KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

PINETOWN DISTRICT

TEACHING AND LEARNING SUPPORT – CURRICULUM FET (GRADES 10-12)

Grade 10 Mathematics Teacher Support Document

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
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1. ALGEBRA

1.1 Algebra part 1: Expressions

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2. Euclidean Geometry


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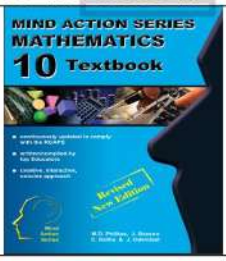
TOPIC: ALGEBRA PART 1 (Lesson 1)	Weighting	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		

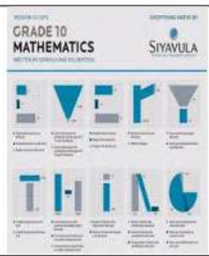
Sub-topics	**REAL NUMBERS: Rational and Irrational Numbers**
RELATED CONCEPTS/ TERMS/VOCABULARY	**Terminating decimals and recurring decimals** **Non- terminating and non-recurring**

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Natural numbers (N), Whole numbers (No), Integers (Z), fractions

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Integers are negative numbers
- Integers are not rational number
- Negative numbers are non-real numbers

METHODOLOGY

REAL NUMBERS

-are all rational and irrational numbers put together.

1. Rational Numbers

A rational number(Q) is a number that can be expressed in the form $\frac{a}{b}$ where $b \neq 0$ and where a and b are integers.

Examples:

a) Integers

e.g. 5 can be written as $\frac{5}{1}$ where 5 and 1 are integers.

b) Mixed fractions

e.g. $2\frac{3}{7}$

c) Terminating decimals

e.g. $0{,}25 = \frac{25}{100} = \frac{1}{4}$

d) Recurring decimals have an infinite pattern & can be expressed as a fraction

e.g. 0, 3 = 0, 3333333
0, 12 = 0, 12121212

Converting Recurring Decimals to Fractions

Examples:

a) Show that $0,\dot{3}$ is rational.

Let $x = 0,333333....$ (eqn 1)

$10x = 3,333333...$ (eqn 2) (multiply both sides by 10, one digit rec)

(eqn 2 - eqn 1) $10x - x = 3,33333 - 0,33333$ (subtract equations)

$9x = 3,0000000$

$9x = 3$

$x = \frac{1}{3}$ … a rational number!

b) Show that $1,\dot{7}\dot{5}$ is a rational number

Let $x = 1,75757575......$(eqn 1

$100x = 175,75757575...$(eqn 2) (multiply by both sides by 100, two digits recurring)

(Eqn 3 – eqn 1): $100x - x = 175,75757575 - 1,75757575$

$99x = 174$

$x = \frac{174}{99}$ … a rational number!

2. Irrational number(Q/)

- Numbers that cannot be written in the form $\frac{a}{b}$ where $b \neq 0$
- Therefore, recurring numbers that neither terminate nor recur with a pattern

Examples:

a) 5,739129…
b) -4,883291103…
c) π

π is = 3,142857143….

However, π can be approximated as an improper fraction $\frac{22}{7}$

ACTIVITIES/ ASSESSMENT

1. Given the numbers: -3; $\frac{3}{4}$; $\sqrt{2}$; $\sqrt{9}$; 0; 2; $\sqrt{-4}$. Write down all:

a) rational numbers

b) irrational numbers

c) real numbers

2. Show that the decimals below are rational.

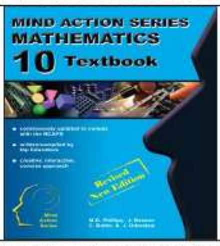
a) $0,\dot{4}$ b) $0,\dot{2}\dot{1}$ c) $0,1\dot{4}$ d) $0,\dot{1}2\dot{4}$ e) $-1,1\dot{2}\dot{4}$

TOPIC: ALGEBRA PART 1 (Lesson 2)	**Weighting** 30 ± 3	Grade	10
Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Surds lie between Integers**		
RELATED CONCEPTS/ TERMS/VOCABULARY	**Rational and Irrational numbers** **Inequality signs**		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Integers, Greater than sign, Less than sign, Square root, Cube root, etc.

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Integers are negative numbers
- Integers are not rational number
- Negative numbers are non-real numbers

METHODOLOGY

A **Surd** is an expression that include a square root, cube root, or any other root symbol.
Surds are used to write irrational numbers.

Examples:

1. **Determine <u>without</u> the use of a calculator, between which 2 consecutive integers $\sqrt{11}$ lies. Consecutive integers are two integers that follow one another on the number line, e.g. 2 and 3, 3 and 4, 4 and 5, 5 and 6, etc.**

- **Find an integer smaller and bigger than 11 that can be squared rooted … 9 and 16**
- **Now create an inequality … $9 < 11 < 16$**
- **Square root all integers … $\sqrt{9} < \sqrt{11} < \sqrt{16}$**
- **Solve … $3 < \sqrt{11} < 4$**

ACTIVITIES/ ASSESSMENT

1. Determine between which two integers $\sqrt{16}$ lies.

2. Between which two integers does $\sqrt[3]{5}$ lie?

3. Write down two consecutive integers such that $-\sqrt{12}$ lies between them.

4. Without using a calculator, determine between which two integers the following irrational numbers lie. Then verify your answers by using a calculator.

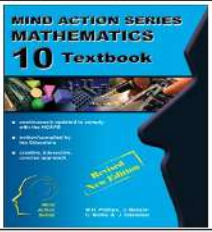
(1) $\sqrt{50}$ (2) $\sqrt{29}$ (3) $\sqrt[3]{45}$ (4) $-\sqrt{54}$ (5) $\sqrt[3]{30}$ (6) π

TOPIC: ALGEBRA PART 1 (Lesson 3)	Weighting	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Rounding Decimal Numbers to an Appropriate degree of accuracy**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Decimal place, digit			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Rational and Irrational Numbers

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Integers are not rational number
- Negative numbers are non-real numbers

METHODOLOGY

Rounding off a decimal number to a given number of decimal places is the quickest to approximate a number.

Examples:

1. Round off 2,6525272 to 3 decimal places

Count 3 spaces after the decimal ("coma") and put a vertical line (|) between the 3rd and the 4th digit.
Add 1to the 3rd digit (round up) if the 4th digit is equal to 5 or greater than 5
Leave the 3rd digit unchanged if the fourth digit is less than 5.
If the 3rd digit is 9 and needs to be rounded up, the 9 becomes a 0 and add 1 to the 2nd digit

In 2,6525272, the 3rd digit is 2 and the 4th digit is 5
Therefore, add 1 to the 3rd digit to be 3
Answer is 2,653

2. Round off $\sqrt{3}$ to 2 decimal places

If the number is not in decimal form, first write it as a decimal.
$\sqrt{3} = 1{,}7320508$
2nd digit is 3 and 3rd digit is 2 which is less than 5
Answer is 1,73

3. Round off $1{,}\dot{1}\dot{2}$ to 3 decimal places

$1{,}\dot{1}\dot{2} = 1{,}12121212\ldots$
Answer is 1,121

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4. Round off $\frac{1}{5}$ to 2 decimal places

$\frac{1}{5} = 0{,}2$

If there not enough number of digits after the decimal, add 0

Answer is 0,20

ACTIVITIES/ ASSESSMENT

Round off the following numbers to the number of decimal places indicated:

1. 12,07963 (3 decimal places)

2. $\sqrt{5}$ (2 decimal places)

3. 9,998 (1 decimal places)

4. $0{,}2\dot{6}$ (2 decima places)

5. π (4 decimal places)

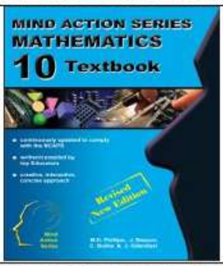
TOPIC: ALGEBRA PART 1 (Lesson 4)	Weighting	30 ± 3	Grade	10

Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Multiplication of Binomial by Trinomial**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Expression, coefficient, constant, exponent, like terms, unlike terms		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Monomial, binomial, trinomial, base, variable, meaning of brackets, operation signs

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- $x + y = xy$
- $(x + y)^2 = x^2 + y^2$

METHODOLOGY

In the expression **2*x* + 3*y* + 6**: 2*x*, 3*y*, and 6 are called *terms*. A <u>term</u> can be a number, a variable, or a product of numbers and variables. Terms in an expression are separated by + and –.

In the term $2x$, 2 is called the *coefficient*. A <u>coefficient</u> is a number that is multiplied by a variable (x) in an algebraic expression.

<u>Like terms</u> are terms with the same variable raised to the same power. The coefficients do not have to be the same.

Like Terms	$12x$ and $-2x$	a and $\frac{2a}{7}$	5 and 1.8
Unlike Terms	x^2 and $2x$ *The exponents are different*	$3x$ and $6y$ *The variables are different*	7 and x *Only one term contains a variable*

1. **Multiplying a monomial and a binomial**: $a(b + c) = ab + ac$ (distributive law)

A **monomial** is an expression with one term

A **binomial** is an expression with two terms

Example:

Multiply: $-2ab(2c - 3d)$

$= (-2ab)(2c) + (-2ab)(-3d)$

$= -2abc + 6abd$

2. **Multiplying two binomials**

To multiply a binomial by a binomial, you can apply the Distributive Property more than once:

Example:

Find the product of $(x+4)(x-3)$

$= x(x-3) + 4(x-3)$ …………..distribution

$= (x)(x) + (x)(-3) + 4(x) + 4(-3)$ multiply

$= x^2 - 3x + 4x - 12$ ……………….combine like terms

$= x^2 + x - 12$

Another method for multiplying binomials is called the **FOIL** method.

- Multiply the **F**irst terms. $x.x = x^2$
- Multiply the **O**uter terms. $x.-3 = -3x$
- Multiply the **I**nner terms. $4.x = 4x$
- Multiply the **L**ast terms. $4.-3 = -12$

$= x^2 - 3x + 4x - 12$……combine like terms

$= x^2 + x - 12$

<u>Box Method</u>.

Multiply $(2x-3)(4x+1)$

Draw a box.
Write a polynomial on the top and side of a box.
It does not matter which goes where.

	$2x$	-3
$4x$	**F**irst: $8x^2$	**I**nner: $-12x$
1	**O**uter: $+2x$	**L**ast: -3

Combine Like Terms $(-12x + 2x)$
$= 8x^2 - 10x - 3$

3. **Multiplying a binomial and a trinomial**

A **trinomial** is an expression with three terms.

To multiply polynomials with more than two terms, you can use the Distributive Property several times.

Example

Find the product of $(2x+5)(3x^3+4x-1)$

$= 2x(3x^2 + 4x - 1) + 5(3x^2 + 4x - 1$

$= 6x^3 + 8x^2 - 2x + 15x^2 + 20x - 5$

$= 6x^3 + 8x^2 + 15x^2 - 2x + 20x - 5$ …combine like terms

$= 6x^3 + 23x^2 + 18x - 5$

ACTIVITIES/ ASSESSMENT

Expand and Simplify;

1. $(x-2)(x+5)$

2. $(x-2y)(x^2 - 2xy + 3y^2)$

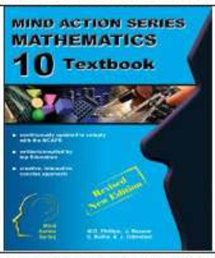
3. $(a-3b)(a-3b)^2$

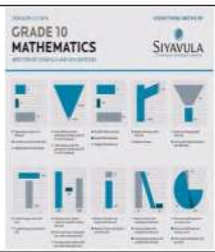
TOPIC: ALGEBRA PART 1 (Lesson 5)		**Weighting**	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**FACTORISATION: Common factor and Difference of TWO squares**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Polynomial, Perfect square				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Highest Common Factor, Expression, Brackets

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- If the expression is of 3rd degree, learners treat 3rd degree term as the highest common factor
- Put minus (-) on both brackets when factorising difference of two squares

METHODOLOGY

Factorisation is the opposite process of expanding brackets.
In any factorising problem, the first step is to look for the *highest common factor*.
The **highest common factor**, abbreviated HCF, is an expression of the highest degree that divides each term of the polynomial.

Common Factor

Examples:

1. $5m^3 - 10m^2 + 15m = 5m(m^2 - 2m + 3)$

2. $2a^2b + 3a^2b^2 - 5abc = ab(2a + 3ab - 5c)$

3. $a(m - n) - b(m - n) = (m - n)(a - b)$

4. $3x(2a - 1) - 5(1 - 2a) = 3x(2a - 1) + 5(2a - 1) \ldots$ $[(\mathbf{1 - 2a}) = -(\mathbf{2a - 1})]$
$= (2a - 1)(3x + 5)$

Difference of two squares

To spot a difference of two squares, look for expressions:

- consisting of two terms;
- with terms that have different signs (one positive, one negative);
- with each term a perfect square.

Therefore $(ax + b)(ax - b)$ can be expanded to $a^2x^2 - b^2$
$a^2x^2 - b^2$ can therefore be factorised as $(ax + b)(ax - b)$

Examples:

1. $x^2 - 9 = (x + 3)(x - 3)$

2. $4x^2 - 25 = (2x + 5)(2x - 5)$

3. $16x^2 - 9y^2$ =(4x+3y)(4x-3y)

4. $81 - z^2 = (9 + z)(9 - z)$

5. $32a^2 - 98) = 2(16a^2 - 49)$
$= 2(4a + 7)(4a - 7)$

ACTIVITIES/ ASSESSMENT

Factorise fully:

1. $12x + 32y$

2. $2xy^2 + xy^2z + 3xy$

3. $4(y - 3) + k(3 - y)$

4. $4x^2 - 1$

5. $49x^4 - 16$

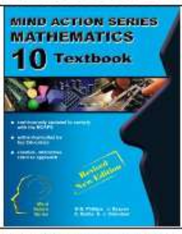
6. $16k^2 - (b - 5)^2$

TOPIC: ALGEBRA PART 1 (Lesson 6)	Weighting 30 ± 3	Grade	10
Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**FACTORISATION: Quadratic Trinomial**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Factors		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trinomial, Expression, Brackets

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Finding factors of the third term without considering the second term
- Finding numbers that sum to third term instead of product od numbers.

METHODOLOGY

A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where x is a variable and a, b and c are non-zero constants.

Quadratic Trinomial when $\boldsymbol{a} = \mathbf{1}$: $(x^2 + bx + c)$
To factorise $x^2 + bx + c$
Open two Brackets with x in front of each: (x)(x)
Find factors of **c** that will add up to **b**

Examples:

1.Factorise $x^2 + 5x + 6$
(x)(x)
Factors of 6 that will add up to 5 are **2 and 3.**
2 x 3 = 6 and 2 + 3 = 5
Then use 2 and 3 to factorise.
$x^2 + 5x + 6 = (x + 2)(x + 3)$

2. $x^2 + 2x - 8$
(x)(x)
Factors of -8 will that add up to +2 are **+4 and -2**
+4 x -2 = -8 and +4 + (– 2) = +2
Then use 4 and -2 to factorise
$x^2 + 2x - 8 = (x + 4)(x - 2)$

3. Factorise $3x^2 - 21x - 24$
$= 3(x^2 - 7x - 8)$ … first take out a common factor
$= 3(x \quad)(x \quad)$
Factors of -8 that will add up to -7 are **-8 x 1**
-8 x 1 = -8 and -8 + 1 = -7

$= 3(x + 1)(x - 3)$

Quadratic Trinomial where a ≠ 1

To factorise $ax^2 + bx + c$

Multiply **a** and **c** and find the factors of **ac** that will add to **b**

Can factorise by grouping terms (easiest method)

Examples:

1. Factorise $2x^2 + x - 6$

a x c = 2 x -6 = -12 and b = 1

Possible Factors of -12 that will add up to 1 are 2 and -6, -2 and 6, **-3 and 4**, 3 x -4

$$\mathbf{1x = -3x + 4x}$$

$= 2x^2 - 3x + 4x - 6$	Substitute $-3x + 4x$ *in plce of* $1x$
$= (2x^2 - 3x) + (4x - 6)$	Group terms in twos
$= x(2x - 3) + 2(2x - 3)$	Take out common factors in each group
$= (2x - 3)(x + 2)$	Take out a common factor

2. Factorise $4x^2 - 19x + 12$

a x c = 4 x 12 = 48 and b = -19

Possible Factors of 48 to give a sum of -19 will be negative: -2 and -24, **-3 and -16**, -4 and -12

$$\mathbf{-19x = -3x + (-16x)}$$

$$4x^2 - 19x + 12 = 4x^2 - 3x - 16x + 12$$
$$= x(4x - 3) - 4(4x - 3)$$
$$= (4x - 3)(x - 4)$$

- **If the sign of the last term of a trinomial is positive and the middle term is positive, the signs in the brackets will be positive. i.e. (… + …)(… + …)**
- **If the sign in the last term of a trinomial is positive and the middle term is negative, the signs in the brackets will be negative. i.e. (… - …)(… - …)**
- **If the sign of the last term of a trinomial is negative, the signs in the brackets are different, i.e. both positive or both negative. i.e. (… + …)(… - …) or (… - …)(… + …)**

ACTIVITIES/ ASSESSMENT

Factorise Fully

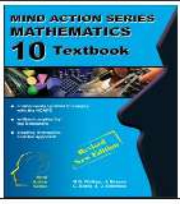
1. $x^2 + 8x + 15$
2. $2x^2 + 5x - 3$
3. $4x^2 + 10x - 6$
4. $2x^2 - 22x + 20$
5. $6x^2 - 15x - 9$
6. $4p^2 + 7pq - 2p^2$
7. $10m^2 13mn - 3n^3$
8. $a^3 - 6a^2b + 9ab^2$
9. $(a + b)^2 + 8(a + b) - 32$

TOPIC: ALGEBRA PART 1 (Lesson 7)		**Weighting**	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**FACTORISATION: Grouping in Pairs**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Similar terms, common bracket				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Common factor

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Taking out a negative: $-x + y = x - y$

METHODOLOGY

Group terms with common factors or with similar brackets.

$ax + ay + px + py$

- Group terms that look similar ((i.e., those that could potentially have common factors)
  $(ax + ay) + (px + py)$
- Factorize each pair separately:
  $a(x + y) + p(x + y)$
- and then take out the common bracket
  $(x + y)(a + p)$

Swapping around or "taking out a negative"

$y - x = -(x - y)$ and $-x - y = -(x + y)$

Examples

Factorise fully:

1. $5x + 10y - ax - 2ay)$

$= (5x + 10y) + (-ax - 2ay)$

$= 5(x + 2y) - a(x + 2y)$

$= (x + 2y)(5 - a)$

2. $p^3 - 3p^2 - p + 3$

$= (p^3 - 3p^2) + (-p + 3)$ OR $(p^3 - p) + (-3p + 3)$

$= p^2(p - 3) - (p - 3)$

$= (p - 3)(p^2 - 1)$

$= (p - 3)(p - 1)(p + 1)$ difference of two squares

ACTIVITIES/ ASSESSMENT

Factorise the following fully:

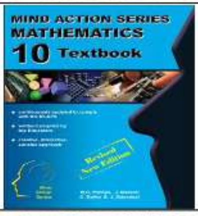
1. $a^2 + a - 6ax - 6x$
2. $6m^2 + 3m - 6p - 12mp$
3. $45q - 18z + 5cq - 2cz$
4. $6x^3 - 2x^2 - 54x + 18$
5. $3ax - 3ay - x + y$
6. $3ax + 3ay - x - y$
7. $x^3 - 2x^2 - 2x + 4$
8. $x^3 + 2x^2 - 2x - 4$
9. $2x^3 - 3x^2 - 6x + 9$
10. $3x^4 - 3x^2 - 27x^2 + 27$
11. $a - b + ab - 1$
12. $6a^2px - 4ap^2y - 6a^2py + 4ap^2x$

TOPIC: ALGEBRA PART 1 (Lesson 8)	Weighting	30 ± 3	Grade	10	
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**FACTORISATION: Sum and Difference of Cubes**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Cube, Sum, Difference, Brackets, factors				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Binomial, Trinomial, product

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- $x^3 + y^3 = (x + y(x^2 + xy + y^2)$
- $x^3 - y^3 = (x - y)(x^2 - xy + y^2)$

METHODOLOGY

Sum of two Cubes

$x^3 + y^3$

To factorise:

- Open two brackets
- First bracket must have two term, cube root of each term: $(x + y)$
- Second bracket must have three terms: $(x^2 - xy + y^2)$
  Multiply the factors of the first bracket to get the middle term of the second bracket
  The sign of the middle term is always negative when factorizing a sum of two cubes.

Therefore, $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Difference of two Cubes

$x^3 - y^3$

To factorise:

- First bracket, cube root of each term: $(x - y)$
- Second bracket, three terms with middle term the being the product of factors of the first term and the sign of the middle term is always negative. $(x^2 + xy + y^2)$

Therefore, $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Examples:

Factorise the following:

1. $x^3 + 8$

$= x^3 + 2^2 = (x + 2)(x^2 - 2x + 4)$

2. $8a^4 - 64a$

$= 8a(a^3 - 8)$

$= 8a(a - 2)(a^2 + 2a + 4)$

2. $\frac{1}{27}x^3 - 216$

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$= \left(\frac{1}{3}\right)^3 x^3 - 6^3 = (\frac{1}{3}x - 6)(\frac{1}{9}x^2 + 2x + 36$

ACTIVITIES/ ASSESSMENT

Factorise Fully

1. $a^3 - 125$
2. $g^3 + 64$
3. $64x^3 + y^3$
4. $2x^3 - 2y^2$
5. $-x^3 - 27$
6. $27x^3y^3 + w^3$
7. $8 - (a-1)^3$
8. $1 - (x-y)^3$
9. $x^3 + \frac{1}{x^3}$
10. $\frac{1}{64q^3} - h^3$

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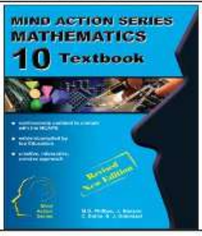
TOPIC: ALGEBRA PART 1 (Lesson 9)		**Weighting**	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics ;	**Simplification of Algebraic Fractions Using Factorisation**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Simplify, Factorise				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Factorisation: Common factor, Difference of two squares, Trinomial, Sum and Difference of Cubes

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

$2-x=x-2$

Open two brackets for $4x^2-12x$

METHODOLOGY

Note that:

$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}\quad (b\neq 0;\ d\neq 0)$

$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}\quad (b\neq 0; c\neq 0; d\neq 0)$

Examples:

Simplify the following:

1. $\frac{12x^3y+3xy}{9y^2}$
$=\frac{3xy(4x^2+1)}{9y^2}$
$=\frac{x(4x^2+1)}{3y}$

2. $\frac{x^3-12x+27}{4x^2-12x}$
$=\frac{(x-3)(x-9)}{4x(x-3)}$
$=\frac{x-9}{4x}$

3. $\frac{27p^3-8}{27p^2+18p+12}$
$=\frac{(3p-2)(9p^2+6p+4)}{3(9p^2+6p+4)}$
$=\frac{3p-2}{3}$

ACTIVITIES/ ASSESSMENT

Simplify the following algebraic fraction:

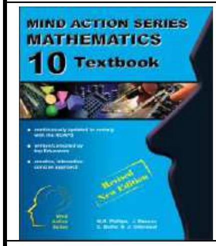
1. $\frac{4a^2-8a}{4a}$
2. $\frac{4m^2+4m}{4m^2}$
3. $\frac{9p^3-81p^2}{6p^2}$
4. $\frac{6+9k}{9k^2-4}$
5. $\frac{2x^2-5x-3}{2x-6}$
6. $\frac{x^3-8}{2-x}$

TOPIC: ALGEBRA PART 1 (Lesson10)	Weighting	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Simplification of Algebraic Fractions Using Factorisation**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Simplify, Factorise			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Factorisation: Common factor, Difference of two squares, Trinomial, Sum and Difference of Cubes

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Swapping denominator and numerator when changing $\div$ to $\times$

METHODOLOGY

Examples:

Factorise the following algebraic fractions:

1. $\frac{x-2}{x^2+4x-5} \div \frac{x^2-4}{x^2+5x}$

$= \frac{x-2}{x^2+4x-5} \times \frac{x^2+5x}{x^2-4}$

$= \frac{x-2}{(x+5)(x-1)} \times \frac{x(x+5)}{(x-2)(x+2)}$

$= \frac{x}{(x-1)(x+2)}$

2. $\frac{x^2-4xy+4y^2}{2x-4} \div \frac{x-2y}{x-2} \times \frac{x^2-4}{4}$

$= \frac{x^2-4xy+4y^2}{2x-4} \times \frac{x-2}{x-2y} \times \frac{x^2-4}{4}$

$= \frac{(x-2y)(x-2y)}{2(x-2)} \times \frac{x-2}{x-2y} \times \frac{(x-2)(x+2)}{4}$

$= \frac{(x-2y)(x-2)(x+2)}{8}$

ACTIVITIES/ ASSESSMENT

Factorise the following algebraic fractions:

1. $\frac{x^2+x-6}{2x-8} \times \frac{x^2-16}{x^2-2x}$

2. $\frac{x^2+x}{x^2} \div \frac{x^2+2x=1}{x^2}$

3. $\frac{6+k-k^2}{3+3k} \div (k-3) \div \frac{2k+4}{k^2-1}$

4. $\frac{2k^2+k-6}{2k^2-6} \times \frac{k^4-9}{k+2} \div (2k-3)$

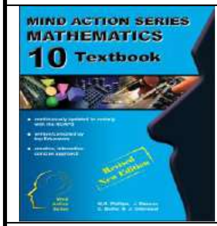
5. $\frac{14-7x}{x^2+x-2} \times \frac{x+2}{3x-6}$

TOPIC: ALGEBRA PART 1 (Lesson 11)	Weighting	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Addition and Subtraction of Algebraic Fractions with Denominator of Degree at Most 3.**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Factorisation, simplification			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Addition and Subtraction of fractions, Lowest Common Denominator (LCD)

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Learners do not continue with the denominator

METHODOLOGY

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad (b \neq 0)$

Whenever you add or subtract algebraic fraction, the first thing you need to do is to determine the multiples of the denominators (LCD).

