KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

LAST PUSH LEARNER REVISION DOCUMENT

MATHEMATICS: PAPER 1&2

GRADE 12

2025

TABLE OF CONTENTS

TOPIC	1. Algebra, Equations and Inequalities: [± 25]

GUIDELINES, SUMMARY NOTES, & STRATEGIES

CONCEPT	HOW TO LEARN IT	RELEVANT FORMULAE AND KEY WORDS
Surd equations	Isolate the surd and square both sides. Remember to check solutions of a surd equation.	
Simultaneous equations	Solve equations with two unknowns, one of which is linear and the other non-linear, algebraically.	Involves making y or x subject of the formula in the linear equation and then substituting into the second equation.
Quadratic formula	Solve quadratic equations (by factorisation, completing the square and using quadratic formula)	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Solve exponential equations	Apply the laws of exponents to expressions involving rational exponents.	
Inequalities	Remove brackets, standard form, factorise, critical values, method and solve.	
Nature of roots	The nature of roots and the conditions for which the roots are real, non-real, equal, unequal, rational and irrational.	$\Delta = b^2 - 4ac$ $\Delta < 0$ non-real/imaginary $\Delta \geq 0$ real $\Delta = 0$ real and equal (1 root) $\Delta > 0$ real and unequal (2 roots) $\Delta > 0$ and a perfect square e.g $\Delta = 25$ $\Delta > 0$ and not a perfect square eg $\Delta = 30$

		ACTIVITIES		
1.	**Solve for x in the following equations:**			
	1.1	$x(x-2)-1=-1$	(3)	**L1**
	1.2	$5x^2-4x=0$	(2)	**L1**
	1.3	$x^2-3x-10=0$	(3)	**L1**
	1.4	$(3-x)(2-x)=0$	(2)	**L1**
	1.5	$x^2+2x+1=0$	(2)	**L1**
	1.6	$x^2+4x=0$	(2)	**L1**
	1.7	$x^2-2x+3+\frac{2}{x^2-2x}=0$	(4)	**L2**
	1.8	$x^2-121=0$	(3)	**L1**
	1.9	$x^2-x-12=0$	(3)	**L1**
	1.10	$x(x-3)=0$	(2)	**L1**
	1.11	$(x+3)(2-5x)=0$	(2)	**L1**

	1.12	$x=\frac{a^2+a-2}{a-1}$ if $a=888\ 888\ 888\ 888$	(2)	**L2**
	1.13	$3x^2+5x=0$	(2)	**L1**
	1.14	$(x+2)^2-9=0$	(3)	**L2**
	1.15	Given: $2k=(x-5)(x-k)$, determine: (a) k if $x=2$ (b) x if $k=2$	 (2) (4)	 **L2** **L2**
	1.16	$(2x-6)(x+5)=0$	(2)	**L1**
	1.17	$3x(x-5)(3x-1)=0$	(3)	**L1**
2.	**Solve for x (leave your answer correct to TWO decimal places not unless otherwise stated)**			
	2.1	$3x^2+6x+1=0$	(4)	**L2**
	2.2	$x(5x-3)=1$	(4)	**L1**
	2.3	$2x^2-\frac{1}{2}=3x$	(4)	**L2**
	2.4	$x^2+3=5x$	(4)	**L2**
	2.5	$5x^2+2x=9$	(4)	**L1**
	2.6	$2x^2+1=4x$	(4)	**L2**
	2.7	$7x^2+5x-8=0$	(3)	**L1**
	2.8	$4x^2+3x-5=0$	(3)	**L1**
	2.9	$2x^2-3x-7=0$	(4)	**L1**
	2.10	$x^2-7x-7=0$	(3)	**L1**
	2.11	$2x^2-3x=4$	(4)	**L2**
	2.12	$7x^2-11x+3=0$	(3)	**L1**
3.	**INEQUALITIES**			
	Solve for x in the following inequalities.			
	3.1	$x^2-5x+4<0$	(3)	**L2**
	3.2	$4+5x>6x^2$	(4)	**L2**
	3.3	$(x-8)(x+2)\le 0$	(3)	**L2**
	3.4	$2x+3>x^2$	(4)	**L2**
	3.5	$3x^2+5x\ge 2$	(4)	**L2**
	3.6	$18-3x-x^2\ge 0$	(4)	**L2**
	3.7	$\frac{7}{x^2-2x-8}>0$	(4)	**L3**
	3.8	$(x-1)^2-9\ge 0$	(4)	**L2**
	3.9	$4+5x>6x^2$	(4)	**L2**
	3.10	$6x^2+7x>5$	(4)	**L2**
	3.11	$x^2-2x\le 15$	(4)	**L2**
	3.12	$3^x(x-5)<0$	(2)	**L2**
	3.13	$x^2\ge 5x$	(4)	**L2**

4.	**EXPONENTS AND SURDS**			
	Solve for x in the following equations.			
	4.1	$5^{-x}.5^{x-2}=\frac{25^{2x}}{5}$	(3)	**L2**
	4.2	$2^{x+4}+2^{x}=8704$	(3)	**L2**
	4.3	$9^{x}+9=10.3^{x}$	(4)	**L2**
	4.4	$\sqrt{7x-12}-x=0$	(4)	**L2**
	4.5	$\left(\frac{1}{6}\right)^{3x+2}.216^{3x}=\frac{1}{216}$	(4)	**L3**
	4.6	$x+3\sqrt{x+2}=2$	(3)	**L2**
	4.7	$\sqrt{2-x}-x=-2$	(4)	**L2**
	4.8	$2^{2x}+2^{x}-6=0$	(3)	**L2**
	4.9	$2^{x}+2^{x+2}=40$	(4)	**L2**
	4.10	$\sqrt{10-x^{2}}-x+2=0$	(5)	**L2**
	4.11	$6-\sqrt{x+4}=x+4$	(6)	**L2**
	4.12	$2^{2x}-2^{x+2}-32=0$	(5)	**L2**
	4.13	$\sqrt{-2x+4}-x=2$	(4)	**L2**
	4.14	$3^{x+2}=42-5.3^{x}$	(3)	**L2**
	4.15	$x-\sqrt{5x-1}=5$	(5)	**L2**
	4.16	$5^{2x}-5^{x}=0$	(4)	**L2**
	4.17	$\frac{x}{\sqrt{20-x}}=1$	(5)	**L3**
	4.18	$1=\frac{-6}{\sqrt{x+2}}+\sqrt{x+2}$	(5)	**L3**
	4.19	$\sqrt{5^{x}}+\sqrt{5^{x+2}}=150$	(4)	**L3**
	4.20	$x^{x}=2^{2048}$	(4)	**L3**
	4.21	$\sqrt{2(x+10)}-10=x-12$	(4)	**L2**
	4.22	$3\sqrt{x+12}-x=8$	(5)	**L2**
5.	**SIMPLIFICATION**			
	5.1	Show that $\left(1+x^{m}+x^{-n}\right)^{2}-\left(1-x^{m}-x^{-n}\right)^{2}$ is divisible by 2 for all real values of m and n.	(3)	**L3**
	5.2	Given: $\left(\sqrt{5}\right)^{x}-\left(\sqrt[3]{2}\right)^{y}=17$; where x and $y\in$ N , determine a value of $P=xy$.	(4)	**L3**
	5.3	Given: $f(x)=x(x+a)-b$ and $g(x)=2x^{2}+b-dx$, where $a\neq 0$ and $b\neq 0$. If $x-a$ is a factor of $f(x)$ and $x-b$ is a factor of $g(x)$, prove that: $d-1=4a^{2}$.	(5)	**L4**

	5.4	If $x^3-y^3=(x-y)(x^2+xy+y^2)$, show that $\left(3-\frac{1}{\sqrt{5}}\right)\left(9+\frac{3}{\sqrt{5}}+\frac{1}{5}\right)$ can be expressed as $a-\frac{1}{25}\sqrt{b}$ where $a,b\in\mathbb{Q}$.	(4)	**L4**
	5.5	Consider the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)...$ Each factor in the product is of the form $\left(1+\frac{1}{n}\right)$ for $n\geq 2$. Determine ALL the values of n for which the product will be an interger value.	(3)	**L4**
	5.6	Given: $P=(1-a)$ and $T=(1+a)(1+a^2)(1+a^4)...(1+a^{512})$ Determine the values of P x T in terms of a.	(3)	**L4**
	5.7	Simplify fully: $\sqrt[n]{\frac{10^n+2^{2+n}}{5^{2n}+4.(5^n)}}$ where $n\neq 0$	(4)	**L3**
	5.8	Simplify fully: $\frac{8^{n-3}.10^{n+2}}{8^{n-1}.5^{1+n}}$	(4)	**L2**
	5.9	Simplify the following expression and express your answer in the form $a+b\sqrt{c}$, where $a,b,c\in\mathbb{R}$ and c is not a perfect square: $\frac{3+\sqrt{5}}{2-\sqrt{5}}+\frac{5-\sqrt{5}}{3+\sqrt{5}}$	(4)	**L3**
	5.10	Simplify the following expression fully: $\left(\frac{9^{\frac{3}{2}}.\sqrt{50}}{3.\sqrt{2}}\right)\div\left(\frac{3^{\frac{5}{2}}.\sqrt{8}}{\sqrt{5}}\right)$	(4)	**L3**
	5.11	Simplify the following expression fully: $\frac{\sqrt{m^{2022}-m^{2020}}}{\sqrt{25m^{2024}-25m^{2022}}}$	(4)	**L4**
6.		**NATURE OF ROOTS**		
	6.1	For which values of k will the roots of $6x^2+6=4kx$ be real and equal?	(3)	**L2**
	6.2	The roots of a quadratic equation is given by $x=\frac{-2\pm\sqrt{4-20k}}{2}$. Determine the value(s) of k for which the equation will have real roots.	(2)	**L1**
	6.3	Determine the value(s) of k for which the roots of $x\left(\frac{1}{2}x+2\right)=-k^2$ are equal.	(5)	**L2**
	6.4	Given: $(x+5)^2=1-p^2$ Calculate the values of p for which the roots of the equation are non-real.	(5)	**L3**
	6.5	Consider the equation: $(x+k)^2=2k+1$ (a) Solve for x in terms of k.	(2)	**L2**

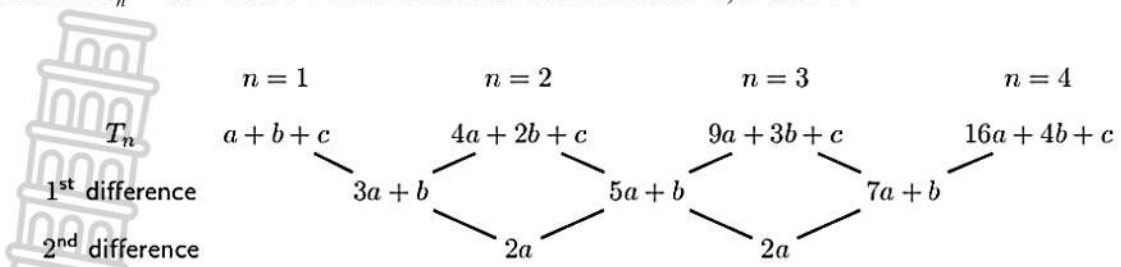
		(c) Hence, or otherwise, write down two positive values of k for which the roots are rational and real.	(2)	**L2**
	6.6	Prove that the roots of the following equation are nonreal for all real values a and b, $a \neq 0$. $a^2x^2 + abx + b^2 = 0$	(3)	**L3**
	6.7	Given that: • $f(x) = x^2 + 3x$ • $2x - [t(x)]^{\frac{1}{2}} = 0$ For which values of k will the equation $f(-x) + \frac{t(2k)}{4}$.	(5)	**L4**
	6.8	Given: $f(x) = 3(x-1)^2 + 5$ and $g(x) = 3$ (a) Is it possible for the graphs of f and g to intersect? Give a reason for your answer. (b) Determine the value(s) of k for which $f(x) = g(x) - k$ has TWO unequal real roots.	(2) (4)	**L2** **L3**
	6.9	The solutions of a quadratic equation are given by $x = \frac{-2 \pm \sqrt{2p+5}}{7}$. Determine the value(s) of p for which this equation will have: (a) Two equal solutions (b) No real solutions	(1) (1)	**L1** **L1**
7.		**SIMULTANEOUS EQUATIONS**		
		Solve for the UNKNOWN in the following simultaneous equations:		
	7.1	$10^x.20^y = 50$	(4)	**L3**
	7.2	A rectangle having sides of $(y-3)$ metres and $(x+2)$ has a perimeter of 24 metres and an area of 32 square metres. Calculate the values of x and y.	(6)	**L3**
	7.3	$x + y = 3$ and $x^2 - 2y^2 = 18$	(5)	**L2**
	7.4	$x + 2y = 5$ and $2y^2 - xy - 4x^2 = 8$	(6)	**L2**
	7.5	$y - x = 2$ $x^2 + 2xy - 4 = 0$	(5)	**L2**
	7.6	$2y - x = 3$ and $y^2 + 3x - 2xy = 0$	(6)	**L2**
	7.7	$2x + y = 3$ $y^2 + xy = 2$	(5)	**L2**
	7.8	$x - y = 3$ $x^2 - xy = 2y^2 + 7$	(6)	**L2**
	7.9	$x + y = 9$ and $2x^2 - y^2 = 7$	(5)	**L2**
	7.10	$2x - y + 1 = 0$ and $x^2 + xy - y = 3x - 2$	(5)	**L2**
	7.11	$x - 2y - 3 = 0$ and $4x^2 - 5xy + y^2 = 0$	(6)	**L2**
	7.12	$6y + 2x = 4$ and $x^2 + xy = 4$	(6)	**L2**
	7.13	$\sqrt{3^x}.9^y = 27$ and $x + 4y^2 = 6$	(6)	**L3**

	7.14	$2y = 3 + x$ and $y^2 + 3xy = 2x^2 + 50$	(6)	**L2**
	7.15	$(x-5)^2 + (3x-y)^2 = 0$	(4)	**L3**
	7.16	$3^{x+y} = 9$ $x^2 - 5x + y^2 = 9$	(6)	**L2**
	7.17	$2x + 3y = 4x + 7$ $4x + 7 = 7x + 7$	(4)	**L1**
8.		**MIXED PROBLEMS**		
	8.1	Solve for x in terms of y: $(x+1)(x-3) = (y+1)(y-3)$, where $x \neq y$	(5)	**L3**
	8.2	Solve for x, in terms of y, if $3^{x+2y} = \frac{9^{3y}}{27}$.	(3)	**L2**
	8.3	Given $k + 5 = \frac{14}{k}$ *a*) Solve for k *b*) Hence or otherwise, solve for x if $\sqrt{x+5} + 5 = \frac{14}{\sqrt{x+5}}$	(3) (3)	**L2** **L2**
	8.4	If $x - 6 = 0$ is one of the solutions of the equation $x + \frac{40}{x} = 16$, determine ONE value of y for which $2y + 3 + \frac{40}{2y+3} = 16$.	(3)	**L2**
	8.5	Calculate the maximum value of S if $S = \frac{6}{x^2+2}$	(2)	**L2**
	8.6	If $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}$, calculate two values of $\frac{x}{y}$.	(5)	**L3**
	8.7	Consider: $5x^2 - kx + 16 = (x+2).Q(x) + 10$ where k is a constant and $Q(x)$ is a polynomial in terms of x. Calculate k.	(4)	**L3**
	8.8	Prove that $x^2 + 2xy + 2y^2$ cannot be negative for $x,\ y \in R$.	(4)	**L4**
	8.9	Determine the value of: $\frac{\left(2^{2p-1}\right)^3}{\sqrt{7^k}}$ if $2^{6p} = 81$ and $7^k = 729$	(4)	**L3**

SEQUENCES AND SERIES

Definition: Second difference are equal where the first term forms an arithmetic sequence.

General term: $T_n = an^2 + bn + c$ **. To calculate the values of** a, b **and** c**:**



Arithmetic number patterns:	**Geometric number patterns:**
Definition: All first differences are equal, i.e. you always add or subtract a constant difference **NB:** $T_2 - T_1 = T_3 - T_2$ (same difference; d) General term: $Tn = a + (n-1)d$ $d = T_2 - T_1$ This formula can be used to determine the value of any specific term of an arithmetic sequence. Sum of n Terms: $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}(a + l)$ Where l is the last term or T_n	Definition: There exists constant ratio, i.e. you multiply by the same ratio. **NB:** $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ (common ratio; r) General term: $T_n = ar^{n-1}; r = \frac{T_2}{T_1}$ Sum of n terms: $S_n = \frac{a(r^n - 1)}{r - 1}$ or $S_n = \frac{a(1 - r^n)}{1 - r}$ $S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + ... + a_n$ (n terms) **Sum to infinity:** $S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

NB: A given sum formula can be used to determine the terms of a sequence. $T_n = S_n - S_{n-1}$

ACTIVITIES

1. **REDHILL HIGH SCHOOL SEPT 2022**

1.1 Consider the quadratic sequence: 72 ; 100 ; 120 ; 132 ; .....

1.1.1 Determine Tn, the nth term of the quadratic sequence. (4) **L2**

1.1.2 A term in the quadratic sequence is equal to the twelfth term of the sequence of the first differences. Determine the position of this term in the quadratic sequence. (5) **L3**

1.1.3 Determine the maximum value of the quadratic sequence. (3) **L2**

1.1.4 Hence, determine the maximum value of the following quadratic sequence: -23 ; 5 ; 25 ; 37 ; ..... (1) **L3**

2. **MPUMALANGA PRE-SEPT EXAM 2024**

2.1 The first term of a quadratic sequence, $T_n = n^2 + an + b$, is 9 and the term of the first difference is 11.

2.1.1 Determine the values a and b, hence the general term. (3) **L2**

2.1.2 What is the value of the first term of the sequence that is greater than 240 (4) **L2**

2.2 Given the arithmetic sequence: $3; -1; -5; ... -85; -89$

2.2.1 Calculate the number of terms in the sequence. (3) **L2**

2.2.2 Calculate the sum of all negative terms in this sequence (3) **L2**

2.3 Consider the sequence: $3; -1; -5; ... -85; -89 ...; -389$

Determine the number of terms in this sequence that will be exactly divisible by 3. (4) **L3**

3. **GP MAY/JUN 2025**

3.1 The first three terms of an arithmetic pattern are: $-5; 2; 9; ...$

3.1.1 Write down the next two terms of the pattern (2) **L1**

3.1.2 Show that the sum of the first n terms of the pattern is given by: (3) **L1**

$$S_n = \frac{1}{2}n(7n-17)$$

3.2 The first four terms of a quadratic pattern are: $x; 3x-5; 4x-3; 5x+1; \ldots$

3.2.1 Determine the value of x. (3) **L2**

3.2.2 If the pattern continues indefinitely, prove that all the terms of the pattern are positive. (5) **L4**

3.3 Consider the geometric series: $\frac{1}{2}(p-3)+\frac{3}{4}(p-3)^2+\frac{9}{8}(p-3)^3+\ldots$; for $p \neq 3$

3.3.1 Determine the values of p for which the series converges. (4) **L2**

3.3.2 If the sum to infinity of the series is 1, determine the value of p. (3) **L2**

4. **KZN SEPT 2024**

4.1 The first three terms of a geometric sequences are given: $81; m; \frac{m}{3}; \ldots..$

4.1.1 Determine the values of m. (2) **L1**

4.1.2 Calculate: $\sum_{t=1}^{9} 81\left(\frac{1}{3}\right)^{t-1}$ (2) **L2**

4.2 Given the arithmetic sequences: $\frac{12}{5}; 3; \frac{18}{5}; \ldots\ldots\ldots\ldots..; \frac{333}{5}$

4.2.1 Calculate the number of terms in this sequence. (4) **L2**

4.2.2 How many terms of this sequence are integers? (4) **L3**

4.3 Evaluate: $1^2-2^2+3^2-4^2+5^2-6^2+7^2-8^2+\ldots\ldots\ldots\ldots..+399^2-400^2$ (5) **L4**

5. **DBE NOV 2024**

5.1 The sequence of the first differences of a quadratic pattern is: 1 ; 3 ; 5 ; …

5.1.1 If T_{99} of this quadratic pattern is 9 632, calculate the value of T_{98}. (3) **L2**

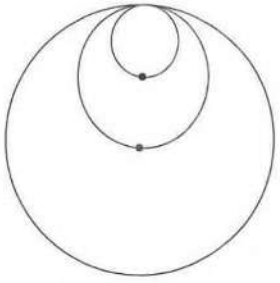
5.1.2 If it is further given that the third term of the quadratic pattern is 32, determine the general term, T_n, of the quadratic pattern. (5) **L3**

5.2 The first term of an arithmetic series is 7. The common difference of the series is 5 and the series contains 20 terms.

5.2.1 Calculate the sum of this series. (2) **L2**

5.2.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14 400. Using sigma notation, write down an equation for the sum of the terms added to the original series. (4) **L3**

5.3 A circle with radius 6 cm is drawn. A second, smaller circle is drawn through the centre of the first circle and also touches the first circle internally, as shown in the diagram. A third, smaller circle is drawn through the centre of the second circle and touches the second circle internally. The process of drawing circles continues and forms a geometric pattern.



5.3.1 Write down the radius of the 3rd circle (2) **L1**

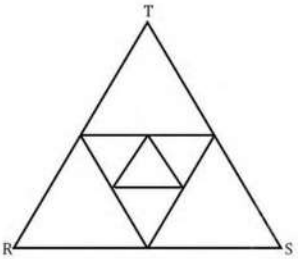
5.3.2 Calculate the sum of the areas of the first 10 circles. (4) **L3**

5.3.3 Which circle has a diameter of $\frac{3}{128}$ cm (4) **L3**

6. **GAUTENG MAY/JUN 2025**

6.1 If $\sum_{k=2}^{n} 2\left(3^{k-1}\right) = 59\,046$, determine the value of n. (5) **L3**

6.2 An equilateral triangle RST with side of length $2p$ units is drawn. A second triangle is drawn by joining the midpoints of the sides of the previous triangle as shown on the sketch, and this continues indefinitely.



6.2.1 Write down, in terms of p, the length of the second triangle. (1) **L1**

6.2.2 Calculate, in terms of p, the perpendicular height of ΔRST. (2) **L3**

6.2.3 Show that the sum of the areas of all the triangles formed will not exceed $48\sqrt{3}p^2$. (5) **L4**

7. **IEB NOVEMBER 2024**

7.1 Given: $\sum_{n=0}^{\infty}\left(\frac{x+2}{2}\right)^n$

7.1.1 Write down an expression for the sum of the first three terms. (2) **L1**

7.1.2 For which values of x will the given sum to infinity converge? (3) **L2**

7.1.3 If the sum converges find $\sum_{n=0}^{\infty}\left(\frac{x+2}{2}\right)^n$ in terms of x. Simplify your answer. (3) **L2**

8. **IEB MAY 2023**

8.1 In the geometric sequence $S_5 = \frac{605}{243}$ and $T_n = w.3^{-n}$.

8.1.1 Determine the value w. (6) **L2**

8.1.2 Determine the value of p if: $\sum_{n=1}^{\infty}\left(2^{-n}\right)+\sum_{n=1}^{p}(2n+1)=484$ (7) **L3**

9. **IEB NOV 2023**

9.1 Consider the arithmetic sequence given: 22; 25; 28; 31; …; 262

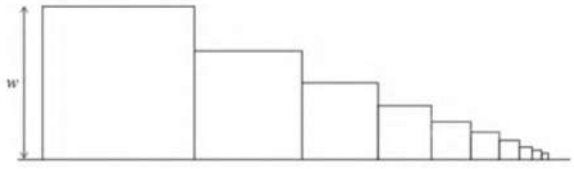
9.1.1 Determine the number of terms in the sequence. (3) **L2**

9.1.2 Represent the sum of all the terms using sigma notation. (2) **L2**

9.1.3 If the pattern continues, determine the smallest number of terms that must be added to the sequence above so that the sum of all the terms will be more than 15 000. (5) **L3**

10. **SAHETI SCHOOL SEPT 2022**

10.1 Mpho is creating a mosaic pattern by placing square tiles next to each other along a straight line indefinitely.



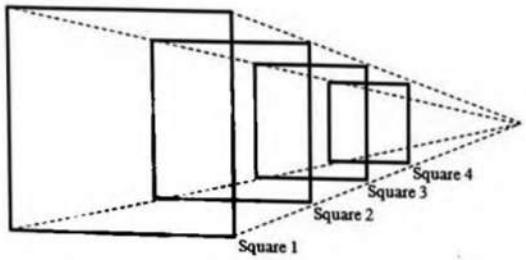
The area of each tile is half the area of the previous tile, and the sides of the largest tile have length w centimetres.

10.1.1 Find, in terms of w, the length of the sides of the second largest tile. (3) **L2**

10.1.2 Assume the tiles are in contact with adjacent tiles, but do not overlap. Show that, no matter how many tiles are in the pattern, the total length of the series of tiles will be less than $3.5w$. (4) **L3**

11. **KZN JUNE 2025**

11.1 Given the folowing geometric series: $36+18+9+\frac{9}{2}+...$ representing the areas of the squares in cm^2, as drawn below.



11.1.1 Does the series converge ? Justify your answer. (2) **L1**

11.1.2 Calculate S_∞. (2) **L2**

11.1.3 Which square will have a side length of $\frac{3}{8}$cm? (3) **L3**

11.1.4 Calculate the sum of the diagonals of the first ten squares. (4) **L4**

11.2 The seventh term of an arithmetic sequence is 4 and the twelfth term is 14.

11.2.1 Determine the common difference and the first term of the sequence. (3) **L3**

11.2.2 Which term of the sequence will be the additive inverse of the first term? (2) **L2**

12. **KZN MAR 2024**

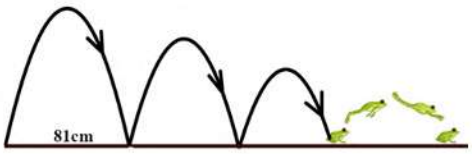
12.1 Consider the arithmatic sequence: 8; 15; 22, …

12.1.1 Determine the 36^{th} term. (2) **L1**

12.1.2 Calculate the sum of the first 36 terms. (2) **L2**

12.1.3 If it is given that $T_{72}+T_{72-m}=786$, determine the value of m. (4) **L3**

12.2 The diagram represents a frog making a series of jumps. With every next jump, he has only enough energy left to jump $\frac{2}{3}$ the distance of his previous jump.



12.2.1 If his first jump is $81\,cm$ long, calculate the length of his second jump. (1) **L1**

12.2.2 Determine the length of his ninth jump. (2) **L1**

12.2.3 If the frog continues to jump in this way, will he be able to catch a trapped insect that is 230 *cm* away from his tarting position? Show all your calculations. (3) **L2**

13. **KZN JUN 2023**

13.1 The following sequence is a combination of arithmetic and geometric sequence: $3;3;9;6;15;12;...$

13.1.1 Write down the next TWO terms (2) **L1**

13.1.2 Determine $T_{20}-T_{21}$ (5) **L3**

13.2 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$

13.2.1 Prove, without the use of a calculator, that the sum of the series to infinity is $16+8\sqrt{2}$ (4) **L3**

14. **MPUMALANGA SEPT 2024**

14.1 Given the following series: 220+213+206+ … –11

14.1.1 Calculate the sum of the series. (5) **L2**

14.2 Write the series in sigma notation. (3) **L2**

14.2 A ball is dropped from a height of 15 m. It bounced back and loses 10% of its previous height on each bounce. Show that the total distance the ball will bounce cannot exceed 290 m. (4) **L3**

14.3 The sum of the first n terms of a sequence is $S_n = 2^{n-5} + 3$.

Determine the 70th term. Leave your answer in the form $a.b^p$ where a, b and p are all integers. (4) **L2**

ST ANDREW'S COLLEGE/ SEPTEMBER 2021

15.1 The sum of the first eight terms of an arithmetic sequence is 124. The fourth term is 14. Determine the first 3 terms of the sequence. (6)

15.2 In an arithmetic sequence, the n^{th} term is T_n and the sum of the first n terms is S_n.

$T_{10} - T_9 = 6$ and $S_{10} - S_9 = 57$. Find the value of T_1. (5) **L3**

16. **UMGUNGUNDLOVU TERM 1 DOCUMENT 20205**

16.1 Grace who currently weighs 52 kg, desperately seeks to gain weight. She follows a specific diet and training programme, to obtain her goal. She gains 2 kg per week for the first two. Thereafter, her weekly weight gain is 20% less than the weight gained in the previous week. Grace decided to follow this diet and fitness programme strictly, and to continue this pattern of weight gained indefinitely.
Calculate the maximum weight Grace will gain while following this programme. (5) **L4**

17. **IEB MAY 2022**

17.1 The numbers $7; x; y$ are in a geometric sequence. If the sum of these three numbers is $(-2x - 7)$, determine the possible value(s) of x and y. (7) **L3**

17.2 An arithmetic series has a common difference of 5. The sum of the first n terms is 160 and the sum of the first $2n$ terms is 725. Determine the value of n. (6) **L3**

18. **DBE MAY/JUNE 2025**

18.1 Given the arithmetic series: $5 + 7 + 9 + \ldots + 93$

18.1.1 Determine the general term of the series, T_n, in the form $T_n = pn + q$. (2) **L1**

18.1.2 The given series represents the number of kilometres that an athlete ran each week in preparation for an ultramarathon. The athlete ran 93 km in the last week of the training programme. How long, in weeks, was the training programme? (2) **L2**

18.1.3 The athlete used this opportunity to raise funds for her high school. The community sponsored her R10 for each kilometre run during the training programme. Calculate the total amount that the athlete raised for her school. (3) **L3**

18.2 The general term of a geometric sequence is $T_n = 2^{n+2}$

18.2.1 Write down the first term. (1) **L1**

18.2.2 Write down the common ratio. (1) **L1**

18.2.3 Calculate T_{20} (Write your answer as a power of 4.) (2) **L1**

18.2.4 Calculate $\sum_{n=1}^{\infty} \frac{1}{T_n}$ (3) **L2**

18.2.5 Consider the first 21 terms of the sequence $T_n = 2^{n+2}$. Calculate the sum of the terms in this sequence that are not powers of 4. (4) **L3**

19. **LIMPO SEP 2024**

19.1 The quadratic pattern has the following properties:

$T_1 = x$ $T_2 = 7$ $T_4 = 7x$ $T_3 - T_2 = 6$

Determine the value of x (4) **L2**

19.2 Given the geometric series: $2 + \frac{2}{3} + \frac{2}{9} + \ldots\ldots$

19.2.1 Determine the sum to infinity. (3) **L2**

19.2.2 Show that the sum of the first n terms of the series is given by $3-3\left(\frac{1}{3}\right)^n$. (3) **L2**

19.2.3 Calculate the smallest value of n for which the sum of the first n terms is greater than 2,99 (5) **L3**

FUNCTIONS AND INVERSES

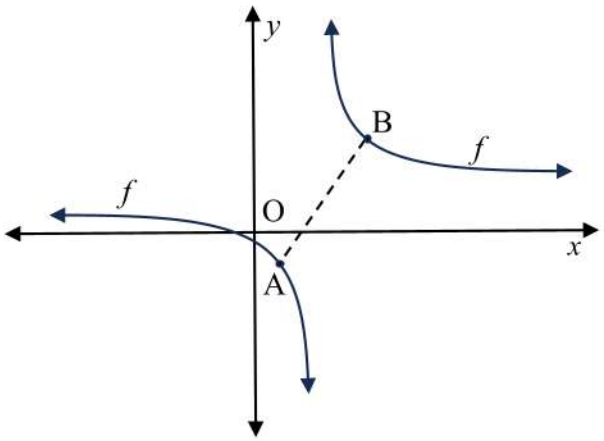
Straight Line	**Parabola**	**Hyperbola**	**Exponential**
m … **gradient** and c … ***y*-intercept**	$y=a(x+p)^2+q$ **Axis of symmetry with equation** $x=-p$ **Maximum** or **minimum value** $(-p;q)$ Turning point	$y=\frac{a}{x+p}+q$ **Vertical asymptote with equation** $x=-p$ **Horizontal asymptote with equation** $y=q$	$y=a.b^{x+p}+q$ $b>0$ **and** $b\neq 1$ **Horizontal asymptote with equation** $y=q$
$m<0$ **… graph is decreasing** $m>0$ **… graph is increasing**	$a<0$ **… graph faces downwards(concave down) and has a minimum turning point** $a>0$ **… graph faces upwards (concave up) and has a maximum turning point**	$a<0$ **… graph is on the second and the fourth quadrant** $a>0$ **… graph is on the first and the third quadrant**	$a<0$ **… graph is below the asymptote** $a>0$ **… graph is above the asymptote**
Domain: $x\in R$ **Range:** $y\in R$	**Domain:** $x\in R$ **Range:** $y>q$ **if** $a>0$ $y<q$ **if** $a<0$	**Domain:** $x\in R,\ x\neq -p$ **Range:** $y\in R,\ y\neq q$	**Domain:** $x\in R$ **Range:** $y>q$ **if** $a>0$ $y<q$ **if** $a<0$
$y-y_1=m(x-x_1)$	$y=ax^2+bx+c$ **Axis of symmetry:** $x=\frac{-b}{2a}$ $y=a(x-x_1)(x-x_2)$ x_1 **and** x_2 **are x-intercepts**	**Axis of symmetry/lines of symmetry:** $\left\{\begin{matrix}y=x+c\\y=-x+c\end{matrix}\right\}$ **substitute point of intersection of asymptotes** **OR** $\left\{\begin{matrix}y=(x-p)+q\\y=-(x-p)+q\end{matrix}\right\}$	

INVERSE FUNCTION		
Straight line	**Parabola**	**Exponential**
$y=mx+c$	$y=ax^2$	$y=b^x$
Inverse is a function $x=my+c$ $y=\frac{x}{m}-\frac{c}{m}$	**Inverse is not a function** $x=ay^2$ $y=\pm\sqrt{\frac{x}{a}}$ **Restrict domain of** $y=ax^2$ **so that the inverse is a function**	**Inverse is a function** $x=b^y$ $y=\log_b x$

	Restrictions: $\left\{\begin{matrix} x \geq 0 \\ x \leq 0 \end{matrix}\right\}$	
Domain: $x \in R$ **Range:** $y \in R$	**Domain:** $x \geq 0$ **or** $x \leq 0$ **Range:** $y > 0$ **if** $a > 0$ $y < 0$ **if** $a < 0$	**Domain:** $x > 0$ **Range:** $y \in R$

	REVISION QUESTIONS			
	KZN MAR 2023			
1.	Given $f(x) = \frac{2}{x}$ and $g(x) = x - 1$			
	1.1	Determine the coordinates of the point(s) where the two graphs intersect.	(4)	**L3**
	1.2	On the same set of axes, draw the two functions. Indicate the coordinates of the point(s) of intersection of the two graphs .	(3)	**L1**
	1.3	Use your graphs to determine the value(s) of x for which: $\frac{2}{x} > x - 1$.	(3)	**L2**
	1.4	If $h(x) = \frac{2}{x+3} - 4$, describe the transformation that takes $f(x)$ to $h(x)$.	(2)	**L3**
	NC SEPT 2022			
2.	Consider: $f(x) = \frac{-2}{x-1} + 2$			
	2.1	Write down the equations of the asymptotes of f.	(2)	**L1**
	2.2	Determine the coordinates of the x-intercepts of f.	(3)	**L2**
	2.3	Sketch the graph of f, clearly show ALL intercept(s) with axes , as well as asymptotes.	(3)	**L2**
	2.4	Graph h is a straight line that passes through the point of intersection of the asymptotes of f. The angle of inclination of h is $135°$. Determine the equation of h in the form $h(x) = mx + c$.	(2)	**L3**
	2.5	Show that $f\left(\frac{1}{x}\right) - f(x) = \frac{2(x+1)}{x-1}$	(3)	**L4**
	IEB MAY 2024			
3.	You are given the following functions: • $g(x) = 3x + c$ • $f(x) = -2x^2 + 15x$			
	If g is a tangent to f. then calculate the coordinates of the point where g cuts the x-axis.		**[5]**	**L2**

4. The graph of $f(x)=\frac{x+1}{x-2}$ has been sketched below.



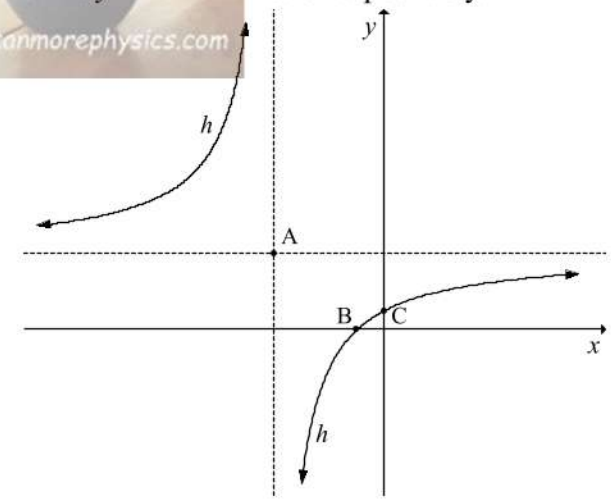
4.1 Determine the equation of line AB, if AB has a minimum length . (3) **L2**

4.2 Hence, determine the coordinates of point A. (Give your answer in the simplest surd form where applicable) . (4) **L3**

4.3 Determine the range of $-f(x)$. (2) **L2**

KZN JUN 2025

5. The sketch below shows the graph of $h(x)=\frac{-9}{x+4}+3$. The asymptotes of h intersect at A. The graph h intersects the x-axis and y-axis at B and C respectively.



5.1 Write down the coordinates of A . (1) **L1**

5.2 Calculate the coordinates of B. (2) **L2**

5.3 Calculate the coordinates of C. (2) **L2**

5.4 Describe the translation from h to $j(x)=\frac{-9}{x}$. (2) **L2**

5.5 Determine the coordinates of the points on j that are closest to the origin. (4) **L3**

KZN JUN 2023 Amended

6. The graph of $h(x)=\frac{a}{3-x}+q$ passes through points $(2;-5)$ and $(0;-1)$

6.1 Write down the equation of the vertical asymptote of h. (1) **L1**

6.2 Write down the domain of h . (2) **L1**

6.3 Determine the values of a and q . (6) **L3**

6.4 Draw the graph of h, showing the intercepts with the axes as well as the asymptotes. (3) **L1**

KZN G11 JUN 2025

7. The straight lines $y = x + 4$ and $y = x - 2$ are the axes of symmetry of the function

$$g(x) = \frac{a}{x+p} + q\,.$$

7.1 Determine the equations of the asymptotes of g. (3) **L2**

7.2 It is given that g passes through $(-7;2)$. Show that $g(x) = \frac{-4}{x+3} + 1$ (3) **L2**

7.3 Calculate the coordinates of the x-intercept of g. (3) **L2**

7.4 Calculate the coordinates of the y-intercept of g. (2) **L1**

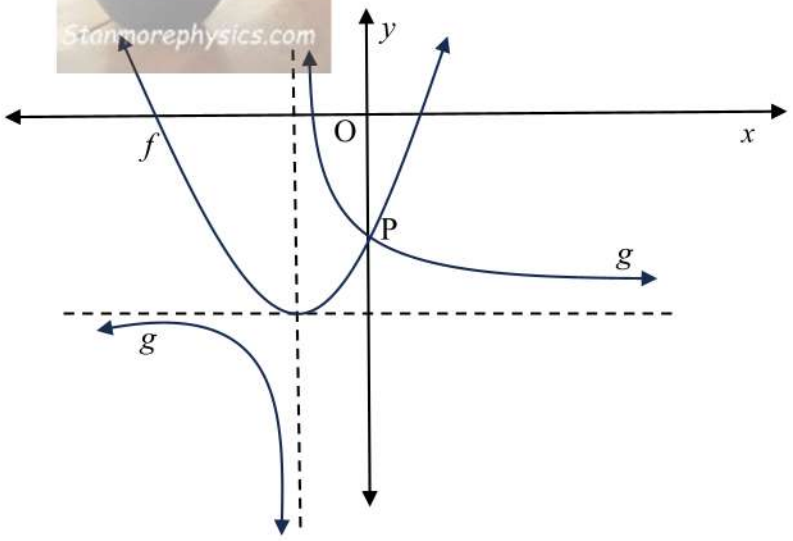
7.5 Draw a neat sketch of g. Clearly show all intercepts with the axes and the asymptotes. (3) **L2**

7.6 It is given that $m(x) = g(x-7) + 2$. Determine the domain of m. (2) **L2**

7.7 Determine the value(s) of x for which $m(x) \geq 0$ (2) **L2**

NW SEPT 2018

8. The graphs of $f(x) = a(x+p)^2 + q$ and $g(x) = \frac{2}{x+1} - 3$ are sketched below. P is the y-intercept of f and g. The horizontal asymptote of g is also a tangent to f at the turning point of f.



