PREPARATORY EXAMINATION

2025

MATHEMATICS
PAPER 1 (10611)

MATHEMATICS: Paper 1

10611E

X05

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION 2025

NAME OF SCHOOL	

CANDIDATE'S NAME	

DATE	D D M M Y Y Y Y		BOOK NUMBER	OF	BOOK(S)

TEACHER			PAPER NUMBER	1

SUBJECT NAME	**MATHEMATICS (10611)**

ANSWER ALL THE QUESTIONS IN THE QUESTION PAPER.

MARKER			MODERATOR'S INITIALS IN RELEVANT BLOCK						RE-MARK/RE-CHECK		
Question	Marks	Marker's Code & Initials	Marks						Question	Marks	Initials
1									1		
2									2		
3									3		
4									4		
5									5		
6									6		
7									7		
8									8		
9									9		
10									10		
11									11		
		TOTAL							TOTAL		

TIME: 3 hours

MARKS: 150

34 pages + 1 information sheet

P.T.O.

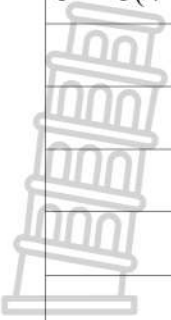## INSTRUCTIONS AND INFORMATION

1.   This question paper consists of 11 questions. Answer ALL questions in the spaces provided.

2.   Show ALL calculations clearly.

3.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4.   Round-off ALL final answers appropriately according to the given context, unless stated otherwise.

5.   Indicate units of measurement, where applicable.

6.   Diagrams are NOT necessarily drawn to scale, unless stated otherwise.
      Show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

7.   No pages may be torn from this question paper.

8.   Candidates may not retain a question paper or remove it from the examination room. Question papers must be returned to the invigilator at the end of the examination session.

9.   Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.

10.   Indicate the questions you have answered by drawing a circle around the relevant numbers on the front cover of the question paper where marks are to be recorded.

11.   Draw a neat line through any work/rough work that must NOT be marked.

12.   In the event that you use the additional space provided:

      12.1   Write down the number of the question.

      12.2   Leave a line and rule off after your answer.

13.   Write neatly and legibly.

P.T.O.

QUESTION 1

1.1	Given: $f(x) = (x^2 - 3)(3x - 1)(x + 2)$	
	Solve $f(x) = 0$ if:	
1.1.1	x is an integer	
		(2)
1.1.2	x is a rational number	
		(1)
1.1.3	x is a real number	
		(1)
1.2	Solve for x:	
1.2.1	$-15x^2 - 9x + 4 = 0$ (Correct to TWO decimal places)	
		(3)
1.2.2	$(3x - 2)^2 \geq 3x$	
		(4)

1.2.3	$5^x = 5(4 + 5^{2-x})$
	(4)

1.3	Solve for x and y: $\log_x 16 = 4$ and $y + \sqrt{x+7} = x+1$; $x \geq 0$
	(6)

1.4	Calculate TWO numerical values for p so that $x^2 + p(2x+7) + 8$ is a perfect square.
	(4)
	[25]

P.T.O.

QUESTION 2

The following sequence of numbers forms a quadratic number pattern:

$-3; \ -2; \ -3; \ -6; \ -11; \ldots$

2.1	The first differences of the given sequence also form a sequence. Determine an expression for the nth term of the first differences.	
		(3)
2.2	Calculate the first difference between the 35th and 36th terms of the quadratic sequence.	
		(1)

P.T.O.

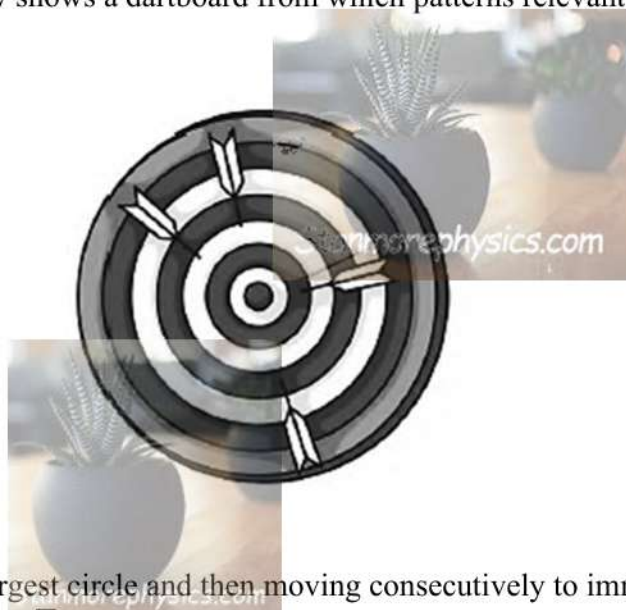| 2.3	Determine an expression for the nth term of the quadratic sequence.	
		(4)
2.4	Explain why the sequence will never contain a POSITIVE term.	
		(2)
		[10]

QUESTION 3

3.1	The graphic below shows a dartboard from which patterns relevant to the different circles can be derived.

The radii of the largest circle and then moving consecutively to immediate inward circles is given by: 18 cm; $6\sqrt{3}$ cm; 6 cm ...

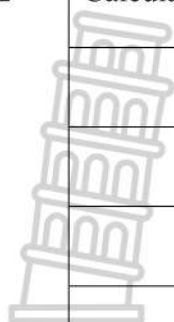The innermost circle has a radius of $\frac{2}{3}$ cm.

3.1.1	Show that the areas of the circles form a converging geometric sequence.

(3)

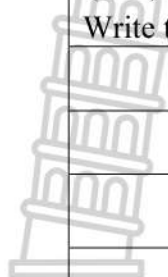| 3.1.2	Calculate the number of circles in the sequence forming the dartboard.	
		(4)

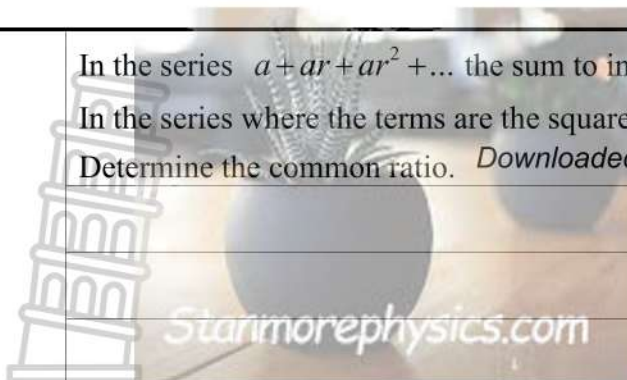| 3.2	Given the series:
	$(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \ldots + (81 \times 82)$
	Write the series in sigma notation. (It is not necessary to calculate the value of the series.)

