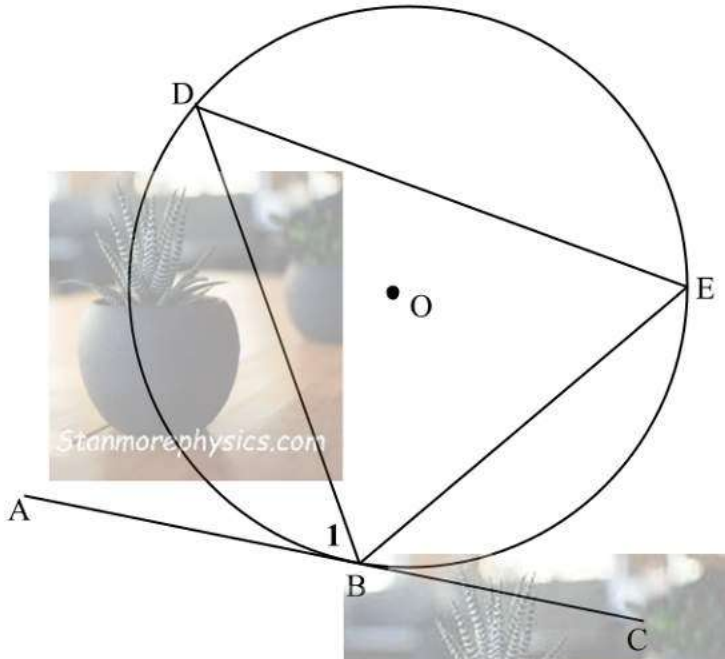


Prove that $(2AO - ED)^2 = BC^2 - AE^2$ (5)

[12]

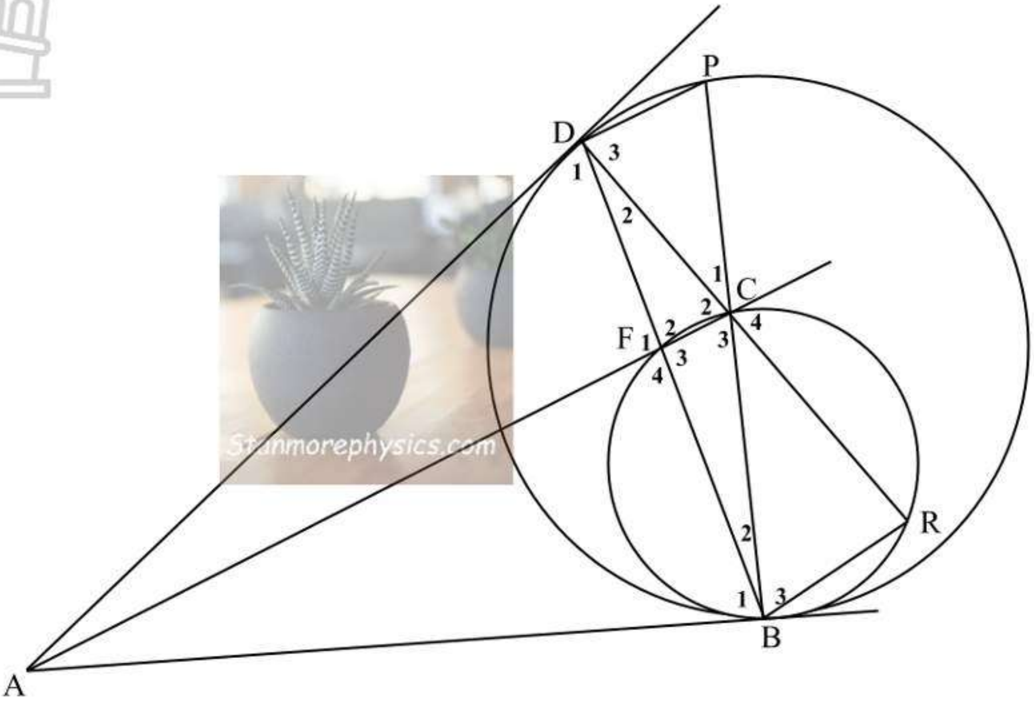
QUESTION 10

10.1 In the diagram below, O is the centre of the circle passing through B, D and E. AC is a tangent to the circle at B. BD, DE and BE are joined.



Use the diagram above to prove the theorem which states that the angle between a tangent to the circle and the chord drawn from the point of contact is equal to the angle that the chord subtends in the alternate segment, i.e. $\hat{B}_1 = D\hat{E}B$. (5)

10.2 In the diagram, two circles touch internally at B. A is a point outside the circle. AB and AD are tangents to the larger circle at B and D respectively. AB is also a tangent to the smaller circle at B. BC is a diameter of the smaller circle. BC produced meets the larger circle at P. The line from A to C intersects DB and the smaller circle at F. DC produced meets the smaller circle at R. DP and BR are drawn.



