DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

PROVINSIALE EKSAMEN

PROVINCIAL EXAMINATION

GRAAD 12/GRADE 12

WISKUNDE/MATHEMATICS

VRAESTEL 2/PAPER 2

JUNIE/JUNE 2025

PUNTE/MARKS: 150

TYD/TIME: 3 uur/hours

Hierdie vraestel bestaan uit 13 bladsye, 1 inligtingsblad en 'n antwoordeboek van 23 bladsye./
This question paper consists of 13 pages, 1 information sheet and an answer book of 23 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of this question paper.
9. Write neatly and legibly.

QUESTION 1

The number of WhatsApp messages sent by 11 learners on a particular day, are as follows:

14	25	31	36	37	41	51	52	55	79	112

1.1 Calculate the mean number of messages sent. (2)

1.2 Calculate the standard deviation. (1)

1.3 Determine the number of learners who sent the messages that are within one standard deviation of the mean. (3)

1.4 Calculate the interquartile range. (3)

1.5 Identify an outlier . (1)
[10]

QUESTION 2

A traffic department set up a camera to record the speed of cars travelling into the town. The findings are shown in the table below.

Speed (km/h)	Frequency
$60 \leq x < 70$	43
$70 \leq x < 80$	69
$80 \leq x < 90$	110
$90 \leq x < 100$	49
$100 \leq x < 110$	20
$110 \leq x < 120$	9

2.1 How many cars were recorded by the camera? (1)

2.2 Complete the cumulative frequency column in the ANSWER BOOK. (2)

2.3 Draw the cumulative frequency curve (ogive) in the ANSWER BOOK. (3)

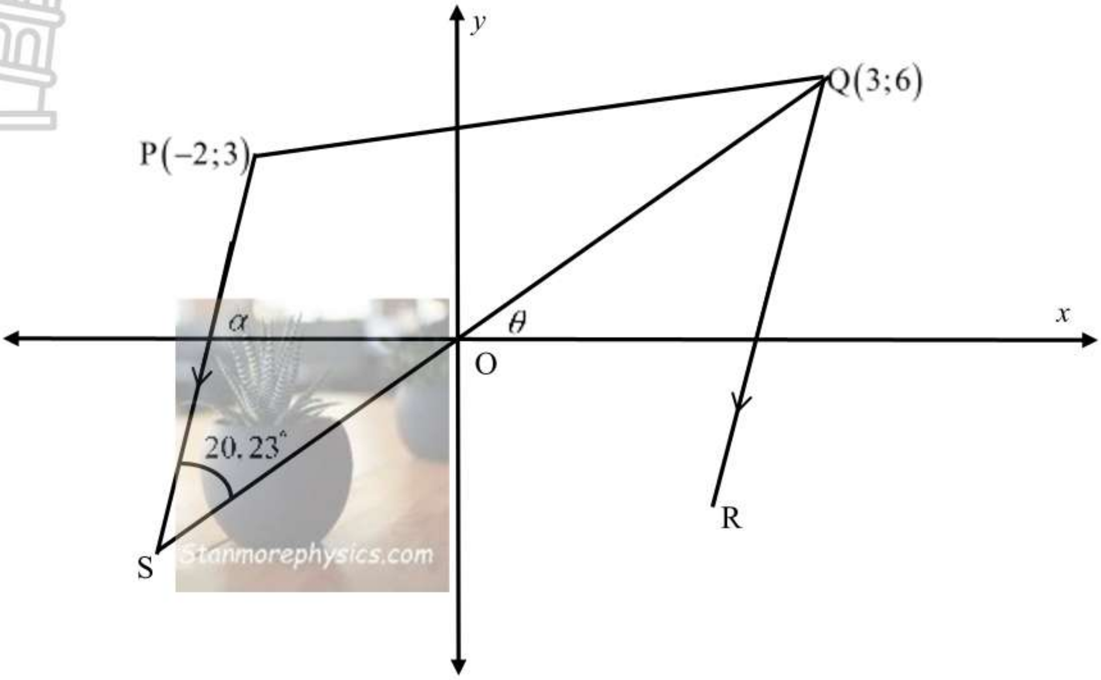
2.4 Use the ogive to estimate the semi-interquartile range. (3)

2.5 If the speed limit of the zone where the camera is installed is 80 km/h, how many cars drove above and equal to the speed limit? (2)

[11]

QUESTION 3

In the diagram below, ΔPQS is drawn with vertices $P(-2\,;3)$ and $Q(3;6)$ and S. Line QS passes through the origin at O. PR ∥ QR and $P\hat{S}Q = 20{,}23°$.



3.1 Calculate the gradient of QS. (2)

3.2 Calculate the size of θ. (2)

3.3 Determine the:

3.3.1 Gradient of PS, correct to the nearest integer (3)

3.3.2 Equation of PS in the form $y = mx + c$ (3)

3.4 If it is further given that the equation of QS is $y = 2x$, determine the coordinates of S. (4)

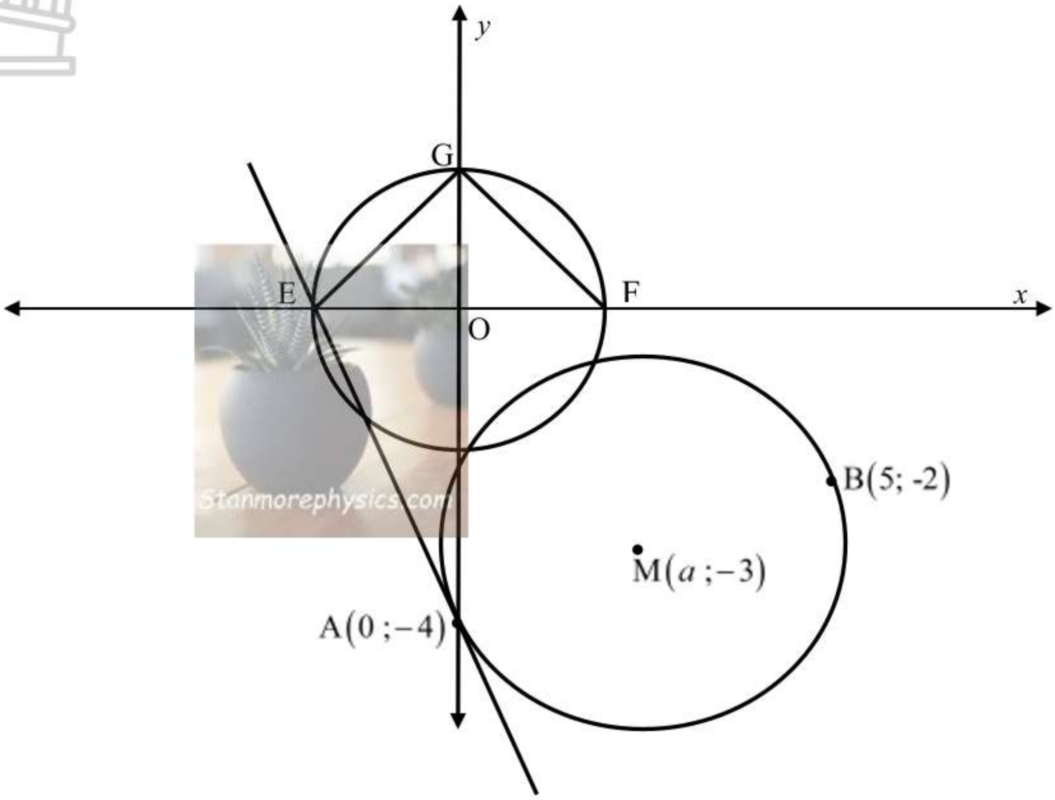
3.5 If $S(-21\,;\,-42)$, determine the coordinates of M, the midpoint of SQ. (2)