(4)

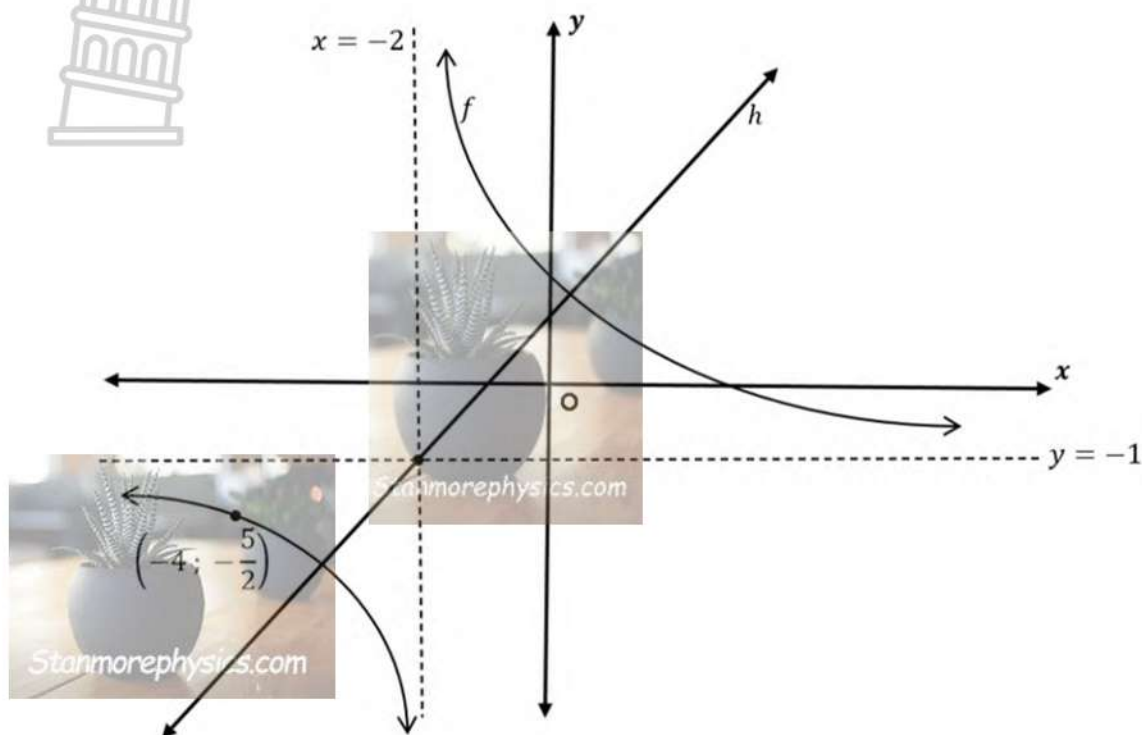| 3.3 | In the series $a + ar + ar^2 + ...$ the sum to infinity is 1. |
| | In the series where the terms are the squares of the above series, the sum to infinity is $\frac{5}{6}$. Determine the common ratio. |

(5)

[16]

QUESTION 4

The graphs of the functions $f(x) = \dfrac{a}{x+p} + q$ and $h(x) = mx + c$ are sketched below.
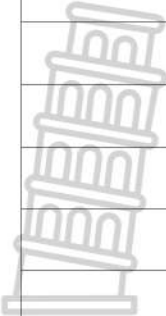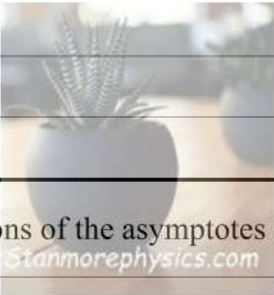

4.1	Write down values of p and q.	
		(2)
4.2	Calculate the value of a.	
		(1)
4.3	Write down the range of f.	
		(1)
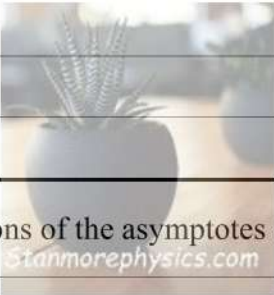
4.4   Determine the equation of the line of symmetry of f for $m < 0$ in the form $y = \ldots$

(3)

4.5   Write down the equations of the asymptotes of $f\left(x + 4\frac{1}{2}\right)$.

(2)

[9]

P.T.O.

QUESTION 5

Given:

- $f(x) = 2x^2 - x - 15$

- $g(x) = 3x - \dfrac{9}{2}$

- $h(x) = \log_{\frac{1}{3}} x$

5.1	Write down the coordinates of the turning point of f.	
		(2)

5.2	Show that the x-coordinates of the x-intercepts of f are $-\dfrac{5}{2}$ and 3 respectively.	
		(1)

P.T.O.

5.3	Sketch the graphs of f, g and h below. Clearly label ALL the intercepts with the axes and the asymptotes (where necessary) on the graph.	
		(5)
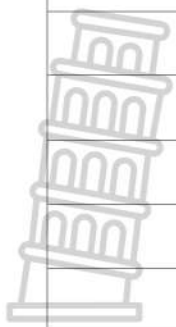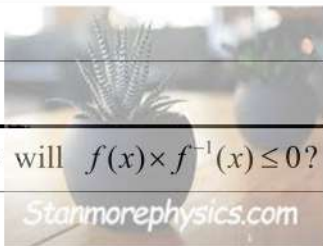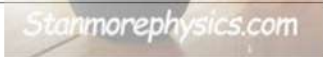
5.4	If point H is a point on g and point R is on f such that the abscissa of both points is 4, determine the length of line HR.	
		(3)
5.5	Write down the:	
5.5.1	Domain of h^{-1}	
		(1)
5.5.2	Range of h^{-1}	
		(1)
5.6	Determine the value(s) of k if the roots of $2x^2 - x + k = 0$ are equal.	
		(2)

P.T.O.

5.7	Determine the maximum value of $g(x) - f(x)$.	
		(3)

[18]

QUESTION 6

Given: $f(x) = 3^x$	
6.1	Determine the equation for f^{-1} in the form $f^{-1}(x) = \ldots$
	(1)
6.2	Sketch the graphs of f and f^{-1}. Clearly show ALL intercepts with the axes and the asymptote.
	(4)
6.3	Write down the equation of the line of symmetry between the two graphs you have drawn.
	(1)
6.4	For which values of x will $f(x) \times f^{-1}(x) \leq 0$?
	(1)

P.T.O.

6.5	Write down the range of $h(x) = 3^{-x} - 4$.	
		(1)
6.6	Write down an equation for g, if the graph of g is the image of the graph of f after f has been translated two units to the right and reflected about the x-axis.	
		(2)
		[10]

QUESTION 7

7.1	A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.	
7.1.1	Determine the scrap value of the machine at the end of 5 years.	
		(2)
7.1.2	After 5 years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of a new machine at the end of 5 years.	
		(2)

P.T.O.

7.1.3	The business estimates that it will need approximately R90 000 by the end of 5 years. A sinking fund for approximately R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period. Calculate the value of the monthly payments into the sinking fund.

(3)

7.2   Kasala receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly. She needs an amount of R18 000 per month to maintain her current lifestyle and plans to withdraw the first amount at the end of the month.

For how many months will she be able to live off her investment?

(5)

[12]

QUESTION 8

8.1	Given: $f(x) = -2x^2 + 1$.	
8.1.1	Determine $f'(x)$ from first principles.	
		(4)
8.1.2	Hence, calculate the gradient of the tangent to f at $x = -\dfrac{1}{2}$.	
		(1)

P.T.O.