Prove that:

10.2.1 AC ⊥ DB (2)

10.2.2 AC // DP. (4)

10.2.3 ABCD is a cyclic quadrilateral. (4)

10.2.4 CP = CD. (4)

10.2.5 BR // AD. (4)

[23]

TOTAL: 150 MARKS

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

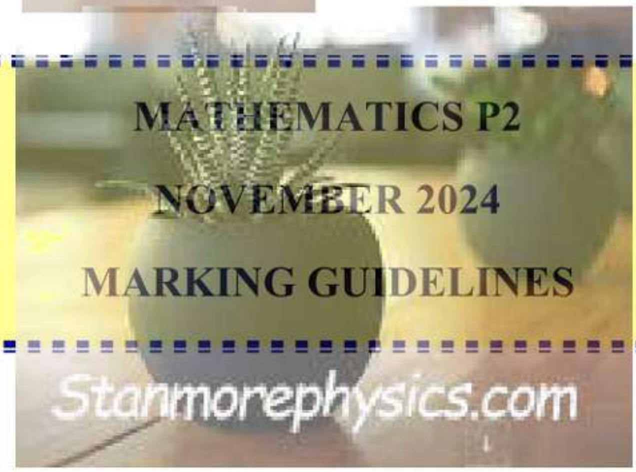
FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11



MATHEMATICS P2

NOVEMBER 2024

MARKING GUIDELINES

MARKS: 150

These marking guidelines consists of 17 pages.

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION 1

1.1.1	$\bar{x} = \frac{1968}{12}$ $= 164$	✓ $\frac{1968}{12}$ ✓ answer (2)
1.1.2	$\sigma = 65{,}18$	✓ $\sigma = 65{,}18$ (1)
1.1.3	$\bar{x} = 164$ Interval: $(164 - 65{,}18\ ;\ 164 + 65{,}18)$ $(98{,}82\ ;\ 229{,}18)$ $\therefore$ 8 days lie within one standard deviation $\therefore$ Percentage of days $= \frac{8}{12} \times 100$ $= 66{,}67\%$	✓ (98,82 ; 229,18) ✓ 8 days ✓ answer (3)
1.2.1	Range $= 220 - 26$ $= 194$	✓ answer (1)
1.2.2	Data is skewed to the right **OR** Data is positively skewed	✓ skewed to the right **OR** ✓ positively skewed (1)
1.2.3	(a) The mean will increase	✓ increase (1)
1.2.3	(b) The median will not change.	✓ not change (1)
1.2.4	$Q_1 = 64$ But there is 25% of the days on the left of Q_1 $\therefore \frac{25}{100} \times 30 = 7{,}5$ $\therefore$ for 7 days there were less than 64 visits a day.	✓25% ✓ 7 days (2)
		[12]

QUESTION 2

2.1

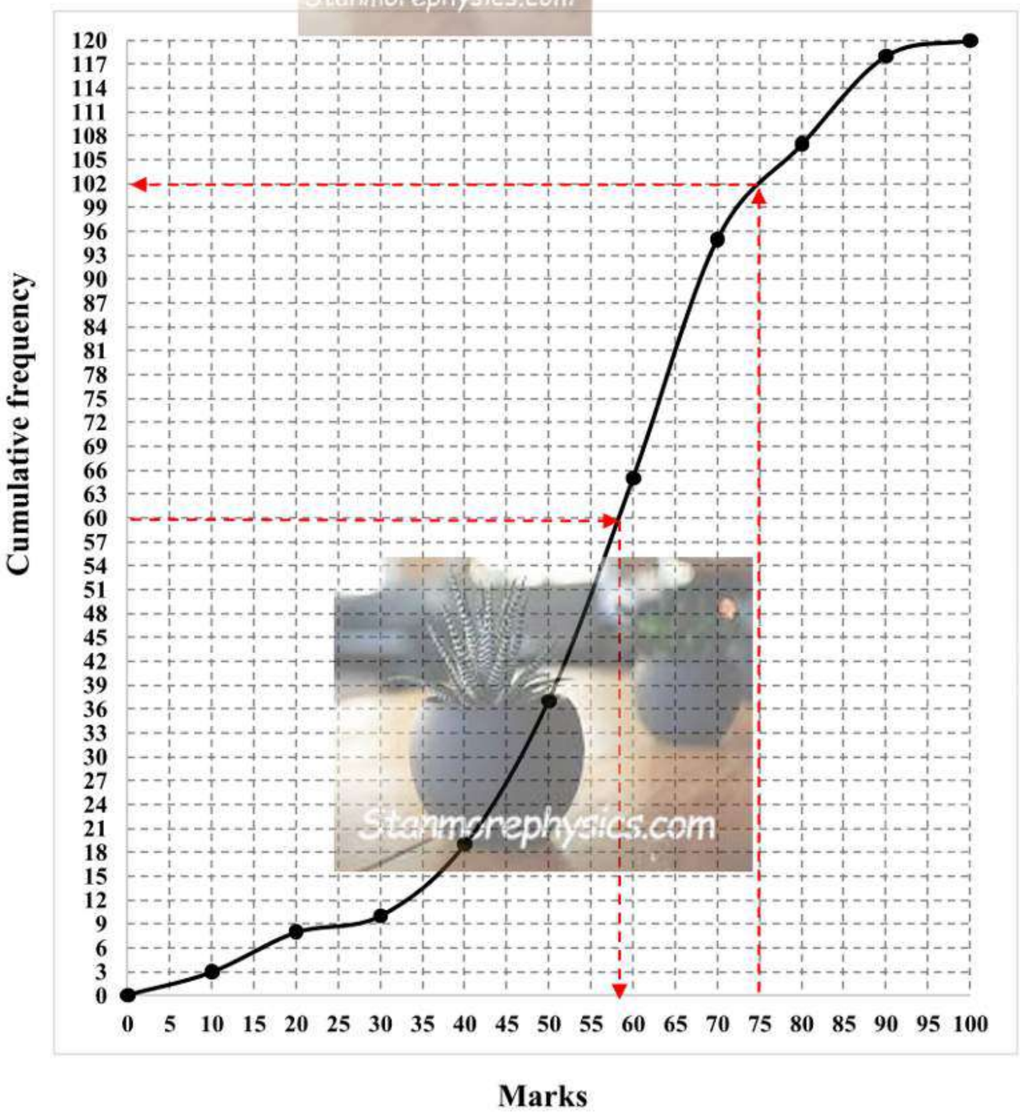
Marks	f	CF
$0 \leq x < 10$	3	3
$10 \leq x < 20$	5	8
$20 \leq x < 30$	2	10
$30 \leq x < 40$	9	19
$40 \leq x < 50$	18	37
$50 \leq x < 60$	28	65
$60 \leq x < 70$	30	95
$70 \leq x < 80$	12	107
$80 \leq x < 90$	11	118
$90 \leq x < 100$	2	120