3.6 Write down the coordinates of R if PQRS is a parallelogram. (3)

[19]

QUESTION 4

In the diagram, the centre of the smaller circle is at the origin and the circle cuts the x-axis at E and F respectively. The centre of the larger circle is at $M(a;-3)$. The equation of tangent AE to the larger circle is given by $y = -\frac{5}{2}x - 4$.



4.1 Determine:

4.1.1 The coordinates of E (2)

4.1.2 The equation of the smaller circle (3)

4.2 Determine the equation of the radius AM in the form $y = mx + c$. (3)

4.3 Hence, calculate the value of a. (3)

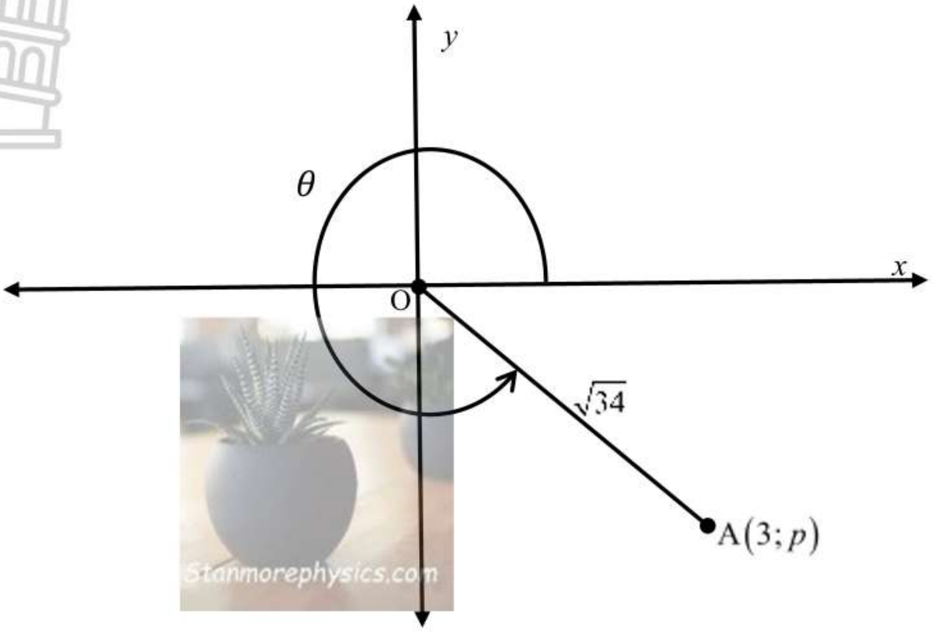
4.4 If $a = \frac{5}{2}$, determine the equation of the larger circle. (4)

4.5 Calculate the area of ΔEFG. (5)

[20]

QUESTION 5

5.1 In the diagram, A$(3\,;p)$ is a point in the Cartesian plane in the fourth quadrant. OA $= \sqrt{34}$ and θ is the reflex angle.



Determine, **without using a calculator,** the value of:

5.1.1 p (2)

5.1.2 $\cos(450° - 2\theta)$ (3)

5.1.3 $\cos(30° - \theta)$ (3)

5.2 Simplify $\frac{2\cos(90°+x).\cos(180°+x)}{\cos(60°+x).\sin x + \sin(60°+x).\cos x}$ to a single trigonometric function. (6)

5.3 Given: $f(x) = \cos(x + 45°)\,.\cos(45° - x)$ and $g(x) = 1 - 2\sin x$

5.3.1 Prove that $f(x) = \frac{1}{2}\cos 2x$ (4)

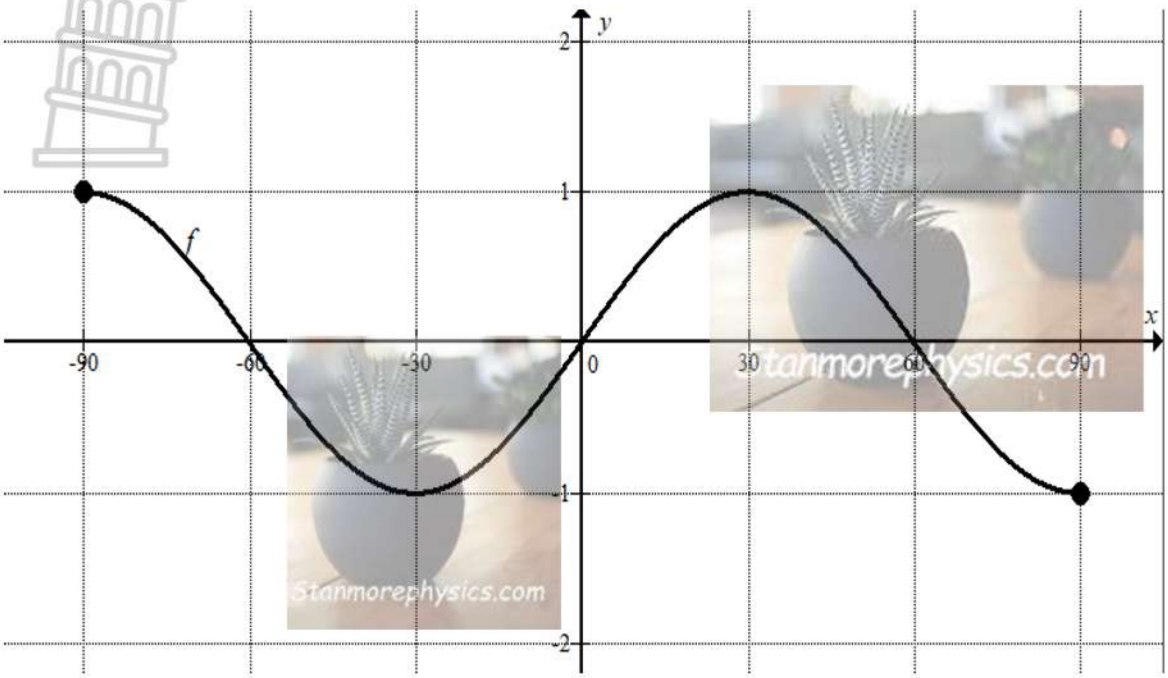
5.3.2 If $f(x) = g(x)$, determine the general solution. (6)