LCD

- Product of all denominators
- If denominator needs to be factorised, first factorise
- Do not repeat the denominator
- In denominators with same variable and different exponent, take variable with highest exponent

Examples:

Simplify the following:

1. $\frac{2x}{3} + \frac{1}{6}$

LCD: 6 (6 is divisible by both 3 and 6)

$= \frac{2x}{3} \times \frac{2}{2} + \frac{1}{6}$ …denominators in each term must be the same (same as LDC)

$= \frac{4x}{6} + \frac{1}{6}$

$= \frac{4x+1}{6}$ …keep the denominator and add the numerators

2. $1 + \frac{1}{4x^2y} - \frac{(x-2)}{3x^3}$

4 does not divide 3 and 3 does not divide 4

LCD: $12x^3y$ (product of 4 and 3, variable with highest exponent and variable appearing once)

$= \frac{1}{1} \times \frac{12x^3y}{12x^3y} + \frac{1}{4x^2y} \times \frac{3x}{3x} - \frac{(x-2)}{3x^3} \times \frac{4y}{4y}$ …denominators in each term must be the same as LCD

$= \frac{12x^3y}{12x^3y} + \frac{3x}{12x^3y} - \frac{(4xy-8y)}{12x^3y}$

$= \frac{12x^3y+1-4xy+8y}{12x^3y}$ …keep the denominator and add the numerators

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3. $\frac{3}{x+3} - \frac{2}{x^2+3}$

LCD: $(x+3)(x^2+3)$

$= \frac{3}{(x+3)} \times \frac{(x^2+3)}{(x^2+3)} - \frac{2}{(x^2+3)} \times \frac{(x+3)}{(x+3)}$ …denominators in each term to be the same as the LCD

$= \frac{3(x^2+3)}{(x+3)(x^2+3)} - \frac{2(x+3)}{(x+3)(x^2+3)}$

$= \frac{3x^2+9-(2x+6)}{(x+3)(x^2+3)}$ …keep the denominator and add the numerators

$= \frac{3x^2+9-2x-6}{(x+3)(x^2+3)}$ …add like terms

$= \frac{3x^2-2x+3}{(x+3(x^2+3)}$

ACTIVITIES/ ASSESSMENT

Simplify the following:

1. $\frac{x-3}{3} - \frac{x+5}{4}$

2. $\frac{3}{x} + \frac{2}{x^2}$

3. $1 + \frac{3}{2xy} + \frac{7x+1}{4x^2y}$

4. $\frac{x+2}{x^2+2} - \frac{6}{x+2}$

5. $x - \frac{2x}{3x-2}$

6. $\frac{5}{2x} - \frac{x+1}{x-2}$

7. $\frac{2x-1}{x+3} - \frac{x+2}{x-3}$

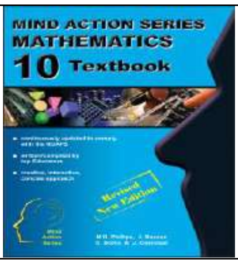
8. $\frac{4}{(2x+1)^2} - \frac{x+1}{2x+1}$

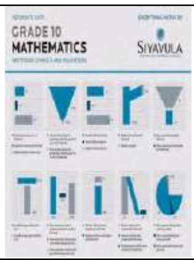
TOPIC: ALGEBRA PART 1 (Lesson 12)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Addition and Subtraction of Algebraic Fractions with Denominator of Degree at Most 3.**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Like terms, numerator, denominator				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Addition and Subtraction of fractions, Lowest Common Denominator (LCD)
Factorisation

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Determining the LCD without factorising the denominators
Inability of determining the LCD

METHODOLOGY

Factorise denominators before finding LCD.

Examples:

Simplify fully.

1. $\frac{x-1}{x^2+4x} - \frac{2x+1}{4x}$

$= \frac{x-1}{x(x+4)} - \frac{2x+1}{4x}$ …factorise the denominator

LCD: $4x(x+4)$

$= \frac{x-1}{x(x+4)} \times \frac{4}{4} - \frac{2x+1}{4x} \times \frac{(x+4)}{(x+4)}$ …denominators in each term to be the same as LCD

$= \frac{(4)(x+1)}{4x(x+4)} - \frac{(2x+1)(x+4)}{4x(x+4)}$

$= \frac{4x+4}{4x(x+4)} - \frac{(2x^2+8x+x+4)}{4x(x+4)}$ …multiply on the numerator

$= \frac{4x+4-2x^2-8x-x-4}{4x(x+4)}$ …keep the denominator and simplify the numerator

$= \frac{-2x^2+4x-8x-x+4-4}{4x(x+4)}$ …group like terms on the numerator

$= \frac{-2x^2-5x}{4x(x+4)}$

2. $\frac{5}{x^2-x-6}+\frac{3x}{3-x}+\frac{4}{2+x}$

$2+x=x+2$ *but* $3-x\neq x-3$

$=\frac{5}{(x+2)(x-3)}+\frac{3x}{-(x-3)}+\frac{4}{(x+2)}$ … $3-x=-(x-3)$

$=\frac{5}{(x+2)(x-3)}-\frac{3x}{(x-3)}+\frac{4}{(x+2)}$ …LCD: $(x+2)(x-3)$

$=\frac{5}{(x+2)(x-3)}-\frac{3x}{(x-3)}\times\frac{(x+2)}{(x+2)}+\frac{4}{(x+2)}\times\frac{(x-3)}{(x-3)}$ …denominators in each term to be the same as the LCD

$=\frac{5}{(x+2)(x-3)}-\frac{3x(x+2)}{(x-3)(x+2)}+\frac{4(x-3)}{(x+2)(x-3)}$

$=\frac{5}{(x+2)(x-3)}-\frac{3x^2+6x}{(x+2)(x-3)}+\frac{4x-12}{(x+2)(x-3)}$ …multiply numerators

$=\frac{5-3x^2-6x+4x-12}{(x+2)(x-3)}$ …keep the denominator and add the numerators

$=\frac{-3x^2-6x+4x+5-12}{(x+2)(x-3)}$ …group like terms on the numerator

$=\frac{-3x^2-2x-7}{(x+2)(x-3)}$

ACTIVITIES/ ASSESSMENT

Simplify the following as far as possible.

1. $\frac{x}{x+3}-\frac{2}{3+x}$
2. $\frac{x}{x-2}+\frac{1}{2-x}$
3. $\frac{x}{4-x}-\frac{2x-24}{x^2-4x}$
4. $\frac{2x}{x-3}+\frac{x+1}{9-x^2}$
5. $\frac{x+4}{x^2-2x}-\frac{x}{(2-x)^2}+\frac{x+2}{x-2}$
6. $\frac{x+2}{4x^2-2x+1}-\frac{8x+1}{8x^3+1}$

TEST: ALGEBRAIC EXPRESSIONS

Marks: 25 Duration: 30 Min

QUESTION 1[9 Marks]

1. Consider: $A = \sqrt{\frac{9}{11-x}}$

   If $x \in \{-14;\ -11;\ -5; 0; 5; 11; 14\}$, which value(s) of x will make A:

1.1 Rational (1)
1.2 Irrational (1)
1.3 Undefined (1)
1.4 Non-real (1)

2. Determine between which two consecutive integers does $\sqrt{11}$ lie. (2)

3. Write $0,\dot{7}$ as a common fraction. Clearly show all your working. (3)

[9]

QUESTION 2 [6 Marks]

1. Determine the product of the following and simplify fully:

$(x-2)(x^2+5x-1)$ (3)

2. Factorise $y^2(y-2)+x^2(2-y)$ fully (3)

[6]

QUESTION 3 [10 Marks]

Simplify the following expressions fully:

1. $\frac{x^2-4}{2x^2+5x+2} \div \frac{x^3-8}{6x+3}$ (5)

2. $\frac{x}{x+y} + \frac{x^2}{y^2-x^2}$ (5)

[10]

MEMORANDUM/ MARKING GUIDELINE

QUESTION1

1.1 $x = -14$; -5 1.2 $x =$ -11; 0; 5 $x = 11$ $x = 14$ (4)

2. $\sqrt{9} < \sqrt{11} < \sqrt{16}$

$= 3 < \sqrt{11} < 4$ …between 3 and 4 (2)

3. Let $x = 0{,}7777$ …(1) √

$10x = 7{,}7777$ …(2) √

$9x = 7$ (2)-(1)

$= \frac{7}{9}$ √ (3)

QUESTION 2

1. $= x^3 + 5x^2 - x - 2x^2 - 10x + 2$ √

$= x^3 + 5x^2 - 2x^2 - x - 10x + 2$ √

$= x^3 + 3x^2 - 11x + 2$ √ (3)

2. $= y^2(y-2) - x^2(y-2)$ √

$= (y-2)(y^2 - x^2)$ √

$= (y-2)(y-x)(y+x)$ √ (3)

QUESTION 3

1. $= \frac{x^2-4}{2x^2+5x+2} \times \frac{6x+3}{x^3-8}$ √ changing $\div$ *to* $\times$ and swapping numerator and denominator.

$= \frac{(x-2)(x+2)}{(2x+1)(x+2)} \times \frac{3(2x+1)}{(x-2)(x^2+2x+4)}$ √ √ √ factorisation

$= \frac{3}{x^2+2x+4}$ √ (5)

2. $= \frac{x}{x+y} + \frac{x^2}{(y-x)(y+x)}$ √ factorisation

LCD: $(y+x)(y-x)$ √

$= \frac{x}{(y+x)} \times \frac{(y-x)}{(y-x)} + \frac{x^2}{(y+x)(y-x)}$

$= \frac{xy-x^2}{(y+x)(y-x)} + \frac{x^2}{(y+x)(y-x)}$ √ same denominators

$= \frac{xy-x^2+x^2}{(y+x)(y-x)}$ √ Simplification

$= \frac{xy}{(y+x)(y-x)}$ √ (5)

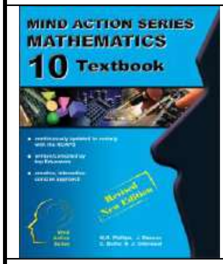
TOPIC: ALGEBRA PART 2 (Lesson 1)	Weighting	30 ± 3	Grade	10

Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**EXPONENTS: Revision of Laws of Exponents and Definitions (negative exponents and exponent 0)**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Base, exponent, power		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Laws of Exponents

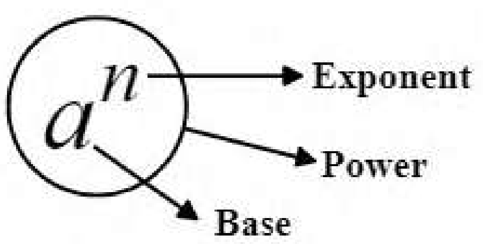
RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Product with same bases, multiply exponents instead of adding them.
Subtracting smaller exponent from the bigger one even if the bigger exponent is on the denominator

METHODOLOGY



PRODUCT RULE

To multiply when two bases are the same, write the base and ADD the exponents: $\boldsymbol{x^m \times x^n = x^{m+n}}$

Examples:

1. $x^3.x^8 = x^{3+8} = x^{11}$ 2. $2^4 \times 2^2 = 2^{4+2} = 2^6$ 3. $(x^2y)(x^3y^4) = x^{2+3}y^{1+4} = x^5y^5$

QUOTIENT RULE

To divide when two bases are the same, write the base and SUBTRACT the exponents: $\boldsymbol{\frac{x^m}{x^n} = x^{m-n}}$

Examples:

1. $\frac{x^5}{x^2} = x^{5-2} = x^3$ 2. $\frac{3^4}{3^7} = 3^{4-7} = 3^{-3}$ 3. $\frac{x^2y^5}{xy^3} = x^{2-1}y^{5-3} = xy^2$

POWER RULE:

To raise a power to another power, write the base and MULTIPLY the exponents: $\boldsymbol{(x^m)^n = x^{m\times n}}$

Examples:

1. $(x^3)^2 = x^{3\times 2} = x^6$ 2. $(3^2)^4 = 3^{2.4} = 3^8$ 3. $(y^3z^5)^2 = y^{3\times 2}z^{5\times 2} = y^6z^{10}$

EXPANDED POWER RULE: $(xy)^m = x^m y^m$ $\quad \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

1. $(2a)^3 = 2^3a^3 = 8a^3$ $\quad$ 2. $(6x^3)^2 = 6^2x^{3\times2} = 36x^6$ $\quad$ 3. $\left(\frac{2x^2}{3y}\right)^3 = \frac{2^3x^{2\times3}}{3^3y^3} = \frac{8x^6}{27y^3}$

ZERO EXPONENT RULE

Any base (except 0) raised to the zero power is equal to one: $x^0 = 1$

1. $y^0 = 1$ $\quad$ 2. $6^0 = 1$ $\quad$ $(7a^3b^{-1})^0 = 0$

NEGATIVE EXPONENTS:

If a factor in the numerator or denominator is moved across the fraction bar, the sign of the exponent is changed. $x^{-m} = \frac{1}{x^m}$ $\quad \frac{1}{x^{-m}} = x^m$ $\quad \left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

1. $x^{-3} = \frac{1}{x^3}$ $\quad$ 2. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ $\quad$ 3. $-4x^5y^{-2} = \frac{-4x^5}{y^2}$ $\quad$ 4. $\left(\frac{x^2}{y}\right)^{-3} = \left(\frac{y}{x^2}\right)^3 = \frac{y^3}{x^6}$

Examples:

Simplify the following and leave the answer in exponential Form:

1. $-4x^2.(-5)^2x$

$= -4.(-5)(-5).x^{2+1}$

$= -100x^3$

2. $\frac{5^7\times3^3\times3^6\times5^8}{3^7\times5^{20}\times3}$

$= 5^{7+8-20} \times 3^{3+6-7-1}$

$= 5^{-5} \times 3^1 = \frac{3}{5^5}$

3. $\frac{3x^2\times4xy^7}{2x^2y\times6x^2y}$

$=\frac{12x^3y^7}{12x^4y^2}$

$= x^{3-4}y^{7-2}$

$= x^{-1}y^5 = \frac{y^5}{x}$

ACTIVITIES/ ASSESSMENT

Simplify the following:

1. $2x^5 \times (-3x^3y)^2$

2. $\frac{7x^2y^5}{14x^3y} \times \frac{36x^3y}{6y}$

3. $\frac{(-2xy^3)^3}{4xy^4\times2x^2y^5}$

4. $\left(\frac{3p^3q^2}{9p^5}\right)^3$

5. $\left(\frac{6x^7}{12x^9}\right)^{-2}$

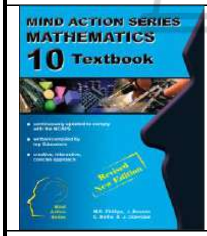
TOPIC: ALGEBRA PART 2 (Lesson 2)	Weighting	30 ± 3	Grade	10

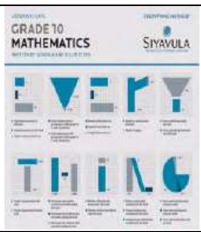
Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**EXPONENTS: Use Laws of Exponents to Simplify Expressions**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Prime numbers		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Fractions, laws of exponents

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

$5^2 = 5+5$ or $5 \times 2 = 10$, $3^2 = 3 \times 2 = 6$

METHODOLOGY

Terms can often be simplified by reducing the powers to a prime base:

1. $9^x = (3^2)^x = 3^{2x}$
2. $25^{2x} = (5^2)^{2x} = 5^{4x}$
3. $8^{2x-1} = (2^3)^{2x-1} = 2^{6x-3}$
4. $10^x = (2 \times 5)^x = 2^x 5^x$
5. $250^{4-3p} = (2 \times 5^3)^{4-3p} = 2^{4-3p} 5^{12-9p}$

Examples:

Simplify the following:

1. $\frac{16^2.(3^2)^4}{2^7.2^5.81}$

$= \frac{(2^4)^2.3^8}{2^{12}.3^4}$

$= 2^{8-12}.3^{8-4}$

$= 2^{-4}.3^4 \quad = \frac{3^4}{2^4}$

2. $\frac{(2^x)^4.9^x}{12^{2x}}$

$= \frac{2^{4x}.(3^2)^x}{(3 \times 2^2)^{2x}}$

$= \frac{2^{4x}.3^{2x}}{3^{2x}2^{4x}}$

$= 2^{4x-4x}.3^{2x-2x} = 1$

3. $\frac{16^{x-1}}{2^{4x}.32}$

$= \frac{(2^4)^{x-1}}{2^{4x}2^5}$

$= \frac{2^{4x-4}}{2^{4x+5}}$

$= 2^{4x-4-4x-5} = 2^{-9} = \frac{1}{2^9}$

ACTIVITIES/ ASSESSMENT

Simplify each of the following:

1. $81^x.27^{2x}$
2. $\frac{32^x}{4^x}$
3. $\frac{(2^x)^3.2^x}{16^x}$
4. $\frac{25\,.\,9^x}{3^x\,.\,3^x\,.\,5}$
5. $\frac{5^x.25^{x-1}}{5.125^x}$
6. $\frac{9^a\,.\,4^{a-1}}{3^{2a-1}\,.\,2^{2a}}$

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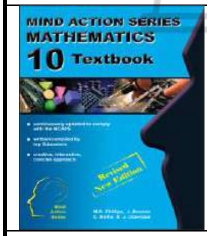
TOPIC: ALGEBRA PART 2 (Lesson 3)	Weighting	30 ± 3	Grade	10

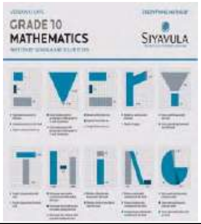
Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**EXPONENTS: Use Laws of Exponents to Simplify Expressions**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Prime numbers Factorisation		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Fractions, laws of exponents

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Write $x^{m+n} as x^m + x^n$

METHODOLOGY

Bases to the power of more than one term can be expanded and written as a product.

1. $2^{x+1} = 2^x \times 2$
2. $3^{2x} = 3^x . 3^x$
3. $5^{1-2x} = 5^1 . 5^{-2x}$

Examples:

1. $\frac{2^{x+2} + 2^{x+3}}{12 . 2^x}$

$= \frac{2^x . 2^2 + 2^x . 2^3}{12 . 2^x}$ **expand**

$= \frac{2^x(4+8)}{12 . 2^x}$ **common factor**

$= \frac{12}{12} = 1$

2. $\frac{8^a . 2^a + 2 . 16^{a+1}}{11 . 2^{a+1}}$

$= \frac{(2^3)^a . 2^a + 2 . (2^4)^{a+1}}{11 . 2^a . 2^1}$

$= \frac{2^{3a} . 2^a + 2^{4a+4+1}}{11 . 2^a . 2}$

$= \frac{2^{4a} + 2^{4a} . 2^5}{2^a . 11 \times 2}$

$= \frac{2^{4a}(1+32)}{2^a . 22} = \frac{3}{2} 2^{3a}$

3. $\frac{9^x + 3^x - 2}{9^x - 4}$

$= \frac{3^{2x} + 3^x - 2}{3^{2x} - 4}$ …prime bases

$= \frac{(3^x+2)(3^x-1)}{(3^x+2)(3^x-2)}$ …factorise

Trinomial on numerator

Difference of two squares

On denominator

$= \frac{(3^x-1)}{(3^x-2)}$

ACTIVITIES/ ASSESSMENT

Simplify:

1. $\frac{2^{x+2} - 2^{x=1}}{2^x + 2^{x+2}}$
2. $\frac{3^{x+1} + 3^{x+2}}{8 . 3^{x+1}}$
3. $\frac{4^x + 3 . 2^{2x+1}}{7 . 2^{2x+1}}$
4. $\frac{12^x + 4^x . 3^{x+1}}{2^{2x+4} . 3^x}$
5. $\frac{2 . 3^x + 3^{x-2}}{5 . 2^{x+1} - 7 . 3^{x-1}}$
6. $\frac{9^x - 3^x - 6}{3^x - 3}$

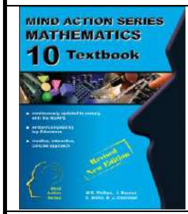
COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

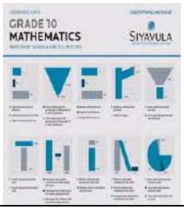
TOPIC: ALGEBRA PART 2 (Lesson 4)	**Weighting**	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**EXPONENTS: Use Laws of Exponents to Solve equations**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Equation, Decimal fraction, Exponent, Base			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Laws of Exponents, Prime numbers

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Writing $2 \times 3^x = 6^x$ and $(2^2)^{x+3} = 2^{2x+3}$

METHODOLOGY

In an exponential equation, the exponent is the unknown.

Examples:

Solve the following equations

1. $4\,.\,25^{x+3} = 4$

$(5^2)^{x+3} = 1$ …divide by 4 on both side and write 25 as prime number

$5^{2x+6} = 5^0$ …write 1 as base 5 to exponent 0 (any number to exponent 0 = 1)

$2x + 6 = 0$ …equate exponents

$2x = -6$

$x = -3$

2. $(0{,}5)^{x-1} = \left(\frac{1}{4}\right)^x$

$\left(\frac{1}{2}\right)^{x-1} = \left(\frac{1}{2^2}\right)^x$ … write 4 as a prime number

$(2^{-1})^{x-1} = (2^{-2})^x$ …use definition $\frac{1}{a^n} = a^{-n}$

$2^{-x+1} = 2^{-2x}$

$-x + 1 = -2x$

$-x + 2x = -1$

$x = -1$

3. $9^x + 3^{2x+1} = 36$

$3^{2x} + 3^x.\ 3 = 36$

$3^{2x}(1 + 3) = 36$ …take out common factor and divide by 4 on both sides

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$3^{2x} = 9$
$3^{2x} = 3^2$
$2x = 2$
$x = 1$

ACTIVITIES/ ASSESSMENT

Solve the following equations;

1. $3^x = 3$
2. $2^{3x} = 1$
3. $4^x = 16$
4. $8^x.2 = 128$
5. $\frac{1}{8}.2^{2x} = 1$
6. $3\left(\frac{1}{3}\right)^{x-1} = \frac{1}{3}$
7. $49^{x+2} = 49$
8. $\left(\frac{2}{3}\right)^{x-2} = \frac{8}{27}$
9. $4\,.\,2^x = (0{,}5)^{x-2}$
10. $3^x\,.\,9^{x-1} = 81$
11. $8^{-x} = 2\,.\,4^{x-1}$
12. $2^{x+1} + 2^{x+2} = 24$
13. $7^x + 14\,.\,7^x = 147$
14. $2\,.\,3^{2x+1} + 3\,.\,9^x = 243$

TEST: EXPONENTS

Marks: 25 **Duration: 30 Min**

QUESTION 1[14 Marks]

Simplify the following expressions (Give your answer with positive exponents)

1.1 $\frac{2^{2x-1}-2\,.\,4^{x+1}}{4^{x}}$ (3)

1.2 $\frac{9x^{2}y^{3}\times 6x^{7}y^{5}}{12xy^{6}}$ (3)

1.3 $\frac{2^{n+1}\,.\,9^{n-2}}{6^{n-1}\,.\,3^{n+1}}$ (4)

1.4 $\left(\frac{x^{3}y^{-2}}{z^{-2}}\right)^{2} \div \left(\frac{x^{-2}y^{3}}{z^{3}}\right)^{-2}$ (4)

[14]

QUESTION 2 [11 Marks]

Solve for x:

2.1 $4^{x} = 1$ (2)

2.2 $3^{x} = \frac{1}{81}$ (3)

2.3 $5^{x-1} = 0,04$ (3)

2.3 $3^{t} \times 9^{t+3} = 27^{3}$ (3)

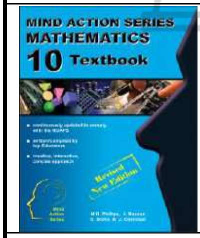
[11]

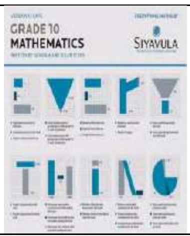
TOPIC: ALGEBRA PART 3 (Lesson 1)	Weighting	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**EQUATIONS: Revise linear equations**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Solving equation, solution			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

LDC, like terms, bracket, variable, algebraic fraction

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Forgetting to change sign when transposing to the other side of the equal sign.

METHODOLOGY

A linear equation is an equation where the highest exponent of the variable is 1 and with at most one solution.

Solving an equation means:

- finding the value of the variable that makes the equation true.
- Determining the value of the variable that will make the left-hand side of the equation equal to the right-hand side of the equation.

The solution is the value of the variable that satisfies the equation. It is also called the root of the equation.

Examples:

1. $2x + 2 = 1$

$2x = 1 - 2$ ..,like terms together

$2x = -1$

$x = -\frac{1}{2}$ …divide both sides by 2

2. $\frac{2-x}{3x+1} = 2$ LCD: $(3x + 1)$

$\frac{2-x}{3x+1} \times (3x + 1) = 2 \times (3x + 1)$

Multiply terms on both side by LCD

$2 - x = 6x + 2$

$-x - 6x = 2 - 2$ …like terms together

$-7x = 0$

$x = 0$ …divide both sides by -7

1. $3 - 4(k + 2) = 3k + 16$

$3 - 4k - 8 = 3k + 16$ …expand brackets

$-4k - 3k = 16 - 3 + 8$ …like terms together

$-7k = 21$

$k = -3$ …divide both sides by -7

2. $\frac{4x-1}{3} - \frac{7x+2}{6} = \frac{x}{2}$ LCD: 6

$6 \times \frac{4x-1}{3} - 6 \times \frac{7x+2}{6} = 6 \times \frac{x}{2}$

Multiply all terms by LCD

$2(4x - 1) - (7x + 2) = 3x$

$8x - 2 - 7x - 2 = 3x$ …expand brackets

$8x - 7x - 3x = 2 + 2$

$-2x = 4$

$x = -2$ …divide both sides by -2

NOTE: An equation must always be balanced, whatever you do on the left-hand side, you must also do on the right-hand side.