✓reading from the histogram (f) (all values)

✓✓cumulative frequency
1 mark if 3 – 5 are correct
2 marks if all values are correct

(3)

2.2



✓plotting cumulative frequency at upper limit

✓ shape

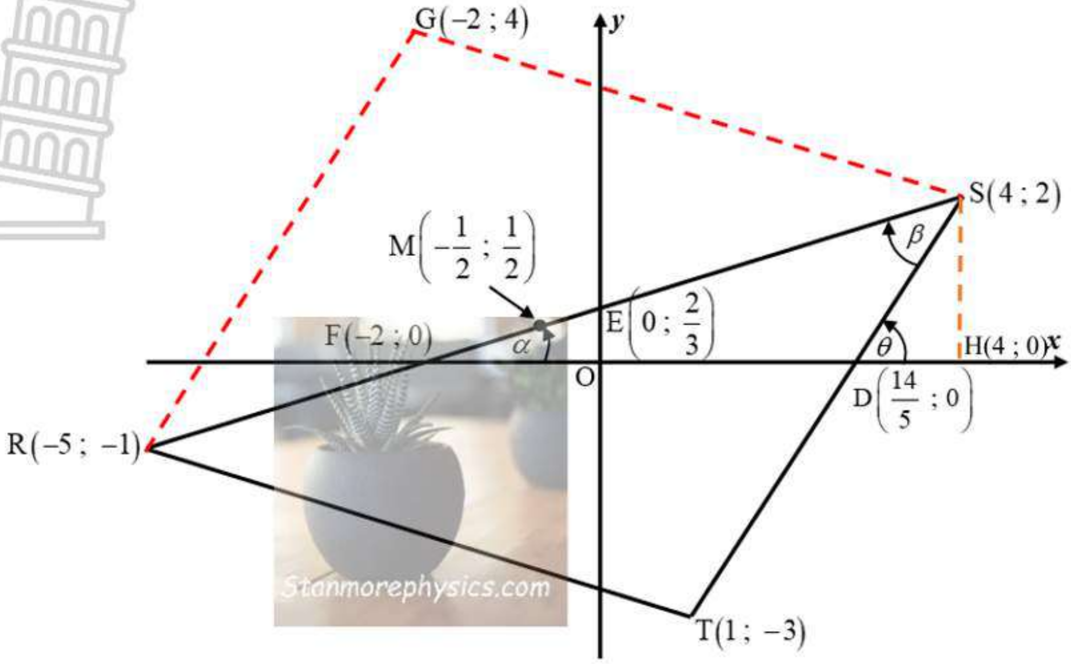
✓ grounding at (0 ; 0)

(3)

2.3.1	$120-102=18$ $\therefore 18$ learners	✓ 102 (accept reading from 101 to 103) ✓18 learners (accept 17 to 19 learners) (2)
2.3.2	Median position: $\frac{1}{2}\times 120=60$ $\therefore Q_2=58$	✓ 58 (accept 57 to 59) (1)
		[9]

QUESTION 3

3.1.1	$P(-1\,;1)\quad Q(2\,;5)\quad R(5\,;y)$ $m_{PQ}=\frac{5-1}{2-(-1)}$ $=\frac{4}{3}$ $m_{PQ}=m_{QR}$ $\frac{4}{3}=\frac{y-5}{5-2}$ $3y-15=12$ $y=9$	✓ m_{PQ} ✓ condition ✓ substitution ✓ answer (4)
3.1.2	$m_{PQ}\times m_{QR}=-1$ $\frac{4}{3}\times\frac{y-5}{3}=-1$ $4y-20=-9$ $4y=11$ $y=\frac{11}{4}$ **OR** $m_{PQ}=\frac{-1}{m_{QR}}$ $\frac{y-5}{3}=-\frac{3}{4}$ $4y-20=-9$ $4y=11$ $y=\frac{11}{4}$	✓ condition ✓ substitution ✓ answer (3)