5.4 If $\cos\theta = 2m$ and $\cos 2\theta = 7m$, determine the value(s) of m. (5)

[29]

QUESTION 6

In the diagram below, the graph of $f(x) = \sin 3x$ is drawn for the interval $x \in [-90°\,; 90°]$.



6.1 Write down the period of f. (1)

6.2 On the grid given in the ANSWER BOOK, draw the graph of $g(x) = 2\cos(x - 30°)$ on the same set of axes. (3)

6.3 Use the graphs and write down the value(s) of x for which:

6.3.1 $f(x) > g(x)$ (2)

6.3.2 $f(x).g(x) < 0$ (3)

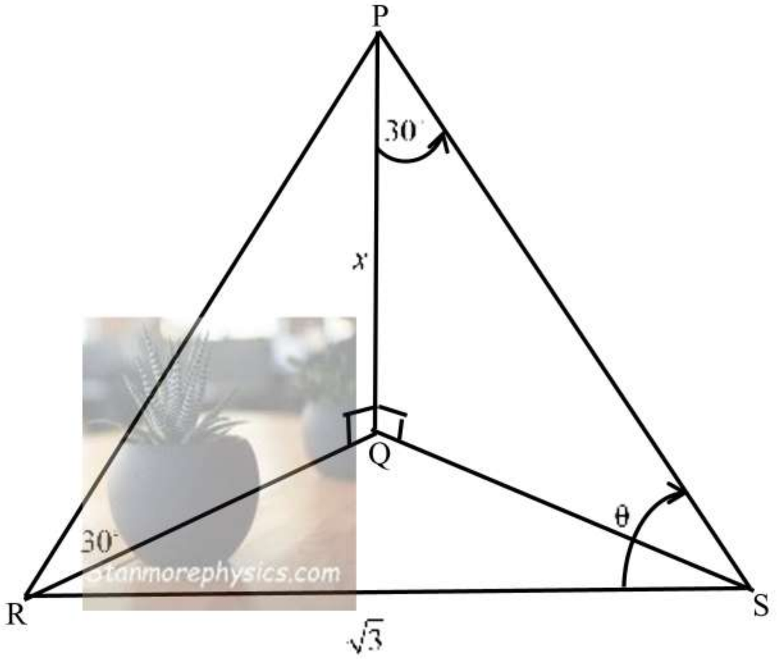
6.3.3 $f(x) - g(x) = 1$ (1)

6.4 Graph h is obtained when g is translated 60° to the right. Determine the equation of h. Write you answer in its simplest form. (2)

[12]

QUESTION 7

In the diagram, PQ is a vertical line with length x units. RS $= \sqrt{3}$ units, $P\hat{S}R=\theta$ and $P\hat{R}Q=S\hat{P}Q=30°$.

