



Province of the  
**EASTERN CAPE**  
EDUCATION

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Provincie van die Oos-Kaap; Departement van Onderwys  
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## NATIONAL SENIOR CERTIFICATE

**GRADE 12**



**MARKS:** 150

**TIME:** 3 hours



\* M A T H E 2 \*

This question paper consists of 14 pages, including an information sheet and an answer book of 22 pages.

**QUESTION 1**

The ages (in years) of visitors that visited a bed and breakfast (B&B) on a specific day were recorded as follows.

15	17	20	20	20	21	22	24
29	29	30	$11t$	$11t + 2$	36	38	55

- 1.1 Identify the mode of the data. (1)
  - 1.2 It is further given that interquartile range of the ages of visitors is 14. Calculate the value of  $t$ . (3)
  - 1.3 Calculate the mean ages of the visitors if  $t = 3$ . (2)
  - 1.4 Calculate the standard deviation of the data. (1)
  - 1.5 How many visitors have ages that are less than one standard deviation of the mean? (3)
- [10]**

**QUESTION 2**

An athlete's finishing times and the daily temperatures were tracked during 10 years of Comrades Marathon to investigate the relationship between temperature and race completion time. The table below shows the data from these 10 years.

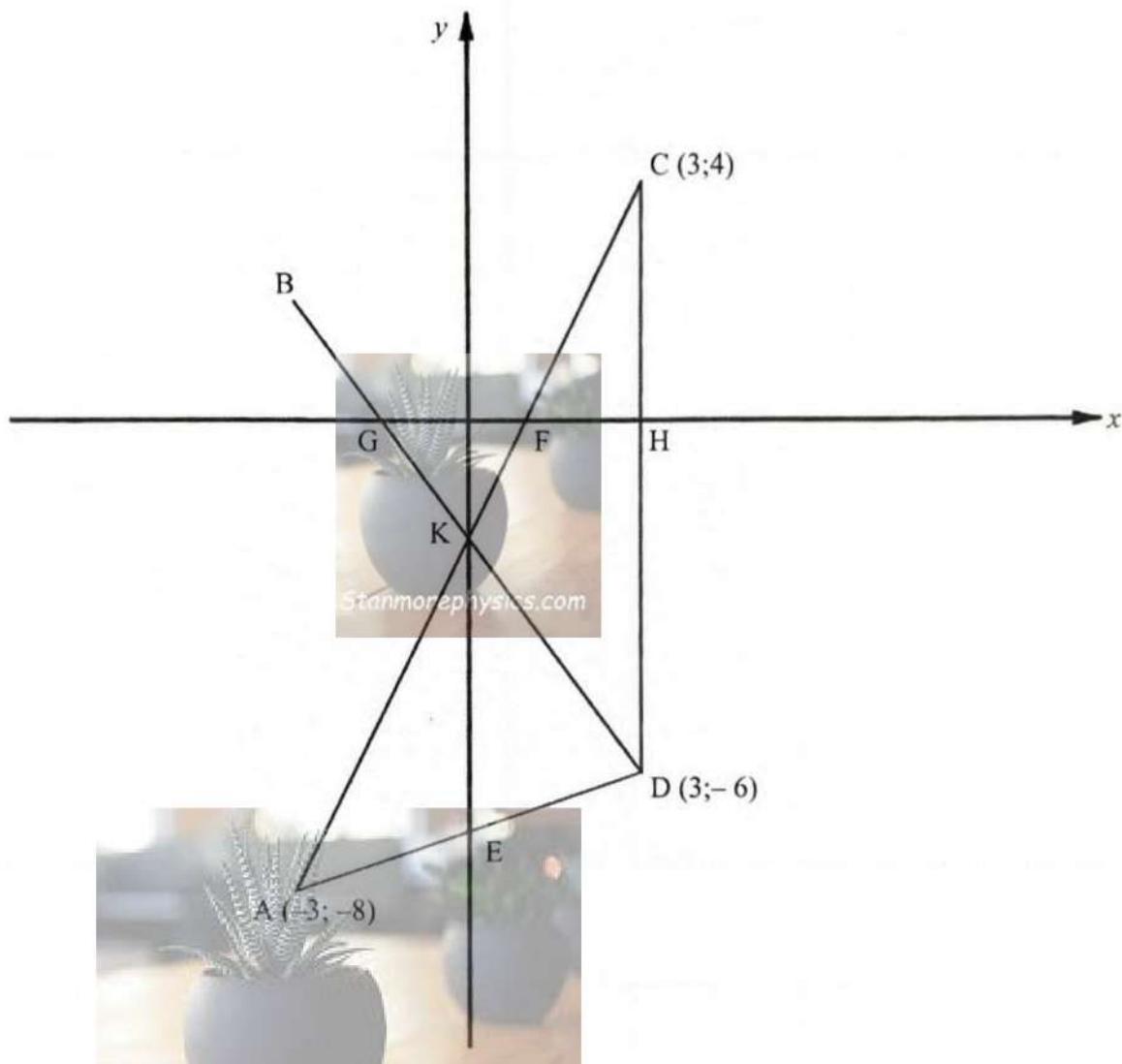
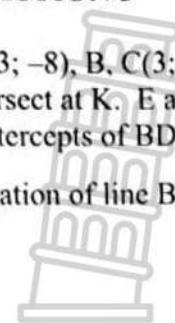
Year	1	2	3	4	5	6	7	8	9	10
Temperatures (degrees Celsius) $x$	41	9	30	15	25	20	20	35	39	16
Finishing times of athlete (in minutes) $y$	72	30	66	29	45	43	41	66	68	31

- 2.1 Calculate the correlation coefficient of the data. (1)
  - 2.2 Comment on the strength of the relationship between temperatures and finishing time. (1)
  - 2.3 Calculate the equation of the least squares regression line of the data. (3)
  - 2.4 Use the equation of the least squares regression line to predict the temperatures if an athlete takes 57 minutes to finish the race. Round the answer off to the nearest whole number. (2)
  - 2.5 Draw the least squares regression line on the grid provided in the SPECIAL ANSWER BOOK. (2)
- [9]**

**QUESTION 3**

A(-3; -8), B, C(3; 4) and D(3; -6) are vertices of a quadrilateral. Diagonals BD and AC intersect at K. E and K are y-intercepts of AD and BD respectively. G, F and H are x-intercepts of BD, CA and CD respectively.

Equation of line BD is given by  $y = -\frac{4}{3}x - 2$



- 3.1 Calculate the length of AC. Leave your answer in simplest surd form. (2)
- 3.2 Calculate the gradient of AC. (2)
- 3.3 Determine the equation of line AD in the form  $y = \dots$  (3)
- 3.4 Determine the size of  $\hat{C}KD$ . (5)
- 3.5 If it is further given that  $AC = 2KC$ , determine the area of  $\triangle CKD$ . (5)

3.6 Write down the equation of CD. (1)

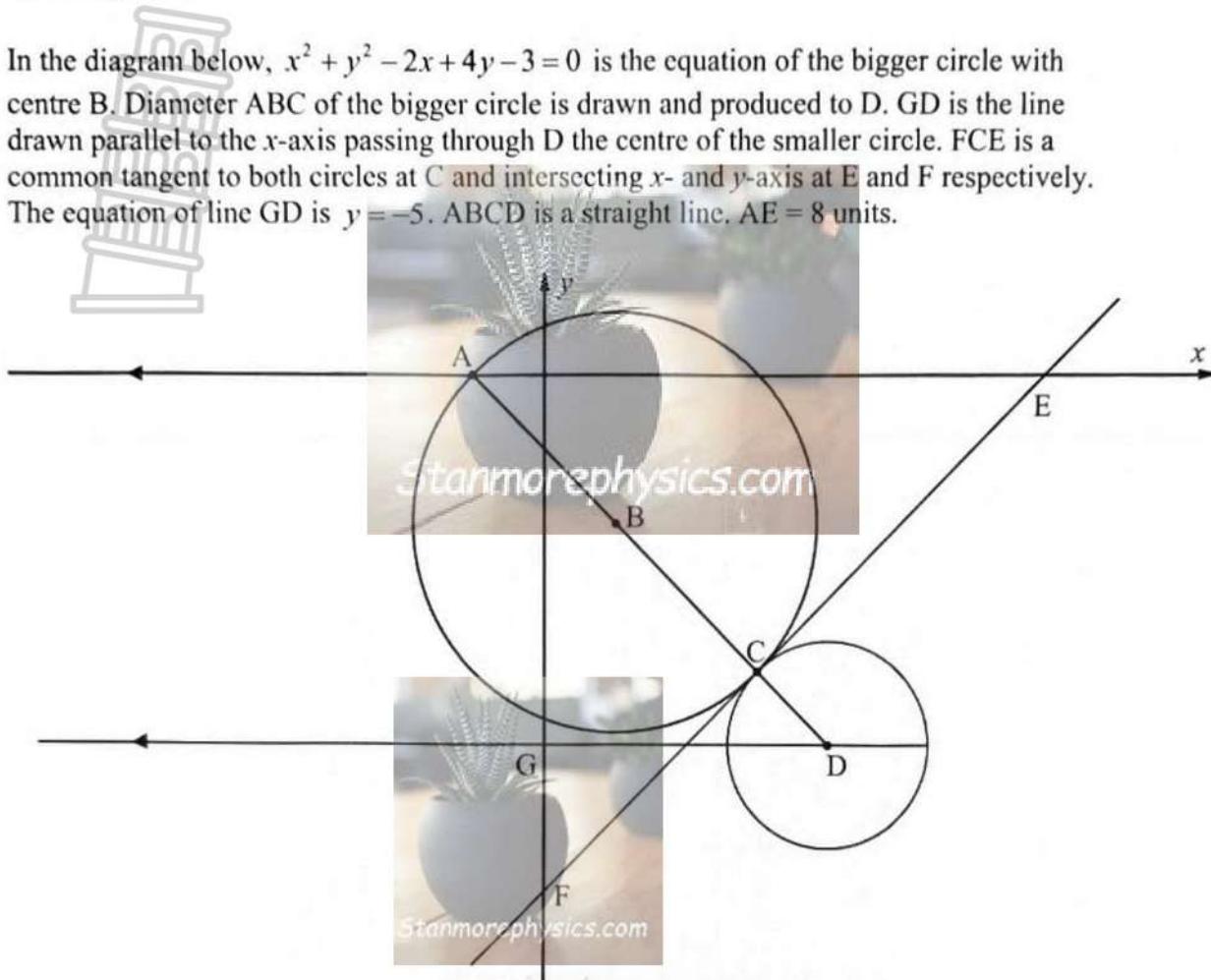
3.7 Determine the numerical value of  $\frac{\text{Area of } \Delta CKD}{\text{Area of EKCD}}$  (5)  
[23]



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**QUESTION 4**

In the diagram below,  $x^2 + y^2 - 2x + 4y - 3 = 0$  is the equation of the bigger circle with centre B. Diameter ABC of the bigger circle is drawn and produced to D. GD is the line drawn parallel to the x-axis passing through D the centre of the smaller circle. FCE is a common tangent to both circles at C and intersecting x- and y-axis at E and F respectively. The equation of line GD is  $y = -5$ . ABCD is a straight line. AE = 8 units.