8.1 Write down the equation of the vertical asymptote of g. (1) **L1**

8.2 Determine the coordinates of P. (2) **L1**

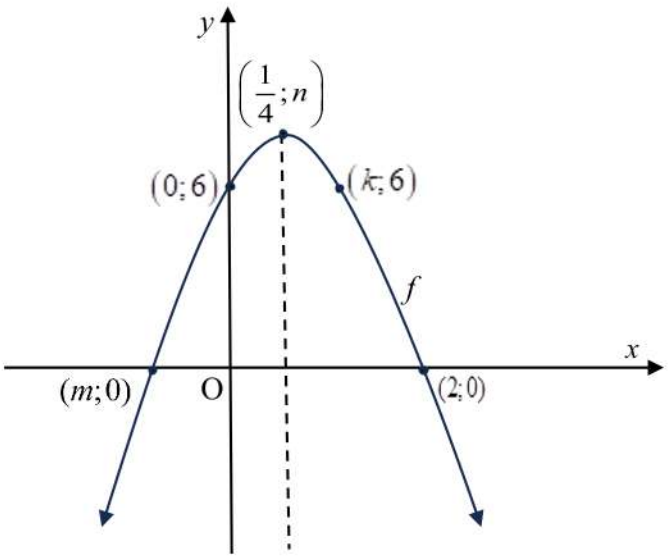
8.3 Determine the equation of f. (3) **L2**

8.4 One of the axes of symmetry of g is a decreasing function. Write down the equation of this axis of symmetry, $h(x)$. (2) **L2**

8.5 For which values of k will $g(x) = h(x) + k$ have two real roots that are of opposite signs? (2) **L3**

8.6 Give the domain of $m(x)$ if $m(x) = g(2x) + 5$. (3) **L3**

9. The diagram shows the graph of a parabola f(x) which intersect the x-axis at $(m;0)$ and at $(2;0)$. It is further given that $\left(\frac{1}{4};n\right)$ is the turning point of the parabola while $(0;6)$ and also $(k;6)$ point on the curve off.



Determine:

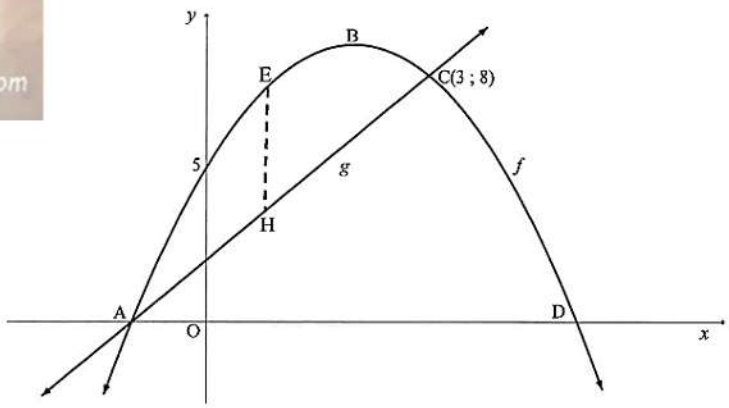
9.1 the value of k. (1) **L1**

9.2 The value of m. (2) **L2**

9.3 The value of n (Show all your working). (5) **L3**

DBE NOV 2024

10. In the diagram below, the graphs of $f(x) = -x^2 + 4x + 5$ and g, a straight line, are drawn. C(3;8) is a point of intersection of f and g. EH is drawn parallel to the y - axis, with E a point on f and H a point on g.



10.1 Calculate the coordinates of B, the turning point of f. (3) **L1**

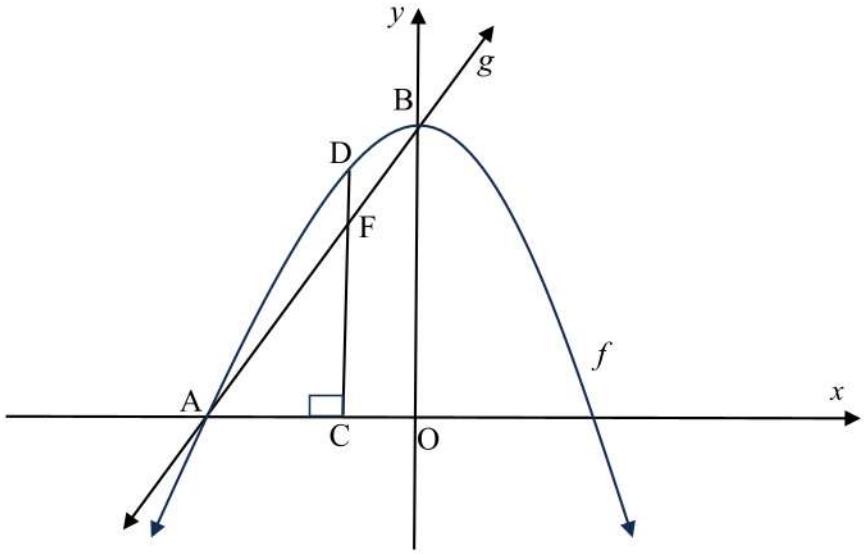
10.2 Show that the equation of the line through A and C is given by $g(x) = 2x + 2$ (3) **L2**

10.3 Calculate the maximum length of EH for $f > g$. (4) **L2**

10.4 Given: $k(x) = f(x+m) = -x^2 - 2mx - m^2 + 4x + 4m + 5$. (5) **L4**

Determine the value of m such that g is a tangent to k.

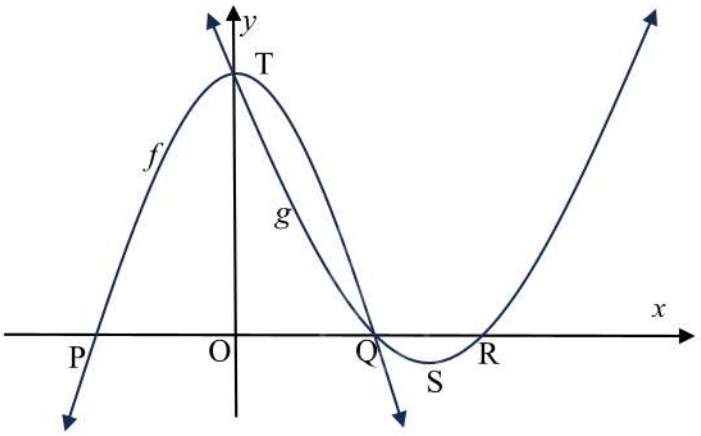
11 In the sketch, A is an x-intercept and B is the turning point of the parabola $f(x)=ax^2+bx+c$. The straight-line AB is defined as $g(x)=3x+6$. D$\left(-\frac{1}{3};\frac{35}{6}\right)$ is a point on f and CD is perpendicular to the x-axis. B is the y-intercept of f and g.



11.1	Determine the coordinates of B.	(1)	**L1**
11.2	Determine the coordinates of A.	(2)	**L1**
11.3	Determine the equation of the parabola f, in the form $y=...$	(4)	**L2**
11.4	Calculate the length of CF.	(2)	**L2**
11.5	A new parabola is obtained when the y-axis is moved 2 units to the left. Write down the coordinates of the turning point of the new parabola.	(2)	**L2**
11.6	A new graph p is obtained when f is translated 6 units downwards. Write down the equation of p in the form $p(x)=...$	(1)	**L2**
11.7	Determine the equation of p^{-1}, the inverse of, p in the interval $x\le 0$. Write down your answer in the form $y=...$	(2)	**L3**
11.8	Is p^{-1} a function? Motivate your answer.	(2)	**L2**
11.9	Graph g is a tangent to $h(x)=\frac{k}{x}$. Determine the value of k.	(5)	**L4**

DBE NOV 2015

12.1 The graph of $f(x)=-2x^2+18$ and $g(x)=ax^2+bx+c$ are sketched below. Point P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g



12.1.1	Write down the coordinates of T.	(1)	**L2**
12.1.2	Determine the coordinates of Q.	(3)	**L3**
12.1.3	Given that $x=4,5$ at S, determine the coordinates of R.	(2)	**L2**
12.1.4	Determine the value(s) x for which $g''(x)<0$.	(2)	**L2**
12.2	The function defined as $y=\frac{a}{x+p}+q$ has the following properties	(4)	**L3**

- The domain is $x \in R,\ x \neq -2$
- $y = x + 6$ is an axis of symmetry
- The function is increasing for all $x \in R,\ x \neq -2$

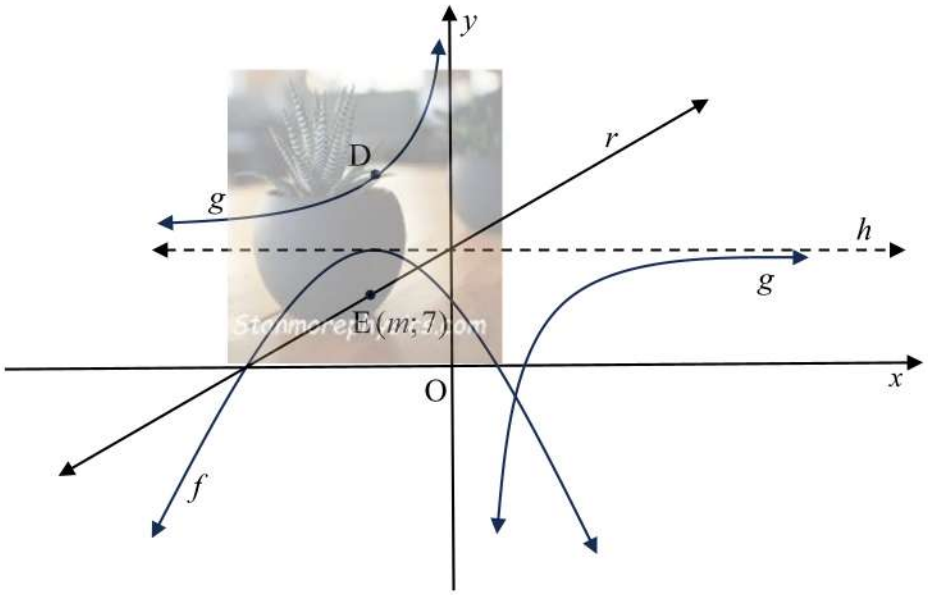
Draw a neat sketch graph of this function. Your sketch must include asymptote if any.

NW SEPT 2022 (Adapted)

13. Below are the graphs of $f(x) = -2(x+p)^2 + q$ and $g(x) = \frac{-3}{x} + n$.

- $h(x) = n$, an asymptote of g, is also a tangent of f.
- The line $r(x) = x + 8$ is an axis of symmetry of g.

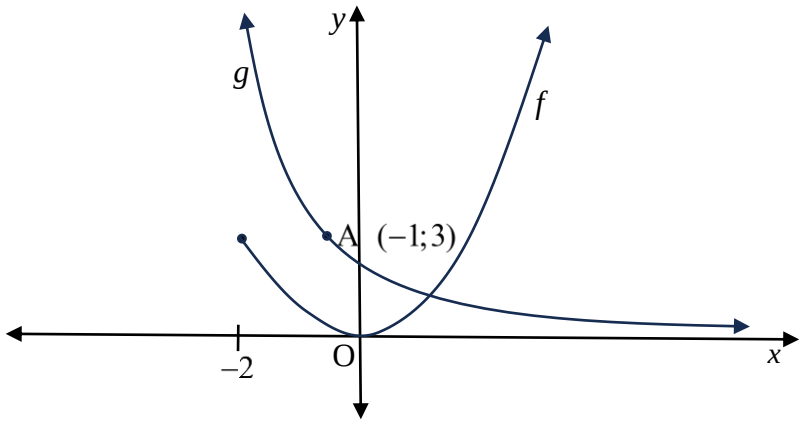
$r(x) = x + 8$ also intersects the axis of symmetry of f in the point $\text{E}(m;7)$.



13.1	Write down the domain of g.	(2)	**L1**
13.2	Calculate the value of m.	(2)	**L2**
13.3	Write down the value of n.	(1)	**L1**
13.4	Given: $f(x) = -2(x+p)^2 + q$. Write down the values of p and q.	(2)	**L2**
13.5	If it is given that $f(x) = -2x^2 - 4x + 6$, calculate the x-intercepts of f.	(3)	**L2**
13.6	The axis of symmetry of f intersects the graph of g at point D. Determine the coordinate of D.	(2)	**L2**
13.7	Determine the equation of the tangent to g at point D. Write down your answer in the form $y = bx + c$.	(5)	**L3**
13.8	Determine the equation of $k(x)$ in the form $k(x) = \frac{a}{x+t} + s$ if k is the reflection of g about the line $x = 2$.	(3)	**L3**
13.9	Determine the value(s) of k for which the equation of $g(x+4) + k = 0$ will have a root that is less than -5.	(3)	**L4**

NW SEPT 2018

14. The diagram below shows the curves of $f(x)=\frac{1}{4}x^2$, where $x\geq -2$ and $g(x)=a^x$, where $a>0$. The point $A(-1;3)$ lies on the graph of g.



14.1 Show that $g(x)=\left(\frac{1}{3}\right)^x$ (1) **L1**

14.2 For which value(s) of x is the graph of f strictly decreasing? (2) **L2**

14.3 Determine the inverse of f in the form $y=...$ (2) **L1**

14.4 Sketch the graph of f^{-1}. (2) **L2**

14.5 Write down the range of f^{-1}. (2) **L2**

14.6 Determine the inverse of g in the form $y=...$ (2) **L2**

14.7 For which values of x will $g^{-1}(x)\geq -1$? (2) **L3**

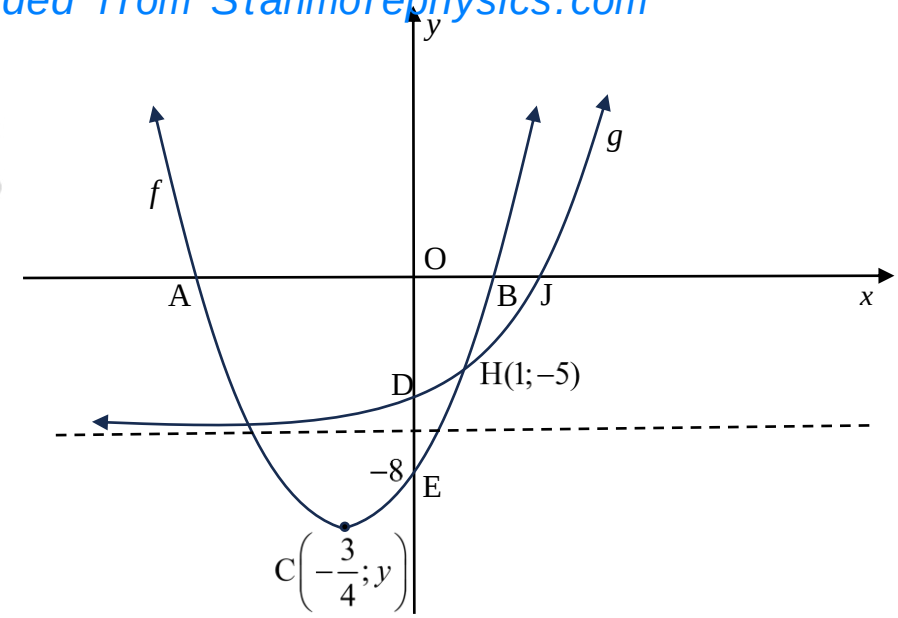
GRD 11 KZN JUN 2025

15. The function of $g(x)=b^x+q$ has the following properties:

- $q>0$
- $b>0$
- Draw a neat sketch of g. Indicate clearly on your sketch the asymptote and the y - intercept. **[3]** **L3**

EC SEPT 2020

16. The diagram below shows the graphs of $f(x)=ax^2+bx+c$ and $g(x)=b^x+q$. A and B are the x-intercept, E is the *y*-intercept and $C\left(-\frac{3}{4};y\right)$ is the turning point J is the *x*-intercept and D is the *y*-intercept of *g*. $y=-8$ is the equation of the asymptote of *g*. $H(1;-5)$ is one of the points of the intersection of *f* and *g*.



16.1 Write down the coordinates of D. (1) **L1**

16.2 Write down the value of q. (1) **L1**

16.3 Show that $a=2$, $b=-3$ and $c=-10$. (6) **L3**

16.4 Write down the range of g (2) **L2**

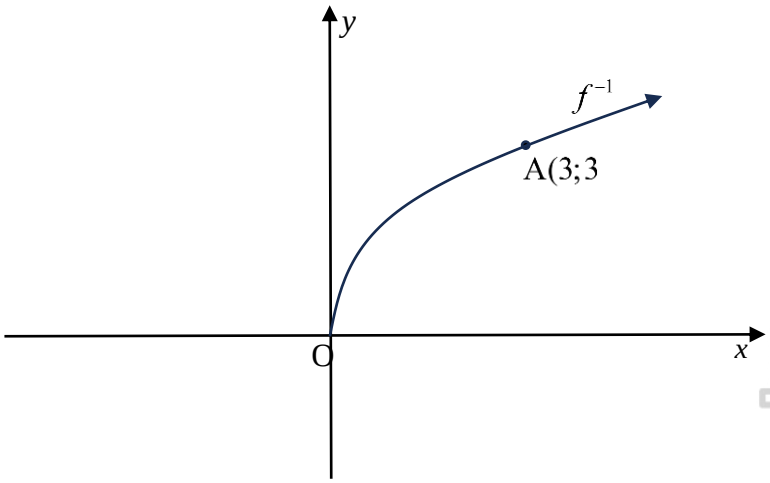
16.5 The line with equation $y+9x=-28$, is the tangent of f at a point T.
Determine the coordinates of T. (5) **L3**

16.6 Given that $h(x)=g(x)+8$, write down $h^{-1}(x)$ in the form $y=...$ (2) **L2**

16.7 Given that $p(x)=f(x)+1$, determine the values of x for which $x.p(x)<0$. (4) **L3**

KZN MAR 2025

17.1 The graph of $f^{-1}(x)=\sqrt{3x},\ x\geq 0$ is drawn in the sketch. f^{-1} passes through the point A(3;3).



17.1.1 Determine the equation of f in the form $y=...$ (3) **L1**

17.1.2 For which values of x will $f(x)\leq f^{-1}(x)$? (3) **L2**

17.2 Consider $g(x)=\left(\frac{1}{3}\right)^x$

17.2.1 Determine the equation of g^{-1} in the form $y=...$ (2) **L2**

17.2.2 Draw a sketch graph of g^{-1}, indicating any intercepts with the axes as well as one more point on the graph . (3) **L2**

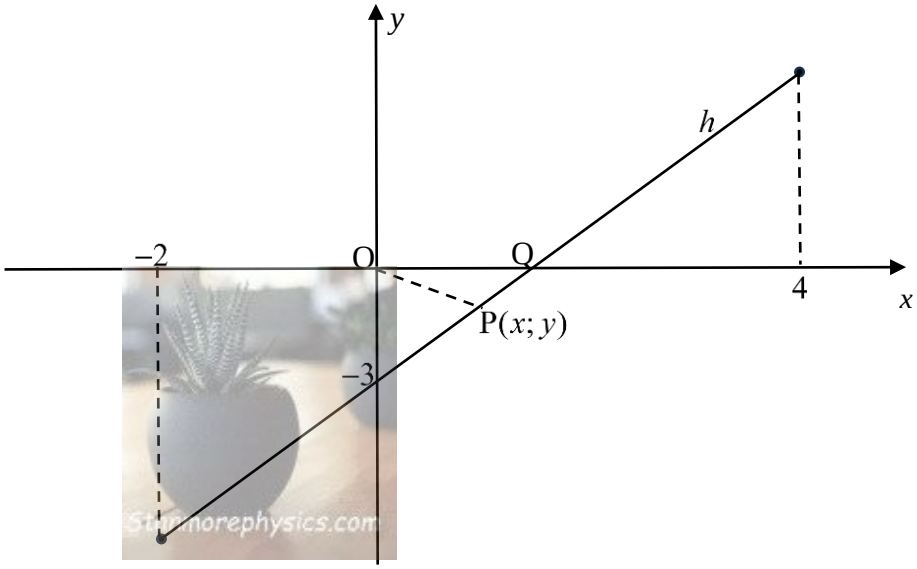
17.2.3 The graph of $h(x)=a\left(\frac{1}{3}\right)^x+7$ passes through the point $(-2;10)$.

Calculate the value of a . (2) **L3**

17.2.4 Describe the transformation from h to g . (3) **L2**

DBE NOV 2015

18. Given $h(x)=2x-3$ for $-2\le x\le 4$. The x - intercept of h is Q .



18.1 Determine the coordinates of Q . (2) **L1**

18.2 Write down the inverse of h^{-1}. (3) **L1**

18.3 Sketch the graph of h^{-1} in your answer book, clearly indicating the y - intercept and the end points . (3) **L2**

18.4 For which values of x is $h(x)=h^{-1}(x)$? (3) **L3**

18.5 $P(x;y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5) **L2**

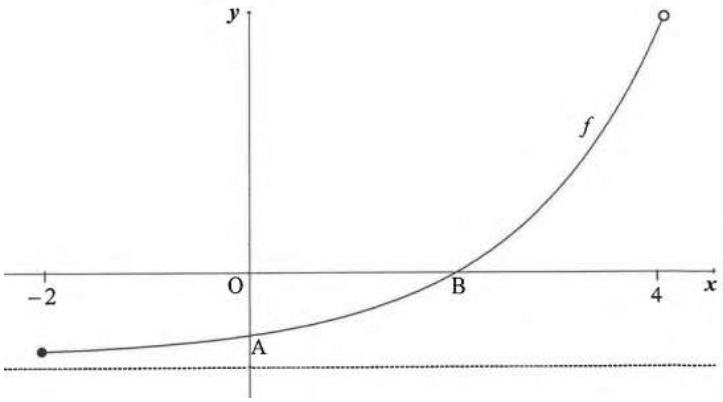
18.6 Given $h(x)=f'(x)$ where f is a function defined for $-2\le x\le 4$

18.6.1 Explain why f has a local minimum . (2) **L2**

18.6.2 Write down the value of the minimum gradient of the tangent to the graph of f . (1) **L2**

DBE NOV 2023

19. Sketched below is the graph of $f(x)=2^x-4$ for $x\in[-2;4)$. A and B are respectively y and x-intercepts of f.



19.1 Write down the equation of the asymptote of f. (1) **L1**

19.2 Determine the coordinates of B. (2) **L2**

19.3 Determine the equation of k, a straight line passing through A and B in the form $k(x)=...$ (3) **L2**

19.4 Calculate the vertical distance between k and f at $x=1$. (3) **L3**

19.5 Write down the equation of g if it is given that $g(x)=f(x)+4$. (1) **L2**

19.6 Write down the domain of g^{-1}. (2) **L2**

19.7 Write down the equation of g^{-1} in the form $y=...$ (2) **L2**

DBE NOV 2021

20. The graph of $f(x)=\log_4 x$ is drawn below. B $(k;2)$ is a point *f*.

20.1 Calculate the value of k. (2) **L1**

20.2 Determine the values of x for which $-1\le f(x)\le 2$ (2) **L2**

20.3 Write down the equation of f^{-1}, the inverse of f, in the form of $y=...$ (2) **L2**

20.4 For which values of x will $x.f^{-1}(x)<0$? (2) **L1**

TOPIC	**4. FINANCIAL MATHEMATICS (15 ± 3)**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

SIMPLE INTEREST AND COMPOUND INTEREST (A > P)

- On **Simple interes**t, the interest is calculated on the **original** amount invested or borrowed.
  $A=P(1+in)$
- On C**ompound interest**, the interest is calculated on the accumulated amount.
  $A=P(1+i)^n$

DEPRECIATION (A < P)

- For depreciation we use:
  $A=P(1-in)$ **Straight line depreciation**
  $A=P(1-i)^n$ **Reducing balance depreciation**

COMPOUNDING PERIOD	**INTEREST (i)**	**Period (n)**
Monthly	$\frac{i}{12}$	$n\times 12$
Quarterly	$\frac{i}{4}$	$n\times 4$
Half yearly/Semi-annually	$\frac{i}{2}$	$n\times 2$
The above is true provided n is in years		

EFFECTIVE AND NOMINAL INTEREST RATES

- For the annual effective rate, we use the formula: $1+i_{eff}=\left(1+\frac{i_{nom}}{m}\right)^{m}$
- When working with different compounding periods use the formula: $(1+i_{new})^{m}=(1+i_{nom})^{n}$

  where n is the number of time periods at the original compounding rate and m is the number of time periods at the new compounding rate

ANNUITIES

- An annuity is a series of equal payments made at regular time intervals.
- The annuity formulae are used under the following conditions:
  - ✓ All payments are equal
  - ✓ The payments are made at regular intervals
  - ✓ The interest rate remains fixed and the compounding period for interest is the same as the payment intervals

THE FUTURE VALUE

- We can use the following formula to calculate the future value of an annuity:

$$F=\frac{x\left[(1+i)^{n}-1\right]}{i}$$

F is the future value.
x is the payment.
i is the interest rate per interval.
n is the number of payments.

THE PRESENT VALUE

- We can use the following formula to calculate the present value of an annuity:

$$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$$

P is the present value.
x is the payment.
i is the interest rate per interval.
n is the number of payments.

SINKING FUND

- $A=P(1-i)^{n}$ (Scrap value of old asset)
- $A=P(1+i)^{n}$ (Cost of new asset)
- Sinking fund = new – old
- Calculate x
- Withdrawals (calculate x_{new}) treat it separately and add it back

THE OUTSTANDING BALANCE ON A LOAN

Outstanding Balance = Loan with interest to date – Repayments with interest to date

$$OB=P(1+i)^{n}-\frac{x\left[(1+i)^{n}-1\right]}{i} \quad \textbf{OR} \quad P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$$

Note:

- When using the P formula, use the remaining number of payments.
- When using OB = A – F , use n as number of payments made.

DELAYED/ DEFERRED ANNUITIES

- When the first payment of a loan is made more than one period after the loan was received, this payment is referred to as a ***deferred annuity.***
- Apply the compound interest to the loan to move it to the same point on the timeline as the present value of the annuity

THE LAST/FINAL PAYMENT

Last payment = Outstanding balance after the last full payment multiplied by $(1+i)^{1}$

MISSED PAYMENTS

To calculate the new payment:

- We calculate the outstanding balance immediately after the last payment made.
- We then apply the compound interest to this outstanding balance, till one period before payments resume. The result is the present value of the new annuity consisting of all the remaining payments.

1 MIND THE GAP STUDY GUIDE

1.1 Mary borrowed a certain sum of money from a bank at a compound interest rate of 15% calculated quarterly. After 3 years she now owes R 7 000. How much did she borrow? (3) **L2**

1.2 R 1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R 23 000? (4) **L2**

1.3 R2 000 was invested in a fund paying interest compounded monthly. After 18 months the value of the fund was R2 860, 00. Calculate the interest rate. (4) **L2**

2 IEB NOV 2011

2.1 Sapna bought a new computer. It depreciated in value from R 12 000 to R 7 500 over a period of 3 years. Calculate the rate of depreciation per year, using depreciation on a reducing balance. (4) **L2**

2.2 Mr. Kekana takes a bank loan of R110 400 to be paid back over 60 months at an interest rate of 10% p.a. compounded monthly. Calculate the value of his monthly payments. (4) **L2**

2.3 Ayanda's father agreed to buy her a car costing R 120 000 for her 21st birthday.
He had one condition: she had to supply the 8% deposit required.

2.3.1 Determine the amount that Ayanda had to supply as a deposit. (1) **L1**

2.3.2
- Ayanda decided to invest every year's birthday money in a savings account, starting on her 18th birthday.
- At that stage, the bank gave an interest rate of 8,5% p.a. compounded monthly.
- On her 18th birthday, she received R 2 650.
- On her 19th birthday, she received R 3 340, but the interest rate then changed to 12% p.a. compounded quarterly.
- On her 20th birthday she received R 2 400.
- Six months after her 20th birthday, she withdrew R 1 200 to pay towards a holiday.

Summarise this information on a timeline. (2) **L2**

2.3.3 Dtermine whether Ayanda was able to supply the deposit. (5) **L2**

3 DBE FEB/MAR 2014

Susan buys a car for R350 000. She secures a loan at an interest rate of 7% p.a., compounded monthly. The monthly instalment is R 6 300. She pays the first instalment one month after the loan was secured.

3.1 Calculate the effective annual interest rate on the loan. Leave your answer correct to TWO decimal places. (3) **L2**

3.2 How many months will it take to repay the loan? (5) **L2**

3.3 Calculate the value of the final instalment. (5) **L3**

3.4 The value of the car depreciates at *i* % p.a. After 3 years its value is R 252 000. Calculate *i*. (3) **L2**

4 GP SEP 2024

4.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (3) **L2**

4.2 Micaela buys furniture to the value of R 10 000. She borrows the money on 1 February 2023 from a financial institution that charges interest at a rate of 9,5% p.a. compounded monthly. Micaela agrees to pay monthly instalments of R 450. The loan agreement allows Micaela to start paying equal monthly instalments from 01 August 2023.

4.2.1 Calculate the total amount owing to the financial institution on 1 July 2023. (3) **L2**

4.2.2 How many months will it take Micaela to pay back the loan? (4) **L2**

4.2.3 What is the balance of the loan immediately after Micaela has made the 25th payment? (3) **L3**

5 FS SEP 2024

5.1 The value of a vehicle worth R 150 000 depreciates at 13% p.a. Calculate the value of the vehicle in 6 years if depreciation is calculated on the reducing-balance method. (3) **L2**

5.2 A loan of R300 000 is taken out at an interest rate of 5,3% p.a. compounded quarterly. The loan was taken out on the 1st of March 2016. The first payment was made on 1 December 2016 and is repaid in 72 equal quarterly payments.

5.2.1 What is the outstanding balance of the loan on 1 September 2016? (3) **L2**

5.2.2 Determine the quarterly repayments required to pay back the loan. (3) **L2**

5.3 Gert landed a job which remunerated him R 27 562,50 quarterly. He then decided to open an investment account and deposit 11% of his salary at the end of every quarter into the investment account, earning an interest of 7, 9% p.a. compounded monthly for eight years.
What amount will be in the account at the end of eight years? (6) **L3**

6 IEB NOV 2018

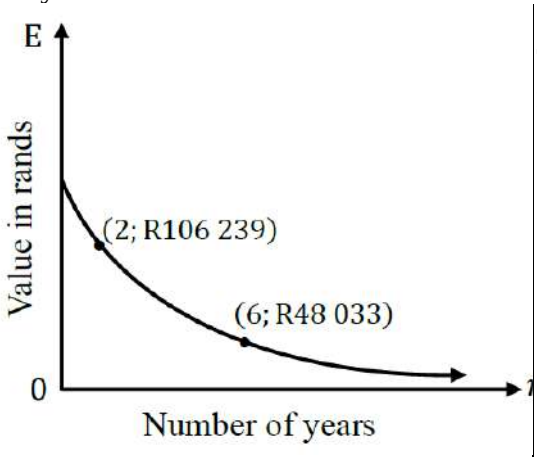
6.1 Riyan opened a bank account 15 years ago, with the intention of saving money for when he retires. The bank offered him an interest rate of 16% per annum compounded monthly for the first 5 years and thereafter changed the interest rate to 11% per annum (compounded annually). Riyan made an immediate deposit of R 300 000 upon opening the account. He then withdrew R 500 000 at the end of 13 years.
Calculate how much money he would have in this account at the end of the 15th year. (5) **L2**

6.2 If instead, Riyan had taken a retirement annuity over the same period of 15 years, and the insurance company had offered him 8% per annum compounded monthly, what would his **monthly payments** have been if he were to save an amount of R 1 270 000 at the end of the 15th year? (4) **L2**

7 CAPE WINELANDS SEP 2024

7.1 Determine how long it will take for an investment of R 20 000 to grow to R 45 000 in an account earning interest at 7,5% p.a., compounded monthly.
Give your answer to the nearest month. (3) **L2**

7.2



The depreciation of office equipment is represented by the graph alongside.
E is the value of the equipment in Rands and n is the number of years the equipment is being used.

7.2.1 What does the y-intercept of the graph indicate? (1) **L1**

7.2.2 According to this model, will the value of the equipment ever become R0? Explain your answer. (1) **L1**

7.2.3 Calculate the annual rate of depreciation. (3) **L2**

7.3 Herman plans to purchase a house for R 2 464 000. He will take out a loan for the full amount for 30 years at 10,2% p.a., compounded monthly.

7.3.1 Calculate what his instalments will be if he is required to repay the bank equal amounts at the end of each month. (4) **L2**

7.3.2 Instead of paying the bank-stipulated amount calculated in QUESTION 7.3.1, Herman decides to repay the loan by R22 500 instead, at the end of every month. Calculate the balance outstanding on the loan immediately after he pays his 84th payment. (3) **L3**

8 NC SEP 2024

8.1 On the 1 April 2019, Lebere deposited R 5 000 into a savings account at an interest rate of r% per annum, compounded quarterly.

8.1.1 On 1 April 2023, the amount in the savings account was R 12 980. Calculate r. (3) **L2**

8.1.2 He used R 7 580 of the money to buy a new laptop. The value of the laptop depreciated at a rate of 22% per annum, according to the reducing balance method. After how many years will its book value be R 1 707? (3) **L2**

8.2 Andy takes up a loan of R 850 000. The loan is repaid over 20 years at 8,5% p.a., compounded monthly.

8.2.1 Calculate the value of Andy's monthly instalment. (4) **L2**

8.2.2 Calculate how much Andy will still owe the bank after his 26th payment. (3) **L2**

8.2.3 Hence, calculate the total interest Andy will have paid over the first 26 months. (2) **L2**

9 DEB FEB/MAR 2015

9.1 For each of 3 years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year.
At the end of 2012 the population of these two towns were equal.

Determine the ratio of the population of town X (call it P_X) to the population of town Y (call it P_Y) at the beginning of 2010. (4) **L2**

10 DBE NOV 2012

10.1 A business buys a machine that costs R 120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

10.1.1 Determine the scrap value of the machine at the end of 5 years. (3) **L2**

10.1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3) **L2**

10.1.3 The business estimates that it will need R 90 000 by the end of five years. A sinking fund for R 90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately, and the last payment will be made at the end of the 5-year period. Calculate the value of the monthly payment into the sinking fund. (5) **L3**

10.2 Lorraine receives an amount of R 900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly. She needs an amount of R 18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.
For how many months will she be able to live from her investment? (6) **L3**

11 DBE NOV 2016

11.1 On 1 June 2016 a bank granted Thabiso a loan of R 250 000 at an interest rate of 15% p.a. compounded monthly, to buy a car. Thabiso agreed to repay the loan in monthly instalments commencing on 1 July 2016 and ending 4 years later on 1 June 2020.

However, Thabiso was unable to make the first two instalments and only commenced with the monthly instalments on 1 September 2016.

11.1.1 Calculate the amount Thabiso owed the bank on 1 August 2016, a month before he paid his first monthly instalment. (2) **L2**

11.1.2 Having paid the first monthly instalment on 1 September 2016, Thabiso will still pay his last monthly instalment on 1 June 2020. Calculate his monthly instalment. (4) **L2**

11.1.3 If Thabiso paid R 9 000 as his monthly instalment starting on 1 September 2016, how many months sooner will he repay the loan? (5) **L2**

11.1.4 If Thabiso paid R 9 000 as a monthly instalment starting on 1 September 2016, calculate the final instalment to repay the loan. (4) **L3**

12 DBE EXEMPLAR NOV 2014

12.1 Siphokazi bought a house. She paid a deposit of R 102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

12.1.1 Determine the selling price of the house. (1) **L1**

12.1.2 The period of the loan is 20 years, and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4) **L2**

12.1.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2) **L2**

12.1.4 Calculate the balance of her loan immediately after her 85^{th} instalment. (3) **L2**

12.1.5 She experienced financial difficulties after the 85^{th} instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89^{th} month. (2) **L2**

12.1.6 She decides to increase her payments to R 8 500 per month from the end of the 90^{th} month. How many months will it take to repay her bond after the new payment of R 8 500 per month? (4) **L2**

13 DBE NOV 2024

13.1 Tshepo was granted a loan of R 100 000 on 1 March 2022 at an interest rate of 13,5% p.a., compounded monthly. Tshepo agreed to repay the loan over 5 years in monthly instalments of R2 300,98, starting on 1 April 2022.

13.1.1 Calculate the total interest that he will pay over the 5 years. (2) **L1**

13.1.2 Tshepo paid R 22 300,98 (his monthly instalment and an additional R 20 000) on 1 March 2024 into the loan account. He continues to pay the original monthly instalment thereafter. How many months earlier will Tshepo repay the loan? (7) **L3**

14 DBE MAY/JUN 2024

14.1 Six years ago, Thabo bought a phone for R 13 000. The value of the phone is depreciated annually according to the reducing-balance method. The value of the phone is now R 8 337,75. Calculate the annual rate of depreciation. (3) **L2**

14.2 Eric and Thandi need to save R 80 000 each to go on a holiday at the end of December 2027.

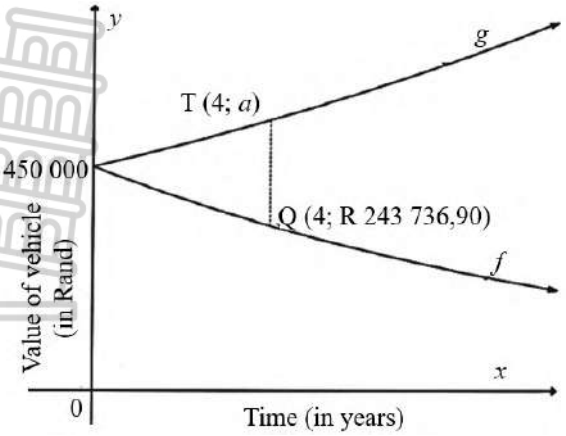
- Thandi decides that she will start saving at the end of January 2025. She will make 36 monthly deposits into a savings account that pays interest at 8,6% p.a., compounded monthly. The deposit will be made at the end of each month.
- Eric calculates that if he makes 48 deposits of R 1 402,31, starting at the end of January 2024, he will have enough money to go on holiday. He will make his deposits into a savings account at the end of each month. The savings account pays interest at 8,6% p.a., compounded monthly.

Calculate the difference between the total amount that Eric and Thandi will deposit into their respective savings accounts over the given period. (4) **L2**

14.3 Lesibana was granted a loan of R225 000. The rate of interest for the loan is 9% p.a., compounded monthly. Lesibana will make monthly payments of R5 500, starting (6) **L2**

exactly four months after the loan was granted. How many payments will Lesibana make to settle the loan?

15 **DBE NOV 2015**



The graph of f shows the book value of a vehicle x years after the time Joe bought it. The graph of g shows the cost price of a similar new vehicle x years later.

15.1 How much did Joe pay for the vehicle? (1) **L1**

15.2 Using the reducing-balance method, calculate the % annual rate of depreciation of the vehicle. (4) **L2**

15.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a. (3) **L2**

15.4 A vehicle that costs R 450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13th month, and the last payment will be made at the end of the 48th month. The sinking fund earns interest at a rate of 6,2% p.a. compounded monthly.
Calculate the monthly payment to the fund. (5) **L3**

TOPIC	**DIFFERENTIAL CALCULUS**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

TEACHING APPROACHES (CALCULUS)

1. FIRST PRINCIPLES:

The learners:

- ✓ Need to understand what is meant by determining the gradient from first principles and know the first principles formula.
- ✓ must be able to copy the first principle formula from the formula sheet correctly.
- ✓ Be able to simplify the first principles expression (It seems as if learners handled this question better when they determine $f(x+h)$ separately and then bring it back to the formula).
- ✓ Need to be mindful of the notation and apply it correctly when they simplify the first principle expression.
- ✓ At this stage, learners can also determine the equation of the tangent at a point.

2. RULES FOR DIFFERENTIATION

- ✓ The learners:
  - i. need to revise how to simplify surds, rational, irrational exponents.
  - ii. Must know how to simplify expressions before differentiation.
  - iii. Must know how to tell which variable they are required to differentiating with respect to.
- ✓ Must expose themselves to variety of questions having different notations including where a variable is given as constant.
- ✓ Following instructions is once more important, on how the answer should be provided

whether with a + positive or – negative.

- ✓ Must always use of correct notation.