The general steps for solving linear equations are:

- Expand all brackets.
- Rearrange the terms so that all terms containing the same variable are on the same side of the equation and all the constants are on the other side.
- Group like terms together and simplify.
- Factorise if necessary.
- Find the solution and write down the answer.
- Check the answer by substituting the solution back into the original equation.

ACTIVITIES/ ASSESSMENT

Solve the following equations:

1. $2y - 3 = 7$

2. $x + 12 = 6 - 3x$

3. $4(p - 3) = 3 - 6(p - 2)$

4. $7(m - 2) - 2(7 - 3m) + 2 = 0$

5. $\frac{y+2}{3} - \frac{y-3}{6} = 5$

6. $\frac{m+2}{4} - \frac{m-6}{3} = \frac{1}{2}$

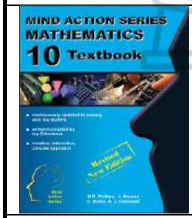
7. $\frac{x+2}{4} - 2(x + 1) = 2$

TOPIC: ALGEBRA PART 3 (Lesson 2)	Weighting	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**EQUATIONS: Solve quadratic equations**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Zero-factor law, standard form			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Facrorisation: common factor, difference of two squares, quadratic trinomial

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Multiplying out factors that are equal to 0. Dividing common variable common factors.

METHODOLOGY

A quadratic equation is an equation where the exponent of the variable is at most 2 with at most two real solutions.

The **standard form** of a quadratic equation is $ax^2 + bx + c = 0$.

The quadratic expression on the left-hand side has to be factorised.

Zero-factor law states that if $a \times b = 0 \; then \; a = 0 \; or \; b = 0$

Examples:

Solve each of the following equations:

1. $x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$ …factorise

$x - 7 = 0 \; or \; x + 2 = 0$ …apply the zero-factor law

$x = 7 \; or \; x = -2$

2. $8x - 16 - x^2 = 0$

$0 = x^2 - 8x + 16$ …standard form

$0 = (x-4)(x-4$ …factorise

$x - 4 = 0$ …no need to write both

$x = 4$

3. $(3x-1)(x+2) = 0$

No need to expand brackets because factors are equal to 0.

$3x - 1 = 0 \; or \; x + 2 = 0$

$3x = 1 \; or \; x = -2$

$x = \frac{1}{3}$

4. $x(x-5) + 6 = 0$

$x^2 - 5x + 6 = 0$…expand brackets

$(x-2)(x-3) = 0$ …factorise

$x - 2 = 0 \; or \; x - 3 = 0$

$x = 2 \; or \; x = 3$

5. $5x^2 + 2x = 3$

$5x^2 + 2x - 3 = 0$ …standard form

$(5x-3)(x+1) = 0$ …factorise

$5x - 3 = 0 \; or \; x + 1 = 0$

$5x = 3 \; or \; x = -1$

$x = \frac{3}{5}$

6. $(3x-1)(x+2) = 10$

$3x^2 + 6x - x - 2 - 10 = 0$

Expand brackets because factors are not equal to 0

$3x^2 + 5x - 12 = 0$ …standard form

$(3x-4)(x+3) = 0$

$3x - 4 = 0 \; or \; x + 3 = 0$

$x = \frac{4}{3} \; or \; x = -3$

7. $2x^2 = 4x$

$2x^2 - 4x = 0$ …standard form

8. $(x+5)^2 = 9$

Expand: $(x+5)(x+5) - 9 = 0$

$2x(x-2)=0$ ...common factor
$2x=0\ or\ x-2=0$
$x=0\ or\ x=2$

OR difference of two squares: $(x+5-3)(x+5+3)=0$
OR square root on both sides: $\sqrt{(x+5)^2}=\pm\sqrt{9}$

General steps for solving quadratic equations:

- Rewrite the equation in the required form: $ax^2+bx+c=0$
- Divide the entire equation any common factor of the coefficients to obtain an equation of the form $ax^2+bx+c=0$.
- Factorise $ax^2+bx+c=0$.
- Apply the zero-factor law
- Solve both factors.
- You may check the answer by substituting it back to the original equation.

ACTIVITIES/ ASSESSMENT

Solve the following equations:

1. $x^2+2x-15=0$

2. $p^2-7p-18=0$

3. $-3a^2+27a-54=0$

4. $3x(2x+5)=0$

5. $12x^2-4x=0$

6. $2x^2-x=10$

7. $(x+3)(x+6)=0$

9. $(m-3)(m-2)=12$

10. $2x^2=5x$

11. $4y^2-9=0$

12. $5(x-1)^2=16$

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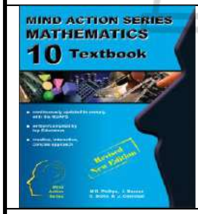
TOPIC: ALGEBRA PART 3 (Lesson 3)	**Weighting**	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**EQUATIONS: Solve quadratic equations**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Undefined, Restriction, linear, quadratic			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Sign-change rule, Algebraic fractions, LCD, Factorisation

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Sign-change rule, Determining LCD

METHODOLOGY

Equations with fractions

When solving equations with fractions (variables in the denominator) it is important to remember that **division by 0 is not permissible.**
You must always check that your solutions **do not make any one of the denominators 0**.
That is the reason why we must state restrictions when solving equations with variables in the denominators.

Examples:

1. $10+\frac{16}{x}=\frac{4(x+4)}{x}$ Restriction $x \neq 0$ and LCD: x

$x\times 10+x\times\frac{16}{x}=x\times\frac{4x+16}{x}$ …multiply every term by LCD

$10x+16=4x+16$ …linear equation
$10x-4x=16-16$
$6x=0$
$x=0$

However, since $x=0$, the equation does not have a solution.

2. $\frac{4}{x-5}-\frac{10}{x}=\frac{2}{x^2-5x}$

$\frac{4}{x-5}-\frac{10}{x}=\frac{2}{x(x-5)}$ …factorise denominator, LCD: $x(x-5)$

Restrictions: $x\neq 0$ and $x\neq 5$

$x(x-5)\times\frac{4}{x-5}-x(x-5)\frac{10}{x}=x(x-5)\frac{2}{x(x-5)}$ …multiply every term by LCD

$4x-10x+50=2$ …linear equation
$-6x=-48$
$x=8$

This is a valid solution since it is not stated as a restriction.

3. $\frac{2x}{x-3}+\frac{5x-3}{9-x^2}=\frac{x}{x+3}$

$\frac{2x}{x-3}+\frac{5x-3}{(3-x)(3+x)}=\frac{x}{x+3}$ …factorise denominator

$\frac{2x}{x-3}-\frac{5x-3}{(x-3)(x+3)}=\frac{x}{x+3}$ …$3-x=-(x-3)$ and $3+x=x+3$

LCD: $(x-3)(x+3)$. Therefore, Restrictions: $x \neq 3$ and $x \neq -3$

$(x-3)(x+3)\times\frac{2x}{x-3}-(x-3)(x+3)\frac{5x-3}{(x-3)(x+3)}=(x-3)(x+3)\times\frac{x}{x+3}$ …multiply terms by LCD

$2x(x+3)-(5x-3)=x(x-3)$

$2x^2+6x-5x+3=x^2-3x$ quadratic equation

$2x^2-x^2+6x-5x+3x+3=0$

$x^2+4x+3=0$ …standard form

$(x+3)(x+1)=0$

$x=-3$ or $x=-1$

Since $x \neq -3$, the solution is $x=-1$

General steps for solving quadratic equations with fractions

- Apply the sign-change rule if necessary and then factorise the denominators.
- State the restrictions.
- Multiply every term by the lowest common denominator (LCD).
- Identify the equation as linear or quadratic and solve the equation.

ACTIVITIES/ ASSESSMENT

Solve the following equations. Remember to verify whether or not the solution to the equation is viable by considering the restrictions.

1. $\frac{3}{x}-\frac{1}{x}=\frac{2}{x}+\frac{5}{2}$

2. $\frac{7x}{x^2+x}-\frac{5}{x}=\frac{3}{x+1}$

3. $\frac{x+6}{x^2-4}+\frac{2}{2-x}=\frac{-1}{x+2}$

4. $\frac{x-2}{x-1}-\frac{5}{x+2}=\frac{7}{x-1}$

5. $\frac{3x+4}{x+6}=\frac{3x+2}{x-3}-\frac{23x}{x^2+3x-18}$

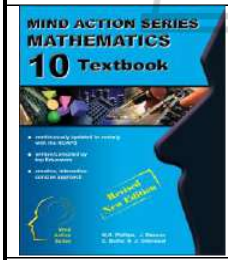
TOPIC: ALGEBRA PART 3 (Lesson 4)	Weighting	30 ± 3	Grade	10

Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**EQUATIONS: Solve simultaneous linear equations**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Algebraically, Graphically, System of equations, Simultaneously		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Variables, linear equations, substitution

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Forget to solve for the second variable, multiplying the second term in the bracket

METHODOLOGY

When solving for **two** unknown variables, two equations are required and these equations are known as **simultaneous equations.**
The solutions are the values of the unknown variables which **satisfy both equations simultaneously.**
To solve for **two unknown variables**, two independent equations are given.
Simultaneous equations can be solved **algebraically** using **substitution and elimination methods.**
System of simultaneous equations can be solved **graphically**.

Substitution Method

- Use the simplest of the two given equations to express one of the variables in terms of the other.
- Substitute into the second equation. By doing this we reduce the number of equations and the number of variables by one.
- We now have one equation with one unknown variable which can be solved.
- Use the solution to substitute back into the first equation to find the value of the other unknown variable. method of
- Substitution is a method you can use for other systems of equations as well.

Examples:

1. Solve for $x \text{ and } y$ simultaneously: $x - y = 1 \text{ and } 3 = y - 2x$

Method 1: Make one of the variables the subject of the formula in one of the equations.

First equation: $x = y + 1$	**OR**	Second equation: $y = 2x + 3$
Substitute into second equation		Substitute into first equation
$3 = y - 2(y + 1)$		$x - (2x + 3) = 1$
$3 = y - 2y - 2$		$x - 2x - 3 = 1$
$2y - y = -3 - 2$		$-x = 1 + 3$
$y = -5$		$x = -4$
Substitute $y = -5$ into original equations	**OR**	Substitute $x = -4$
$3 = -5 - 2x$		$3 = y - 2(-4)$

$8 = -2x$ $\quad$ $3 = y + 8$

$x = -4$ $\quad$ $y = -5$

Method 2:

Make y the subject of the formula in both equations.

$x - 1 = y$ and $y = 3 + 2x$

$x - 1 = 3 + 2x$ …equate

$-1 - 3 = 2x - x$

$-4 = x$, substitute to original equations

$-4 - y = 1$ OR $3 = y - 2(-4)$

$y = -5$

Method 3:

Make x the subject of the formula in both equations.

$x = y + 1$ and $x = \frac{y}{2} - \frac{3}{2}$

$y + 1 = \frac{y}{2} - \frac{3}{2}$ LCD:2

$2 \times y + 2 \times 1 = 2 \times \frac{y}{2} - 2 \times \frac{3}{2}$ …multiply every term by LCD

$2y + 2 = y - 3$

$y = -5$, substitute into original equations

$x - (-5) = 1$ OR $3 = -5 - 2x$

$x = -4$

ACTIVITIES/ ASSESSMENT

Solve the following equations simultaneously:

1. $3x - y = 10$ and $x + y = 6$

2. $3x - 2y = 8$ and $4x + 2y = 6$

3. $4x + 3y = 100$ and $4y - 9x = 12$

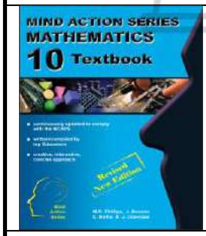
TOPIC: ALGEBRA PART 3 (Lesson 5)	Weighting	30 ± 3	Grade	10

Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**EQUATIONS: Solve simultaneous equations**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Algebraically, Graphically, System of equations, Simultaneously		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Variables, linear equations, substitution

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Forget to solve for the second variable, multiplying the second term in the bracket

METHODOLOGY

Elimination Method

Elimination is the preferred method when solving two linear equations simultaneously.

Examples:

Solve for x and y simultaneously:

1. $3x - 2y = 8$ and $4x + 2y = 6$

Write equations beneath each other and label them

$3x - 2y = 8$ …(1)

$4x + 2y = 6$ …(2)

(1) + (2): $7x = 14$ …add like the terms of (1) and (2)

$x = 2$, substitute into either (1) or (2) to get y.

$3(2) - 2y = 8$

$6 - 2y = 8$

$-2y = 2$

$y = -1$

2. $4x + 3y = 100$ and $4y - 9x = 12$

$4x + 3y = 100$ …(1)

$-9x + 4y = 12$ …(2)

If we want the terms in *x* to be eliminated when added, the coefficients have to be additive inverses. Let us therefore consider the lowest common multiple (LCM) of 4 and 9 (the coefficients of *x*) in order to attain that. The LCM is 36. We want the coefficient of *x* of equation (1) to be 36 and that of equation (2) to be -36.

(1) x 9: $36x + 27y = 900$

(2) x 4: $-36x + 16y = 48$

(1) + (2): $43y = 948$

$y = 22{,}05$, substitute into either equations (1) or (2)

$4x + 3(22{,}05) = 100$

$4x = 100 - 66{,}14$

$x = 8{,}47$

If we want the terms in y to be eliminated when added, the coefficients have to be additive inverses. Let us therefore consider the lowest common multiple (LCM) of 3 and 4 (the coefficients of y) in order to attain that. The LCM is 12. We want the coefficient of *x* of equation (1) to be 12 and that of equation (2) to be 12.

(1) x 4: $12y + 16x = 400$

(2) x 3: $12y - 27x = 36$

(1) - (2): $43x = 364$

$x = 8{,}47$, substitute into either equation (1) or (2)

$4(8{,}47) + 3y = 100$

$3y = 100 - 25{,}41$

$y =$

ACTIVITIES/ ASSESSMENT

Solve for x and y simultaneously:

1. $x + 2y = 5 \text{ and } x - y = -1$

2. $7x - 3y = 41 \text{ and } 3x - y = 17$

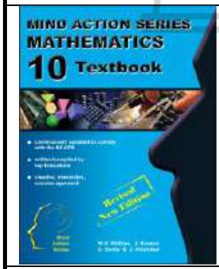
3. $2a - 2b = 5 \text{ and } 3a - 2b = 20$

TOPIC: ALGEBRA PART 3 (Lesson 6)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**EQUATIONS: Solve word sums/problems involving linear, quadratic or simultaneous equations**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Variables, Algebraic Expressions, Algebraically				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Linear equations, system of equations

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Reading the whole question, assigning variable to unknown, translating words to algebraic expressions

METHODOLOGY

To solve **word problems**, we need to write a set of equations that represent the problem mathematically. The solution of the equations is then the solution to the problem.

Problem Solving Strategy:

- Read the whole question.
- What are we asked to solve for?
- Assign a variable to the unknown quantity, for example x.
- Translate the words into algebraic expressions by rewriting the given information in terms of the variable.
- Set up an equation or system of equations to solve for the variable.
- Solve the equation algebraically using substitution.
- Check the solution.

Examples:

1. A shop sells bicycles and tricycles. In total there are 7 cycles (cycles include both bicycles and tricycles) and 19 wheels. Determine how many of each there are, if a bicycle has two wheels and a tricycle has three wheels.

Let the number of bicycles be b and the number of tricycles be t

$b + t = 7$ and $2b + 3t = 19$

$b = 7 - t$ $\quad 2(7 - t) + 3t = 19$

$b = 7 - 5$ $\quad 14 - 2t + 3t = 19$

$b = 2$ $\quad t = 5$

Therefore, 2 bicycles and 5 tricycles

2. The product of two consecutive negative integers is 1122. Find the two integers.

Let the first integer be n and the second integer be $n + 1$.

$n(n + 1) = 1122$

$n^2 + n = 1\,122$

$n^2 + n - 1\,122 = 0$ …standard form

$(n - 33)(n + 34) = 0$…factors

$n = 33$

The numbers are 33 and 34

3. A curtain manufacturing company sold a client 12 meters of material. The cost of the good material was R8 per meter. Some of the material was inferior and was sold for R7 per meter. The total cost of the material was R100. How many meters of material was inferior?
   (Let the good material = *x*)

4. A rectangle has an area of 8 square metres. Its breath is 2 metres less than its length. Determine the dimensions of the rectangle.
   Let the length of a rectangle be l, therefore, the breath = $l - 2$
   $l(l-2) = 8$
   $l^2 - 2l - 8 = 0$…standard form
   $(l-4)(l+2) = 0$…factors
   $l = 4m$ and $b = 4 - 2 = 2m$…$b = l - 2$

Therefore, length = 4cm and breadth = 2cm

ACTIVITIES/ ASSESSMENT

Word Problems

1. Kadesh bought 20 shirts at a total cost of R 980. If the large shirts cost R 50 and the small shirts cost R 40, how many of each size did he buy?

2. The sum of 27 and 12 is equal to 73 more than an unknown number. Find the unknown number.

3. The length of a rectangle is twice the breadth. If the area is 128 cm^2, determine the length and the breadth.

4. If 4 times a number is increased by 6, the result is 15 less than the square of the number. Find the number.

5. If a third of the sum of a number and one is equivalent to a fraction whose denominator is the number and numerator is two, what is the number?

6. A shop owner buys 40 sacks of rice and mealie meal worth R 5250 in total. If the rice costs R 150 per sack and mealie meal costs R 100 per sack, how many sacks of mealie meal did he buy?

7. Lisa has 170 beads. She has blue, red and purple beads each weighing 13 g, 4 g and 8 g respectively. If there are twice as many red beads as there are blue beads and all the beads weigh 1,216 kg, how many beads of each type does Lisa has?

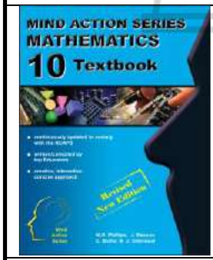
TOPIC: ALGEBRA PART 3 (Lesson 7)	Weighting	30 ± 3	Grade	10

Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**EQUATIONS: Solve word sums/problems involving linear, quadratic or simultaneous equations**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Variables, Algebraic Expressions, Algebraically		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Linear equations, system of equations

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Reading the whole question, assigning variable to unknown, translating words to algebraic expressions

METHODOLOGY

Corrections of previous activity with learners

ACTIVITIES/ ASSESSMENT

Word Problems

1. Two jets are flying towards each other from airports that are 1200 km apart. One jet is flying at 250 km·h^{-}1and the other jet at 350 km·h^{-1}. If they took off at the same time, how long will it take for the jets to pass each other?

2. Two boats are moving towards each other from harbours that are 144 km apart. One boat is moving at 63 km·h^{-1} and the other boat at 81 km·h^{-1}. If both boats started their journey at the same time, how long will they take to pass each other?
3. Zwelibanzi and Jessica are friends. Zwelibanzi takes Jessica's civil technology test paper and will not tell her what her mark is. He knows that Jessica dislikes word problems so he decides to tease her.
Zwelibanzi says: "I have 12 marks more than you do and the sum of both our marks is equal to 148. What are our marks?"

4. Kadesh bought 20 shirts at a total cost of R 980. If the large shirts cost R 50 and the small shirts cost R 40, how many of each size did he buy?

5. The diagonal of a rectangle is 25 cm more than its width. The length of the rectangle is 17 cm more than its width. What are the dimensions of the rectangle?

6. The sum of 27 and 12 is equal to 73 more than an unknown number. Find the unknown number.

7. A group of friends is buying lunch. Here are some facts about their lunch:
   - a milkshake costs R 7 more than a wrap
   - the group buys 8 milkshakes and 2 wraps
   - the total cost for the lunch is R 326

   Determine the individual prices for the lunch items.

8. The two smaller angles in a right-angled triangle are in the ratio of 1: 2. What are the sizes of the two

angles?

9. The length of a rectangle is twice the breadth. If the area is 128 cm2, determine the length and the breadth.

10. If 4 times a number is increased by 6, the result is 15 less than the square of the number. Find the number.

11. The length of a rectangle is 2 cm more than the width of the rectangle. The perimeter of the rectangle is 20 cm. Find the length and the width of the rectangle.

12. Stephen has 1 litre of a mixture containing 69% salt. How much water must Stephen add to make the mixture 50% salt? Write your answer as a fraction of a litre.

13. The sum of two consecutive odd numbers is 20 and their difference is 2. Find the two numbers.

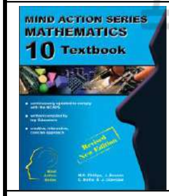
14. The denominator of a fraction is 1 more than the numerator. The sum of the fraction and its reciprocal is $\frac{5}{2}$. Find the fraction.

TOPIC: ALGEBRA PART 3 (Lesson 8)	**Weighting**	30 ± 3	Grade	10	
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**EQUATIONS: Solve literal equations (changing the subject of the formula)**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Variables, formula				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Equations

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Transpose coefficient instead of dividing by it. Forgetting to put a square root on both sides of the equation.

METHODOLOGY

A literal equation is one that has several letters or variables.
A literal equation is one in which letters of the alphabet are used as coefficients and constants.
These equations, **usually referred to as formulae**, are used a great deal in Mathematics, Science and Technology.
In this section we solve literal equations in terms of **one variable.**
To do this, we use the principles we have learnt about solving equations and apply them to **rearranging literal equations.**
Solving literal equations is also known as changing the subject of the formula.

Examples:

1. Make t the subject of the formula $v = u + at$

$$v - u = at \text{ ...subtract } u \text{ on both sides}$$
$$\frac{v-u}{a} = t \text{ ...divide both sides by } a \text{ where } a \neq 0$$

2. The area of a circle is given by $A = \pi r^2$. Make r the subject of the formula.

$$\frac{A}{\pi} = r^2 \text{ ...divide both sides by } \pi$$
$$\sqrt{\frac{A}{\pi}} = \sqrt{r^2} \text{ ...square root both sides}$$
$$r = \sqrt{\frac{A}{\pi}} \text{ ...not } \pm \text{ } as \text{ } r \text{ is positive.}$$

3. The surface area of a cylinder is given by $A = 2\pi r(h + r)$. Show that $h = \frac{A-2\pi r^2}{2\pi r}$

$$\frac{A}{2\pi r} = h + r \text{ ...divide both sides by } 2\pi r$$
$$\frac{A}{2\pi r} - r = h \text{ ...subtract } r \text{ on both sides}$$

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$$\frac{A-2\pi r^2}{2\pi r} = h \text{ ...LCD: } 2\pi r$$

4. Solve for x in terms of y if $x^2 = 2xy - 8y^2$ …quadratic equation

$$x^2 - 2xy + 8y^2 = 0 \text{ ...standard form}$$

$$(x - 4y)(x - 2y) = 0 \text{ ...factorise}$$

$$x = 4y \text{ or } x = 2y$$

ACTIVITIES/ ASSESSMENT

1. Make a the subject of the formula $v = u + at$

2. Solve for x in $3ax = 2bx + c$

3. Make g the subject of the formula $F = \frac{mv^2}{gr}$

4. Make v the subject of the formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

5. Make u the subject of the formula $v^2 = u^2 + 2as$

6. Solve for b in $c = \sqrt{a^2 + b^2}$

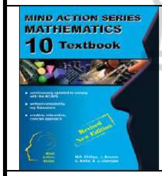
TOPIC: ALGEBRA PART 3 (Lesson 9)	Weighting	30 ± 3	Grade	10

Term	1	**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**LINEAR INEQUALITIES with graphical solutions**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Inequality sign, Interval notation		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Linear equations. Number line

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Meaning of brackets, meaning of inequality signs, changing the inequality sign when dividing or multiplying by a negative.

METHODOLOGY

A linear inequality is similar to a linear equation in that the largest exponent of a variable is 1.
Consider the following true statement: $-5 < 3$

If we **add or subtract** any value to both sides, the statement above **will still be true.**

$-5 + 2 < 3 + 2$ $\qquad$ $-5 - 2 < 3 - 2$

$-3 < 5$ $\qquad$ $-7 < 1$

But if we multiply or divide both sides by -1: $-5 \times -1 < 3 \times -1$

$5 < -3$ which is not true.

If the direction of the inequality sign is reversed: $5 > -3$, the statement would become true.

Rules that are always applicable when working with inequalities

- Change the direction of the inequality sign whenever you **multiply or divide by a negative number.**
- **Do not change** the direction of the inequality sign if you multiply or divide by a positive number
- Do not change the direction of the inequality sign if you **add or subtract** by a number or expression.

Note that we cannot multiply or divide by a variable.

Inequalities can be represented by **a number line and/or interval notation.**
Interval notation is a way of writing subsets of the real number line by using square brackets, round brackets or a mixture of the two depending on the intervals/inequalities.
A closed interval is the one that includes its end points e.g. $-3 \leq x \leq 1$
An open interval is the one that does not include its end points e.g. -3<x<1 inequality/notation

INEQUALITY/INTERVAL	INTERVAL NOTATION	INTERVAL TYPE
$a \leq x \leq b$	$[a, b]$	Closed interval
$a < x < b$	(a, b)	Open interval
$a \leq x < b$ $a < x \leq b$	$[a, b)$ $(a, b]$	Half-open interval
$a \leq x < \infty$ $-\infty < x \leq b$	$[a, \infty)$ $(-\infty, b]$	Infinite and closed

$a < x < \infty$ $-\infty < x < b$ $-\infty < x < \infty$	(a, ∞) $(-\infty, b)$ $(-\infty, \infty)$	Infinite and open

Examples:

Solve the following inequalities and then represent solutions on a number.