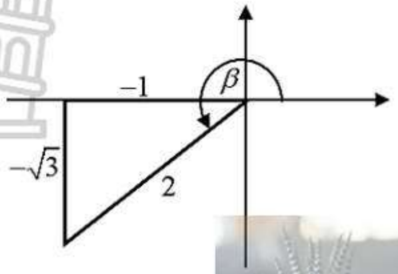
3.2		
3.2.1	$RT=\sqrt{(-5-1)^2+(-1+3)^2}$ $=\sqrt{40}$ $=2\sqrt{10}$ or 6,32 units	✓ substitution of R and T into correct formula ✓ answer (2)
3.2.2	$m_{ST}=\frac{2-(-3)}{4-1}$ $=\frac{5}{3}$	✓ substitution of S and T into correct formula ✓ answer (2)
3.2.3	$M\left(\frac{4-5}{2}\ ;\ \frac{2-1}{2}\right)$ $M\left(-\frac{1}{2}\ ;\ \frac{1}{2}\right)$	✓ x-value ✓ y-value (2)
3.2.4	$m=\frac{5}{3}$ point: $M\left(-\frac{1}{2}\ ;\ \frac{1}{2}\right)$ $y-y_1=m(x-x_1)$ $y-\frac{1}{2}=\frac{5}{3}\left(x+\frac{1}{2}\right)$ $y-\frac{1}{2}=\frac{5}{3}x+\frac{5}{6}$ $y=\frac{5}{3}x+\frac{4}{3}$ **OR** $y=mx+c$ $\frac{1}{2}=\frac{5}{3}\left(-\frac{1}{2}\right)+c$ $c=\frac{4}{3}$ $y=\frac{5}{3}x+\frac{4}{3}$	✓ $m=\frac{5}{3}$ ✓ subst $M\left(-\frac{1}{2}\ ;\ \frac{1}{2}\right)$ into equation of line ✓ answer (3)

3.2.5	By using transformation: $T \rightarrow S: (1\ ;\ -3) \rightarrow (4\ ;\ 2)$ $(x\ ;\ y) \rightarrow (x+3\ ;\ y+5)$ $\therefore R \rightarrow G:\ G(-5+3\ ;\ -1+5)$ $\therefore G(-2\ ;\ 4)$	✓x value ✓y value (2)
	OR	
	$T \rightarrow R:\ (1\ ;\ -3) \rightarrow (-5\ ;\ -1)$ $(x\ ;\ y) \rightarrow (x-6\ ;\ y+2)$ $\therefore S \rightarrow G:\ G(4-6\ ;\ 2+2)$ $\therefore G(-2\ ;\ 4)$	✓x value ✓y value (2)
	OR	
	Midpoint RS: $M\left(-\frac{1}{2}\ ;\ \frac{1}{2}\right)$ $x_M = \frac{x_G + x_T}{2}$ and $y_M = \frac{y_G + y_T}{2}$ $-\frac{1}{2} = \frac{x_G + 1}{2}$ and $\frac{1}{2} = \frac{y_G - 3}{2}$ $2x_G + 2 = -2$ and $2y_G - 6 = 2$ $x_G = -2$ and $y_G = 4$ $\therefore G(-2\ ;\ 4)$ **Answer only: Full marks**	✓x value ✓y value (2)
3.2.6	$m_{ST} = \frac{5}{3}$ $\tan\theta = \frac{5}{3}$ $\theta = 59,04°$ $m_{RS} = \frac{1}{3}$ $\tan\alpha = \frac{1}{3}$ $\alpha = 18,43°$ $\beta = \theta - \alpha$ $\beta = 59,04° - 18,43°$ $\beta = 40,61°$	✓ $\tan\theta = m_{ST}$ ✓ size of θ ✓ $m_{RS} = \frac{1}{3}$ ✓ size of α ✓ answer (5)

3.2.7	$RS_{equation}: y-2=\frac{1}{3}(x-4)$ $y=\frac{1}{3}x+\frac{2}{3}$ $\therefore E\left(0\,;\,\frac{2}{3}\right)$ $ST_{equation}: y-2=\frac{5}{3}(x-4)$ $y=\frac{5}{3}x-\frac{14}{3}$ $\therefore D\left(\frac{14}{5}\,;\,0\right)$ Area SEOD = Area ΔFSD − Area ΔFEO $=\frac{1}{2}(FD\times SH)-\frac{1}{2}(FO\times EO)$ $=\frac{1}{2}\left(\frac{24}{5}\times 2\right)-\frac{1}{2}\left(2\times\frac{2}{3}\right)$ $=\frac{24}{5}-\frac{2}{3}$ $=\frac{62}{15}$ or $4\frac{2}{15}$ or 4,13 square units	✓ coordinates of E ✓ equation of ST ✓ coordinates of D ✓ substitution into area ΔFSD ✓ substitution into area ΔFEO ✓ answer (6)
	OR $RS_{equation}: y=\frac{1}{3}x+\frac{2}{3}$ $\therefore E\left(0\,;\,\frac{2}{3}\right)$ $ST_{equation}: y=\frac{5}{3}x-\frac{14}{3}$ $\therefore D\left(\frac{14}{5}\,;\,0\right)$ Area SEOD = Area SEOH − Area ΔSDH $=\frac{1}{2}(EO+SH)\times OH-\left(\frac{1}{2}\times DH\times SH\right)$ $=\frac{1}{2}\left(\frac{2}{3}+2\right)\times 4-\left(\frac{1}{2}\times\frac{6}{5}\times 2\right)$ $=\frac{16}{3}-\frac{6}{5}$ $=\frac{62}{15}$ or $4\frac{2}{15}$ or 4,13 square units	✓ coordinates of E ✓ equation of ST ✓ coordinates of D ✓ substitution into area SEOH ✓ substitution into area ΔSDH ✓ answer (6)