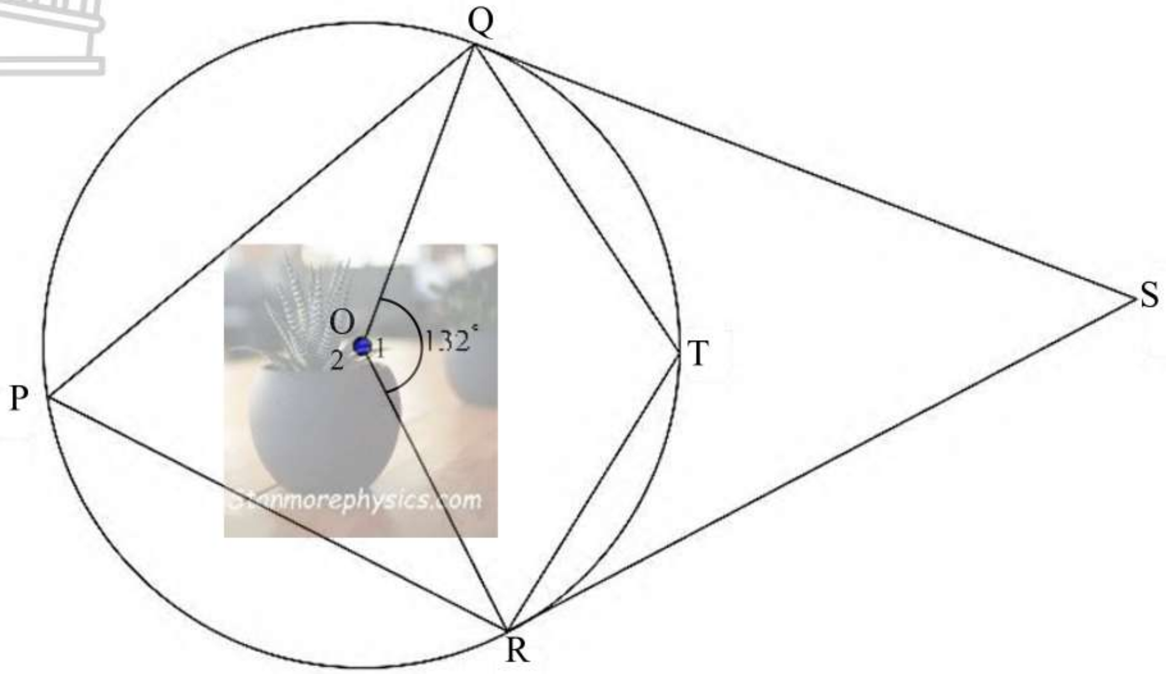


7.1 Show that $\cos\theta = \frac{9-8x^2}{12x}$ (6)

7.2 If $x = 1$, show that the area of $\Delta PSR = \sin\theta$ units^2. (2)
[8]

QUESTION 8

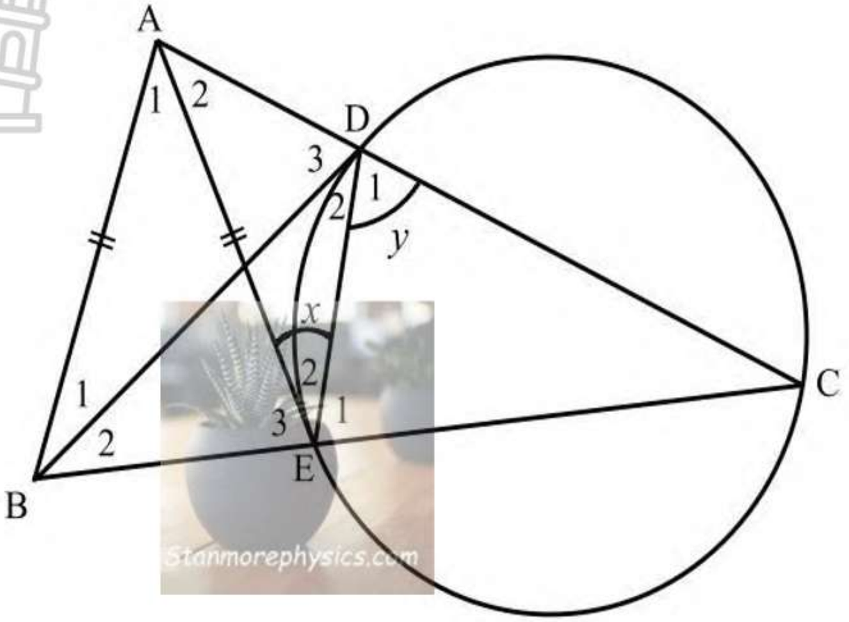
In the diagram, QS and RS are tangents to the circle at Q and R respectively. O is the centre of the circle. $Q\hat{O}R = 132°$.



8.1	Calculate, with a reason, the size of $Q\hat{T}R$.	(3)
8.2	Prove that QORS is a cyclic quadrilateral.	(3)
8.3	Hence, calculate the size of $Q\hat{S}R$.	(2)
		[8]

QUESTION 9

In the diagram below, chords CD and CE of the circle areproduced to A and B respectively. AE is a tangent to the circle and AB = AE. $\text{E}_2 = x$ and $\widehat{\text{D}}_1 = y$.

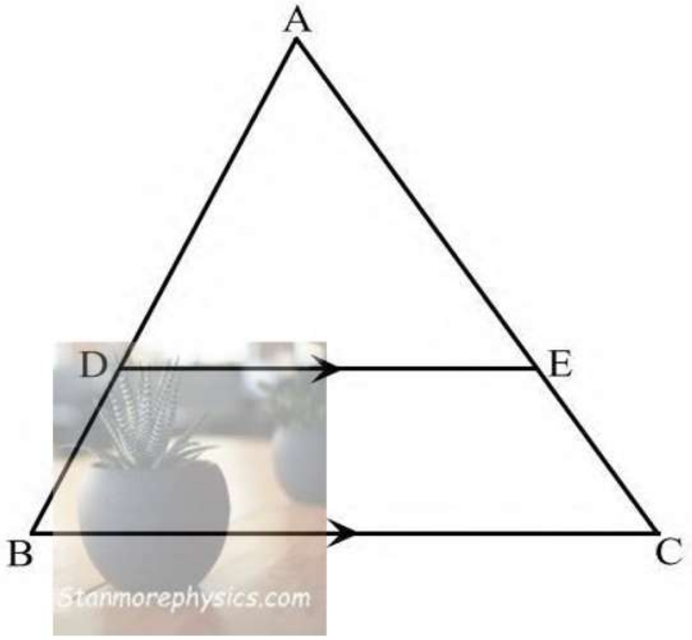


9.1 Prove that ABED is a cyclic quadrilateral. (6)

9.2 Prove that AB is a tangent to a circle passing through B, C and D. (3)
[9]

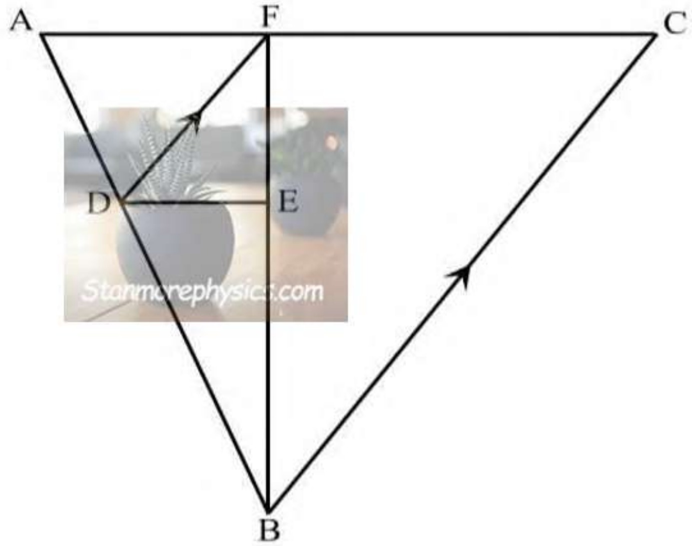
QUESTION 10

10.1 In the diagram, ΔABC is drawn. Line DE intersects AB and AC at D and E respectively such that DE ∥ BC.



Use the diagram and prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

10.2 In the diagram is ΔABC with DF∥BC and $\frac{AF}{FE} = \frac{FC}{EB}$.

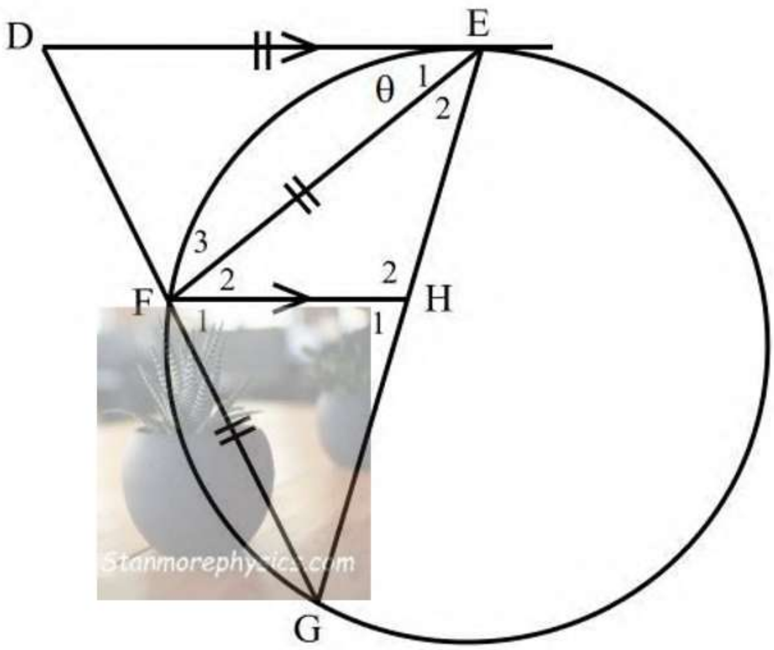


Prove, with reasons, that DE∥AF. (6)