8.2   Determine the derivative of $f(x) = \sqrt[3]{x^2} + \dfrac{1}{4x^4}$.

(3)

8.3   Given: $h(x) = ax^2$, $a > 0$.

Determine the value of a if it is given that $h^{-1}(8) = h'(4)$.

(5)

[13]

P.T.O.

QUESTION 9

The graph of the function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.
The following properties of f are given below:

*   Point A(1 ; 0) is a stationary point.

*   Point B(4 ; 0) is an x-intercept.

*   Point P is the point of inflection.

*   Point C is the turning point.

*   Point D is the y-intercept.
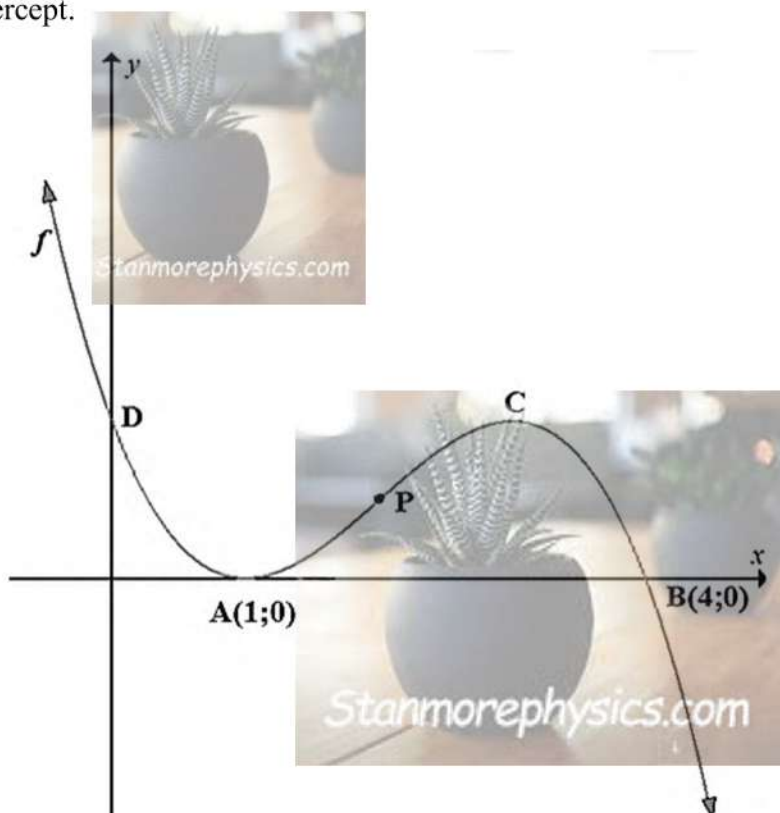
9.1	Show that $a = 12$, $b = -18$ and $c = 8$.	
		(3)

9.2	Kusal states that a tangent to the curve of f can be drawn through points C and D. Validate this statement using an appropriate calculation.	
		(3)

9.3	Points A, P and B are joined to form $\triangle APB$. Calculate the area of $\triangle APB$.	
		(4)

9.4	For which values of x is f concave down?	
		(1)

9.5	Write down the values of x for which f is strictly increasing.	
		(1)
9.6	Write down the coordinates of the turning points of $h(x) = f(x) - 3$.	
		(2)
		[14]

P.T.O.

QUESTION 10

A metal box (without a top) is to be constructed, from a square metal sheet with sides of 20 cm, by cutting square pieces of equal size from the corners of the sheet and then folding up the sides. Calculate the dimensions of the box with the largest volume that can be constructed in this manner.

(7)

[7]

P.T.O.
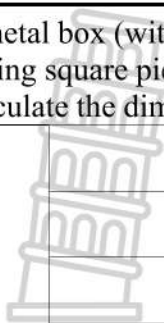
QUESTION 11

11.1	A six-sided die is rolled and the number of dots landing face up is noted. Consider the following events: *Downloaded from Stanmorephysics.com* • Event *A*: The number observed is 2 at the most. • Event *B*: The number that landed face up is an even number. • Event *C*: The number 6 is facing up. Use the given information to:
11.1.1	Determine, separately, the probability of event *A* and event *B*.
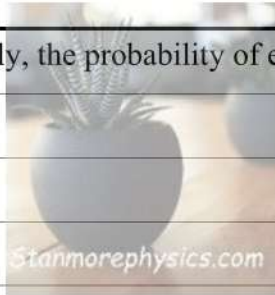
(2)

11.1.2	Use $P(A)$ and determine $P(A')$.

(1)

11.1.3	Are the events mutually exclusive? Give a reason for your answer.

(2)

P.T.O.

11.1.4	Calculate $P(A \text{ or } C)$.
	(2)
11.1.5	Are B and C independent events? Give a reason for your answer.
	(3)

11.2 The probability of getting the first answer in a quiz correct, is 0,6. If the first answer is correct, the probability of getting the next answer correct rises to 0,7. However, if the first answer is incorrect, the probability of getting the next answer correct is 0,4.
With the use of a tree diagram, determine the probability of getting the second answer correct.

(3)

11.3 The discus cage on an athletics field must be moved to a different position. Five boys from a group of fifteen boys must be selected to perform this task. How many different groups of five boys can be selected?

(3)

[16]

Additional space

P.T.O.

P.T.O.

Additional space

Additional space

TOTAL: 150

END

MATHEMATICS (Paper 1)   10611/25   1

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad\qquad A = P(1 - i)^n \qquad\qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad\qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad\qquad T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \; ; \; r \neq 1 \qquad\qquad S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x\left[(1 + i)^n - 1\right]}{i} \qquad\qquad P = \frac{x\left[1 - (1 + i)^{-n}\right]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \qquad\qquad y = mx + c$$

$$y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2 \qquad\qquad \text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{area}\,\Delta ABC = \frac{1}{2} ab.\sin C$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\sin\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$
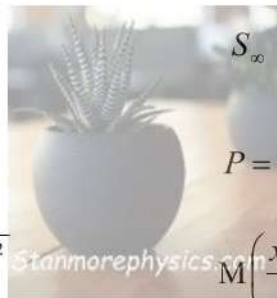
GAUTENG PROVINCE
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PREPARATORY EXAMINATION

2025

MARKING GUIDELINES

| MATHEMATICS PAPER 1 |

26 pages

Approved: _____ 6 September 2025

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

AMENDMENT TO MARKING GUIDELINES

PREPARATORY EXAMINATIONS – 2025

ATTENTION

THE CHIEF INVIGILATOR

SUBJECT / *VAK*	MATHEMATICS/*WISKUNDE*
PAPER / *VRAESTEL*	1
DATE OF EXAMINATION	5 SEPTEMBER 2025

The errata for the Marking Guidelines of MATHEMATICS P1 has reference.

There is a contention that **Question 11.3** (3 marks) was not answerable by most candidates in the province. This matter was addressed at the Marking Standardisation Meeting.   To ensure that candidates are not disadvantaged and prejudiced in way, you are advised to ask your Mathematics Educator(s) to **ignore Question 11.3** when marking.

In other words, the paper must be marked out of a total of 147 instead of 150, and then the learners' marks must be converted to a mark out of 150. E.g., Should a learner attain 85/147 then that mark is recalculated as 87/150.