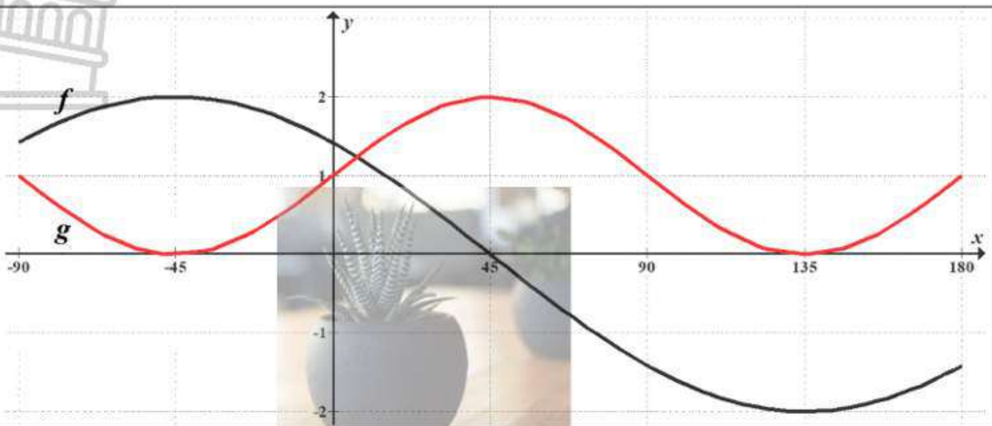
	OR	
	$RS_{equation}: y=\frac{1}{3}x+\frac{2}{3}$	
	$\therefore E\left(0\,;\,\frac{2}{3}\right)$	✓ coordinates of E
	$ST_{equation}: y-2=\frac{5}{3}(x-4)$	
	$y=\frac{5}{3}x-\frac{14}{3}$	✓ equation of ST
	$\therefore D\left(\frac{14}{5}\,;\,0\right)$	✓ coordinates of D
	Area SEOD = Area ΔSFH $-$ (Area ΔEFO + Area ΔSDH)	
	$=\frac{1}{2}(FH\times SH)-\left(\frac{1}{2}\times FO\times EO+\frac{1}{2}\times DH\times SH\right)$	
	$=\frac{1}{2}(6\times 2)-\left(\frac{1}{2}\times 2\times\frac{2}{3}+\frac{1}{2}\times\frac{6}{5}\times 2\right)$	✓ subst into area of ΔSFH ✓ subst into area of ΔEFO & ΔSDH
	$=6-\frac{23}{15}$	
	$=\frac{62}{15}$ or $4\frac{2}{15}$ or 4,13 square units	✓ answer (6)
	OR	
	$RS_{equation}: y-2=\frac{1}{3}(x-4)$	
	$y=\frac{1}{3}x+\frac{2}{3}$	
	$\therefore E\left(0\,;\,\frac{2}{3}\right)$	✓ coordinates of E
	$ST_{equation}: y-2=\frac{5}{3}(x-4)$	
	$y=\frac{5}{3}x-\frac{14}{3}$	✓ equation of ST
	$\therefore D\left(\frac{14}{5}\,;\,0\right)$	✓ coordinates of D
	Area SEOD = Area SKOD $-$ Area ΔSKE	
	$=\frac{1}{2}(SK+OD)\times KO-\frac{1}{2}(KE\times KS)$	
	$=\frac{1}{2}\left(4+\frac{14}{5}\right)\times 2-\frac{1}{2}\left(2-\frac{2}{3}\right)\times 4$	✓ subst into area SKOD ✓ subst into area ΔSKE
	$=\frac{34}{5}-\frac{8}{3}$	
	$=\frac{62}{15}$ or $4\frac{2}{15}$ or 4,13 square units	✓ answer (6)
		[29]

