KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

JUNE EXAMINATION

2025

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 19 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer **ALL** the questions in the **ANSWER BOOK** provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly.

QUESTION 1

The box-and-whisker diagram below illustrates the distribution of the Mathematics June examination marks, out of 150, for a class of 32 grade 12 learners. The median of the marks is 65 and the mean is 71,75.

1.1 If none of the learners had a mark of 90 out of 150, how many learners had marks of higher than 90? (1)

1.2 Describe the skewness of the data. (1)

1.3 Calculate the range of the data. (2)

1.4 There is only one candidate who had a mark of 125 out of 150. On checking the answer book of this candidate, it was discovered that a mistake was made when adding his marks. The mistake was corrected, and his total mark then changed to 142 out of 150.
Determine the resulting value of each of the following:

1.4.1 the median (1)

1.4.2 the mean (3)

[8]

QUESTION 2

A group of teenagers were surveyed on how many hours they spent using social media over a period of 7 days. The results are tabulated below.

NUMBER OF HOURS SPENT USING SOCIAL MEDIA OVER A PERIOD OF 7 DAYS	NUMBER OF LEARNERS
$0 \le x < 10$	4
$10 \le x < 20$	5
$20 \le x < 30$	9
$30 \le x < 40$	13
$40 \le x < 50$	18
$50 \le x < 60$	11
$60 \le x < 70$	7

2.1 How many teenagers were surveyed? (1)

2.2 Write down the modal class. (1)

2.3 Calculate the estimated mean. (3)

2.4 Complete the cumulative frequency table provided in the ANSWER BOOK. (2)

2.5 Draw a cumulative frequency curve (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

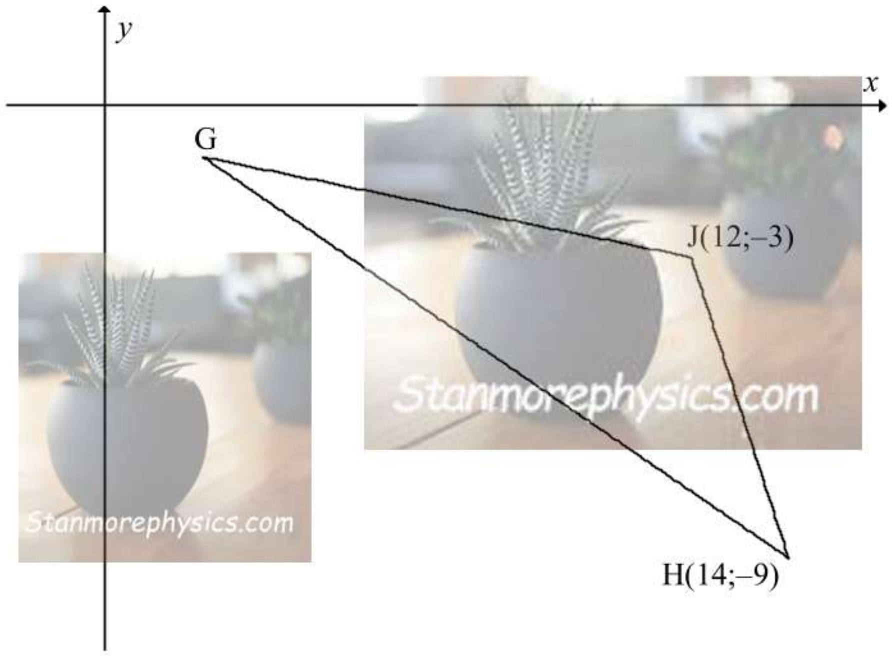
2.6 Use the cumulative frequency curve (ogive) to estimate the number of teenagers from this group who spent on average between 2 and 4 hours per day using social media. (3)

[13]

QUESTION 3

3.1 In the diagram below, G, $J(12;-3)$ and $H(14;-9)$ are vertices of ΔGHJ.

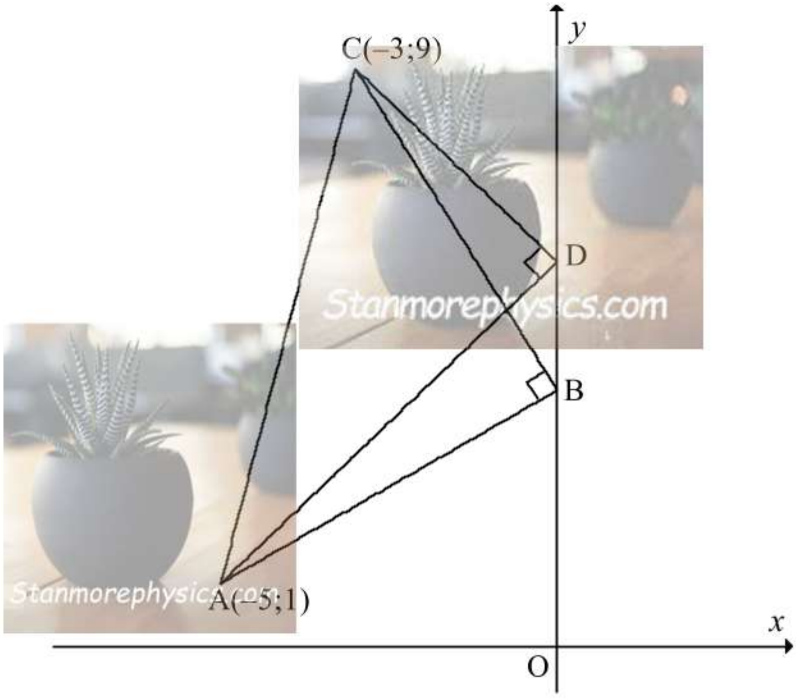
The equation of line GH is $y=-\frac{2}{3}x+c$.



3.1.1 Calculate the angle of inclination of line JH. (4)

3.1.2 Calculate the size of $\hat{H}$. (3)

3.2 In the diagram, $A(-5;1)$ and $C(-3;9)$ are vertices of ΔADC and ΔABC.
B and D are points on the y-axis such that $A\hat{B}C = A\hat{D}C = 90°$.