3. CUBIC FUNCTIONS $f(x) = ax^3 + bx^2 + cx + d$

The learners need to know and follow these steps when sketching a cubic function:

- ✓ Before learners can sketch a cubic function, they at least need to know the shape of their graph as guided by value of a where a could be $a > 0$ and $a < 0$.
- ✓ The learners must be able to Factories a third-degree polynomial using any other method to determine the ***x*-intercepts** (the *x*-intercepts are known as the: zero, roots, $f(x) = 0$. It would be an advantage if they can be able factories using a calculator.
- ✓ They must also be able to find the ***y*- intercept**, which is when $x = 0$, or given by the value of *d*.
- ✓ Learners must be able to use the first derivative to find the coordinates of the turning points, which are also known as the Stationery points or local minima and local maxima. In simple terms, this is finding $f'(x) = 0$, solve for *x*, and then find the corresponding *y*-values to give the coordinate of the turning point.
- ✓ Examiners often require learners to write the intercepts with the axes, stationary points and points of inflection in coordinate form $(a\ ;\ b)$ .Make sure that the learners are aware of this.

4. INTERPRETATION OF A CUBIC FUNCTION:

The learners must be able to:

- ✓ Tell what the domain is, that $x \in R$
- ✓ Understand the relationship between the graph of a function and the graph of its derivative is important in that it explains to the learners why the second derivative is zero at a point of inflection.
- ✓ Understand that the point of inflection is determined by equating the second derivative to zero and solving for *x*. An alternative method is to add up the *x*-coordinates of the turning points and divide by 2 (i.e. determining the midpoint of the two turning points).
- ✓ Tell for which values of *x* will $f(x)$ be concave up: $f''(x) > 0$ & Concave down: $f''(x) < 0$
- ✓ Tell where *f* is increasing or decreasing: increasing ($f'(x) > 0$, decrease $f'(x) < 0$).
- ✓ Determine the values of *x*, for which: $x.f(x) > 0$, $f'(x) > 0$, $f'(x).f(x) < 0$
- ✓ when will f have three real roots, two real roots or one real root?

5. OPTIMIZATION

The learners need to develop the conceptual understanding on Optimization

- **Calculus of motion**
  - ✓ In this regard, the equation will be given.
  - ✓ The learners need to know that, Velocity is the derivative of displacement, and
  - ✓ Acceleration (2^{nd} derivative) is the derivative of velocity
- **Rates of change**
  - ✓ Knowledge of formulae for the surface area and volume of right prisms is required from learners.
  - ✓ A list of relevant formulae will only be provided for the surface area and volume of cones, spheres and pyramids. Learners must select the correct one to use.

REVISION QUESTIONS

1							
1.1	**EC SEPT 24** Determine $f'(x)$ from first principles $f(x) = x^2 - 3$.	(4)	**L2**	1.2	**FS SEPT 24** Determine $f'(x)$ from first principles if it is given that $f(x) = -1 + 4x^2$.	(5)	**L2**
1.3	**GP SEPT 24**	(4)	**L2**	1.4	**KZN SEPT 24**	(5)	**L2**

	If $f(x)=-2x^2+3$, determine $f'(x)$ from first principle.				Given: $f(x)=-x^2+x$. Determine $f'(x)$ from first principles.		
1.5	**MP SEPT 24** Given: $f(x)=\frac{-2}{x}$, determine $f'(x)$ from first principles.	(5)	**L2**	1.6	**NW SEPT 20** Given: $f(x)=-x^2+7x+9$, determine $f'(x)$ from first principles.	(3)	**L2**
1.7	**MIND ACTION GR12** Determine $f'(x)$ from first principles if $f(x)=4$	(3)	**L2**	1.8	**MATHS HANDBOOK GR12** Determine $f'(x)$ from first principles if $f(x)=x$	(5)	**L1**
1.9	**FS SEPT 16** Determine $f'(x)$ from first principles if $f(x)=x^3$ and hence find $f'(-2)$	(5)	**L2**	1.10	**KZN JUN 20** Determine $f'(x)$ from first principles given $f(x)=x^2-bx$.	(5)	**L2**
2	**Rules for Differentiation:**						
2.1	**EC SEPT 24** Determine $D_x\left(\frac{x^2-5x}{x^3}-\sqrt{x}\right)$	(4)	**L2**	2.2	**FS SEPT 23** Determine $\frac{dy}{dx}$ if $y=3x^4-\frac{7}{x}+2.\sqrt[3]{x^2}$	(4)	**L3**
2.3	**KZN SEPT 24** Determine the derivative of $f(x)=\frac{2x^2+3}{\sqrt{x}}$	(4)	**L3**	2.4	**LP SEPT 24** Determine $f'(x)$ if $f(x)=(x-1)(x^6+x^5+x^4+x^3+x^2+x+1)$	(3)	**L1**

2.5	**LP SEP 23** Determine $\frac{dy}{dt}$ given that $y=2t^5+\sqrt[4]{t^7}$	(3)	**L2**	2.6	**KZN MAR 20** Determine $f'(x)$ if $f(x)=\frac{x^3-8}{2-x}$	(4)	**L2**
2.7	**MP SEP 16** $\frac{dy}{dx}$ if $y=-2\sqrt{x}+x-\frac{1}{\sqrt{x}}$	(4)	**L2**	2.8	**MP SEP 17** Determine $D_x\left[\left(x^2-2\right)\left(\frac{1}{x^2}+3\right)\right]$	(4)	**L2**
2.9	**KZN SEPT 23** Determine $\frac{dy}{dx}$ if $\sqrt{y+x}=x+3$	(3)	**L2**	2.10	**MP SEP 24** Determine $Dx\left[\sqrt[5]{\frac{32}{x^3}}\right]$	(3)	**L2**
2.11	**WC SEPT 18** Differential with respect to x, $xy=\left(x-\frac{1}{x^2}\right)\left(x+\frac{1}{x^2}\right)$	(4)	**L3**	2.12	**EC SEPT 20** Determine $D_t\left[\frac{1}{2}gt^2-\frac{5}{t}+3g\right]$	(4)	**L3**
3							
3.1	**EC SEPT 24** Suppose that $g(x)$ represents the rate of change of $h(x)=-3x^3-3x^2+1$. Calculate the largest value of $g(x)$.	(3)	**L2**	3.2	**NC SEPT 24** The line $y=-4x+k$ is a tangent to $f(x)=\frac{9}{x}-3x$ at $S(a,b)$. Determine the value(s) of a.	(4)	**L3**
3.3	**EC SEPT 16** Given $s(t)=t^3$. Show that the gradient of any tangent to s will never be negative.	(2)	**L3**	3.4	**FS SEPT 17**	(6)	**L2**

					Given $f(x)=x^3-2x^2$ Determine the equation of the tangent to f at the point where $x=2$		
3.5	**FS SEPT 17** The line $g(x)=-\frac{1}{8}x+p$ is a tangent to the graph of $f(x)=5-2x^2$ at the point A. Determine the coordinates of A.	(5)	**L3**	3.6	**NSC NOV 16** $g(x)=-8x+20$ is a tangent to $f(x)=x^3+ax^2+bx+18$ at $x=1$. Calculate the values of a and b.	(5)	**L3**
3.7	**KZN SEPT 17** Given: $f(x)=x^2-\frac{4}{x^2}$						
	3.7.1	Determine the gradient of the tangent to f at the point where $x=2$				(3)	**L2**
	3.7.2	Determine the equation of the tangent to f at $x=2$				(3)	**L2**
3.8	**NW SEPT 17** The graph $h(x)=ax^3+px$ passes through the point $(3;-2)$. The gradient of the tangent to h at $(0;0)$ is 3.						
	3.8.1	Determine the value of *a* and *p*.				(4)	**L3**
	3.8.2	Determine the gradient of the tangent to h at $x=2$.				(2)	**L2**
3.9	**FS SEPT 24** The equation of the cubic function f is given as $f(x)=-3x^3+15x^2-21x+9$.						
	3.9.1	Determine the x and y -intercepts of the graph.				(4)	**L2**

	3.9.2	Determine the coordinates of the turning points of f.		(4)	**L2**
	3.9.3	Sketch the graph of f clearly indicating the intercepts with the axes and the turning points.		(4)	**L2**
	3.9.4	For which value(s) of:			
		a)	x is $f'(x)>0$	(2)	**L3**
		b)	k will $f(x)=k$ have exactly three different real roots.	(2)	**L4**
3.10	**PLATINUM MATHS GR12** Consider the graph $f(x)=-x^3-3x^2+4$ and $g(x)=\frac{23}{9}x^2-\frac{19}{3}x$				
	3.10.1	State the point where the graphs share a common tangent.		(5)	**L4**
	3.10.2	Determine the equation of the common tangent at this point.		(4)	**L3**
4.1	**FS SEPT 23** Given: $f(x)=x^3-12x-16$				
	4.1.1	Calculate the coordinates of the turning points of the graph of f		(5)	**L2**
	4.1.2	Calculate the x -intercepts of f		(3)	**L2**
	4.1.3	$y=15x+p$ is a tangent to the graph of f .Calculate the x -coordinates of the point(s) of contact		(4)	**L3**
	4.1.4	For which value(s) of x will the given function be concave up?		(3)	**L2**
4.2	Given: $f(x)=3x^3-3x^2+6x-2$ For which values of x will f be concave down.			(4)	**L4**

4.3	**NSC NOV 23** Given: $f(x)=-x^3+6x^2-9x+4=(x-1)^2(-x+4)$			
	4.3.1	Determine the coordinates of the turning points of f.	(4)	**L1**
	4.3.2	Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points.	(4)	**L2**
	4.3.3	Use the graph to determine the value(s) of k for which $-x^3+6x^2-9x+4=k$ will have three real and unequal roots.	(2)	**L3**
	4.3.4	The line $g(x)=ax+b$ is the tangent to f at the point of inflection of f. Determine the equation of g.	(6)	**L4**
	4.3.5	Calculate the value of θ, the acute angle formed between g and the $x-$axis in the first quadrant.	(2)	**L3**
4.4	**NSC JUN 17** Given: $f(x)=x^3-x^2-x+1$			
	4.4.1	Write down the coordinates of the y-intercept of f.	(1)	**L1**
	4.4.2	Calculate the coordinates of the x-intercepts of f.	(5)	**L2**
	4.4.3	Calculate the coordinates of the turning point of f.	(6)	**L2**
	4.4.4	Sketch the graph of f. Clearly indicate all intercepts with the axes and the turning points.	(3)	**L2**
	4.4.5	Write down the values of x for which $f'(x)<0$.	(2)	**L2**
4.5	**KZN JUN 18** $f(x)=-x^3+3x^2+9x-27=-(x+3)(x-3)^2$ is the equation of a cubic function.			
	4.5.1	Write down the intercepts of f.	(3)	**L2**
	4.5.2	Calculate the co-ordinates of the stationary points of f.	(5)	**L2**
	4.5.3	Sketch the graph of f on a system of axes. (Clearly indicate the coordinates of the stationary points and the intercepts with the axes).	(4)	**L2**
	4.5.4	Determine the value(s) of x for which the graph is concave down.	(2)	**L2**
	4.5.5	Determine the equation of the tangent to the graph of f at $x=0$.	(3)	**L2**
	4.5.6	If $f(x)=k$ has 3 unequal real roots, determine the values(s) of k.	(3)	**L2**
	4.5.7	Write down the equation of t if t is the graph of f shifted 3 units horizontally to the left.	(2)	**L3**
4.6	**NSC NOV 17** Given: $f(x)=x(x-3)^2$ with $f'(1)=f'(3)=0$ and $f(1)=4$			
	4.6.1	Show that f has a point of inflection at $x=2$.	(5)	**L3**
	4.6.2	Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points.	(4)	**L2**
	4.6.3	For which values of x will $y=-f(x)$ be concave down?	(2)	**L3**
	4.6.4	Use your graph to answer the following question:		
	4.6.4.1	Determine the coordinates of the local maximum of h if $h(x)=f(x-2)+3$.	(2)	**L2**
	4.6.4.2	Claire claims that $f'(2)=1$. Do you agree with Claire? Justify your answer.	(2)	**L3**
4.7	**GP SEPT 19** A cubic function has following essential properties:	$f(4)=f(1)=0$ $f(0)=8$ $f'(3)=f'(1)=0$ $f(3)=8$		
	4.7.1	Sketch the graph of f, clearly indicating the turning point(s) and the points of intersection of the graph with the axes.	(3)	**L3**
	4.7.2	Show that the defining equation of f is $f(x)=-2x^3+12x^2-18x+8$.	(4)	**L3**
	4.7.3	Show that the defining equation of f is $f(x)=-2x^3+12x^2-18x+8$.	(3)	**L2**

4.8 Use the information given below to sketch then graph of $f(x) = ax^3 + bx^2 + cx + d$

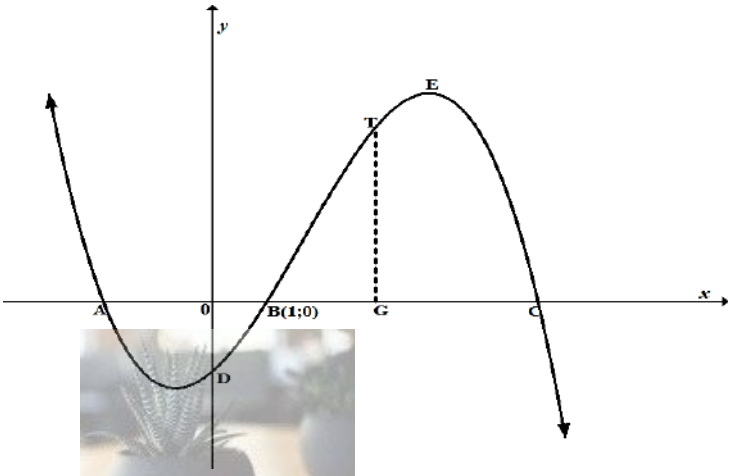
$f(1) = f(4) = 0$ $\quad f'(1) = f'(3) = 0$

$f'(x) > 0$ for $1 < x < 3$ $\quad f(0) = f(3) = 4$

(5) **L3**

4.9 **MP SEPT 23**

In the diagram, the graph of $f(x) = -x^3 + 5x^2 + 8x - 12$ is drawn. A, B and C are the x-intercepts of f. E is a turning point of f. T is a point f and G is a point on the x-axis such that TG is perpendicular to the x-axis. D is the y-intercept of f.

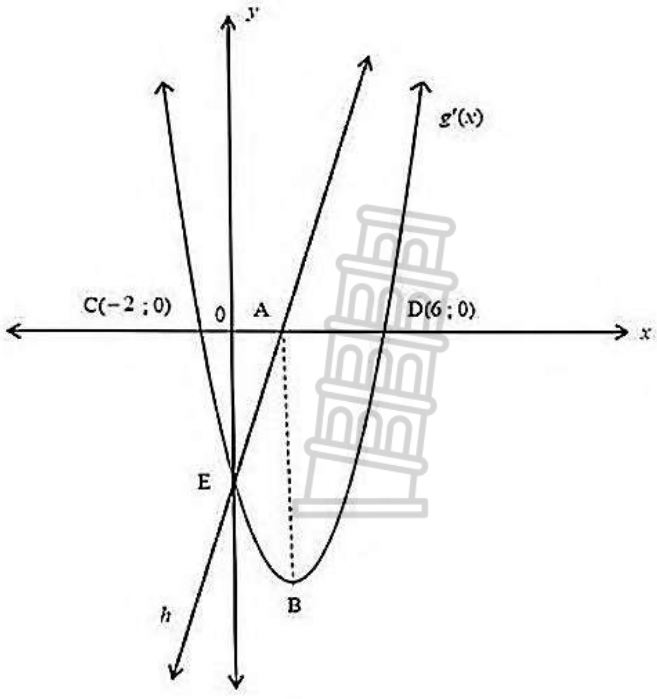


4.9.1	Calculate the coordinates of C if B (1;0).	(4)	**L2**
4.9.2	Determine the coordinates of E.	(5)	**L2**
4.9.3	For which value(s) of x will f be concave up?	(2)	**L2**
4.9.4	Calculate the length of OG if the tangent to the curve at T is parallel to the tangent to the curve at D.	(5)	**L4**
4.9.5	Determine the value of m if $y = mx + c$ intersects f perpendicularly at $x = 5$.	(3)	**L3**

4.10 **GP SEPT 24**

The graphs of $g'(x) = ax^2 = bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $g'(x) = ax^2 = bx + c$ is the derivative of a cubic function g.

- The graphs of h and g' have a common $y-$intercept at point E.
- $C(-2,0)$ and $D(6,0)$ are the $x-$intercepts of the graph of g'.
- Point A is the $x-$intercept of h and point B is the turning point of g'.
- Line AB is parallel to the $y-$axis.



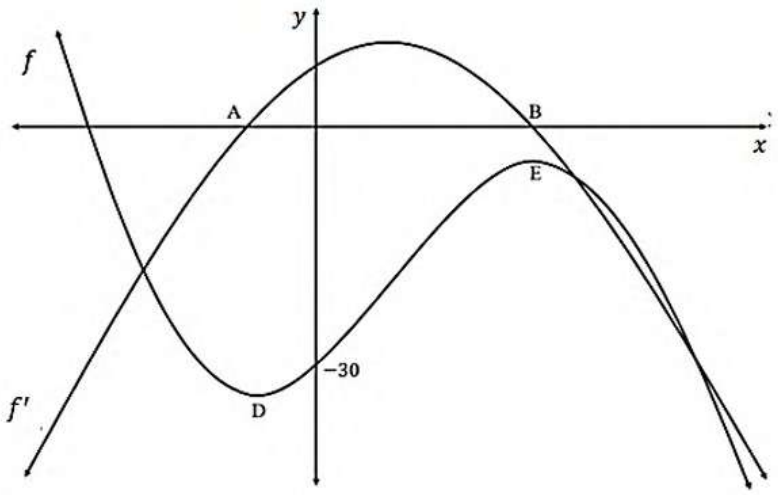
4.10.1	Write down the coordinates point E.	(1)	**L2**
4.10.2	Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$.	(4)	**L3**
4.10.3	Write down the x-coordinates of the turning point of g	(2)	**L2**

	4.10.4	Write down the x-coordinates of the point of inflection of the graph of g.	(1)	**L2**
	4.10.5	Explain why g has a local maximum at $x=-2$.	(2)	**L3**

4.11 **WC SEPT 2024**

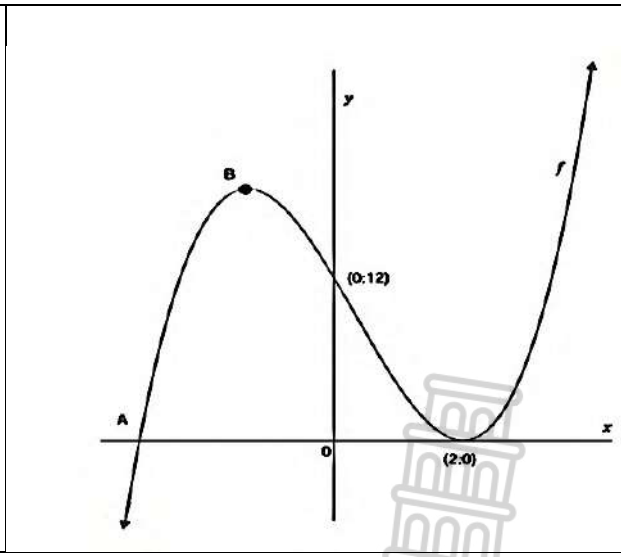
The diagram below shows the curves of $f(x)=ax^3+bx^2+cx+d$ and $f'(x)=-3x^2+6x+9$.

- The graph of $f'(x)=-3x^2+6x+9$ intersect the $x-$axis at A and B.
- D and E are the stationary points of the cubic graph $f(x)=ax^3+bx^2+cx+d$.
- -30 is the $y-$intercept of f.



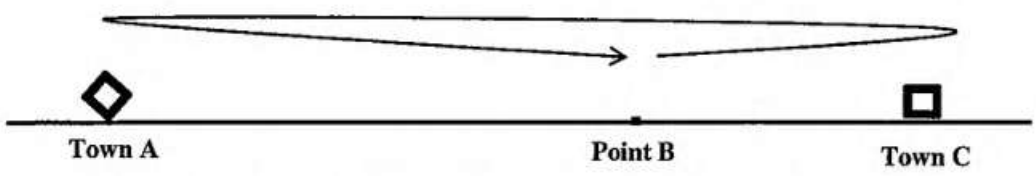
	4.11.1	Determine the $x-$coordinates of D and E. Show all your calculations	(3)	**L2**
	4.11.2	Determine the equation of f.	(5)	**L3**
	4.11.3	Determine the value(s) of x for which f is increasing.	(2)	**L2**
	4.11.4	For which value(s) of x is the graph of f concave -down?	(3)	**L2**

4.12 Given the following graph $f(x)=x^3+bx^2+cx+12$ with a $y-$intercept of (0,12), $x-$intercept and turning point at (2,0), turning point B and $x-$intercept A.



	4.12.1	Prove that the value of $b=-1$ and $c=-8$.	(5)	**L3**
	4.12.2	For which values of k will the equation $x^3-x^2-8x+15+k=0$ have three roots?	(4)	**L4**
	4.12.3	Is the graph f concave up or down at $x=-3$? Show ALL your working.	(3)	**L3**
	4.12.4	Determine the coordinates of the point of inflection.	(2)	**L2**

5.1

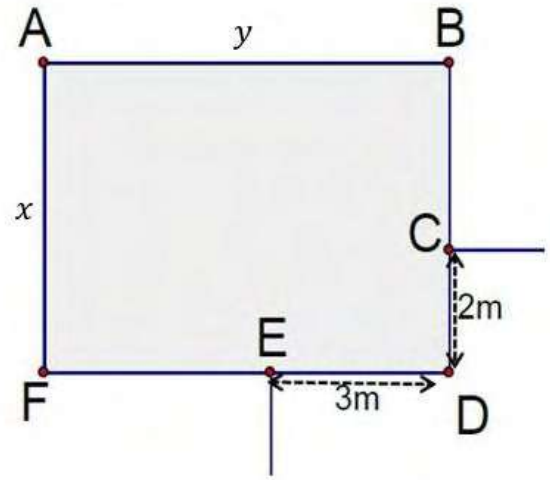


The athlete stars at point B which lies between towns A and C. To complete one cycle, he runs from point B to town passes point B on his way to town A and then back to point B. The road

between the towns is in a straight line. The displacement S, in kilometres, from point B after t hours, is given by: $S(t) = -t^3 + 12t^2 - 32t$.

	5.1.1	How many hours will it take the athletes to complete a full cycle and return to point B?	(3)	**L2**
	5.1.2	Calculate the distance between point B and town C.	(5)	**L4**
	5.1.3	Calculate the maximum speed that the athlete has reached while training.	(4)	**L3**

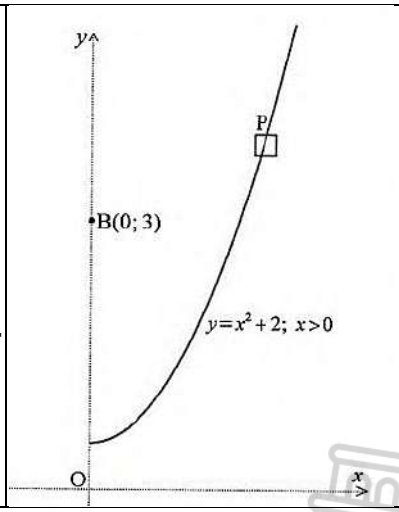
5.2 A man wants to add a rectangular room (shaded area) as shown in the sketch to his home. The line segments ED &CD, which are 3m and 2m long respectively, represent an existing corner of the house where doors will be fitted. He has enough material to build 18m of additional wall.



	5.2.1	Express y in terms of x.	(3)	**L2**
	5.2.2	What should the dimensions of the room be, so that it will have the biggest possible surface area? (Show ALL steps of working).	(5)	**L3**

5.3 An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram. Benny sits at a vantage point B (0,3) and observes a car, P, travelling along the road.

Calculate the distance between Benny and the car when the car is closest to Benny.



(7) **L4**

5.4 **NW SEPT 16**

A marathon athlete trans between towns A and C. He starts at point B which lies between towns A and C. The athlete runs from point B to town C and back to point B. The road between the towns is in a straight line. The displacement S, in kilometres, from point B after t hours, is given by:

$s(t) = -t^3 + 12t^2 - 32t$

	5.4.1	How many hours will it take the athlete to return to point B?	(3)	**L1**
	5.4.2	Calculate the distance between point B and town C.	(5)	**L2**
	5.4.3	Calculate the maximum speed that the athlete has reached while training.	(4)	**L3**

5.5 **WC SEPT 24**

The flowerpot below is in the shape of a cone.

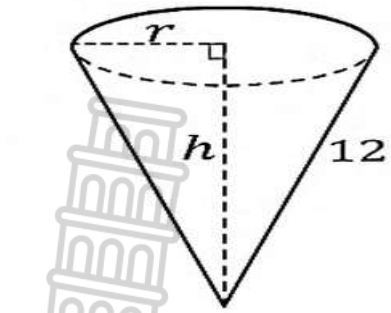
- r is the radius of the base and h is the perpendicular height.
- The slant height of the cone is 12cm.

Formulae for volume

$V = lbh$

$V = \pi r^2 h$

$V = \frac{1}{3}\pi r^2 h$

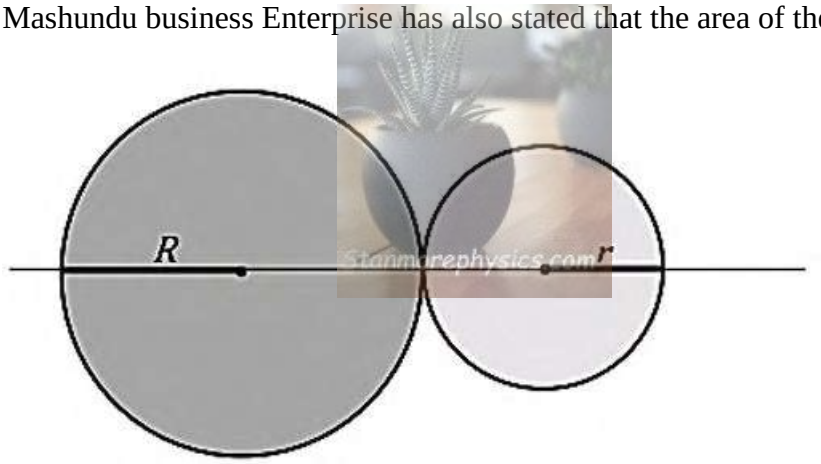


$V = \frac{4}{3}\pi r^3 h$

5.5.1	Show that the volume of the water needed to fill the entire flowerpot can be expressed as: $V = 48\pi h - \frac{1}{3}\pi h^3$	(3)	**L3**
5.5.2	The gardener wants to maximize the volume of water in the flowerpot. Determine the value of *h* for which the volume is a maximum.	(3)	**L4**

5.6 **LP SEPT 24**

Mashundu business Enterprise has asked you design an advertisement disc that consists of two circles and has the shape shown in the figure below. The larger circle has radius R and the smaller circle has radius *r*. The values of R and *r* must vary, and R+*r* =200 mm. To minimise costs, Mashundu business Enterprise has also stated that the area of the shape be a minimum.



5.6.1	Show that the area(A) of the figure is given by $A = 2\pi\left(R^2 - 200R + 20000\right)$	(3)	**L2**
5.6.2	Determine the values R and *r* if the area of the figure is a minimum.	(4)	**L3**
5.6.3	Hence, explain why the shape suggested by the company is not possible if you want to maintain a minimum area.	(2)	**L3**

TOPIC	6. PROBABILITY [±15 MARKS]

GUIDELINES, SUMMARY NOTES, & STRATEGIES

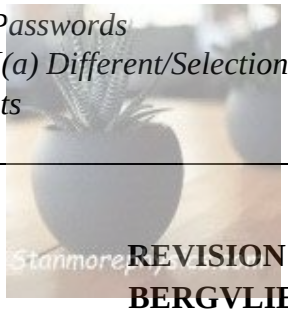
The probability scale: $0 \le P \le 1$. If P (an event) = 0, the event is impossible; If P (an event) = 1, the event is certain to happen.

The **definition of probability**: $P(E) = \frac{n(E)}{n(S)}$

Addition Rule for any 2 events A and B: **P (A or B) = P(A) + P(B)−P (A and B)**
Mutually exclusive events A and B: **P (A or B) = P(A) + P(B)**
NOTE: Since P (A and B) = 0
Independent events A and B: **P (A and B) = P(A) × P(B)**
The complementary rule: **P (not A) = 1−P(A)**
Venn-Diagram, Tree diagram and Contingency Table

The fundamental counting principle: If one operation can be done in *m* ways and a second operation can be done in *n* ways then the total possible number of different ways in which both operations can be done is $m \times n$.

- *Pin codes and Passwords*
- *Arrangements [(a) Different/Selection (b) Identical)]*
- *Re-arrangements*



REVISION QUESTIONS

1 BERGVLIET HS 2015

The Personnel Department of a company analysed the qualification level and gender of its employees. Qualification Level is the highest qualification achieved by the employee. The results are given in the table below:

Qualification Level	Male	Female	Total
Senior Certificate	24	26	50
Diploma	20	17	(a)
Degree	(b)	15	(c)
Total	60	(d)	(e)

1.1 Write down the values of *a, b, c, d* and *e.* (4) **L1**

1.2 What is the probability that a person in the company is a female with a degree? (2) **L2**

1.3 If Event A is a person who is male, and Event B is a person who has a Senior Certificate, are the events independent? Justify your answer with the necessary calculations. (5) **L3**

2 AUTHOR

Consider the letters of the word "INDEPENDENT". Determine, using all letters:

2.1 the number of unique arrangements of the letters that can be formed? (3) **L2**

2.2 the number of unique arrangements of letters that can be formed starting with the letter "N"? (3) **L2**

2.3 the number of unique arrangements of letters that can be formed in starting andending with the same letter ? (3) **L2**

3 **AUTHOR**

3.1 In a survey done at a School , the following information was obtained.

	Failed	Passed	Total
Male	**200**	**B**	1000
Female	**C**	**D**	400
Total	300	1100	1400

3.1.1 Calculate the probability that a person selected at random will be male (1) **L1**

3.1.2 Calculate the probability that a person selected at random failed the test (1) **L1**

3.1.3 Are the events being male and failing the test independent events? Justify using necessary calculations. (3) **L3**

3.1.4 Determine the values of **B**, **C** and **D**. (3) **L1**

3.1.5 Calculate the probability of choosing a female who failed. (2) **L2**

3.2 9 cars of different makes of which 4 are black and 5 are red are to be parked in a straight line.

3.2.1 In how many different ways can all the cars be parked? (2) **L1**

3.2.2 If the 4 black cars must be parked next to each other and the 5 red cars must be parked next to each other, determine in how many different ways the cars can be parked. (3) **L2**

4 **GAUTENG TRIAL 2023**

4.1 When Marge turned eight, her friends Emily, Klara, Cory, Liza, Shirley and Penny were invited to her birthday party. Marge and her friends sat in a row and played a game. In how many ways can they be seated if:

4.1.1 They sit in alphabetical order (1) **L1**

4.1.2 Emily and Klara do NOT want to sit next to each other? (3) **L3**

4.2 The probability that a certain rugby team has all its players fit to play is 70%. The probability that they will win a game if all their players are fit is 90%. When they are not fit the probability of them winning becomes 45%. Calculate the probability of them winning the FIRST game. (2) **L2**

5 **KZN SEPT 2019**

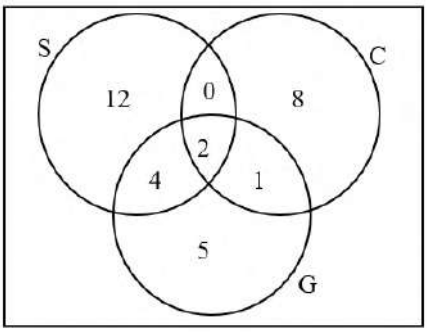
A bag contains 12 blue balls, 10 red balls and 18 green balls. 2 balls are chosen at random without replacement. Determine the probability:

5.1 if the two balls chosen at random are green. (3) **L3**

5.2 if the two balls chosen at random are blue and red. (3) **L3**

6 **BERGVLIET 2015**

In a survey of learners at a particular school on which of the sports of soccer (S), chess (C) or golf (G) they liked, data was collected and presented in the form of a Venn diagram as shown below.



6.1 How many learners were surveyed in total? (1) **L1**

6.2 Based on this survey, what is the probability that a learner, chosen at random, likes all three sports? (2) **L2**

6.3 What is the probability that a learner, chosen at random, likes both soccer and chess but not golf? (2) **L3**

7 **KZN TRIALS 2023**

Nonhle who is a Grade 12 learner has 8 textbooks from eight different subjects: Mathematics, English, Accounting, History, Tourism, Afrikaans, Geography and Drama which she wants to arrange in a line on a shelf.

7.1 In how many ways can the textbooks be arranged? (1) **L1**

7.2 In how many ways can the textbooks be arranged if the Mathematics textbook and the Accounting textbook must be on each end of the shelf? (3) **L2**

7.3 If the Mathematics textbook and the Accounting textbook must be on each end of the shelf, what is the probability that the History textbook and the Tourism textbook are not next to each other? (4) **L3**

8 **ST. CYPRIAN 2017**

There are 10 yoghurts in a refrigerator. Three are strawberry flavoured (S), two are blueberry flavoured (B) and the remainder are apricot flavoured (A). Two yoghurts are chosen at random, one after the other without replacement.

8.1 Draw a tree diagram to represent this information. (2) **L2**

8.2 Use your answer to 8.1 to determine the probability that:

8.2.1 both yoghurts are the same flavour. (2) **L2**

8.2.2 at least one is strawberry flavoured. (3) **L3**

9 **GAUTENG JUNE 2022**

9.1 In a survey, 1 530 people were asked whether they had ever broken a limb. The results of the survey were as follows:

	Broken a limb	Not Broken a limb	Total
Male	463	b	782
Female	a	c	d
Total	913	617	1530

9.1.1 Calculate the values of a, b, c, and d. (4) **L2**

9.1.2 If a person is chosen at random, what is the probability that it will be a female who has not broken a limb? (2) **L2**

9.1.3 Is having a broken limb dependent on gender? Motivate your answer. (3) **L2**

9.2 Two learners are selected at random from a group of 10 boys and 12 girls. Determine the probability that …

9.2.1 they are both girls. (2) **L2**

9.2.2 one is a boy and one is a girl. (3) **L3**

9.3 A group of 80 Olympic swimmers entered the following swimming events:100m backstroke, 100m freestyle and 100m butterfly as follows:

- 6 entered all three events
- 21 entered none of these events
- 10 entered the 100m backstroke and the 100m freestyle
- 11 entered the 100m freestyle and the 100m butterfly
- Of the 21 who entered the 100m backstroke, 10 entered nothing else 27 entered the 100m butterfly

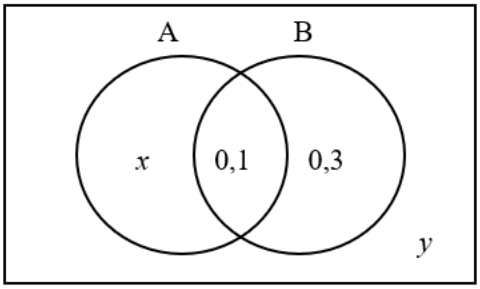
9.3.1 Draw a Venn Diagram of the above situation (6) **L3**

9.3.2 What is the probability that a swimmer chosen at random took part in none of these events? (1) **L2**

9.3.3 What is the probability that a swimmer chosen at random is swimming in at least two of these events (2) **L2**

10 **GAUTENG SEPT 2023**

10.1 Machine A and machine B are two different coin-pressing machines that operate at the same time. The probability that machine A ONLY presses a R5 coin, is x and the probability that machine B ONLY presses a R5 coin, is 0,3. The probability that both the machines press R5 coins at the same time is 0,1.



10.1.1 If A and B are independent events, determine the values of x and y. (4) **L2**

10.1.2 Determine the probability that exactly one of the machines is pressing a R5 coin (1) **L2**

10.2 The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny the probability that Jenny plays tennis is $\frac{4}{5}$. If it is not sunny, the probability that Jenny plays tennis is $\frac{2}{5}$

Determine the probability that Jenny plays tennis.(Hint use a tree diagram) (5) **L3**

11 **GAUTENG SEPT 2021**

Events A, B and C occur as follows, where A and B are independent events:

- $P(A) = 0,38$
- $P(B) = 0,42$
- *P(A and B)=0,1596*
- $P(C) = 0,28$

11.1 Are A and B mutually exclusive events? Motivate your answer. (2) **L2**

11.2 By using an appropriate formula, show that the value of P(A or B)=0,64 (2) **L2**

11.3 Calculate the number of people in the sample space. (2) **L2**

11.4 Determine n(not C) (2) **L3**

12 **GAUTENG SEPT 2021**

12.1 Each of the digits: 1 ; 1 ; 2 ; 3 ; 4 ; 7 is written on a separate card.
The cards are then placed next to each other to create a 6-digit number.

12.1.1 How many numbers start and end with the same digit? (1) **L2**

12.1.2 Find the probability that the number is 112347 or 743211. (4) **L4**

12.2 n people (numbered 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; … ; n) are arranged randomly in a line.

12.2.1 Find the number of ways, in terms of n, that person 1 and person 2 are standing next to each other. (You do not need to simplify your answer.) (3) **L4**

13 **ST. CYPRIAN 2017**

13.1 The probability that an event A occurs is P(A) = 0,4. B is an event independent of A and P(A *or* B) = 0,7.

13.1.1 Are A and B mutually exclusive? Motivate your answer. (1) **L1**

13.1.2 Determine:

(a) P (B) (4) **L2**

(b) P (not A or not B) (2) **L2**

13.2 A teacher has 5 different poetry books, 4 different dramas and 3 different novels.
She must arrange these 12 books from left to right on a shelf.

13.2.1 write down the probability that a novel will be the first book placed on the shelf. (1) **L2**

13.2.2 Calculate the number of different ways these 12 books can be placed on the shelf if any book can be placed in any position (2) **L3**

13.2.3 Calculate the probability that a poetry book is placed in the first position, the three novels are placed next to each other, and a drama is placed in the last position. (4) **L4**

14 **DBE NOV. 2023**

14.1 A and B are independent events. $P(A)=\frac{1}{3}$ and $P(B)=\frac{3}{4}$

Determine

14.1.1 P(A and B) (2) **L2**

14.1.2 P (at least ONE event occurs) (2) **L3**

14.2 The probability that it will snow on the Drakensberg Mountains in June is 5%.

- When it snows on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0°C is 72%.
- If it does not snow on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0°C is 35%.

14.2.1 Represent the given information on a tree diagram. Clearly indicate the probabilities associated with EACH branch. (3) **L3**

14.2.2 Calculate the probability that the temperature in Central South Africa will NOT drop below 0°C in June 2024. (3) **L3**

14.3 Ten learners stand randomly in a line, one behind the other.

14.3.1 In how many different ways can the ten learners stand in the line? (1) **L2**

14.3.2 Calculate the probability that there will be 5 learners between the 2 youngest learners in the line. (4) **L4**

15 **IEB NOV 2023**

After some research you decide that the best option for your online security is to have two unique passwords before opening important documents.

15.1. The first ten-digit password consists of two parts.

15.1.1 The first part is made up of six numbers. The numbers that can be used are the numbers from 1 to 9. How many unique six-digit codes can be created if **repetition is allowed**? (2) **L2**

15.1.2 The second part is a four-digit code compiled from the digits 1 to 9, **repetition is not allowed**. How many even ten-digit passwords can be created by combining these two parts? (3) **L2**

15.2 The second nine-digit password has to be a number greater than 600 000 000 with the last digit being divisible by 3. How many nine-digit passwords are possible if you can use the digits from 1 to 9, **no repetition is allowed**? (4) **L3**

16 **IEB NOV 2014**

16.1 In a sample space S, the number of elements in S, n(S) = 30 and there are two events A and B such that n(A) = 15, n(B) = 20 with n(A and B) = 6.

16.1.1 Draw a Venn diagram to represent this situation. (3) **L2**

16.1.2 Write down the value of n(A or B). (1) **L1**

16.1.3 An element is randomly selected from S.

- Write down the probability that the element is in both events A and B. That is, P(A and B). (1) **L1**
- Showing all working, determine whether the events A and B are independent (3) **L3**

16.2 Steve needs to set up a format for passwords onto his website. He has decided on having letters from the alphabet (of 26 letters), followed by digits 0 to 9. Letters and digits can be repeated.