QUESTION 4

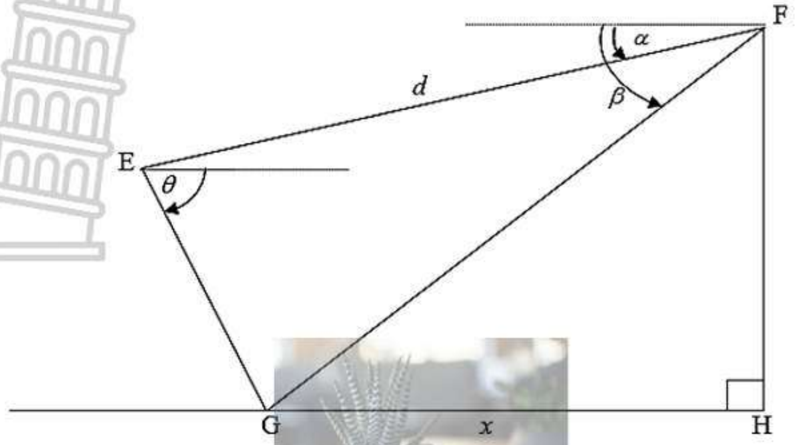
4.1.1	$2\cos\beta + 1 = 0$ $\cos\beta = -\frac{1}{2}$  $y^2 = r^2 - x^2$ $y^2 = (2)^2 - (-1)^2$ $y^2 = 3$ $y = \pm\sqrt{3}$ $y = -\sqrt{3}$ $\sin\beta = \frac{-\sqrt{3}}{2}$	✓ diagram in quadrant 3 ✓ substitution of x and r ✓ value of y ✓ answer (4)
4.1.2	$2\tan^2\beta - \cos^2\beta = 2\left(\frac{-\sqrt{3}}{-1}\right)^2 - \left(\frac{-1}{2}\right)^2$ $= 5\frac{3}{4}$ or $\frac{23}{4}$	✓ substitution ✓ answer (2)
4.2	$\frac{\cos(\theta - 90°).\cos(-\theta).\tan(360° - \theta)}{\sin(180° + \theta)}$ $= \frac{\sin\theta.\cos\theta.-\tan\theta}{-\sin\theta}$ $= \cos\theta.\frac{\sin\theta}{\cos\theta}$ $= \sin\theta$	✓ $\sin\theta$ ✓ $\cos\theta$ ✓ $-\tan\theta$ ✓ $-\sin\theta$ ✓ quotient identity ✓answer (6)
4.3	$\frac{2\sin 510° - \cos 340°.\cos 20°}{\cos^2 110°} = \frac{2\sin 30° - \cos 20°.\cos 20°}{(-\cos 70°)^2}$ $= \frac{2\left(\frac{1}{2}\right) - \cos^2 20°}{\cos^2 70°}$ $= \frac{1 - \cos^2 20°}{\sin^2 20°}$ $= \frac{\sin^2 20°}{\sin^2 20°}$ $= 1$	✓ $\sin 30°$ ✓ $\cos 20°$ ✓ $\frac{1}{2}$ ✓ $\sin^2 20°$ ✓ square identity ✓ answer (6)

4.4.1	LHS $= \frac{\sin x}{1+\cos x} + \frac{\cos x}{\sin x}$ $= \frac{\sin^2 x + \cos x + \cos^2 x}{(1+\cos x)(\sin x)}$ $= \frac{\sin^2 x + \cos^2 x + \cos x}{(1+\cos x)(\sin x)}$ $= \frac{1+\cos x}{(1+\cos x)(\sin x)}$ $= \frac{1}{\sin x}$ $=$ RHS	✓ numerator ✓ LCD $= (1+\cos x)(\sin x)$ ✓ identity ✓ simplification (4)
4.4.2	(i): $1+\cos x = 0$ $\cos x = -1$ $x = 180° + k.360° ; k \in Z$ (ii): $\sin x = 0$ $x = 0° + k.360°$ or $x = 180° + k.360° ; k \in Z$ OR $x = 0° + k.180° ; k \in Z$	✓ $\cos x = -1$ ✓ 180° ✓ $\sin x = 0$ ✓ 0° ✓ $k.360°; k \in$ Z (5)
		[27]

QUESTION 5

5.1	Amplitude = 2	✓ answer (1)
5.2	$p = 45°$	✓ answer (1)
5.3	Period $= 180°$	✓ answer (1)
5.4		Graph of g: ✓ shape ✓ x and y intercepts ✓ turning points (3)
5.5.1	$45° < x < 180°; x \neq 135°$ **OR** $x \in (45°\ ;\ 135°) \cup (135°\ ;\ 180°)$	✓ endpoints ✓ notation ✓ rejection (3) ✓ endpoints ✓ notation ✓ exclude 135° (3)
5.5.2	$f(x) = -g(x) + 2$ $f(x) + g(x) = 2$ $x = -45°$ and $x = 45°$	✓ $f(x) + g(x) = 2$ ✓ $x = -45°$ ✓ $x = 45°$ (3)
5.6	$f(x) = 2\cos(x + 45°)$ $h(x) = 2\cos(x + 45° + 45°)$ $h(x) = 2\cos(90° + x)$ $h(x) = -2\sin x$	✓ adding 45° ✓ answer (2)
		[14]

QUESTION 6

6.1	 In ΔEFG: $\theta+\alpha+\beta-\alpha+\hat{FGE}=180°$.........∠s of Δ $\hat{FGE}=180°-(\theta+\beta)$	✓ S ✓ answer (2)
6.2	In ΔEFG: $\frac{FG}{\sin F\hat{E}G}=\frac{EF}{\sin E\hat{F}G}$ $\frac{AC}{\sin(\alpha+\theta)}=\frac{d}{\sin[180°-(\theta+\beta)]}$ $FG=\frac{d.\sin(\alpha+\theta)}{\sin(\theta+\beta)}$ In ΔFGH: $FH^2=FG^2-GH^2$ (Pythagoras) $=\left[\frac{d.\sin(\alpha+\theta)}{\sin(\theta+\beta)}\right]^2-x^2$ $FH=\sqrt{\left[\frac{d.\sin(\alpha+\theta)}{\sin(\theta+\beta)}\right]^2-x^2}$	✓ use of sine rule ✓ substitution into sine rule ✓ expression of FG ✓ substitution into Pythagoras ✓ simplification (5)
		[7]