1. $-2x > 6$

$x < -3$ …divide both sides by -2

All real number less than -3

2. $4(2x - 1) < 5x + 2$

$8x - 4 < 5x + 2$

$8x - 5x < 2 + 4$

$3x < 6$

$x < 2$

Number line:

Interval Notation: $(-\infty, -3)$ $\quad$ $(-\infty, 2)$

3. $\frac{2-x}{3x+1} \geq 2$

$(3x + 1) \times \frac{2-x}{3x+1} \geq 2(3x + 1)$ …LCD: $(3x + 1)$

$2 - x \geq 6x + 2$

$-5x \geq 0$

$x \leq 0$ …divide by -5 (inequality sign changes)

4. $\frac{4}{3}x - 6 \leq 7x + 2$

$3 \times \frac{4}{3}x - 3 \times 6 \leq 3 \times 7x + 3 \times 2$

LCD: 3

$4x - 18 \leq 21x + 6$

$-17x \leq 24$

$x \geq -\frac{24}{17}$

Number line:

Interval Notation: $(-\infty, 0]$ $\quad$ $[-\frac{24}{17}, \infty)$

ACTIVITIES/ ASSESSMENT

1. Solve the following equations and represent solutions on a number line and in interval notation.

a) $x + 18 \leq 9 - 2x$

b) $x - 3 < 2x = 5$

b) $5(x - 1) > 7(x - 1)$

d) $4(x - 3) - 2(2x - 1) \geq 0$

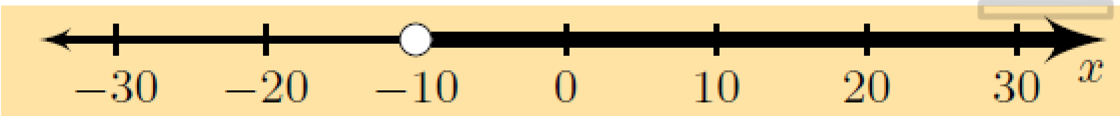
e) $\frac{x}{3} - \frac{x}{2} > 1$

f) $6 \quad \frac{3x}{4} - x \leq 1\frac{1}{2}$

g) $\frac{y+5}{3} + y \leq 1$

h) $\frac{3y+2}{4} - \frac{y-6}{3} > 0$

2. Look at the **number line** and write down the inequality it represents.

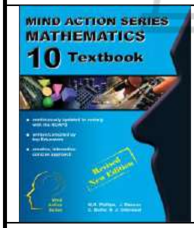


TOPIC: ALGEBRA PART 3 (Lesson 10)	Weighting	30 ± 3	Grade	10
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**LINEAR INEQUALITIES with graphical solutions**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Number line, Interval Notation, compound			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Greater than sign, Less than sign, Greater than and equal sign, Less than and equal to sign

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Signs not changed after dividing or multiplying by a negative number.

METHODOLOGY

Compound Linear Inequalities.

Examples:

Solve for x and represent the answer in a number line and in interval notation.

1. $5 \leq x + 3 < 8$

   $5 - 3 \leq x < 8 - 3$ …subtract 3 from all terms

   $2 \leq x < 5$

Number Line:

Interval Notation: $[2, 5)$

2. $3 < 3x - 1 < 5$

   $3 + 1 < 3x < 5 + 1$ …add 1 to all terms

   $\frac{4}{3} < x < 3$ …divide all terms by 3

Number line:

Interval notation: $(\frac{4}{3}, 3)$

3. $2 < 3 - \frac{1}{2}x \leq 5$

   $2 - 3 < -\frac{1}{2}x \leq 5 - 3$

   $-2 \times -1 < x < -2 \times 2$

   $2 < x \leq -4$ …Note that the inequality signs had to be reversed

   $-4 \leq x < 2$

Number line:

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Interval Notation: $[-4, 2)$

ACTIVITIES/ ASSESSMENT

1. Solve for x and represent your answer on a number line and in interval notation.

a) $-2 \leq x - 1 < 5$

b) $-3 \leq x = 2 \leq 4$

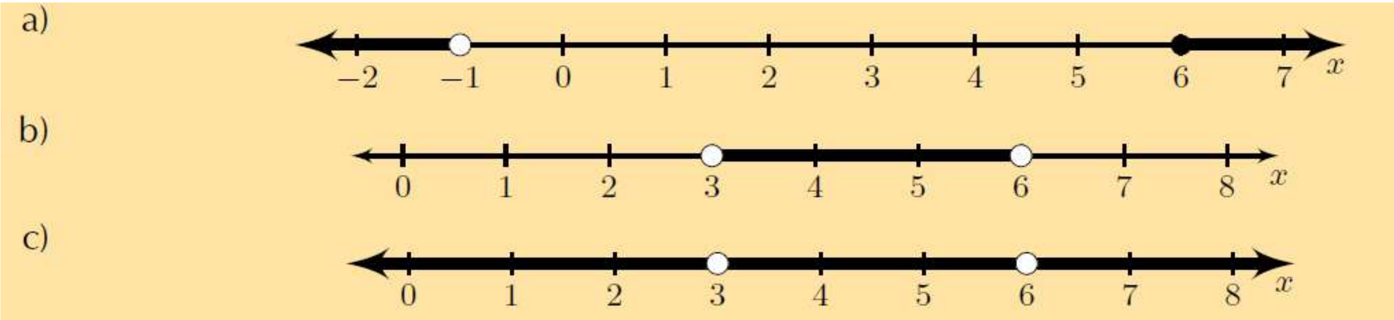
c) $-5 \leq 2x + 1 \leq 5$

d) $-8 < 3x - 2 < 4$

e) $-9 < 1 - 5x \leq 5$

f) $-2 \leq 2 - x < 2$

2. Look at the number line and write down the inequality it represents.



TEST: EQUATIONS AND INEQUALITIES

MARKS: 25 **DURATION: 30 Min**

QUESTION 1 [15 Marks]

1. Solve for x:

1.1 $2x^2 - 7x + 3 = 0$ (3)

1.2 $2(1 - 3x) = -2$ (4)

1.3 $\frac{x+1}{3} - \frac{x-2}{5} - 2 = 0$ (3)

2. Solve for x and y simultaneously:

$4x + y = -5$ and $-3x + 4y = 18$ (5)

QUESTION 2 [10 Marks]

Solve for the following inequalities:

2.1 $-5 < \frac{3x-1}{2} \leq 10$ and represent the answer in interval notation. (4)

2.2 Given: $\frac{x}{4} + 15 \leq \frac{5x}{3} - 2$ where $x \epsilon R$

2..2.1 Solve for x (3)

2.2.2 Represent the answer to 4.1 in a number line. (1)

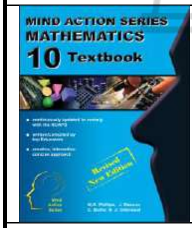
2.3 The formula $F = 32 + \frac{9C}{5}$ is used for converting temperatures for degrees Celsius (℃) to degrees Fahrenheit ($°F$). Make C the subject of this formula. (2)

TOPIC: EUCLIDEAN GEOMETRY (Lesson 1)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Revise lines and angles**				
RELATED CONCEPTS/ TERMS/VOCABULARY	**Parallel lines, intersect, vertex**				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Types of angles

RESOURCES

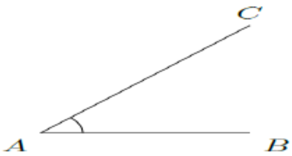


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Assuming that lines are parallel

METHODOLOGY

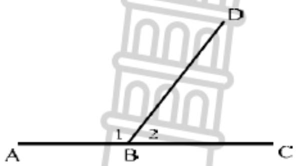
An angle is formed when **two straight lines** meet at a point, also known as a **vertex.**



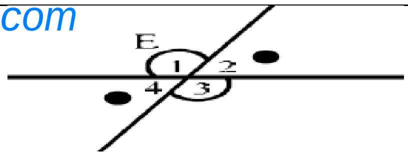
Angles are labelled with a caret on a letter, for example, $\hat{A}$.
Angles can also be labelled according to the **line segments** that make up the angle, for example $C\hat{A}B$ or $B\hat{A}C$.
The "∠" symbol is a short method of writing angle in geometry and is often used in phrases such as "sum of $\angle s$ in Δ".
Angles are measured in degrees which is denoted by °, a small circle raised above the text, similar to an exponent.

Types of Angles

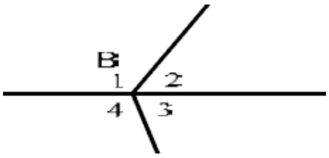
1. **Adjacent angles** on a straight line are supplementary.
   Supplementary angles add up to 180°
   Angles that **share a vertex** and a common side.
   ABC is a straight line. ∴ $\hat{B}_1 + \hat{B}_2 = 180°$



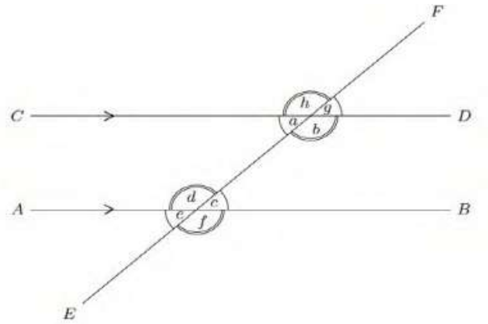
2. If two lines intersect, **vertically opposite angles** are equal.
   Two lines intersect if they **cross each** other at a point.
   Vertically opposite angles are **angles opposite each other** when two lines intersect.
   They share a vertex and are equal.
   $\hat{E}_1 = \hat{E}_3$ and $\hat{E}_2 = \hat{E}_4$



3. The **angles around a point** add up to 360° (Revolution).
   $\hat{B}_1 + \hat{B}_2 + \hat{B}_3 + \hat{B}_4 = 360°$



PARALLEL LINES
Parallel lines are always the **same distance apart** and they are **denoted by arrow symbols** as shown below.



$CD \parallel AB$. EF is a **transversal line**.
A transversal line **intersects** two or more parallel lines.

The **properties of the angles** formed by the above intersecting lines.

1. Corresponding Angles
   Corresponding angles **lie either both above or both below the lines** and on the **same side of the transversal**.
   If the **lines are parallel**, the **corresponding angles will be equal**.
   $\hat{h} = \hat{d}, \quad \hat{a} = \hat{e}, \quad \hat{g} = \hat{c}$ and $\hat{b} = \hat{f}$

2. Alternate Angles
   Alternate angles **lie on opposite sides of the transversal** and between the lines.
   If the **lines are parallel**, the **alternate angles will be equal**.
   $\hat{a} = \hat{c}$ and $\hat{d} = \hat{b}$

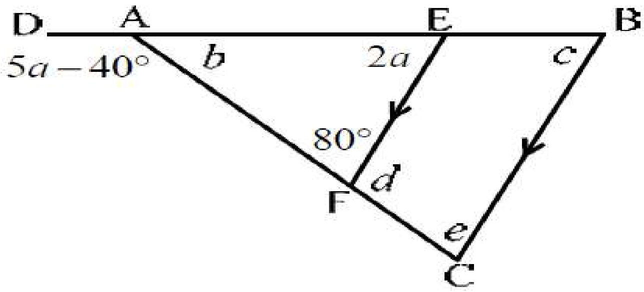
3. Co-interior Angles
   Co-interior angles lie on the same side of the transversal between the lines.
   If the **lines are parallel**, the **co-interior angles are supplementary**

If two lines are intersected by a transversal such that **corresponding angles are equal**; or **alternate angles are equal**; or co-interior angles are supplementary, then the **two lines are parallel**.

ACTIVITIES/ ASSESSMENT

Exercise 1: Mind Action Series (Pg. 165)

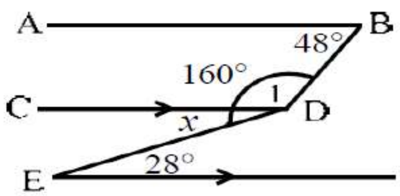
1. In ΔABC, EF∥BC. BA is produced to D.



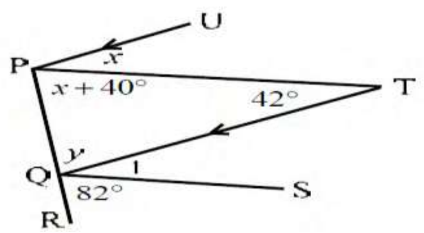
a) Calculate a and hence show that AE = AF.

b) Calculate, with reasons, the value of $b, c, d, and\ e$.

2. In the diagram below, CD∥EF, D$\hat{E}$F = 28°, A$\hat{B}$D = 48° and B$\hat{D}$E = 160°. Prove that AB∥CD.



3. In the diagram below, PU∥ QT , $\hat{T}$ = 42° , R$\hat{Q}S = 82°$, P$\hat{Q}$T = y, U$\hat{P}$T = x and Q$\hat{P}$T = x + 40°



a) Prove that PT∥QS

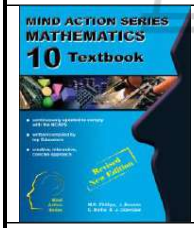
b) Calculate y

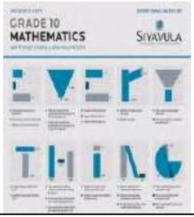
TOPIC: EUCLIDEAN GEOMETRY (Lesson 2)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Revise Triangles: Classification, Congruency and Similarity**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Types of triangles

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Using congruency conditions without understanding

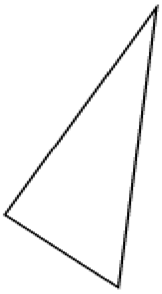
METHODOLOGY

PROPERTIES OF TRIANGLES

A triangle is a three-sided polygon. Triangles can be classified **according to sides** and also be classified **according to angles.**

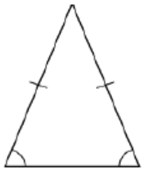
TYPES OF TRIANGLES according to **sides**

a) Scalene Triangle:



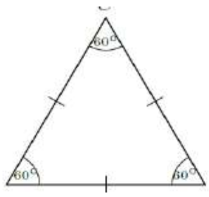
No sides are equal in length.

b) Isosceles Triangle:



Two sides are equal.
Angles opposite equal sides are equal.

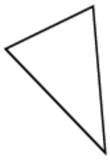
c) Equilateral Triangle:



All three sides are equal.
All three interior angles are equal

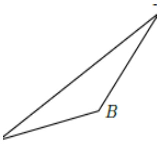
TYPES OF TRIANGLES according to **angles**

a) Acute-angled Triangle



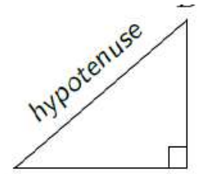
All 3 interior angles are less than 90°

b) Obtuse-angled Triangle

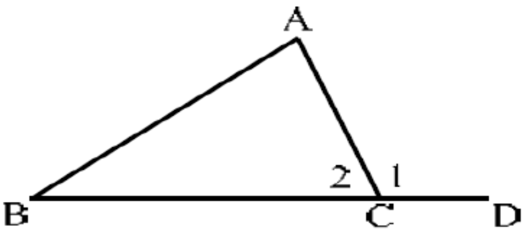


One interior angle is greater than 90°
The other two are acute

c) **Right-angled Triangle**



One interior angle is equal to 90°
The other two are acute



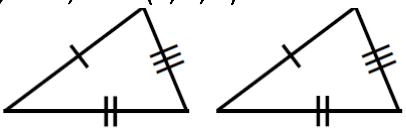
In triangle ABC, BC is produced to D.

$\hat{A} + \hat{C} + \hat{C}_2 = 180°$ (Sum of $\angle s\ of\ a\ \Delta$)

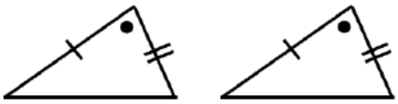
$\hat{C}_1 = \hat{A} + \hat{B}$ (ext. $\angle\ of\ \Delta$)

2. CONGRUENT TRIANGLES – 4 Conditions

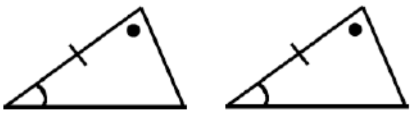
a) If three sides of a triangle are equal in length to the corresponding sides of another triangle, then the two triangles are congruent. Side, Side, Side (S, S, S)



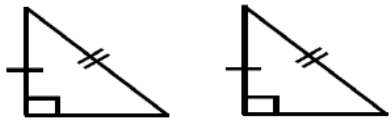
b) If two sides and the **included** angle of a triangle are equal to the corresponding two sides and **included** angle of another triangle, then the two triangles are congruent. Side, Angle, Side (S, A, S)



c) If one side and two angles of a triangle are equal to the corresponding one side and two angles of another triangle, then the two triangles are congruent. Angle, Side, Angle (A, S, A)



d) If the hypotenuse and one side of a right-angled triangle are equal to the hypotenuse and the corresponding side of another right-angled triangle, then the two triangles are congruent.
90°, Hypotenuse, Side (R, H, S).

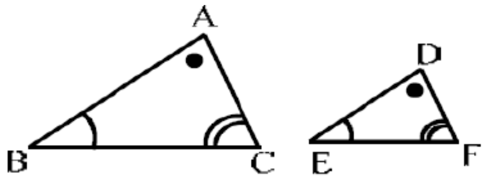


We use ≡ to indicate that triangles are congruent.

NOTE: The order of letters when labelling congruent triangles is very important.

3. SIMILAR TRIANGLES
Two triangles are similar if **one triangle is a scaled version of the other**. This means that their **corresponding angles are equal** in measure and the **ratio of their corresponding sides are in proportion**. The two triangles have **the same shape**, but different scales.
Congruent triangles are similar triangles, but **not all similar triangles are congruent**.
We use /// to indicate that two triangles are similar.

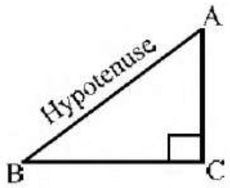
a) If all three pairs of corresponding angles of two triangles are equal, then the triangles are similar.
Angle, Angle, Angle (A, A, A)



b) If all three pairs of corresponding sides of two triangles are in proportion, then the triangles are Similar. $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$. Side, Side, Side (S, S, S)

NOTE: The order of letters for similar triangles is very important.
Always label similar triangles in corresponding order.

4. PYTHAGORAS THEOREM

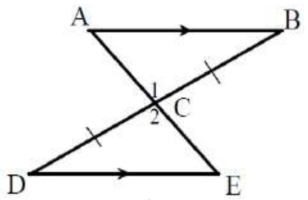


$AB^2 = BC^2 + AC^2$

ACTIVITIES/ ASSESSMENT

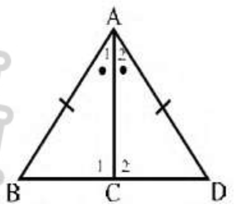
Exercise 1: Mind Action Series (Pg. 166)

1. AB||DE and DC = CB



a) Prove that AC = CE and b) AB = DE

2.

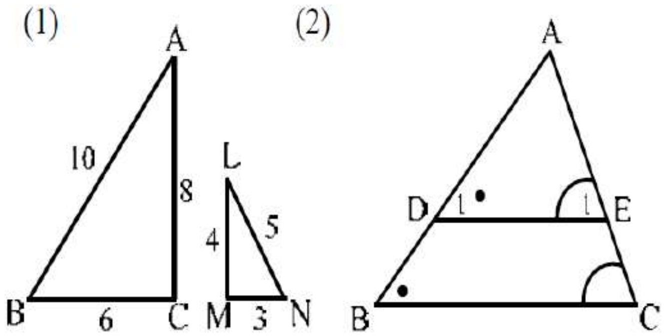


Prove that Δ ABC≡ Δ ADC using different conditions of congruency.

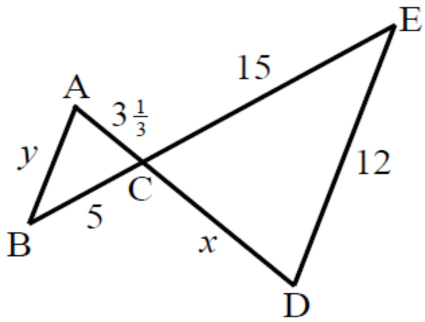
3. In the diagram below, sides PR and QS of triangles PQR and SQR intersect at T. PD = SR and

$\hat{P} = \hat{S} = 90°$. Prove that $\Delta PQR \equiv \Delta SRQ$

4. Show that the following triangles are similar:



5. If $\Delta ABC /// \Delta DEC$, calculate *x* and *y*.



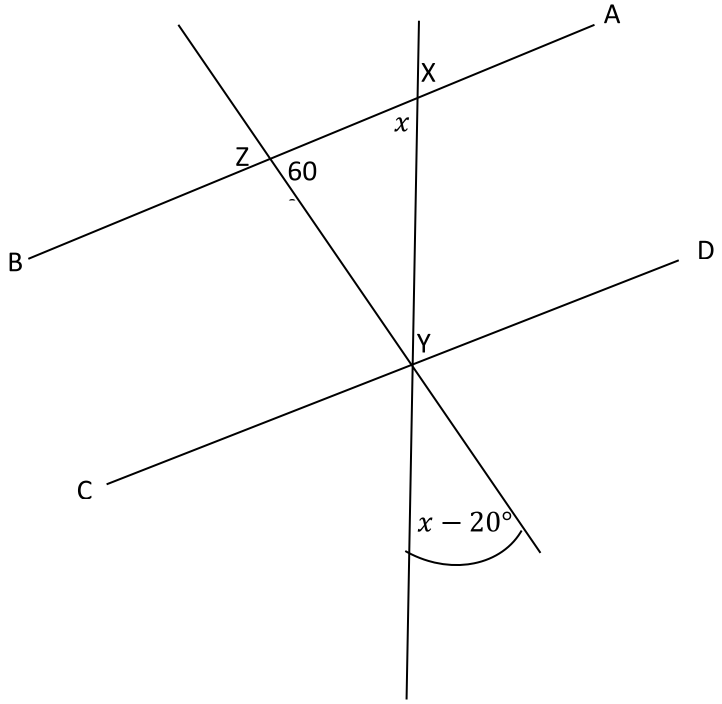
COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

TEST 1: LINES, ANGLES AND TRIANGLES

MARKS: 25 **DURATION: 30 Min**

QUESTION 1

1.1 In the diagram below, AB and DC are two parallel lines cut by two transversal lines at X, Y and Z respectively.



1.1.1 Determine giving reasons, the value of x in the diagram: (6)

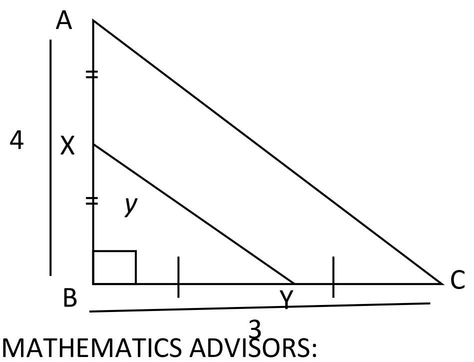
1.1.2 Name one pair of co-interior angles (1)

1.1.3 Name one pair of alternate angles (1)

1.1.4 Complete: If two parallel lines are cut by a transversal, then the co-interior angles are ....(1)

1.1.5 Complete: The size of angle XYD = Reason .... (2)

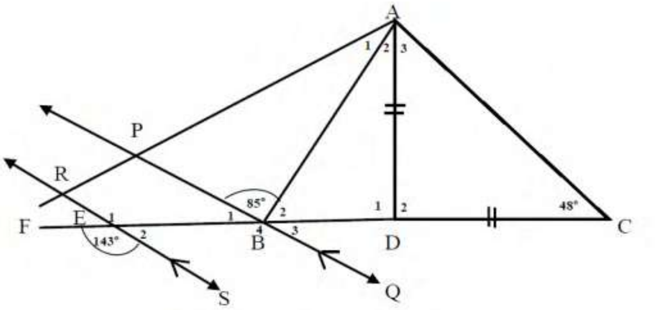
1.2 Determine with reasons, the value of y (XY) in the diagram below. Given that AB = 4 units and BC = 3 units. X and Y are the midpoints of AB and BC respectively. (5)



COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

QUESTION 2

In the diagram below, AD = CD and PQ∥RS. AR and FC are straight lines. RS and FC intersect at E also PQ intersects FC at B.



2.1 Determine the sizes of the following angles, giving appropriate reasons:

2.1.1 $\widehat{D}_1$ (2)

2.1.2 $\widehat{B}_1$ (2)

2.1.3 $\widehat{A}_2$ (2)

2.2 Show that $R\widehat{E}F = \widehat{B}_3$ (3)

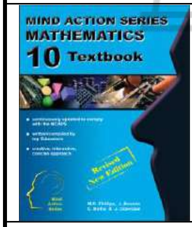
[9]

TOPIC: EUCLIDEAN GEOMETRY (Lesson 3)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Properties of Quadrilaterals (sides, angles and diagonals)**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Polygon, Straight line, diagonals, bisect, intersect, adjacent				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Parallel lines, interior angles

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Treating parallelogram as a rectangle or as a rhombus.

METHODOLOGY

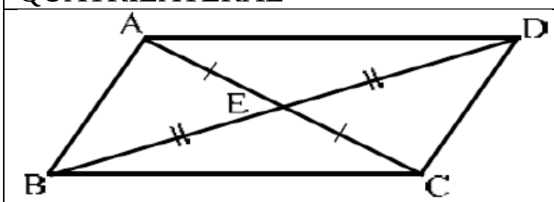
A quadrilateral is a closed shape (polygon) consisting of four straight line segments.
A **polygon** is a two-dimensional figure with three or more straight sides.
The **interior angles** of a quadrilateral add up to 360°.

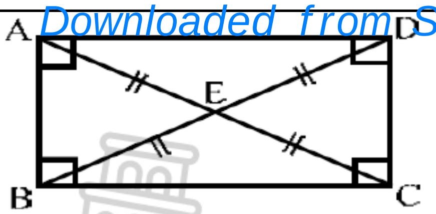
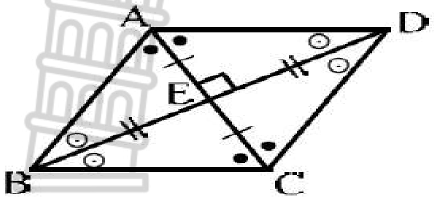
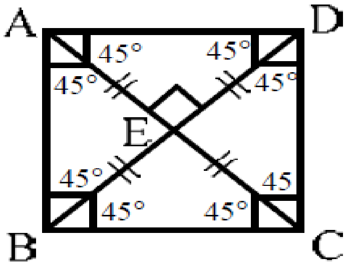
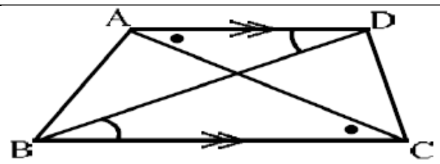
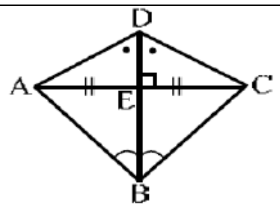
1. PARALLELOGRAM
   A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
2. RECTANGLE
   A rectangle is a parallelogram that has all four angles equal to 90°.
3. RHOMBUS
   A rhombus is a parallelogram with all four sides of equal length.
4. SQUARE
   A square is a rhombus with all four interior angles equal to 90°
   A square has all the properties of a rhombus.
   OR
   A square is a rectangle with all four sides equal in length.
5. TRAPEZIUM
   A trapezium is a quadrilateral with one pair of opposite sides parallel.