- 4.1 Determine the coordinates of B, the centre of the larger circle. (3)
  - 4.2 Determine the coordinates of A, the x-intercept of the larger circle. (3)
  - 4.3 Calculate the gradient of AB. (2)
  - 4.4 Calculate the length of CE. (4)
  - 4.5 Determine the coordinates of D. (4)
  - 4.6 Determine equation of the circle with centre D in the form  $(x - a)^2 + (y - b)^2 = r^2$  (3)
- [19]

**QUESTION 5**

5.1 Given:  $\tan \theta = -\frac{5}{4}$  where  $90^\circ < \theta < 270^\circ$ . With the aid of a diagram and without using a calculator, determine the value of:

5.1.1  $\cos \theta$  (2)

5.1.2  $2\sin^2 \theta$  (2)

5.1.3  $\cos(90^\circ - 2\theta)$  (4)

5.2 Prove that:  $\frac{3\cos 2x + 3\cos^2 x + 9\sin^2 x}{4 - 4\sin^2 x} = \frac{3}{2\cos^2 x}$  (4)

5.3 Simplify without using a calculator:

$$\frac{\cos x \cdot \cos(90^\circ - x) \sin(48^\circ - x) + \sin^2 x \cos(48^\circ - x)}{\sin(-x) \cos 24^\circ \cdot \cos 66^\circ} \quad (7)$$

5.4 Given:  $[\cos(60^\circ - x) + \cos(60^\circ + x)]^2$

5.4.1 Simplify to a single trigonometric ratio of  $x$ :

$$[\cos(60^\circ - x) + \cos(60^\circ + x)]^2 \quad (3)$$

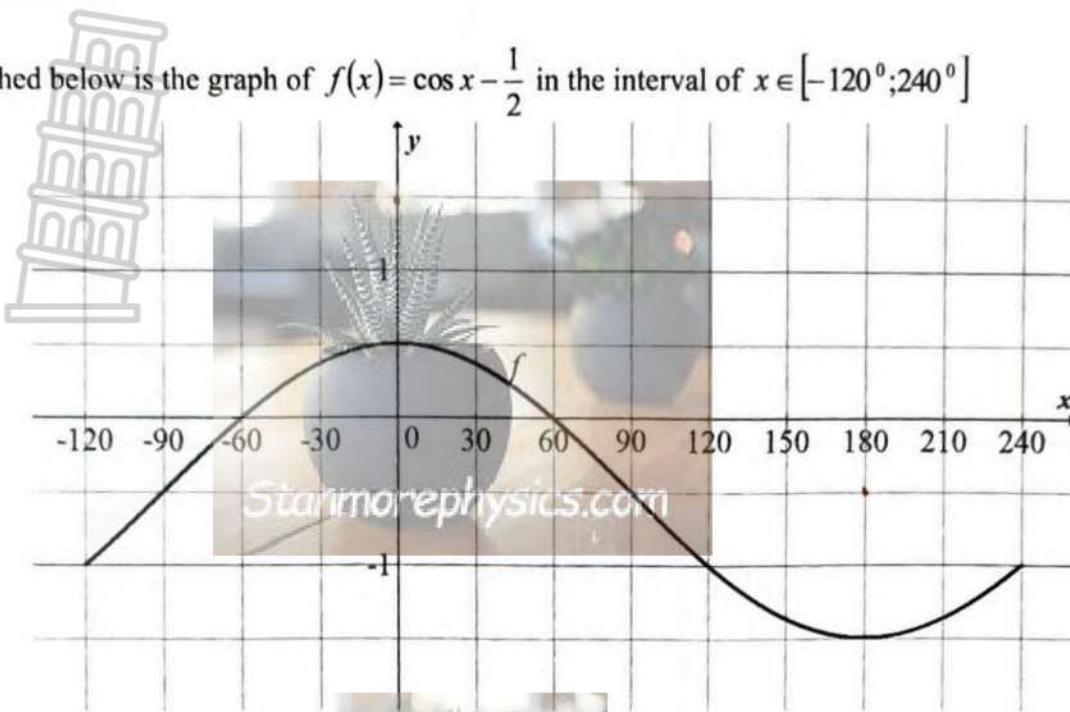
5.4.2 Hence, or otherwise, determine the general solution of:

$$[\cos(60^\circ - x) + \cos(60^\circ + x)]^2 = \frac{3}{4} \quad (4)$$

[26]

**QUESTION 6**

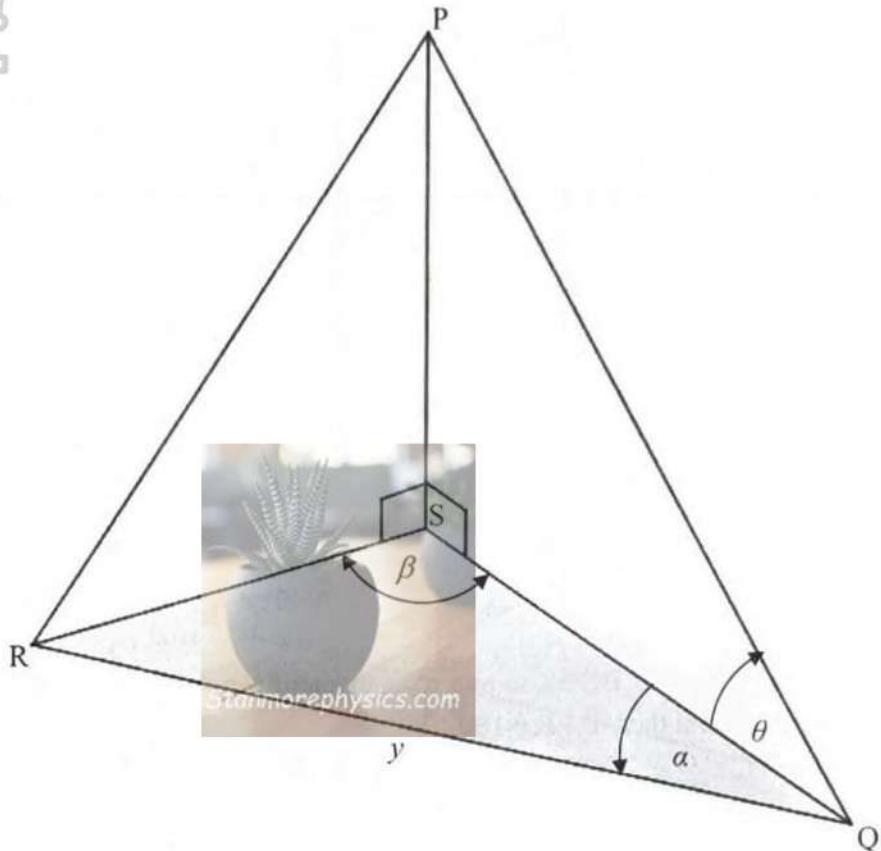
Sketched below is the graph of  $f(x) = \cos x - \frac{1}{2}$  in the interval of  $x \in [-120^\circ; 240^\circ]$



- 6.1 Determine the range of  $f(x) + 1$ . (2)
- 6.2 On the same set of axis, sketch the graph of  $g(x) = \sin(x + 30^\circ)$  in the interval of  $x \in [-120^\circ; 240^\circ]$  on the grid provided in the SPECIAL ANSWER BOOK. Clearly indicate intercepts with the axis. (3)
- 6.3 Write down the value(s) of  $x$  where  $g$  has a minimum value. (2)
- 6.4 For which values of  $x$  is  $f'(x) < 0$ . (2)
- 6.5 Write down the amplitude of  $f$ . (1)
- 6.6 Describe the transformation of  $f$  to  $h(x) = -\cos x$  (2)  
[12]

**QUESTION 7**

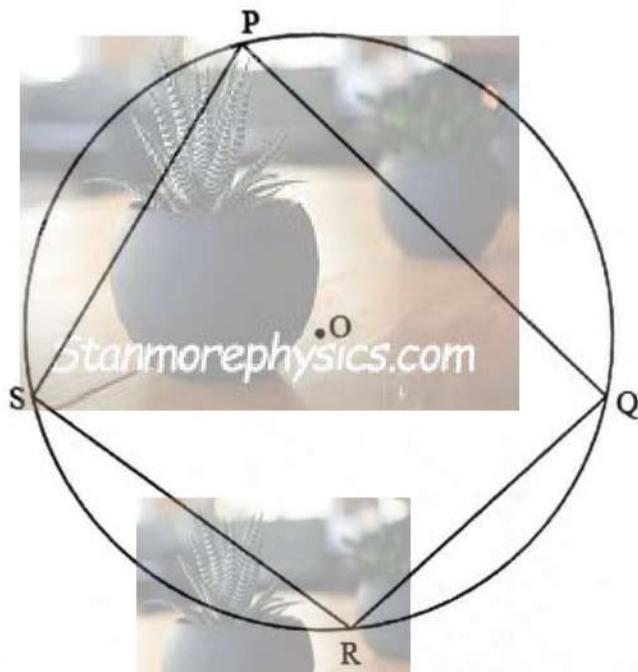
In the diagram below, PS is a vertical tower standing on a horizontal plane QRS.  $QR = y$ ,  $\hat{Q}_1 = \alpha$ ,  $\hat{Q}_2 = \theta$  and  $\hat{QSR} = \beta$ .