[12]

QUESTION 11

In the diagram below, E, F and G are points on the circle. DE is to the circle at E. DFG is a secant such that DE=EF=FG. FH∥DE and $\hat{E}_1 = \theta$.



11.1 State, with reasons, THREE other angles each equal to θ. (3)

11.2 Prove that $DE^2 = DF.DG$ (5)

11.3 Prove, with reasons, that $\frac{DF^2}{DE^2} + \frac{DF}{DE} = 1$ (4)

[12]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ *area* $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$ $\quad \mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})}$

P(A of B) = P(A) + P(B) - P(A en B) $\quad \hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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GRAAD 12/GRADE 12

WISKUNDE/MATHEMATICS

VRAESTEL 2/PAPER 2

JUNIE/JUNE 2025

NASIENRIGLYNE/MARKING GUIDELINES

PUNTE/MARKS: 150

TYD/TIME: 3 uur/hours

Hierdie nasienriglyne bestaan uit 17 bladsye./
These marking guidelines consist of 17 pages

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

1.1	$\bar{x} = 48,45$	✓✓Answer (2)
1.2	$\sigma = 25.97$	✓Answer (1)
1.3	[48,45 – 25,97 ; 48,45 + 25,97] [22,48 ; 74,42] 8 learners	✓22.48 ✓74.42 ✓8 learners (3)
1.4	IQR=55-31 = 24	✓55 ✓31 ✓Answer (3)
1.5	112 $Q_3 + 1.5.IQR = 55 + 1.5(24) = 91$ $112 > 91$	✓Answer (1)
		[10]

QUESTION/*VRAAG* 2

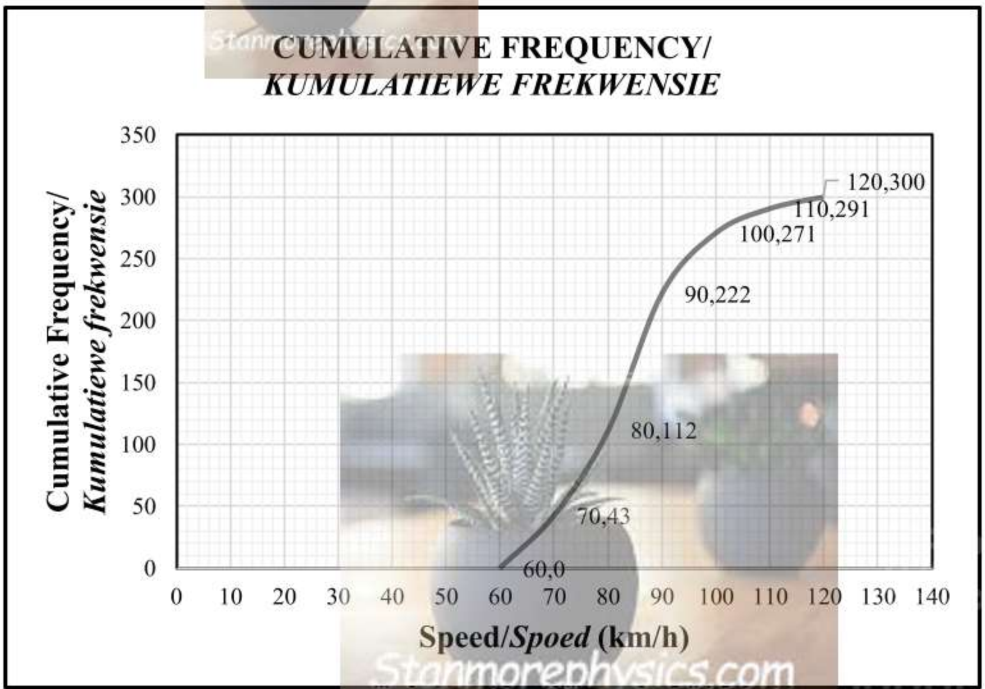
2.1	43 + 69 + 110 + 49 + 20 + 9 = 300 cars	✓300 (1)

2.2

Speed/*Spoed* km/h	Frequency/ *Frekwensie*	F
$60 \leq x < 70$	43	43
$70 \leq x < 80$	69	112
$80 \leq x < 90$	110	222
$90 \leq x < 100$	49	271
$100 \leq x < 110$	20	291
$110 \leq x < 120$	9	300

✓43,112, 222
✓271,291, 300
(2)

2.3



✓ grounding
✓Upper boundaries
✓shape
(3)

2.4	$Q_1 = 75$ $Q_3 = 90$ accept Q1 from 72-75, accept Q3 from 90-93 $SIQR = \frac{90-75}{2} = 7.5$ CA from Ogive	✓75 ✓90 ✓Answer (3)
2.5	$300 - 112 = 188\,cars$ (If read from ogive-accept from 187-189) 300- 112 Could also just add using the table.	✓✓188 (2)
	CA from learners' ogive all the way	**[11]**