16.2.1 Calculate the number of passwords that can be created using 2 letters followed by 2 digits. (2) **L2**

16.2.2 Steve thinks that he will need to cater for 3 million different passwords. He will stick with 2 letters but will need more digits. Determine the least number of the digits he will need. (4) **L4**

DBE NOV 2024

17. A company generates a 4-character code using the 26 letters of the alphabet and the 10 digits from 0 to 9 .The code is in the form:

letter	digit	letter	digit

17.1 Determine how many different codes can be formed if letters and digits maybe repeated. (2) **L1**

17.2 Determine how many different codes can be formed if :

- The letters D, F, I, Q, U, and V may NOT be used
- The code may NOT start with a W or a Z
- Letters or digits may NOT be repeated

The code ends with an odd digit (4) **L4**

17.3 The company wishes to increase the number of 4-character codes formed in Question 17.2 by allowing the letters D, F, I, Q, U, and V to be used. Calculate the percentage increase in the number of different codes formed. (2) **L3**

17.4 The trees in an orange orchard are harvested twice a year. During the first harvest, 70% of the oranges are picked while the rest are left. At the second harvest, 35% of the remaining oranges are picked while the rest are not picked. Assume no oranges were added between harvests.

17.4.1 Calculate the probability that a randomly selected orange will not be picked. (3) **L3**

17.4.2 If it is further given that all the oranges that are picked are packaged with:

- 9% from each harvest selected for export
- 31% sold to the local market and
- the rest are sent to a factory to be made into juice.

What percentage of oranges will be sent to the factory to be made into juice? (4) **L3**

17.4.3 There are 120 oranges in an export box. If 172 export boxes are produced, then how many oranges were there in the total crop? (4) **L3**

18 **DBE May/June 2025**

18.1 Consider the three-digit numbers from 501 up to 999

18.2.1 How many three digit numbers have exactly one 5 in them ? (4) **L3**

18.2.2 Calculate the probability of a three-digit number not satisfying the condition given in 18.2.1 ? (3) **L4**

18.3 A game at a fun park requires a player to roll a six-sided die and pick a card from a deck of 52 cards.

A player wins if an odd number appears on the uppermost face of the dice and the player also draws a picture card from the deck.

- A deck of cards has 4 suites (hearts, diamonds, spades and clubs).
- There are 4 picture cards (king, queen, jack and ace) in every suite.
- A player pays R10 to play a game and in an hour, 260 people can each play one game.

  If the owner wants to make a 70% profit per hour, calculate the maximum amount that the owner must pay out to each winner. (6) **L4**

18.4 A four-digit code is made from the digits 0 to 6.
How many four-digit codes can be made if the code has to be greater than 2 000, less than 3 000, and must be even?
You may not repeat digits. (3) **L3**

18.5 Consider the word MILLION

18.5.1 Determine the number of seven-letter words that can be made. (2) **L2**

18.5.2 Determine the probability that the vowels will be next to each other. (3) **L3**

HERSCHEL 2017

19. Consider the digits 1 to 8 and answer the following questions:

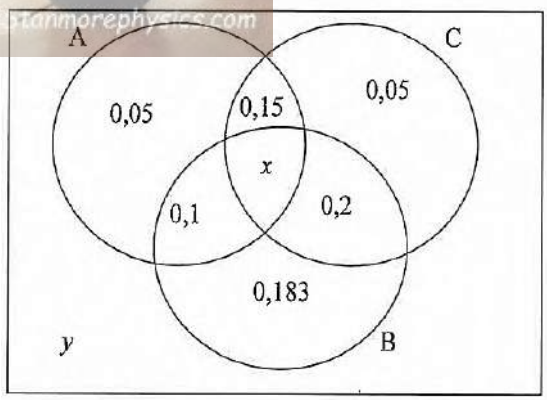
19.1 How many 4-digit numbers can be formed if repetition is allowed? (2) **L2**

19.2 What is the probability of choosing a 4-digit number that starts with a 5, ends with a 6, and has no repeated digits? (2) **L3**

19.3 How many numbers between 4 000 and 5 000 can be formed if digits are allowed to be repeated? (2) **L3**

20. **DBE NOV 2022**

A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below



If t given that the probability that at least one of the events will occur is 0.893, calculate the value of:

20.1 y, the probability that none of the events will occur. (1) **L1**

20.2 x, the probability that all three events will occur. (1) **L1**

20.3 Determine the probability that at least two of the events will take place. (2) **L2**

20.4 Are events B and C independent? Justify your answer. (5) **L3**

TOPIC: STATISTICS/DATA HANDLING

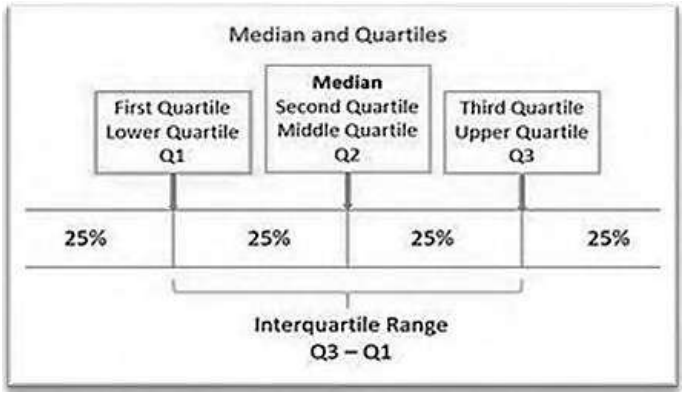
GUIDELINES, SUMMARY NOTES, & STRATEGIES

KEY CONCEPTS

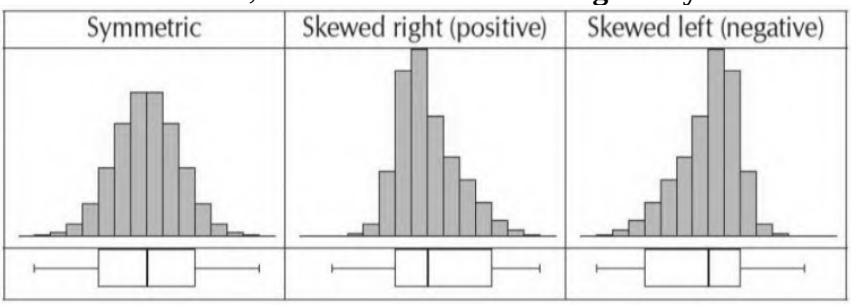
Data Handling is a process during which data (information) is collected, recorded, and presented.

- ❖ **Ungrouped data** – a set of random data elements gathered for analysis.
- ❖ **Grouped data** – data elements aggregated into different classes, groups, or intervals.

- **Measures of central tendency** – single numbers around which all data items seem to be spread.
  - ❖ The **Mean**, also known as the average, is the sum of all the data values in a set, divided by number of all elements in the set.
  - ❖ The **Median,** (Q_2) it presents the middle value in a data set.
  - ❖ The **Mode** is the most frequent data item in a set. In grouped data, the modal group will have the highest frequency. Data sets may have no mode, two modes (bimodal), three modes (trimodal), etc.
- **Measures of dispersion** – numbers that describe the spread of the data.
  - ❖ The **Range** is the difference between the maximum and the minimum data values in a given data set.
  - ❖ The **Inter-Quartile-Range** (IQR) is the difference between the third and first quartiles, i.e. $IQR = Q_3 - Q_1$
  - ❖ **Standard Deviation** (σ) is a measure of how dispersed data is around the mean. The square of the standard deviation is the **variance.**
- **Five Number Summary** – five numbers that separate a data set into quarters.
  - ❖ Minimum value
  - ❖ Lower quartile (Q_1) position, $\frac{1}{4}(n+1)$
  - ❖ Median (Q_2) position, $\frac{1}{2}(n+1)$
  - ❖ Upper quartile (Q_3) position, $\frac{3}{4}(n+1)$
  - ❖ Maximum value



- **Box – and – Whisker Diagram** (drawn using the five number summary), divides the data set into FOUR quarters.
  - ❖ It is important in analysing the distribution of data in a given set.
  - ❖ If **mean = median**, then the distribution is **symmetric.**
  - ❖ If **mean > median**, then the distribution is **positively skewed.**
  - ❖ If **mean < median**, then the distribution is **negatively skewed**.



- Outliers – An outlier is a data entry that is far removed from the other entries in the data set e.g. a data entry that is much smaller or much larger than the rest of the data values.

They are determined as follows:

- ❖ Lower outliers are numbers that lies $< Q_1 - 1.5xIQR$
- ❖ Upper outliers are numbers that lies $> Q_3 + 1.5xIQR$

- Graphical representations
  - ❖ **Histogram** – represents grouped data as condensed bars whose widths and lengths represent class intervals and frequency respectively.

- ❖ **Ogive (Cumulative Frequency Curve)** – an *S*-shaped **smooth curve** drawn by plotting using the **upper limits of class intervals** of a grouped data against **cumulative frequency** of a set.
- ❖ **Scatter plot** – A scatter plot is a graph that helps you to see whether there is a **correlation** (relationship) between any set of two numeric data.

- Bi-variate data summaries:
  - ❖ **Regression line (line of best fit)** – a line drawn on the scatter plot that shows a general trend that bivariate data seems to follow.
  - ❖ **Least squares regression line** – is a straight line that passes through the mean point relating bi-variate data
  - ❖ **Correlation Coefficient** (**r**) – indicates the strength of the relationship between the variables in bivariate data and range from -1 to 1
- Using a Least Squares Regression Line to Make Predictions:
  - ❖ When a value for one of the variables that was not originally in the data is found, you are making a prediction. The required value can be read off from the scatter plot or by using the equation of the regression line.
  - ❖ Predictions made from the equation of the line can be made through the process of INTERPOLATION and EXTRAPOLATION.
  - ❖ **Interpolation** is a method of predicting/estimating new data value(s) within the known range of data values.
  - ❖ **Extrapolation** on the other hand is a method of estimating new data value(s) beyond a discrete set of known data values.
  - ❖ Note that data values that are the result of **extrapolation from statistical data are often less valid than those that are the result of interpolation. This is because the values are often estimated outside the tabulated or observed range of data.**

REVISION QUESTIONS

KZN NOV 2024 (Gr 11)

1 The table below shows the number of tourists who stayed at a hotel in Durban from 20 to 31 December 2023.

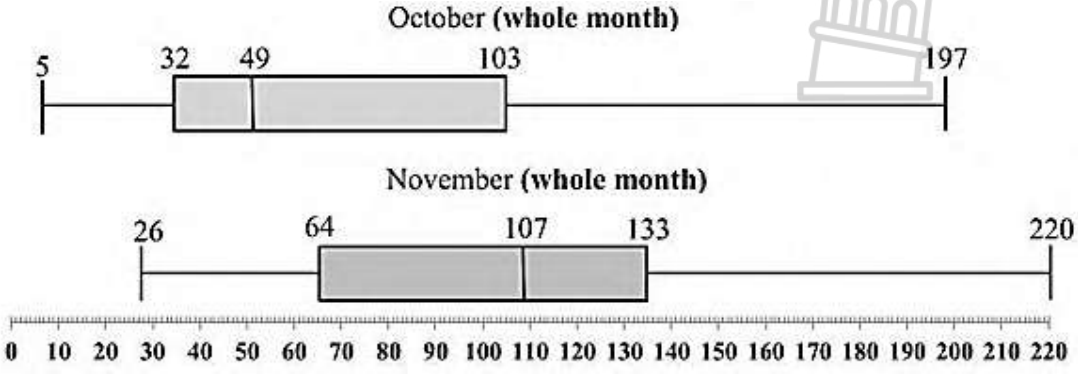
Date in December	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
Number of tourists	**286**	**68**	**150**	**147**	**176**	**255**	**132**	**174**	**172**	**197**	**172**	**39**

1.1 Mean of the data. (2) **L1**

1.2 The standard deviation for the given data. (1) **L2**

1.3 Calculate the percentage of days on which the number of tourists who stayed at this hotel was within ONE standard deviation of the mean. (3) **L2**

1.4 The number of tourists who stayed at this hotel during the entire month of October and the entire month of November are summarised in the box and whisker diagram below.



1.4.1 Calculate the range of the number of tourists who stayed at the hotel during November. (1) **L1**

1.4.2 Comment on the skewness of the data for October. (1) **L1**

1.4.3 The maximum for October was incorrectly recorded. The correct value is higher than the recorded value. If the correction is made, what effect will it have on the:

(a) Mean

(b) Median (2) **L2**

1.5 There was only one day during November on which 64 tourists stayed at the hotel. On how many days during November did less than 64 tourists stay at the hotel? (2) **L2**

EC NOV 2024 (Gr 11)

2 After utilising GeoGebra to teach Geometry, the participants mark's out of 100 are displayed in the table below.

16	28	41	41	42	52	54
55	58	59	60	62	64	99

2.1 Write down the mode of the data. (2) **L1**

2.2 Identify any outlier. (2) **L1**

2.3 Determine the median of the data. (3) **L2**

2.4 Determine the interquartile range of the data. (3) **L2**

2.5 Draw a box and whisker diagram to represent the information given above. (2) **L2**

2.6 Comment on the skewness of the data by using the box and whisker diagram above. (1) **L1**

WC SEPT 2024

3 During a congress that took place in July, the ages of the 100 members who attended, were recorded in the following table:

AGE (x) (IN YEARS)	**NUMBER OF PEOPLE**
$15 < x \leq 25$	8
$25 < x \leq 35$	14
$35 < x \leq 45$	22
$45 < x \leq 55$	37
$55 < x \leq 65$	a
$65 < x \leq 75$	3

3.1 Calculate the value of a. (1) **L1**

3.2 Write down the modal class of the given data. (1) **L1**

3.3 Complete the cumulative frequency column using the above information. (2) **L1**

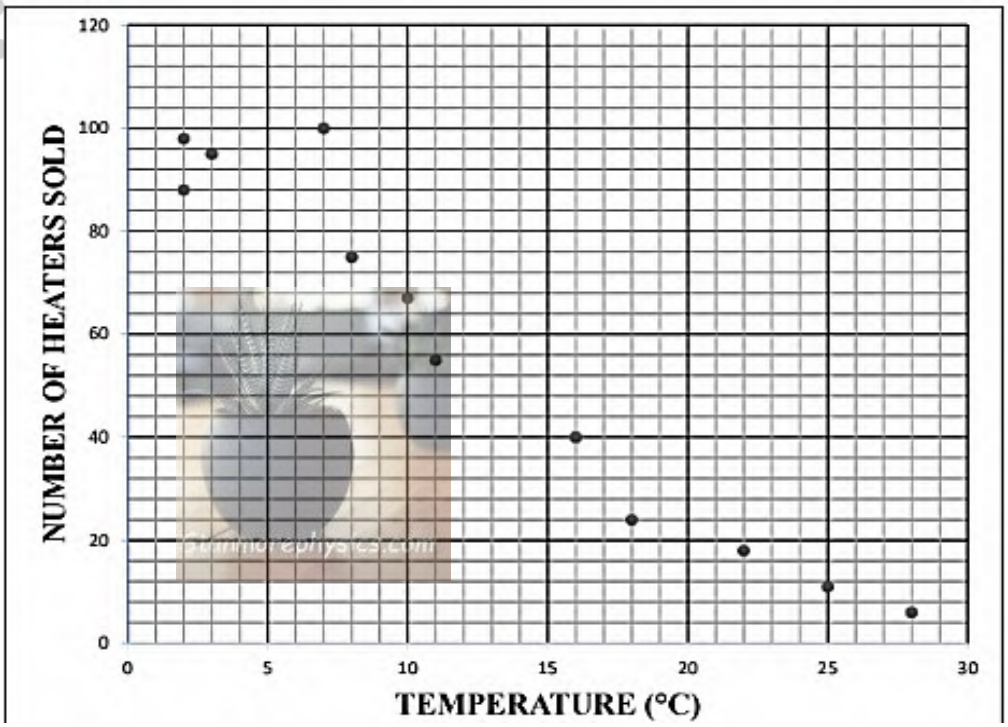
3.4 Draw a cumulative frequency graph (ogive) to represent the data given above. (3) **L2**

3.5 The congress organisers discovered that a mistake was made, and that 3 people counted in the range $45 < x \leq 55$ should have been in the range $35 < x \leq 45$. Calculate the estimated average age of the people who attended the congress after the mistake was corrected. (2) **L2**

WC SEPT 2024

4. The scatter plot below shows the number of heaters a company sold per month and the average temperature of that month in a certain year. Both the temperature in °C and the number of heaters sold, are given in the table below.

MONTH	J	F	M	A	M	J	J	A	S	O	N	D
TEMPERATURE (°C)	2	7	8	10	18	22	28	25	16	11	2	3
NUMBER OF HEATERS SOLD	98	100	75	67	24	18	6	11	40	55	88	95



4.1 Describe the correlation between the number of heaters sold and the average temperature per month. Verify your answer by referring to the correlation coefficient. (2) **L2**

4.2 Determine the equation of the least squares regression line for the data. (2) **L1**

4.3 Predict the number of heaters sold for a month where the average temperature is 20°C. (3) **L2**

4.4 Draw the least squares regression line on the grid given above. (1) **L1**

4.5 Calculate the standard deviation of the number of heaters sold. (2) **L2**

NC SEPT EXAM 2024

5 Twenty-five rugby players were asked about the number of times they visited the gymnasium during the December festive season. The responses were as follows:

Number of gym visits	Number of players
0	**1**
x	**1**
$x+5$	**3**
10	**1**
13	**6**
14	**3**
15	**2**
18	**4**
20	**1**
22	**3**

5.1 The mean of gym visits is 13,96. Use calculations to show that $x = 4$. (2) **L1**

5.2 Calculate the standard deviation of the data. (2) **L1**

5.3 How many players visited the gym more than one standard deviation above the mean? (2) **L2**

5.4 Determine the interquartile range of the data. (3) **L2**

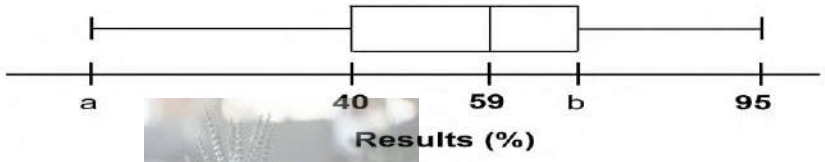
5.5 During January, each player increased his number of gym visits with k visits. What impact will this have on the:

5.5.1 Standard deviation. (1) **L1**

5.5.2 Interquartile range. (1) **L2**

IEB NOV 2023

6. The percentage results obtained by 26 learners in a mathematics test is displayed in the box and whisker plot below:



If the range of the data is **80** and the interquartile range (IQR) is **30**:

6.1 Determine the value of ***a***. (1) **L1**

6.2 Determine the value of ***b***. (1) **L1**

6.3 Determine whether the minimum result obtained (using your answer to 6.1) is an outlier or not. Use the formula $Q_1 - 1{,}5 \times IQR$ (2) **L2**

6.4 The table illustrates the approximate time spent (x) in minutes by 7 learners studying for a mathematics test and their mark obtained as a percentage.

Time spent (x) in minutes	**0**	**90**	**90**	**80**	**90**	**120**	**150**
Mark obtained (y) as a %	**15**	**59**	**60**	**73**	**85**	**90**	**95**

6.4.1 Predict, using the equation of the least squares regression line in the form $y = a + bx$ for this data, what mark a learner who studies for 180 minutes will obtain? (3) **L3**

6.4.2 Is this prediction in 6.4.1 a reliable one? Explain. (2) **L2**

DBE MAY/JUNE 2025

7 An insurance broker signed contracts with 15 people. The premium (in rands) payable on each contract is given below.

134	**215**	**325**	**326**	**362**	**429**	**515**	**531**	**598**	**610**	**624**	**728**	**1024**	**1200**

7.1 Calculate the mean. (2) **L1**

7.2 Write down the standard deviation of the data. (1) **L1**

7.3 Calculate how many monthly premiums are ONE standard deviation of the mean. (2) **L2**

7.4 The insurance company decided to increase the monthly premiums.

- Monthly premiums that were less than R500 increased by 18%
- Monthly premiums that were equal to or more than R500 increased by ***k***%

After these increases were applied to the above data, the new mean monthly premium was R686,44. Calculate the value of ***k***. (4) **L4**

DBE MAY/JUNE 2025

8 The manager of a supermarket decided to do a survey on the number of items that a customer ordered online and the time (in minutes) that a packer took to have the order ready for delivery. The supermarket received 10 online orders on a certain day. The information for these 10 orders is shown in the table below.

Number of items (x)	**10**	**3**	**20**	**14**	**17**	**9**	**12**	**18**	**15**	**19**
Time (in minutes) (y)	**5**	**5**	**9**	**7**	**6**	**6**	**8**	**11**	**10**	**12**

8.1 Draw a scatter plot using the information provided above. (3) **L2**

8.2 Determine the equation of the least squares regression line. (3) **L2**

8.3 Write down the correlation coefficient of the data. (1) **L1**

8.4 The supermarket received an online order for items. Predicts how long (in minutes) it will take a parker to pack the order and have it ready for delivery. (2) **L2**

8.5 Explain why the y-intercept of the least regression line in QUESTION 8.2 does NOT make sense in this context. (1) **L2**

EC NOV 2024 G11

9 The weight of the boxers who underwent fitness and health checks is shown in the frequency table below.

Weight	**Frequency**	**Cumulative Frequency**
$35 \leq x < 55$	1	
$55 \leq x < 75$	3	
$75 \leq x < 115$	9	
$115 \leq x < 135$	3	
$135 \leq x < 155$	1	

9.1 Complete the cumulative frequency column provided in the table above. (2) **L2**

9.2 Write down the total number of boxers. (1) **L1**

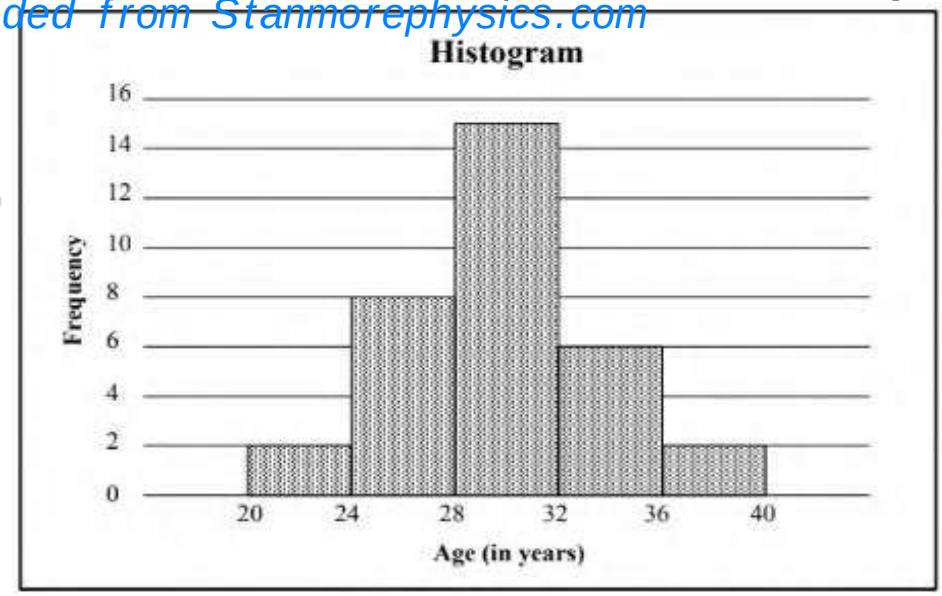
9.3 Estimate the mean for the data. (3) **L2**

9.4 Draw a cumulative frequency graph (ogive) for the data. (3) **L2**

9.5 It is further given that, for a boxer to qualify for the next upcoming match, he must have a mass that is in the interval of $75 \leq x < 115$.
Using the cumulative frequency graph (ogive) and estimate the number of boxers that will qualify for the upcoming match. (2) **L3**

NW SEPT 2024

10 During the Rugby World Cup of 2023, the ages (in years) of the players of the Springbok rugby squad were recorded. The data is represented in the histogram below.



10.1 How many players were in this rugby squad? (1) **L1**

10.2 Calculate the estimated mean age of these rugby players. (2) **L2**

10.3 Use the histogram to:

(a) Complete the cumulative frequency column. (2) **L2**

(b) Draw an ogive (cumulative frequency graph) of the data above. (3) **L2**

10.4 Write down the estimated median of the above data. (2) **L2**

10.5 It was discovered that the frequency of the age data for ***k*** player(s) in the modal age interval was recorded incorrectly. The mistake is corrected and the frequency of TWO other intervals are increased. The number of players in the squad remains unchanged. Determine the minimum value of ***k***, if the data of the new histogram is symmetrical. (3) **L3**

NW SEPT 2024

11 Mrs Mochini wants to use mathematical modelling to predict the final results of her grade 12 Mathematics learners. She decides to use the Preparatory and Final Mathematics examination results of the previous year to help her in developing such a possible model.
She records in the table below, 10 learners' previous year's results (in %) as follows:

Preparatory (x)	**38**	**65**	**78**	**23**	**67**	**93**	**39**	**83**	**51**	**66**
Final examination (y)	**57**	**72**	**81**	**27**	**59**	**94**	**41**	**85**	**54**	**79**

11.1 Determine the equation of the least squares regression line. (3) **L2**

11.2 A learner obtained 46% for the Preparatory examination:

(a) Calculate the possible final examination results that Mrs Mochine can expect from this learner. (2) **L2**

(b) Is the answer in QUESTION 11.2(a) a good indication of the expected final examination result? Motivate your answer. (2) **L3**

11.3 The point $(\overline{x}\ ;\ q)$ lies on the regression line of QUESTION 11.1.
Only ONE of the options below correctly reflects the value of q. Write only the letter of the correct option as your answer.

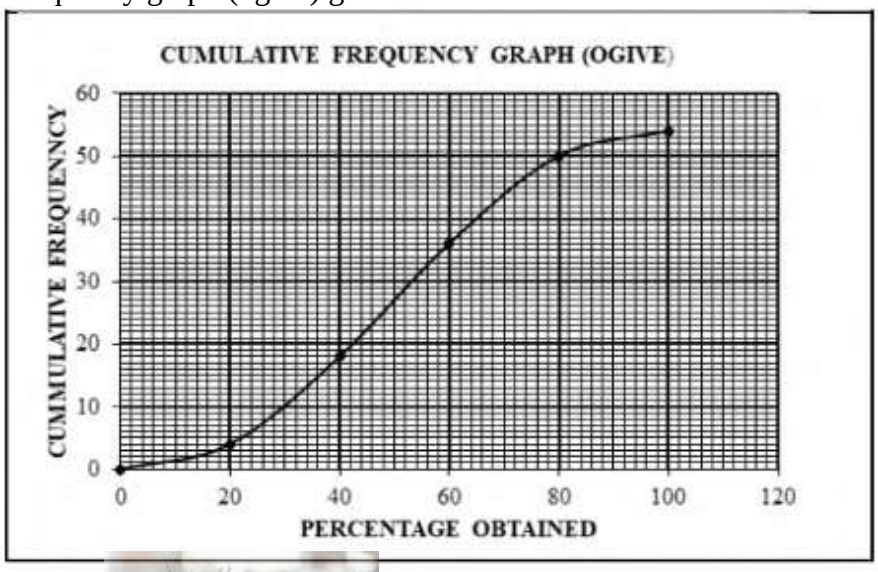
A. $\sqrt{\overline{x}}$ C. σ_x

B. $\dfrac{\sum y}{10}$ D. σ_y

(1) **L2**

EC JUNE 2024

12 In a certain school, the analysis of mathematics matric results in percentages were represented in the cumulative frequency graph (ogive) given below.



Use the above graph to answer the following questions.

12.1 Complete the frequency table provided below.

Percentage obtained	**Frequency**	**Cumulative frequency**
$0 \leq x < 20$		4
$20 \leq x < 40$		18
$40 \leq x < 60$		36
$60 \leq x < 80$		50
$80 \leq x < 100$		54

(2) **L2**

12.2 Write down the total number of matriculants who wrote mathematics in this school. (1) **L1**

12.3 Write down the modal class. (1) **L1**

12.4 Estimate the median percentage for mathematics of this school. (2) **L2**

12.5 If the requirement for a learner to be admitted in a certain institution is 70% and more in mathematics, determine how many matriculants will qualify for admission. (2) **L3**

KZN SEPT 2024

13 The Human Resources Department of a company in KZN wants to create a model to be used in determining the monthly salaries of its employees. Twelve of their current employees were surveyed and the information is displayed in the table below:

Employees' experience in the number of years. (x)	26	1	3	5	6	6	10	14	12	33	20	8
Salary in R1000s per month. (y)	20	9	10.5	11	10	12	16	15	12	23	18	9

13.1 Calculate the **mean** monthly salaries of these 12 employees. Round your answer off to the nearest rand. (2) **L2**

13.2 Calculate the standard deviation of the monthly salaries of these employees. Round your answer off to the nearest rand. (1) **L1**

13.3 How many of the employees earn a monthly salary that is more than one standard deviation above the mean? (2) **L2**

13.4 Determine the equation of the least squares regression line for the data give in the table above. (3) **L2**

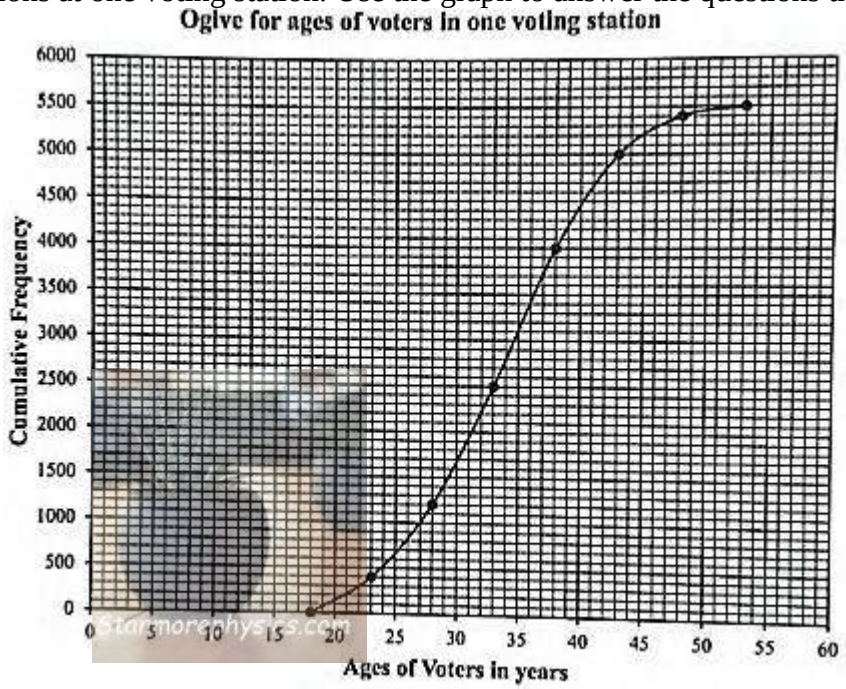
13.5 Calculate the correlation coefficient between the experience in years and monthly salary of an employee. (1) **L1**

13.6 Predict what the monthly salary will be of an employee who has been working for this company for 30 years.
Round your answer off to the nearest rand. (2) **L2**

13.7 Is the prediction that is made in question 13.6 likely to be reliable? Give a reason for your answer. (2) **L3**

KZN SEPT 2024

14 The cumulative frequency graph (ogive) drawn below shows that ages of people who voted the local Government elections at one voting station. Use the graph to answer the questions that follow:



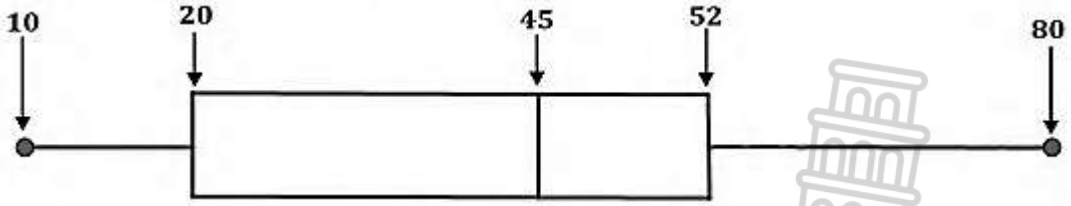
14.1 How many people voted at this voting station? (1) **L1**

14.2 Determine the interquartile range of the ages of the voters. (3) **L2**

14.3 What percentage of the voters was 25 years or younger? (2) **L2**

MP PRE-TRIAL 2024

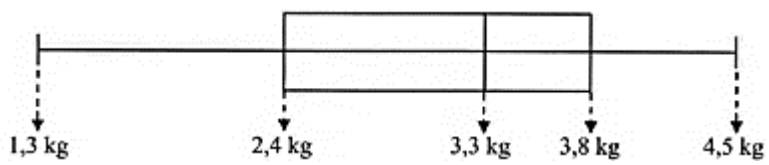
15 The data set contains a total of nine numbers. The second and third numbers of the data set are the same and the fourth number is 32. The seventh and eighth numbers are different. The eighth number is one more than the 75th percentile. The mean for the data is 40.



15.1 Write down a possible list of nine numbers which will result in the above box and whisker plot. (6) **L4**

KZN SEPT 2023 G11

16 On a certain day 20 babies were born to one of the hospitals. The weight at birth of these babies was summarised in the box and whisker diagram below.



Use the diagram to answer the questions below:

16.1 Calculate the interquartile range of the data. (2) **L2**

16.2 One of the doctors said that the mean weight of the babies was 3.5kg. Could this possibly be correct? Clearly motivate your answer, using the box and whisker diagram. (3) **L2**

FS PRE-TRIAL 2023

17 During the month of June patients visited a number of medical facilities for treatment. The table shows the number of patients treated on certain dates during the month of June.

Dates in the month of June	3	5	8	12	15	19	22	26
Number of patients	270	275	376	420	602	684	800	820

17.1 Draw a scatter plot of the given data (3) **L2**

17.2 Determine how the equation of the least squares regression line of patients (y) against date (x) (3) **L2**

17.3 Estimate how many patients have been treated on the 24th of June. (2) **L2**

17.4 Draw the least squares regression line on the scatter plot. (3) **L2**

17.5 Calculate the correlation coefficient of the data. Comment on the strength of the relationship between the variables. (3) **L2**

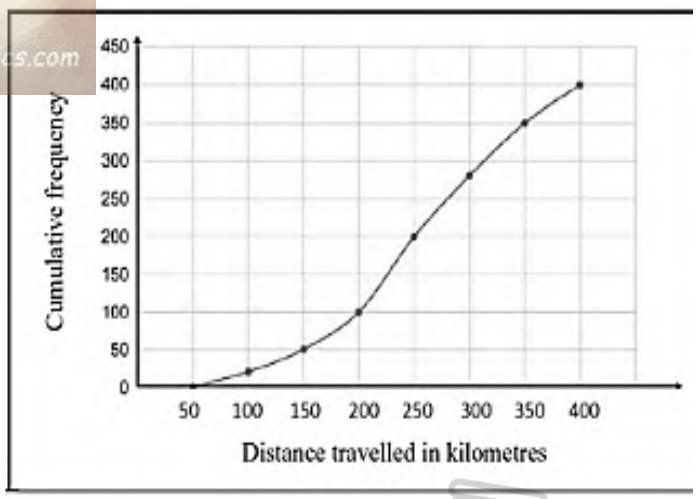
17.6 Given that the mean for the patients treated on certain dates is 528,63 calculate how many patients were within one standard deviation of the mean. (3) **L2**

[17]

GAUTENG JUNE 2024

18 In a survey, a group of people were asked about the total distance they had travelled from their homes in the last week of December 2023. The data collected is represented in the frequency table and ogive (cumulative frequency curve) below.

Distance travelled (x kilometres)	**Frequency**
$50 \leq x < 100$	20
$100 \leq x < 150$	30
$150 \leq x < 200$	**A**
$200 \leq x < 250$	**B**
$250 \leq x < 300$	80
$300 \leq x < 350$	70
$350 \leq x < 400$	50



18.1 How many people participated in this survey? (1) **L1**

18.2 Determine the values of A and B in the table above. (2) **L2**

18.3 Use the ogive/table to estimate the number of people who travelled between 100 km and 300 km. (2) **L2**

18.4 If all people who travelled more than 350 km were removed from the survey, how would this affect the median of the data? (1) **L2**

18.5 If the estimated mean of the data below is 16.4, then what is the value of t?

Class Interval	**Frequency**
$0 < x \leq 10$	13
$10 < x \leq 20$	t
$20 < x \leq 30$	12
$30 \leq x < 40$	4

(4) **L3**

TOPIC | **Analytical Geometry** [40 ± 3]

GUIDELINES, SUMMARY NOTES, & STRATEGIES

1. Distance formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

When given the distance between two points with both points given but one value being a variable. Use this formula through substitution.

2. Midpoint: $\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

Remember, midpoint might be given and be requested to calculate any end point.

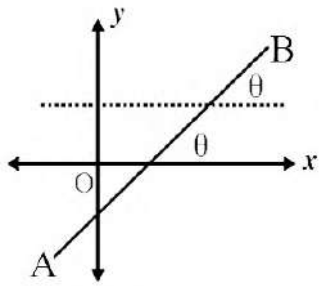
3. Gradient: $m = \frac{y_1 - y_2}{x_1 - x_2}$

3.1 **Collinear** points and **parallel** lines have same (equal) gradient. i.e., $m_1 = m_2$

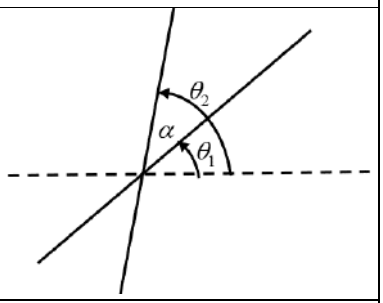
3.2 The product of the gradient of **perpendicular** lines is -1. i.e., $m_1 \times m_2 = -1$

4. Inclination: $\tan\theta = m$

4.1 The inclination of a line is the angle formed with the horizontal in an anti-clockwise direction. On the cartesian plane, the inclination of a line is calculated by finding the angle formed at the x-axis measured in anticlockwise direction. θ is the angle of inclination of line AB.



4.2 The angle between two lines: $\alpha = \theta_2 - \theta_1$



4.3 Sometimes you will be given the angle of inclination then requested to determine the gradient.

5. Equation of a straight line: $y = mx + c$ or $y - y_1 = m(x - x_1)$

Take note of the following form: $ax + by + c = 0$

6. Point of intersection

Equate the equations of those two lines or solve using simultaneous equations in a case where a line intersects a circle.