- 7.1 Write down the size of  $\hat{SRQ}$  in terms of  $\beta$  and  $\alpha$ . (1)
- 7.2 Show that  $PS = \frac{y \cdot \tan \theta \cdot \sin(\beta + \alpha)}{\sin \beta}$  (3)
- 7.3 Hence or otherwise, determine the length of PS if  $y=116\text{m}$ ,  $\theta=57^\circ$ ,  $\beta=102^\circ$  and  $\alpha=27^\circ$  (2)
- 7.4 Determine the angle of elevation from R to P. (4)  
[10]

**QUESTION 8**

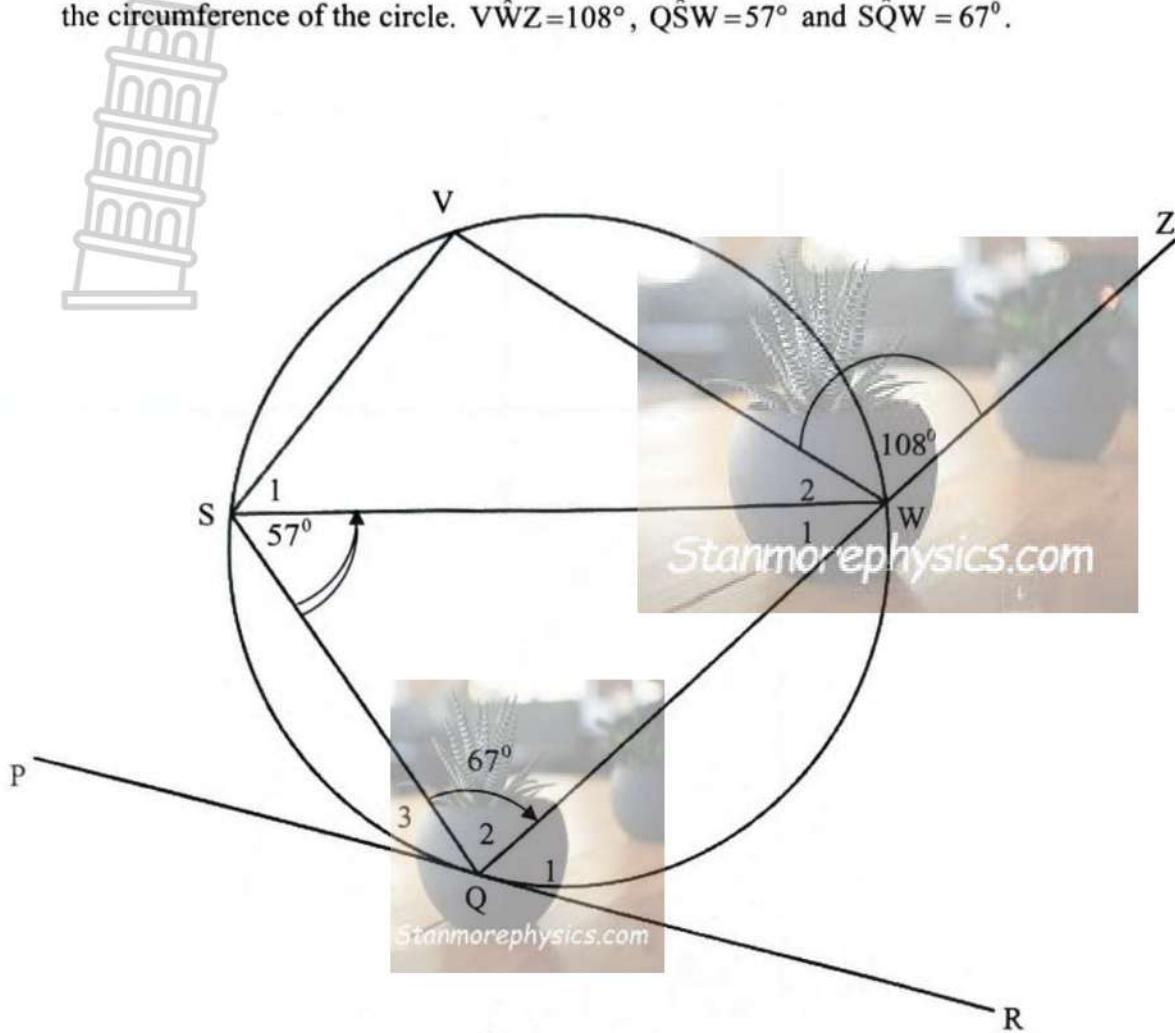
- 8.1 In the diagram below, O is the centre of the circle. The circle passes through the points P, Q, R and S.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if PQRS is a cyclic quadrilateral then  $P + R = 180^\circ$

(5)

- 8.2 PR is a tangent to the circle at Q. QWZ is a straight line. Q, S, V and W are points on the circumference of the circle.  $\hat{VWZ} = 108^\circ$ ,  $\hat{QSW} = 57^\circ$  and  $\hat{SQW} = 67^\circ$ .



8.2.1 Determine the size of  $\hat{V}$  (2)

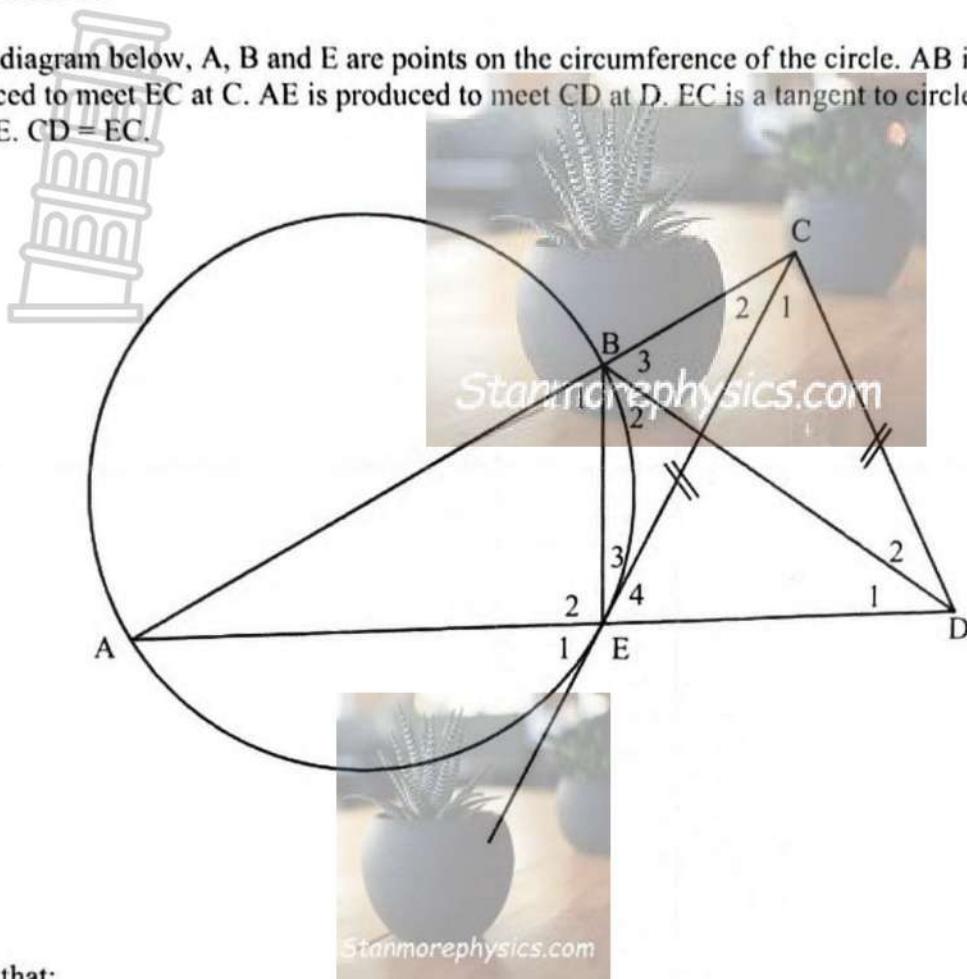
8.2.2 Determine the size of  $\hat{S}_1$  (2)

8.2.3 Calculate the size of  $\hat{WQR}$  (2)

[11]

**QUESTION 9**

In the diagram below, A, B and E are points on the circumference of the circle. AB is produced to meet EC at C. AE is produced to meet CD at D. EC is a tangent to circle at point E.  $CD = EC$ .



Prove that:

9.1  $BCDE$  is a cyclic quadrilateral (5)

9.2  $\Delta CEB \parallel \Delta CAE$  (3)

9.3  $CD = \frac{EB \cdot AC}{AE}$  (2)

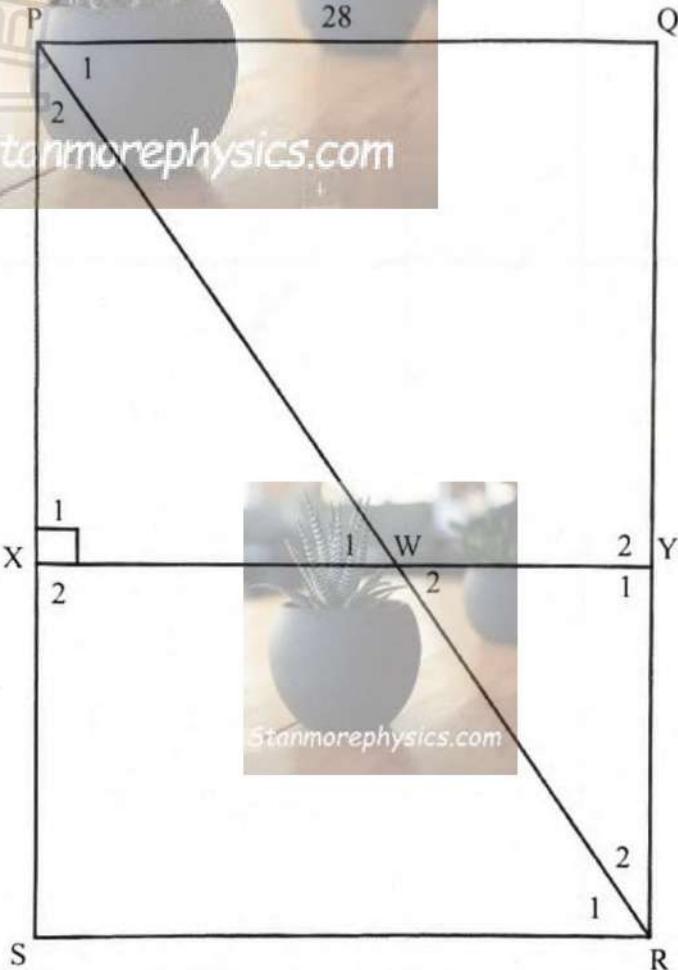
9.4 CD is a tangent to circle ABD (3)

9.5 
$$\frac{EB^2}{AE^2} = \frac{BC}{AC}$$
 (6)

[19]

**QUESTION 10**

In the diagram below, PQRS is a rectangle. PR intersect XY at W.  $XY \perp PS$ .  
 $QY:YR = 4:3$ ,  $PQ = 28$  units and  $PR = 42$  units.



- 10.1 Prove that  $XY \parallel PQ$ . (2)
- 10.2 Determine, with reasons, the length of WR. (4)
- 10.3 Determine the length of XW. (5)  
**[11]**

**TOTAL: 150**



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Porafensiye Ya Kapa Botjahabala: Letapha la Thuto

# NATIONAL SENIOR CERTIFICATE *NASIONALE SENIOR SERTIFIKAAT*

**GRADE/GRAAD 12**



**MATHEMATICS P2/WISKUNDE V2  
MARKING GUIDELINE/NASIENRIGLYN**

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**MARKS/PUNTE: 150**

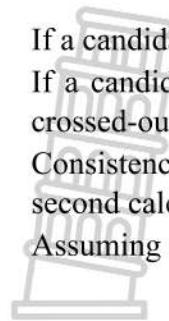
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This marking guideline consists of 20 pages.  
*Hierdie nasienriglyn bestaan uit 20 bladsye.*

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**NOTE**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone a question, mark the crossed-out version.
- Consistency accuracy applies in ALL aspects of the marking guideline. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.