QUESTION/*VRAAG* 3

3.1	$m=\frac{6-0}{3-0}$ Not allowed to use the value of S in q 3.1 - 3.3 $=2$	✓Substitution ✓Answer (2)
3.2	$\tan\theta=2$ $\theta=\tan^{-1}(2)$ $=63.43^\circ$	✓Substitution ✓Answer (2)
3.3.1	$\alpha=63.43^\circ+20.23^\circ$ $(ext\ \angle\ of\ \Delta)$ $\alpha=83.66^\circ$ $m=\tan 83.66^\circ$ $m=9$	✓ $\alpha=83.66^\circ$ ✓substitution ✓Answer (3)
3.3.2	$y=9x+c$ $3=9(-2)+c$ $3+18=c$ $c=21$ $y=9x+21$	✓✓substitution ✓Answer (3)
3.4	$2x=9x+21$ Mistake in question paper. S given in paper differs from S calculated in 3.4 $-7x=21$ $x=-3$ $y=2(-3)$ $y=-6$ $S(-3;-6)$	✓equating ✓ $x=-3$ ✓substitution ✓ $y=-6$ (4)
3.5	$S(-21;-42)$ $M\left(\frac{-21+3}{2};\frac{6+(-42)}{2}\right)$ $M(-9;-18)$ $S(-3;-6)$ $M\left(\frac{-3+3}{2};\frac{-6+6}{2}\right)$ $M(0;0)$	✓x-value ✓y-value (2)

3.6	$-9=\frac{-2+x}{2}$ $\quad -18=\frac{3+y}{2}$ $-18+2=x$ $\quad -36-3=y$ $R(-16\,;-39)$	$0=\frac{-2+x}{2}$ $x=2$ $0=\frac{3+y}{2}$ $y=-3$ $R(2;-3)$ Answer only: full marks	✓substitution ✓x-value ✓y-value (3)
			[19]

QUESTION/*VRAAG* 4

4.1.1	$0=\frac{-5}{2}x-4$ $4=\frac{-5}{2}x$ $8=-5x$ $x=-\frac{8}{5}$ $E\left(\frac{-8}{5};0\right)$	✓0 ✓ $-\frac{8}{5}$ (2)
4.1.2	$x^2+y^2=r^2$ $(0)^2+\left(-\frac{8}{5}\right)^2=r^2$ $r^2=\frac{64}{25}$ $x^2+y^2=\frac{64}{25}$ NB It was not given that AMB is a diameter. BUT the sketch is misleading. The option is therefor added to the memo, BUT it is actualluy INCORRECT to assume that AMB is a diameter.	✓substitution ✓ $r^2=\frac{64}{25}$ ✓answer (3)
4.2	CORRECT $M_{AE}=-\frac{5}{2}$ $M_{AM}=\frac{2}{5}$ (tan ⊥ *radii*) $y=\frac{2}{5}x+c$ $c=-4$ $y=\frac{2}{5}x-4$ $M_{AM}=\frac{-2-(-4)}{5-0}$ $=\frac{2}{5}$ $c=-4$ $y=\frac{2}{5}x-4$ xx ASSUMED	✓gradient of AM ✓ $y=\frac{2}{5}x+c$ ✓Answer (3)
4.3	$-3=\frac{2}{5}a-4$ CORRECT $1=\frac{2}{5}a$ $a=\frac{5}{2}$ OR $a=\frac{5+0}{2}$ xx ASSUMED $a=\frac{5}{2}$	✓substitution ✓simplification ✓Answer (3) OR ✓✓substitution ✓Answer (3)

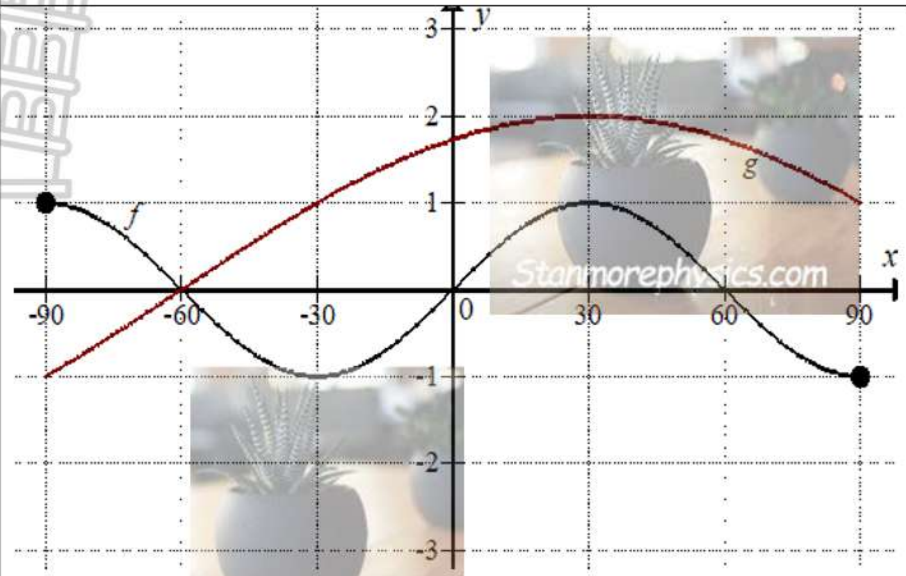
4.4	$\left(x-\frac{5}{2}\right)^2+\left(y-(-3)\right)^2=r^2$ $\left(x-\frac{5}{2}\right)^2+\left(y+3\right)^2=r^2$ $\left(0-\frac{5}{2}\right)^2+\left(-4+3\right)^2=r^2$ $r^2=\frac{29}{4}$ $\left(x-\frac{5}{2}\right)^2+\left(y+3\right)^2=\frac{29}{4}$ AO 1 out of 4	✓$\left(x-\frac{5}{2}\right)^2+\left(y+3\right)^2=r^2$ ✓Substitution ✓$r^2=\frac{29}{4}$ ✓Answer (4)
4.5	$\hat{G}=90^\circ$ $(\angle\ in\ semi-circle)$ $G\left(0\ ;\frac{8}{5}\right)$ $F\left(\frac{8}{5};0\right)$ $EG=\sqrt{\left(\frac{-8}{5}-0\right)^2+\left(\frac{8}{5}-0\right)^2}=\frac{8\sqrt{2}}{5}$ $GF=\frac{8\sqrt{2}}{5}$ Area of ΔEFG=$\frac{1}{2}\left(\frac{8\sqrt{2}}{5}\right)\left(\frac{8\sqrt{2}}{5}\right)$ $=\frac{64}{25}=2.56\ units^2$ OR Area of ΔEFG=$\frac{1}{2}$.EF.OG Area of ΔEFG=$\frac{1}{2}\left(2.\frac{8}{5}\right)\left(\frac{8}{5}\right)$ $=\frac{64}{25}=2.56\ units^2$	✓Angle G. ✓length of EG ✓length of GF ✓Substitution ✓Answer (5) OR ✓✓method ✓✓substitution ✓Answer (5)
		[20]