3.2.1 Calculate the coordinates of M, the midpoint of AC. (2)

3.2.2 Calculate the length of the radius of the circle passing through A, C and D. (3)

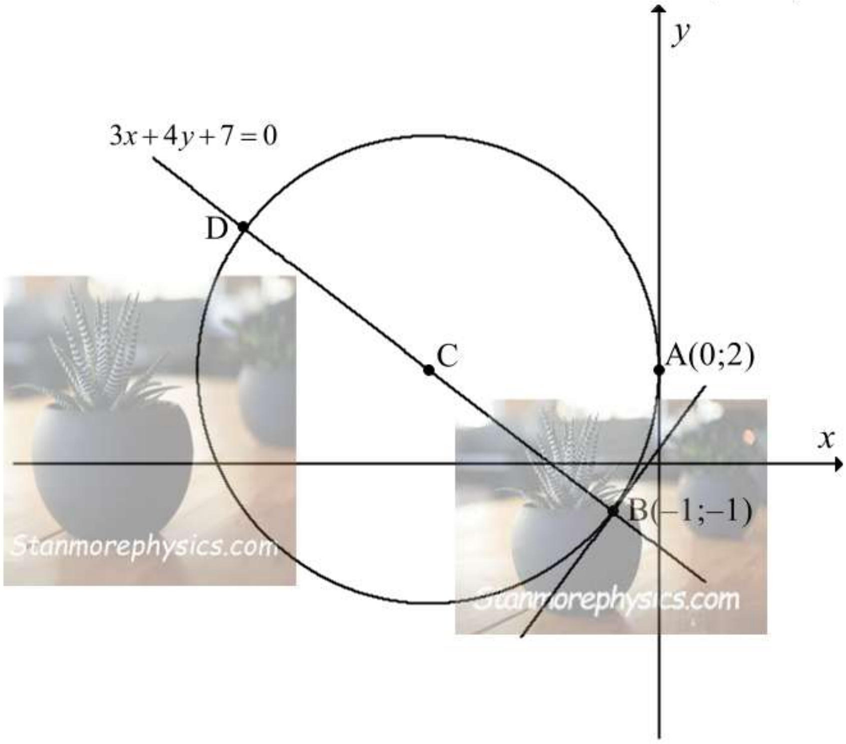
3.2.3 Calculate the coordinates of D. (5)

3.2.4 Write down the coordinates of B. (2)

[19]

QUESTION 4

4.1 In the diagram below, the circle with centre C touches the y-axis at $A(0\,;2)$.
A straight line with equation $3x+4y+7=0$ cuts the circle at $B(-1;-1)$ and D.



4.1.1 Determine the equation of the tangent to the circle at B. (4)

4.1.2 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (5)

4.1.3 Determine the coordinates of the image of B, after reflection of the circle in the line $y=2$. (2)

4.2 A circle with equation $x^2-4x+y^2+6y-51=0$ is drawn in a Cartesian plane.

4.2.1 Determine the coordinates of the centre of the circle and the length of its radius. (4)

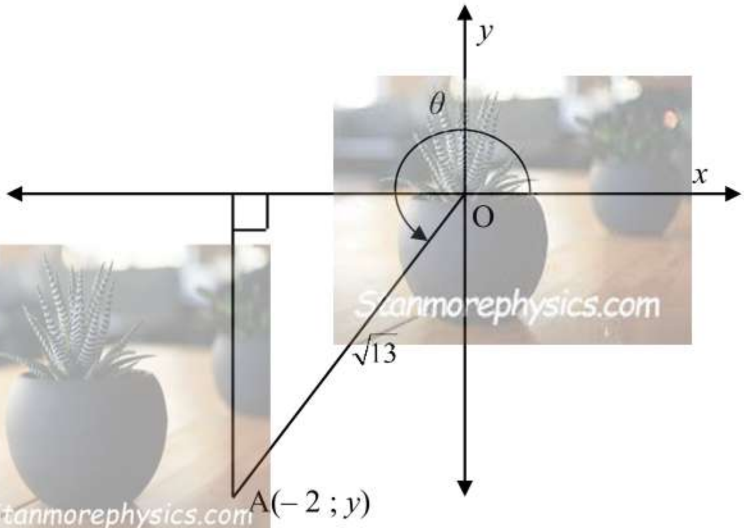
4.2.2 Another circle with equation $x^2+y^2=r^2$ is drawn in the same Cartesian plane and touches the circle with equation $x^2-4x+y^2+6y-51=0$ internally.

Calculate the value of r. Give your answer correct to 2 decimal digits. (4)

[19]

QUESTION 5

5.1 Given: $A(-2;y)$, a point in a Cartesian plane, with $OA=\sqrt{13}$ and θ the angle between OA and the positive x-axis.



5.1.1 Without the use of a calculator, determine the value of:

(a) y (2)

(b) $\sin\theta$ (1)

5.1.2 Use a calculator and determine the size of angle θ. (2)

5.2 Simplify the following without the use of a calculator:

$$\frac{\tan(-60^\circ).\cos(-156^\circ).\cos 294^\circ}{\sin 852^\circ}$$ (7)

5.3 Given: $\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\cos x-\sin x}{\cos x+\sin x}=2\tan 2x$.

5.3.1 Prove the given identity. (4)

5.3.2 For which values of x is $2\tan 2x$ undefined? (2)

5.4 If $\cos 40^\circ = p$, determine the value of the following in terms of p, without the use of a calculator:

$\cos 10^\circ+\cos 70^\circ$ (4)

5.5 Solve for x, in the interval $x\in(-180^\circ;180^\circ]$, if $4\sin x\cos x=3\sin^2 x$. (6)

5.6 If $\tan\theta = m$, in any right-angled triangle:

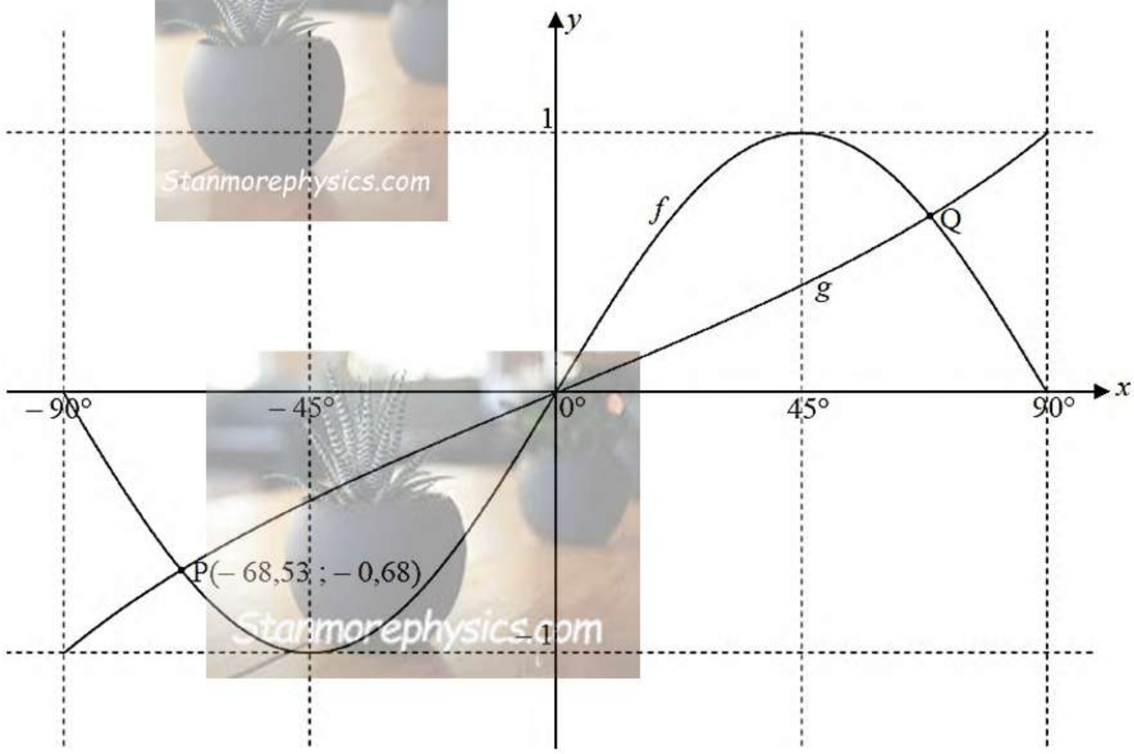
5.6.1 Show that $\sin 2\theta = \dfrac{2m}{m^2+1}$. (3)