NOTE: A trapezium is sometimes called a trapezoid.

6. KITE
   A kite is a quadrilateral with two pairs of adjacent sides equal.

PROPERTIES OF QUADRILATERALS

QUATRILATERAL	PROPERTIES
	• Both pairs of opposite sides are parallel. • Both pairs of opposite sides are equal in length. • Both pairs of opposite angles are equal. • Both diagonals bisect each other.
	• Both pairs of opposite sides are parallel. • Both pairs of opposite sides are of equal length. • Both pairs of opposite angles are equal. • Both diagonals bisect each other.

	• **Diagonals are equal in length.** • **All interior angles are equal to 90°**
	• Both pairs of opposite sides are parallel. • Both pairs of opposite sides are equal in length. • Both pairs of opposite angles are equal. • Both diagonals bisect each other. • **All sides are equal in length.** • **The diagonals bisect each other at 90°** • **The diagonals bisect both pairs of opposite angles.**
	• Both pairs of opposite sides are parallel. • Both pairs of opposite sides are equal in length. • Both pairs of opposite angles are equal. • Both diagonals bisect each other. • All sides are equal in length. • The diagonals bisect each other at 90° • The diagonals bisect both pairs of opposite angles. • **All interior angles equal 90°.** • **Diagonals are equal in length.**
	• One pair of opposite side are parallel. • The diagonals of a trapezium intersect but don't bisect each other. • Diagonals lie between parallel lines and therefore, the alternate angles are equal.
	• Diagonal between equal sides bisects the other diagonal. • One pair of opposite angles are equal (the angles between unequal sides). • Diagonal between equal sides bisects the interior angles and is an axis of symmetry. • Diagonals intersect at 90°

ACTIVITIES/ ASSESSMENT

1. The following are properties of some quadrilaterals.
   a) Having a pair of parallel sides.
   b) Having two pairs of parallel sides.
   c) Having four right angles.
   d) Having four equal sides.
   e) Having equal diagonals.

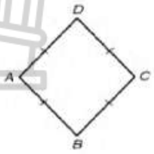
In the table below, mark a "√" in the box if the quadrilateral has the property.

	PROPERTIES				
QUADRILATERALS	a)	b)	c)	d)	e)
Kite					
Trapezium					

Parallelogram					
Rectangle					
Square					
Rhombus					

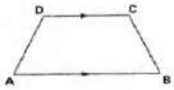
2. Referring to the figure below, use the names of the quadrilaterals to complete the sentences.

1.



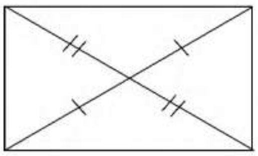
This is a ________________.

2.



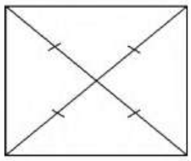
This is a ________________.

3.



This is a ________________.

4.



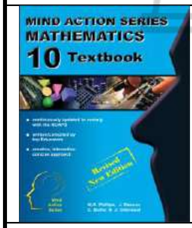
This is a ________________.

TOPIC: EUCLIDEAN GEOMETRY (Lesson 4)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Properties of Quadrilaterals (sides, angles and diagonals)**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Parallel, interior angles, bisect				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Properties of quadrilaterals, naming quadrilaterals, Naming triangles

RESOURCES



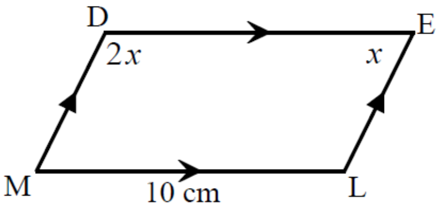
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Difference between parallelogram and rhombus

METHODOLOGY

Examples:

1. DELM is a parallelogram.



a) Calculate the value of x and hence the sizes of the interior angles.
b) If DE = 2DM and ML = 10 cm, determine the length of the other sides of DELM.

SOLUTION:

a) $2x + x = 180°$ co-int $\angle s$; DM‖EL

$3x = 180°$

$x = 60°$

$\hat{E} = 60°$ and $\hat{M} = 60°$ opp $\angle s$ of parm equal

$\hat{D} = 2(60°) = 120°$

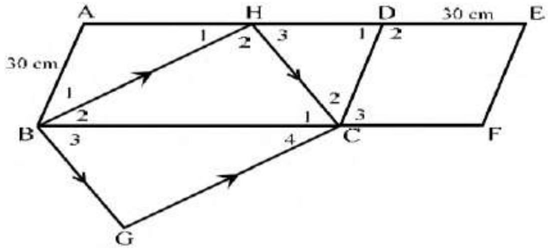
$\hat{L} = 120°$ opp $\angle s$ of parm equal

b) DE = 10 cm opp sides of a parm

DM = 5 cm DE = 2DM

EL = 5 cm opp sides of a parm

2. ABCD is a parallelogram. BH bisects $A\hat{B}C$ and HC bisects $B\hat{C}D$. $A\hat{B}C = 60°$, $\hat{F} = 60°$, BH||GC and BG||HC. AD is produced to E such that AB = DE = 30 cm. BC is produced to F.



Prove that:

a) BGCH is a rectangle

b) DCFE is a rhombus

SOLUTION:

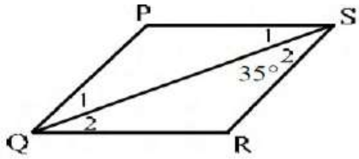
a) BCGH is a parallelogram	BG||HC
$A\hat{B}C = 60°$	given
$\hat{B}_1 = \hat{B}_2 = 30°$	BH bisects $A\hat{B}C$
$B\hat{C}D = 120°$	co-int ∠ ; AB||DC
$\hat{C}_1 = \hat{C}_2 = 60°$	HC bisects $B\hat{C}D$
$\hat{H}_2 = 90°$	int $\angle s\ of\ \Delta$
∴ BGCH is a rectangle	BGCH is a parm with an interior $\angle = 90°$
b) $\hat{F} = 120°$	given
$\hat{C}_1 + \hat{C}_2 = 120°$	proved
∴ $\hat{F} = \hat{C}_1 + \hat{C}_2$	
∴ DC||EF	corr $\angle s =$
AD||BC	ABCD is a parallelogram
ADE and BCE are straight lines	given
∴ DE||CF	
∴ DCFE is a parallelogram	opp sides ||
DC = 30 cm	opp sides of a parm

DC = DE = 30 cm

∴ DCEF is a rhombus | DCEF is a parm with adjacent sides equal

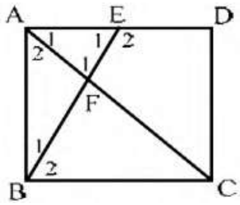
ACTIVITIES/ ASSESSMENT

1. PQRS is a rhombus with $\hat{S}_2 = 35°$.



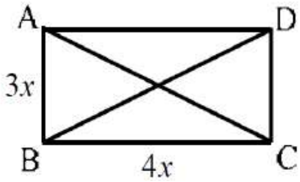
Calculate the size of all other interior angles.

2. ABCD is a square. AEˆB = 55°.



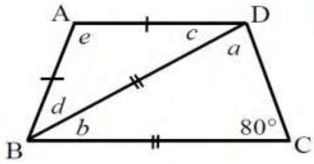
Calculate $\hat{F}_1$

3. In rectangle ABCD, AB = $3x$ and BC = $4x$.



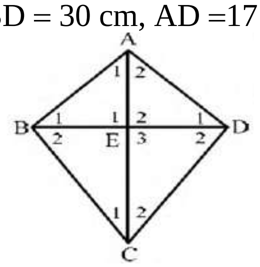
Find the length of AC and BD in terms of x.

4. ABCD is a trapezium with AD||BC. AB = AD and BD = BC. $\hat{C} = 80°$.



Determine the unknown angles.

5. ABCD is a kite. The diagonals intersect at E. BD = 30 cm, AD =17 cm and DC = 25 cm.



Determine:

(1) AE

(2) AC

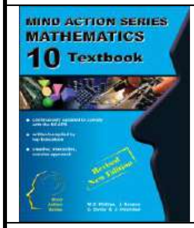
(3) $\hat{B}_1$ if $\hat{A}_1 = 20°$

TOPIC: EUCLIDEAN GEOMETRY (Lesson 5)	**Weighting**	30 ± 3	Grade	10	
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Properties of Quadrilaterals (sides, angles and diagonals)**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Triangle, quadrilaterals, parallel				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Types of quadrilaterals and Properties of quadrilaterals

RESOURCES



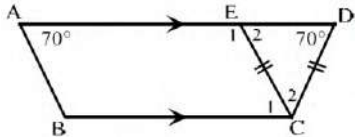
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Properties of a kite, stating reasons

METHODOLOGY

Examples:

In trapezium ABCD, AD||BC with $\hat{A} = \hat{D} = 70°$ and EC = DC.



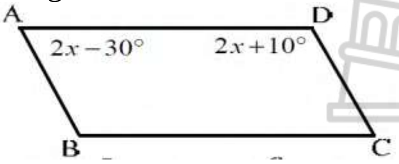
Prove that ABCE is a parallelogram.

Solution:

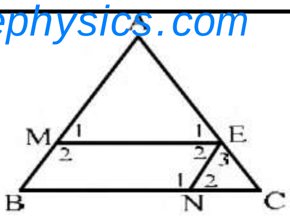
$\hat{E}_2 = 70°$	∠s opp = sides
$\hat{C}_1 = 70°$	alt ∠ s; AD||BC
$\therefore \hat{A} = \hat{C}_1$	
$\hat{E}_1 = 110°$	adj. ∠s on a line
$\hat{B} = 110°$	co-int ∠s; AD||BC
$\therefore \hat{C}_1 = \hat{B}$	
Therefore, ABCE is a parallelogram	opp ∠ s of quad equal

ACTIVITIES/ ASSESSMENT

1. Determine the sizes of the interior angles of parallelogram ABCD.

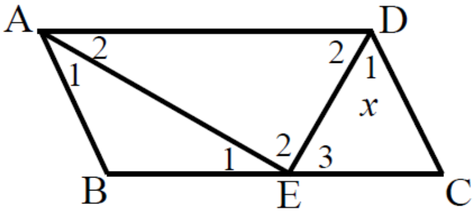


2. In ΔABC, $\hat{A} = 80°$ and $\hat{C} = 35°$.



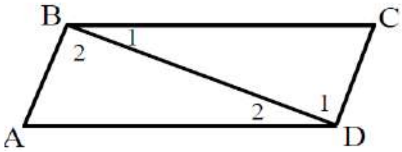
Calculate the interior angles of parallelogram MENB.

3. In parallelogram ABCD, AB = BE = DE.



Calculate the size $\widehat{D}_1$ if $\widehat{D}_1 = x$ and $\hat{A}_1 = 28°$.

4. ΔABD and ΔBCD are two isosceles triangles. $\hat{C} = 75°$ and $A\widehat{D}B = 30°$.



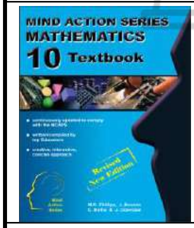
Prove that ABCD is a parallelogram.

TOPIC: EUCLIDEAN GEOMETRY (Lesson 6)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**The opposite Sides and Angles of a parallelogram are equal**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Diagonals, Parallel				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Properties of parallelogram, congruent triangles

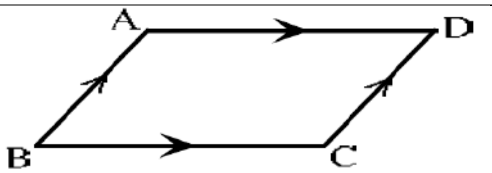
RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

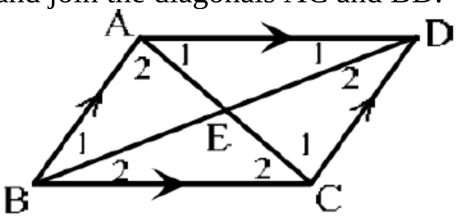
Naming angles as sides

METHODOLOGY



Required to prove: AB = CD; AD = BC; $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

Draw parallelogram ABCD and join the diagonals AC and BD.

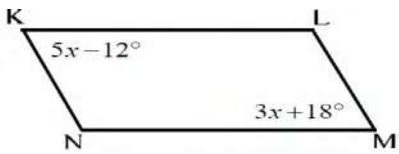


In ΔABC and ΔCDA:

(a) $\hat{A}_1 = \hat{C}_2$ — alt ∠s; AD||BC
(b) $\hat{A}_2 = \hat{C}_1$ — alt ∠s; AB||DC
(c) AC = AC — common side
∴ ΔABC ≡ ΔCDA — SAA
∴ AB = CD and AD = BC
Also $\hat{B} = \hat{D}$
Similarly, it can be proved that ΔABD ≡ CΔDB
∴ $\hat{A} = \hat{C}$

ACTIVITIES/ ASSESSMENT

1. KLMN is a parallelogram.



Calculate the size of the interior angles.

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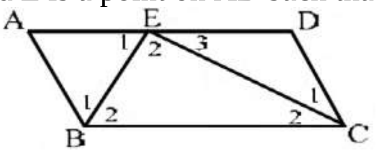
2. In parallelogram ABCD, AB = 50 cm and E is a point on AD such that AB = AE and CD = DE .

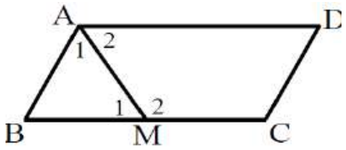


Determine:

a) DE

b) the perimeter of ABCD.

3. ABCD is a parallelogram. AM bisects $\hat{A}$. AB = AM. $\hat{C}$ =120° .



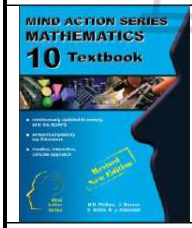
Calculate the sizes of all interior angles.

TOPIC: EUCLIDEAN GEOMETRY (Lesson 7)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**The diagonals of a parallelogram bisect each other**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Diagonals,				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Properties of a parallelogram, congruent triangles

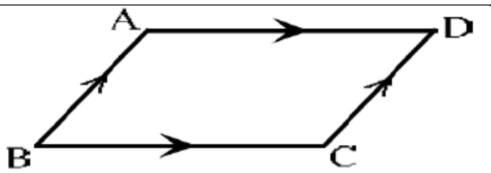
RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

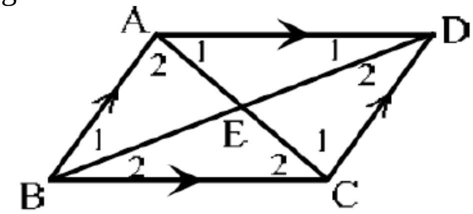
Choosing triangles to prove congruency.

METHODOLOGY



Required to prove that: AE = EC and BE = ED

Draw parallelogram ABCD and join the diagonals AC and BD.



In ΔABE and CDE:

a) $\hat{A}_2 = \hat{C}_1$ alt ∠ s; AB||DC

b) $\hat{B}_1 = \hat{D}_2$ alt ∠ s; AB||DC

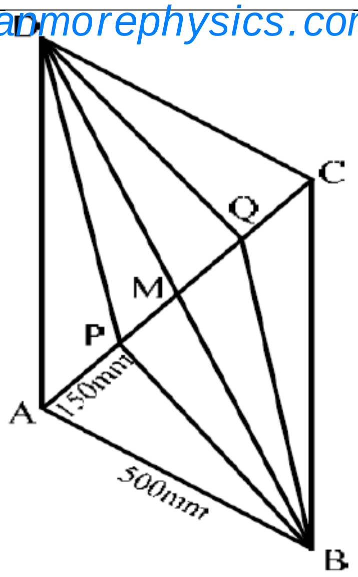
c) AB = DC opp sides of a parm

∴ ΔABC ≡ ΔCDA SAA

∴BE = ED and AE = EC

Example:

Diagonals AC and BD of parallelogram ABCD intersect at M. AP = QC and AC = 600 mm, AB = 500 mm and AP =150 mm.



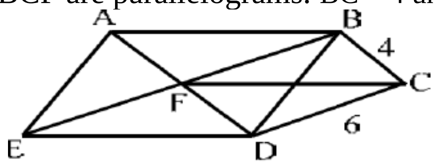
Prove that PBQD is a parallelogram

Solution:

AM = MC	diagonals of a parm
But AC = 600 mm	given
AM = MC = 300 mm	
AP = QC =150 mm	given
PM = MQ =150 mm	
Also, BM = MD	diagonals of parm
∴ PM = MQ and BM =MD	
∴ PBQD is a parallelogram	diagonal of quad bisect

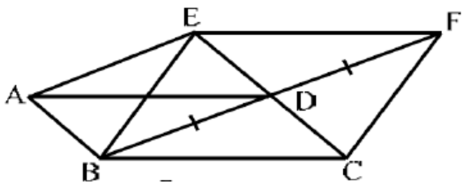
ACTIVITIES/ ASSESSMENT

1. In the diagram, BCDF, EDCF and ABCF are parallelograms. BC = 4 units and CD = 6 units.



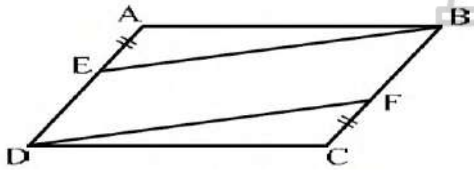
Prove that ABDE is a parallelogram.

2. Parallelograms ABCD and ABDE are given with DF = DB.



Prove that BCFE is a parallelogram.

3. ABCD is a parallelogram with AE = FC.



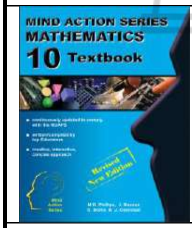
Prove that BEDF is a parallelogram.

TOPIC: EUCLIDEAN GEOMETRY (Lesson 8)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Diagonal, parallel				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Congruent triangles

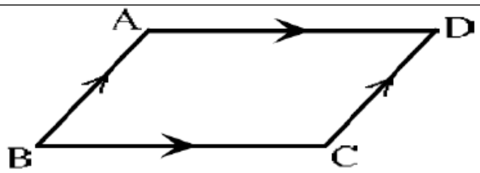
RESOURCES



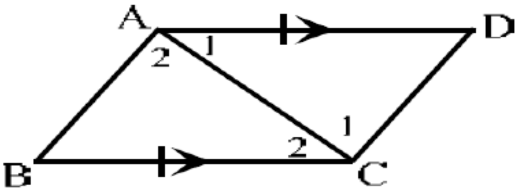
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Prove congruency and reasons for congruency

METHODOLOGY



Required to prove: ABCD is a parallelogram



In ΔABC and ΔCDA:

a) $\hat{A}_1 = \hat{C}_2$ alt s; AD||BC

b) AC = AC common side

c) AD = BC given

∴ ΔABC ≡ ΔCDA SAS

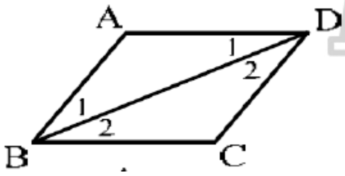
∴ $\hat{A}_2 = \hat{C}_1$

∴ AB||DC alt ∠s=

∴ ABCD is a parallelogram since the opposite sides are parallel

ACTIVITIES/ ASSESSMENT

1. In parallelogram ABCD, AB = AD and $\hat{C}$ =110° .



Calculate the size of all interior angles.

2. ΔABDand ΔBCD are two isosceles triangles. $\hat{C}$ = 75° and $A\hat{D}B$ = 30° .

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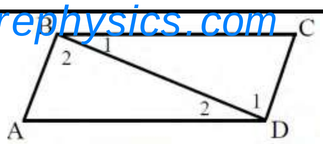
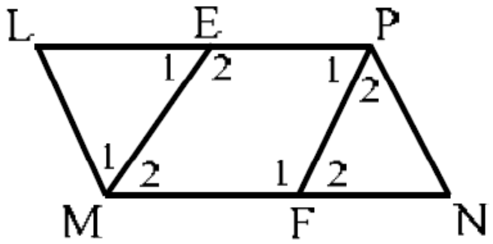


Prove that ABCD is a parallelogram.

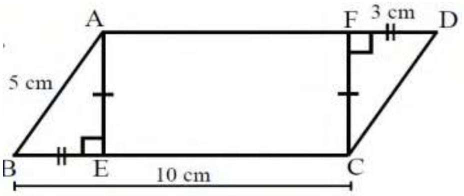
3. In quadrilateral LMNP, $\hat{E}_1 = 62°$, $\hat{P}_1 = 68°$, $\hat{P}_2 = 56°$, FP = FN and LE = LM.



Prove that:

a) LP||MN
b) LMNP is a parallelogram.

4. In quadrilateral ABCD, AB = 5 cm, BC = 10 cm, FD = 3 cm, BE = FD and AE = FC.
AE ⊥ BC and CF ⊥ AD



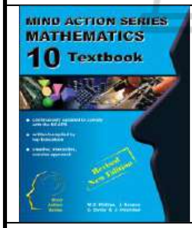
Prove that ABCD is a parallelogram.

TOPIC: EUCLIDEAN GEOMETRY (Lesson 9)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Midpoint Theorem**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Midpoint				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Parallel lines, types of angles, properties of parallelogram

RESOURCES

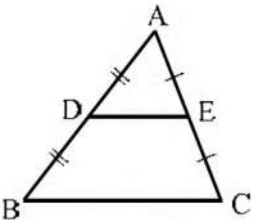


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

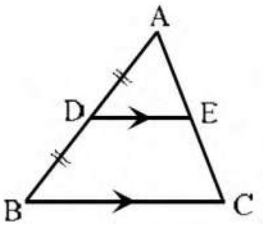
Assuming that any point between the line is the midpoint.

METHODOLOGY

The **midpoint** is the middle point in a line segment (bisects the line segment)
The Midpoint Theorem can be stated in the following two ways:

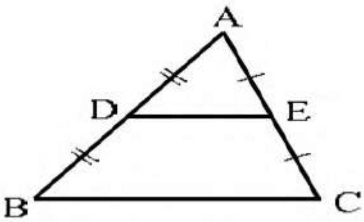


If AD = DB and AE = EC
Then DE∥BC and DE = $\frac{1}{2}$ BC

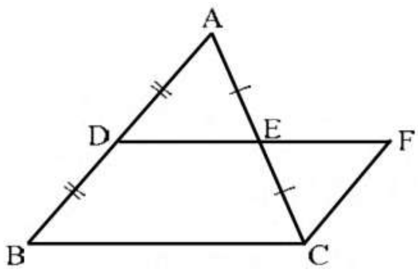


If AD = BD and DE∥BC
Then AE = EC and DE = $\frac{1}{2}$ BC

Prove that the line **joining the mid-points** of two sides of a triangle is **paralle**l to the third side and equal to **half the length of the third side.**



Extend DE to F so that DE = EF and join CF.



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1. Prove that BCFD is a parallelogram.

In Δ EAD and Δ ECF:

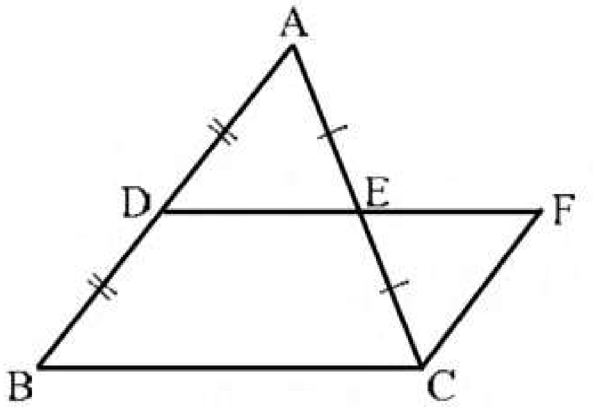
$A\hat{E}D = C\hat{E}F$	vert opp ∠ =
AE = CE	given
DE = EF	by construction
∴ Δ EAD ≡ Δ ECF	SAS
∴ $A\hat{D}E = C\hat{F}E$, these are alternate interior angles	
∴ BD∥ FC	
BD = DA	(given)
DA = FC	(Δ EAD ≡ Δ ECF0
∴ BD = TC	
∴ BCFD is a parallelogram	(one pair of opp sides = and ∥)

2. Use properties of parallelogram BCFD to prove that DE = $\frac{1}{2}$ BC

DF = BC	(opp sides of a parm)
And DF = 2(DE)	(by construction)
2 DE = BC	
∴ DE = $\frac{1}{2}$BC	

ACTIVITIES/ ASSESSMENT

1. In ΔABC, AD = DB and AE = EC. DE is produced to F.
   DB||FC and BC = 32 mm.



a) Prove that DBCF is a parallelogram.

b) Calculate the length of DE.

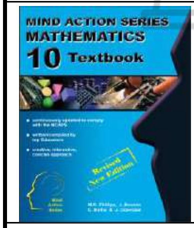
Siyavula: Exercise 7 – 7 pg. 264 – 266 No. 5,7,8,10, 11 (c, 11(d)

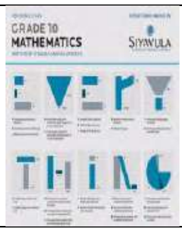
TOPIC: EUCLIDEAN GEOMETRY (Lesson 10)		Weighting	30 ± 3	Grade	10
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Midpoint Theorem**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Line joining the midpoints of two sides of a triangle.				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Midpoint, parallel lines

RESOURCES



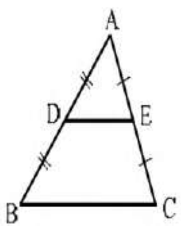


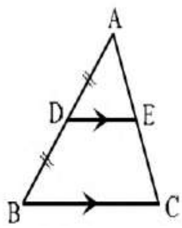
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Assuming that any point between the line is the midpoint.