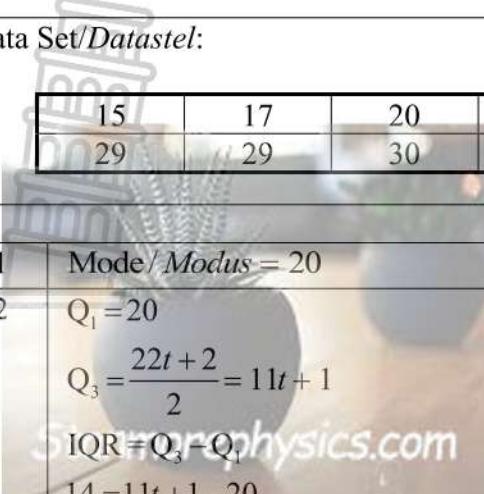
<b>GEOMETRY</b>	
<b>S</b>	A mark for a correct statement. (A statement mark is independent of a reason).
<b>R</b>	A mark for the correct reason. (A reason mark may only be awarded only if the statement is correct).
<b>S/R</b>	Award a mark if a statement and a reason are both correct.

**NOTA:**

- As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n poging van 'n vraag doodtrek en dit nie oordoen nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyn toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat nie.

<b>MEETKUNDE</b>	
<b>S</b>	'n Punt vir korrekte bewering. ('n Punt vir 'n bewering is onafhanklik van die rede.)
<b>R</b>	'n Punt vir 'n korrekte rede. ('n Punt word slegs vir die rede toegeken as die bewering korrek is.)
<b>S/R</b>	Ken 'n punt toe as die bewering en rede beide korrek is.

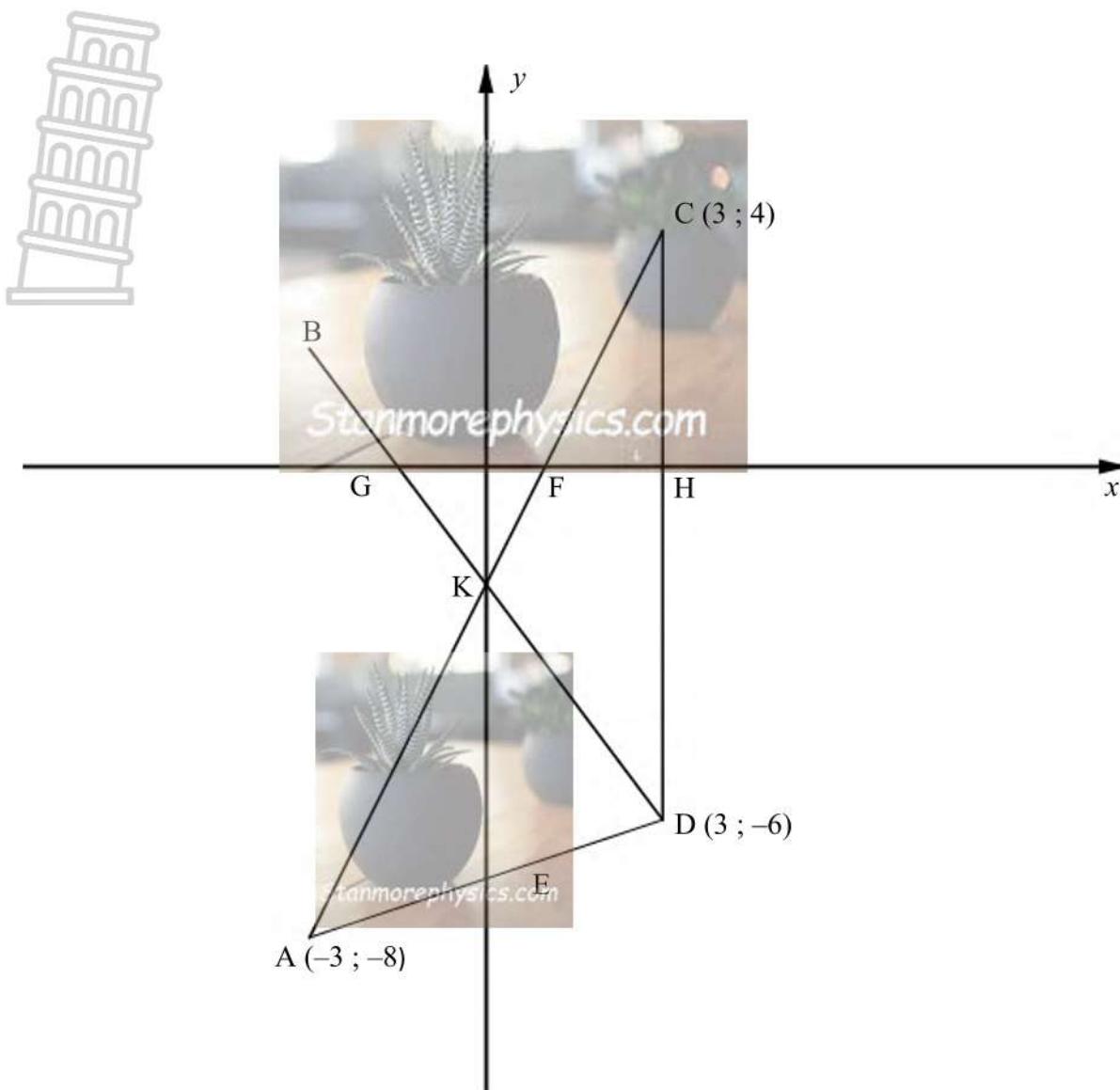
## QUESTION/VRAAG 1

Data Set/Datasset:							
							
15	17	20	20	20	21	22	24
29	29	30	$11t$	$11t + 2$	36	38	55
1.1	Mode / Modus = 20		✓ answer/antwoord		(1)		
1.2	$Q_1 = 20$ $Q_3 = \frac{22t + 2}{2} = 11t + 1$ $IQR = Q_3 - Q_1$ $14 = 11t + 1 - 20$ $\therefore t = 3$		✓ $Q_1$ ✓ $Q_3 = 11t + 1$ ✓ answer/antwoord				
1.3	$\bar{x} = \frac{444}{16}$ $\therefore = 27,75$ (Answer only full marks/Slegs antwoord – volpunte)		✓ 444 ✓ answer/antwoord		(2)		
1.4	Standard deviation / Standaardafwyking = 9,86		✓ answer/antwoord		(1)		
1.5	$\bar{x} - sd = 27,75 - 9,86$ $= 17,89$ 2 visitors 2 besoekers		✓ 27,75 – 9,86 correct substitution korrekte vervanging ✓ 17,89 ✓ answer/antwoord		(3)		[10]

## QUESTION/VRAAG 2

Years / Jaar		1	2	3	4	5	6	7	8	9	10															
Temperatures (degrees Celsius)	$x$	41	9	30	15	25	20	20	35	39	16															
Temperature (grade Celsius)	$x$																									
Finishing times of athletes (in minutes)	$y$	72	30	66	29	45	43	41	66	68	31															
Eindtyd van atleet (in minute)	$y$																									
2.1	$r = 0,96$					✓ answer/antwoord	(1)																			
2.2	Very strong positive correlation <i>Baie sterk positiewe korrelasie</i>					✓ very strong positive correlation <i>Baie sterk positiewe korrelasie</i>	(1)																			
2.3	$a = 10,92$ $b = 1,53$ $y = 10,92 + 1,53x$					✓ correct value of $a$ <i>korrekte waarde van a</i> ✓ correct value of $b$ <i>korrekte waarde van b</i> ✓ answer/antwoord	(3)																			
2.4	$57 = 10,92 + 1,53x$ $x = 30$					✓ substitution/vervanging ✓ answer/antwoord	(2)																			
2.5	<p style="text-align: center;"><b>Temperatures versus Finishing Time</b> <b>Temperature teenoor Eindtyd</b></p> <table border="1"> <caption>Data points estimated from the scatter plot</caption> <thead> <tr> <th>Temperature (in Degrees Celsius) / Temperatuur (in Grade Celsius)</th> <th>Finishing Time (in minutes) / Eindtyd (in minute)</th> </tr> </thead> <tbody> <tr><td>9</td><td>30</td></tr> <tr><td>15</td><td>30</td></tr> <tr><td>20</td><td>42</td></tr> <tr><td>25</td><td>45</td></tr> <tr><td>30</td><td>66</td></tr> <tr><td>35</td><td>68</td></tr> <tr><td>40</td><td>70</td></tr> <tr><td>41</td><td>72</td></tr> </tbody> </table>	Temperature (in Degrees Celsius) / Temperatuur (in Grade Celsius)	Finishing Time (in minutes) / Eindtyd (in minute)	9	30	15	30	20	42	25	45	30	66	35	68	40	70	41	72	<ul style="list-style-type: none"> <li>✓ Any two correct points <i>Enige twee korrekte punte</i></li> <li>✓ straight line joining points for <math>x \in [9; 41]</math> <i>Reguitlyn verbind punte vir <math>x \in [9; 41]</math></i></li> </ul>	(2)					
Temperature (in Degrees Celsius) / Temperatuur (in Grade Celsius)	Finishing Time (in minutes) / Eindtyd (in minute)																									
9	30																									
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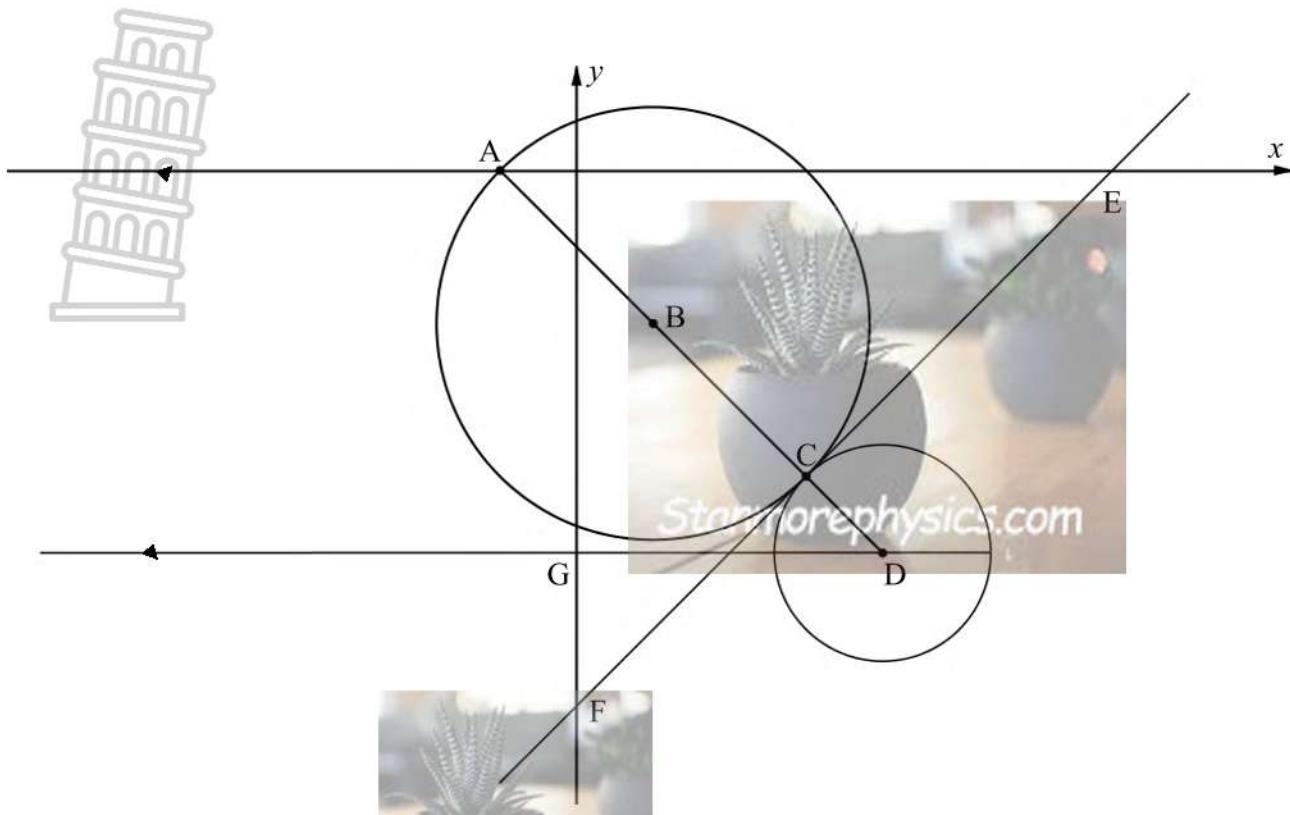
## QUESTION/VRAAG 3