Use the formula: $\dfrac{a}{147} \times 100 = b.$ Then, $\dfrac{b}{100} \times 150 = c$

*C is the mark that is entered into SASAMS **out of 150**.*

_____
Mr. Jonathan Williams

DIRECTOR: EXAMINATIONS MANAGEMENT

    5 SEPTEMBER 2025

INSTRUCTIONS AND INFORMATION

NOTES:

- If a candidate answered a question *TWICE*, mark only the *FIRST* attempt.
- If a candidate crossed *OUT* an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy *(CA)* applies to ALL aspects of the marking guidelines.
- It is UNACCEPTABLE for candidates to assume values/ answers in order to solve a question.
- (A) - denotes an accuracy mark.

QUESTION 1

1.1.1	$f(x) = (x^2 - 3)(3x - 1)(x + 2)$		
	$0 = (x^2 - 3)(3x - 1)(x + 2)$		
	$\therefore x^2 = 3$ or $x = \dfrac{1}{3}$ or $x = -2$		
	$\therefore x = \pm\sqrt{3}$ or $x = \dfrac{1}{3}$ or $x = -2$	✓ all 3 x-values	
	$x = -2$	✓ answer	
	NOTE: Answer only, full marks.		(2)
1.1.2	$x = \dfrac{1}{3}$ or $x = -2$	✓ both x-answers	(1)
1.1.3	$x = -2$ or $x = \dfrac{1}{3}$ or $x = \pm\sqrt{3}$	✓ all 4 x-answers	(1)
1.2.1	$-15x^2 - 9x + 4 = 0$		
	$x = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4(-15)(4)}}{2(-15)}$	✓ substitution into correct formula	
	$\therefore x = -0{,}90$ or $x = 0{,}30$	✓✓ answers	
	NOTE: Accept $-0{,}89$ or $0{,}29$.		
	Must show substitution into the formula to obtain full marks		
	Any other valid method.		(3)
1.2.2	$(3x - 2)^2 \geq 3x$		
	$9x^2 - 12x + 4 \geq 3x$		
	$9x^2 - 15x + 4 \geq 0$	✓ standard form	
	$(3x - 4)(3x - 1) \geq 0$	✓ factors	
	$x \leq \dfrac{1}{3}$ or $x \geq \dfrac{4}{3}$	✓✓ answer	
	NOTE: Any other valid method.		(4)

4

1.2.3	$5^x = 5(4 + 5^{2-x})$		
	$5^x = 20 + 5.5^{2-x}$	✓ simplificaction	
	$5^x = 20 + \dfrac{5.5^2}{5^x}$		
	$\therefore 5^{2x} = 20.5^x + 125 \cdots\cdots (\times 5^x)$	✓ standard form	
	$\therefore 5^{2x} - 20.5^x - 125 = 0$	✓ factors	
	$(5^x - 25)(5^x + 5) = 0$		
	$\therefore 5^x = 25 \quad$ or $\quad 5^x = -5$	✓ answers with rejection	
	$5^x = 5^2 \quad$ or $\quad NA$		
	$\therefore x = 2 \quad$ or $\quad 5^x > 0$		
	NOTE: Does not have to indicate that $5^x > 0$ if rejection is indicated.		
	OR	**OR**	
	$5^x = 5(4 + 5^{2-x})$		
	$5^x = 20 + 5.5^{2-x}$	✓ simplificaction	
	$5^x = 20 + \dfrac{5.5^2}{5^x}$		
	let $5^x = k$		
	$\therefore k = 20 + \dfrac{125}{k}$		
	$k^2 = 20k + 125$		
	$k^2 - 20k - 125 = 0$	✓ standard form	
	$(k + 5)(k - 25) = 0$	✓ factors	
	$\therefore 5^x \neq -5 \quad$ or $\quad 5^x = 25$		
	$\qquad\qquad\qquad 5^x = 5^2$		
	$\qquad\qquad x = 2$	✓ answers with rejection	
			(4)

1.3	$\log_x 16 = 4 \ldots(1)$ and $y + \sqrt{x+7} = x+1\ldots(2)$	✓ exponential form	
	$x^4 = 16$		
	$x^4 = 2^4$		
	$x = 2\ldots$ sub into (2)	✓ x-value	
		✓ substitution into (2)	
	$y + \sqrt{2+7} = 2+1$		
	$y + \sqrt{9} = 3$		
	$\sqrt{9} = 3 - y$		
	$9 = 9 - 6y + y^2$	✓ square both sides	
	$0 = y^2 - 6y$	✓ standard form	
	$0 = y(y-6)$	✓ y-values and rejection	
	$y = 0$ or $y \neq 6$		
	OR	**OR**	
	$\log_x 16 = 4 \ldots(1)$ and $y + \sqrt{x+7} = x+1\ldots(2)$	✓ exponential form	
	$x^4 = 16$		
	$x^4 = 2^4$		
	$x = 2\ldots$ sub into (2)	✓ x-value	
		✓ substitution into (2)	
	$y + \sqrt{2+7} = 2+1$		
	$y + \sqrt{9} = 3$		
	$3 = 3 - y$	✓ using +3	
	$0 = y$	✓ simplification	
		✓ y- value	
	NOTE: Full CA for this question. Do NOT stop marking after two errors.		(6)
1.4	$x^2 + p(2x+7) + 8$	✓ simplification	
	$= x^2 + 2px + 7p + 8$	✓ condition for = roots; $\Delta = 0$	
	$\Delta = (2p)^2 - 4(1)(7p+8)$		
	$0 = 4p^2 - 28p - 32$	✓ factors	
	$0 = (p-8)(p+1)$	✓ both answers	
	$p = 8$ or $p = -1$		
	OR	**OR**	

6

$x^2 + p(2x + 7) + 8$

$= x^2 + 2px + 7p + 8$

$= x^2 + 2px + p^2 - p^2 + 7p + 8$

$= (x + p)^2 - p^2 + 7p + 8$

$\therefore -p^2 + 7p + 8 = 0$

$p^2 - 7p - 8 = 0$

$(p - 8)(p + 1) = 0$

$\therefore p = 8 \quad \text{or} \quad p = -1$

NOTE: Only CA if the square was completed.