METHODOLOGY

Use the information below to do activities:



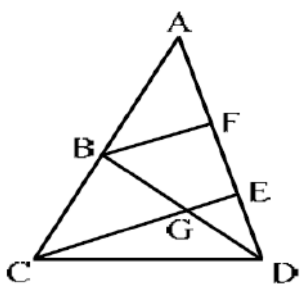


If AD = DB and AE = EC
Then DE∥BC and DE = $\frac{1}{2}$ BC

If AD = BD and DE∥BC
Then AE = EC and DE = $\frac{1}{2}$ BC

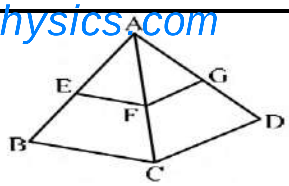
ACTIVITIES/ ASSESSMENT

1. In ΔACD, AB = BC, GE =15 cm, AF = FE = ED.



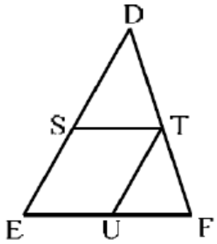
Calculate the length of CE.

2. In ΔABC, AE = EB and EF||BC. In Δ ACD, FG||CD.



Prove that AG = GD.

3. In ΔDEF, DS = SE, EU = EF and ST||EF.



Prove that SEUT is a parallelogram.

SIYAVULA: pg. 266 – 267 Ex. 7 – 7 No. 13, 14, 15 and 16

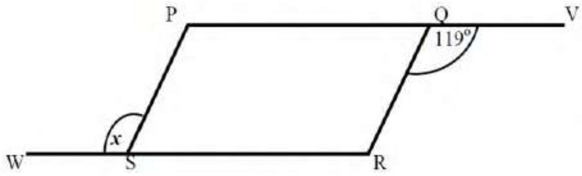
TEST 2: EUCLIDEAN GEOMETRY

TOTAL: 25 **DURATION: 30 Min**

QUESTION 1

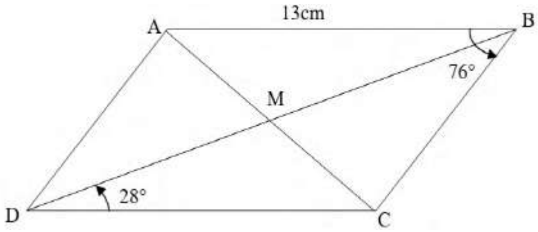
In the question, you must give reason to justify each of your statement.

1.1 Given: PQRS is a parallelogram with $R\hat{Q}V = 119°$. PV and WR are straight lines.



Calculate the magnitude of x (4)

1.2 In the diagram below, the diagonals of a parallelogram ABCD intersect at M.

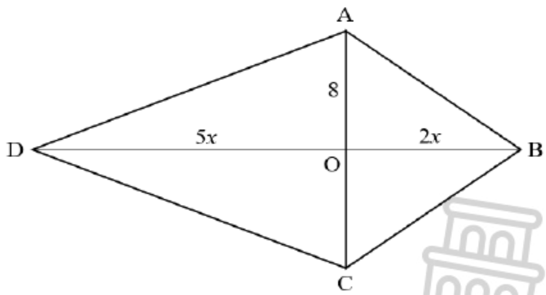


1.2.1 Write down giving reasons the length of DC. (2)

1.2.2 Calculate, giving reasons the size of $A\hat{D}M$. (2)

1.3 In the diagram below, ABCD is a kite having diagonals AC and BD intersecting at O.

AO = 8cm, BO = $2x$ cm and DO = 5 x CM.

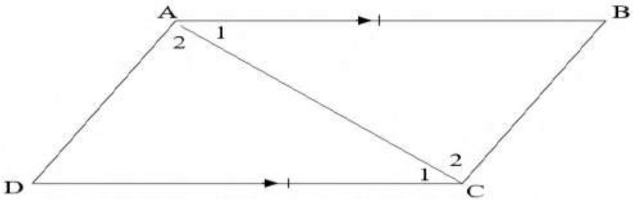


Write down, giving reasons, the length of OC. (2)

[10]

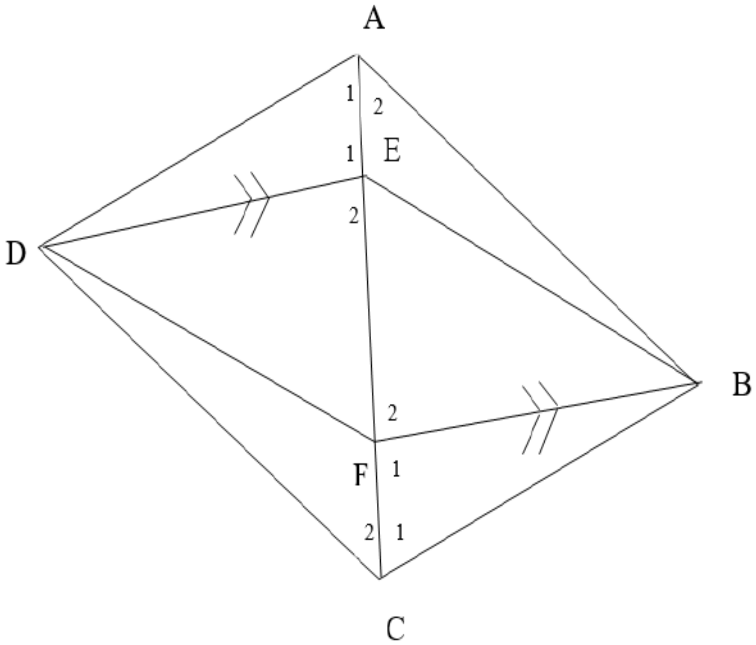
QUESTION 2

2.1 In the diagram below, ABCD is a quadrilateral having AB = DC and AB∥DC.



Use the diagram to prove that ABCD is a parallelogram. (5)

2.2 ABCD is a parallelogram. DE∥FB. Let $\hat{E}_1 = x$.



Prove:

2.2.1 $\hat{E}_1 = \hat{F}_1$ (3)

2.2.2 ΔAED ≡ ΔCFB (4)

2.2.3 DEBF is a parallelogram (3)

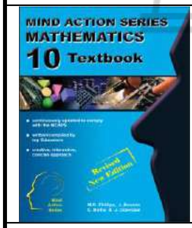
[15]

TOPIC: EUCLIDEAN GEOMETRY (Lesson 11)		Weighting	30 ± 3	Grade	10
Term	4	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Lines, Angles and Triangles - problems**				
RELATED CONCEPTS/ TERMS/VOCABULARY	**Intersect, parallel lines, transversal line, congruent**				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Types of angles, Congruent triangles, similar triangles

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Giving reason for congruent triangles

METHODOLOGY

Summary:

1. Parallel lines cut by a transversal line:

Co-interior angles are supplementary
Alternate angles are equal
Corresponding angles are equal
Vertically opposite angles are equal (any intersecting lines)
Adjacent angles are supplementary (any intersecting lines)

2. Congruent triangles:

SAS
AAS/ASA
SSS
RHS

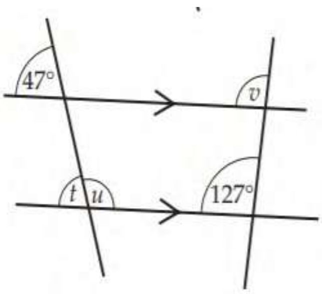
3. Similar triangles

AAA

ACTIVITIES/ ASSESSMENT

1. Calculate the size of the lettered angles. Give reasons for your answers

a)



b)

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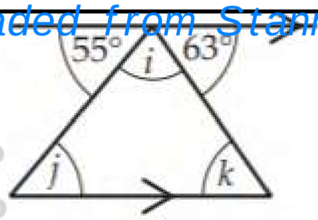
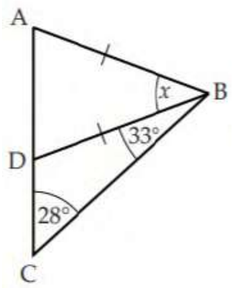
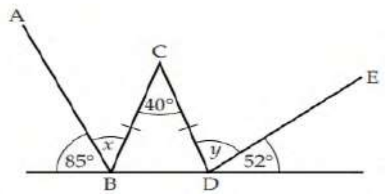


c)

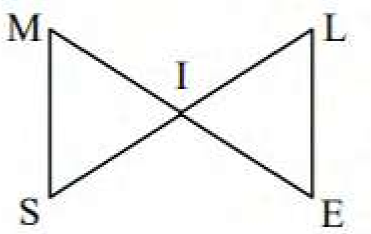


d)

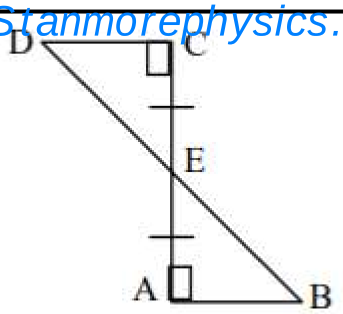


2.

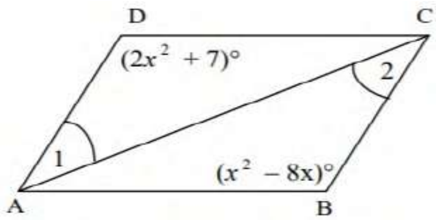
a) If I is the midpoint of ME and SL, show that ΔMIS ≡ ΔEIL.



b) Prove that ΔCED ≡ ΔAEB



c) Given that $\Delta ADC \equiv \Delta CBA$, calculate the value of x



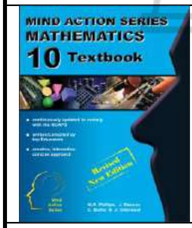
SIYAVULA: pg.270 – 271 no. 4 (a), (c) and pg. 274 No. 14

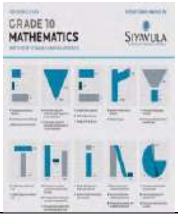
TOPIC: EUCLIDEAN GEOMETRY (Lesson 12)		Weighting	30 ± 3	Grade	10
Term	4	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Quadrilaterals and midpoint theorem** Consolidation and Extension Exercise				
RELATED CONCEPTS/ TERMS/VOCABULARY	Midpoint, similar, congruent				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Properties of quadrilaterals, midpoint theorem, Proofs of theorems

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Reading from the diagram.

METHODOLOGY

Summary:

A **quadrilateral** is a closed shape consisting of four straight line segments.

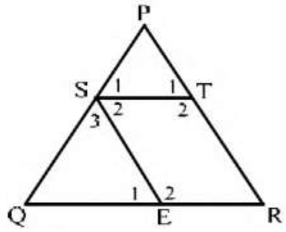
1. A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.
   – Both pairs of opposite sides are equal in length.
   – Both pairs of opposite angles are equal.
   – Both diagonals bisect each other.
2. A **rectangle** is a parallelogram that has all four angles equal to 90°
   – Both pairs of opposite sides are parallel.
   – Both pairs of opposite sides are equal in length.
   – The diagonals bisect each other.
   – The diagonals are equal in length.
   – All interior angles are equal to 90°.
3. A **rhombus** is a parallelogram that has all four sides equal in length.
   – Both pairs of opposite sides are parallel.
   – All sides are equal in length.
   – Both pairs of opposite angles are equal.
   – The diagonals bisect each other at 90°.
   – The diagonals of a rhombus bisect both pairs of opposite angles.
4. A **square** is a rhombus that has all four interior angles equal to 90°.
   – Both pairs of opposite sides are parallel.
   – The diagonals bisect each other at 90°.
   – All interior angles are equal to 90°.
   – The diagonals are equal in length.
   – The diagonals bisect both pairs of interior opposite angles (i.e. all are 45°
5. A **trapezium** is a quadrilateral with one pair of opposite sides parallel.
6. A **kite** is a quadrilateral with two pairs of adjacent sides equal.
   – One pair of opposite angles are equal (the angles are between unequal sides).
   – The diagonal between equal sides bisects the other diagonal.
   – The diagonal between equal sides bisects the interior angles.
   – The diagonals intersect at 90°.

• The **mid-point theorem** states that the line joining the mid-points of two sides of a triangle is **parallel**

to the third side and equal to **half the length** of the third side.

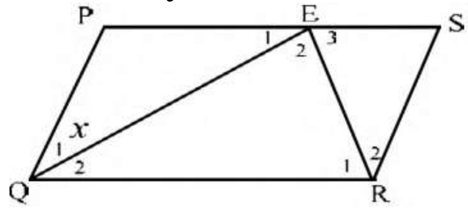
ACTIVITIES/ ASSESSMENT

1. In Δ PQR, PQ = PR and STRE is a parallelogram. $\hat{Q} = x$ and $\hat{P} = 2\hat{Q}$.



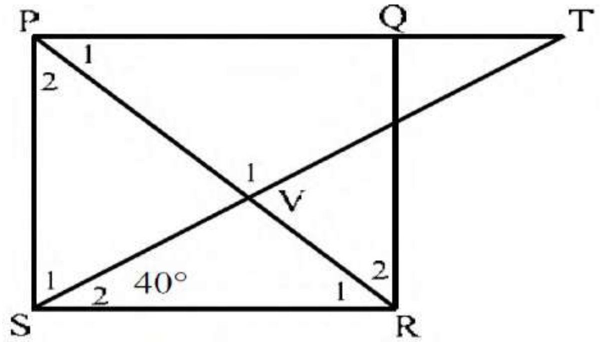
Calculate the sizes of the angles of STRE.

2. PQRS is a parallelogram. PQ = PE, QE = QR, ER = SR and P$\hat{Q}$ E = x.



Calculate the size of Q$\hat{E}$R.

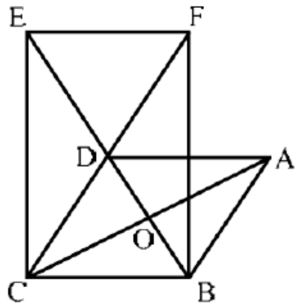
3. PSRQ is a square. Diagonal PR and line SVT intersect at V and $\hat{S}_2 = 40°$.



3.1 Calculate $\hat{V}_1$

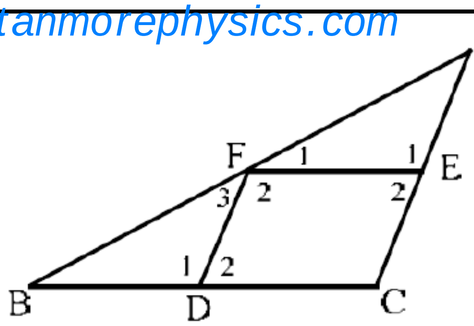
3.2 Prove that $\frac{PR}{PS} = \sqrt{2}$

4. ABCD is a parallelogram. FD = DC and DE = 2DO. DO = x.



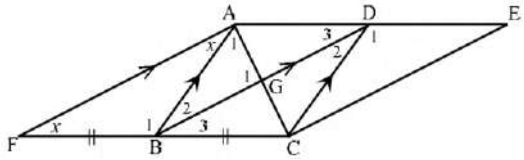
Prove that BCEF is a parallelogram.

5. FDCE is a parallelogram. CE is produced to A such that CE = EA and CD = DB.



Prove that ΔBDF ≡ ΔFEA

6. AFCE is a parallelogram. AB||DC and AF||BD. $\hat{F}$ = F$\hat{A}$B = x and FB = BC.

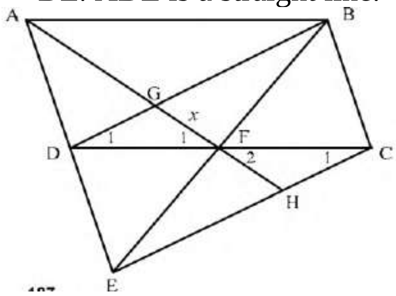


Prove that:

a) ABCD is a rhombus

b) $\hat{A}_1 = 90° - x$

7. ABCD is a parallelogram. GF = FH = x and AD = DE. ADE is a straight line.



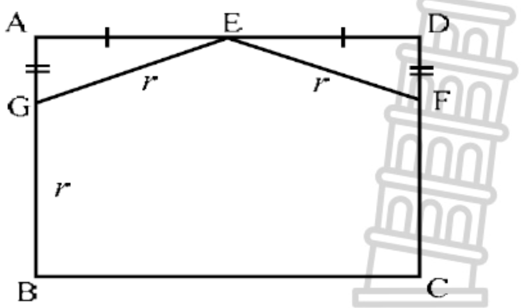
Prove that:

a) DECB is a parallelogram

b) AG = 2GF

c) FH = $\frac{1}{3}$AF

8. ABCD is a parallelogram with AE = ED and AD = a. GE = EF = BG = r and AB = b.
DF = $\frac{1}{2}\sqrt{4r^2 - a^2}$.



a) Prove that ABCD is a rectangle

b) Express AG in terms of b and r

c) Prove that r = $\frac{a^2+4b^2}{8b}$

SIYAVULA: pg. 276 No. 17, pg. 277 no. 21, pg. 279 No. 30, 31 and 280 No. 34

TOPIC: EUCLIDEAN GEOMETRY (Lesson 13)		**Weighting**	30 ± 3	Grade	10
Term	4	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Quadrilaterals and midpoint theorem** Consolidation and Extension Exercise				
RELATED CONCEPTS/ TERMS/VOCABULARY	Midpoint, bisect, perpendicular, congruent, similar				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Properties of quadrilaterals, midpoint theorem, Proofs of theorems

RESOURCES

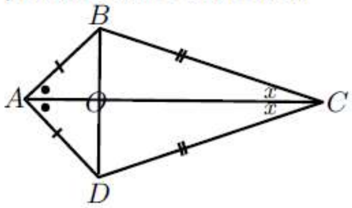
GRADE 10 ASSESSMENT BOOKLET

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

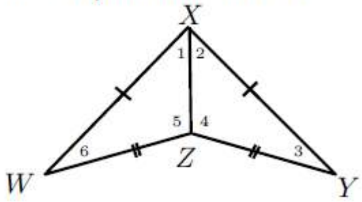
METHODOLOGY

ACTIVITIES/ ASSESSMENT

1. Use the sketch of quadrilateral $ABCD$ to prove the diagonals are perpendicular to each other.

2.2


2. Explain why quadrilateral $WXYZ$ is a kite. Write down all the properties of quadrilateral $WXYZ$.

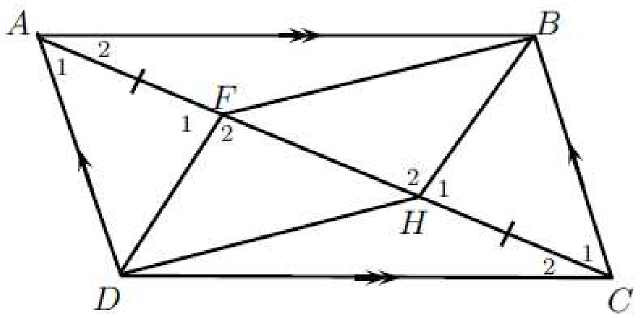


Assess whether the following statements are true or false. If the statement is false, explain why:

(a) A trapezium is a quadrilateral with two pairs of opposite sides that are parallel.
(b) Both diagonals of a parallelogram bisect each other.
(c) A rectangle is a parallelogram that has one corner angles equal to $90°$.
(d) Two adjacent sides of a rhombus have different lengths.
(e) The diagonals of a kite intersect at right angles.
(f) All squares are parallelograms.
(g) A rhombus is a kite with a pair of equal, opposite sides.
(h) The diagonals of a parallelogram are axes of symmetry.
(i) The diagonals of a rhombus are equal in length.
(j) Both diagonals of a kite bisect the interior angles.

$ABCD$ is a parallelogram with diagonal AC.
Give 3.2 : $AF = HC$, show that:

(a) $\triangle AFD \equiv \triangle CHB$
(b) $DF \parallel HB$
(c) $DFBH$ is a parallelogram

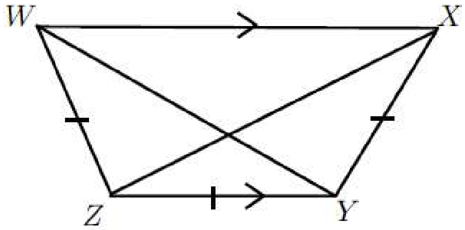


Question 4

4.1 Given that $WZ = ZY = YX$, $\hat{W} = \hat{X}$ and $WX \parallel ZY$, prove that:

(a) XZ bisects $\hat{X}$

(b) $WY = XZ$



4.2 $LMNO$ is a quadrilateral with $LM = LO$ and diagonals that intersect at S such that $MS = SO$. Prove that:

(a) $M\hat{L}S = S\hat{L}O$

(b) $\triangle LON \equiv \triangle LMN$

(c) $MO \perp LN$

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

TOPIC: EUCLIDEAN GEOMETRY (Lesson 14)	Weighting	30 ± 3	Grade	10
Term	4	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**DBE/November Past papers: 2015 and 2016**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Lines, Angles, Triangles, Quadrilaterals, Midpoint Theorem			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Parallel, Perpendicular, bisect, bisect, congruent triangles, similar triangles, properties of quadrilateral

RESOURCES

PAST PAPERS

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Mixing proofs of theorems

METHODOLOGY

DBE/NOVEMBER 2016

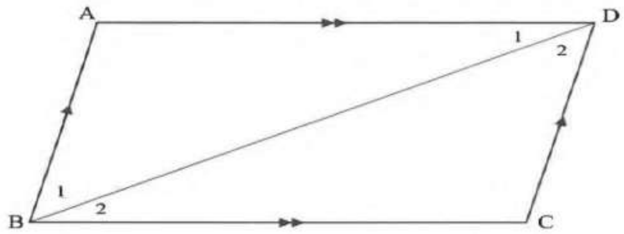
Give reasons for your statements in QUESTIONS 8 and 9.

QUESTION 8

8.1 Complete the following statement:

If the opposite angles of a quadrilateral are equal, then the quadrilateral ... (1)

8.2 Use the sketch below to prove that the opposite sides of a parallelogram are equal.



(6)

8.1 is a parallelogram

8.2 In Δ ABD and Δ CDB

$\hat{D}_1 = \hat{B}_2$ [alt. angles, $AD \parallel BC$]

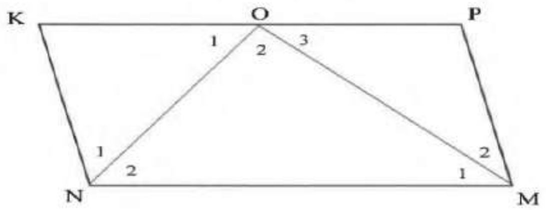
$\hat{B}_1 = \hat{D}_2$ [alt. angles, $AB \parallel DC$]

BD = BD [common side]

∴ Δ ABD ≡ Δ CDB [A,A,S]

∴AB=DC, AD=BC

8.3 In the sketch below, KPMN is a parallelogram. ON bisects $K\hat{N}M$ and OM bisects $N\hat{M}P$.



8.3.1 Show that $N\hat{O}M = 90°$. (3)

8.3.2 Prove that O is the midpoint of KP. (6)
[16]

8.3.1 Let $\hat{N}_1 = \hat{N}_2 = x$ [ON bisects $K\hat{N}M$]

Let $\hat{M}_1 = \hat{M}_2 = y$ [OM bisects $N\hat{M}P$]

$\therefore 2x + 2y = 180°$ [co-int. KN ∥ PM]
$\therefore x + y = 90°$
$\hat{O}_2 + x + y = 180°$ [int. angles of Δ]

$\therefore \hat{O}_2 + 90° = 180°$

$\therefore \hat{O}_2 = 90°$

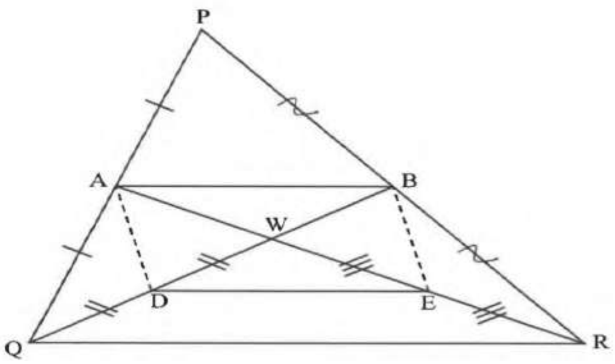
8.3.2 $\hat{N}_2 = \hat{O}_1$ [alt. angle KP ∥ NM]
$\hat{O}_1 = \hat{N}_1$ [AB = DE]
KO = KN [opp. sides =]
$\hat{O}_3 = \hat{M}_1$ [alt angle KP ∥ MN]
$\hat{O}_3 = \hat{M}_2$
$\therefore OP = PM$ [sides opp. = angles]
but KN = PM [opp. sides =]
∴ KO = OP
∴ O is the midpoint

QUESTION 9

9.1 Complete the following statement:

The line through the midpoint of two sides in a triangle is parallel to and ... the third side. (1)

9.2 In ΔPQR, A and B are the midpoints of sides PQ and PR respectively. AR and BQ intersect at W. D and E are points on WQ and WR respectively such that WD = DQ and WE = ER.