5.6.2 Hence, or otherwise, calculate the maximum value of $\dfrac{(m+1)^2}{m^2+1}$. (3)

[34]

QUESTION 6

In the diagram below, the graphs of $f(x) = a\sin 2x$ and $g(x) = \tan bx$ for $x \in [-90° ; 90°]$ are drawn. $P(-68,53° ; -0,68)$ and Q are points of intersection of f and g.



6.1 Write down the:

6.1.1 value of a (1)

6.1.2 value of b (1)

6.1.3 coordinates of Q (2)

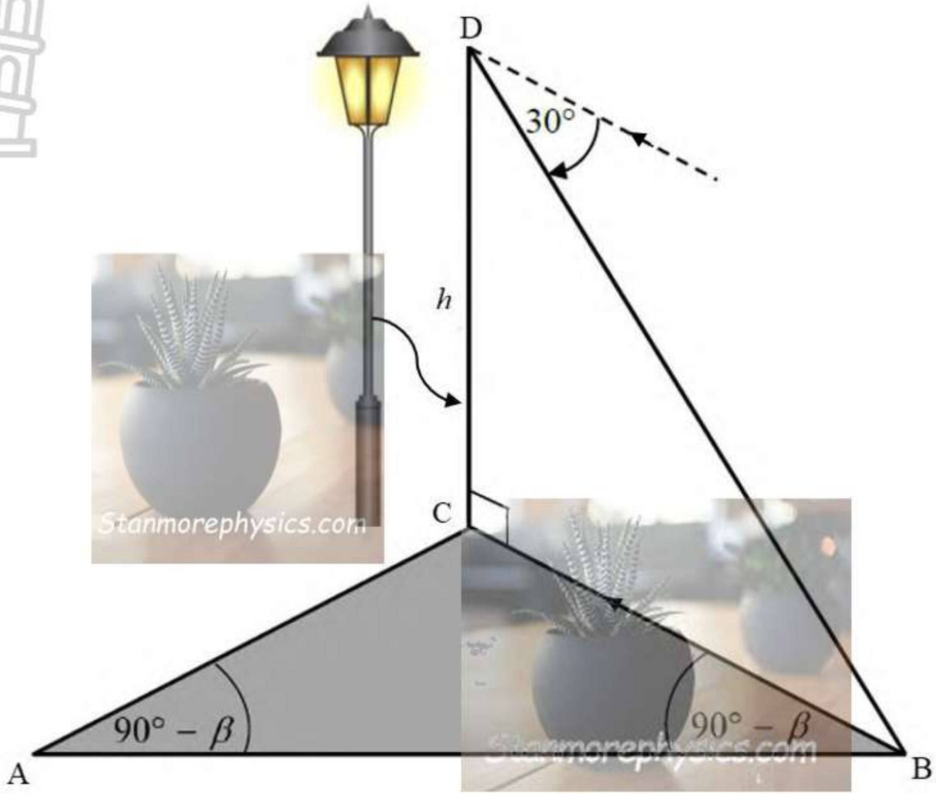
6.1.4 value of k if $f(x+k) = 2\sin^2 x - 1$. (2)

6.2 For which value(s) of x, in the given interval, will $x.\sqrt{g(x) - f(x)} > 0$? (3)

[9]

QUESTION 7

In the diagram, A, B and C lie in the same horizontal plane. CD is a vertical lamp post. The angle of depression from D to B is $30°$. $A\hat{B}C = B\hat{A}C = 90° - \beta$ and $CD = h$ metres.

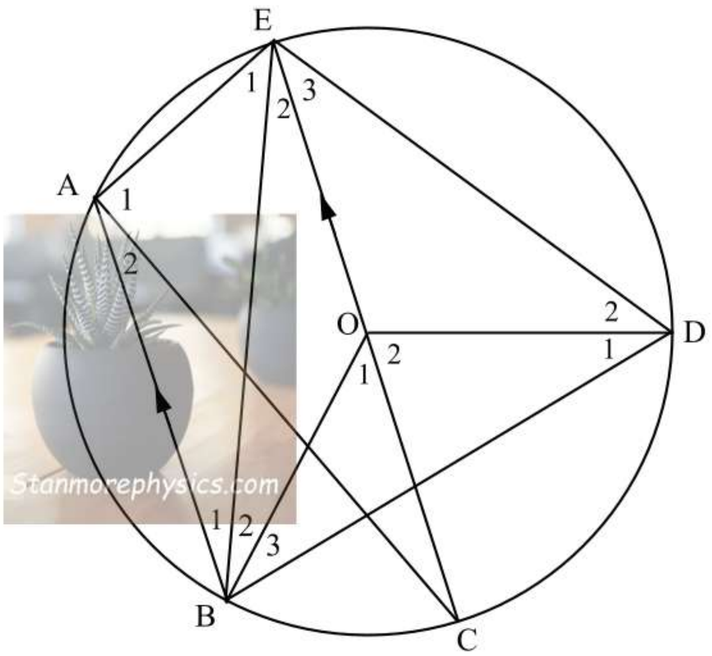


Show that $AB = 2\sqrt{3}.h\sin\beta$. **[7]**

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 8, 9 AND 10.

QUESTION 8

In the diagram, O is the centre of circle ABCDE. CE is a diameter. AB || EC. BE, AC, BO and OD have also been drawn.
$\hat{C} = 26°$.