✓ complete the square

✓ $-p^2 + 7p + 8 = 0$

✓ factors

✓ answers

(4)

[25]

QUESTION 2

2.1	$-3; \ -2; \ -3; \ -6; \ -11; \ldots$ first difference: $\ +1; \ -1; \ -3; \ -5; \ldots$ $\therefore a = 1 \quad$ and $\quad d = -2$ $\therefore T_n = a + (n-1)d$ $T_n = 1 + (n-1)(-2)$ $T_n = 1 - 2n + 2$ $\therefore T_n = -2n + 3$	✓ 1st difference ✓ substitute into correct formula ✓ answer	(3)
2.2	$T_n = -2n + 3$ $\therefore T_{35} = -2(35) + 3$ $\therefore T_{35} = -67$ **NOTE**: Substitution must be $n = 35$. Answer only, full marks.	✓ answer	(1)
2.3	$-3; \ -2; \ -3; \ -6; \ -11;$ $\quad \searrow \nearrow \searrow \quad \searrow \nearrow \ \searrow \nearrow$ $\quad +1; \ -1; \ -3; \ -5;$ 1st differences $\quad\quad \searrow \nearrow \ \searrow \nearrow \ \searrow \nearrow$ $\quad\quad -2 \quad -2 \quad -2$ 2nd differences $T_n = an^2 + bn + c$ $but \cdots 2a = -2$ $\therefore a = -1$ $\therefore T_n = -n^2 + bn + c$ $\therefore T_1 = -(1)^2 + b(1) + c$ $\therefore -3 = -1 + b + c$ $\therefore -2 = b + c \cdots\cdots (1)$ $\therefore T_2 = -(2)^2 + b(2) + c$ $\therefore -2 = -4 + 2b + c$ $\therefore 2 = 2b + c \cdots\cdots (2)$ $(2) - (1) \therefore b = 4$ $in \cdots (1) \therefore -2 = 4 + c$ $\therefore c = -6$ $\therefore T_n = -n^2 + 4n - 6$	✓ 2nd difference ✓ value of a ✓ value of b ✓ value of c	(4)

8

2.4	$T_n = -n^2 + 4n - 6$		✓ complete the square

$T_n = -n^2 + 4n - 6$

$T_n = -[n^2 - 4n + 4 - 4 + 6]$

$T_n = -[(n-2)^2 + 2]$

$\therefore T_n = -(n-2)^2 - 2$

$\therefore T_n(\max) = -2$

$\therefore$ NO positive terms.

✓ complete the square

✓ $T_n(\max) = -2$

OR

The turning point of a maximum quadratic function is $(2 \; ; \; -2)$.

$\therefore$ no values ABOVE the x-axis, hence no positive values.

NOTE: Accept a sketch indicating the correct turning point and shape (ie. $a < 0$). Award full marks.

OR

✓ turning point

✓ explanation concluding no positive terms

(2)

[10]

9

QUESTION 3

3.1.1	$18cm$; $6\sqrt{3}$cm ; $6cm$.. Area of circle 1: 324π Area of circle 2: 108π Area of circle 3: 36π $\dfrac{108\pi}{324\pi} = \dfrac{1}{3}$; $\dfrac{36\pi}{108\pi} = \dfrac{1}{3}$ $\therefore$ Consant ratio $= \dfrac{1}{3}$ $-1 < r < 1$ $\therefore$ ratio is converging $\therefore$ sequence converges **NOTE:** If the candidate uses the radii to establish the pattern, $r = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$; award 1 mark.	✓ areas of 3 outermost circles ✓ value of r ✓ $-1 < r < 1$	(3)
3.1.2	18; $6\sqrt{3}$; 6 $r = \dfrac{1}{\sqrt{3}}$ $T_n = ar^{n-1}$ $\dfrac{2}{3} = 18\left(\dfrac{1}{\sqrt{3}}\right)^{n-1}$ $\dfrac{1}{27} = \left(\dfrac{1}{\sqrt{3}}\right)^{n-1}$ $3^{-3} = \left(3^{-\frac{1}{2}}\right)^{n-1}$ $3^{-3} = 3^{-\frac{1}{2}n+\frac{1}{2}}$ $\therefore -3 = -\dfrac{1}{2}n + \dfrac{1}{2}$ $\therefore n = 7$ **OR**	✓ ratio of radii ✓ substitution into correct formula ✓ $3^{-3} = 3^{-\frac{1}{2}n+\frac{1}{2}}$ ✓ answer **OR**	(4)

10

	$r = \dfrac{1}{\sqrt{3}}$ $T_n = ar^{n-1}$ $\dfrac{2}{3} = 18\left(\dfrac{1}{\sqrt{3}}\right)^{n-1}$ $\dfrac{1}{27} = \left(\dfrac{1}{\sqrt{3}}\right)^{n-1}$ $n-1 = \log_{\frac{1}{\sqrt{3}}} \dfrac{1}{27}$ $\therefore n-1 = 6$ $\therefore n = 7$ Downloaded from Stanmorephysics.com **NOTE:** Candidates may use the areas of the circles to do this question. Answer only – award ZERO marks	✓ ratio of radii ✓ substitution in correct formula ✓ correct use of logs ✓ answer	
3.2	$(1\times 2) + (5\times 6) + (9\times 10) + (13\times 14) + \ldots + (81\times 82)$ $1 ; 5 ; 9 ; 13 ; \ldots 81$ (1st terms in the bracket) $a = 1$ and $d = 4$ $T_n = a + (n-1)d$ $T_n = 1 + (n-1)4$ $T_n = 1 + 4n - 4$ $\therefore T_n = 4n - 3$ Number of terms: $4n - 3 = 81$ $4n = 84$ $\therefore n = 21$ $2 ; 6 ; 10 ; 14 ; \ldots 82$ (2nd factor in the bracket is 1 more than 1st factor) $\therefore T_n = 4n - 3 + 1$ $\therefore T_n = 4n - 2$ $\therefore (1\times 2) + (5\times 6) + (9\times 10) + (13\times 14) + \ldots + (81\times 82)$ $= \displaystyle\sum_{n=1}^{21} (4n-3)(4n-2)$ $= \displaystyle\sum_{n=1}^{21} (16n^2 - 20n + 6)$ **NOTE**: Accept answer mark either as a quadratic equation or in factors form.	✓ $T_n = 4n - 3$ ✓ $n = 21$ ✓ $T_n = 4n - 2$ ✓ answer	(4)

3.3	$a + ar + ar^2 + \ldots = 1 \cdots\cdots (1)$		
	$a^2 + a^2r^2 + a^2r^4 + \ldots = \dfrac{5}{6} \cdots\cdots (2)$		
	$S_\infty = \dfrac{a}{1-r} = 1$		
	$\therefore a = 1 - r \cdots\cdots (3)$	✓ a as subject	
	$S_\infty = \dfrac{a^2}{1-r^2} = \dfrac{5}{6}$		
	$\therefore 6a^2 = 5 - 5r^2 \cdots\cdots (4)$		
	(3) in (4)	✓ $6a^2 = 5 - 5r^2$	
	$\therefore 6(1-r)^2 = 5 - 5r^2$		
	$6(1 - 2r + r^2) = 5 - 5r^2$		
	$6 - 12r + 6r^2 = 5 - 5r^2$	✓ substitution	
	$\therefore 11r^2 - 12r + 1 = 0$		
	$(11r - 1)(r - 1) = 0$	✓ standard form	
	$\therefore r = \dfrac{1}{11}$ or $r = 1 (NA); \ (-1 < r < 1)$		
	$\therefore r = \dfrac{1}{11}$	✓ answers with rejection	
			(5)
			[16]

QUESTION 4

4.1	$p = 2$ $q = -1$ **NOTE:** Do NOT accept answers in terms of x and y. *Downloaded from Stanmorephysics.com*	✓ answer ✓ answer	(2)
4.2	$f(x) = \dfrac{a}{x+p} + q$ $-\dfrac{5}{2} = \dfrac{a}{-4+2} - 1$ $a = 3$ **NOTE:** Answer only, full marks.	✓ answer	(1)
4.3	$y \in \Re \; ; \; y \neq -1$ **NOTE:** Both conditions must be stated.	✓ answer	(1)
4.4	$m = -1$ $y - (-1) = -1(x - (-2))$ $y + 1 = -x - 2$ $y = -x - 3$	✓ $m = -1$ ✓ substitution ✓ answer	(3)
4.5	$f\left(x + 4\dfrac{1}{2}\right)$ moves $4\dfrac{1}{2}$ units to the left. $x = -6\dfrac{1}{2}$ $y = -1$ **NOTE:** Answers only, full marks.	✓ x-asymptote ✓ y- asymtote	(2)
			[9]