7. Equation of the circle: $(x - a)^2 + (y - b)^2 = r^2$

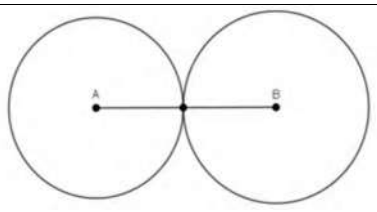
$(a; b)$ is the centre and r is the radius,

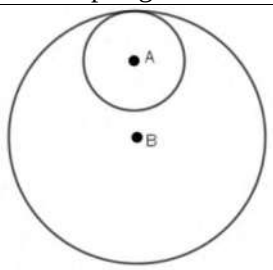
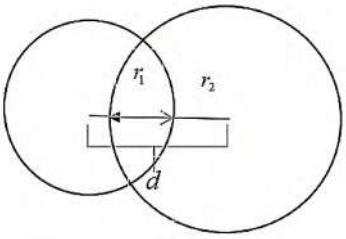
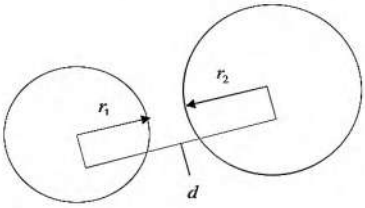
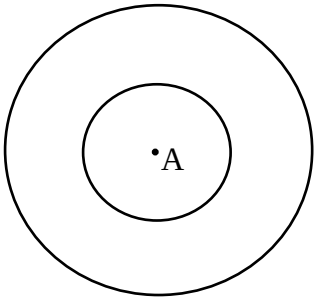
so, when centre is the origin then: $x^2 + y^2 = r^2$

8. Interpretation of circles

8.1 Touching at one point

Externally: $d_{AB} = r_A + r_B$ where A and B are centres

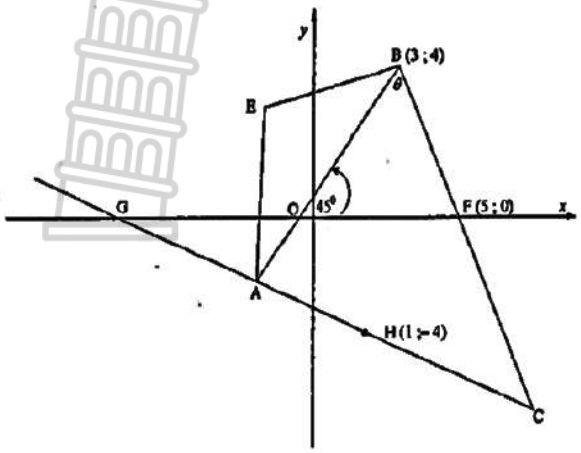


		Internally: $d_{AB} = r_A - r_B$ where A and B are centres	
	8.2 Intersecting at two points	$d < r_1 + r_2$	
	8.3 Not touching at all	$d > r_1 + r_2$	
		Same centre (concentric circles)	

9. Equation of a tangent	9.1	Tangent is always perpendicular to the radius at the point of contact. Meaning: $m_{rad} \times m_{tan} = -1$
	9.2	Tangents from the same point outside the circle are equal in length.
10. Calculation of Areas	10.1	Area of a triangle (known height): $A = \frac{1}{2}$base×height
	10.2	Area of a triangle (known angle): use area rule $A = \frac{1}{2}bc\sin A$ or $A = \frac{1}{2}ac\sin B$ or $A = \frac{1}{2}ab\sin C$
	10.3	Area of regular Quads: use standard formulas from grade 10 Area of a Rectangle: $A = l \times b$, Area of a Square: $A = s \times s$, Area of a Kite: $A = \frac{1}{2} \times d_1 \times d_2$, Area of a Rhombus: $A = \frac{d_1 \times d_2}{2}$, Area of a Parallelogram: $A =$ base $\times$ height and Area of a Trapezium: $A = \frac{1}{2}$(the sum of $\|\|$ sides) $\times$ height .
	10.4	10.4 Area of irregular Quads: use the difference of areas of known figures

EC SEPT 2024

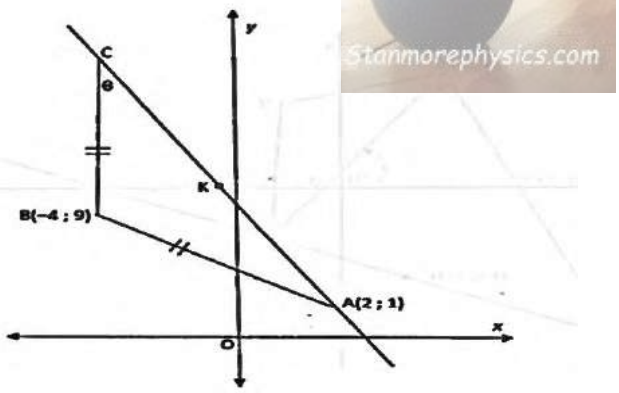
1 Quadrilateral AEBC is drawn. Coordinate of B are (3;4). G, O and F (5;0) are *x*-intercepts of lines AC, AB, and BC, respectively. $H(1;-4)$ is a point on line AC. $A\hat{B}C=\theta$. Area of $\Delta OBF=12$ square units and inclination of line AB is $45°$. $HC=2AH$



1.1 Calculate the length of BF. (2) **L1**
1.2 Calculate the gradient of BF. (2) **L1**
1.3 Calculate the size of θ (3) **L2**
1.4 Prove that $HF \parallel AB$ (4) **L2**
1.5 It is further given that; EC bisects AB perpendicularly. What type of quadrilateral is AEBC? (1) **L3**
1.6 Hence or otherwise calculate the length of AC. (4) **L3**
1.7 Calculate the area of quadrilateral AOFC. (3) **L2**

FS SEPT 2024

2 In the diagram, ABC is an isosceles triangle with $A(2;1)$ and $B(-4;9)$. AB=BC, and BC is parallel to the y-axis

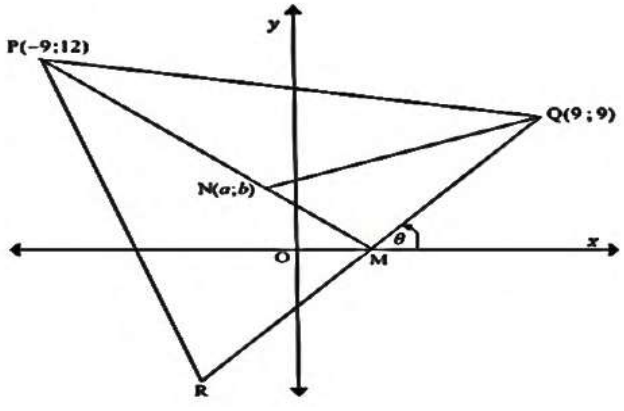


Calculate

2.1 The length of AB (2) **L1**
2.2 The coordinate of C (2) **L2**
2.3 The coordinate of K, the midpoint of AC (2) **L2**
2.4 The equation of AC in the form $y=mx+c$ (3) **L2**
2.5 The size of θ (3) **L3**
2.6 The area of triangle ABC (4) **L2**
2.7 The coordinate of D if ABCD is a rhombus (2) **L2**

GP SEPT 2024

3 In the diagram below, $P(-9;12)$, $Q(9;9)$ and R are vertices of ΔPQR. M is the midpoint of QR and $N(a;b)$ is a point on PM in the second quadrant. The equation of QR is given by $zy-3x+9=0$. The angle of inclination of QR is θ



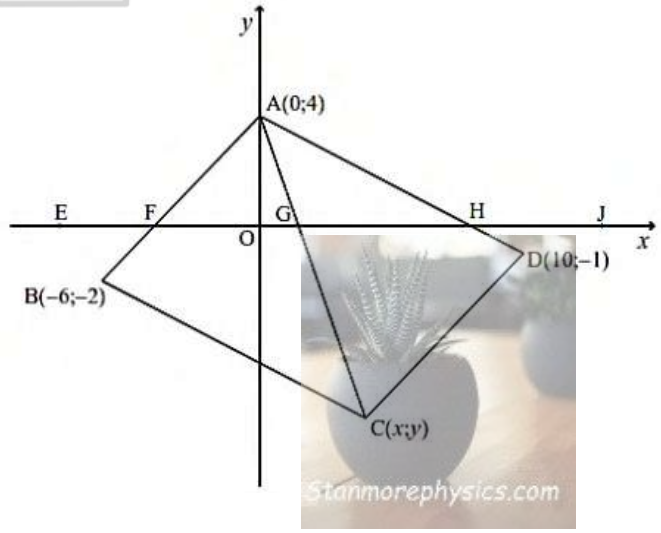
3.1 Calculate the coordinate of M, the *x*-intercept of line PM. (2) **L1**
3.2 Determine the equation of PM in the form $y=mx+c$ (4) **L2**
3.3 Calculate the size of θ (2) **L1**
3.4 Show that $b=3-a$, if P, N and M are collinear, (1) **L2**
3.5 Hence, determine the value of a and b if $NQ=5\sqrt{5}$ units (5) **L2**
3.6 Determine the equation of a circle having centre at O, the (4) **L3**

origin and passing through point R

3.7 The acute angle between the line QR and the line with equation $y = mx + 4$ is 45°. Determine the possible value(s) of m (4) **L4**

KZN SEPT 2024

4 ABCD is a parallelogram with $A(0;4), B(-6;-2), C(x;y)$ and $D(10;-1)$ as shown below.AC is drawn. F, G and H are the x-intercept of AB, AC and AD respectively. E is a point on the x-axis to the left of F and J a point on the x-axis to the right of H.



4.1 Determine the gradient of AB (2) **L1**

4.2 Determine the equation of CD (3) **L2**

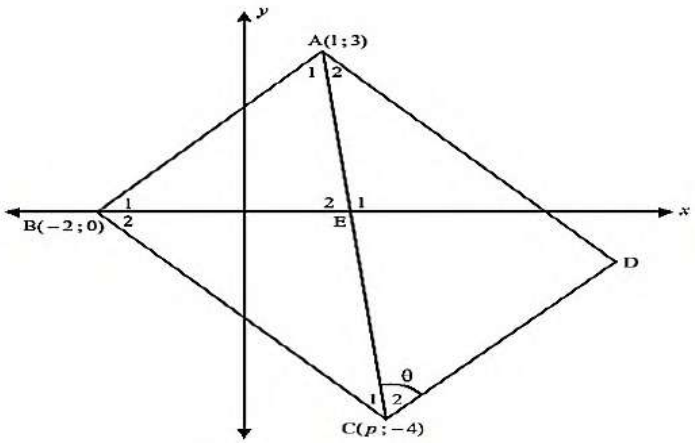
4.3 Determine the coordinates of M, the midpoint of AC (3) **L2**

4.4 Determine the coordinate of C. (2) **L2**

4.5 Determine the size of $B\hat{C}D$ (6) **L3**

MP SEPT 2024

5.1 ABC is a triangle with vertices $A(1;3), B(-2;0)$ and $C(p;-4)$ where $p > 0$.The length of AC is $\sqrt{50}$ units



5.1.1 Determine the gradient of AB (2) **L1**

5.1.2 Show, by calculation that $p = 2$ (4) **L2**

5.1.3 Determine the equation of the perpendicular bisector of AB (4) **L2**

5.1.4 Write down the coordinate of D such that ABCD is a rectangle (2) **L2**

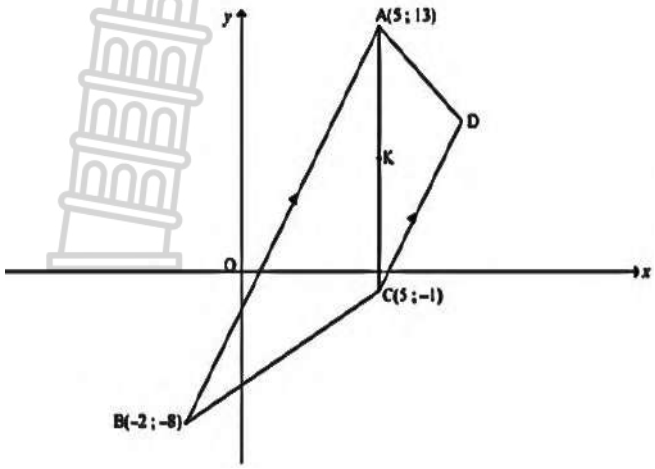
5.1.5 Determine the equation of the circle passing through A, B and C (4) **L3**

5.1.6 Calculate the size of θ rounded off to the nearest whole number (5) **L3**

5.2 Three straight lines AB, RS and $x = -3$ intersect each other. The equation of AB is $3x + by = -2$ with $b \neq 0$, and the equation of RS is $y = -\frac{2}{3}x + 2$. Calculate the value of b (4) **L4**

NC SEPT 2024

6 ABCD is a trapezium with vertices $A(5;13), B(-2;-8), C(5;-1)$ and D. K is the midpoint of AC and $BA \parallel CD$.



6.1 Write down the coordinates of K. (2) **L1**

6.2 Calculate:

6.2.1 The gradient of AB. (2) **L1**

6.2.2 p, if $D(p;p)$ (2) **L2**

6.3 ABED is parallelogram with E a point in the fourth quadrant. Calculate the coordinate of E. (2) **L3**

6.4 Calculate the area of obtuse ΔABC (3) **L2**

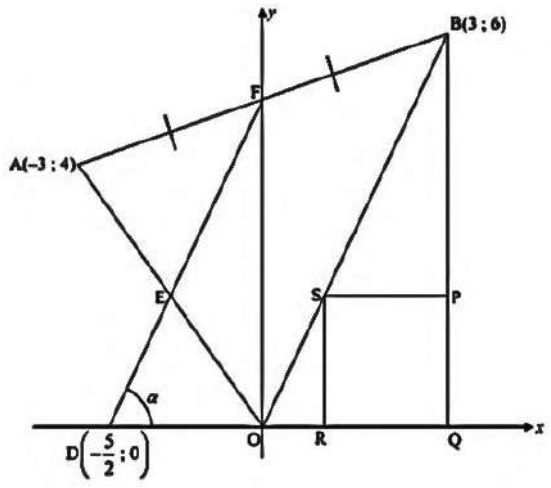
6.5 ΔABC is reflected in the line $x=5$ to form ΔAFC.

6.5.1 Calculate the perimeter of ΔABF. (5) **L3**

6.5.2 Determine the equation of a circle, centred at O, the origin, passing through point F. (2) **L3**

NW SEPT 2024

7 In the diagram, $A(-3;4), B(3;6)$ and $O(\text{origin})$ are vertices of $\Delta AB0$. F is the midpoint of AB and is joined with $D\left(-\frac{5}{2};0\right)$. The angle of inclination of FD is α. The lines AO and DF intersect at E. A quadrilateral PQRS is drawn with QR on the x-axis and S is a point on OB. The side QP is produced to B.



7.1 Calculate the:

7.1.1 coordinates of F (2) **L1**

7.1.2 gradient of DF (2) **L2**

7.1.3 size of α (2) **L2**

7.2 Write down the equation of OB (1) **L2**

7.3 Give a reason why $DF \parallel OB$ (1) **L2**

7.4 It is given that PQRS is a square with an area of $9x^2$ squared units. (6) **L4**

7.5 Prove that EDOS forms a parallelogram. (4) **L3**

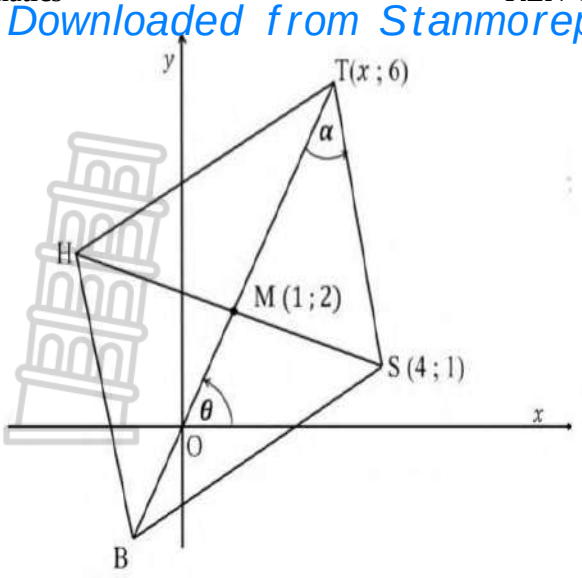
WC SEPT 2024

8 In the diagram below HS and TB are the diagonals of parallelogram HTSB.

- HS and TB intersect at $M(1;2)$.
- TB intersect the x- and y-axis at the origin.
- $T(x,6)$ and $S(4;1)$ are vertices of HTSB.
- $M\hat{T}S = \alpha$ and the inclination angle of TB is θ.

8.1 Determine:

8.1.1 the equation of the line TB in the form $y=mx+c$. (2) **L2**



8.1.2 the size of θ. (2) **L2**

8.1.3 the coordinates of H. (3) **L2**

8.2 Show that the x-coordinate of T is 3 if it is further given that $TS = \sqrt{26}$. (3) **L2**

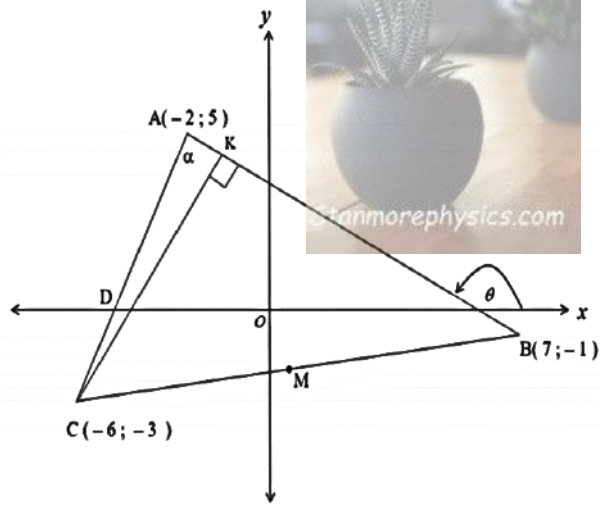
8.3 Calculate α, the size of $M\hat{T}S$. (4) **L3**

8.4 Calculate the area of ΔBTS. (4) **L2**

8.5 Determine the perpendicular height of the parallelogram if TS is the base. (3) **L3**

NDENGETHO HIGH SCHOOL JUNE 2025

9 Refer to the diagram below and answer the quaestions that follow:



Determine:

9.1 The coordinates of M, the midpoint of CB. (2) **L1**

9.2 The length of AB. Leave your answer in surd form. (2) **L2**

9.3 The gradient of AB. (2) **L2**

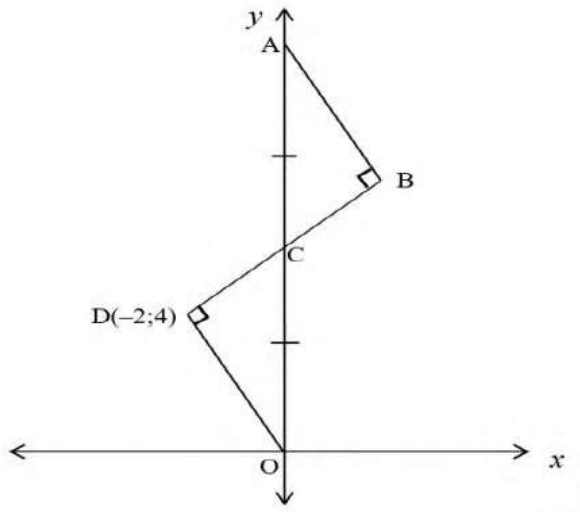
9.4 θ, the angle of inclination of AB. (2) **L2**

9.5 α, the acute angle between lines CA and AB. (4) **L3**

9.6 The area of ΔACK if $AK : KB = 4 : 35$ and $CK = 8{,}88$ mm. (5) **L4**

NDENGETHO HIGH SCHOOL JUNE 2025

10 In the diagram below, right-angled triangles ABC and ODC are drawn.O is the origin.A and C lie on the y-axis.C is the midpoint of OA. D is the point $(-2;4)$.



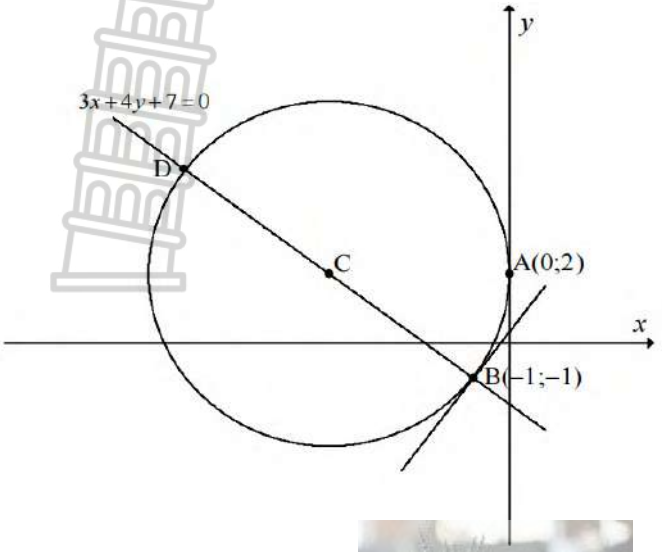
10.1 Determine the equation of OD (2) **L1**

10.2 Determine the equation of BD in the form $y = mx + c$ (3) **L2**

10.3 Determine the coordinates of A. (2) **L3**

KZN JUN 2025

11.1 In the diagram below, the circle with C touches the y-axis at $A(0;2)$. A straight line with equation $3x+4y+7=0$ cuts the circle at $B(-1;-1)$ and D.



11.1.1 Determine the equation of the tangent to the circle at B. (2) **L2**

11.1.2 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (5) **L2**

11.1.3 Determine the coordinates of image of B, after reflection of the circle in the line $y=2$. (2) **L3**

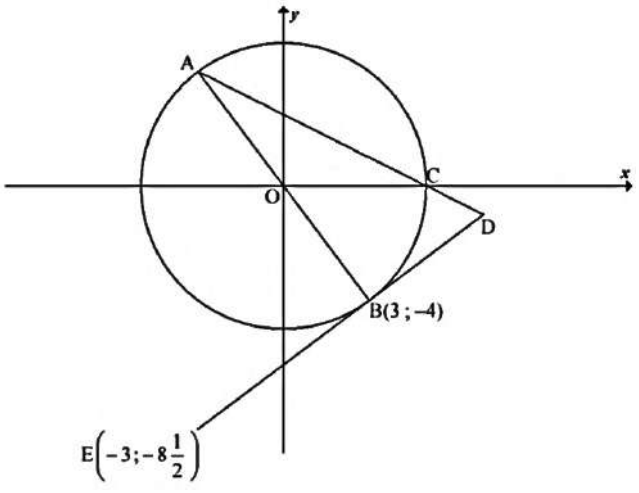
11.2 A circle with equation $x^2-4x+y^2+6y-51=0$ is drawn in a Cartesian plane.

11.2.1 Determine the coordinates of the centre of the circle and the length of its radius. (4) **L2**

11.2.2 Another circle with equation $x^2+y^2=r^2$ is drawn in the same Cartesian plane and touches the circle with equation $x^2-4x+y^2+6y-51=0$ internally. Calculate the value of r. Give your answer correct to 2 decimal digits. (4) **L2**

GP (SECOND PUSH) JUN 2025

12 The diagram below shows a circle with centre O at the origin. AB is a diameter of the circle. The straight line ACD meets the tangent EBD to the circle at D. The coordinates of B and E are $\left(-3;-8\frac{1}{2}\right)$ respectively.



12.1 Determine the coordinates of A. (2) **L2**

12.2 Determine the equation of the circle passing through A, B and C. (3) **L2**

12.3 Write down the length of AB. (2)

12.4 If it is given that AD is $\sqrt{125}$ units, calculate the length of BD. Give reasons. (3) **L2**

12.5 Calculate the area of ΔABD. (3) **L2**

12.6 Another circle passes through A, B and E. Determine, with reasons, the equation of this circle. Write the answer in the form $(x-a)^2+(y-b)^2=r^2$. (6) **L4**

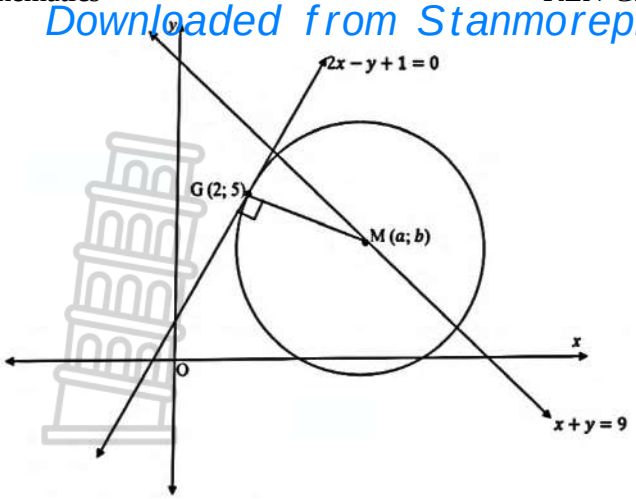
GP JUN 2025

13.1 In the figure below, the line $2x-y+1=0$ is a tangent to the circle, with centre $M(a;b)$, at $G(2;5)$. The centre of the circle lies on the line $x+y=9$.

13.1.1 Determine the gradient of GM (2) **L2**

13.1.2 Determine the equation of GM in the form $y=mx+c$. (2) **L2**

13.1.3 Calculate the coordinates of M. (4) **L2**



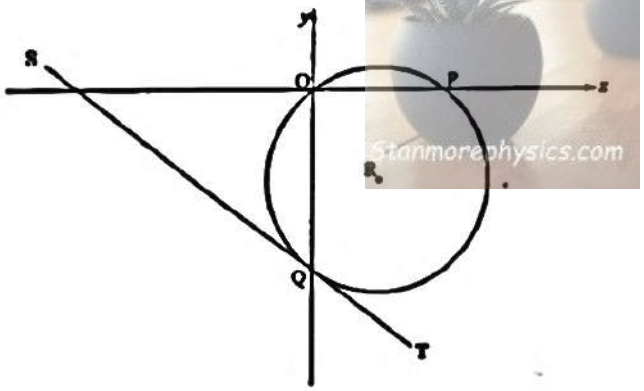
13.1.4 Hence, or otherwise, calculate the length of the radius of the circle. (2) **L3**

13.1.5 Write down the equation of the circle in the form $x^2 + y^2 + Cx + Dy + E = 0$. (3) **L2**

13.2 Determine the equation of the inverse of the tangent to the circle $x^2 + y^2 - 26x + 105 = 0$ at $(7;2)$. Give your answer in the form $y = mx + c$ (7) **L4**

EC SEPT 2024

14.1 In the diagram below, R is the centre of the circle OPQ. Point Q is the *y*-intercept of the circle. SQT is the tangent of the circle at Q. The equation of SQT is $y = -\frac{3}{4}x - 8$



14.1.1 Calculate the coordinates of Q (2) **L2**

14.1.2 Determine the equation of QR in the form $y = mx + c$ (3) **L2**

14.1.3 Calculate the coordinates of P, the x-intercept of line QR. (2) **L2**

14.1.4 Calculate the coordinates of R, the centre of the circle. (3) **L2**

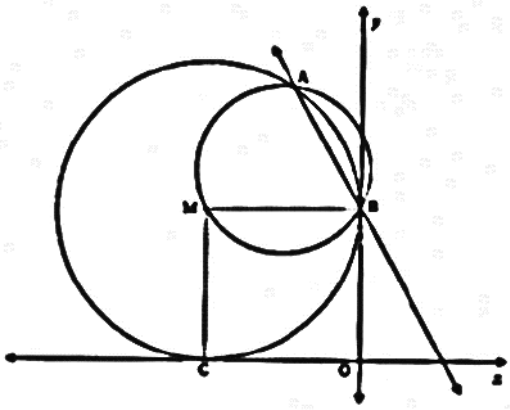
14.1.5 Write down the equation of the circle centred at R in the form: $(x-a)^2 + (y-b)^2 = r^2$. (3) **L2**

14.1.6 If $y = k$ is a tangent to the circle, determine the value(s) of k (3) **L3**

14.2 Calculate the maximum length of the radius of the circle having equation $x^2 + y^2 - 2x\sin\theta - 4y\sin\theta = -2$. (5) **L4**

FS SEPT 2024

15 In the diagram, a circle centred at M touches the x-axis at C and the y-axis at point B. A second circle with equation $x^2 + y^2 + x - 3y + 2 = 0$ passes through A and M and intersects circle M at A and B. The equation of the common chord AB is given by $y = -x + 1$



15.1 Determine the coordinates of the centred and the radius of the circle which passes through B, M and A. (4) **L2**

15.2 Calculate the coordinate of A (5) **L2**

15.3 Show that the equation of the circle centred at M, is $x^2 + y^2 + 2x - 2y + 1 = 0$ (5) **L2**

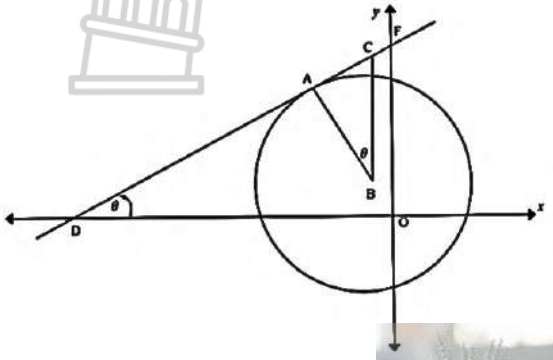
15.4 The straight line with equation $y = -x + k$ is a tangent to the circle with centre M.

15.4.1 Show that this equation can be written as: $2x^2 + (4-2k)x + (k^2 - 2k + 1) = 0$ (3) **L3**

	y5.4.2 Calculate the numerical value(s) of k	(5)	**L4**

GP SEPT 2024

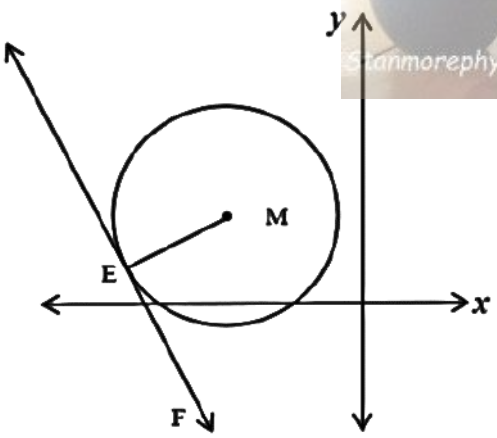
16 In the diagram, the equation of the circle centred at B is given by $(x+1)^2+(y-1)^2=20$. DF is a tangent to the circle at A with D and F, the *x*- and *y*-intercepts, respectively. $C(-1;6)$ is a point on DF with BC parallel to the y-axis. $C\hat{B}A = A\hat{D}O = \theta$



16.1	Write down the coordinates of B.	(1)	**L2**
16.2	Show that $AC=\sqrt{5}$.	(3)	**L2**
16.3	Write down the value of $\tan\theta$	(1)	**L1**
16.4	Show that the equation of AB is given by $y=-2x-1$	(3)	**L2**
16.5	Determine the coordinate of A	(4)	**L2**
16.6	Calculate the ratio of the area of ΔABC to the area of ΔODF. Simplify your answer.	(6)	**L4**

NDENGETHO HIGH SCHOOL JUN 2025

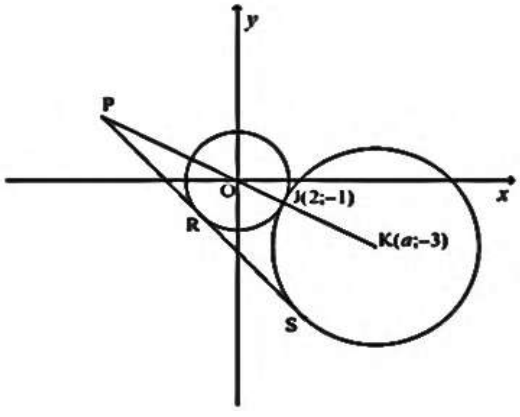
17 The diagram shows a circle with centre M. The equation of the circle is $x^2+8x+y^2-4y+15=0$. The tangent to the circle from F touches the circle at point E.



17.1	Determine the coordinates of the centre of the circle, M, and the length of the radius. Show your working	(5)	**L3**
17.2	If the coordinates of M are $(-4;2)$, and the length of the tangent EF is $y=-2x-11$, determine the equation of line ME and the coordinates of point E	(7)	**L4**

KZN SEPT 2024

18 The diagram below shows two circle touching at $J(2;-1)$. The smaller circle has its centred at the origin and the bigger circle has centre $K(a;-3)$. The length of the radius of the bigger circle is TWICE the length of the smaller circle. SR is a tangent to both circles, touching the bigger circle at S and the smaller circle at R. KO and SR are both produced to intersect in point P.



18.1	Calculate the length of the radius of the smaller circle.	(2)	**L2**
18.2	Show that $a=6$	(3)	**L2**
18.3	Determine the equation of the bigger circle.	(2)	**L2**
18.4	Does the point $(10;-4)$ lie outside, inside or on the bigger circle?	(3)	**L2**
18.5	Calculate the length of PS.	(5)	**L3**

18.2 The length of the diameter of the circle with equation $x^2 - 4x + y^2 + 5y = -d$ is 24.

Determine:

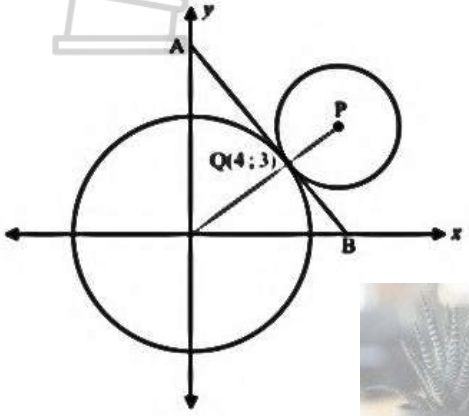
18.2.1 The coordinates of the centre of the circle, (4) **L3**

18.2.2 The value of d (3) **L2**

MP SEPT 2024

19 Two circles in the diagram below represent two interlocking gears, which touches at the point $Q(4;3)$.

The circle have the following equations: $x^2 + y^2 = 25$ and $x^2 - 12x + y^2 - 9y + 50 = 0$



19.1 Show that the coordinates of P are $\left(6;4\frac{1}{2}\right)$. (3) **L2**

19.2 Determine the equation of the common tangent AB. (4) **L3**

19.3 If the larger gear makes one full revolution, how many times will the smaller gear turn completely? (4) **L4**

19.4 Determine the area of ΔAOB (3) **L2**

19.5 Another tangent to the circle with centre O, drawn from A, touches the circle at C, and C is the reflection of Q by the y-axis. Determine the length of CQ (2) **L4**

STUDY AND MASTER

20 Determine the equations of the two tangents to the circle defined by the equation $x^2 + y^2 = 20$, given that both tangents are parallel to the line with the equation $y = -2x + 4$. (7) **L3**

STUDY AND MASTER

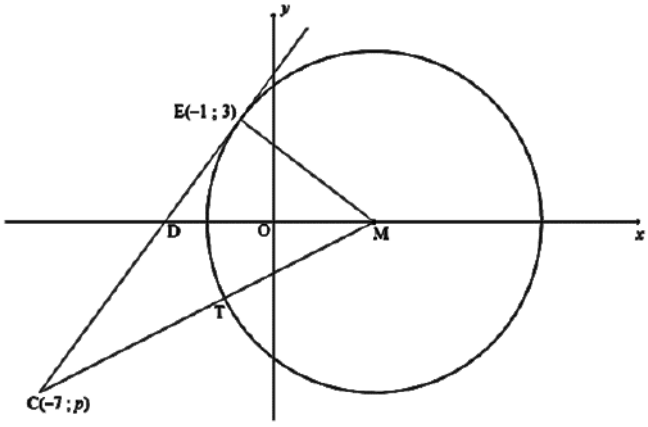
21 A circle with centre $P(m;n)$ touches the y-axis at point Q and passes through point $R(-1;-1)$. If P is on line $2x + y + 4 = 0$:

21.1 Calculate the possible coordinates of P. (7) **L4**

21.2 Write down the equation(s) of the circle(s) in the form $(x-a)^2 + (y-b)^2 = r^2$. (4) **L3**

DBE MAY/JUN 2025

22 In the diagram, M is the centre of the circle having equation $(x-3)^2 + y^2 = 25$. E (–1; 3) and T are points on the circle. EC is a tangent to the circle at E and cuts the x-axis at D. ED = $\frac{15}{4}$ units. MT is produced to meet the tangent at C (–7; p).



22.1 Write down the size of $C\hat{E}M$. (1) **L1**

22.2 Determine the equation of the tangent EC in the form $y = mx + c$. (4) **L2**

22.3 Calculate the length of DM. (3) **L2**

22.4 Show that $p = -5$. (1) **L2**

22.5 Calculate the coordinates of S if SEMC is a parallelogram and $x_s < 0$. (3) **L2**

22.6 If the radius of the circle, centred at M, is increased by 7 units, determine whether S lies inside or outside the new circle. Support your answer with the necessary calculations. (3) **L4**

22.7 If ET is drawn, calculate the size of $E\hat{T}M$. (5) **L3**

LEARNING CHANNEL TEXTBOOK

23 Determine the equation of the diameter of the circle $x^2+y^2+4x-6y=12$ through the point $(1;-1)$ on the circle. (4) **L3**

24 Determine the length of the tangent drawn from the point $A(2a;2b)$ to the circle $(x-a)^2+(y-b)^2=a^2$. (4) **L4**

25 $P(-1;3)$; $Q(-5;-5)$ and $R(4;-2)$ are the vertices of ΔPQR. A line through $O(0;0)$ parallel to PQ cuts the perpendicular bisector of PQ in M. Determine an equation of the circle centre M and radius equal to the length of OR. (7) **L4**

26 ABCD is a trapezium with co-ordinates $A(-4;3)$, $B(x;6)$, $C(4;y)$ and $D(-2;-1)$ where $x<4$. If $AD \parallel BC$ and $BC=2AD$. Show that $x=0$ and find y. (7) **L4**

TOPIC	**TRIGONOMETRY** **[± 50 MARKS]**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

1. Definitions of trig ratios:

In a right angled triangle: $\sin\theta=\frac{opposite}{hypotenuse}$; $\cos\theta=\frac{adjacent}{hypotenuse}$ and $\tan\theta=\frac{opposite}{adjacent}$

SOH CAH TOA helps you to remember these definitions.

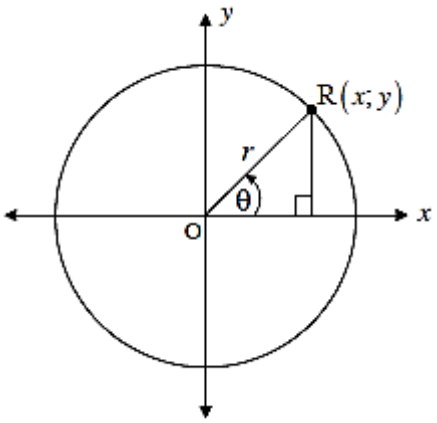
In a Cartesian plane:

$\sin\theta=\frac{y}{r}$;

$\cos\theta=\frac{x}{r}$;

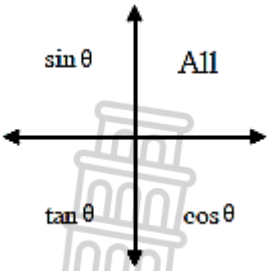
$\tan\theta=\frac{y}{x}$

and $r^2=x^2+y^2$



2. CAST Rule:

All trig ratios are positive in the 1st quadrant. **All**
Only $\sin\theta$ is positive in the 2nd quadrant. **Students**
Only $\tan\theta$ is positive in the 3rd quadrant. **Take**
Only $\cos\theta$ is positive in the 4th quadrant. **Care**



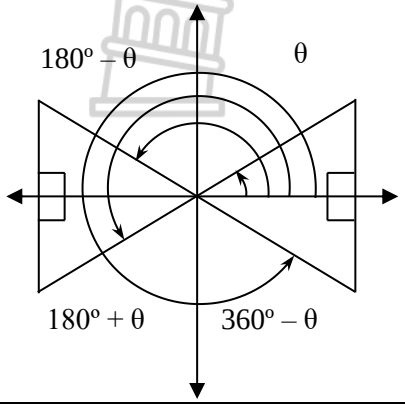
3. Reduction Formulae:

If θ is an acute angle, i.e. in the 1st quadrant,
$180°-\theta$ will lie in the 2nd quadrant,
$180°+\theta$ will lie in the 3rd quadrant,
and $360°-\theta$ will lie in the4th quadrant.

$\sin\theta=\sin(180°-\theta)=-\sin(180°+\theta)=-\sin(360°-\theta)$

$\cos\theta=-\cos(180°-\theta)=-\cos(180°+\theta)=\cos(360°-\theta)$

$\tan\theta=-\tan(180°-\theta)=\tan(180°+\theta)=-\tan(360°-\theta)$



For $90°-\theta$ and $90°+\theta$ the ratio changes to its co-function.
The co-function of cos is sin and the co-function of sin is cos.

$\sin(90°-\theta)=\cos\theta$ and $\cos(90°-\theta)=\sin\theta$ $\sin(90°+\theta)=\cos\theta$; and $\cos(90°+\theta)=-\sin\theta$

Trigonometric identities:

Square identity: $\sin^2\theta+\cos^2\theta=1$

Quotient identity: $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$

Compound Angles:

$\sin(\theta\pm\beta)=\sin\theta\cos\beta\pm\cos\theta\sin\beta$

$\cos(\theta\pm\beta)=\cos\theta\cos\beta\pm\sin\theta\sin\beta$

Double Angles:

$\cos 2\theta=\cos^2\theta-\sin^2\theta$

$\cos 2\theta=2\cos^2\theta-1$

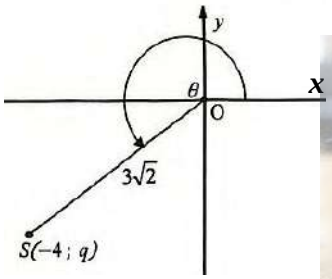
$\cos 2\theta=1-2\sin^2\theta$

$\sin 2\theta=2\sin\theta\cos\theta$

1. **EC SEPT 2024**

In the diagram below, point $S(-4\,;\,q)$ and reflex angle θ are shown. O is the point at the origin and $OS=3\sqrt{2}$.

Without using a calculator, determine the value of:



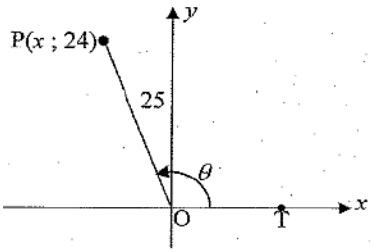
1.1 q (2) **L2**

1.2 $\sin(\theta+45°)$ (4) **L2**

1.3 $\cos(2\theta-360°)$ (4) **L3**

2. **NW SEPT 2024**

In the diagram below, the point P(x ; 24) is 25 units from the origin O. T is a point on the x-axis such that . $T\hat{O}P=\theta$.