8.1 Write down, with reasons, three other angles each equal to $26°$. (5)

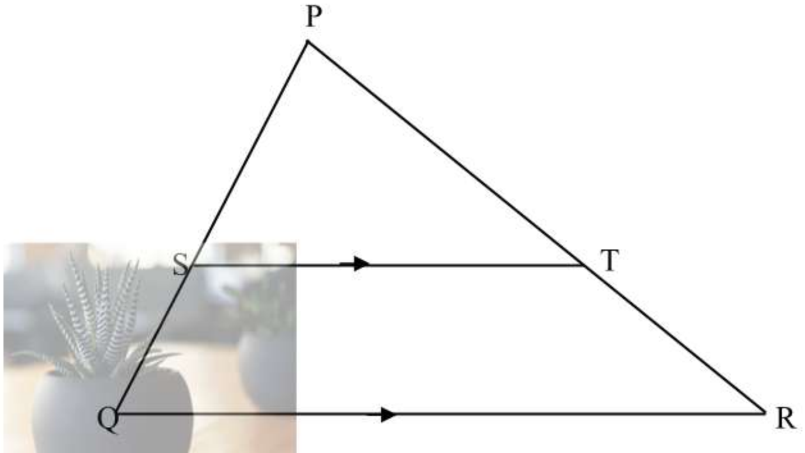
8.2 Calculate the size of $\hat{O}_1$. (2)

8.3 Calculate the size of $B\hat{D}E$. (3)

[10]

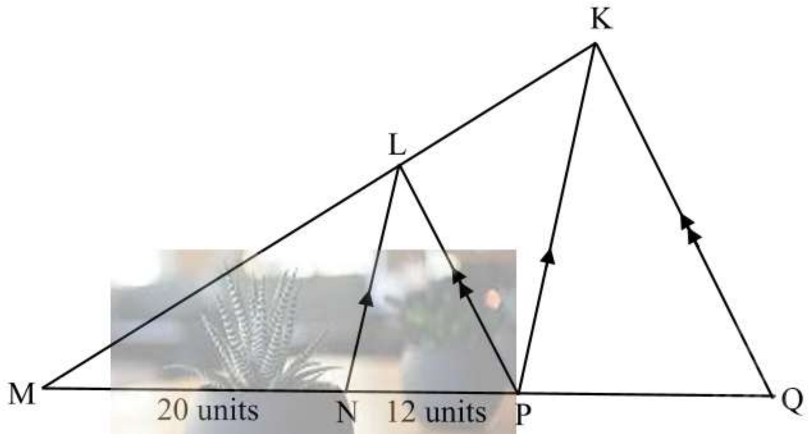
QUESTION 9

9.1 In the diagram ΔPQR is drawn. Line ST intersects PQ and PR at S and T respectively, such that ST || QR.



Prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

9.2 In the diagram below, L is a point on side KM of ΔKMQ. N and P are points on side MQ, such that NL || PK and LP || KQ. MN = 20 units and NP = 12 units.



9.2.1 Determine the ratio $\frac{KL}{LM}$ in its simplest form. (2)

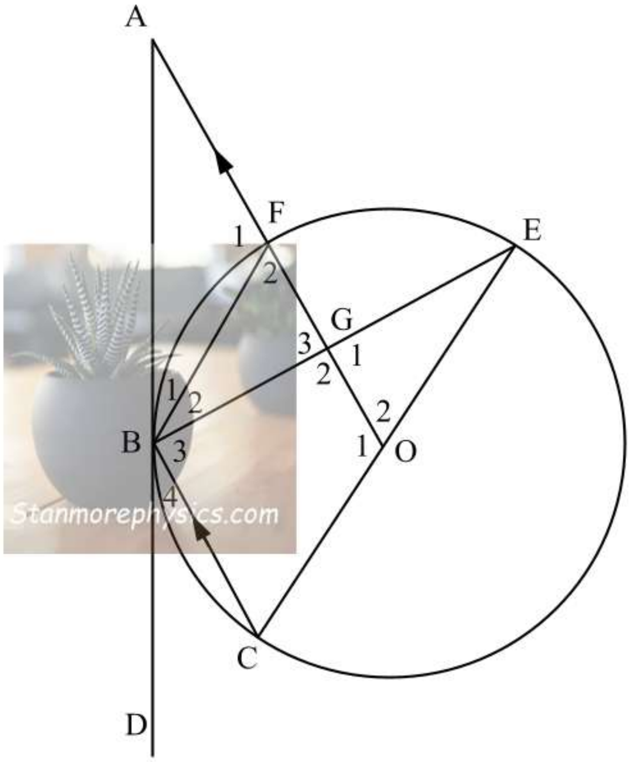
9.2.2 Calculate the length of PQ. (3)

9.2.3 Determine the ratio $\frac{KQ}{LP}$ in its simplest form. (3)

[14]

QUESTION 10

In the diagram, O is the centre of circle BCEF. ABD is a tangent to the circle at B, and COE is a diameter. Lines AFO, BF, BE and BC are drawn. AFO cuts BE in G. AO || BC.



10.1 Prove that:

10.1.1 BG = EG (3)

10.1.2 AEOB is a cyclic quadrilateral. (4)

10.1.3 ΔOEG|||ΔBAG (3)

10.2 If BC = 10 units and AG = 35 units, calculate the length of EO. (7)

[17]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin \text{A}} = \frac{b}{\sin \text{B}} = \frac{c}{\sin \text{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta \text{ABC} = \frac{1}{2} ab.\sin \text{C}$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\text{P(A)} = \frac{n(\text{A})}{n(\text{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

JUNE 2025 EXAMINATION

MARKING GUIDELINES

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 150

These marking guidelines consist of 13 pages.