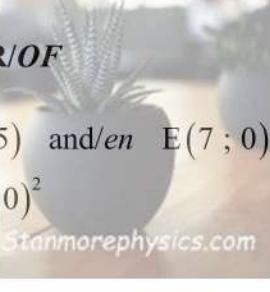
3.1	$AC = \sqrt{(-3-3)^2(-8-4)^2} = 6\sqrt{5}$	✓ correct substitution/korrekte vervanging ✓ correct answer/korrekte antwoord	(2)
3.2	$m_{AC} = \frac{4+8}{3+3} = 2$	✓ correct substitution/korrekte vervanging ✓ answer/antwoord	(2)
3.3	$m_{AD} = \frac{-6+8}{3+3} = \frac{1}{3}$ $y + 8 = \frac{1}{3}(x + 3)$ $y = \frac{1}{3}x - 7$	✓ $m_{AD}$ ✓ substitution of $m_{AD}$ and D(3 ; -6) or A(-3 ; -8) vervanging van $m_{AD}$ en D(3 ; -6) of A(-3 ; -8) ✓ Equation/Vergelyking	(3)

3.4	$\tan B\hat{G}F = m_{BD} = -\frac{4}{3}$ $B\hat{G}F = 126,87^\circ$ $K\hat{G}F = 53,13^\circ$ [∠s on a str. line / ∠e op 'n reguitlyn] $\tan C\hat{F}H = 2$ $C\hat{F}H = 63,44^\circ$ $G\hat{F}K = 63,44^\circ$ [vert. opp ∠s / regoorst. ∠e] $\therefore C\hat{K}D = 116,57^\circ$ [ext ∠ of Δ GKF / buite ∠ van Δ GKF] <b>Stanmorephysics.com OR/OF</b> $m_{AC} = 2$ $\tan C\hat{F}H = 2$ $C\hat{F}H = 63,44^\circ$ $G\hat{F}K = 63,44^\circ$ [vert. opp ∠s / regoorst. ∠e] $\tan B\hat{G}F = m_{BD} = -\frac{4}{3}$ $B\hat{G}F = 126,87^\circ$ $B\hat{K}C = 63,43^\circ$ [ext ∠ of Δ GKF / buite ∠ van Δ GKF] $C\hat{K}D = 116,57^\circ$ [∠s on a str. line / ∠e op 'n reguitlyn]	✓ $B\hat{G}F = 126,87^\circ$ ✓ $K\hat{G}F = 53,13^\circ$ ✓ $C\hat{F}H = 63,44^\circ$ ✓ $G\hat{F}K = 63,44^\circ$ ✓ answer/antwoord <b>OR/OF</b>	
3.5	$K(0 ; -2)$ $KD = \sqrt{(0-3)^2 + (-2+6)^2}$ $= 5$ $CK = 3\sqrt{5}$ Area of $\Delta CKD$ / Oppervlakte van $\Delta CKD$ $= \frac{1}{2} \times 2\sqrt{5} \times 5 \times \sin 116,57^\circ$ $= 15$	✓ $K(0 ; -2)$ ✓ $KD = 5$ ✓ $CK = 3\sqrt{5}$ ✓ correct substitution korrekte vervanging ✓ answer/antwoord	(5)
3.6	$x = 3$	✓ answer/antwoord	(1)
3.7	$E(0; -7)$ or $EK = 5$ $CD = 10$ height of / hoogte van trapezium = 3 $Area of / van EKCD = \frac{1}{2}(5+10) \times 3$ $= \frac{45}{2}$ $\frac{Area of / van \Delta CKD}{Area of / van EKCD} = 15 \times \frac{2}{45}$ $= \frac{2}{3}$	✓ $E(0; -7)$ or $EK = 5$ ✓ $CD = 10$ ✓ height / hoogte = 3 ✓ $\frac{45}{2}$ ✓ answer/antwoord	(5)
			[23]

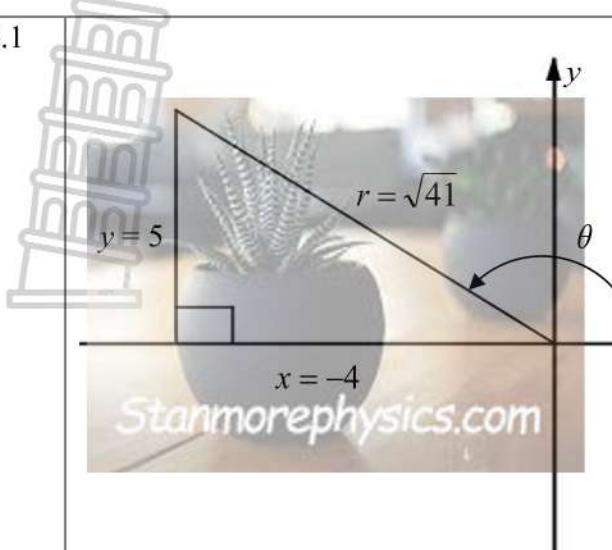
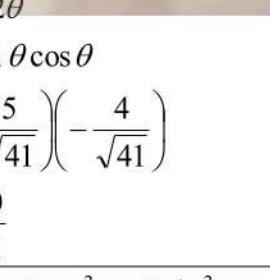
## QUESTION/VRAAG 4

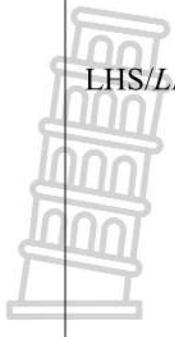
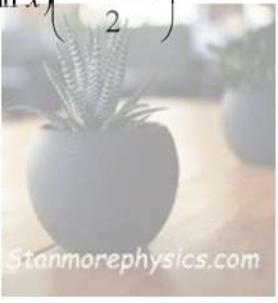


4.1	$x^2 + y^2 - 2x + 4y - 3 = 0$ $(x-1)^2 + (y+2)^2 = 8$ $\therefore B(1; -2)$	✓ completing square <i>voltooiing van vierkant</i> ✓ x-coordinate/x-koördinaat ✓ y-coordinate/y-koördinaat	(3)
4.2	$x^2 + 0^2 - 2x + 4.0 - 3 = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x \neq 3 \text{ or } x = 1$ $\therefore A(-1; 0)$	✓ $y = 0$ ✓ standard form/ <i>standaardvorm</i> ✓ correct x-coordinate <i>korrekte x-koördinaat</i>	(3)
4.3	$m_{AB} = \frac{-2-0}{1+1} = -1$	✓ substitution into correct formula <i>vervanging in korrekte formule</i> ✓ answer/ <i>antwoord</i>	(2)
4.4	$\hat{ACE} = 90^\circ$ [diameter $\perp$ tan / middellyn $\perp$ raaklyn] $AB = 2\sqrt{2}$ radius $\therefore AC = 4\sqrt{2}$ diameter / middellyn $CE^2 = 8^2 - (4\sqrt{2})^2$ $CE = 4\sqrt{2}$	✓ $\hat{ACE} = 90^\circ$ ✓ $AC = 4\sqrt{2}$ (diameter / middellyn) ✓ correct use of Pyth. Theorem <i>korrekte gebruik van Pyth. Stel.</i> ✓ $CE = 4\sqrt{2}$	(4)

4.5	<p>D(<math>x; -5</math>)      GD <math>\parallel</math> <math>x</math>-axis  <math>m_{AD} = m_{AB}</math>      collinear points / samelynige punte  <math display="block">\frac{-5-0}{x+1} = -1</math>  <math display="block">-x-1 = -5</math>  <math display="block">\therefore x = 4</math>  <math>D(4; -5)</math></p> <p style="text-align: center;"><b>OR/OF</b></p> <p>D(<math>x; -5</math>)      GD <math>\parallel</math> <math>x</math>-axis / <math>-as</math>  <math>m_{BD} = m_{AB}</math>      collinear points / saamlynige punte  <math display="block">\frac{-2+5}{1-x} = -1</math>  <math display="block">x-1 = 3</math>  <math display="block">\therefore x = 4</math>  <math>D(4; -5)</math></p> <p style="text-align: center;"><b>OR/OF</b></p> <p><math>\hat{A}CE = 90^\circ</math> ;   D(<math>x; -5</math>) and/en E(<math>7; 0</math>)  <math>DE^2 = (x-7)^2 + (-5-0)^2</math>  <math>= x^2 - 14x + 74</math>  <math>CE^2 = 32</math>  <math>CD^2 = (3-x)^2 + (-4+5)^2</math>  <math>= 10 - 6x + x^2</math>  <math>x^2 - 14x + 74 = -6x + x^2 + 10 + 32</math>  <math>-8x = -32</math>  <math>x = 4</math></p> <p style="text-align: center;"></p> <p>D(<math>4; -5</math>)</p>	<p>✓ D(<math>x; -5</math>)  ✓ <math>m_{AD} = m_{AB}</math>  ✓ <math>\frac{-5-0}{x+1} = -1</math>  ✓ <math>\therefore x = 4</math></p> <p style="text-align: center;"><b>OR/OF</b></p> <p>✓ D(<math>x; -5</math>)  ✓ <math>m_{BD} = m_{AB}</math>  ✓ <math>\frac{-2+5}{1-x} = -1</math>  ✓ <math>\therefore x = 4</math></p> <p style="text-align: center;"><b>OR/OF</b></p> <p>✓ standard form of <math>DE^2</math>  <i>standaardvorm van <math>DE^2</math></i></p> <p>✓ standard form of <math>CD^2</math>  <i>standaardvorm van <math>CD^2</math></i></p> <p>✓ simplification  <i>vereenvoudiging</i></p> <p>✓ <math>x = 4</math></p>	(4)
4.6	$CD^2 = (7-4)^2 - (4\sqrt{2})^2$ $= 2$ $(x-4)^2 + (y+5)^2 = 2$	<p>✓ <math>CD^2 = 2</math>  ✓ LHS of the equation  <i>LK van die vergelyking</i>  ✓ RHS of the equation  <i>RK van die vergelyking</i></p>	(3)
			<b>[19]</b>