2.1 Calculate the value of x. (2) **L1**

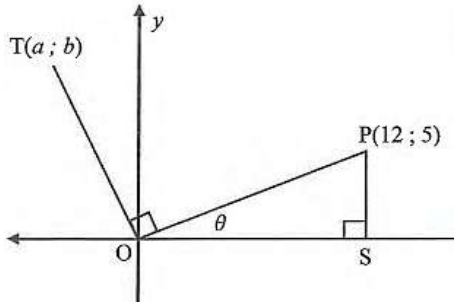
2.2 **Without using the calculator**, determine the value of $\tan(360°-\theta)$ (2) **L2**

2.3 Calculate the size of $P\hat{O}T$. (2) **L2**

3. **GP SEPT 2023**

In the diagram below, P is a point $(12\,;\,5)$ and $T(a\,;\,b)$. $OT\perp OP$; $PS\perp$ x-axis and $P\hat{O}S=\theta$.

Without using the calculator, determine the value of:



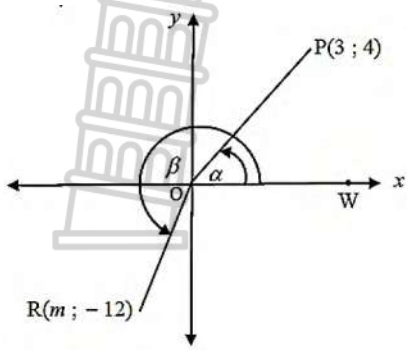
3.1 $\tan\theta$ (2) **L1**

3.2 $\sin\theta$ (2) **L2**

3.3 a, if $TO=19,5$ units. (3) **L2**

4. **LP SEPT 2023**

In the diagram below $P(3;4)$ and $R(m;-12)$ are two points as indicated. $P\hat{O}W=\alpha$ and $R\hat{O}W=\beta$. Answer the following questions without using a calculator.



4.1	Write down the value of $\tan\alpha$	(1)	**L1**
4.2	Determine the value of $\sin(90°+\alpha)$	(3)	**L2**
4.3	Determine the value of *m* if it is given that $12+13\sin\beta=0$.	(4)	**L2**
4.4	Determine the value of $\cos(\alpha+\beta)$.	(3)	**L3**

5.

If $\sqrt{5}\sin\theta+2=0$ and $\theta\in[90°;270°]$, determine **without the use of a calculator** the value of the following:

5.1	$\cos 2\theta$	(3)	**L2**
5.2	$\cos(\theta-135°)$	(4)	**L2**

6.

If $\sin 2\theta=\dfrac{-4\sqrt{2}}{9}$ and $2\theta\in[90°;270°]$, determine the value of the following **without the use of a calculator.**

6.1	$\cos 2\theta$	(3)	**L1**		6.2	$\sin\theta$	(4)	**L2**		6.3	$\tan\theta$	**(4)**	**L2**

7. **MP SEPT 2023**

If θ is a reflex angle, and $\tan\theta=-\dfrac{3}{4}$, determine without the use of a calculator and with the aid of a sketch, the value of:

7.1	$\cos 2\theta$	(3)	**L2**		7.2	$\cos(\theta+30°)$	(3)	**L2**

8. **NC SEPT 2020**

If $13\cos\theta=5$, and $\theta\in[180°;360°]$, **without the use of a calculator**, determine the value of the following:

8.1	$\sin\theta$	(2)	**L1**		8.2	$\sin(\theta+45°)$	(5)	**L2**

9. **KZN SEPT 2024**

If $\tan 58°=n$, determine the following in terms of *n* without using a calculator.

9.1	$\sin 58°$	(3)	**L2**		9.2	$\sin 296°$	(4)	**L2**		9.3	$\cos 2°$	(3)	**L3**

10.

If $\cos 21°=p$, determine the following in terms of *p*.

10.1	$\tan 201°$	(3)	**L2**		10.2	$\sin 42°$	(3)	**L2**		10.3	$\cos 51°$	(3)	**L2**

11. **NC SEPT 2024**

Given that $\sin 10°=\sqrt{k}$, write each of the following in terms of *k* **without using a calculator**.

11.1	$\sin 190°$	(2)	**L2**		11.2	$\cos 20°$	(3)	**L2**		11.3	$\cos 50°$	(4)	**L2**

12. **FS SEPT 2024**

If $\sin\dfrac{x}{2}=p$, express $\sin x-1$ in terms of *p*. (4) **L3**

13. Given that $\cos 42°=\sqrt{k}$, determine the value of $\sin^2 69°$ in terms of *k*. (3) **L3**

14. If $\sin 38°=p$, determine the value of the following in terms of *p* **without the use of a calculator.**

14.1	$\cos 218°$	(3)	**L2**		14.2	$\cos 14°$	(3)	**L3**		14.3	$\sin 26°\cos 26°$	(2)	**L3**

15. **WC SEPT 2023**

Given $\sin 64° = p$. **Without using the calculator** determine the value of the following in terms of *p*.

15.1	$\tan 64°$	(2)	**L1**	15.2	$\cos 334°.\sin 244°$	(3)	**L2**	15.3	$8\sin 16°.\cos 16°.\cos 32°$	(3)	**L2**

16. **DBE NOV 2024**

If $\cos\alpha = p$, express the following expression in terms of *p*.

$$\frac{\cos\left(\frac{\alpha}{2}-45°\right).\sin\left(\frac{\alpha}{2}-45°\right)}{2}$$

(4) **L3**

17. **IEB NOV 2023**

Given that $\cos\theta = \frac{a^2-b^2}{a^2+b^2}$ where $0 < b < a$ and $\sin\theta < 0$.

Determine the value of $\tan\theta$ in terms of a and b. (4) **L4**

2021 HIGH FLYER DOC

18. Given $\cos A + \sin A = k$, and k is acute

18.1 Determine the value of $\tan A + \frac{1}{\tan A}$ in terms of *k*. (4) **L4**

18.2 Prove that: $\sin A + \cos A = \sqrt{1+2k}$ (3) **L4**

19. Simplify the following **without the use of a calculator.**

19.1 $2\cos^2 15° - 1 + \frac{2\sin 140°}{\cos 310°}$ (5) **L3**

19.2 $\cos 420°\cos 15° + \sin 300°\cos 105°$ (5) **L3**

19.3 $\frac{\sin 210°.\cos 510°}{\cos 315°.\sin(-135°)}$ (7) **L2**

19.4 $\sin 20° + \cos 120°.\tan 405° + \cos 110°$ (4) **L2**

19.5 $\frac{\cos 105°.\cos 15°}{\cos(45°-x).\cos x - \sin(45°-x).\sin x}$ (5) **L3**

19.6 $\frac{\cos 330^\circ.\sin 140^\circ}{\sin(-160^\circ).\tan 405^\circ.\sin 290^\circ}$ (10) **L2**

19.7 $\sqrt{4^{\sin 150°}.2^{3\tan 225°}}$ (5) **L3**

19.8 $\frac{\left(\sqrt{2}\cos 15° + 1\right)\left(\sqrt{2}\cos 15° - 1\right)\sin(-2x)}{4\sin x\cos x}$ (4) **L3**

20. Simplify the following expressions.

20.1 $\frac{\sin^2(180°+x).\sin(-x)}{-\sin(90°+x).\tan x} - 1$ (6) **L2**

20.2 $\frac{1}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)} - \frac{(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)}$ (5) **L3**

20.3 $\frac{\sin(-2x).(1-\sin^2 x)}{\sin(90°+x).\tan x}$ (6) **L2**

20.4 $\frac{\sin(360°-2x).\sin(-x)}{\sin(90°+x)} + 2\cos^2(180°+x)$ (6) **L2**

20.5 $\dfrac{\tan(180°+x).\cos x}{\sin(180°+x).\cos x-\cos(540°+x).\cos(90°+x)}$ (6) **L2**

20.6 $\dfrac{\frac{1}{2}\cos(90°+\theta)-\sin(\theta-90°)}{\cos^2(180°-\theta)-2\cos(-\theta)+\cos^2(\theta+90°)}$ (7) **L3**

20.7 $\tan(-\theta).\sin(90°+\theta)+\dfrac{\sin 2\theta}{2\cos(360°+\theta)}$ (6) **L2**

20.8 $\dfrac{\sin 3x+\sin x}{\cos 2x+1}$ (6) **L4**

DBE MAY/JUN 2024

21. Given the expression: $\dfrac{\sin 150°+\cos^2 x-1}{2}$

21.1 **Without using a calculator,** simplify the expression to a single trigonometric term in terms of $\cos 2x$. (6) **L3**

21.2 Hence, determine the general solution for $\dfrac{\sin 150°+\cos^2 x-1}{2}=\dfrac{1}{25}$ (5) **L3**

22. Prove the following identities

22.1 $\dfrac{3\sin x+2\sin 2x}{2+3\cos x+2\cos 2x}=\tan x$ (4) **L3**

22.2 $\tan^2 x\left(\dfrac{1}{\tan^2 x}-1\right)=\dfrac{\cos 2x}{\cos^2 x}$ (3) **L3**

22.3 $\dfrac{1-\cos 2x}{\sin 2x}=\tan x$ (3) **L2**

22.4 $\dfrac{\cos\theta-\cos 2\theta+2}{3\sin\theta-\sin 2\theta}=\dfrac{1+\cos\theta}{\sin\theta}$ (5) **L2**

22.5 $\dfrac{\cos 2x+\cos^2 x+3\sin^2 x}{2-2\sin^2 x}=\dfrac{1}{\cos^2 x}$ (4) **L3**

22.6 $\dfrac{\sin\theta.\tan\theta}{\tan 2\theta.(1-\tan^2\theta)}=\dfrac{\sin\theta}{2}$ (5) **L3**

22.7 $\tan x(1-\cos^2 x)+\cos^2 x=\dfrac{(\sin x+\cos x)(1-\sin x.\cos x)}{\cos x}$ (5) **L3**

22.8 $\dfrac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}=1-\sin x$ (4) **L3**

22.9 $\dfrac{\tan x+1}{\sin x\tan x+\cos x}=\sin x+\cos x.$ (5) **L2**

22.10 $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}=-\sin x$ (3) **L3**

22.11 $\dfrac{\sin 2x}{\cos x\,(1-\cos 2x)\left(1+\dfrac{1}{\tan^2 x}\right)}=\sin x$ (5) **L3**

22.12 $\dfrac{2\cos 2\theta.\cos\theta}{\cos^2\theta-\sin^2\theta}+2\tan\theta.\sin\theta=\dfrac{2}{\cos\theta}$ (3) **L3**

22.13 $\dfrac{\sin(90°+\theta).\sin(\theta-180°)-\cos^3(180°+\theta)}{\cos\theta}-2\sin^2\theta=\cos 2\theta$ (5) **L2**

22.14 $\dfrac{\sin 2\theta+\cos 2\theta+1}{\cos 2\theta}=\dfrac{2\cos\theta}{\cos\theta-\sin\theta}$ (4) **L3**

22.15 $\frac{\cos(360°-2x)\cdot\cos(180°+x)}{\sin 2x-\cos(90°-x)}=\frac{\cos x+1}{\sin x}$ (7) **L2**

22.16 $\frac{\sin 5x.\cos 3x-\cos 5x.\sin 3x}{\tan 2x}-1=-2\sin^2 x$ (4) **L2**

22.17 $\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}=\tan x$ (4) **L3**

23. **DBE NOV 2024**

Given the identity: $\cos(x-y)=\cos x.\cos y+\sin x.\sin y$

23.1 Use the compound angle identity to derive a formula for $\cos(x+y)$. (2) **L2**

23.2 Hence, or otherwise, show that :

$$\frac{\cos(90°-x).\cos y+\sin(-y).\cos(180°+x)}{\cos x.\cos(360°+y)+\sin(360°-x).\sin y}=\tan(x+y)$$ (6) **L3**

24 **2021 HIGH FLYER DOC**

Given the following identity: $\frac{\cos x-\sin x.\sin 2x}{\cos 2x}=\cos x$

24.1 Prove the identity (4) **L3**

24.2 For which values of x is the identity undefined? Give your answer in general solution form. (5) **L2**

25. **2021 HIGH FLYER DOC**

Given the following identity: $\frac{\sin 7x+\sin x}{2\cos 3x}=\sin 4x$

25.1 Prove the identity (4) **L3**

25.2 For which values is the identity above undefined? (3) **L2**

26. Determine the general solution for the following equations.

26.1 $\cos 2x-\frac{1}{3}=\frac{1}{3}\sin x$ (6) **L4**

26.2 $\sin\theta.\sin\frac{3\theta}{2}+\cos\frac{3\theta}{2}.\cos\theta=-\frac{\sqrt{3}}{2}$ (4) **L3**

26.3 $\cos(x-60°)=-\frac{\sqrt{3}}{2}\sin x$ (5) **L3**

26.4 $2\sin\alpha\cos\alpha=2\cos^2\alpha-2\sin^2\alpha$ (5) **L3**

26.5 $\cos 2x-5\cos x-2=0$ (6) **L2**

26.6 $\frac{\cos 2x+1}{\sin 2x}=1$ (7) **L3**

26.7 $\sqrt{6\sin^2 x-11\cos(90°+x)+7}=2$; where $x\in(0°\,;360°)$. (6) **L2**

26.8 $\cos 54°.\cos x+\sin 54°.\sin x=\sin 2x$ (5) **L3**

26.9 $6\cos 2\theta+\cos\theta+6=0$ (5) **L3**

26.10 $1-\sin x.\cos x-3\cos^2 x=0$ (5) **L3**

26.11 Solve $\sin x+1=\cos 2x$ in the interval $x\in[180°\,;360°]$ (6) **L3**

26.12 $4\sin x+6\sin x.\cos x-3\cos x=2$ (5) **L3**

DBE MAY/JUN 2023

Given: $3\tan 4x=-2\cos 4x$

27.1 **Without using a calculator**, show that $\sin 4x=-0,5$ is the only solution to the above equation. (4) **L3**

27.2 Hence, determine the general solution of x in the equation $3\tan 4x=-2\cos 4x$ (3)

28. Given that $\cos x = 2m$ and $\cos 2x = 7m$, determine the value(s) of m. (5) **L4**

TOPIC TRIGONOMETRIC GRAPHS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

- The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.
- Candidates must be able to use and interpret functional notation. Learners must understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$ and $a.f(x)$ where $a \in$ R.

1.

1.1 Consider the functions $f(x) = \sin(x - 60°)$ and $g(x) = \cos 2x$.
On the same set of axes, draw the graphs of f and g for $x \in [-90°;180°]$.
Clearly show all intercepts with the axes, turning points and end points. (4) **L2**

1.2 Consider: $f(x) = -3\tan 2x$.
Draw the graph of f for the interval $x \in [-180°:180°]$.
Clearly show all asymptotes, intercepts with the axes and endpoints of the graph (3) **L2**

1.3. Given: $f(x) = \cos(x + 45°)$ and $g(x) = -2\sin x$ where $-180° \le x \le 180°$

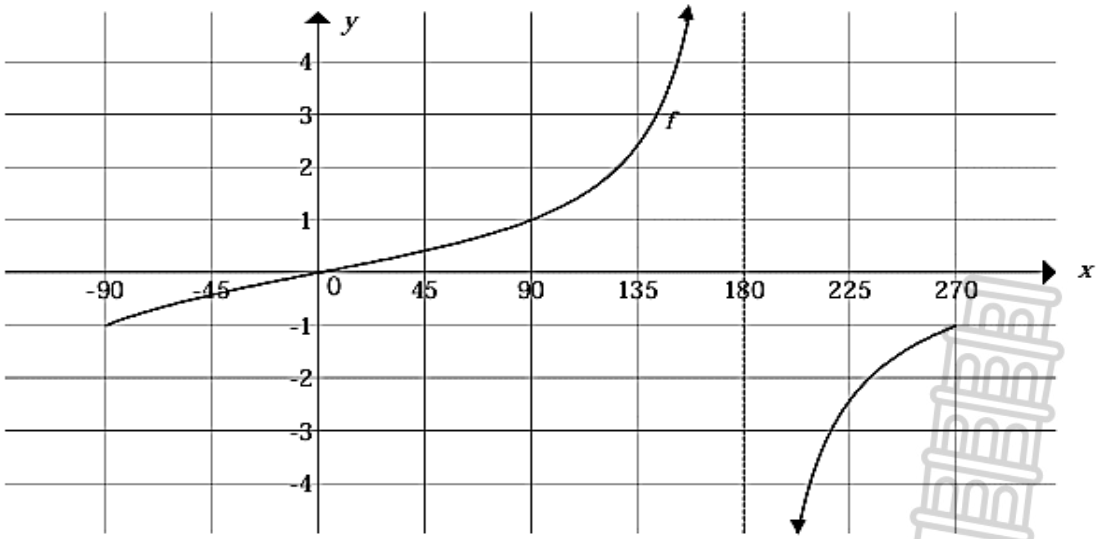
1.3.1 Draw the graph of f and g on the same set of axes, showing all the intercepts with the axes, turning points and endpoints. (2) **L1**

1.3.2 What is the period of g ? (3) **L2**

1.3.3 Determine by means of calculation, the values of x if $f(x) = g(x)$ in the interval above. (5) **L3**

2. NC Sept 2024

The graph of $f(x) = \tan\frac{1}{2}x$ for the interval $x \in [-90°;270°]$, is drawn below.



2.1 Write down the period of f. (1) **L1**

2.2 Write down the equation of the asymptote of f in the interval $x \in [-90°;270°]$ (2) **L2**

2.3 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = 2\cos x$ for the interval $x \in [-90°;270°]$. Clearly show all intercepts with the axes, turning points and end points. (3) **L2**

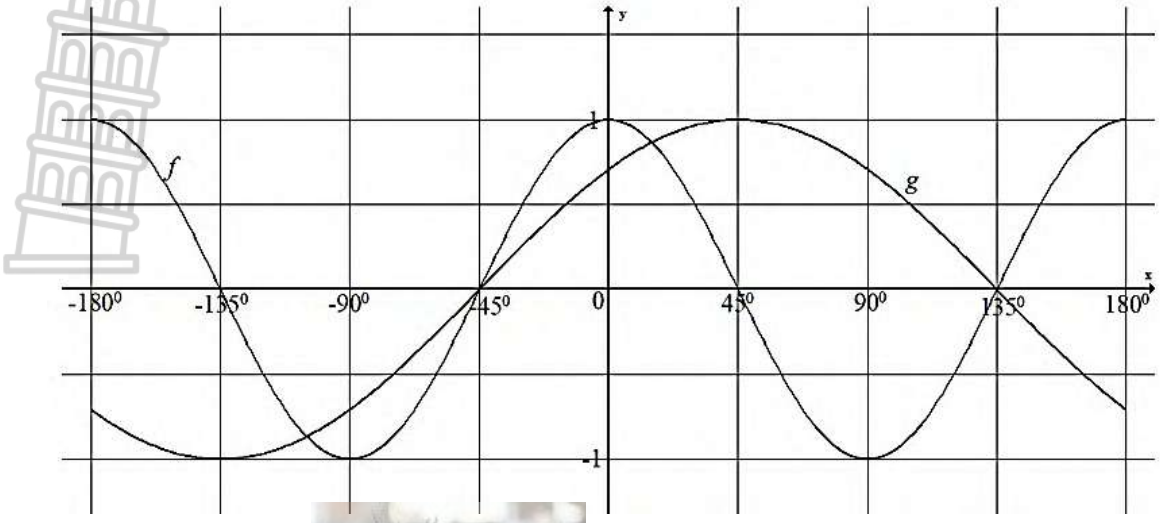
2.4 Give 2 values of x, in the interval $x \in [-90°;270°]$, for which $g(x) - f(x) = 1$ (2) **L2**

2.5 Write down the range of g(x) – 3 in the interval $x \in [-90°;270°]$. (2) **L2**

2.6 Determine the maximum value of, $\left[5-2\sin(90°-x)\right]$ for $x\in\mathbb{R}$. (3) **L2**

3.

In the diagram below the graphs of $f(x)=a\cos bx$ and $g(x)=\sin(x+p)$ are drawn for $x\in[-180°\ ;\ 180°]$.



3.1 Write down the values of *a*, *b* and *p*. (3) **L2**

3.2 For which values of *x* in the given interval does the graph of f increase as the graph of g increases? (2) **L1**

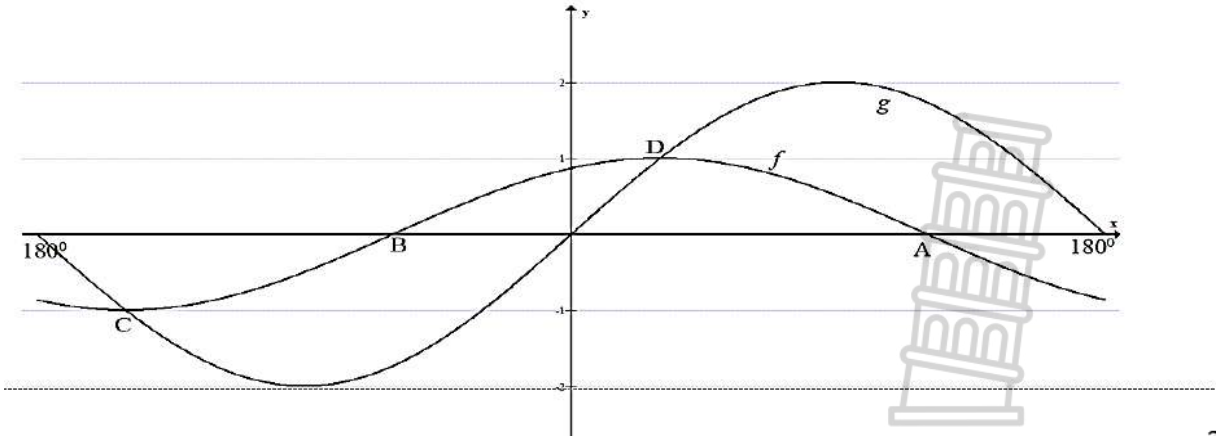
3.3 Write down the period of $f(3x)$. (2) **L1**

3.4 Determine the minimum value of *h* if $h(x)=2f(x)-5$. (2) **L2**

3.5 Describe how the graph *g* must be transformed to form the graph *h*, where $h(x)=-\cos x$ (2) **L3**

3.6 Determine the value(s) of x for which $f'(x).g(x)<0$ (6) **L3**

4. In the diagram, the graphs of $f(x)=\cos(x-30°)$ and $g(x)=2\sin x$ are drawn for the interval $x\in[-180°\ ;\ 180°]$, A and B are the *x*-intercepts of f. The two graphs intersect at C and D, the minimum and maximum turning points respectively of f.



4.1 Write down the coordinates

a) A and B (2) **L1**

b) C and D (2) **L1**

4.2 Determine the values of *x* in the interval $x\in[-180°\ ;\ 180°]$, for which:

a) $g(x)>f(x)$ (2) **L2**

b) $f(x-20°)>g(x-20°)$ (2) **L2**

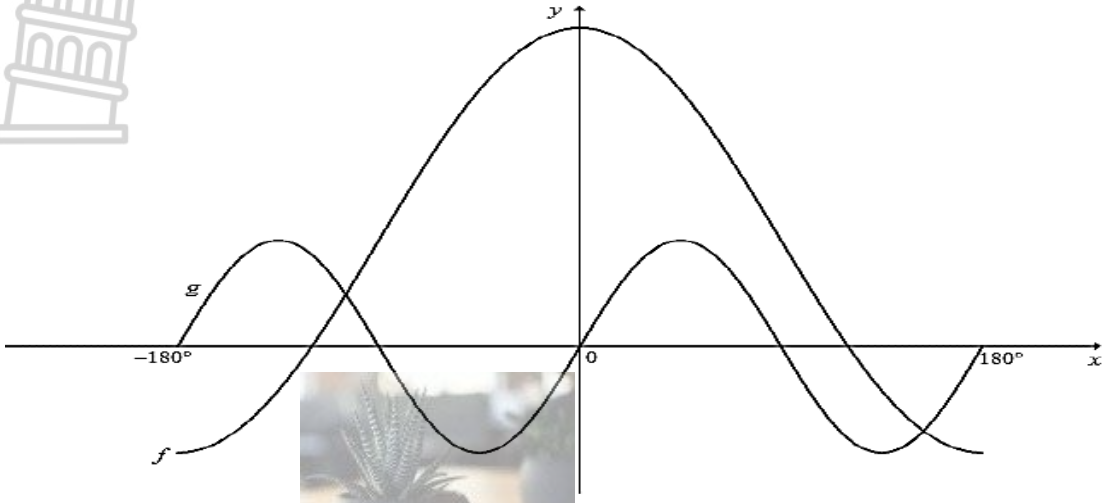
c) $x.f(x)<0$ (3) **L2**

d) $x.\sqrt{f(x)-g(x)}>0$ (2) **L3**

4.3 Determine the range of $y=2^{\cos(x-30°)+1}$ (4) **L2**

5. NSC May/Jun 2025

In the diagram, the graphs of $f(x)=2\cos x+1$ and $g(x)=\sin 2x$ are drawn for the interval $x\in[-180°;180°]$.



5.1 Write down the range of f. (1) **L1**

5.2 Write down the period of g. (1) **L1**

5.3 For which values of x, in the interval $x\in[-180°;180°]$, is f increasing? (1) **L2**

5.4 Use the graphs to determine the values of x, in the interval $x\in[-180°;180°]$, for which:

5.4.1 $g(x).f'(x)<0$ (2) **L2**

5.4.2 $\cos x\le -\frac{1}{2}$ (3) **L2**

5.5 Graph g is shifted 45° to the right to obtain a new graph h. Determine the equation of h in its simplest form. (2) **L2**

TOPIC	**TRIGONOMETRY: PROBLEMS IN TWO AND THREE DIMENSIONS**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

THE SINE RULE

In any ΔABC it is true that:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

Important: Use the Sine Rule when given **two angles and a side** in a triangle, also when **two sides and a non-included angle** are given.

It is advisable that when calculating **sides** have the **sides as numerators**: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ and when calculating **angles, have the angles as numerators:** $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.

THE COSINE RULE

In any ΔABC it is true that: $a^2=b^2+c^2-2bc.\cos A$, $b^2=a^2+c^2-2ac.\cos B$ and $c^2=a^2+b^2-2ab.\cos C$

Important: Use the Cosine Rule when given **two sides and an included angle,** also when you are given **all the three sides.**

THE AREA RULE

In any ΔABC it is true that:

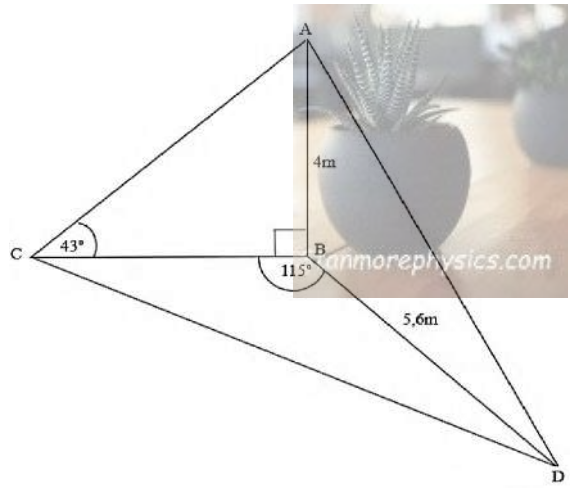
Area of $\Delta ABC = \frac{1}{2}bc.\sin A = \frac{1}{2}ac.\sin B = \frac{1}{2}ab.\sin C$

Important: To use the Area Rule, you need **two sides and an included angle** of the triangle**.**

STRATEGIES

Note: When solving 3D problems separate all the triangle so that they will be 2D and easy to solve. It is also advisable that write all your findings back to the diagrams to help you with the next sub-question.

1. **FS SEPT 2024**



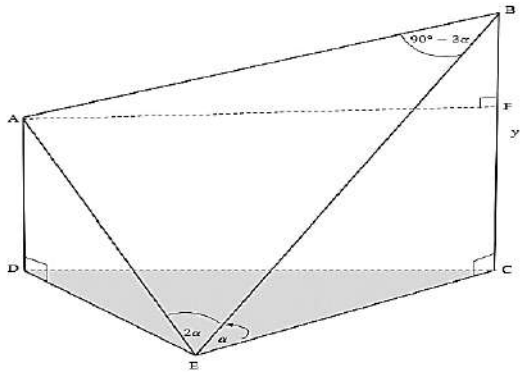
B, C, and D are points in the same horizontal plane. AB is a vertical pole of length 4 m, BD = 5.6 m, $C\hat{B}D = 115°$ and the angle of elevation of A from C is $43°$.

1.1 Calculate the length of CB (3) **L2**

1.2 Calculate the length of CD (3) **L2**

1.3 Determine the area of ΔBCD (3) **L2**

2. **GP SEPT 2024**



The diagram alongside shows two vertical poles, AD and BC. Point E lies on the same horizontal plane as bases D and C of poles AD and BC.

$A\hat{E}B = 2\alpha$; $B\hat{E}C = \alpha$; $A\hat{B}E = 90° - 3\alpha$ and BC = y metres

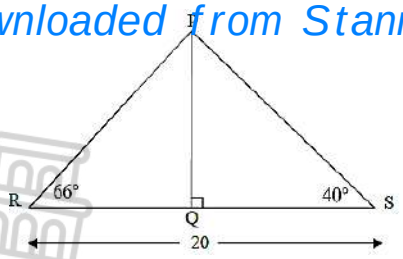
2.1 Determine BE in terms of α and y (2) **L1**

2.2 Show that $AB = 2y$ (5) **L2**

2.3 It is further given that $AF = \frac{7}{4}BC$. Determine $B\hat{A}F$, the angle of elevation of B from A. Give your answer to the nearest degree. (3) **L2**

3. **NC SEPT 2024**

In the diagram alongside, ΔPRS is drawn with RS = 20 units, $\hat{R} = 66°$ and $\hat{S} = 40°$. Q is a



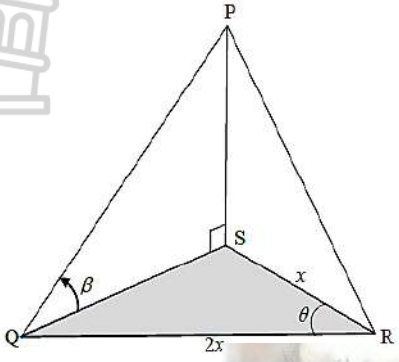
point on RS such that $PQ \perp RS$.

Calculate the length of:

3.1 PS (3) **L1**

3.2 PQ (2) **L2**

4. **NC SEPT 2024**



In the diagram, PS is a vertical flagpole. Q, R and S lie in the same horizontal plane. PQ and PR are two cables, anchored at Q and R.

$P\hat{Q}S = \beta$ and $Q\hat{R}S = \theta$. SR = x and QR = $2x$.

4.1 Show that

$$PQ = \frac{x\sqrt{5-4\cos\theta}}{\cos\beta}$$ (4) **L2**

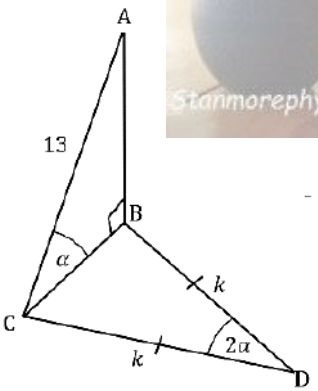
4.2 The area of ΔQRS is $57{,}36\ m^2$ and $\theta = 35°$.

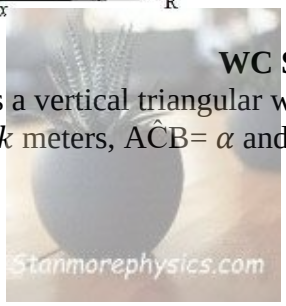
Calculate the value of x. (2) **L2**

5. **WC SEPT 2024**

In the diagram below, ABC is a vertical triangular wall on the floor CBD.

CA = 13 meters, CD = BD = k meters, $A\hat{C}B = \alpha$ and $B\hat{D}C = 2\alpha$



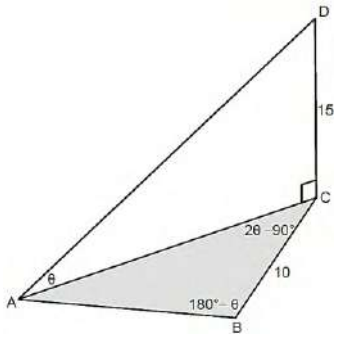


5.1 Show that $CB = 13\cos\alpha$ (1) **L1**

5.2 Show that $k = \frac{13}{2\tan\alpha}$ (4) **L3**

5.3 Calculate the area of the floor ΔBCD if $\alpha = 35°$ (2) **L2**

6. **IEB NOV 2024**



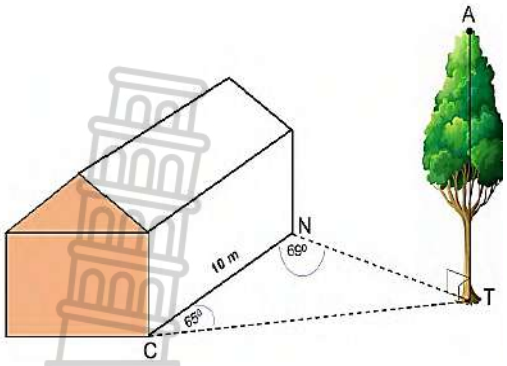
In the diagram:

- ΔABC is in the horizontal plane, with $BC = 10$ units
- $A\hat{C}B = 2\theta\text{-}90°$ and $A\hat{B}C = 180° - \theta$
- The angle of inclination from A to D is θ.
- ΔACD is in the vertical plane, with $CD = 15$ units

6.1 Show that AC = 10tanθ. (4) **L3**

6.2 Hence, determine the size of θ if $\theta \in [0°; 90°]$ (4) **L3**

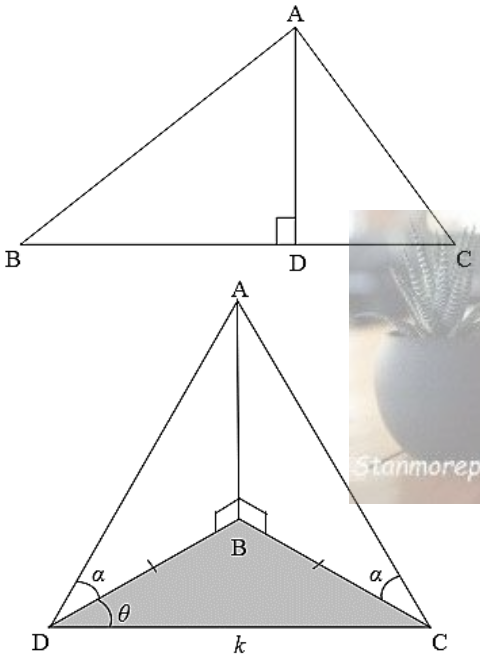
7. **IEB NOV 2023**



In the diagram alongside, a house is in the shape of a prism with a triangular shaped roof with one side CN = 10m.
A vertical tree is growing at a point T in the garden.
$N\hat{C}T=65°$ and $C\hat{N}T=69°$.
The angle of elevation of the top of the tree from C is $43{,}5°$

7.1 Is it possible for the tree to hit the house if it falls? Show all working. (8) **L4**

8. **NSC MAY/JUNE 2024**



8.1 In the diagram, ΔABC is drawn. AD is drawn such that $AD \perp BC$

8.1.1 Use the diagram above to determine AD in terms of $\sin\hat{B}$ (2) **L1**

8.1.2 Hence, prove that the area of $\Delta ABC=\frac{1}{2}(BC)(AB)\sin\hat{B}$ (1) **L2**

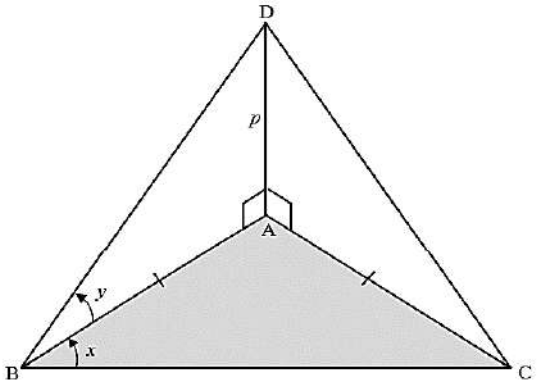
8.2 In the diagram, points B, C and D lie in the same horizontal plane. $A\hat{D}B=A\hat{C}B=\alpha$, $C\hat{D}B=\theta$ and $DC=k$ units. BD=BC

8.2.1 Prove that AD = AC (2) **L2**

8.2.2 Prove that $BD=\dfrac{k}{2\cos\theta}$ (3) **L3**

8.2.3 Determine the area of ΔBCD in terms of k and a single trigonometric ratio of θ. (3) **L3**

9. **NSC MAY/JUNE 2025**



In the diagram, A, B and C lie in the same horizontal plane with AB = AC. D is directly above A such that 2AD = BC.
Also, $AD=p$, $A\hat{B}C=x$ and $D\hat{B}A=y$

9.1 Determine AB in terms of p and y. (2) **L2**

9.2 Show that $\cos x=\tan y$ (4) **L3**

9.3 If $x=60°$, calculate the size of y. (2) **L2**

EUCLIDEAN GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

DIFFERENT WAYS EUCLIDEAN GEOMETRY CAN BE TESTED

1. COMPLETING A STATEMENT OF A THEOREM IN WORDS.

- Know by heart all the theorems and be able to complete the statement.

2. DETERMINING THE VALUE OF AN ANGLE

- Know all the theorems about **lines**, **triangles** and **circles (Centre** group, **non-centre** group, **tangent** group and **cyclic quad** group**)**.
- Every statement must come with a reason and reasons must be stated according to the list of acceptable reasons from the exam guidelines

3. PROOFS IN RIDERS

Know how theorems and their converses are being formed in diagrams.

- When given 3 points on the circumference look out for a possibility of a triangle. If one side is produced, then you may expect exterior angle of a triangle. If there is a tangent on the circle, then there is a possibility of having a Tan Chord Theorem
- When given 4 or 5 points on the circumference then there is a possibility that 4 points may be joined and then there is a cyclic quad. In a case that one side is produced then you may expect exterior angles of a cyclic quad.
- Start with a given angle linking with what is required to prove
- Visualization: Mind picture of diagrams of theorems

DIRECT AND INDIRECT PROOFS IN RIDERS.

- In Geometry we mostly use angles to prove in questions.

1. Direct proof question: Prove $A = B$
2. Indirect proof question: Prove that a line // to another line.
Remember in Euclidean geometry- we mostly use angles to prove. This question is not asking about the angles directly. Here we need to prove sides but using angles i**ndirectly. Why indirectly?** Because we mostly use angles to prove.
∴ First, we need to change this question to be direct, and then prove. If we say it must be direct we mean that it must ask to prove angles 1st, then conclude by stating the sides that are parallel

4. SIMILARITY AND PROPORTIONALITY THEOREMS

PROPORTIONALITY THEOREM

- Identify parallel lines, and use ratios for proportion.
- **Useful strategies in solving problems involving ratio in areas of triangles:**

CASE 1: If triangles share a **common angle** use area rule. Area = $\frac{1}{2}$a.bsinC

CASE 2: If triangles share a common vertex or height use Area = $\frac{1}{2}$b*h*

CASE 3: If none of the cases above apply then identify a common triangle and relate the two triangles in question to it, then use any of the two methods mentioned above. **OR**

Required Area = Area of big Δ − other known Area

SIMILARITY THEOREM

CASE 1: Prove that triangles are similar e.g. $\Delta ABC ||| \Delta DEF$

- Angles and / or sides in proportion can be used to prove that two triangles are similar.
- Always name the triangles you are referring to when proving similar triangles

CASE 2: Prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. First prove: $\Delta ABC|||\Delta PQR$ and then deduce the proportion of the sides.

CASE 3: Prove that: KN. PX = NR. YP. Find two triangles in which KN, PX, NR and Y, (or sides equal to these), and thus prove that: $\Delta KNR|||\Delta YPX$, then deduce what you were asked to prove. Identify triangles. This method is used when proved similarity don't give asked ratios.

CASE 4: Prove: Proportion with square, with division, with + in between, there is a possibility that two similarities were used or Pythagoras theorem was used.

e.g. $\frac{CF^2}{EF^2} = \frac{BD}{DE}$

5. EXAMINABLE PROOFS

Five grade 11 proofs to be known for exam purposes:

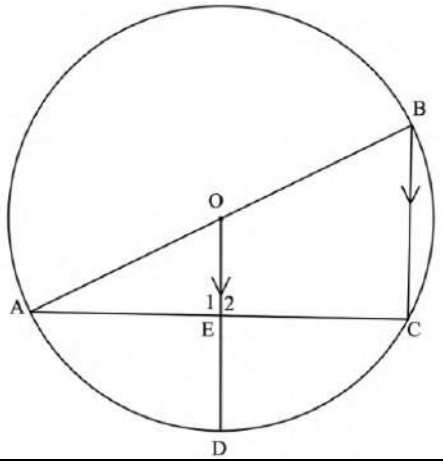
5.1 Line from the centre ⊥ chord
5.2. line from centre to midpoint of chord
5.3. Angle at the centre is 2× angle at the circumference.
5.4. Opposite angles of a cyclic quad are supplementary.
5.5. Tan chord theorem.