Prove that ADEB is a parallelogram. (5)
[6]

9.1 half the length of

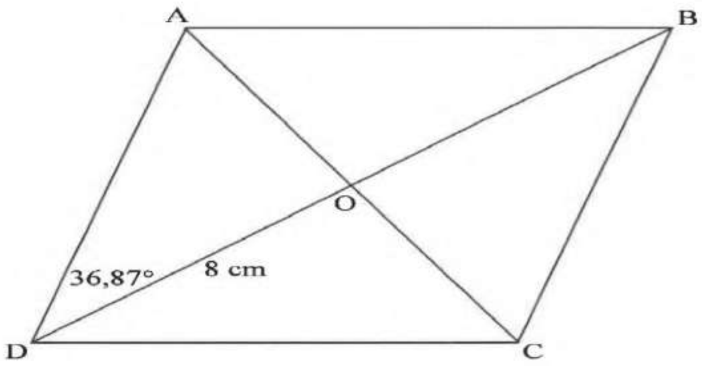
9.2 AB ∥ QR [line joining midpoint]
AB = 12 QR [line joining midpoint]
DE ∥ QR [line joining midpoint]
DE = $\frac{1}{2}$QR
∴ AB ∥ DE and
AB = DE
∴ ADEB is a parm. [one pair of opp. sides = and ∥]

DBE/NOVEMBER 2015

Give reasons for your statements in QUESTIONS 8 and 9.

QUESTION 8

In the diagram, ABCD is a rhombus having diagonals AC and BD intersecting in O. $A\hat{D}O = 36,87°$ and DO = 8 cm.



8.1 Write down the sizes of the following angles:

8.1.1 $C\hat{D}O$ (1)

8.1.2 $A\hat{O}D$ (1)

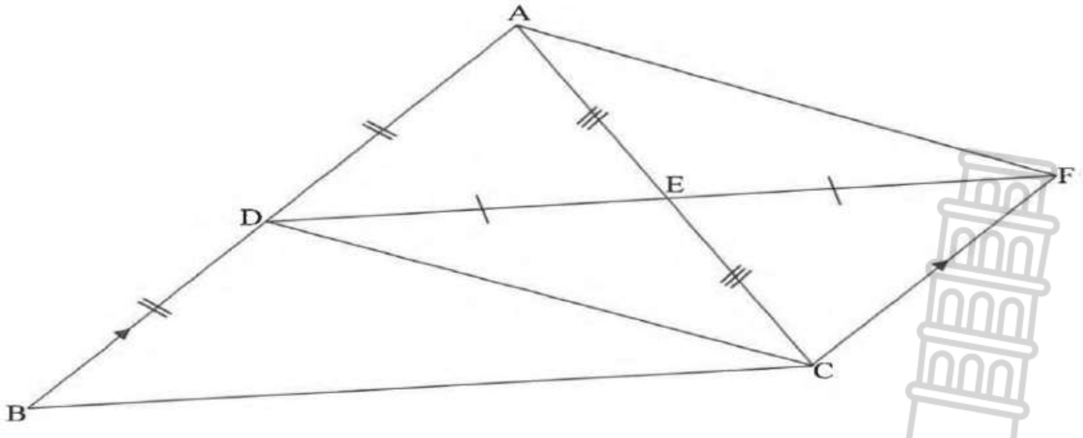
8.2 Calculate the length of AO. (2)

8.3 If E is a point on AB such that OE | | DA, calculate the length of OE. (4)

[8]

QUESTION 9

9.1 In the diagram below, D is the midpoint of side AB of ΔABC. E is the midpoint of AC. DE is produced to F such that DE = EF. CF | | BA.

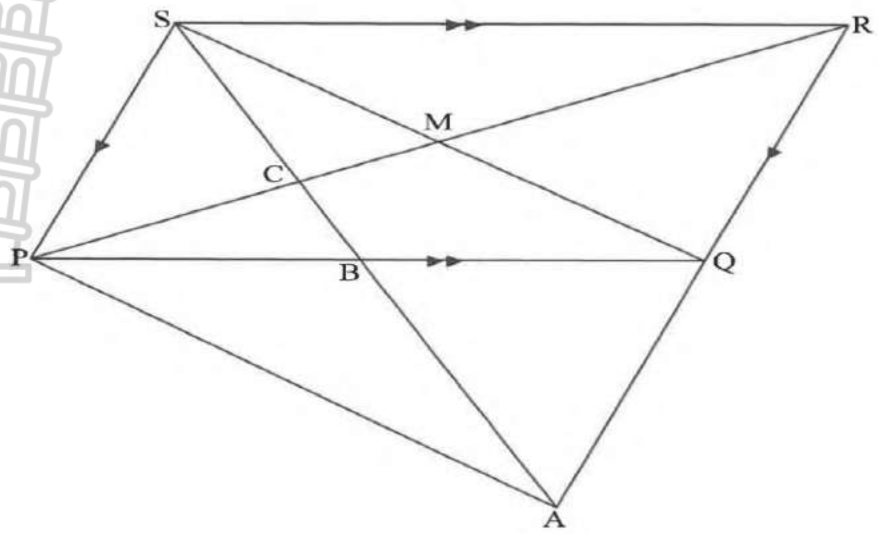


9.1.1 Write down a reason why ΔADE ≡ ΔCFE. (1)

9.1.2 Write down a reason why DBCF is a parallelogram. (1)

9.1.3 Hence, prove the theorem which states that $DE = \frac{1}{2}BC$. (2)

9.2 In the diagram below, PQRS is a parallelogram having diagonals PR and QS intersecting in M. B is a point on PQ such that SBA and RQA are straight lines and SB = BA. SA cuts PR in C and PA is drawn.



9.2.1	Prove that SP = QA.	(4)
9.2.2	Prove that SPAQ is a parallelogram.	(2)
9.2.3	Prove that AR = 4MB.	(4)
		[14]

TOPIC: EUCLIDEAN GEOMETRY (Lesson 15)	Weighting	30 ± 3	Grade	10
Term	4	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**DBE/NOVEMBER PAST PAPERS – 2017 AND 2018**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Lines, Angles, Triangles, Quadrilaterals, Midpoint Theorem			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Parallel, perpendicular, bisect, congruent triangles, similar triangles, properties of quadrilaterals

RESOURCES

PAST PAPERS

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Mixing proofs of theorems

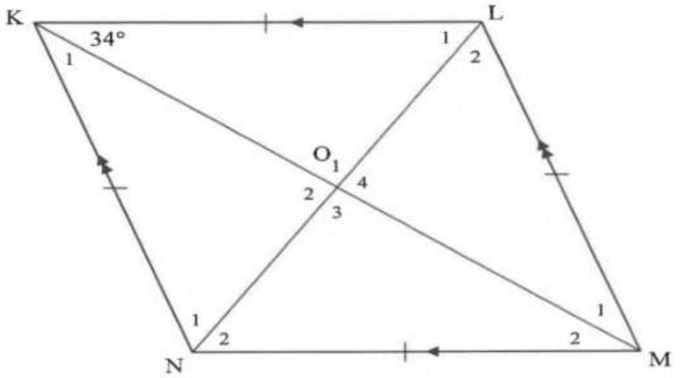
METHODOLOGY

DBE/NOVEMBER 2017

Give reasons for ALL statements in QUESTIONS 8 and 9.

QUESTION 8

8.1 KLMN is a rhombus with diagonals intersecting at O. $L\hat{K}M = 34°$.

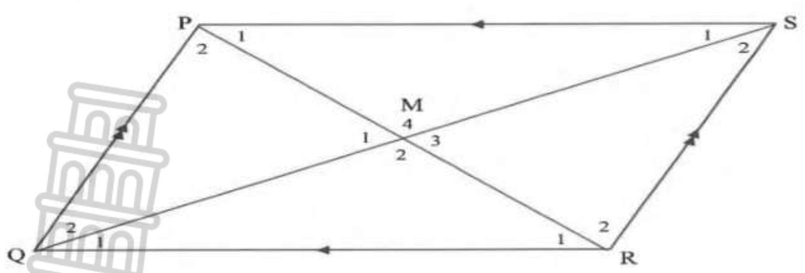


8.1.1 Write down the size of $\hat{O}_1$. (1)

8.1.2 Calculate the size of $\hat{L}_1$. (2)

8.1.3 Calculate the size of $K\hat{N}M$. (2)

8.2 Given parallelogram PQRS with diagonals PR and QS intersecting at M.



Prove that the diagonals bisect each other. (4)

8.1.1 $\hat{O}_1 = 90°$ — diagonals bisect at 90°

8.1.2 $\hat{L}_1 = 180° - (34° + 90°)$ — Sum of angles of Δ

$= 56°$

8.1.3 $\hat{L}_1 = \hat{L}_1 = 56°$ — diagonals bisect the $\angle s$

$\hat{L}_1 + \hat{L}_2 = \hat{N}_1 + \hat{N}_2$ — opp. $\angle s$ of rhombus

$\therefore K\hat{N}M = 112°$

8.2 Given parallelogram PQRS with diagonals PR and QS

R.P.T: PM = MR

Proof: In ΔPMS and ΔRMQ

1. $\hat{P}_1 = \hat{R}_1$ — alt. $\angle s$, PS∥QR
2. $\hat{S}_1 = \hat{Q}_1$ — alt. $\angle s$, PS∥QR
3. PS = QS — opp. Sides of parm =

$\therefore \Delta PMS \equiv \Delta RMQ$ — AAS

PM = MR and MS = MQ

OR

Given parallelogram PQRS with diagonals PR and QS

P.T.P: QM = MS

Proof: In ΔPQM and ΔRSM

1. $\hat{P}_2 = \hat{R}_2$ — alt. $\angle s$, QP∥ SR
2. $\hat{S}_2 = \hat{Q}_2$ — alt. $\angle s$, SR∥ PQ
3. PQ = SR — opp. Sides of parm =

$\therefore \Delta PQM \equiv \Delta RSM$ — AAS

QM = MS and PM = MR

8.3 DB = 2DE	DE = DB
DE = FC	opp. Side of //gram
But FC = 2KC	FK = KC
DE = 2KC	DE = FC
DB = 2(2KC)	DB = 2DE
DB = 4KC	

ACTIVITIES/ ASSESSMENT

DBE/NOVEMBER 2018

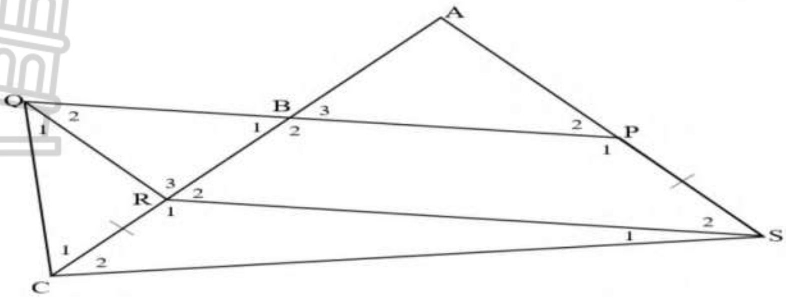
Give reasons for ALL geometry statements in QUESTIONS 7 and 8.

QUESTION 7

7.1 Complete the statement so that it is TRUE:

The line drawn from the midpoint of the one side of a triangle, parallel to the second side, ... (1)

7.2 ACS is a triangle. P is a point on AS and R is a point on AC such that PSRQ is a parallelogram. PQ intersects AC at B such that B is the midpoint of AR. QC is joined. Also, CR = PS, $\hat{C}_1 = 50°$ and BP = 60 mm.



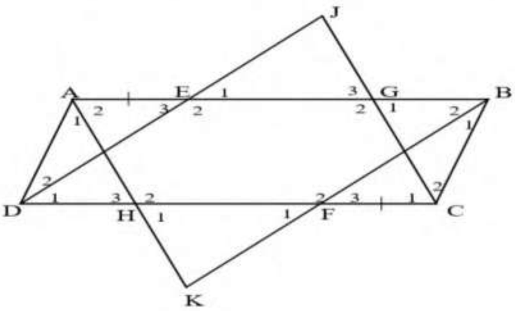
7.2.1 Calculate the size of $\hat{A}$. (5)

7.2.2 Determine the length of QP. (3)

[9]

QUESTION 8

8.1 ABCD is a parallelogram. E and F are points on AB and DC respectively such that AE = CF. DE is produced to J and CJ is drawn. BF is produced to K and AK is drawn.

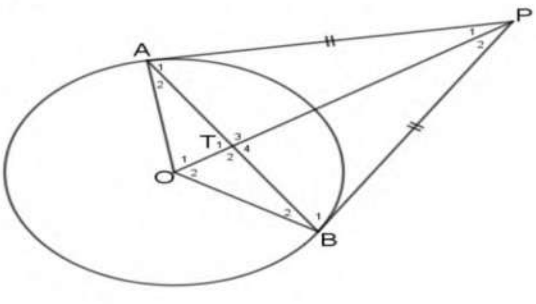


Prove that:

8.1.1 DJ || BK (5)

8.1.2 $\hat{E}_1 = \hat{F}_1$ (4)

8.2 In the diagram below O is the centre of the circle. A and B lie on the circumference of the circle. AP = BP.



Prove that:

8.2.1 AT = BT (5)

8.2.2 $O\hat{T}A = 90°$ (1)

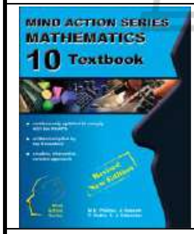
[15]

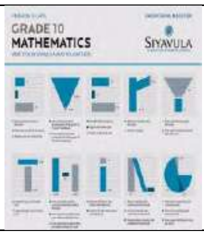
TOPIC: TRIGONOMETRY (Lesson 1)	Weighting	40 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Definition of Trigonometric ratios, using right-angled triangle: Sine, Cosine, Tangent**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Adjacent, opposite, trigonometric identities			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Ratio, right-angled triangle, hypotenuse

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing opposite side and adjacent side.

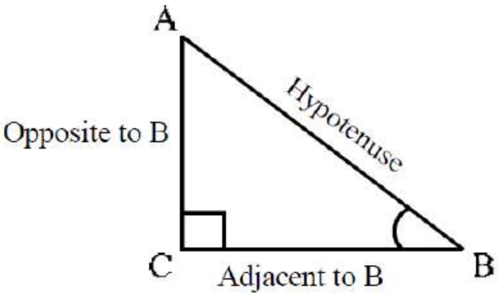
METHODOLOGY

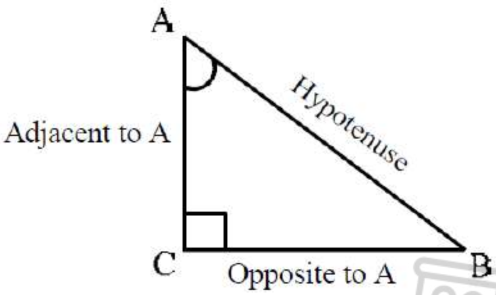
Trigonometry (pronounced: trig-oh-nom-eh-tree) deals with the relationship between the **angles and the sides of a right-angled triangle.**

We will label the three sides as ADJACENT, OPPOSITE and HYPOTENUSE.

The longest side (opposite the right-angle) is called the **hypotenuse.**

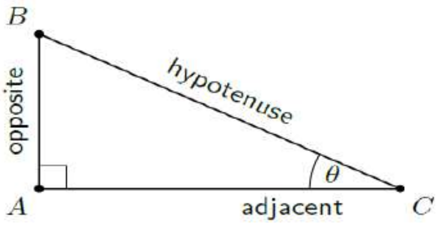
The opposite and adjacent sides are dependent on which angle is used as the reference point.





There are **three special ratios** in the study of Trigonometry, namely the **sine, cosine and tangent** ratios.

Consider a right-angled triangle *ABC* with an angle marked θ (said 'theta').



In a right-angled triangle, we refer to the three sides according to how they are placed in relation to the angle θ . The side opposite to the right-angle is labelled the **hypotenuse**, the side opposite θ is labelled "**opposite**", the side next to θ is labelled "**adjacent**".

We define the trigonometric ratios: sine (**sin**), cosine (**cos**) and tangent (**tan**), of an angle, as follows:

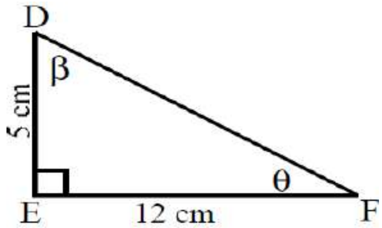
$$\sin\theta = \frac{opposite\ side}{hypotenuse} \qquad \cos\theta = \frac{adjacent\ side}{hypotenuse} \qquad \tan\theta = \frac{opposiye\ side}{adjacent\ side}$$

These ratios, also known as **trigonometric identities**, relate the lengths of the sides of a right-angled triangle to its interior angles. These three ratios form the **basis of trigonometry.**

NOTE: The definitions of opposite, adjacent and hypotenuse are only applicable when working with **right-angled triangles**! Always check to make sure your triangle has a right-angle before you use them, otherwise you will get the wrong answer.

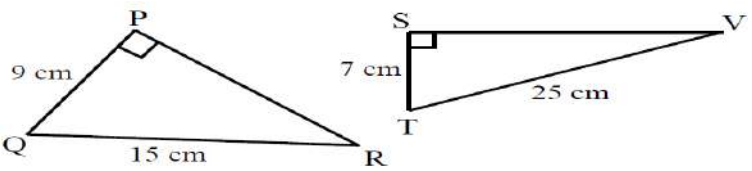
You may also hear people saying "**Soh Cah Toa**". This is a mnemonic technique for remembering the trigonometric ratios:

Examples:
1. Given the following triangle:



a) Label the hypotenuse, opposite and adjacent sides of the triangle with respect to θ.
b) Determine the length of the hypotenuse.
c) State which sides of the triangle you would use to find $\sin\theta$, $\cos\theta$ and $\tan\theta$.
Redraw the triangle and:
d) Label the hypotenuse, opposite and adjacent sides of the triangle with respect to β.
e) Write down the values of $\sin\beta$, $\cos\ \beta$ $\tan\ \beta$

SOLUTION:
a) DF is the hypotenuse (opposite the right angle), DE is opposite to θ and EF is adjacent to θ
b) $DF^2 = DE^2 + EF^2$ Pythagoras Theorem
$DF^2 = 5^2 + 12^2$
$DF^2 = 169$
DF = 13
c) $\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{12}{13}$ $\tan\theta = \frac{5}{12}$
d) DF is the hypotenuse (opposite the right angle), EF is opposite to an β d DE is adjacent to β
e) $\sin\beta = \frac{12}{13}$ $\cos\beta = \frac{5}{13}$ $\tan\beta = \frac{12}{5}$



2.
a) tan Q

b) cos V

SOLUTION:

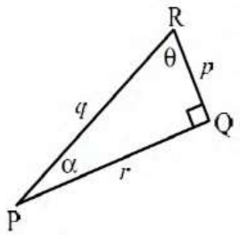
$PR^2 = 15^2 - 9^2$
$PR^2 = 144$
PR = 12 cm
$\therefore \tan Q = \frac{opp}{adj} = \frac{12}{9} = \frac{4}{3}$

$SV^2 = 25^2 - 7^2$
$SV^2 = 576$
SV = 24 cm
$\cos V = \frac{adj}{hyp} = \frac{24}{25}$

ACTIVITIES/ ASSESSMENT

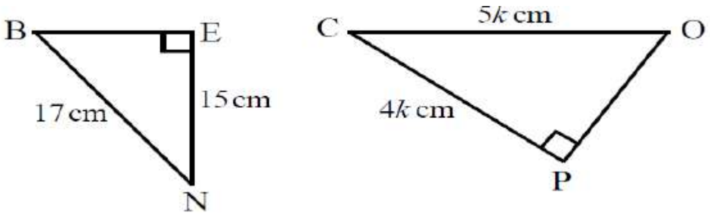
1. Redraw the triangles below and indicate which sides are opposite, adjacent and hypotenuse with respect to θ.

2. State the following in terms of *p*, *q* and *r*:



a) $\sin\theta$ b) $\cos\theta$ c) $\tan\theta$ d) $\sin\alpha$ e) $\cos\alpha$ f) $\tan\alpha$

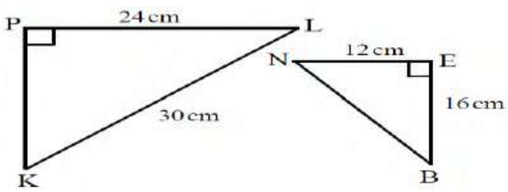
3.



Determine the value of:

a) sin N
b) tan C

4.



a) Determine cos L and cos B
b) What do you notice?
c) What can you deduce about the two triangles and why?

TOPIC: TRIGONOMETRY (Lesson 2)	Weighting	40 ± 3	Grade	10

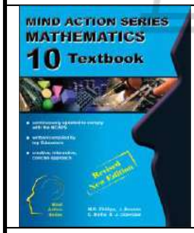
Term		**Week no.**	
Duration	1 hour	**Date**	

Sub-topics	**Definition of the reciprocals of the trigonometric ratios, and using right-angled triangles. cosecant, secant and cotangent**
RELATED CONCEPTS/ TERMS/VOCABULARY	Reciprocal

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Multiplicative inverse, whole number, numerator, denominator

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Reciprocal of nine as secant and of cosine as cosecant or of cos as cot.

METHODOLOGY

A **reciprocal, or multiplicative inverse** is simply one of a pair of numbers that, when multiplied together, equal 1.

If you can reduce the number to a fraction, finding the **reciprocal** is simply a matter of transposing the **numerator and the denominator**.

e.g. reciprocal of $\frac{2}{3}$ *is* $\frac{3}{2}$ and $\frac{2}{3} \times \frac{3}{2} = 1$

To find the reciprocal of a **whole number**, just turn it into a fraction in which the original number is the denominator and the numerator is 1.

e.g. reciprocal of 7 is $\frac{1}{7}$ and $7 \times \frac{1}{7} = 1$

reciprocal of $\boldsymbol{\sin\theta}$ *is* $\frac{1}{\sin\theta}$ and reciprocal of $\sin\theta$ is $\boldsymbol{cosec\theta}$

$\therefore cosec\theta = \frac{1}{\sin\theta}$

reciprocal of $\boldsymbol{\cos\theta}$ *is* $\frac{1}{\cos\theta}$ and reciprocal of $\cos\theta$ is $\boldsymbol{sec\theta}$

$\therefore sec\theta = \frac{1}{\cos\theta}$

reciprocal of $\boldsymbol{\tan\theta}$ *is* $\frac{1}{\tan\theta}$ and reciprocal of $\tan\theta$ is $\boldsymbol{cot\theta}$

$\therefore cot\theta = \frac{1}{\tan\theta}$

TRIGONOMETRIC RATIO	RECIPROCAL RATIO	CONCLUSION
$\sin\theta = \frac{opp}{hyp}$	$cosec\theta = \frac{hyp}{opp}$	$cosec\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{adj}{hyp}$	$sec\theta = \frac{hyp}{adj}$	$sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{opp}{adj}$	$cot\theta = \frac{adj}{opp}$	$cosec\theta = \frac{1}{\tan\theta}$

CALCULATING THE TRIGONOMETRIC RATIOS OF A GIVEN ANGLE

The **sine, cosine and tangent ratio** can be calculated with the use of a calculator which must be on the DEG (degree) mode.
e.g. to calculate cos 35 ° correct to three decimal place, on your calculator, press the button "cos" and then type in "35" (some calculators will expect you to close the brackets first). Then press " = " and you will get 0,819152... = 0,819
Now verify the ratios of sin 60° = 0,866 and tan 60° = 1,703 rounded to three decimal places.

When doing calculations involving the **reciprocal ratios** you need to **convert the reciprocal ratio to** one of the **standard trigonometric ratios**: sin, cos and tan as this is the only way to calculate these ratios on your calculator.
e.g. to calculate cot 49° correct to 2 decimal places, first write cot in terms of tan ((since there is no cot button on your calculator): $\cot 49° = \frac{1}{\tan 49°}$. On your calculator, press 1 ÷ tan (49), and press "=" you will get 0,869286…= 0,87

CALCULATOR WORK USING SUBSTITUTION

Examples:
Determine the decimal value of the following if A = 23,8° and B = 18,1°
(Round off your answers to one decimal place)

1. sin (A+B)
= sin (23,8°+18,1°)
= sin 41,9°
= 0,667832… = 0,7

2. tan 2B
= tan 2(18,1°)
= tan 36,2°
= 0,731889… = 0,7

3. $\sec^2(2A-10°)$
$= \frac{1}{cos^2(2A-10°)}$
$= \frac{1}{(\cos 2(18,1°)-10°)^2}$
$= \frac{1}{(cos 37,6°)^2}$
= 1,593059… = 1,6

ACTIVITIES/ ASSESSMENT

1. Calculate with the use of a calculator the following rounded off to two decimal places, where appropriate.
   a) cos 34°
   b) 7 tan 58°
   c) cosec 140°
   d) tan 35° + cot 35°
   e) $\frac{\sin 60°}{20}$
   f) $\frac{1}{3}\cos^2 23°$
   g) $\sqrt{4\ sec\ 99°}$

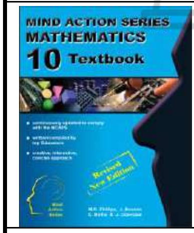
2. Determine the decimal value of the following if A = 35° and B = 52° . Round off your answers to two decimal places).
   a) cos (A + B)
   b) cos A +cos B
   c) 3 sin 2B
   d) 3 tan $\frac{1}{3}$ A
   e) 2 sin (2A – B)
   f) $\sqrt{\cos 3A + \sin B}$

TOPIC: TRIGONOMETRY (Lesson 3)	Weighting	40 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Trigonometric ratios of the Special Angles $30°, 45° and\ 60°$**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Bisect, perpendicular bisector			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Equilateral triangle, isosceles, right-angled triangle, Pythagoras theorem, midpoint

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

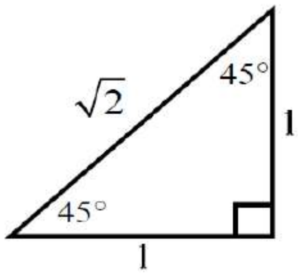
Subtracting the biggest side from the smallest even if the side is not the hypotenuse.