## QUESTION/VRAAG 5

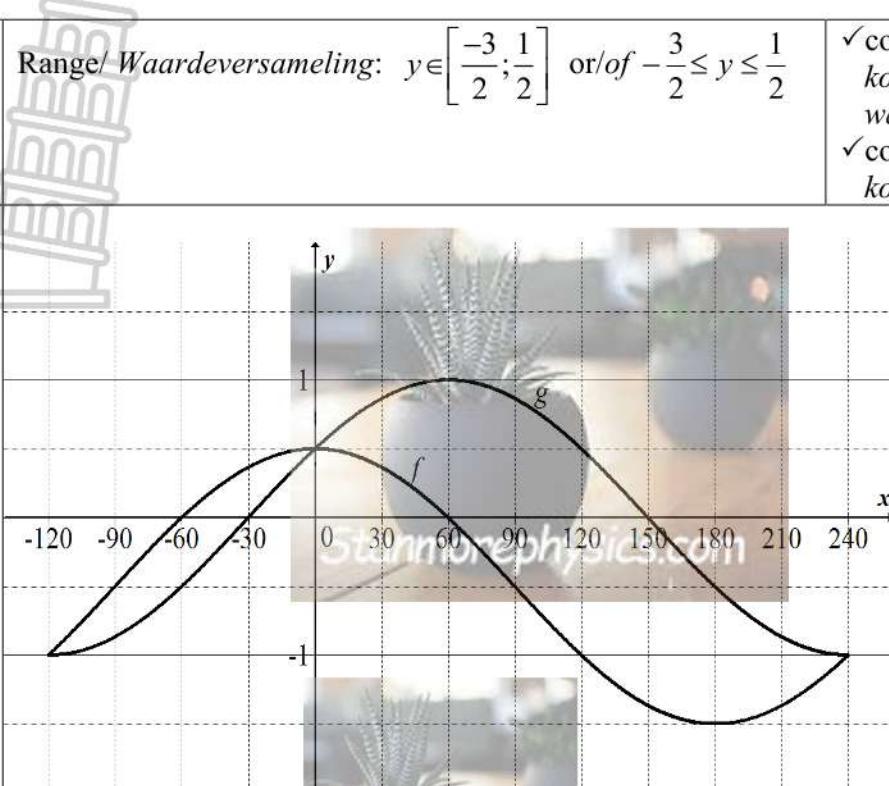
5.1			
5.1.1	$r = \sqrt{3^2 + (-4)^2} = 5$ $\cos \theta = -\frac{4}{\sqrt{41}}$ 	✓ value of $r$ / waarde van $r$  ✓ answer/antwoord	(2)
5.1.2	$2\sin^2 \theta = 2\left(\frac{5}{\sqrt{41}}\right)^2$ $= \frac{50}{41}$ 	✓ correct substitution korrekte vervanging ✓ correct answer korrekte antwoord	(2)
5.1.3	$\begin{aligned} \cos(90^\circ - 2\theta) &= \sin 2\theta \\ &= 2\sin \theta \cos \theta \\ &= 2\left(\frac{5}{\sqrt{41}}\right)\left(-\frac{4}{\sqrt{41}}\right) \\ &= -\frac{40}{41} \end{aligned}$ 	✓ reduction/reduksie ✓ identity/identiteit  ✓ correct substitution korrekte vervanging  ✓ answer/antwoord	(4)
5.2	$\begin{aligned} \text{LHS/LK} &= \frac{3\cos 2x + 3\cos^2 x + 9\sin^2 x}{4 - 4\sin^2 x} \\ &= \frac{3(2\cos^2 x - 1) + 3\cos^2 x + 9(1 - \cos^2 x)}{4(1 - \sin^2 x)} \\ &= \frac{6\cos^2 x - 3 + 3\cos^2 x + 9 - 9\cos^2 x}{4\cos^2 x} \\ &= \frac{6}{4\cos^2 x} \\ &= \frac{3}{2\cos^2 x} \end{aligned}$	✓ $2\cos^2 x - 1$ ✓ $1 - \cos^2 x$ ✓ $1 - \sin^2 x$  ✓ $\frac{6}{4\cos^2 x}$	(4)

	<b>OR/OF</b>	<b>OR/OF</b>
	 $\begin{aligned} \text{LHS/LK} &= \frac{3\cos 2x + 3\cos^2 x + 9\sin^2 x}{4 - 4\sin^2 x} \\ &= \frac{3(1 - 2\sin^2)x + 3(1 - \sin^2 x) + 9\sin^2 x}{4(1 - \sin^2 x)} \\ &= \frac{6}{4\cos^2 x} \\ &= \frac{3}{2\cos^2 x} \end{aligned}$	$\checkmark 1 - 2\sin^2 x$ $\checkmark 1 - \sin^2 x$ $\checkmark \cos^2 x$ $\checkmark \frac{6}{4\cos^2 x}$
5.3	$\begin{aligned} &\frac{\cos x \cdot \cos(90^\circ - x) \sin(48^\circ - x) + \sin^2 x \cos(48^\circ - x)}{\sin(-x) \cos 24^\circ \cdot \cos 66^\circ} \\ &= \frac{\cos x \cdot \sin x \sin(48^\circ - x) + \sin^2 x \cos(48^\circ - x)}{(-\sin x) \cos 24^\circ \sin 24^\circ} \\ &= \frac{\sin x [\sin(48^\circ - x) \cos x + \sin x \cos(48^\circ - x)]}{(-\sin x) \left(\frac{\sin 48^\circ}{2}\right)} \\ &= \frac{\sin(48^\circ - x + x)}{-\frac{1}{2} \sin 48^\circ} \\ &= \frac{\sin 48^\circ}{-\frac{1}{2} \sin 48^\circ} \\ &= -2 \end{aligned}$ 	$\checkmark \sin x$ $\checkmark -\sin x \checkmark \sin 24^\circ$ $\checkmark$ taking out common $\sin x$ <i>uitstaal van sin x as gemene faktor</i> $\checkmark \frac{1}{2} \sin 48^\circ$ $\checkmark$ compound angle <i>saamgestelde hoek</i>
		$\checkmark$ answer/ <i>antwoord</i>
	<b>OR/OF</b>	<b>OR/OF</b>
	$\begin{aligned} &\frac{\cos x \cdot \cos(90^\circ - x) \sin(48^\circ - x) + \sin^2 x \cos(48^\circ - x)}{\sin(-x) \cos 24^\circ \cdot \cos 66^\circ} \\ &= \frac{\cos x \cdot \sin x \sin(48^\circ - x) + \sin^2 x \cos(48^\circ - x)}{(-\sin x) \cos 24^\circ \sin 24^\circ} \\ &= \frac{\sin x [\sin(48^\circ - x) \cos x + \sin x \cos(48^\circ - x)]}{(-\sin x) \sin 24^\circ \cos 24^\circ} \\ &= \frac{\sin(48^\circ - x + x)}{-\frac{1}{2} \sin 48^\circ} \\ &= \frac{2 \sin 24^\circ \cos 24^\circ}{-\sin 24^\circ \cdot \cos 24^\circ} \\ &= -2 \end{aligned}$	$\checkmark \sin x$ $\checkmark -\sin x \checkmark \sin 24^\circ$ $\checkmark$ taking out common $\sin x$ <i>sin x as gemene faktor</i> $\checkmark$ compound angle <i>saamgestelde hoek</i> $\checkmark 2 \sin 24^\circ \cos 24^\circ$ $\checkmark$ answer/ <i>antwoord</i>
		(7)

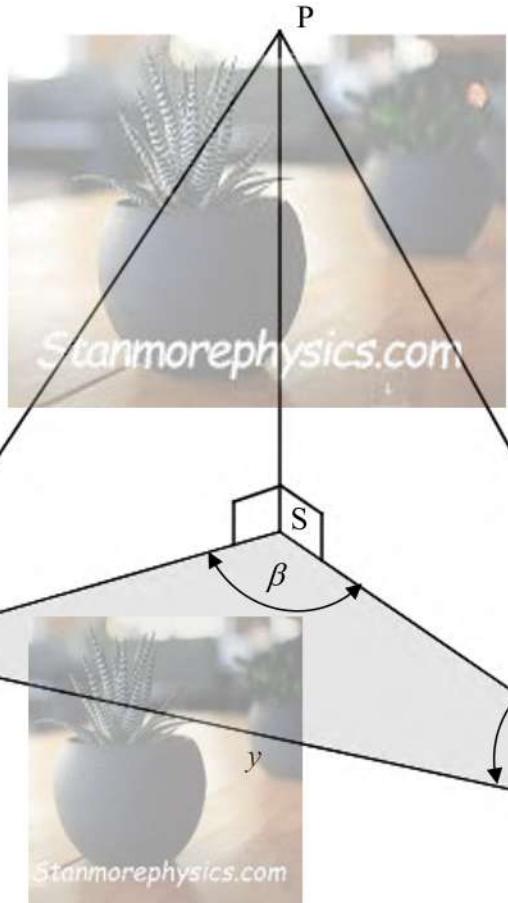
5.4.1	$\begin{aligned} & [\cos(60^\circ - x) + \cos(x + 60^\circ)]^2 \\ &= [\cos 60^\circ \cos x + \sin 60^\circ \sin x + \cos x \cos 60^\circ - \sin x \sin 60^\circ]^2 \\ &= [2 \cos 60^\circ \cos x]^2 \\ &= \left[ 2 \cdot \frac{1}{2} \cos x \right]^2 \\ &= \cos^2 x \end{aligned}$	✓ expansion/ uitbreiding  ✓ simplification/ vereenvoudiging  ✓ answer/antwoord	(3)
5.4.2	$\begin{aligned} \cos^2 x &= \frac{3}{4} \\ \cos x &= \pm \frac{\sqrt{3}}{2} \\ x &= \pm 30^\circ + 360^\circ k \quad \text{or / of} \quad x = \pm 150^\circ + 360^\circ k, \quad k \in \mathbb{Z} \end{aligned}$	✓ $\cos^2 x = \frac{3}{4}$  ✓ $\cos x = \pm \frac{\sqrt{3}}{2}$  ✓ $\pm 30^\circ + 360^\circ k, k \in \mathbb{Z}$  ✓ $\pm 150^\circ + 360^\circ k$	(4)
			[26]