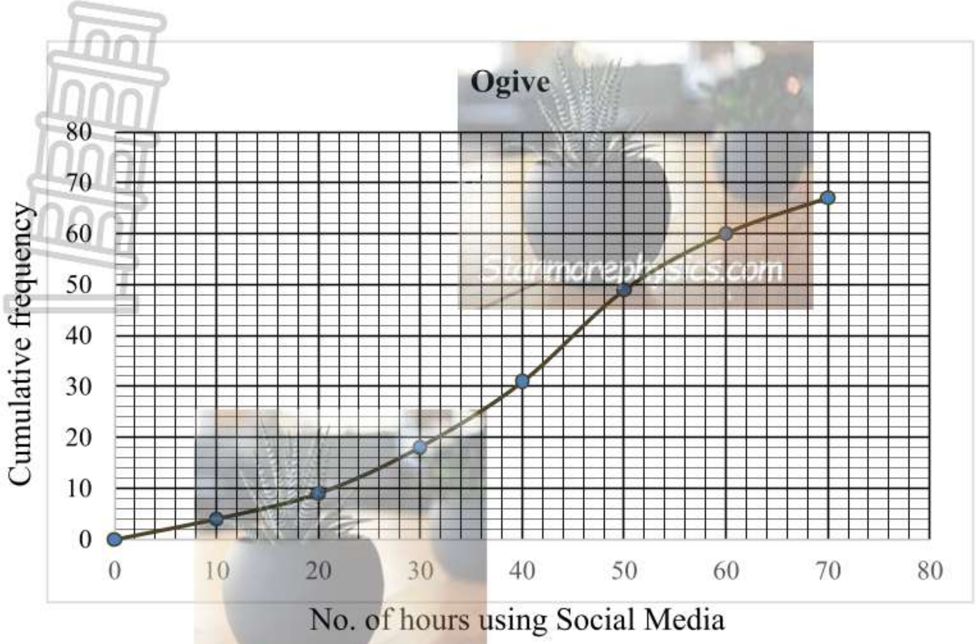
QUESTION 1

1.1	8 learners	✓A answer (1)
1.2	Skewed to the right **OR** positively skewed	✓A answer (1)
1.3	$\text{Range} = 125 - 34$ $= 91$ Answer only: Full marks	✓A $125 - 34$ ✓A answer (2)
1.4.1	$\text{median} = 65$ OR the value does not change	✓A answer (1)
1.4.2	Total of learners' marks = $71,75 \times 32 = 2296$ New total of learners' marks = $2296 + (142 - 125) = 2313$ New mean $= \frac{2313}{32} = 72,28$	✓A $71,75 \times 32 = 2296$ ✓CA new total of learners' marks ✓CA answer (3)
		[8]

QUESTION 2

2.1	67	✓A answer (1)
2.2	$40 \leq x < 50$	✓A answer (1)
2.3	$\bar{x} = \frac{5\times4 + 15\times5 + 25\times9 + 35\times13 + 45\times18 + 55\times11 + 65\times7}{67}$ $\bar{x} = \frac{2645}{67}$ $= 39,48$ Answer only: Full marks	✓A numerator ✓CA denominator ✓CA answer (3)
2.4	(see table below)	✓A 9; 18; 31 ✓CA 49; 60; 67 (2)

NUMBER OF HOURS ON SOCIAL MEDIA	FREQUENCY	CUMULATIVE FREQUENCY
$0 \leq x < 10$	4	4
$10 \leq x < 20$	5	9
$20 \leq x < 30$	9	18
$30 \leq x < 40$	13	31
$40 \leq x < 50$	18	49
$50 \leq x < 60$	11	60
$60 \leq x < 70$	7	67
TOTAL	67	

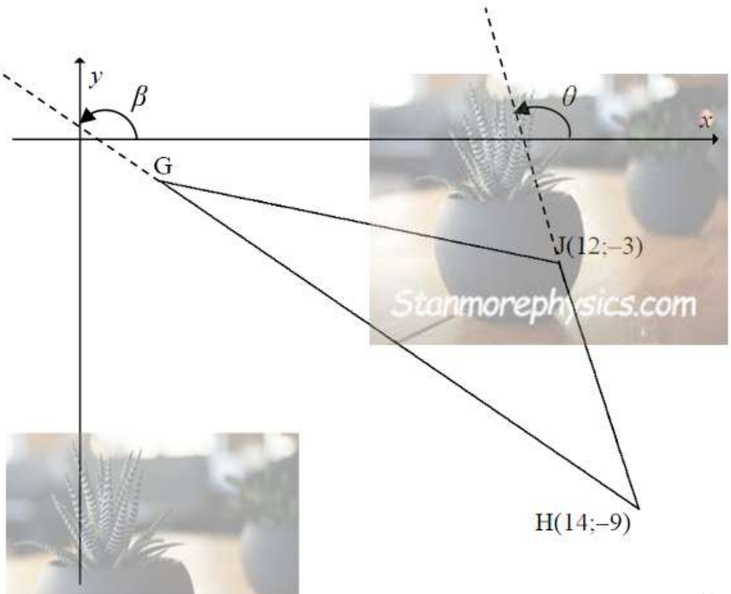
<table>
<tr><td>2.5</td><td>
</td><td>✓A plotting all points correctly (at upper limits)

✓CA shape of an ogive

✓A grounding at (0 ; 0)

(3)</td></tr>
<tr><td>2.6</td><td>Between 14 and 28 hours per week
No. of teenagers $=16-6$
$=10$</td><td>✓CA 16 (accept: 15 to 17)
✓CA 6 (accept 5 to 7)
✓CA answer
(3)</td></tr>
<tr><td colspan="3">[13]</td></tr>
</table>

QUESTION 3



3.1.1	$m_{JH}=\frac{-3-(-9)}{12-14}$ $=-3$ $\tan\theta=m_{JH}=-3$ $\therefore\theta=180°-71,57°=108,43°$	✓A substitution in gradient formula ✓CA gradient of JH ✓CA $\tan\theta=-3$ ✓CA answer (4)
3.1.2	$m_{GH}=\frac{-2}{3}=\tan\beta$ $\therefore\beta=180°-33,69°=146,31°$ $\hat{H}=146,31°-108,43°$ $=37,88°$	✓A $\tan\beta=\frac{-2}{3}$ ✓A $\beta=146,31°$ ✓CA answer (3)
3.2.1	$M(-4;5)$	✓A x-value ✓A y-value (2)
3.2.2	AC is a diameter, $\therefore$ CM is a radius $CM=\sqrt{(-3-(-4))^2+(9-5)^2}$ $=\sqrt{17}$ **OR** AC is a diameter, $\therefore$ AM is a radius $AM=\sqrt{(-5-(-4))^2+(1-5)^2}$ $=\sqrt{17}$ **OR** AC is a diameter $AC=\sqrt{(-3-(-5))^2+(9-1)^2}$ $=2\sqrt{17}$ $\therefore$ radius $=\sqrt{17}$ Don't penalise if it is not written that AC is the diameter, but it is used.	✓A identifying AC as diameter ✓CA substitution ✓CA answer (3) **OR** ✓A identifying AC as diameter ✓CA substitution ✓CA answer (3) **OR** ✓A identifying AC as diameter ✓A substitution ✓CA answer (3)

3.2.3	$m_{AD} \times m_{CD} = -1$ $\frac{1-y}{-5-0} \times \frac{9-y}{-3-0} = -1$ $(1-y)(9-y) = -15$ $y^2 - 10y + 24 = 0$ $(y-6)(y-4) = 0$ $y = 6$ or $y = 4$ $D(0;6)$	✓A $m_{AD} \times m_{CD} = -1$ ✓A substitution ✓CA standard form ✓CA factors ✓CA answer (5)
3.2.4	$B(0;4)$	✓✓CA answer (2)
		[19]