QUESTION/*VRAAG* 5

5.1.1	$\left(\sqrt{34}\right)^2=(3)^2+p^2$ $34-9=p^2$ $p=-5$	✓substitution ✓Answer (2)
5.1.2	$\cos\left(450^\circ-2\theta\right)$ $=\sin 2\theta$ $=2\sin\theta.\cos\theta$ First mark awarded here $=2\left(\frac{-5}{\sqrt{34}}\right)\left(\frac{3}{\sqrt{34}}\right)$ $=\frac{-30}{34}$ $=\frac{-15}{17}$	✓ $\sin 2\theta=2\sin\theta\cos\theta$ ✓substitution ✓Answer (3)
5.1.3	$\cos\left(30^\circ-\theta\right)$ $\cos 30^\circ.\cos\theta+\sin 30^\circ.\sin\theta$ $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{\sqrt{34}}\right)+\left(\frac{1}{2}\right)\left(\frac{-5}{\sqrt{34}}\right)$ $\frac{3\sqrt{3}-5}{\sqrt{34}}$ Answer $\frac{3\sqrt{102}-5\sqrt{34}}{34}$	✓Expansion ✓Substitution ✓Answer (3)
5.2	$\frac{-2\sin x.-\cos x}{\sin\left(x+60^\circ+x\right)}$ Question paper misleading...does not simplify to a single trigonometric function $\frac{2\sin x.\cos}{\sin\left(2x+60^\circ\right)}$ $\frac{\sin 2x}{\sin 2x.\cos 60^\circ+\cos 2x.\sin 60^\circ}$ $\frac{\sin 2x}{\frac{1}{2}\sin 2x+\frac{\sqrt{3}}{2}\cos 2x}$ $\frac{2\sin 2x}{\sin 2x+\sqrt{3}\cos 2x}$ $\frac{2\tan 2x}{\tan 2x+\sqrt{3}}$	✓ $-\sin x$ ✓ $-\cos x$ ✓compound angle ✓ $\sin 2x$ ✓ $\sin 2x.\cos 60^\circ+\cos 2x.\sin 60^\circ$ ✓Answer (6)

5.3.1	$f(x)=\cos(x+45^\circ).\cos(45^\circ-x)$ $=[\cos x.\cos 45^\circ-\sin x.\sin 45^\circ][\cos x.\cos 45^\circ+\sin x.\sin 45^\circ]$ $=\left[\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right]\left[\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right]$ $=\frac{1}{2}\cos^2 x-\frac{1}{2}\sin^2 x$ $=\frac{1}{2}(\cos^2 x-\sin^2 x)$ $=\frac{1}{2}\cos 2x$	✓first expansion ✓second expansion ✓$\frac{1}{2}\cos^2 x-\frac{1}{2}\sin^2 x$ ✓common factor (4)
5.3.2	$\frac{1}{2}\cos 2x=1-2\sin x$ $1-2\sin^2 x=2-4\sin x$ $2\sin^2 x-4\sin x+1=0$ $\sin x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$ $\sin x=\frac{2-\sqrt{2}}{2}=0,29$ *or* $\sin x=\frac{2+\sqrt{2}}{2}=1,71$ $x=17.03^\circ+k.360^\circ$ *no solution* $x=162.97^\circ+k.360^\circ$ $k\in Z$ CA is learner uses 0,293 etc	✓formulae ✓standard form ✓$\sin x$ value(s) ✓$x=17.03^\circ+k.360^\circ$ ✓$x=162.97^\circ+k.360^\circ$ ✓$k\in Z$ (6)
5.4	$\cos\theta=2m$ *and* $\cos 2\theta=7m$ $2\cos^2\theta-1=7m$ $2(2m)^2-1=7m$ $8m^2-7m-1=0$ $(8m+1)(m-1)=0$ $m=\frac{-1}{8}$ *or* $m=1$ $\therefore m=-\frac{1}{8}$	✓formulae ✓substitution ✓standard form ✓factors ✓Answer with selection (5)
		[29]

QUESTION/*VRAAG* 6

6.1	120°	✓Answer (1)
6.2		g ✓shape ✓x-intercept ✓turning point (3)
6.3.1	$x \in [-90^\circ; -60^\circ)$ *or* $-90^\circ \leq x < -60^\circ$	✓✓Answer (2)
6.3.2	$-90^\circ < x < 0^\circ$ *or* $60^\circ < x < 90^\circ$, $x \neq -60^\circ$ Incorrect , some equal signs omitted	✓ $-90^\circ < x < 0^\circ$ ✓ $60^\circ < x < 90^\circ$ ✓ $x \neq -60^\circ$ (3)
6.3.3	$x = 30^\circ$ Question paper asked wrong way round - the only answer available is a -1. Therefor...actually no solution **Note: Accept no solution**	✓Answer (1)
6.4	$h(x) = 2\cos(x - 30^\circ - 60^\circ)$ $= 2\cos(x - 90^\circ)$ $= 2\sin x$	✓ $2\cos(x - 90^\circ)$ ✓Answer (2)
		[12]

QUESTION/VRAAG 7

7.1	in ΔPQS $\cos 30^\circ = \frac{x}{PS}$ $\frac{\sqrt{3}}{2} = \frac{x}{PS}$ $PS = \frac{2x}{\sqrt{3}}$ in ΔPQR $\sin 30^\circ = \frac{x}{PR}$ $\frac{1}{2} = \frac{x}{PR}$ $PR = 2x$ in ΔPSR $PR^2 = RS^2 + PS^2 - 2RS.PS\cos\theta$ $(2x)^2 = \left(\frac{2x}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 - 2\left(\frac{2x}{\sqrt{3}}\right)\left(\sqrt{3}\right)\cos\theta$ $4x^2 = \frac{4}{3}x^2 + 3 - 4x\cos\theta \qquad \times 3$ $12x^2 = 4x^2 + 9 - 12x\cos\theta$ $12x\cos\theta = 9 - 8x^2$ $\cos\theta = \frac{9-8x^2}{12x}$	✓ $\cos 30^\circ = \frac{x}{PS}$ ✓ $PS = \frac{2x}{\sqrt{3}}$ ✓ $\sin 30^\circ = \frac{x}{PR}$ ✓ $PR = 2x$ ✓Substitution into Cosine rule ✓ $4x^2 = \frac{4}{3}x^2 + 3 - 4x\cos\theta$ (6)
7.2	Area of $\Delta PSR = \frac{1}{2}\left(\sqrt{3}\right)\left(\frac{2(1)}{\sqrt{3}}\right)\sin\theta$ $= \sin\theta$ units^2	✓✓Substitution into Area rule (2)
		[8]