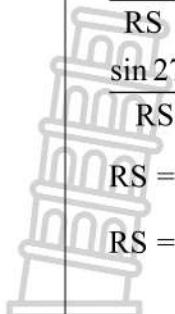
## QUESTION/VRAAG 6

6.1	Range/ Waardeversameling: $y \in \left[ \frac{-3}{2}; \frac{1}{2} \right]$ or/of $-\frac{3}{2} \leq y \leq \frac{1}{2}$	✓ correct critical values korrekte kritieke waarde ✓ correct notation korrekte notasie	(2)
6.2		✓ correct intercepts with the axis/korrekte afsnitte met die asse ✓ correct turning points/korrekte draaipunte ✓ shape/vorm	(3)
6.3	$x = -120^\circ$ and/en $x = 240^\circ$	✓✓ each x-value/ elke x-waarde	(2)
6.4	$0^\circ < x < 180^\circ$	✓ correct critical values/ korrekte kritieke waardes ✓ correct notation/ korrekte notasie	(2)
6.5	Amplitude / Amplitude = 1	✓ answer/antwoord	(1)
6.6	The graph of $f$ is translated/shifted $\frac{1}{2}$ units up and reflected about the $x$ -axis.  <i>Die grafiek van <math>f</math> skuif <math>\frac{1}{2}</math> eenheid op en word gereflekteer om die <math>x</math>-as.</i>	✓ $\frac{1}{2}$ units up $\frac{1}{2}$ eenheid op ✓ reflected about the $x$ -axis gereflekteer om die $x$ -as	(2)
			[12]

## QUESTION/VRAAG 7

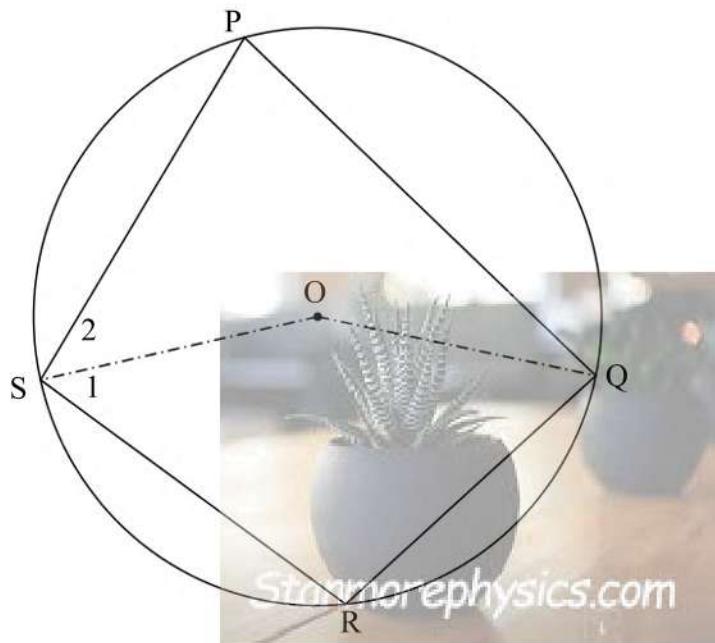


7.1	$\hat{SRQ} = 180^\circ - (\alpha + \beta)$	✓ answer/antwoord	(1)
7.2	$\frac{SQ}{\sin(180^\circ - (\alpha + \beta))} = \frac{y}{\sin \beta}$ $SQ = \frac{y \sin(180^\circ - (\alpha + \beta))}{\sin \beta}$ $SQ = \frac{y \sin(\alpha + \beta)}{\sin \beta}$ $\tan \theta = \frac{PS}{SQ}$ $PS = \tan \theta \cdot SQ$ $PS = \frac{y \cdot \tan \theta \cdot \sin(\alpha + \beta)}{\sin \beta}$	✓ use of sine rule <i>gebruik van sinusreël</i>  ✓ $\sin(\alpha + \beta)$ reduction/ <i>vermindering</i>  ✓ correct ratio of $\tan \theta$ <i>korrekte verhouding van </i> $\tan \theta$	(3)
7.3	$PS = \frac{(116) \cdot \tan 57^\circ \cdot \sin(27^\circ + 102^\circ)}{\sin 102^\circ}$ $PS = 141,92 \text{ units/eenhede}$	✓ substitution/ <i>vervanging</i>  ✓ answer/antwoord	(2)

 <p>7.4</p> $\frac{\sin \alpha}{RS} = \frac{\sin \beta}{y}$ $\frac{\sin 27^\circ}{RS} = \frac{\sin 102^\circ}{116}$ $RS = \frac{116 \sin 27^\circ}{\sin 102^\circ}$ $RS = 53,84 \text{ units}$ $\tan P\hat{R}S = \frac{PS}{RS}$ $\tan P\hat{R}S = \frac{141,92}{53,84}$ $P\hat{R}S = \tan^{-1}\left(\frac{141,92}{53,84}\right)$ $\therefore PRS = 69,22^\circ$	<ul style="list-style-type: none"> <li>✓ substitution into sine rule <i>vervanging in die sinusreeël</i></li> <li>✓ length of RS/<i>lengte van RS</i></li> <li>✓ correct substitution ratio of <math>\tan P\hat{R}S</math> <i>korrekte vervanging verhouding van <math>\tan P\hat{R}S</math></i></li> <li>✓ answer/<i>antwoord</i></li> </ul>	(4) <b>[10]</b>
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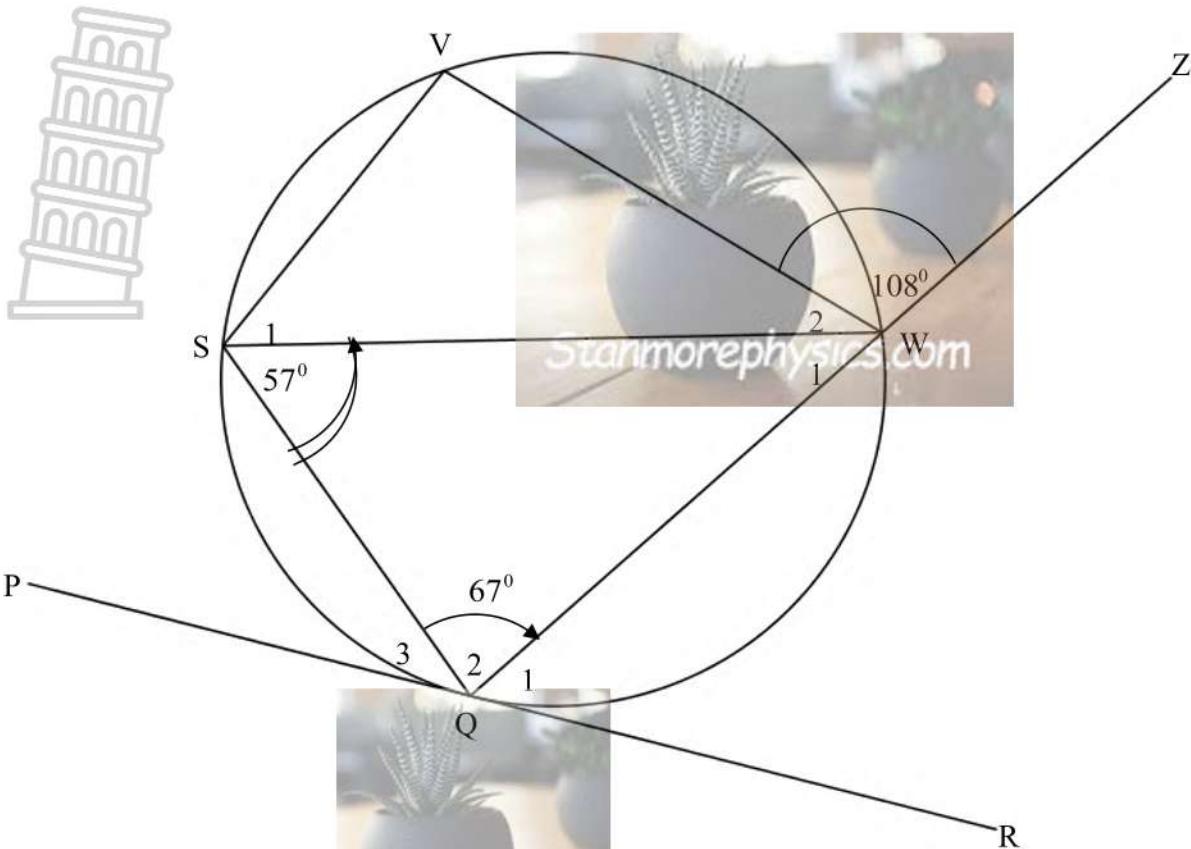


## QUESTION/VRAAG 8



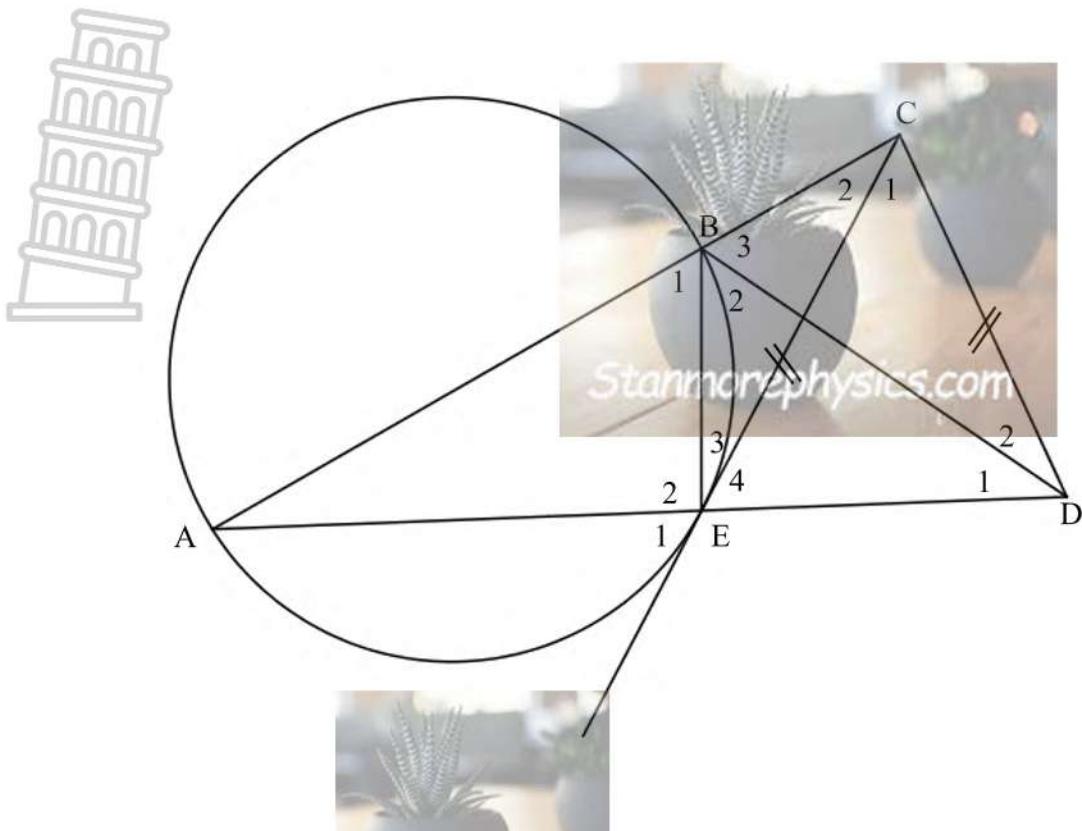
8.1	<p>Constructions: Draw radii OS and OQ</p> <p><i>Proof :</i></p> $\hat{O}_1 = 2\hat{P} \quad [\angle \text{ at centre} = 2\angle \text{ at circumf}]$ $\hat{O}_2 = 2\hat{R} \quad [\angle \text{ at centre} = 2\angle \text{ at circumf}]$ $\hat{O}_1 + \hat{O}_2 = 360^0 \quad [\angle \text{s around a point}]$ $2\hat{P} + 2\hat{R} = 360^0 \quad [\text{Substitution}]$ $\hat{P} + \hat{R} = 180^0$	✓ Construction ✓ S/R ✓ S ✓ S/R ✓ S	(5)
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8.1	<p>Konstruksies: Teken radiusse OS en OQ</p> <p><i>Bewys :</i></p> $\hat{O}_1 = 2\hat{P} \quad [\text{Middelpunts } \angle = 2 \times \text{Omtreks } \angle]$ $\hat{O}_2 = 2\hat{R} \quad [\text{Middelpunts } \angle = 2 \times \text{Omtreks } \angle]$ $\hat{O}_1 + \hat{O}_2 = 360^0 \quad [\angle \text{'e om'n punt}]$ $2\hat{P} + 2\hat{R} = 360^0 \quad [\text{Vervanging}]$ $\hat{P} + \hat{R} = 180^0$	✓ Konstruksie ✓ S/R ✓ S ✓ S/R ✓ S	(5)
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8.2.1	$\hat{V} = 113^\circ$ [opp $\angle$ s of a cyclic quad] [teenoorst. $\angle$ van koordev.]	✓ S   ✓ R	(2)
8.2.2	$\hat{S}_1 = 51^\circ$ [ext $\angle$ of a cyclic quad] [buite $\angle$ van koordev.]	✓ S   ✓ R	(2)
8.2.3	$\hat{WQR} = 57^\circ$ [tan – chord theorem] [raaklyn – koord stelling]	✓ S   ✓ R	(2)
			[11]