QUESTION 4

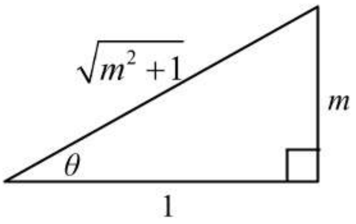
4.1.1	Equation of diameter (radius): $y = -\frac{3}{4}x - \frac{7}{4}$ $\therefore m_{radius} = -\frac{3}{4}$ $\therefore m_{tangent} = \frac{4}{3}$ Equation of tangent: $y = \frac{4}{3}x + c$ Substitute $(-1;-1)$: $-1 = \frac{4}{3}(-1) + c$ $c = \frac{1}{3}$ $\therefore y = \frac{4}{3}x + \frac{1}{3}$	✓A gradient of radius ✓CA gradient of tangent ✓CA substitution of *m* and point ✓CA answer (4)
4.1.2	*y*-coordinate of C = *y*-coordinate of A = 2 For *x*-coordinate of C, substitute *y* = 2 into $3x + 4y + 7 = 0$: $3x + 4(2) + 7 = 0$ $x = -5$ $\therefore C(-5;2)$ $r^2 = [-5-(-1)]^2 + [2-(-1)]^2 = 25$ Equation of the circle: $(x-a)^2 + (y-b)^2 = r^2$ $(x+5)^2 + (y-2)^2 = 25$	✓A y – coordinate of C ✓A substitution in $3x + 4y + 7 = 0$ ✓A x – coordinate of C ✓CA calculation of r^2 ✓CA answer (5)
4.1.3	Vertical distance from B to $y = 2$: 3 units Vertical distance from B′ to $y = 2$: 3 units $\therefore$ B′ is 6 units higher than B. Image of B: $(-1;5)$	✓A *x*-coordinate ✓A *y*-coordinate (2)

4.2.1	$x^2-4x+y^2+6y-51=0$ $(x-2)^2+(y+3)^2=51+4+9$ $(x-2)^2+(y+3)^2=64$ centre: $(2\,;-3)$ radius $=8$ units	✓A LHS of equation ✓A RHS of equation ✓CA coordinates of centre ✓CA length of radius (4)
4.2.2	Coordinates of centre of $x^2+y^2=r^2$: $(0\,;0)$ Distance between centres of two circles $=\sqrt{(2-0)^2+(-3-0)^2}$ $=\sqrt{13}$ When touching internally: Distance between centres $=$ radius of circle$_1$ $-$ radius of circle$_2$ $\sqrt{13}=8-r$ $r=8-\sqrt{13}$ $=4{,}39$ units	✓A substitution ✓CA distance between centres ✓CA $\sqrt{13}=8-r$ ✓CA answer (4)
		[19]

QUESTION 5

5.1.1 (a)	$y^2=r^2-x^2$ [Pythagoras] $=\left(\sqrt{13}\right)^2-(-2)^2$ $=9$ $y=-3$	✓A substitution ✓CA answer (2)
5.1.1 (b)	$\sin\theta=\frac{-3}{\sqrt{13}}$	✓CA answer (1)
5.1.2	ref. $\angle$: $56{,}31°$ $\therefore\theta=180°+56{,}31°=236{,}31°$	✓CA $56{,}31°$ ✓CA answer (2)
5.2	$\frac{\tan(-60°).\cos(-156°).\cos 294°}{\sin 852°}$ $=\frac{-\tan 60°.\cos 204°.\cos 66°}{\sin 132°}$ $=\frac{-\tan 60°.-\cos 24°.\sin 24°}{\sin 48°}$ $=\frac{\tan 60°.\cos 24°.\sin 24°}{2\sin 24°\cos 24°}$ $=\frac{\tan 60°}{2}$ $=\frac{\sqrt{3}}{2}$	✓A $-\tan 60°$ ✓A $\cos 66°$ ✓A $-\cos 24°$ ✓A $\sin 24°$ ✓A $\sin 48°$ ✓A $2\sin 24°.\cos 24°$ ✓CA answer (7)

	OR	**OR**
	$\dfrac{\tan(-60°).\cos(-156°).\cos 294°}{\sin 852°}$	
	$=\dfrac{-\tan 60°.\cos 204°.\cos 66°}{\sin 132°}$	✓A $-\tan 60°$ ✓A $\cos 66°$
	$=\dfrac{-\tan 60°.-\cos 24°.\cos 66°}{\sin 132°}$	✓A $-\cos 24°$ ✓A $\sin 132°$
	$=\dfrac{\tan 60°.\sin 66°.\cos 66°}{2\sin 66°.\cos 66°}$	✓A $\sin 66°$ ✓A $2\sin 66°.\cos 66°$
	$=\dfrac{\tan 60°}{2}$	
	$=\dfrac{\sqrt{3}}{2}$	✓CA answer (7)
	OR	**OR**
	$\dfrac{\tan(-60°).\cos(-156°).\cos 294°}{\sin 852°}$	
	$=\dfrac{-\tan 60°.\cos 204°.\cos 66°}{\sin 132°}$	✓A $-\tan 60°$ ✓A $\cos 66°$
	$=\dfrac{-\tan 60°.-\cos 24°.\sin 24°}{\sin 48°}$	✓A $-\cos 24°$ ✓A $\sin 24°$ ✓A $\sin 48°$
	$=\dfrac{\tan 60°.\frac{1}{2}\sin 48°}{\sin 48°}$	✓A $\frac{1}{2}\sin 48°$
	$=\dfrac{\tan 60°}{2}$	
	$=\dfrac{\sqrt{3}}{2}$	✓CA answer (7)
5.3.1	$\dfrac{\cos x+\sin x}{\cos x-\sin x}-\dfrac{\cos x-\sin x}{\cos x+\sin x}$	
	$=\dfrac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$	✓A numerator ✓A denominator
	$=\dfrac{\cos^2 x+2\sin x\cos x+\sin^2 x-(\cos^2 x-2\sin x\cos x+\sin^2 x)}{(\cos x-\sin x)(\cos x+\sin x)}$	
	$=\dfrac{4\sin x\cos x}{\cos^2 x-\sin^2 x}$	✓A $\dfrac{4\sin x\cos x}{\cos^2 x-\sin^2 x}$
	$=\dfrac{2\sin 2x}{\cos 2x}$	✓A $\dfrac{2\sin 2x}{\cos 2x}$
	$=2\tan 2x$	(4)