QUESTION 7

7.1	$V_{cylinder} = 4000\,m^3$ $V_{cylinder} = \pi r^2 h$ $\pi r^2(15) = 4000$ $r^2 = \frac{4000}{15\pi}$ $r = \sqrt{\frac{4000}{15\pi}}$ or $9,21\ cm$ $H = \sqrt{(10)^2 - \left(\sqrt{\frac{4000}{15\pi}}\right)^2}$ $= \sqrt{100 - \frac{4000}{15\pi}}$ or $3,89\ cm$ $V_{cone} = \frac{1}{3} \times \pi \times \left(\sqrt{\frac{4000}{15\pi}}\right)^2 \times \sqrt{100 - \frac{4000}{15\pi}}$ $= 345,61\ cm^3$ or $345,54\ cm^3$ (if rounded off for r & H) $V_{container} = 4\ 000\ cm^3 + 345,61\ cm^3$ $= 4\ 345,61\ cm^3$ or $4\ 345,54\ cm^3$ (if rounded off for r & H)	✓ substitution of 15 & 4 000 into $V_{cylinder}$ ✓ value of r ✓ using Theorem of Pythagoras to calculate H ✓ value of H ✓ substitution into V_{cone} ✓ V_{cone} ✓ answer (7)
7.2	$SA = \pi r^2 + 2\pi rh + \pi rs$ $= \pi\left(\sqrt{\frac{4000}{15\pi}}\right)^2 + 2\pi\left(\sqrt{\frac{4000}{15\pi}}\right)(15) + \pi\left(\sqrt{\frac{4000}{15\pi}}\right)(10)$ $= 1\ 423,43\ cm^2$ or $1\ 423,85\ cm^2$ (if rounded off for r)	substitution into: ✓$SA_{cylinder}$ ✓ SA_{cone} ✓ answer (3)
		[10]

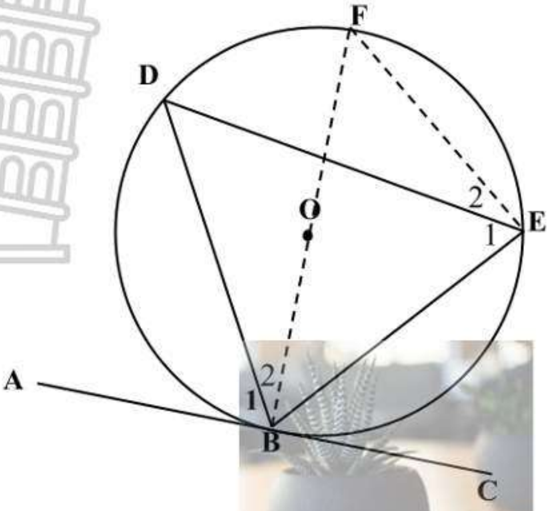
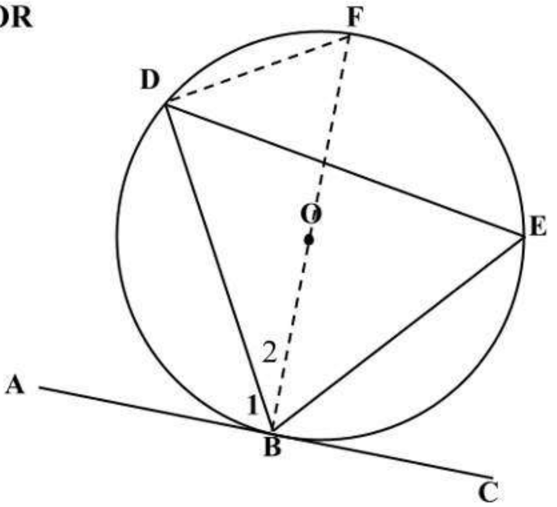
QUESTION 8

8.1	$M\hat{O}L = 100°$	[∠ @ centre = 2×∠ @ circumf]	✓S ✓R (2)
8.2	$MO = LO$ $\hat{M}_2 = \hat{L}_1$ $\hat{L}_1 = \frac{180° - 100°}{2}$ $= 40°$	[radii] [∠s opp = sides] [sum of ∠s of isosceles Δ MOL]	✓ S/R ✓ S/R (2)
8.3	$KJ = JL$ $\hat{J}_1 = 90°$ $\hat{N} = 40°$	[given] [line from centre to midpoint of chord] [sum of angles in a Δ]	✓ S ✓R ✓ S/R (3)
			[7]