QUESTION/*VRAAG* 8

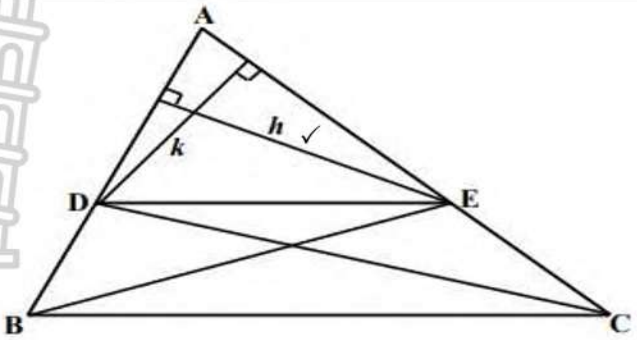
8.1	$\text{Q}\hat{\text{O}}\text{R} = 228^\circ$ ($\angle$ *around a point*) $Q\hat{T}R = 114^\circ$ ($\angle$ *at center* $= 2\angle$ *at circumference*) OR $R\hat{P}Q = 66^\circ$ ($\angle$ *at center* $= 2\angle$ *at circumference*) $Q\hat{T}R = 114^\circ$ (*opp* $\angle$*s of cyclic quad*)	✓S ✓S ✓R OR ✓S✓R ✓S (3)
8.2	$O\hat{Q}S = 90^\circ$ (tan $\perp$ *radii*) $O\hat{R}S = 90^\circ$ (tan $\perp$ *radii*) $\therefore$ QORS is a cyclic quadrilateral (*opp* $\angle$*s* are supplementary) OR (converse of opp $\angle$*s* of cyclic quad)	✓S/R ✓S/R ✓R (3)
8.3	$\text{Q}\hat{\text{S}}\text{R} = 48^\circ$ (Opp $\angle$*s* of cyclic quad) OR (sum of $\angle$*s* of quad)	✓S ✓R (2)
		[8]

QUESTION/*VRAAG* 9

9.1	$c = x$ (tan−*chord theorem*) S R $E_1 = 180^\circ - (x+y)$ (*sum of* $\angle s$ *in* Δ) S/R $E_3 = y$ ($\angle s$ *in str line*) / *ext* $\angle$ *of* ΔDEC S/R $A\hat{B}E = y$ ($\angle s$ *opp* = *sides*) S/R $D_1 = A\hat{B}E = y$ $\therefore$ ABED is a cylic quad (converse of ext $\angle$ of cyclic quad) R	✓S✓R ✓S/R ✓S/R ✓S/R ✓R (6)
9.2	$\hat{B}_1 = x$ ($\angle s$ *in same seg*) S R $\hat{B}_1 = \hat{C} = x$ AB is a tangent to a circle B,C and D (converse of tanchord theorem) *or* ($\angle$ between a line and chord) R	✓S ✓R ✓R (3)
		[9]

QUESTION/*VRAAG* 10

10.1



✓construction

Construct height h and k perpendicular to AD and AE respectively. Join DC and BE.

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2}AD.h}{\frac{1}{2}.BD.h} \checkmark \quad (\text{Same height } h)$$

$$= \frac{AD}{BD}$$

✓S

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED} = \frac{\frac{1}{2}AE.k}{\frac{1}{2}.EC.k} \checkmark \quad (\text{Same height } k)$$

$$= \frac{AE}{EC}$$

✓S

But Area of ΔBDE=Area of ΔCED ✓ (same base DE,same height between ∥ lines.) ✓

✓S ✓R

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED} \checkmark$$

✓S

$$\therefore \frac{AD}{BD} = \frac{AE}{EC}$$

(6)

10.2	$\frac{AD}{DB}=\frac{AF}{FC}$ (Prop theorem $DF \parallel BC$) $AF=\frac{AD.FC}{DB}$ From $\frac{AF}{FE}=\frac{FC}{EB}$ $AF=\frac{FE.FC}{EB}$ $\frac{FE.FC}{EB}=\frac{AD.FC}{DB}$ $\frac{FE}{EB}=\frac{AD}{DB}$ $\therefore DE \parallel AF$ (Converse of prop theorem)	✓S ✓R ✓ $AF=\frac{AD.FC}{DB}$ ✓ $AF=\frac{FE.FC}{EB}$ ✓Equating ✓R (6)
		[12]

QUESTION/*VRAAG* 11

The word "TANGENT" DE was omitted on ENGLISH side

11.1	$F\hat{G}E = \theta$ (*Tan – chord theorem*) $\hat{E}_2 = \theta$ ($\angle s$ *opp = sides*) $\hat{F}_2 = \theta$ (*Alt* $\angle s$, $DE \parallel FH$)	✓S ✓S ✓S (3)
11.2	In ΔDEF and ΔDGE ✓ $\hat{D} = \hat{D}$ common ✓ $\hat{E}_1 = \hat{G}$ tan chord theorem or (proved in 11.1) ✓ $\hat{F}_3 = \hat{E}_1 + \hat{E}_2$ (Sum of $\angle s$ in Δ) ΔDEF /// ΔDGE (AAA) ✓ $\frac{DE}{DG} = \frac{EF}{GE} = \frac{DF}{DE}$ ✓ (from /// Δs) $DE^2 = DF.DG$	✓identifying triangles ✓S/R ✓S ✓R(AAA) ✓S (5)
11.3	$\frac{DF^2}{DE^2} + \frac{DF}{DE} = \frac{DF^2 + DF.DE}{DE^2}$ $= \frac{DF(DF+DE)}{DE^2}$ $= \frac{DF(DF+FG)}{DF.DG}$ ($DE = FG$) $= \frac{DF+FG}{DG}$ ($DE^2 = DF.DG$) $= \frac{DG}{DG}$ ($DG = DF.FG$) $= 1$ OR $\frac{EF}{GE} = DE^2 = DF.DG$ $= DF.(DF + FG)$ $= DF^2 + DF.FG$ $= DF^2 + DF.DE$ $\frac{DE^2}{DE^2} = \frac{DF^2}{DE^2} + \frac{DF.DE}{DE^2}$ $1 = \frac{DF^2}{DE^2} + \frac{DF}{DE}$	✓ $\frac{DF^2 + DF.DE}{DE^2}$ ✓common factor ✓S ✓S (4) ✓ $DF.(DF + FG)$ ✓ $DF^2 + DF.FG$ ✓ $DF^2 + DF.DE$ ✓Dividing both sides by DE^2 (4) **[12]**
		TOTAL: 150