Two grade 12 proofs:

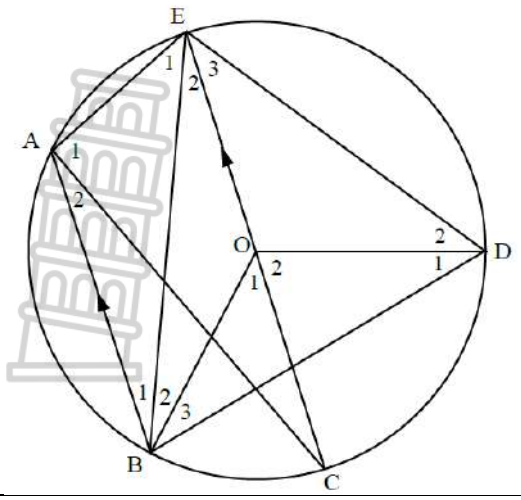
5.6. Line drawn parallel to one side of a triangle, divides the other two sides proportionally:
Proportionality theorem
5.7 If two triangles are equiangular, then their corresponding sides are in proportion:
Similarity theorem

NB!!!!!

- Do not make any assumption e.g. do not assume that a line is a tangent or a diameter, unless you are told that it is.
- Look for key words in the statement such as centre, // lines, tangents, cyclic quads, bisects, etc.
- Continuously update the diagram as you read the statement and as you find the angles.
- When proving theorems, no construction no marks.
- You will not always be told that you have a cyclic quadrilateral. Therefore, check lines joining four points on the circumference.
- For every statement there **must** be a reason.

1.	**GR 11 REV DOC 2025**		
		AB is the diameter of the circle with centre O. OD // BC. OD intersect AC at E. $ED = 4cm$ and $AC = 16cm$	
		Calculate the length of AB	(5) **L3**

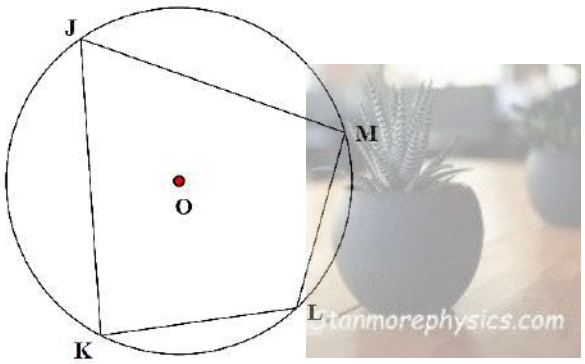
2. KZN JUN 2025



In the diagram, O is the centre of the circle ABCDE. CE is a diameter. AB // EC. BE, AC, BO and OD have also been drawn.
$\hat{C} = 26°$

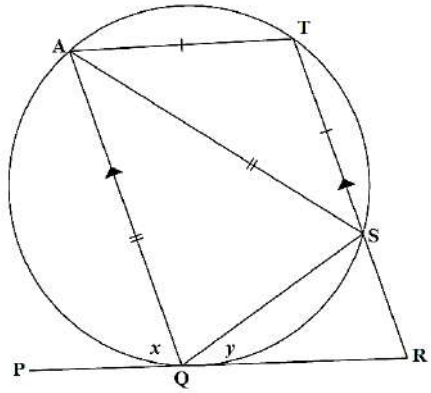
2.1	Write down, with reasons, three other angles each equal to $26°$	(5)	**L2**
2.2	Calculate the size of $\hat{O}_1$.	(2)	**L1**
2.3	Calculate the size of $B\hat{D}E$.	(3)	**L3**

3. KZN PRE JUN 2025



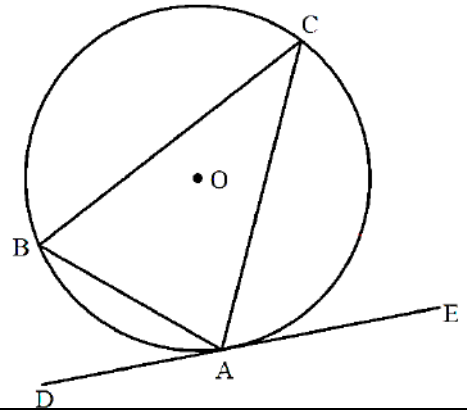
In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.

3.1	Prove the theorem which states that $\hat{J} + \hat{L} = 180°$	(5)	**L3**



3.2	In the diagram, PQR is a tangent at Q. $AS = AQ$ and $AT = TS$. AQ // TS. If $P\hat{Q}A = x$ and $R\hat{Q}S = y$:		
3.2.1	Prove that $Q\hat{S}R = x$.	(3)	**L3**
3.2.2	Show that $S\hat{T}A = 180° - x$	(2)	**L1**
3.2.3	Name, with reasons, three other angles equal to y.	(3)	**L2**
3.2.4	Calculate values of x and y.	(4)	**L4**

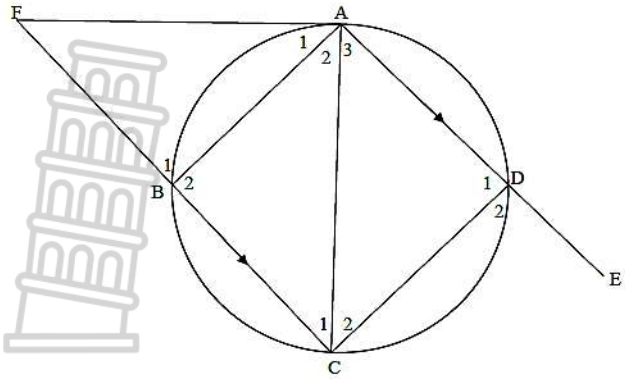
4. MP JUN 2025



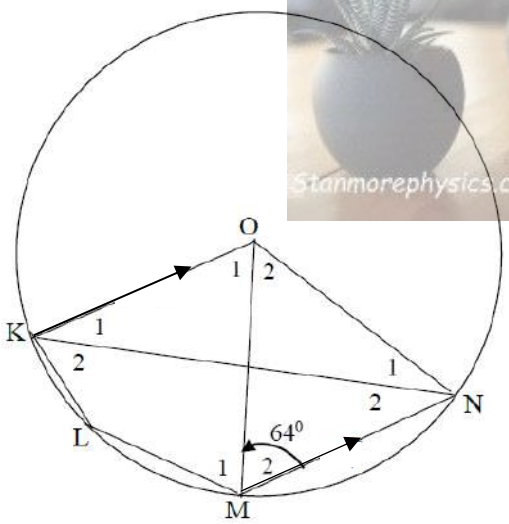
In the diagram alongside, O is the centre of the circle. A, B, C are points on the circle. DE is a tangent to the circle at A

	Prove the theorem which states that $C\hat{A}E = A\hat{B}C$	(4)	**L2**

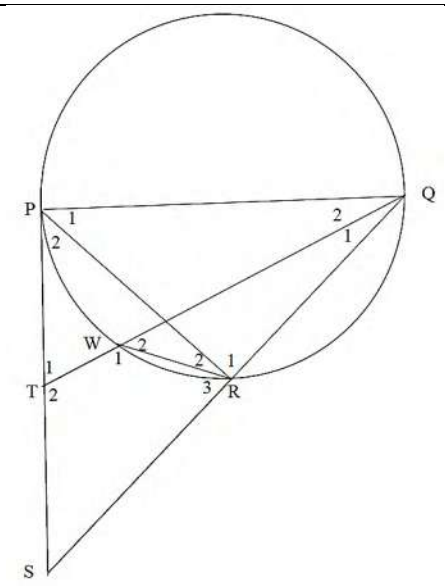
5. MPU JUN 2025



		5.2	In the diagram, FA is a tangent to the circle passing through A, B, C and D. AD // BC. CB produced meet the tangent at F. AD is produced to E. $\hat{A}_1 = x$ and $\hat{D}_2 = y$.	
		5.2.1	Determine, with reasons, two angles equal to *x*. (4)	**L2**
		5.2.2	Determine, with reasons, two angles equal to *y*. (4)	**L2**
		5.2.3	Show that $\hat{F} = \hat{C}_2$. (3)	**L2**
		5.2.4	Express $\hat{A}_2$ in terms of *x* and *y*. (2)	**L1**
		5.2.5	Hence calculate the value of *y* for which AC is a tangent to the circle passing through the points A, B and F. (3)	**L3**
6.			**NW JUN 2025**	

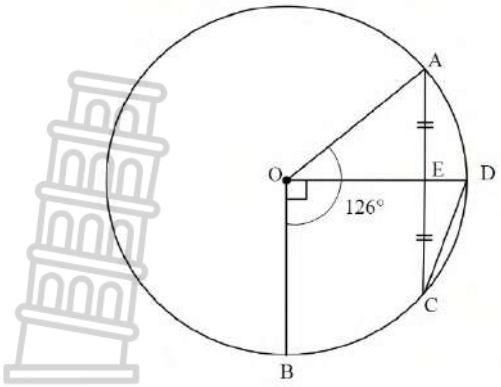


		6.1	In the diagram, O is the centre of the circle. K, L, M and N are points on the circumference of the circle. KO // MN and $\hat{M}_2 = 64°$. Calculate, giving reasons, the size of:	
		6.1.1	$\hat{N}_2$ (3)	**L2**
		6.1.2	$\hat{L}$ (2)	**L1**
		6.1.3	$\hat{N}_1$ (3)	**L2**



		6.2	In the diagram, PQ is the diameter of the circle PWRQ. PS is a tangent to the circle at P. $\hat{QPR} = x$.	
		6.2.1	Give a reason why $\hat{PRQ} = 90°$ (1)	**L1**
		6.2.2	Prove that $\hat{S} = x$. (4)	**L3**
		6.2.3	Prove that RSTW is a cyclic quadrilateral. (3)	**L2**

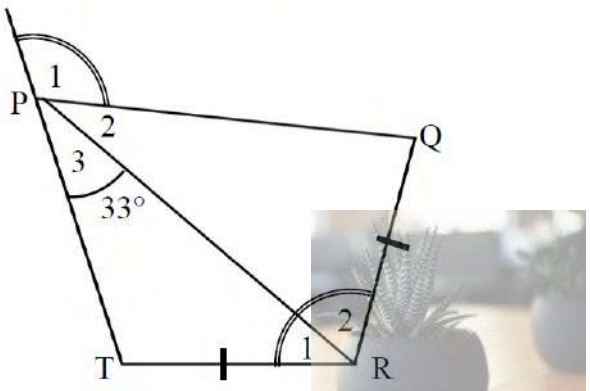
8. NC SEP 2023



8.1 In the diagram, O is the centre of the circle. $BO \perp OD$ and $A\hat{O}B = 126°$. OD bisect the chord AC at E and CD is drawn.

Calculate, giving reasons, the size of:

8.1.1	$O\hat{A}C$	(4)	**L3**
8.1.2	$A\hat{C}D$	(2)	**L2**

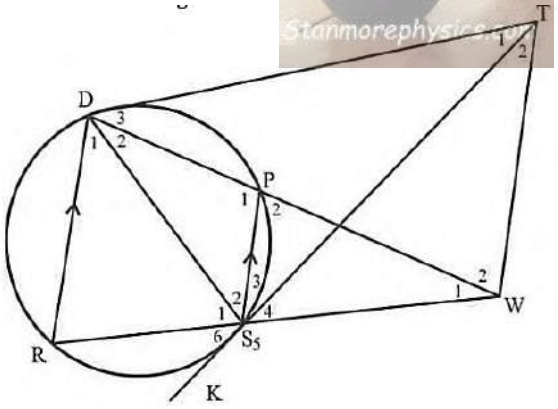


8.2 In the diagram, quadrilateral PQRT is drawn with $TR = RQ$. TP is produced such that $\hat{P}_1 = T\hat{R}Q$. PR is drawn.

$\hat{P}_3 = 33°$ and $\hat{R}_2 = 66°$.

Calculate giving reasons, the size of $\hat{T}$ (6) **L3**

9. KEV SMI GR 11

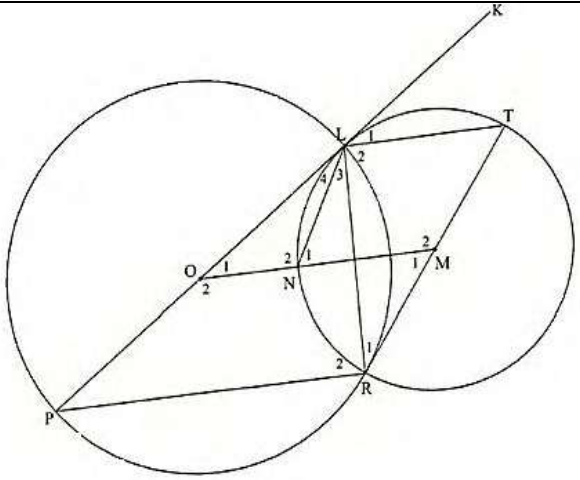


In the diagram alongside, TD is a tangent to the circle at D. RS and DP are produced to meet at W and KST is a straight line. If $\hat{S}_4 = \hat{S}_2$ and DR // PS.

Prove that:

9.1	SWTD is a cyclic quadrilateral	(4)	**L2**
9.2	TK is a tangent to the circle at S	(4)	**L3**
9.3	TW // PS	(3)	**L2**

10. MAY JUN 2025

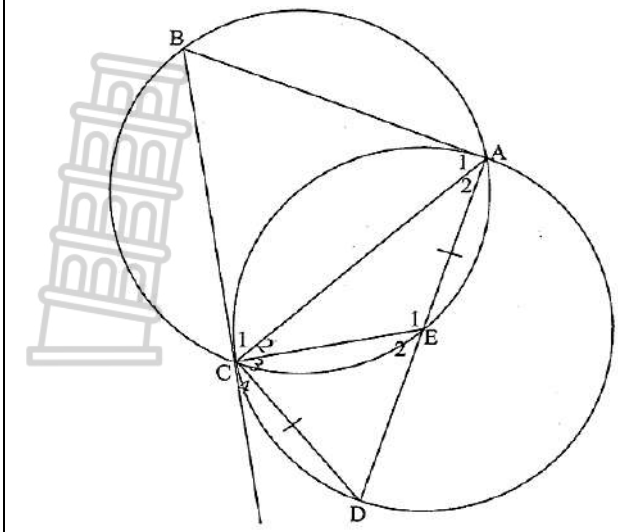


In the diagram alongside, POL is the diameter of the larger circle with centre O. TMR is the diameter of the smaller circle with centre M. The two circles intersect at L and R. PLK is the tangent to the smaller circle at L and TR is a tangent to the larger circle at R. OM intersect the smaller circle at N. Straight lines LT, LR, LN and PR are drawn.

Prove, giving reasons, that:

10.1	LT // PR.	(4)	**L2**
10.2	LORM is a cyclic quadrilateral, if it is also given that LT // OM.	(5)	**L3**
10.3	LN bisect $O\hat{L}R$.	(4)	**L3**

11. NW SEPT 2018

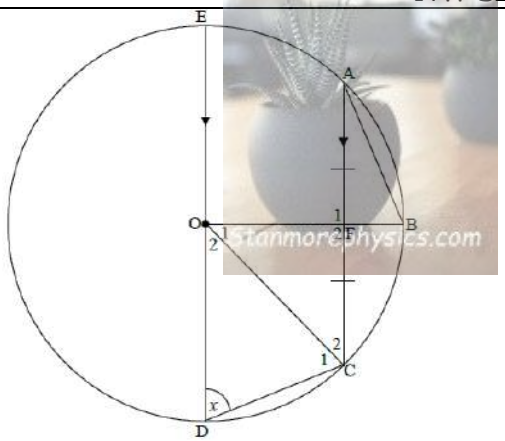


Two equal circles cut each other in A and C. BA and BC are tangents to one circle at A and C respectively and they are chords of the other circle. E is a point on the circumference of one circle and AE produced cuts the other circle in D. Chords AE and CD are equal.

Prove that:

11.1	$\hat{C}_2 = \hat{C}_4$	(4)	**L3**
11.2	$\hat{C}_3 = \hat{A}_1$	(3)	**L2**
11.3	E is the centre of the circle that passes through A, C and D.	(4)	**L3**
11.4	ΔECD is equilateral	(2)	**L2**

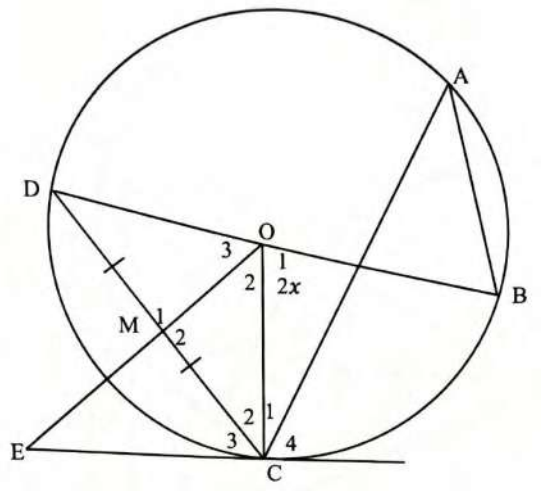
12. NW SEPT 2024



DE is the diameter of the circle having centre O. Points A, B and C lie on the circle. AC // DE, $AF = FC$ and $O\hat{D}C = x$

12.1	Calculate the size of $\hat{F}_1$.		(2)	**L1**
12.2	Determine in terms of *x*, giving reasons, the size of:			
	12.2.1	$\hat{O}_2$	(2)	**L1**
	12.2.2	$\hat{B}$	(6)	**L3**

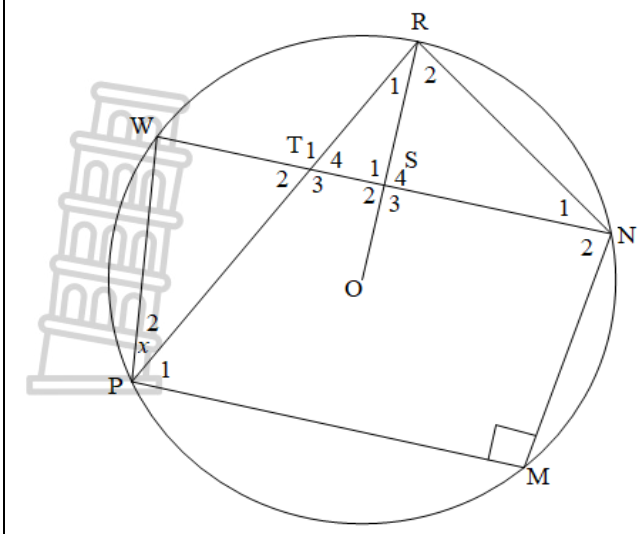
13. MP SEPT 2024



In the diagram, O is the centre of the circle and EC is a tangent to the circle at C. DM = MC, $\hat{O}_1 = 2x$ and OME is a straight line

13.1	Determine, with reasons, three angles equal to *x*.	(6)	**L2**
13.2	Prove that $\hat{O}_2 = 90° - x$	(3)	**L2**
13.3	Prove that EC is a tangent to the circle passing through points M, C and O.	(4)	**L2**
13.4	Prove that DOCE is a cyclic quadrilateral.	(3)	**L3**

14. LP SEPT 2022

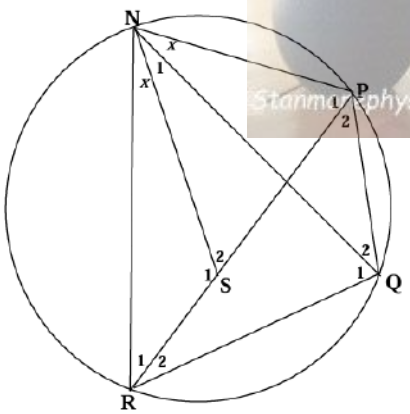


In the diagram, O is the centre of the circle. M, P, W, R and N are points on the circle. $PM \perp MN$. WN intersects PR at T and RO at S. Let $\hat{P}_2 = x$.

14.1	Name, stating reasons, one other angle equal to *x*.	(2)	**L1**
14.2	Prove that PT is the diameter of the circle PWT.	(2)	**L2**
14.3	If it is further given that S is the midpoint of chord WN, prove that TR is a tangent to the circle RSN.	(5)	**L3**

SIMILARITY AND PROPORTIONALITY

15. GP JUN 2023

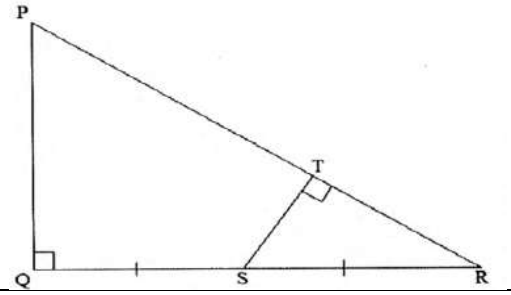


In the diagram alongside, NPQR is a cyclic quadrilateral with S a point on chord PR. N and S are joined $R\hat{N}S = P\hat{N}Q = x$.

Prove that:

15.1	$\Delta NSR \,|||\, \Delta NPQ$	(3)	**L2**
15.2	$\Delta NQR \,|||\, \Delta NPS$	(3)	**L2**

16. MP JUN 2023



In the sketch, $QS = SR$ and $P\hat{Q}T = S\hat{T}R = 90°$.

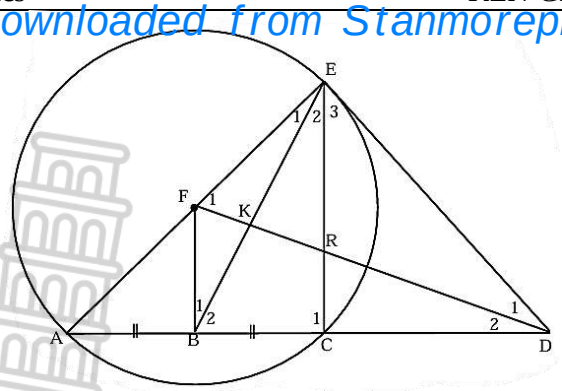
16.1	Prove that $\Delta PQR \,|||\, \Delta STR$	(4)	**L3**
16.2	Hence, prove that $PR \cdot RT = SQ \cdot RQ$	(2)	**L3**

17. EC JUN 2023

In the diagram alongside, ED is a tangent to the circle passing through A, C and E. F is the centre of the circle. AC is extended to meet ED at D and FB bisects AC. Straight-lines FD, BE and EC are drawn.

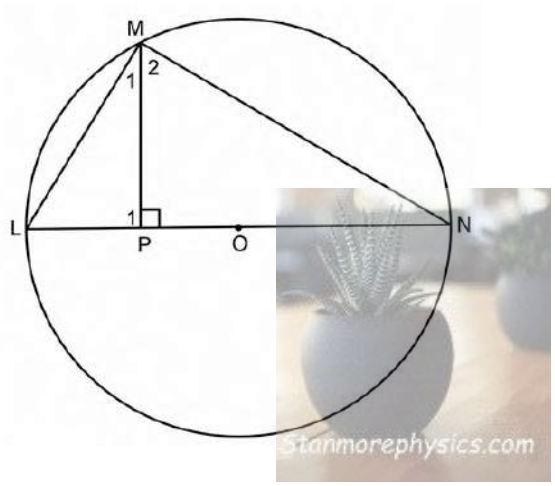
Prove, with reasons, that:

17.1	EFBD is a cyclic quadrilateral	(4)	**L3**
17.2	$\Delta BCE \,|||\, \Delta FED$	(6)	**L3**



17.3 $BC = \dfrac{FA \cdot CE}{ED}$ (3) **L2**

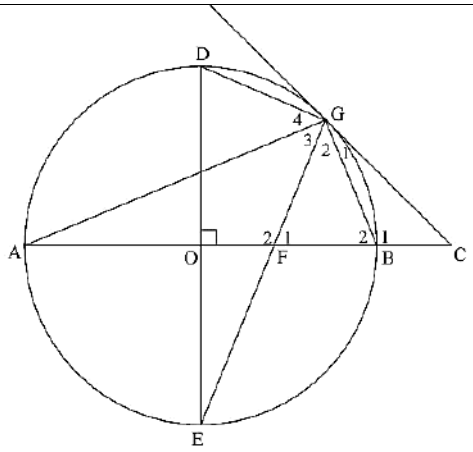
18. **IEB NOV 2024**



Given circle centre O with diameter of 8 units, $LM = 4$ units and $LN \perp MP$.

18.1	Determine: $\hat{M}$	(2)	**L1**
18.2	Prove: $\Delta LMN \,|||\, \Delta LPM$	(3)	**L1**
18.3	Determine: LP	(3)	**L2**

19. **L.P PRE-TRIAL AUGUST 2022**



In the diagram alongside, ED is a tangent to the circle passing through A, C and E. F is the centre of the circle. AC is extended to meet ED at D and FB bisects AC. Straight-lines FD, BE and EC are drawn.

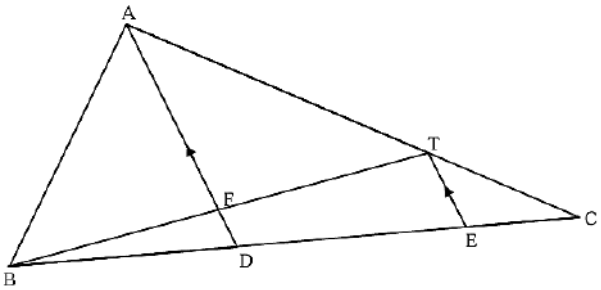
Prove that:

19.1	DGFO is a cyclic quadrilateral	(3)	**L2**
19.2	$GC = CF$	(5)	**L3**

If it is further given that $CO = 11$ units and $DE = 14$ units, calculate:

19.3	The length of BC	(3)	**L2**
19.4	The length of CG	(5)	**L4**
19.5	The size of $\hat{E}$	(4)	**L3**

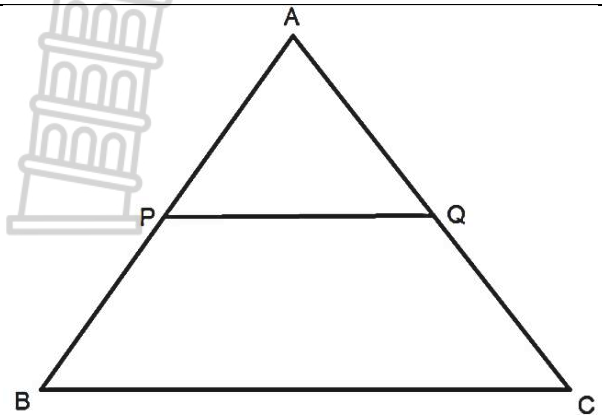
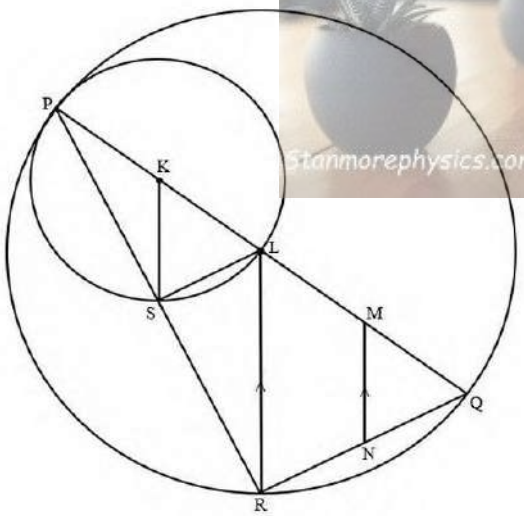
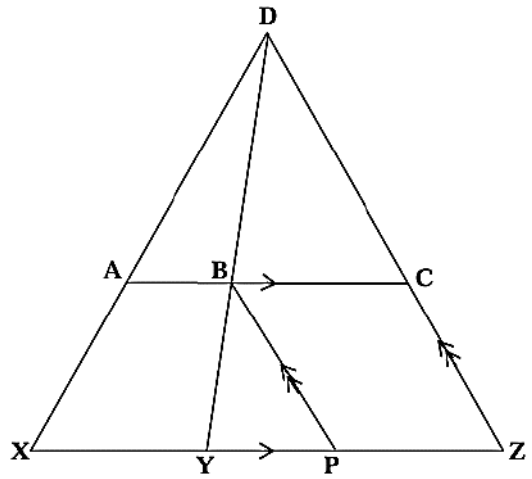
20. **FS SEPT 2020**



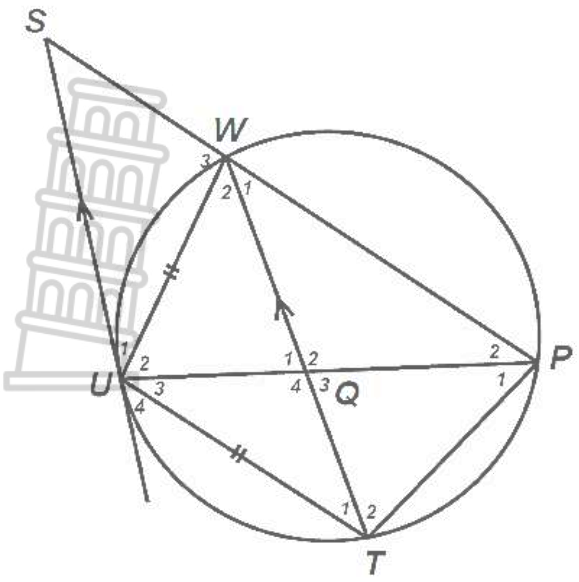
In the figure, D is a point on side BC of $BD = 6$ cm and $DC = 9$ cm. T and E are points on AC and DC respectively such that $TE \,||\, AD$ and $AT : TC = 2 : 1$

20.1.	Show that D is the midpoint of BE	(3)	**L2**
20.2	If $FD = 2$ cm, calculate the length of TE.	(3)	**L1**

Calculate the numerical value of:

		20.3	$\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$	(3)	**L2**
		20.4	$\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$	(3)	**L3**
21.	**NW JUN 2025**				
		In the diagram ΔABC is drawn with $PQ \parallel BC$.			
		21.1	Prover the theorem which states that: $\frac{AP}{PB} = \frac{AQ}{QC}$	(5)	**L2**
		21.2	If $PQ : BC = 4 : 7$ **Prove that:** $\Delta APQ = \frac{16}{49}\Delta ABC$	(5)	**L3**
22.	**NW JUN 2022**				
		In the diagram, two circles PSL and PQR touch internally at P. K and L are the centres and PQ is the diameter of the larger circle. PSR is a straight line.			
		22.1	Determine the size of $P\hat{S}L$	(2)	**L1**
		22.2	$SL \parallel RQ$	(3)	**L2**
		22.3	$2KS = LR$	(3)	**L3**
		22.4	Determine the value of: $\frac{KL}{PQ}$	(2)	**L1**
		22.5	If is given that $PQ = 30$ units and $\frac{QR}{NR} = \frac{7}{9}$, determine the length of LM	(3)	**L2**
23.	**GP JUN 2023**				
		In ΔDXZ alongside, $AC \parallel XZ$. DY is drawn to intersect AC at B.			
			Prove that: $\frac{BC}{YZ} = \frac{DA}{DX}$	(5)	**L3**

24. L.P PRE-JUN 2024

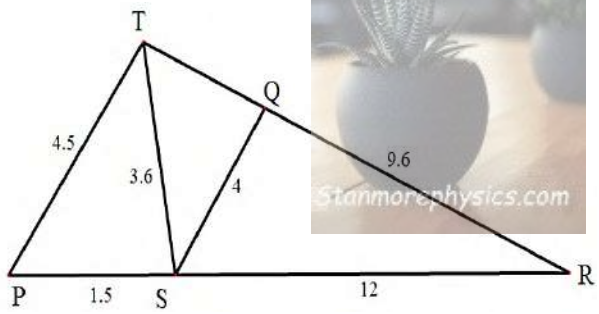


In the diagram, WPTU is a cyclic quadrilateral with $UW = UT$. Chords WT and PU intersect at Q. PW extends to S such that $SU \parallel WT$.

Prove that:

24.1	US is a tangent to circle PWUT at U.	(6)	**L2**
24.2	$\Delta SPU \,|||\, \Delta SUW$	(4)	**L2**
24.3	$SU^2 = SP \cdot SW$	(2)	**L2**
24.4	$SU^2 \cdot QU = PU \cdot SW^2$	(4)	**L3**

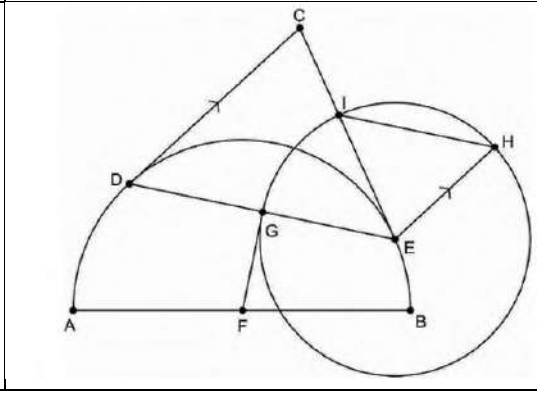
25. FS SEPT 2020



In the diagram, TPR is a triangle with $TP = 4.5$ units. Points Q and S are on TR and PR respectively such that $QR = 9.6$ units, $QS = 4$ units, $TS = 3.6$ units, $PS = 1.5$ units and $SR = 12$ units.

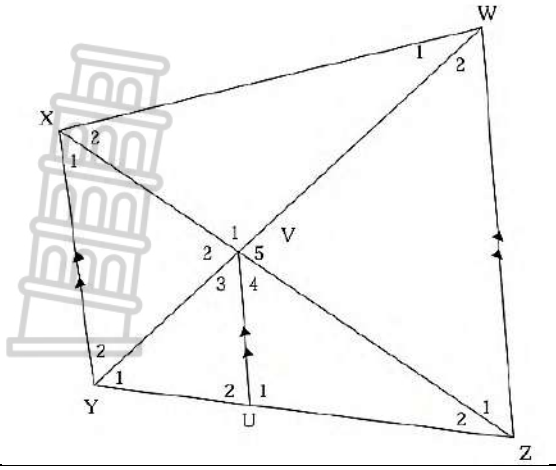
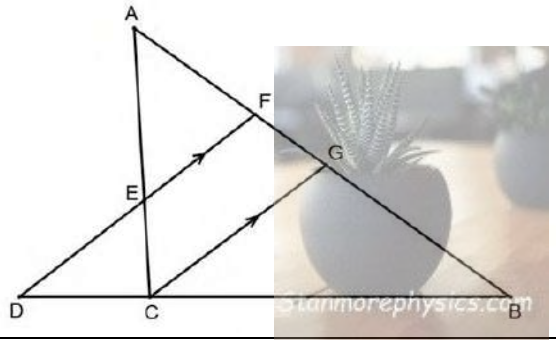
25.1	Prove that PT is a tangent to the circle which passes through the points T,S and R	(7)	**L4**
25.2	Calculate the length of TQ	(5)	**L3**

26. IEB MAY 2021



In the diagram alongside. I, H and G are points on the circle with centre E. AB is a diameter of semi-circle. CD and CE are tangents to the semi-circle at D and E respectively, $CD \parallel HE$ and FG is a tangent to the circle with centre E.