QUESTION 5

5.1	$0 = 2x^2 - x - 15$ $\therefore 0 = 4x - 1$ $\therefore x = \dfrac{1}{4}$ $\therefore f\left(\dfrac{1}{4}\right) = -\dfrac{121}{8}$ $\left(\dfrac{1}{4} \; ; \; -\dfrac{121}{8}\right)$ **OR** $0 = 2x^2 - x - 15$ $\therefore x = -\dfrac{(-1)}{2(2)}$ $\therefore x = \dfrac{1}{4}$ $\therefore f\left(\dfrac{1}{4}\right) = -\dfrac{121}{8}$	✓ x-value ✓ y-value **OR** ✓ x-value ✓ y-value	
			(2)
5.2	$f(x) = 2x^2 - x - 15$ $\therefore 0 = 2x^2 - x - 15$ $\therefore 0 = (2x + 5)(x - 3)$ $\therefore x = -\dfrac{5}{2}$ or $x = 3$ **OR** $\therefore 0 = 2x^2 - x - 15$ $\therefore x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-15)}}{2(2)}$ $\therefore x = -\dfrac{5}{2}$ or $x = 3$	✓ factors = 0 **OR** ✓ substitution into quadratic formula	
			(1)

14

5.3
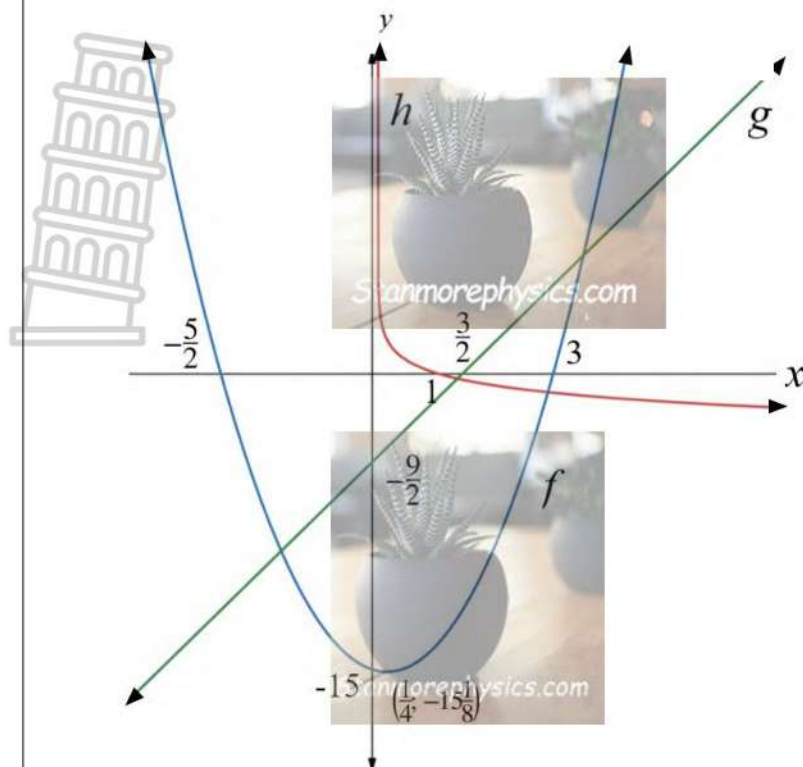
✓ f: x and y intercepts

✓ f: shape

✓ g: y-intercept and positive gradient

✓ h: shape

✓ h: asymptote

NOTE: Candidates must not be penalised if the functions are sketched on 3 different axes.

(5)

15
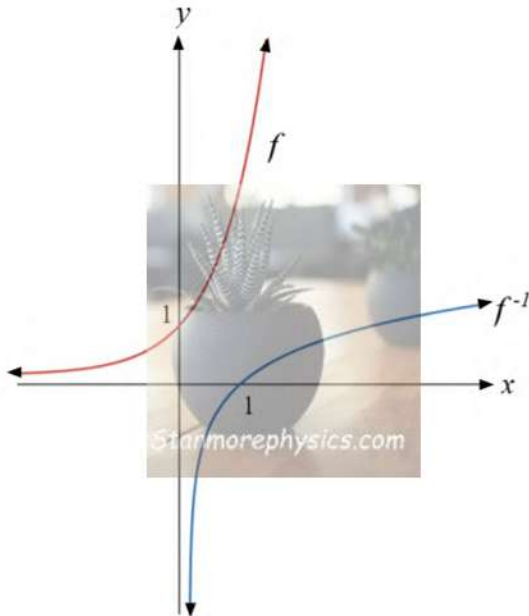
5.4	$HR = f(x) - g(x)$	✓ method	
	$HR = 2x^2 - x - 15 - 3x + 4\frac{1}{2}$		
	$HR = 2x^2 - 4x - 10\frac{1}{2}$		
	$but \quad (x = 4)$		
	$\therefore HR = 2(4)^2 - 4(4) - 10\frac{1}{2}$	✓ substitution	
	$HR = 5\frac{1}{2} \, units$	✓ answer	
	OR	**OR**	
	$f(4) = 2(4)^2 - 4 - 15$		
	$\therefore f(4) = 13$	✓ calculate $f(4)$ and $g(4)$	
	$g(4) = 3(4) - 4\frac{1}{2}$		
	$\therefore g(4) = 7\frac{1}{2}$		
	$g(4) = 13 - 7\frac{1}{2}$	✓ $f(4) - g(4)$	
	$HR = 5\frac{1}{2} \, units$	✓ answer	
	NOTE: Accept $g(x) - f(x)$ however candidate must present answer as a POSTIVE value for HR.		(3)
5.5.1	$x \in \Re$	✓ answer	
			(1)
5.5.2	$y \in \Re; \quad y > 0$	✓ answer	
	NOTE: No penalty for not stating $y \in \Re$.		(1)
5.6	$k = \frac{1}{8}$	✓✓ value of k	
	For equal roots the graph shifts $15\frac{1}{8}$ units upwards and the y-intercept will be $15\frac{1}{8}$ units above where it is now.		
	OR	**OR**	