5.3.2	$2\tan 2x$ is undefined when: $2x = 90° + k.180°,\ k \in Z$ $\therefore x = 45° + k.90°,\ k \in Z$ **OR** LHS is undefined when $\cos x - \sin x = 0$ or $\cos x + \sin x = 0$ $\tan x = 1$ or $\tan x = -1$ $\therefore x = 45° + k.180°,\ k \in Z$ or $\therefore x = 135° + k.180°,\ k \in Z$ Answer only: Full marks	✓A $2x = 90° + k.180°,\ k \in Z$ ✓A $x = 45° + k.90°,\ k \in Z$ (2) **OR** ✓A $\tan x = 1$ or $\tan x = -1$ ✓A $\therefore x = 45° + k.180°,\ k \in Z$ or $\therefore x = 135° + k.180°,\ k \in Z$ (2)
5.4	$\cos 10° + \cos 70°$ $= \cos(40° - 30°) + \cos(40° + 30°)$ $= \cos 40° \cos 30° + \sin 40° \sin 30° + \cos 40° \cos 30° - \sin 40° \sin 30°$ $= 2\cos 40° \cos 30°$ $= 2p\left(\frac{\sqrt{3}}{2}\right)$ $= \sqrt{3}p$	✓A $\cos(40° - 30°) + \cos(40° + 30°)$ ✓A compound ∠ expansions ✓A $2\cos 40° \cos 30°$ ✓A answer (4)
5.5	$4\sin x \cos x = 3\sin^2 x$ $4\sin x \cos x - 3\sin^2 x = 0$ $\sin x(4\cos x - 3\sin x) = 0$ $\therefore \sin x = 0$ or $4\cos x = 3\sin x$ $x = 0° + k.180°$ or $\frac{\sin x}{\cos x} = \frac{4}{3}$ $\tan x = \frac{4}{3}$ $x = 53{,}13° + k.180°,\ k \in Z$ For $x \in (-180°;180°]$: $x = 0°\ ;\ 180°$ or $x = -126{,}87°\ ;\ 53{,}13°$	✓A factorisation ✓CA both equations ✓A $x = 0° + k.180°$ ✓CA $\tan x = \frac{4}{3}$ ✓A $0°\ ;\ 180°$ ✓CA $-126{,}87°\ ;\ 53{,}13°$ (6)
5.6.1	$\tan\theta = m$ $r = \sqrt{m^2+1}$ [Pythagoras]  $\sin 2\theta = 2\sin\theta\cos\theta$ $= 2\left(\frac{m}{\sqrt{m^2+1}}\right)\left(\frac{1}{\sqrt{m^2+1}}\right)$ $= \frac{2m}{m^2+1}$	✓A $r = \sqrt{m^2+1}$ ✓A double angle expansion ✓A substitution into double angle expansion (3)

5.6.2	$\frac{(m+1)^2}{m^2+1}$ $=\frac{m^2+2m+1}{m^2+1}$ $=\frac{2m}{m^2+1}+\frac{m^2+1}{m^2+1}$ $=\sin 2\theta+1$ maximum value of $\sin 2\theta$ is 1 Maximum value of $\frac{(m+1)^2}{m^2+1}$ =1+1 = 2	✓A $\sin 2\theta+1$ ✓A max. value of $\sin 2\theta$ ✓A answer (3)
		[34]

QUESTION 6

6.1.1	$a=1$	✓A answer (1)
6.1.2	$b=\frac{1}{2}$	✓A answer (1)
6.1.3	$Q(68,53°\ ;\ 0,68)$	✓A x-coordinate ✓A y-coordinate (2)
6.1.4	$f(x+k)=2\sin^2 x-1=-(1-2\sin^2 x)=-\cos 2x$ $\therefore f(x+k)$ is obtained by shifting $f(x)$ 45° to the right. $\therefore k=-45°$	✓A $-\cos 2x$ ✓A answer (2)
6.2	$x.\sqrt{g(x)-f(x)}>0$ $\therefore x>0$ and $\sqrt{g(x)-f(x)}>0$ $\therefore g(x)-f(x)>0$ $\therefore g(x)>f(x)$ $68,53°<x\leq 90°$ **OR** $x\in(68,53°;90°]$ Answer only: Full marks	✓A $g(x)>f(x)$ ✓CA ✓A answer (3)
		[9]

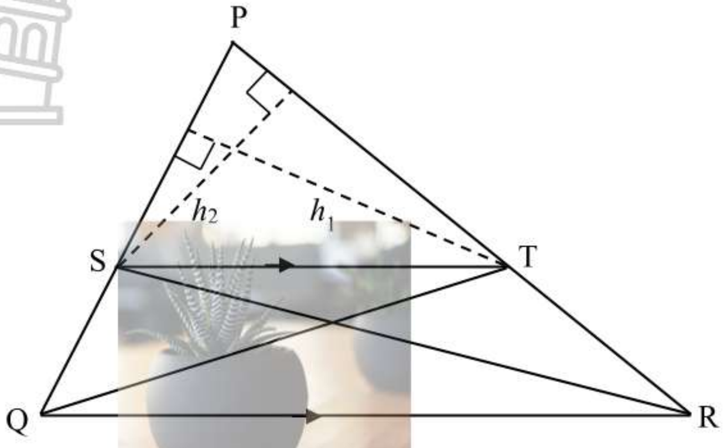
QUESTION 7

7.	$\hat{DBC} = 30°$ [alternate $\angle$s ; $\parallel$ lines] $\frac{h}{BC} = \tan 30°$ $BC = \frac{h}{\tan 30°} = \frac{h}{\frac{1}{\sqrt{3}}} = \sqrt{3}h$ $\hat{ACB} = 180° - (90° - \beta + 90° - \beta) = 2\beta$ $\frac{AB}{\sin \hat{ACB}} = \frac{BC}{\sin A}$ $\frac{AB}{\sin 2\beta} = \frac{\sqrt{3}h}{\sin(90° - \beta)}$ $AB = \frac{\sqrt{3}h.\sin 2\beta}{\sin(90° - \beta)}$ $= \frac{\sqrt{3}h.2\sin\beta\cos\beta}{\cos\beta}$ $= 2\sqrt{3}.h\sin\beta$	✓A $\frac{h}{BC} = \tan 30°$ ✓A $BC = \sqrt{3}h$ ✓A $\hat{ACB} = 2\beta$ ✓A substitution in sine rule ✓A AB subject of formula ✓A $\sin 2\beta = 2\sin\beta\cos\beta$ ✓A $\sin(90° - \beta) = \cos\beta$ (7)
	OR	**OR**
	$\hat{DBC} = 30°$ [alternate $\angle$s ; $\parallel$ lines] $\frac{h}{BC} = \tan 30°$ $BC = \frac{h}{\tan 30°} = \frac{h}{\frac{1}{\sqrt{3}}} = \sqrt{3}h$ $\hat{ACB} = 180° - (90° - \beta + 90° - \beta) = 2\beta$ $AC = BC$ [$\angle$s opp. = sides] $AB^2 = AC^2 + BC^2 - 2AB.BC.\cos \hat{ACB}$ $= (\sqrt{3}h)^2 + (\sqrt{3}h)^2 - 2(\sqrt{3}h)^2.(\sqrt{3}h)^2.\cos 2\beta$ $= 6h^2 - 6h^2.\cos 2\beta$ $= 6h^2(1 - \cos 2\beta)$ $= 6h^2[1 - (1 - 2\sin^2\beta)]$ $= 12h^2\sin^2\beta$ $\therefore AB = 2\sqrt{3}.h\sin\beta$	✓A $\frac{h}{BC} = \tan 30°$ ✓A $BC = \sqrt{3}h$ ✓A $\hat{ACB} = 2\beta$ ✓A substitution in cosine rule ✓A factorisation ✓A $\cos 2\beta = 1 - 2\sin^2\beta$ ✓A $= 12h^2\sin^2\beta$ (7)
		[7]