	Prove that: $\Delta DCE \,|||\, \Delta HEI$	(5)	**L4**

27.	**EC SEPT 2018**			
		In the diagram alongside, WXYZ is a trapezium, with $XY \parallel UV \parallel WZ$. $UZ:YZ = 4:7$. $\hat{V}_3 = \hat{V}_4$		
		27.1 **Prove that:** $\frac{YV}{VW} = \frac{XV}{VZ}$	(3)	**L2**
		27.2 Determine the numerical value of: $\frac{\text{Area of } \Delta XVY}{\text{Area of } \Delta WVZ}$	(4)	**L3**
28.	**IEB MAY 2023**			
		In the diagram, E and F are points on AC and AB respectively such that $\frac{AE}{EC} = \frac{3}{2}$ and $\frac{AF}{FB} = \frac{2}{5}$. $FD \parallel GC$		
		28.1 **Calculate, giving reasons:** $\frac{BC}{CD}$	(5)	**L3**

ANSWERS
ALGEBRA, EQUATIONS AND INEQUALITIES

1.1	$x=0$ or $x=2$
1.2	$x=0$ or $x=\frac{4}{5}$
1.3	$x=5$ or $x=2$
1.4	$x=3$ or $x=2$
1.5	$x=-1$
1.6	$x=0$ or $x=-4$
1.7	$x=1$
1.8	$x=\pm 11$
1.9	$x=4$ or $x=-3$
1.10	$x=0$ or $x=3$
1.11	$x=-3$ or $x=\frac{2}{5}$
1.12	888 888 888 890
1.13	$x=0$ or $x=-\frac{5}{3}$
1.14	$x=1$ or $x=-5$
1.15	(a) $k=6$ (b) $x=1$ or $x=6$
1.16	$x=3$ or $x=-5$
1.17	$x=0$ or $x=5$ and $x=\frac{1}{3}$
2.1	$x=-0,18$ or $x=-1,82$
2.2	$x=0,84$ or $x=-0,24$
2.3	$x=1,65$ or $x=-0,15$
2.4	$x=4,30$ or $x=0,70$
2.5	$x=1,16$ or $x=-1,56$
2.6	$x=1,71$ or $x=0,29$
2.7	$x=0,77$ or $x=-1,48$
2.8	$x=0,80$ or $x=-1,55$
2.9	$x=2,77$ or $x=-1,27$
2.10	$x=7,89$ or $x=-0,89$
2.11	$x=2,35$ or $x=-0,85$
2.12	$x=1,22$ or $x=0,35$
3.1	$1<x<4$
3.2	$-\frac{1}{2}<x<\frac{4}{3}$
3.3	$-2\le x\le 8$
3.4	$-1<x<3$
3.5	$x\le -2$ or $x\ge\frac{1}{3}$
3.6	$-6\le x\le 3$
3.7	$x<-2$ or $x>4$
3.8	$x\le -2$ or $x\ge 4$
3.9	$-\frac{1}{2}\le x\le\frac{4}{3}$
3.10	$x<-\frac{5}{3}$ or $x>\frac{1}{3}$
3.11	$-3\le x\le 5$
3.12	$x<5$
3.13	$x\le 0$ or $x\ge 5$
4.1	$x=-\frac{1}{4}$
4.2	$x=9$
4.3	$x=0$ or $x=2$
4.4	$x=4$ or $x=3$
4.5	$x=-\frac{1}{6}$
4.6	$x\neq 14$ or $x=-1$
4.7	$x=2$ or $x\neq 1$
4.8	$x=1$
4.9	$x=3$
4.10	$x=3$ or $x\neq -1$
4.11	$x=0$ and $x\neq 5$
4.12	$x=2$
4.13	$x=0$ or $x\neq -6$
4.14	$x=1$
4.15	$x=13$ or $x\neq 2$
4.16	$x=0$
4.17	$x=4$ or $x\neq -5$
4.18	$x=7$ or $x\neq 2$
4.19	$x=4$
4.20	$x=256$
4.21	$x\neq -2$ or $x=8$
4.22	$x=4$ or $x\neq -11$
5.1	$(2)(2x^{m}+2x^{-n})$
5.2	$p=4x9=36$
5.3	Proof
5.4	Proof
5.5	n must be an odd number, greater than 2.
5.6	$PxT=1-a^{1024}$
5.7	$\frac{2}{5}$
5.8	5.2^{n-4}
5.9	$-21-\sqrt{5}$
5.10	$\frac{5\sqrt{30}}{12}$

5.11	$\frac{1}{m}$
6.1	$k \le -3$ or $k \ge 3$
6.2	$k \le \frac{1}{5}$
6.3	$k = \pm\sqrt{2}$
6.4	$p < -1$ or $p > 1$
6.5	(a) $x = -k \pm \sqrt{2k+1}$ (b) $k = \frac{3}{2}$ or $k = 4$ or $k = \frac{15}{2}$ or $k = 12$
6.6	Proof
6.7	$k = \frac{3}{4}$ or $k = -\frac{3}{4}$
6.8	(a) No, Min value of $3(x-1)^2 + 5$ is 5 (b) $k < -2$
6.9	(a) $p = -\frac{5}{2}$ (b) $p < -\frac{5}{2}$
7.1	$x = 3$ or $y = -1$
7.2	$y = 7$ or $y = 11$ $x = 6$ or $x = 2$
7.3	$y = -3$ or $y = 6$
7.4	$y = \frac{9}{4}$ or $y = 4$ $x = \frac{1}{4}$ or $x = -3$
7.5	$x = \frac{2}{3}$ or $x = -2$ $y = \frac{8}{3}$ or $y = 0$
7.6	$y = 3$ or $y = 1$ $x = 3$ or $x = -1$
7.7	$x = \frac{7}{2}$ or $x = 1$ $y = -4$ or $y = 1$
7.8	$x = 5$ or $x = \frac{5}{2}$ $y = 2$ or $y = -\frac{1}{2}$
7.9	$y = 31$ or $y = 5$ $x = -22$ or $x = 4$
7.10	$x = 1$ or $x = \frac{1}{3}$ $y = 3$ or $y = \frac{5}{3}$
7.11	$y = -\frac{12}{7}$ or $y = -3$ $x = -\frac{3}{7}$ or $x = -3$
7.12	$y = 0$ or $y = \frac{5}{3}$ $x = 2$ or $x = -3$
7.13	$y = 0$ or $y = 1$ $x = 6$ or $x = 2$
7.14	$y = 20$ or $y = 5$ $x = 35$ or $x = 5$
7.15	$x = 5$ or $y = 15$
7.16	$y = \frac{5}{2}$ or $y = -3$ $x = -\frac{1}{2}$ or $x = 5$
7.17	$x = 0$ or $y = \frac{7}{3}$
8.1	$x = 2 - y$
8.2	$x = 4y - 3$
8.3	(a) $k = -7$ or $k = 2$ (b) $x = -1$
8.4	$y = \frac{3}{2}$
8.5	3
8.6	$\frac{1}{4}$ or 4
8.7	$k = -13$
8.8	Proof
8.9	$\frac{3}{8}$

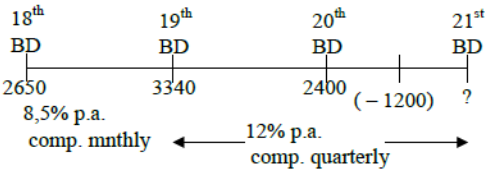
SEQUENCES AND SERIES	
1.1.1	$T_n = -4n^2 + 40n + 36$
1.1.2	$n = 12$
1.1.3	136
1.1.4	41
2.1.1	$T_n = n^2 + 8n$
2.1.2	$n = 12$ or $n \neq -20$
2.2.1	24 terms
2.2.2	$S_n = 1495$

2.3	$n=6$
3.1.1	16; 23
3.1.2	$S_n=\frac{1}{2}n(7n-17)$
3.2.1	$x=5$
3.2.2	Prove
3.3.1	$\frac{7}{3}<p<\frac{11}{3};p\neq 3$
3.3.2	$p=\frac{7}{2}$
4.1.1	$m=27$
4.1.2	121,49
4.2.1	$n=108$
4.2.2	$n=22$
4.3	-80200
5.1.1	9 437
5.1.2	$T_n=n^2-2n+29$
5.2.1	1 090
5.2.2	$\sum_{n=21}^{75}(5n+2)=13310$ *OR* $\sum_{1}^{55}(5n+102)=13310$
5.3.1	1,5 cm
5.3.2	150,80
5.3.3	$n=10$
6.1	$n=10$
6.2.1	$6p$
6.2.2	$6\sqrt{3}p$ units
6.2.3	Show
7.1.1	$1+\left(\frac{x+2}{2}\right)+\left(\frac{x+2}{2}\right)^2+....$
7.1.2	$-4<x<0$
7.1.3	$-\frac{2}{x}$
8.1.1	$w=\frac{5}{3}$
8.1.2	$p\neq 23$ or $p=21$
9.1.1	n=81
9.1.2	$\sum_{n=1}^{81}3n+19$
9.1.3	$n=94-81=13$
10.1.1	$\frac{w^2}{2}$
10.1.2	$3,41w<3,5w$
11.1.1	Yes; $-1<\frac{1}{2}<1$
11.1.2	$72cm^2$
11.1.3	$n=9$
11.1.4	56,13 units
11.2.1	$d=2 \quad a=-8$
11.2.2	$T_9=8$
12.1.1	$T_{36}=253$
12.1.2	$S_{36}=4698$
12.1.3	$m=32$
12.2.1	$54\ cm$
12.2.2	$T_9=\frac{256}{81}=3.16\ cm$
12.2.3	$243\ cm > 230\ cm$
13.1.1	21; 24
13.1.2	$T_{20}-T_{21}=449$
13.2.1	Proof
14.1.1	3 553
14.1.2	$\sum_{n=1}^{34}(-7n+227)$
14.2	< 290
14.3	1.2^{64}
15.1	5; 8; 11
15.2	$a=3$
16.1	12 kg
17.1	$y=7$ or $y=28$
17.2	$n=9$
18.1.1	$T_n=2n+3$
18.1.2	$n=45$
18.1.3	Total raised = R22 050
18.2.1	$T_1=8$
18.2.2	$r=2$
18.2.3	4^{11}
18.2.4	$S_\infty=\frac{1}{4}$
18.2.5	$S_{11}=11184808$
19.1	$x=3$
19.2.1	3
19.2.2	Show
19.2.3	Smallest value is 5

FUNCTIONS

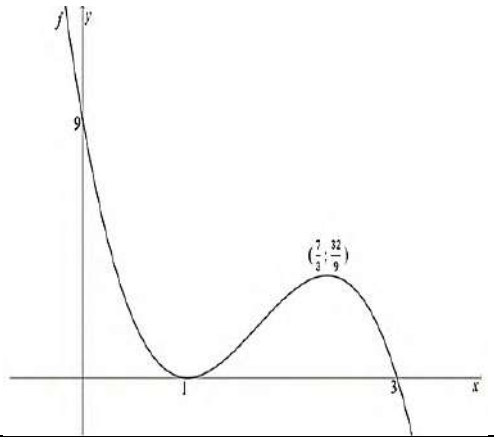
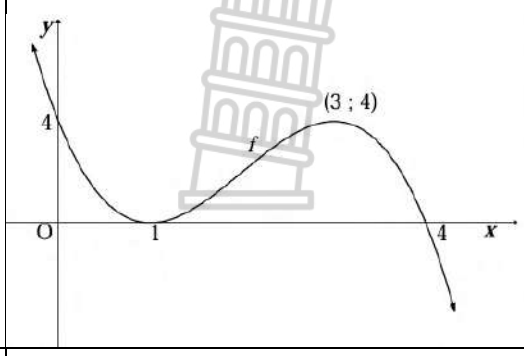
1.1	(2;1) *and* (–1;–2)
1.2	Graph
1.3	$x<-1$ or $0<x<2$

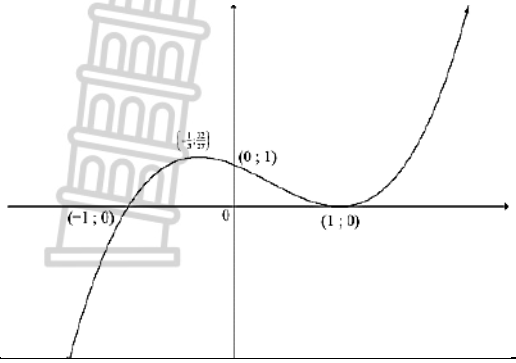
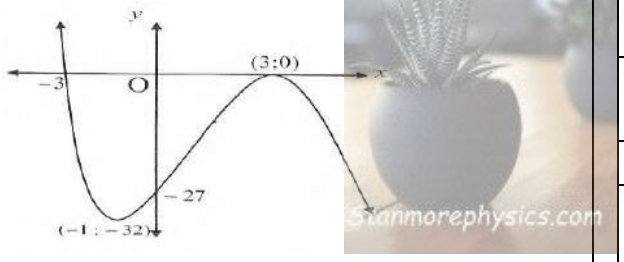
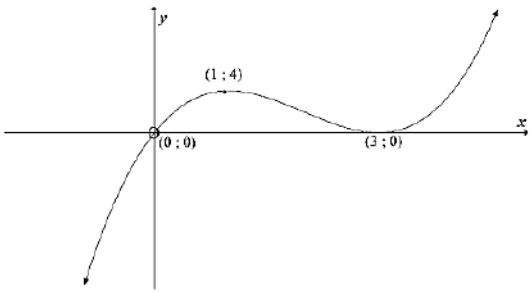
1.4	Translate 3 units to the left and 4 units down
2.1	$x=1$ and $y=2$
2.2	$(2;0)$
2.3	Graph
2.4	$y=-x+3$
2.5	show
3	$(6;0)$
2.7	Graph
3.1	$x<-1$ or $0<x<2$
3.2	Translate 3 units to the left and 4 units down
4.1	$y=x-1$
4.2	$Q(2-\sqrt{3};1-\sqrt{3})$
4.3	$y\in R; y\neq -1$
5.1	$A(-4;3)$
5.2	$B(-1;0)$
5.3	$C(-1;0)$
5.4	Translate h 4 units to the right and 3 units down
5.5	$(3;-3)$ *and* $(-3;3)$
6.1	$x=3$
6.2	$x\in R;\ x\neq 3$
6.3	$q=1$ and $a=-6$
6.4	graph
7.1	$x=-3$ and $y=1$
7.2	Show
7.3	$x=1$
7.4	$y=-\frac{1}{3}$
7.5	Graph
7.6	$x\in R;\ x\neq 4$
7.7	$x<4$ or $x\geq\frac{16}{3}$
8.1	$x=1$
8.2	$P(0;-1)$
8.3	$y=2(x+1)^2-3$
8.4	$y=-x-4$
8.5	$k>3$
8.6	$x\in R; y\neq -\frac{1}{2}$
9.1	$k=\frac{1}{2}$
9.2	$m=-\frac{3}{2}$
9.3	$n=\frac{49}{8}$
10.1	$B(2;\ 9)$
10.2	$y=2x+2$
10.3	Max length of EH=4
10.4	$m=-2$
11.1	$B(0;6)$
11.2	$A(-2;0)$
11.3	$y=-\frac{3}{2}x^2+6$
11.4	CF = 5 units
11.5	$(-2;6)$
11.6	$p(x)=-\frac{3}{2}x^2$
11.7	$y=-\sqrt{-\frac{2}{3}x};\ x\leq 0$
11.8	yes
11.9	$k(x)=-3x^2$
12.1.1	$T(0;18)$
12.1.2	$Q(3;\ 0)$
12.1.3	$R(6;\ 0)$
12.1.4	$x\in R$
12.2	Graph
13.1	$x\in R; x\neq 0$
13.2	$m=-1$
13.3	$n=8$
13.4	$p=1$ and $q=8$
13.5	$x=-3$ or $x=1$
13.6	$D(-1;11)$
13.7	$y=3x+14$
13.8	$y=\frac{3}{x-4}+8$
13.9	$-11<k<-8$
14.1	Show
14.2	$x\in[-2;0)$
14.3	$y=\pm 2\sqrt{x};\ x\geq 0$
14.4	Graph
14.5	$y\geq -2$
14.6	$y=\log_{\frac{1}{3}} x;\ x>0$
14.7	$x\in(0;3]$
15	Graph
16.1	$(0;-7)$
16.2	$q=-8$
16.3	show
16.4	$y>-8$
16.5	$T(-3;-1)$
16.6	$y=\log_3 x$
16.7	$x<-3$ or $0<x<\frac{3}{2}$

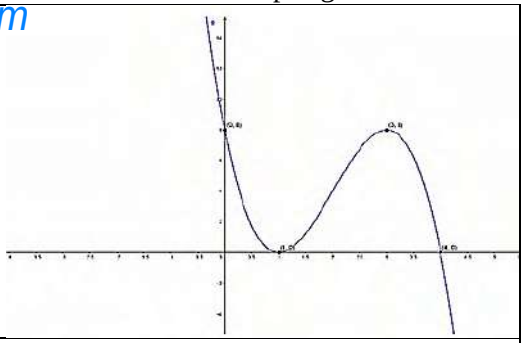
17.1.1	$y=\frac{x^2}{3};\ x\geq 0$
17.1.2	$0\leq x\leq 3$
17.2.1	$y=\log_{\frac{1}{3}} x$
17.2.2	graph
17.2.3	$a=\frac{1}{3}$
17.2.4	Translate of 1 unit to the right and 7 units downwards
18.1	$Q(1,5;\ 0)$
18.2	$-7\leq x\leq 5$
18.3	Graph
18.4	$x=3$
18.5	Min length of OP = 1,34 units
18.6.1	f has a minimum value since $a>0$
18.6.2	$m=5$
19.1	$y=-4$
19.2	$B(2;\ 0)$
19.3	$k(x)=\frac{3}{2}x-3$
19.4	Vertical Distance = $\frac{1}{2}$ units
19.5	$g(x)=2^x;\ x\in[-2;4)$
19.6	$y\in\left[\frac{1}{4};16\right)$
19.7	$g^{-1}(x)=\log_2 x$
20.1	$k=16$
20.2	$\frac{1}{4}\leq x\leq 16$
20.3	$y=4^x$
20.4	$x<0$
FINANCIAL MATHEMATICS	
1.1	R4500,29
1.2	23,69 years~24 years
1.3	24,08%
2.1	14,5%
2.2	R 2 345,67
2.3.1	R9 600
2.3.2	
2.3.3	R9312,82 (Ayanda did NOT have enough)
3.1	7,23%
3.2	67,34 months
3.3	R 2 142,21
3.4	10,37%
4.1	11,72%
4.2.1	R 10 402,15
4.2.2	n =26
4.2.3	R 282,36
5.1	R 65 043,93
5.2.1	R 308 002,67
5.2.2	R 6 664,20
5.3	R 102 128,44
6.1	R 1 269 728,92
6.2	R 3 670,11
7.1	131 months
7.2.1	Original value/value at the start
7.2.2	No, no matter how small the value, the next year it will be a fraction thereof. OR No, E represents an exponential function with an asymptote at E=0.
7.2.3	18%
7.3.1	R 21 988,40
7.3.2	R 2 274 352,74
8.1.1	24,57%
8.1.2	n =6
8.2.1	R 7 376,50
8.2.2	R 811 447,53
8.2.3	R 153 236,88
9.1	1,8:1
10.1.1	R 74 883,86
10.1.2	R 168 306,21
10.1.3	R 1 184,68
10.2	n =66,04 (a little more than 66 months)
11.1.1	R 256 289,06
11.1.2	R 7 359,79
11.1.3	n =35,42~36 payments=10 months
11.1.4	R 3 782,14
12.1.1	R 850 000
12.1.2	R 6 729,95
12.1.3	R 867 188
12.1.4	R 615 509,74
12.1.5	R 634 183,81
12.1.6	n =109,74~110 months
13.1.1	R 38 058,80
13.1.2	12 months earlier
14.1	7,14%
14.2	R 3 097,20
14.3	n =51

15.1	R 450 000
15.2	14,21%
15.3	R 614 490,66
15.4	R 9 397.11

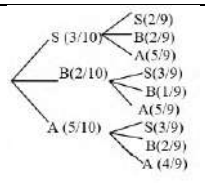
CALCULUS

1.1	$f'(x)=2x$
1.2	$f'(x)=8x$
1.3	$f'(x)=-4x$
1.4	$f'(x)=-2x+1$
1.5	$f'(x)=\frac{2}{x^2}$
1.6	$f'(x)=-2x+7$
1.7	$f'(x)=0$
1.8	$f'(x)=1$
1.9	$f'(x)=3x^2$ and $f'(-2)=12$
1.10	$f'(x)=2x-6$
2.1	$=-x^{-2}+10x^{-3}-\frac{1}{2}x^{-\frac{1}{2}}$
2.2	$\frac{dy}{dx}=12x^3+7x^{-2}+\frac{4}{3}x^{-\frac{1}{3}}$
2.3	$f'(x)=3x^{\frac{1}{2}}-\frac{3}{2}x^{-\frac{3}{2}}$
2.4	$f'(x)=7x^6$
2.5	$\frac{dy}{dt}=10t^4+\frac{7}{4}t^{\frac{3}{4}}$
2.6	$f'(x)=-2x-2$
2.7	$\frac{dy}{dx}=-x^{-\frac{1}{2}}+1+\frac{1}{2}x^{-\frac{3}{2}}$
2.8	$6x+4x^{-3}$
2.9	$\frac{dy}{dx}=2x+5$
2.10	$-\frac{6}{5}x^{-\frac{8}{5}}$
2.11	$1+5x^{-6}$
2.12	$gt+5t^{-2}+3$
3.1	3
3.2	$a=\pm 3$
3.3	Proof
3.4	$y=4x-8$
3.5	$A\left(\frac{1}{32};5\right)$
3.6	$a=-4, b=-3$
3.7.1	$x=1$ or $x=3$ $y=9$
3.7.2	$\left(\frac{7}{3};\frac{32}{9}\right)$ or
3.8.1	$y=5x-7$
3.8.2	$\frac{964}{27}$
3.9.1	$x=1$ or $x=3$ $y=9$
3.9.2	$\left(\frac{7}{3},\frac{32}{9}\right)$ or $(1,0)$
3.9.3	
3.9.4 a)	$1<x<\frac{7}{3}$
b)	$0<k<\frac{32}{9}$
3.10.1	(3,-4)
3.10.2	$y=9x-23$
4.1.1	$(-2,0)$ $(2,-32)$
4.1.2	$x=4$ 0r $x=-2$
4.1.3	$x=3$ or $x=-3$
4.1.4	$x>0$
4.2	$x<\frac{1}{3}$
4.3.1	$(1,1)$ $(3,4)$
4.3.2	
4.3.3	$0<k<4$
4.3.4	$y=3x-4$
4.3.5	$\theta=7157^\circ$
4.4.1	(0,1)

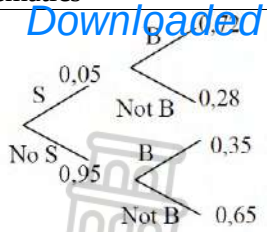
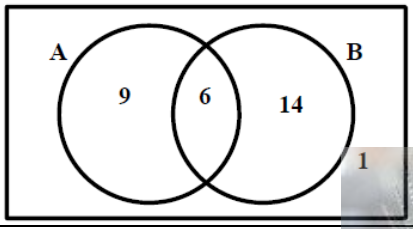
4.4.2	$(1,0)\ (-1,0)$
4.4.3	$(1,0)\ \left(-\frac{1}{3},\frac{32}{27}\right)$
4.4.4	
4.4.5	$-\frac{1}{3}<x<1$
4.5.1	$x=-3$ or $x=3$ or $y=-27$
4.5.2	$(-1;-32),\ (3;0)$
4.5.3	
4.5.4	$x>1$
4.5.5	$y=9x-27$
4.5.6	$-32<k<0$
4.5.7	$f(x+3)=-(x+3)^3+3(x+3)^2+9(x+3)-27$
4.6.1	Proof
4.6.2	
4.6.3	$x>2$
4.6.4.1	$(3,7)$
4.6.4.2	Do not agree $f'(2)=-3$

4.7.1	
4.7.2	Proof
4.7.3	$x>2$
4.8	(6,0)
4.9.1	$x<\frac{5}{3}$
4.9.2	(4,36)
4.9.3	$x<\frac{5}{3}$
4.9.4	$OG=\frac{10}{3}$
4.9.5	$m=\frac{1}{17}$
4.10.1	(0, -4)
4.10.2	$y=\frac{1}{3}x^2-\frac{4}{3}x-4$
4.10.3	$x=-2$ or $x=6$
4.10.4	$x=2$
4.10.5	For $x<-2, g'(x)>0$, the gradient of g is positive and for the values of x immediately right of -2.
4.11.1	$x_D=-1$ or $x_E=3$
4.11.2	$f(x)=-x^3+3x^2+9x-30$
4.11.3	$-1<x<3$ or $-1\le x\le 3$
4.11.4	$x>1$
4.12.1	Proof
4.12.2	$\frac{-419}{27}<k<3$
4.12.3	concave
4.12.4	$\left(\frac{1}{3},\frac{250}{27}\right)$
5.1.1	8 hours
5.1.2	1,69
5.1.3	16 km/h
5.2.1	$y=\frac{23}{2}-2x$
5.2.2	$y=\frac{23}{2}m$
5.3	0,87

5.4.1	8 hours
5.4.2	24,63 km
5.4.3	16 km/h
5.5.1	Proof
5.5.2	6,93
5.6.1	100
5.6.2	Will not get the desired because a shape with two equal circles touches externally.

PROBABILITY	
1.1	$a=37; b=16; c=31; d=58; e=118$
1.2	The events are NOT independent
2.1	554400
2.2	151200
2.3	70560
3.1.1	$\frac{1}{14}$
3.1.2	$\frac{3}{14}$
3.1.3	The events are not independent.
3.1.4	$B=800; C=100; D=300$
3.1.5	$\frac{1}{14}$
3.2.1	$9!$
3.2.2	$5!.4!.2$
4.1.1	1 or 2
4.1.2	3600
4.2	76,5%
5.1	$\frac{51}{260}$ or 0,1962 or 19,62%
5.2	$\frac{2}{13}$ or 0,1538 or 15,38%
6.1	32
6.2	$\frac{2}{32}$
6.3	$\frac{0}{32}$
7.1	40320
7.2	1440
7.3	$\frac{2}{3}$
8.1	

8.2.1	$\frac{14}{45}$
8.2.2	$\frac{8}{15}$
9.1.1	$a=450; b=319; c=298; d=748$
9.1.2	$\frac{298}{1530}$
9.1.3	The events are independent. *P(Male* and *Broken Limb)* $= P(Male) \times P(BrokenLimb)$
9.2.1	$\frac{2}{7} \approx 0,29$
9.2.2	$\frac{40}{77} \approx 0,52$
9.3.1	Venn Diagram
9.3.2	$\frac{21}{80}$
9.3.3	$\frac{16}{80}$
10.2	$\frac{8}{15}$
11.1	No, $(P$ and $B) \neq 0$
11.2	$P(A \text{ or } B) = P(A) + P(B) - (P \text{ and } B)$
11.3	1200
11.4	864
12.1.1	24 ways
12.1.2	$\frac{1}{180}$ *or* 0,01
12.2	$= (n-1)!$ $\therefore 2! \times (n-1)!$
13.1.1	No, because $P(A$ and $B) \neq 0$
13.1.2	(a) 0,55
	(b) 0,33
13.2.1	$\frac{1}{4}$
13.2.2	479001600
13.2.3	$\frac{1}{99}$
14.1.1	$\frac{1}{4}$
14.1.2	$\frac{5}{6}$
14.2.1	

	
14.2.2	0,6315
14.3.1	10!
14.3.2	$\frac{4}{45}$
15.1.1	531 441
15.1.2	531 441
15.2	50 400
16.1.1	
16.1.2	n(A or B) = 29
16.1.3	0,2
16.2.1	67 600
16.2.2	≥ 3,64717...
17.1	67600
17.2	15390
17.3	75,44%
17.4.1	0,195
17.4.2	41,055%
17.4.3	229333
18.2.1	152
18.2.2	0.70
18.3	R19.50
18.4	1x5x4x3=60
18.5.1	1260
18.5.2	$\frac{1}{7}$
19.1	4096
19.2	$\frac{15}{2048}$
19.3	512
20.1	0,107
20.2	0,16
20.3	0,61
20.4	Independent

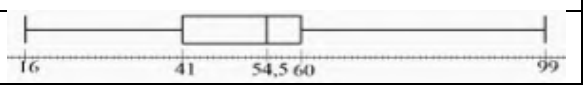
GENERAL TRIGONOMETRY	
1.1	$q=-\sqrt{2}$
1.2	$\frac{-1-2\sqrt{2}}{3\sqrt{2}}$
1.3	$\frac{7}{9}$
2.1	$x=-7$
2.2	$\frac{24}{7}$
2.3	$106,26°$
3.1	$\frac{5}{12}$
3.2	$\frac{5}{13}$
3.3	$a=-7,5$
4.1	$\frac{4}{3}$
4.2	$\frac{5}{3}$
4.3	$m=-5$
4.4	$\frac{33}{65}$
5.1	$-\frac{3}{5}$
5.2	$-\frac{\sqrt{10}}{10}$
6.1	$-\frac{7}{9}$
6.2	$\frac{2\sqrt{2}}{3}$
6.3	$2\sqrt{2}$
7.1	$\frac{7}{25}$
7.2	$\frac{4\sqrt{3}}{10}$
8.1	$-\frac{12}{13}$
8.2	$-\frac{7\sqrt{2}}{26}$
9.1	$\frac{n}{\sqrt{1+n^2}}$
9.2	$\frac{-2n}{1+n^2}$
9.3	$\frac{\sqrt{3}+n}{2\sqrt{1+n^2}}$
10.1	$-\frac{\sqrt{1-p^2}}{p}$
10.2	$2p\sqrt{1-p^2}$

10.3	$\frac{\sqrt{3}p}{2}-\frac{\sqrt{1-p^2}}{2}$
11.1	$-\sqrt{k}$
11.2	$1-2k$
11.3	$\frac{\sqrt{1-k}}{2}+\frac{\sqrt{3k}}{2}$
12	$2p\sqrt{1-p^2}$
13	$\frac{\sqrt{k}+1}{2}$
14.1	$-\sqrt{1-p^2}$
14.2	$2p\sqrt{1-p^2}$
14.3	$\frac{\sqrt{1-p^2}}{2}$
15.1	$\frac{p}{\sqrt{1-p^2}}$
15.2	$-p^2$
15.3	$2p$
16.	$-\frac{1}{4}p$
17.	$-\frac{2ab}{a^2-b^2}$
18.1	$\frac{1}{k}$
18.2	Proof
19.1	$\frac{\sqrt{3}+4}{2}$
19.2	$\frac{\sqrt{2}}{2}$
19.3	$-\frac{\sqrt{3}}{2}$
19.4	$-\frac{1}{2}$
19.5	$-\frac{\sqrt{2}}{4}$
19.6	$\sqrt{3}$
19.7	4
19.8	$-\frac{\sqrt{3}}{4}$
20.1	$-\cos^2 x$
20.2	$-\tan 2\theta$
20.3	$-2\cos^3 x$
20.4	2
20.5	$-\frac{1}{2\cos x}$
20.6	$-\frac{1}{2}\sin\theta$
20.7	0
20.8	$2\sin x$
21.1	$\frac{\cos 2x}{4}$
21.2	$x=40,40°+k.360°;k\in Z$
24.2	$x=\pm45°+k.180°;k\in Z$
25.1	Proof
25.2	$x=\pm30°+k.120°;k\in Z$
26.1	$221.81°+k.360°$ or $30°+k.360°$ or $150°+k.360°$
26.2	$\theta=-300°/300°/420°+k.720°;k\in Z$
26.3	$x=16,10°+k.180°$
26.4	$\alpha=31,72°+k.90°/121,72°+k180°$
26.5	$x=120°/240°+k.360;k\in Z$
26.6	$x=45°+k.180°;k\in Z$
26.7	$199,47°$ or $x=350,53°$
26.8	$x=48°+k.180°/12°-k.360°$
26.9	$x=\pm90°/\pm94,78°+k.360°$
26.10	$x=63,73°/-45°+k.180°$
26.11	$x=180°/-30°+k.360°$
26.12	$x=\pm131,81°/30°/150°+k.360°$
27.1	Proof
27.2	$x=52.5°/82.5°+k.90°$
28.	$m=-\frac{1}{8}$

TRIGONOMETRIC GRAPHS	
1.1	Graph sketching
1.2	Graph sketching
1.3.1	Graph sketching
1.3.2	360°
1.3.3	$x=151,32°$ or $x=-28,68°$
2.1	360°
2.2	$x=180°$
2.3	Graph sketching
2.4	$x=-90°;270°;45°;225°$
2.5	$-5\le y\le -1$
2.6	49
3.1	$a=1;b=2;\ p=-45°$
3.2	$-90°<x<0°$
3.3	60°
3.4	minimum value $=-7$
3.5	shifted 45° to the left and reflected on the $x-$axis

3.6	$90° \le x \le 45°$ or $45° \le x \le 90°$ or $135° \le x \le 180°$
4.1(a)	$A(120°;0)$ $B(-60°;0)$
4.1(b)	$C(-150°;-2)$ $D(30°;2)$
4.2(a)	$-180° < x < -150°$ or $30° < x < 180°$
4.2(b)	$-130° < x < 50°$
4.2(c)	$-60° < x < 0°$ or $120° < x < 180°$
4.2(d)	$0° < x < 30°$
4.3	$1 \le y \le 4$
5.1	$-1 \le y \le 3$
5.2	$180°$
5.3	$-180° < x < 0°$
5.4.1	$-90° < x < 0°$ or $0° < x < 90°$
5.4.2	$-180° < x < 90°$ or $90° < x < 180°$
5.5	$h(x) = -\cos 2x$

ANSWERS 2D/3D TRIGONOMETRY	
1.1	CB = 4,29 m
1.2	CD = 8,37 m
1.3	$10,89\ m^2$
2.1	$BE = \frac{y}{\sin\alpha}$
2.2	Proof
2.3	$B\hat{A}F = 29°$
3.1	PS 19,01 units
3.2	PQ 12,22 units
4.1	Proof
4.2	$x = 10$
5.3	$69.98\ units^2$
6.2	$\theta = 50,77°$
7.1	Tree will hit the house (height =12,3m)
8.1	LB = 3,5752… ≈ 3,58 metres
8.2	AB = 7,38 m
8.3	$\approx 8,57 m^2$
9.1	$AD = AB\sin\hat{B}$
9.3.3	$\frac{1}{4}k^2\tan\theta$
10.1	$AB = \frac{p}{\tan y}$
10.2	$\cos x = \tan y$
10.3	$26,57°$
EUCLIDEAN GEOMETRY	
1.	$AB = 20$
2.2	$O_1 = 52°$
2.3	$B\hat{D}E = 64°$
3.2.4	$x = 72°$ and $y = 36°$
5.1.1	$\hat{C}_1 = x$ (tan. chord theo.) $\hat{A}_3 = x$ (Alt ∠s =; BC // AD)
5.1.5	$y = 90°$
6.1.2	$\hat{L} = 148°$
6.2.1	∠ in a semi-circle
6.2.3	Proof
7.2	Proof
7.3	BG = 14,49
8.1.1	$O\hat{A}C = 54°$
8.1.2	$A\hat{C}D = 18°$
8.1.3	$\hat{T} = 99°$
12.1	$\hat{F}_1 = 90°$
12.2.1	$\hat{O}_2 = 180° - 2x$
12.2.2	$\hat{B} = 135° - x$
14.1	$\hat{N} = x$ (∠s in the same seg)
18.1	$\hat{M} = 90°$
18.3	LP = 2
19.3	BC = 4
2.1	$\hat{A}_2 = 26°$ (Alt ∠s; AB // EC) $\hat{E}_2 = 26°$ (∠s in the same seg) $\hat{B}_1 = 26°$ (∠s in the same seg) $\hat{B}_2 = 26°$ (∠s opp = sides)
3.2.3	$Q\hat{A}S = y$ (tan. chord theo) $A\hat{S}T = y$ (Alt ∠s =; AQ // TS) $T\hat{A}S = y$ (∠s opp = sides)
5.1.2	$\hat{B}_2 = y$ (ext ∠ of a cyclic quad) $B\hat{C}D = y$ (Alt ∠s =; AD // BC)
5.1.4	$\hat{A}_2 = 180° - (x + y)$
6.1.1	$\hat{N}_2 = 32°$
6.1.3	$\hat{N}_1 = 32°$

6.2.2	Proof
7.1	$\hat{A}_2 = 37°$ ($\angle$s in the same seg)
	$\hat{C}_2 = 37°$ ($\angle$s opp = sides)
	$\hat{A}_1 = 37°$ (DA bisect $\hat{A}$)
13.1	$\hat{D} = x$ ($\angle$ at centre = $2 \times \angle$ at circumf) $\hat{C}_2 = x$ ($\angle$s opp = sides) $\hat{A} = x$ ($\angle$s in the same seg)
20.2	$TE = 4$
20.3	$\frac{3}{2}$
19.4	$GC = \sqrt{72} = 8{,}49$
19.5	$\hat{E} = 19{,}76$
22.1	$P\hat{S}L = 90°$
22.4	$\frac{1}{4}$
22.5	$\frac{135}{7}$
25.2	$TQ = 1{,}2$
27.2	$\frac{9}{16}$
20.4	$\frac{1}{15}$
28.1	$\frac{5}{4}$
STATISTICS	
1.1	164
1.2	65.18
1.3	66.67
1.4.1	194
1.4.2	Positively skewed
1.4.3	(a). Mean increases
	(b). Median will not change
1.5	7 days
2.1	41
2.2	99
2.3	54.5
2.4	19
2.5	
2.6	Negatively Skewed
3.1	16
3.2	$45 < x \leq 55$
3.3	8 22 44 81 97 100
3.4	Refer
3.5	44.5

4.1	-0.97
4.2	$\hat{y} = 104.08 - 3.82x$
4.3	$28.4 \approx 28$
4.4	On sketch
4.5	34.13
5.1	4
5.2	5.24
5.3	4
5.4	6.5
5.5.1	Remains the same
5.5.2	Remains the same
6.1	15
6.2	70
6.3	100%
6.4	No, this is extrapolation. Anyone who studies for over 180 min will obtain 100%.
7.1	570.27
7.2	R291,03
7.3	10 premiums
7.4	21%
8.1	Scatter plot
8.2	$\hat{y} = 3.08 + 0.36x$
8.3	r = 0.74
8.4	7.63 min
8.5	No delivery will be made with zero items
9.1	1 4 13 19 22 23
9.2	23
9.3	93.70
9.4	Refer to cumulative frequency graph
9.5	11
10.1	33 players
10.2	29.76
10.3	(a) 2 10 25 31 33
	(b) Refer to cumulative frequency graph
10.4	29.8
10.5	k = 4
11.1	$\hat{y} = 10.85 + 0.9x$
11.2	(a) 52.25%
	(b) 0.94
11.3	B
12.1	4 18 36 50 54
12.2	54 matriculants
12.3	$40 < x \leq 60$
12.4	50%
12.5	12 learners
13.1	R13 792

13.2	R4 404
13.3	2 employees
13.4	$\hat{y} = 0.45 + 8.45x$
13.5	r = 0.94
13.6	R21 804
13.7	Yes. Strong correlation (results reliable).
14.1	5500
14.2	10
14.3	11.28%
15.1	10 20 20 32 45 49 51 53 80
16.1	1.4 kg
16.2	No, this cannot be correct. The data is skewed to the left. Therefore the mean is smaller than the median, and cannot have a value of 3,5. Which is bigger than 3,3.
17.1	Scatter plot
17.2	$\hat{y} = 26.88x + 161.24$
17.3	806.36
17.4	Refer to grid

ANALYTICAL GEOMETRY	
1.1	$BF = \sqrt{20}$
1.2	$m_{BF} = -2$
1.3	$\theta = 71,57^{\circ}$
1.4	$m_{HF} = 1$
1.5	Kite
1.6	$AC = 6\sqrt{5}$
1.7	Area of $AOFC = 42$
2.1	$AB = 10$
2.2	$C(-4;19)$
2.3	$K = (-1;10)$
2.4	$y = -3x + 7$
2.5	$\theta = 18,43°$
2.6	Area of $\Delta ABC = 29,99^{\circ}$
2.7	$D(2;11)$
3.1	$M(3;0)$
3.2	$y = -x + 3$
3.3.	$\theta = 56,31°$
3.4	$b = 3 - a$
3.5	$b = 4$
3.6	$x^2 + y^2 = 90$
3.7	$m = 0,20$ or $m = -5,00$

4.1	$m_{AB} = 1$
4.2	$y = x - 11$
4.3	$M\left(2;-\frac{3}{2}\right)$
4.4	$C(4;-7)$
4.5	$B\hat{C}D = 108,43°$
5.1.1	$m_{AB} = 1$
5.1.2	$p = 2$
5.1.3	$y = -x + 1$
5.1.4	$D(5;-1)$
5.1.5	$\left(x-\frac{3}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 = \frac{25}{2}$
5.1.6	$\theta = 53°$
5.2	$b = \frac{21}{12} = \frac{7}{4}$
6.1	$K(5;6)$
6.2.1	$m_{AB} = 3$
6.2.2	$p = 8$
6.3	$E(1;-3)$
6.4	Area $= 49,01$ units^2
6.5.1	Perimetre $= 58,27$
6.5.2	$x^2 + y^2 = 208$
7.1.1	$F(0;5)$
7.1.2	$m_{DF} = 2$
7.1.3	$\alpha = 63,43°$
7.2	$y = 2x$
7.3	Line have the same gradient
7.4	$S(1;2)$
7.5	EDOS is $\|\|^m$ [both pair opp.sides $\|\|$]
8.1.1	$y = 2x + 0$
8.1.2	$\theta = 63,43°$
8.1.3	$H(-2;3)$
8.2	$x \neq 5; x = 3$
8.3	$\alpha = 37,88°$
8.4	Area of $\Delta BTS = 14$ units^2
8.5	$h = 5,49$ units

9.1	$\text{M}\left(\frac{1}{2};-2\right)$
9.2	$\text{AB}=3\sqrt{13}$
9.3	$m_{AB}=-\frac{2}{3}$
9.4	$\theta=146,31^{\circ}$
9.5	$\alpha=82,88^{\circ}$
9.6	Area of $\Delta\text{AKC}=4,93mm^2$
10.1	$y=-2x$
10.2	$y=\frac{1}{2}x+5$
10.3	$\text{A}(0;10)$
11.1.1	$y=\frac{4}{3}x+\frac{1}{3}$
11.1.2	$(x+5)^2+(y-2)^2=25$
11.1.3	$\text{B}'(-1;5)$
11.2.1	centre $(2;-3)$ radius $=8$units
11.2.2	$r=4,39$units
12.1	$\text{A}(-3;4)$
12.2	$x^2+y^2=25$
12.3	$\text{AB}=10$ units
12.4	$\text{BD}=5$ units
12.5	Area of $\Delta\text{ABD}=25$ square units
12.6	$(x+3)^2+\left(y+\frac{9}{4}\right)^2=\frac{625}{16}$
13.1.1	$m_{GM}=-\frac{1}{2}$
13.1.2	$y=-\frac{1}{2}x+6$
13.1.3	$\text{M}(6;3)$
13.1.4	$GM=r=4,47$
13.1.5	$x^2+y^2-12x-6y+25=0$
13.2	$y=\frac{4}{3}x+\frac{13}{3}$
14.1.1	$Q(0;-8)$
14.1.2	$y=\frac{4}{3}x-8$
14.1.3	$p(6;0)$
14.1.4	$x_R=3$; $y_R=-4$
14.1.5	$(x-3)^2+(y+4)^2=25$
14.1.6	$k=1$ or $k=-9$
14.2	$r=\sqrt{3}$
15.1	$centre\left(-\frac{1}{2};\frac{3}{2}\right)$ radius $=\sqrt{\frac{1}{2}}$
15.2	$\text{A}(-1;2)$
15.3	$x^2+2x+y^2-2y+1=0$
15.4.1	$2x^2+4(4-2k)x+(k^2-2k+1)=0$
15.4.2	$k=\sqrt{2}$ or $k=-\sqrt{2}$
16.1	$\text{B}(-1;1)$
16.2	$\text{AC}=\sqrt{5}$
16.3	$\tan\theta=\frac{1}{2}$
16.4	$y=-2x-1$
16.5	$\text{A}(-3;4)$
16.6	Area ΔABC : Area $\Delta\text{ODF}=20:169$
17.1	radius $=\sqrt{5}$
17.2	$\text{E}(-6;1)$
18.1	$r=\sqrt{5}$
18.2	$a=6$ or $a\neq-6$
18.3	$(x-6)^2+(y+3)^2=20$
18.4	$17<20$, the point lies inside the circle
18.5	$\text{PS}=4\sqrt{10}$
18.2.1	centre $\left(2;-\frac{5}{2}\right)$
18.2.2	$d=-\frac{535}{4}$
19.1	$(x-6)^2+\left(y-\frac{9}{2}\right)^2=\frac{25}{4}$
19.2	$y=-\frac{4}{3}x+\frac{25}{3}$
19.3	circumference of large $=10\pi$ (**Two revolution**)
19.4	Area $\Delta AOB=\frac{625}{24}units^2$
19.5	$\text{CQ}=8$units
20	$y=-2x-10$ and $y=-2x+10$

21.1	$P\left(-2\frac{1}{2};1\right)$ or $P(-1;-2)$
21.2	$\left(x+2\frac{1}{2}\right)^2+(y-1)^2=6\frac{1}{4}$ **or** $(x+1)^2+(y+2)^2=1$
23	$y=-\frac{4}{3}x+\frac{1}{3}$
24	$AC=b$
25	$(y+2)^2+(x+1)^2=20$
26	$y=-2$