10611/25

	$2x^2 - x + k = 0$ For equal roots, $\Delta = 0$ $b^2 - 4ac = 0$ $\therefore (-1)^2 - 4(2)(k) = 0$ $\therefore 1 = 8k$ $\therefore k = \dfrac{1}{8}$	✓ condition for equal roots ✓ value of k	(2)
5.7	$g(x) - f(x)$ $= 3x - 4\dfrac{1}{2} - 2x^2 + x + 15$ $= -2x^2 + 4x + 10\dfrac{1}{2}$ $\therefore \dfrac{-\Delta}{4a}$ $= \dfrac{-\left[4^2 + 8\left(10\dfrac{1}{2}\right)\right]}{-8}$ $\therefore Max = 12\dfrac{1}{2} \, units$ **OR** $g(x) - f(x)$ $= 3x - 4\dfrac{1}{2} - 2x^2 + x + 15$ $= -2x^2 + 4x + 10\dfrac{1}{2}$ $\therefore -4x + 4 = 0$ $\therefore x = 1$ max value $= 12\dfrac{1}{2} \, units$ **NOTE**: Any valid method.	✓ method ✓ substitution ✓ answer **OR** ✓ method ✓ derivative ✓ answer	(3)
			[18]

QUESTION 6

6.1	$f(x) = 3^x$ $\therefore f^{-1} = \log_3 x$	✓ answer	(1)
6.2		✓ shape of f ✓ y-intercept of f ✓ shape of f^{-1} ✓ x-intercept of f^{-1}	(4)
6.3	$y = x$	✓ answer	(1)
6.4	$0 < x \leq 1$ **NOTE**: Accept answer as separate inequalities.	✓ answer	(1)
6.5	$y > -4$	✓ answer	(1)
6.6	$g(x) = -f(x-2)$ $\therefore g(x) = -3^{x-2}$ **NOTE:** Answer only, FULL marks	✓ $g(x) = -f(x-2)$ ✓ answer	(2)
			[10]

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QUESTION 7

7.1.1	$A = P(1 - i)^n$ $\therefore A = 120\,000(1 - 0,09)^5$ $\therefore A = R74883,86$ **NOTE**: Answer only with correct formula, full marks.	✓ substitute correctly into correct formula ✓ answer	(2)
7.1.2	$A = P(1 + i)^n$ $\therefore A = 120\,000(1 + 0,07)^5$ $\therefore A = R168\,306,21$ **NOTE**: Answer only with correct formula, full marks.	✓ substitute correctly into correct formula ✓ answer	(2)
7.1.3	**NOTE**: From Q7.1.1 and Q7.1.2, the (estimated) value of the sinking fund: $R168\,306,21 - R74\,883,86 = R93\,422,35$ [So R90 000 is close to this value] $F_v = \dfrac{x[(1 + i)^{n+1} - 1]}{i} = 90000$ $\therefore \dfrac{x[(1 + \frac{0,085}{12})^{61} - 1]}{\frac{0,085}{12}} = 90\,000$ $\therefore x = R1\,184,68$	✓ value of i and n ✓ substitute correctly into correct formula ✓ answer	(3)

7.2	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i} = 900000$	✓ substitute correctly into correct formula	
	$\therefore \dfrac{18000\left[1-\left(1+\dfrac{0,105}{12}\right)^{-n}\right]}{\dfrac{0,105}{12}} = 900000$	✓ value of i and n	
	$\therefore \dfrac{\left[1-\left(1+\dfrac{0,105}{12}\right)^{-n}\right]}{\dfrac{0,105}{12}} = 50$	✓ simplification	
	$\therefore 1-\left(1+\dfrac{0,105}{12}\right)^{-n} = \dfrac{7}{16}$		
	$\therefore 1-\dfrac{7}{16} = \left(1+\dfrac{0,105}{12}\right)^{-n}$		
	$\therefore \dfrac{9}{16} = \left(1+\dfrac{0,105}{12}\right)^{-n}$		
	$\therefore \left(1+\dfrac{0,105}{12}\right)^{n} = \dfrac{16}{9}$		
	$\therefore n \log\left(1+\dfrac{0,105}{12}\right) = \log\dfrac{16}{9}$	✓ correct use of logs	
	$\therefore n = 66,043$	✓ answer	
	NOTE: Accept 66 or 67 months		(5)
			[12]

QUESTION 8

8.1.1	$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$		
	$= \lim_{h \to 0} \dfrac{-2(x+h)^2 + 1 - (-2x^2 + 1)}{h}$	✓ substitution	
	$= \lim_{h \to 0} \dfrac{-2(x^2 + 2xh + h^2) + 1 + 2x^2 - 1}{h}$		
	$= \lim_{h \to 0} \dfrac{-2x^2 - 4xh - 2h^2 + 1 + 2x^2 - 1}{h}$	✓ simplification	
	$= \lim_{h \to 0} \dfrac{-4xh - 2h^2}{h}$	✓ $= \lim_{h \to 0} \dfrac{-4xh - 2h^2}{h}$	
	$= \lim_{h \to 0} \dfrac{h(-4x - 2h)}{h}$		
	$= \lim_{h \to 0} (-4x - 2h)$	✓ answer	
	$= -4x$		
	NOTE: Penalise for notation error in this question only.		(4)
8.1.2	$f'\left(\dfrac{-1}{2}\right) = -4\left(-\dfrac{1}{2}\right)$		
	$\therefore f'\left(\dfrac{-1}{2}\right) = 2$	✓ answer	
	NOTE: Answer only, full marks.		(1)
8.2	$f(x) = \sqrt[3]{x^2} + \dfrac{1}{4x^4}$		
	$\therefore f(x) = x^{\frac{2}{3}} + \dfrac{1}{4}x^{-4}$	✓ write as exponents	
	$f'(x) = \dfrac{2}{3}x^{-\frac{1}{3}} - 1x^{-5}$	✓✓ answers	
			(3)

21

8.3	$h(x) = ax^2 \cdots\cdots a > 0$		
	$\therefore h'(x) = 2ax$	✓	$h'(x) = 2ax$
	$h'(4) = 2a(4)$	✓	$h'(4) = 8a$
	$\therefore h'(4) = 8a$		
	inverse:		
	$x = ay^2 \quad ; \quad y > 0$		
	$\therefore y^2 = \dfrac{x}{a} \quad ; \quad y > 0$		
	$\therefore y = +\sqrt{\dfrac{x}{a}}$		
	$\therefore h^{-1}(x) = \sqrt{\dfrac{x}{a}}$		
	$\therefore h^{-1}(8) = \sqrt{\dfrac{8}{a}}$	✓	$h^{-1}(8) = \sqrt{\dfrac{8}{a}}$
	$\therefore h^{-1}(8) = h'(4)$		
	$\therefore \sqrt{\dfrac{8}{a}} = 8a$	✓	$\sqrt{\dfrac{8}{a}} = 8a$
	$\left(\sqrt{\dfrac{8}{a}}\right)^2 = (8a)^2$		
	$\dfrac{8}{a} = 64a^2$		
	$\therefore 8 = 64a^3$		
	$\therefore \dfrac{8}{64} = a^3$		
	$\therefore a^3 = \dfrac{1}{8}$		
	$\therefore a = \sqrt[3]{\dfrac{1}{8}}$		
	$\therefore a = \dfrac{1}{2}$	✓	value of a
			(5)
			[13]