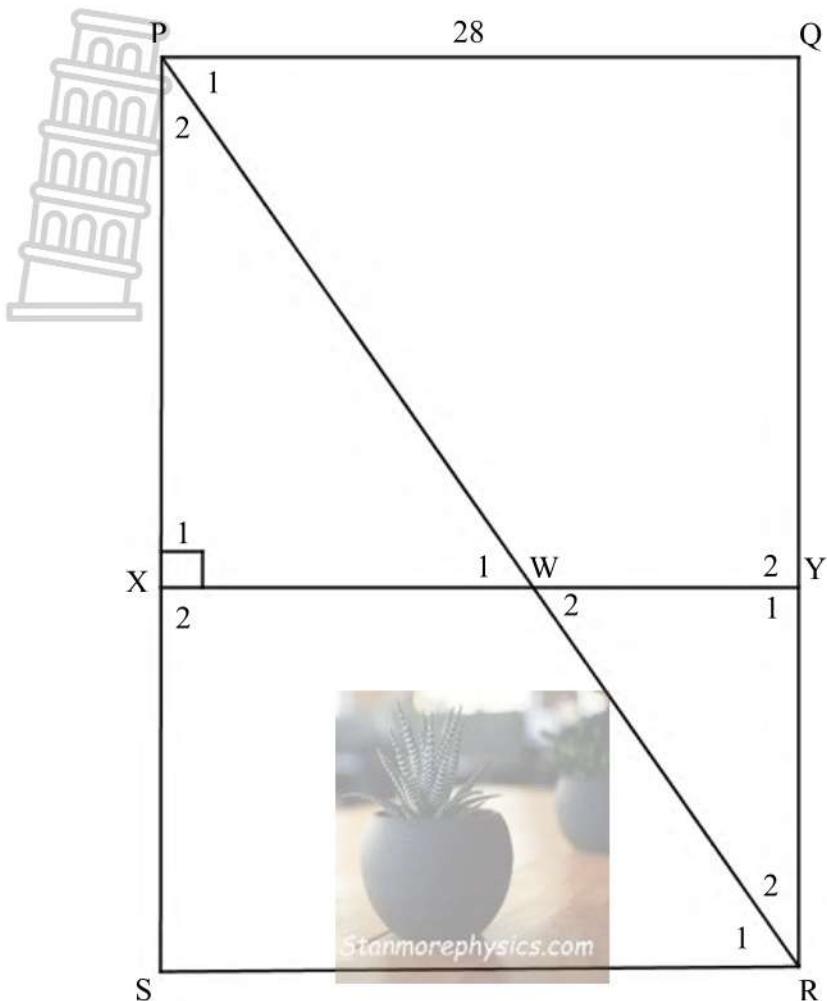
## QUESTION/VRAAG 9



9.1	$\hat{D} = \hat{E}_4$ [∠s opp = sides] / [∠e teenoor = sye] $E_4 = \hat{E}_1$ [vert. opp ∠s] / [regoorst. ∠e] $\hat{E}_1 = \hat{B}_1$ [tan chord theo.] / [raaklyn-koord stelling] $\therefore \hat{D} = \hat{B}$ $\therefore$ BCDE is a cyclic quad [converse ext ∠ of a cyclic quad] $BCDE$ is 'n koordevierhoek [omgekeerde buite ∠ van kv]	✓ S/R ✓ S ✓ S ✓ R ✓ R	(5)
9.2	$\Delta CEB$ and/en $\Delta CAE$ $\hat{E}_3 = \hat{A}$ [tan chord theo]/[raaklyn-koord stelling] $\hat{C}_2 = \hat{C}_2$ [common] / [gemeen] $E\hat{B}\hat{C} = A\hat{E}\hat{C}$ [3rd ∠s] / [3 <sup>de</sup> ∠] $\therefore \Delta CEB \parallel \Delta CAE$ [∠∠∠]	✓ S/R ✓ S ✓ S OR/OF ✓ R	(3)
9.3	$\frac{CE}{AC} = \frac{EB}{AE}$    Δs/e $CE = CD$ [given] / [gegee] $CD = \frac{EB \cdot AC}{AE}$	✓ S ✓ R	(2)

9.4	$\hat{D}_2 = \hat{E}_3$ [∠s same seg] / [∠e in dies. segment] $\hat{E}_3 = \hat{A}$ [proved 9.2] / [bewys in 9.2] $\therefore \hat{A} = \hat{D}_2$ $\therefore CD$ is a tangent to circle ABD [converse tan chord theo] $CD$ is 'n raaklyn aan die sirkel ABD [omgekeerde raaklyn koord stelling]	✓ S      ✓ R ✓ R	(3)
9.5	$\Delta ABCD$ and/en $\Delta ABE$ $\hat{E}_2 = \hat{C}$ [ext ∠ of a cyclic quad]/[buite ∠ van kv] $\hat{A} = \hat{D}_2$ [tan chord theo] / [raaklyn - koord stelling] $\hat{B}_1 = \hat{B}_3$ [3rd ∠] / [3 <sup>de</sup> ∠] $\therefore \Delta EAB \parallel \Delta CDB$ [∠∠∠] $\frac{AE}{CD} = \frac{EB}{CB}$ $CD = \frac{AE \times CB}{EB}$ $\therefore \frac{AE \times CB}{EB} = \frac{EB \times AC}{AE}$ [both/beide = CD] $\frac{EB^2}{AE^2} = \frac{BC}{AC}$ <p style="text-align: center;"><b>OR/OF</b> Stanmorephysics.com</p>	✓ S      ✓ R ✓ S ✓ R ✓ correct ratio korrekte verhouding ✓ equating CD gelyk stel CD	
	$\Delta BCD$ and/en $\Delta ABE$ $\hat{E}_2 = \hat{C}$ [ext ∠ of a cyclic quad]/[buite ∠ van kv] $\hat{A} = \hat{D}_2$ [proved 9.4] / [bewys in 9.4] $\hat{B}_1 = \hat{B}_3$ [3rd ∠] / [3 <sup>de</sup> ∠] $\therefore \Delta EAB \parallel \Delta CDB$ [∠∠∠] $\frac{AE}{CD} = \frac{EB}{CB}$ $CD \times EB = AE \times CB$ $\left( \frac{EB \times AC}{AE} \right) \times EB = AE \times CB$ from/vanaf 9.3 $\frac{EB^2}{AE^2} = \frac{BC}{AC}$ <p style="text-align: center;"><b>OR/OF</b> Stanmorephysics.com</p>	✓ S      ✓ R ✓ S ✓ R ✓ correct ratio korrekte verhouding ✓ substitute CD vervang CD	(6) <b>[19]</b>

## QUESTION/VRAAG 10



10.1	$\hat{P} = 90^\circ$ $\therefore XY \parallel PQ$	$[\angle s \text{ of a rect} = 90^\circ]/[\angle e \text{ van reghoek} = 90^\circ]$ $[\text{co-int. } \angle s \text{ supp}]/[\text{ko-binne } \angle e \text{ is suppl}]$	<input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R	(2)
10.2	$\frac{WR}{PR} = \frac{YR}{RQ}$ $\frac{WR}{42} = \frac{3x}{7x}$ $\therefore WR = 18$	[prop theo, $XY \parallel PS$ / line $\parallel$ to one side of a $\Delta$ ] [eweredigh stelling, $XY \parallel PS$ / lyn $\parallel$ aan een sy van $\Delta$ ]	<input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R  <input checked="" type="checkbox"/> correct substitution <i>korrekte vervanging</i> ✓ answer/antwoord	
	<b>OR/OF</b>  $\frac{PR}{PW} = \frac{QR}{QY}$ $\frac{42}{PW} = \frac{7x}{4x}$ $\therefore PW = 24$ $\therefore WR = 18$	[prop theo, $XY \parallel PS$ / line $\parallel$ to one side of a $\Delta$ ] [eweredigh stelling, $XY \parallel PS$ / lyn $\parallel$ aan een sy van $\Delta$ ]	<b>OR/OF</b> <input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R  <input checked="" type="checkbox"/> correct substitution <i>korrekte vervanging</i> ✓ answer/antwoord	(4)

 <p>10.3</p>	$\hat{P}_2 = \hat{P}_2$	[common] / [gemeen]		
	$\hat{X}_1 = \hat{S} = 90^\circ$	[corresp, $\angle s$ , XY    SR] [ooreenk. $\angle e$ , XY    SR]		
	$\hat{W}_1 = \hat{R}_1$	[3rd $\angle s$ ] / [3 <sup>de</sup> $\angle$ ]		
	$\Delta PXW \parallel\!   \Delta PSR$	[ $\angle\angle\angle$ ]	✓ S for identifying    Δs <i>identifisering van    Δs</i>	
	$\frac{XW}{SR} = \frac{PW}{PR}$	[    Δs]	✓ S	
	$SR = 28$	[opp sides of a rect.] [teenoorst. sye van 'n reghoek]	✓ S/R	
	$\frac{XW}{28} = \frac{24}{42}$	$XW = 16$	✓ substitution into correct ratios <i>vervanging in korrekte verhoudings</i> ✓ answer / antwoord	(5)
				[11]
			<b>TOTAL/TOTAAL: 150</b>	