QUESTION 9

9.1.1	$\hat{R}_1 = \hat{S}_1 = x$ $\therefore \hat{O}_1 = 180° - 2x$ but $\hat{P} = \frac{1}{2}\hat{O}_1$ $\therefore \hat{P} = 90° - x$	[∠s opp = sides] [sum of ∠s in a Δ] [∠ @ centre = 2×∠ @ circumference]	✓S/R ✓S/R ✓R ✓ answer (4)
9.1.2	$PS = SR$ $\therefore \hat{R}_1 + \hat{R}_2 = \hat{P} = 90° - x$ In ΔPSR: $\hat{S}_2 + \hat{P} + S\hat{R}P + \hat{S}_1 = 180°$ $\therefore \hat{S}_2 = 180° - (90° - x + 90° - x + x)$ $\therefore \hat{S}_2 = x$ $\therefore \hat{S}_2 = \hat{S}_1$	[given] [∠s opp = sides] [sum of ∠s in a Δ] [both = to x]	✓S/R ✓S/R ✓S (3)
	OR		
	$PS = SR$ $\hat{R}_1 + \hat{R}_2 = \hat{P} = 90° - x$ $\hat{R}_2 = 90° - 2x$ $\hat{O}_2 = 2x$ $O\hat{T}P = 90°$ $\hat{S}_2 = x$ $\therefore \hat{S}_2 = \hat{S}_1$	[given] [∠s opp = sides] [adj ∠s on a str line] [ext ∠ of Δ] [sum of ∠s in a Δ] [both = to x]	✓S/R ✓S/R ✓S (3)
9.2	$BE = BD - ED$ $BD = 2AO$ $\therefore BE = 2AO - ED$ In ΔBEC: $BE^2 = BC^2 - EC^2$ $\therefore (2AO - ED)^2 = BC^2 - EC^2$ but $EC = AE$ $\therefore (2AO - ED)^2 = BC^2 - AE^2$	[diameter = 2×radius] [Pythagoras] [line from centre ⊥ to chord]	✓ expression for BE ✓ BD = 2AO ✓S/R using Theorem of Pythagoras ✓S ✓R (5)
			[12]

QUESTION 10

10.1			
	Construction: Draw diameter BF & join FE.		✓ construction
	Proof		
	$\hat{B}_1 + \hat{B}_2 = 90°$	[rad ⊥ tan]	✓S ✓R
	$\hat{E}_1 + \hat{E}_2 = 90°$	[angle in semi-circle]	✓S/R
	but $\hat{B}_2 = \hat{E}_2$	[angles in same segment]	✓S/R
	$\therefore \hat{B}_1 = \hat{E}_1$		✓S
	$\Rightarrow \hat{B}_1 = D\hat{E}B$		(5)
	OR		
			
	Construction: Draw diameter BF & join FD.		✓ construction
	Proof:		
	$F\hat{D}B = 90°$	[angle in semi-circle]	✓S/R
	$\therefore \hat{F} + \hat{B}_2 = 90°$	[sum of ∠s in a Δ]	
	but $\hat{B}_1 + \hat{B}_2 = 90°$	[rad ⊥ tan]	✓S ✓R
	$\therefore \hat{F} = \hat{B}_1$		
	but $\hat{F} = \hat{E}$	[angles in same segment]	✓S/R
	$\Rightarrow \hat{B}_1 = D\hat{E}B$		(5)

10.2.1	$\hat{F}_3 = 90°$ $\therefore AC \perp DB$	[angle in the semi-circle]	✓S ✓R (2)
10.2.2	$\hat{B}_1 = \hat{C}_3$ $\hat{B}_1 = \hat{P}$ $\hat{C}_3 = \hat{P}$ AC□DP	[tan chord theorem] [tan chord theorem] [corresp ∠s =]	✓S ✓R ✓S/R ✓R (4)
10.2.3	AD = AB $\hat{D}_1 = \hat{B}_1$ but $\hat{B}_1 = \hat{C}_3$ $\therefore \hat{D}_1 = \hat{C}_3$ ∴ ABCD is a cyclic quad.	[tans from same point] [∠s opp = sides] [proved above] [both = to $\hat{B}_1$] [conv ∠s in same segment]	✓S/R ✓S/R ✓S ✓R (4)
10.2.4	$\hat{D}_3 = \hat{C}_2$ $\hat{C}_2 = \hat{B}_1$ $\hat{B}_1 = \hat{P}$ $\therefore \hat{D}_3 = \hat{P}$ ∴ CP = CD	[alt ∠s = ; AC□DP] [∠s in same segment] [tan chord theorem] [sides opp = ∠s]	✓S/R ✓S/R ✓ S ✓ R (4)
10.2.5	$\hat{B}_2 + \hat{B}_3 = \hat{C}_2$ $\hat{C}_2 = \hat{B}_1$ $\hat{B}_1 = \hat{D}_1$ $\therefore \hat{D}_1 = \hat{B}_2 + \hat{B}_3$ ∴ BR□AD **OR** $\hat{D}_1 + \hat{D}_2 = 90°$ $\hat{R} = 90°$ $\therefore \hat{D}_1 + \hat{D}_2 + \hat{R} = 180°$ ⇒ BR ǁ AD	[ext ∠ of cyclic quad] [∠s in same segment] [proved in Q10.2.3] [alt ∠s =] [opp ∠s of cyclic quad] [angle in the semi-circle] [co-int ∠s supplementary]	✓S ✓R ✓ S ✓ R (4) ✓S ✓R ✓ S ✓ R (4)
			[23]

TOTAL: 150 MARKS