QUESTION 9

9.1	$y = -2(x-1)^2(x-4)$ $= -2(x^2 - 2x + 1)(x-4)$ $= -2(x^3 - 2x^2 + x - 4x^2 + 8x - 4)$ $= -2(x^3 - 6x^2 + 9x - 4)$ $\therefore y = -2x^3 - 12x^2 - 18x + 8$ $\therefore a = 12$ $\therefore b = -18$ $\therefore c = 8$	✓ method ✓ $-2(x^2 - 2x + 1)(x-4)$ ✓ $-2(x^3 - 6x^2 + 9x - 4)$	(3)
9.2	$f(x) = -2x^3 + 12x^2 - 18x + 8$ $\therefore f'(x) = -6x^2 + 24x - 18$ $\therefore f'(x) = x^2 - 4x + 3$ $\therefore 0 = x^2 - 4x + 3$ $0 = (x-3)(x-1)$ $\therefore x = 1 \quad \text{or} \quad x = 3$ but $f(3) = 8$ $\therefore$ tangent passes through points C and D. **NOTE:** Does not have to conclude that the tangent passes through points C and D. Can conclude at $f(3) = 8$. If a candidate calculates the tangent at point D and proves that the tangent does NOT pass through point C, award full marks.	✓ $f'(x)$ ✓ x-values ✓ $f(3) = 8$	(3)
9.3	$f'(x) = -6x^2 + 24x - 18$ $f''(x) = -12x + 24$ $0 = -12x + 24$ $x = 2$ $f(2) = 4$ $\therefore \perp h = 4$ $Area = \dfrac{1}{2}(3)(4)$ $Area = 6 \; units^2$	✓ $f''(x) = 0$ ✓ value for x ✓ $f(2) = 4$ ✓ answer	(4)
9.4	$f''(x) < 0$ $\therefore -12x + 24 < 0$ $\therefore x > 2$ **NOTE**: Valid from Q9.3. Answer only, full marks.	✓ answer	(1)
9.5	$1 < x < 3$	✓ answer	(1)

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9.6	turning points: $(x \; ; \; y-3)$ $(1 \; ; \; -3)$ and $(3 \; ; \; 5)$	✓✓ answers	(2)
			[14]

QUESTION 10
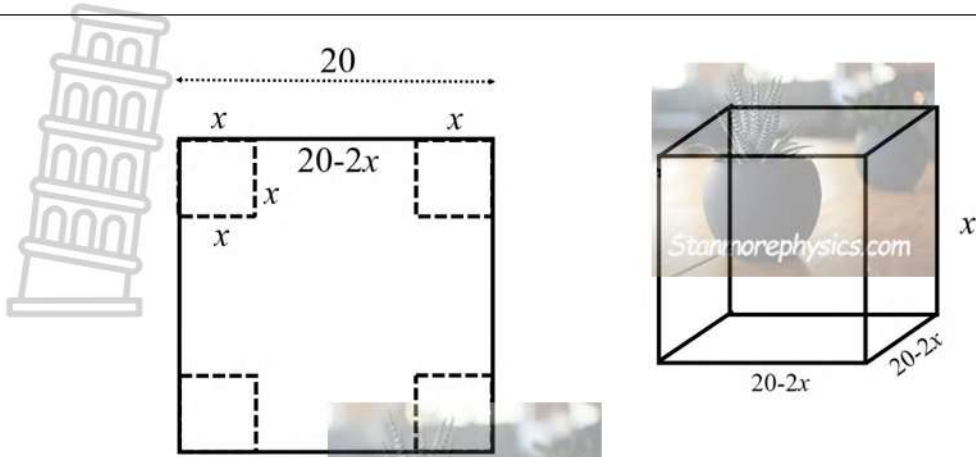
$V(x) = x(20 - 2x)(20 - 2x)$

$\therefore V(x) = 400x - 80x^2 + 4x^3$

$V'(x) = 400 - 160x + 12x^2$

$\therefore 0 = 400 - 160x + 12x^2$

$0 = 4(100 - 40x + 3x^2)$

$0 = 4(3x - 10)(x - 10)$

$\therefore x = \dfrac{10}{3}$ or $x = 10$

$V''(x) = -160 + 24x$

$\therefore V''\left(\dfrac{10}{3}\right) = -160 + 80 < 0$

$\therefore V''(10) = -160 + 240 > 0$

By the 2nd derivative test, the dimensions would be:

$\dfrac{10}{3}\,cm$ by $\dfrac{40}{3}\,cm$ by $\dfrac{40}{3}\,cm$

NOTE: $V'(x) = 0$ must be stated and not implied. Accept valid methods indicating that $x = \dfrac{10}{3}$ at the maximum and $x \neq 10$.

✓ $V(x) = x(20 - 2x)(20 - 2x)$

✓ $V(x) = 400x - 80x^2 + 4x^3$

✓ $0 = 400 - 160x + 12x^2$

✓ x-values

✓ $V''(x) = -160 + 24x$

✓ $V''\left(\dfrac{10}{3}\right) < 0$ and

   $V''(10) > 0$

✓ dimensions

(7)

[7]

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QUESTION 11

11.1.1	$P(A) = \dfrac{2}{6} = \dfrac{1}{3}$ $P(B) = \dfrac{3}{6} = \dfrac{1}{2}$	✓ answer $P(A)$ ✓ answer $P(B)$	(2)
11.1.2	$P(A') = 1 - P(A)$ $\therefore P(A') = 1 - \dfrac{1}{3}$ $\therefore P(A') = \dfrac{2}{3}$	✓ answer	(1)
11.1.3	$A = \{1 \; ; \; 2\}$ $B = \{2 \; ; \; 4 \; ; \; 6\}$ $C = \{6\}$ NO. $P(A, B \text{ and } C) \neq 0$ **NOTE:** The first mark can only be awarded if a reason is provided. Not awarding marks for yes/ no answers only.	✓ NO. ✓ valid explanation	(2)
11.1.4	$P(A \text{ or } C) = P(A) + P(C) - P(A \text{ and } C)$ $\therefore P(A \text{ or } C) = \dfrac{1}{3} + \dfrac{1}{6} - 0$ $\therefore P(A \text{ or } C) = \dfrac{1}{2}$ **OR** $P(A \text{ or } C) = \dfrac{n(A \text{ or } C)}{n(S)}$ $P(A \text{ or } C) = \dfrac{3}{6}$ $\therefore P(A \text{ or } C) = \dfrac{1}{2}$ **NOTE**: Answer only, full marks.	✓ $P(A \cup C) = \dfrac{1}{3} + \dfrac{1}{6} - 0$ ✓ Answer **OR** ✓ substitution ✓ answer	(2)

11.1.5	$P(B) \times P(C)$ $= \frac{1}{2} \times \frac{1}{6}$ $= \frac{1}{12}$ $but \cdots P(B \cap C) = \frac{1}{6}$ $\therefore P(B \cap C) \neq P(B) \times P(C)$ $\therefore$ B and C are not independent events.	✓ $P(B) \times P(C) = \frac{1}{12}$ ✓ $P(B \cap C) \neq P(B) \times P(C)$ ✓ conclusion	(3)
11.2	$P(2^{nd} \cdots ans \cdots corr) = 0,6 \times 0,7 + 0,4 \times 0,4$ $= 0,42 + 0,16$ $= 0,58$	✓ correct 1st and 2nd branches ✓ $= 0,6 \times 0,7 + 0,4 \times 0,4$ ✓ answer	(3)
11.3	**NOTE:** This question is removed from the question paper.		
			[13]

TOTAL: 147

END