QUESTION 8

8.1	$\hat{A}_2 = \hat{C} = 26°$ [alternate $\angle$s; AB ∥ EC] $\hat{E}_2 = \hat{A}_2 = 26°$ [$\angle$s in the same segment] $\hat{B}_1 = \hat{E}_2 = 26°$ [alternate $\angle$s; AB ∥ EC] or [$\angle$s in the same segment] $\hat{B}_2 = \hat{E}_2 = 26°$ [$\angle$s opp. = sides] **Any 3 of the 4 angles mentioned above may be awarded marks.** Two of the three angles, will be awarded 2 marks each: ✓S ✓R The other angle will be awarded only 1 mark: ✓S	✓S ✓R or ✓S ✓S ✓R or ✓S ✓S ✓R or ✓S ✓S ✓R or ✓S A maximum of 5 marks to be awarded; as explained in the text box. (5)
8.2	$\hat{O}_1 = 2 \times \hat{E}_2$ [$\angle$ at the centre = 2 × $\angle$ at the circumference] $= 52°$ **OR** $\hat{E}_2 = \hat{B}_2$ [$\angle$s opp. = radii] $\hat{O}_1 = \hat{B}_2 + \hat{E}_2$ [ext. $\angle$ of ΔOBE] $= 52°$	✓S/R ✓answer (2) **OR** ✓S/R ✓answer (2)
8.3	$\hat{A}_1 = 90°$ [$\angle$ in a semicircle] $B\hat{A}E = 90° + 26° = 116°$ $B\hat{D}E = 180° - 116° = 64°$ [opp. $\angle$s of a cyclic quad.] **OR** $E\hat{O}B = 128°$ [$\angle$s on a straight line] or [sum of $\angle$s of Δ] $B\hat{D}E = 64°$ [$\angle$ at the centre = 2 × $\angle$ at the circumference]	✓S/R ✓ S ✓R (3) **OR** ✓S/R ✓ S ✓R (3)
		[10]

QUESTION 9

9.1	Construction: Join SR and QT, and draw h_1 from S $\perp$ to PT and h_2 from T $\perp$ to PS.  Proof: $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2} PS \times h_1}{\frac{1}{2} SQ \times h_1} = \frac{PS}{SQ}$ $\frac{\text{area } \Delta PST}{\text{area } \Delta RTS} = \frac{\frac{1}{2} PT \times h_2}{\frac{1}{2} TR \times h_2} = \frac{PT}{TR}$ $\text{area } \Delta PST = \text{area } \Delta PST$ [common] $\text{area } \Delta QST = \text{area } \Delta RTS$ [same base; equal heights, because ST ‖ QR] $\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta RTS}$ $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$	✓ construction ✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2} PS \times h_1}{\frac{1}{2} SQ \times h_1}$ ✓ $\frac{PS}{SQ}$ ✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta RTS} = \frac{PT}{TR}$ ✓S ✓R (6)
9.2.1	$\frac{KL}{LM} = \frac{PN}{NM}$ [line ‖ to one side of Δ] $= \frac{12}{20} = \frac{3}{5}$	✓S/R ✓A answer (2)
9.2.2	$\frac{KL}{LM} = \frac{PQ}{MP}$ [line ‖ to one side of Δ] $\frac{3}{5} = \frac{PQ}{32}$ $\therefore PQ = \frac{32 \times 3}{5} = 19,2$ units	✓S ✓CA substitution ✓CA answer (3)

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9.2.3	$\Delta KQM \parallel\mid \Delta LPM$ [$\angle\angle\angle$] $\therefore \frac{KQ}{LP}=\frac{QM}{PM}$ [$\parallel\mid \Delta s$] $=\frac{51,2}{32}=\frac{8}{5}$	✓S ✓S ✓CA answer (3)
		[14]

QUESTION 10

10.1.1	$\hat{B}_3=90°$ [$\angle$ in a semicircle] $\therefore \hat{G}_1=90°$ [corresponding $\angle$ s; AO $\parallel$ BC] $\therefore BG=GE$ [line from centre $\perp$ to chord]	✓ S/R ✓S ✓R (3)
10.1.2	Let $\hat{B}_4=x$ $\hat{E}=\hat{B}_4=x$ [tan-chord theorem] Also: $\hat{A}=\hat{B}_4=x$ [corresponding $\angle$ s; AO $\parallel$ BC] $\hat{A}=\hat{E}$ $\therefore$ AEOB is a cyclic quadrilateral [converse: $\angle$ s in the same segment]	✓S ✓R ✓S ✓R (4)
10.1.3	In ΔOEG and ΔBAG: 1. $\hat{A}=\hat{E}$ [proved above] 2. $\hat{G}_1=\hat{G}_3$ [vertically opposite $\angle$ s] 3. $\hat{O}_2=A\hat{B}G$ [sum of $\angle$ s in Δ OR $\angle$ s in the same segment] $\therefore \Delta OEG \parallel\mid \Delta BAG$ [$\angle;\angle;\angle$]	✓S ✓S ✓R (3)
10.2	$OG=\frac{1}{2}BC=5$ units [midpoint theorem] $\frac{OG}{BG}=\frac{EG}{AG}$ [similar Δ s] $\therefore BG.EG=OG.AG$ $EG^2=OG.AG$ [$BG=GE$] $=5\times 35$ $=175$ $\therefore EG=\sqrt{175}=5\sqrt{7}=13,23$ units $EO^2=EG^2+GO^2$ [Pythagoras] $=\left(\sqrt{175}\right)^2+5^2$ $=200$ $EO=\sqrt{200}=10\sqrt{2}=14,14$ units	✓S/R ✓S ✓S ✓S ✓S/R ✓CA substitution ✓CA answer (7)
		[17]

TOTAL: 150