METHODOLOGY

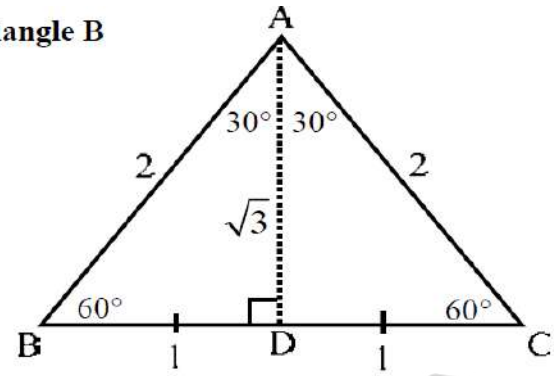
For most angles we need a calculator to calculate the values of sin, cos and tan. However, there are some angles we can easily work out the trigonometric ratios for without a calculator as they produce simple ratios. These angles are **$30°, 45° and\ 60°$** and they are called **special angles.**

The following two **triangles** can be used to determine trigonometric ratios of special angles.

Triangle A



Triangle B



Isosceles right-angled triangle with sides 1 and 1 and then find the third side Using Pythagoras Theorem.

Equilateral triangle with side-lengths 2 units. The dotted line AD is a perpendicular bisector of line BC and divides $\hat{A}$ into two equal angles and then find AD using Pythagoras Theorem.

Examples:

1. Evaluate the following without using a calculator:

a) cos 45°
$= \frac{1}{\sqrt{2}}$

b) cos 60°
$= \frac{\sqrt{3}}{2}$

c) tan 45°
$= \frac{1}{1} = 1$

d) sin 30°
$= \frac{1}{2}$

2. Calculate the following without using a calculator:

a) $\sin 30° + \cos 60°$
$= \frac{1}{2} + \frac{1}{2}$
$= 1$

b) $\tan 60° - \cos 30°$
$= \frac{\sqrt{3}}{1} - \frac{\sqrt{3}}{2}$
$= \frac{2\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

c) $\frac{\sin 60°}{\tan 30°}$

$= \sin 60° \div \tan 30°$

$= \frac{\sqrt{3}}{2} \div \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{1} = \frac{3}{2}$

d) $cos^2 45°$

$= (\cos 45°)^2$

$= \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2}$

θ	$30°$	$45°$	$60°$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

ACTIVITIES/ ASSESSMENT

Siyavula: pg. 118, Exercise 5 – 3 No. 2 and 3

1. Evaluate the following without the use of a calculator:

a) $\cos 60° + \tan 45°$

b) $\tan 60° + \sin 60°$

c) $\frac{\tan 30°}{\tan 60°}$

d) $\sin 30° + cos^2 45°$

e) $sin^2 45° + cos^2 45°$

f) $\frac{\sin 45°.\ \cos 45°}{\tan 60°.t\ an\ 30°}$

g) $\sqrt{2}\sin 45° - \sqrt{3}\tan 60°$

h) $4cos^2 30° + \tan 30° . \sin 60°$

2. Using the fact that $1 = \frac{\sqrt{a}}{\sqrt{a}}$ where $a > 0$, show that $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

2.2 Hence, show that:

a) $\cos 45° = \frac{\sqrt{2}}{2}$

b) $\tan 30° = \frac{\sqrt{3}}{3}$

c) $\tan 60° = \frac{3}{\sqrt{3}}$

d) $\sin 60° = \frac{3}{2\sqrt{3}}$

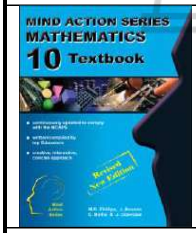
TOPIC: TRIGONOMETRY (Lesson 4)	Weighting	40 ± 3	Grade	10

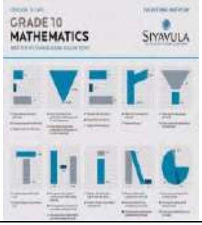
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Finding sides using trigonometric ratios**		
RELATED CONCEPTS/ TERMS/VOCABULARY	sin, cos and tan ratios in terms of opposite side, adjacent side and hypotenuse.		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Opposite side, adjacent side, rounding off

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

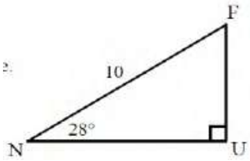
Rounding off incorrectly, using incorrect ratios

METHODOLOGY

Trigonometric ratios can help in finding unknown lengths in right-angled triangles.

Examples:

1. Calculate the length of FU, rounded off to one decimal place in



Solution:

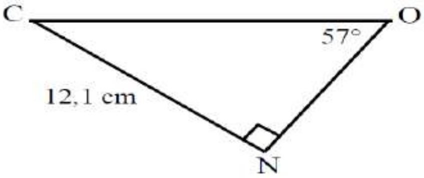
FU is opposite to 28° and 10 is the hypotenuse.

$\therefore \sin 28° = \frac{opp}{hyp} = \frac{FU}{10}$

$10 \times \sin 28°$ = FU …($\times$ by LCD)

FU = 4,7 units

2. Consider the triangle sketched alongside.



Calculate the length of ON correct to one decimal place.

Solution:

ON is adjacent to 57° and CN is opposite to 57°.

$\frac{opp}{adj} = \tan 57°$

$\frac{12,1}{ON} = \tan 57°$

$12,1 = ON \tan 57°$

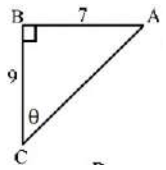
$\frac{12,1}{tan57°} = ON$

ON = 7,9 cm

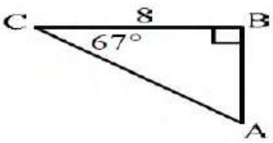
ACTIVITIES/ ASSESSMENT

Round answers off to one decimal place in this exercise.

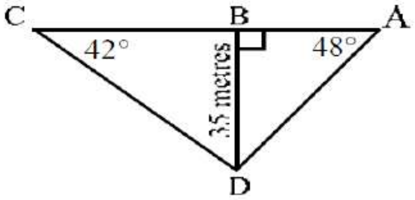
1. Calculate the length of AC.



2. Calculate the length of AC and AB.

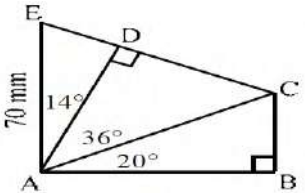


3. In the diagram, BD ⊥ AC.

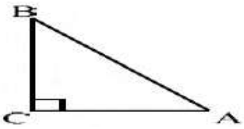


Using the information provided, calculate the length of AC.

4. Using the information provided on the diagram below, calculate the length of BC.



5. In the given diagram, Δ ABC is right-angled at C. It is given that AC = 4 units, $\tan A = \frac{3}{2}$ and $\hat{A} \in (0° ; 90°)$



a) Determine the length of BC without solving for $\hat{A}$.
b) Determine the length of AB

SIYAVULA: pg. 123, Exercise 5 -4 1. A(a), (b), (c), €, (f) and (i)

TEST 1: TRIGONOMETRY

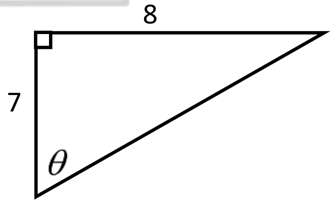
MARK: 25

DURATION: 30 Min

QUESTION 1 [6 Marks]

In each of the following right-angles triangles, write down the value of the required trigonometric ratio (leave your answers as ratios):

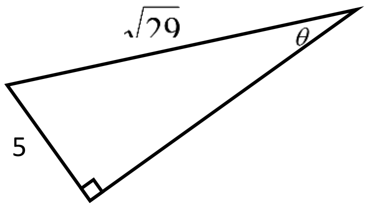
1.1



Find the value of $\sin\theta$

(2)

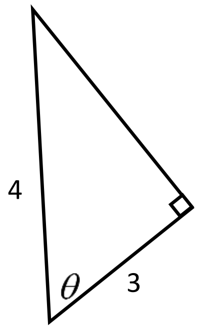
1.2



Find the value of $\cos\theta$

(2)

1.3



Find the value of $\tan\theta$

(2)

QUESTION 2 [5 Marks]

Simplify the following expressions without the use of a calculator and show ALL the workings:

$$\frac{\sin 45^\circ \sin 90^\circ}{\cos 0^\circ \cos 60^\circ}$$ (5)

QUESTION 3 [14 Marks]

In the sketch below, ΔBCD is right angled at C, BD = 3 units, $B\hat{D}C = 30^0$ and $A\hat{B}E = 20^0$. Also, BCDE is a rectangle.

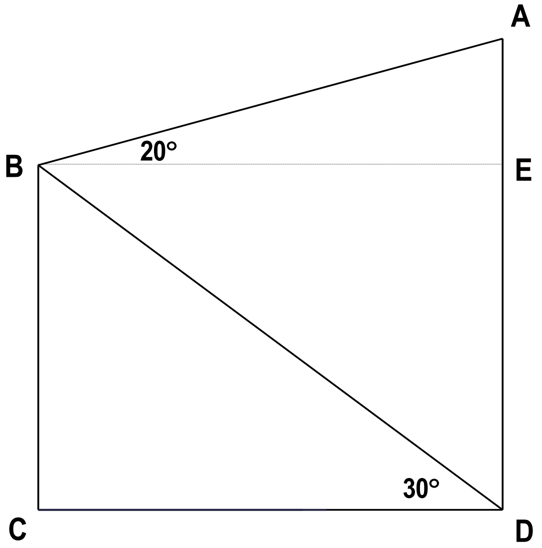
calculate the lengths of

2.1 BC (4)

2.2 CD (4)

2.3 AD (4)

2.4 Angle DBA (2)



TOPIC: TRIGONOMETRY (Lesson 5)	Weighting	40 ± 3	Grade	10

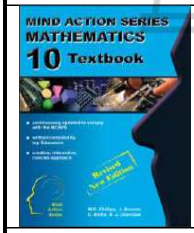
Term		**Week no.**	
Duration	1 hour	**Date**	

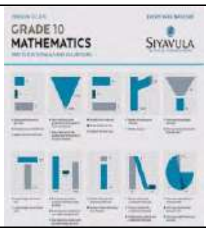
Sub-topics	**Solving simple trigonometric equations without using a calculator**
RELATED CONCEPTS/ TERMS/VOCABULARY	Special angles

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Writing trigonometry ratios as a fraction, numerator divided by denominator

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Mixing up special angles, failing to represent ratios in the diagram

METHODOLOGY

The values of the trigonometric ratios cannot be calculated exactly for most angles. There are, however **three special angles** that lend themselves nicely to ratio calculation. They are 30°, 45° and 60°. Notice that 30° and 60° angles are **complementary** and theses two angles can be done simultaneously since they are complementary. A 45° angle is its own complement.

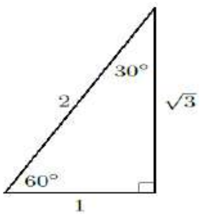
Examples

Solve the following equations without using a calculator where x is an acute angle.

1. $\sin x = \frac{1}{2}$

Here we want to find the angle that gives the number (ratio) $\frac{1}{2}$

$\sin x = \frac{opp}{hyp} = \frac{1}{2}$



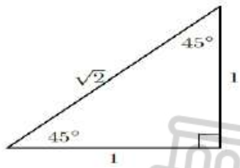
1 is opposite to 30°and the hypotenuse is 2

$\therefore x = 30°$ since $\sin 30° = \frac{qpp}{vyp} = \frac{1}{2}$

2. $\sqrt{2}\cos x = 1$

Rearrange the equation so that cos x is on one side

$\cos x = \frac{1}{\sqrt{2}} = \frac{adj}{hyp}$



$x = 45°$ since $\cos 45° = \frac{1}{\sqrt{2}}$

3. $\frac{1}{4}\sin 4x = \frac{\sqrt{2}}{8}$ … LCD: 8

$8 \times \frac{1}{4} \sin 4x = 8 \times \frac{\sqrt{2}}{8}$ …multiply by LCD all the terms

$2 \sin 4x = \sqrt{2}$

$\sin 4x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = \frac{opp}{hyp}$

$4x = 45°$

$x = 11{,}25°$ …divide by 4

ACTIVITIES/ ASSESSMENT

Without using a calculator, solve the following equations, where the angles are acute.

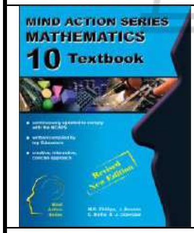
1. $\cos x = \frac{1}{2}$
2. $2 \sin x = \sqrt{3}$
3. $\tan x = \sqrt{3}$
4. $\sqrt{3} \tan x - 1 = 0$
5. $\sin 2x = 0{,}5$
6. $\cos(3x - 15°) = 0{,}5$
7. $2 \cos 2x = \sqrt{2}$
8. $3 \tan x = \sqrt{3}$

TOPIC: TRIGONOMETRY (Lesson 6)	Weighting	40 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Solving simple trigonometric equations**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Calculator usage, rearranging equations			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Rounding off decimals

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Incorrect rounding off, incorrect working with fractions on the reciprocals

METHODOLOGY

To solve for an angle, you will need to use the inverse sine, cosine and tangent function on your calculator. Shift/2nd function then ratio (number) = answer

Examples

Determine the angle correct to two decimal places:

1. $\cos\theta = 0{,}2$

   To solve for θ press shift cos (0,2) = on your calculator.

   $\theta = 78{,}463040\ldots = 78{,}46°$

2. $3\sin x = 2{,}4$

   Rearrange the equation so that $\sin x$ is on one side of the equation.

   $\sin x = \frac{2{,}4}{3}$

   Shift sin $(\frac{2{,}4}{3})$ = 53, 130102.. = 53,13

   $x = 53{,}13°$

3. $1{,}4\sec\alpha = 3$

   There is no sec button on the calculator, so you need to convert sec to cos

   $\frac{1{,}4}{\cos\alpha} = 3$

   Rearrange the equation so that $\cos\alpha$ is on one side

   $1{,}4 = 3\cos\alpha$

   $\text{Cos}\alpha = \frac{1{,}4}{3}$

   Shift cos (1,4 ÷ 3) = 62,181860… = 62,18

   $\alpha = 62{,}18°$

ACTIVITIES/ ASSESSMENT

1. Find the value of the angles correct to two decimal places:

a) $\tan\theta = 1,7$

b) $\sin A = \frac{2}{3}$

c) $4\cos\alpha = 3$

d) $\cos 4\theta = 0,3$

e) $2\sin\beta + 5 = 0,8$

f) $\tan\frac{\theta}{3} = \sin 48°$

g) $\frac{1}{2}\cos 2B = 0,3$

h) $2\sin 3\theta + 1 = 2,6$

i) $\sin x = \tan 45°$

j) $\tan 2x = 3.123$

k) $\tan x = 3\sin 41°$

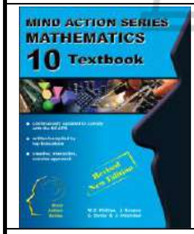
l) $\sin(2x + 45°) = 0,123$

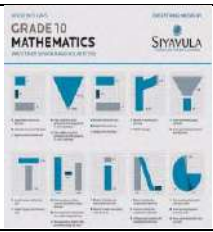
TOPIC: TRIGONOMETRY (Lesson 7)		**Weighting**	40 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Finding angles using trigonometric ratios**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Calculator usage				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Calculator usage, rounding off

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

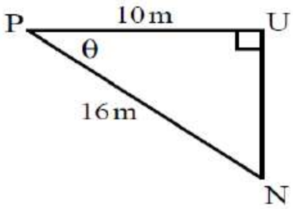
Choosing the correct trigonometric ratio

METHODOLOGY

Examples

Given right-angled triangles, calculate the size of θ correct to one decimal place.

(a)



PU is adjacent to θ and PN is the hypotenuse.

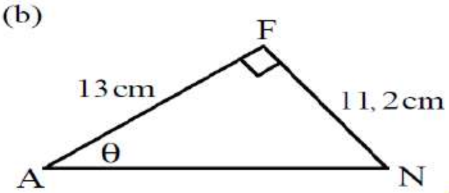
Then, $\frac{adj}{hyp} = cos\theta$

$\therefore \cos\theta = \frac{10}{16}$

On calculator press shift cos($\frac{10}{16}$) OR cos (10÷ 16) =

$\theta = 51{,}3°$

(b)



FN is opposite to θ and AF is adjacent to θ.

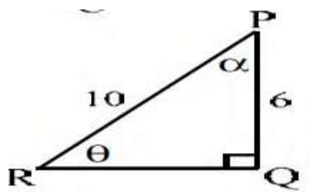
Then, $\frac{opp}{adj} = \tan\theta$

$\therefore \tan\theta = \frac{11{,}2}{13}$

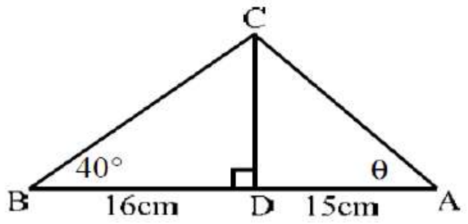
$\theta = 40{,}7°$

ACTIVITIES/ ASSESSMENT

1. Calculate the value of α

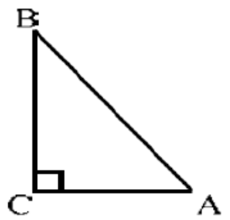


2. In ΔABC , CD ⊥ AB , $\hat{A} = \theta$, $\hat{B} = 40°$, AD =15 cm and DB =16 cm.



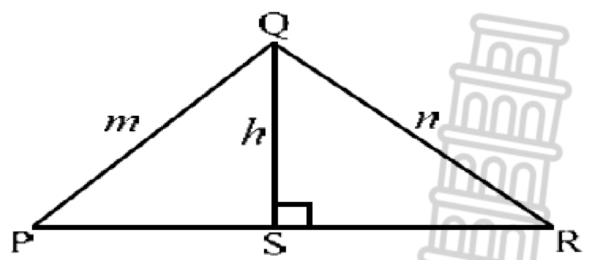
Calculate the size of θ.

3. In the given diagram, Δ ABC is right-angled at C. It is given that AC = 4 units, tan A = $\frac{3}{2}$ and $\hat{A} \in (0° ; 90°)$



Calculate the size of $\hat{B}$

4. In Δ PQR, QS ⊥ PR, QS = h units, PQ = m units, QR = n units.



a) Express sin P in terms of h and m.
b) Express sin R in terms of h and n.
c) Hence show that $m\sin P = n\sin R$.
d) Now use the result in (3) to calculate the size of $\hat{P}$ if it is given that $m = 40$ cm, $n = 30$ cm and $\hat{R} = 80°$.

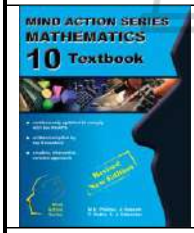
Siyavula: pg. 126, Exercise 5 – 5 No, 1 - 7

TOPIC: TRIGONOMETRY (Lesson 8)	Weighting	40 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Defining ratios in a cartesian plane: $0° \leq \theta \leq 360°$**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Cartesian plane, anticlockwise, radius			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trigonometric ratios, intervals

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

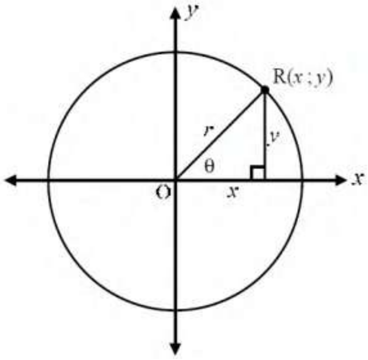
Choosing the correct quadrant in the cartesian plane

METHODOLOGY

Angles in the Cartesian Plane.

We have defined the trigonometric ratios using right-angled triangles. We can extend these definitions to any angle, noting that the definitions do not rely on the lengths of the sides of the triangle, but on the size of the angle only.
In this section, we will extend the trigonometric definitions to include angles in the interval [0°; 360°]

Consider a circle with centre on the origin O and passing through the point R (*x; y*) on the circle. The length from the origin O to point R is the **radius** of the circle which is referred to as the **terminal arm** θ is the angle measured **anti-clockwise** from the positive side of the *x*-axis to OR (the angle formed between the line *OR* and the *x*-axis), and θ is said to be in **standard position**.



We can rewrite all the trigonometric ratios in terms of x, y and r.

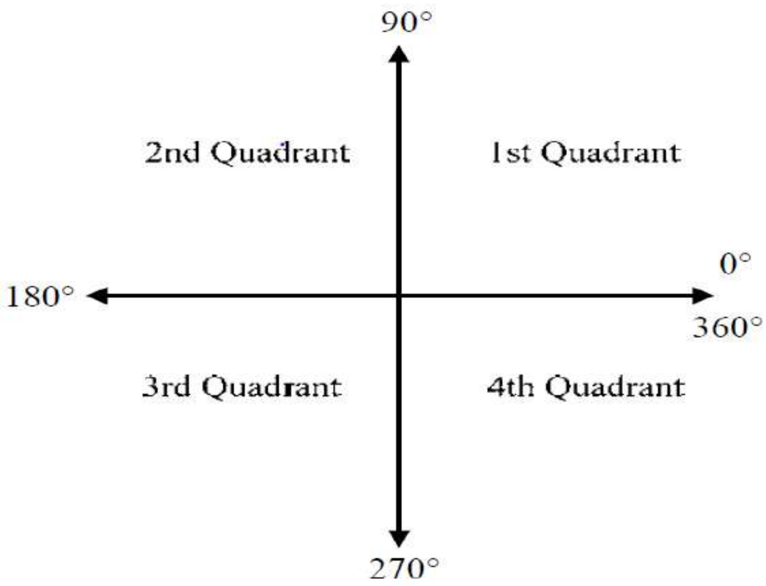
$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} \qquad \operatorname{cosec}\theta = \frac{hyp}{opp} = \frac{r}{y}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r} \qquad \sec\theta = \frac{ophypp}{adj} = \frac{r}{x}$$

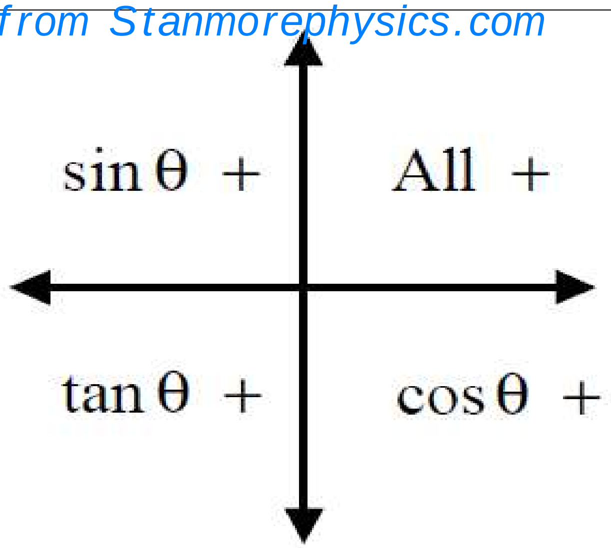
$$\tan\theta = \frac{opp}{hyp} = \frac{y}{x} \qquad \cot\theta = \frac{adj}{opp} = \frac{x}{y}$$

Notice that *r* **is always positive** but the **values of *x* and *y* change** depending on the position of the point in the Cartesian plane. As a result, the trigonometric ratios **can be positive or negative**

The Cartesian plane is divided into **4 quadrants** in an anti-clockwise direction as shown in the diagram below.
Angles in the first quadrant will lie in the interval (0°; 90°)
Angles in the second quadrant will lie in the interval (90° ;180°)
Angles in the third quadrant will lie in the interval (180° ; 270°)
Angles in the fourth quadrant will lie in the interval (270° ; 360°)



- **Quadrant I**
  Both the *x* and *y* values are positive so all ratios are positive in this quadrant.
- **Quadrant II**
  The *y* values are positive therefore sin and cosec are positive in this quadrant (recall that sin and cosec are defined in terms of *y* and *r*).
- **Quadrant III**
  Both the *x* and the *y* values are negative therefore tan and cot are positive in this quadrant (recall that tan and cot are defined in terms of *x* and *y*).
- **Quadrant IV**
  The *x* values are positive therefore cos and sec are positive in this quadrant (recall that cos and sec are defined in terms of *x* and *r*).



Special angles in the Cartesian plane

When working in the Cartesian plane we include two other special angles in right-angled triangles: 0° and 90°.
Now we can extend our knowledge of special angles.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

ACTIVITIES/ ASSESSMENT

In which quadrant does the terminal arm of the angle θ lie if:

1. $\sin\theta > 0$ and $\cos\theta > 0$
2. $\sin\theta < 0$ and $\cos\theta < 0$
3. $\tan\theta > 0$ and $\cos\theta < 0$
4. $\tan\theta < 0$ and $\cos\theta < 0$
5. $\sin\theta < 0$ and $\theta \in [90°; 180°]$
6. $\cos\theta < 0$ and $0° < \theta < 180°$

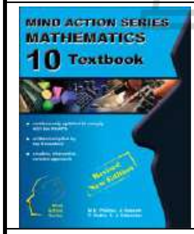
TOPIC: TRIGONOMETRY (Lesson 9)	Weighting	40 ± 3	Grade	10

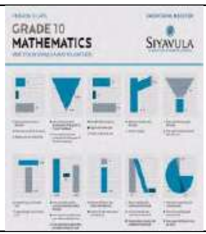
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Ratios in the Cartesian plane: Use diagrams to determine the numerical values of ratios for angles where $\theta \in [0°; 360°]$**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Using quadrants to solve trigonometric ratios		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Quadrants on a cartesian plane, trigonometric ratios

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Choosing the correct/right quadrant

METHODOLOGY

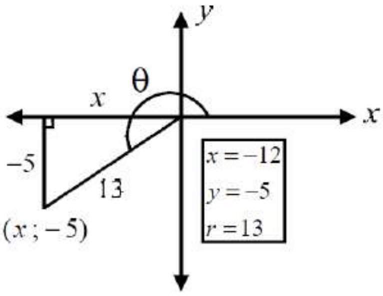
Examples:

1. If $13 \sin\theta = -5$ and $\theta \in [90°; 270°]$, calculate without the use of a calculator and with the aid of a diagram the value of cos θ + sin θ.

Solution:

$\sin\theta = \frac{-5}{13} = \frac{y}{r}$ (r is always positive)

y is negative in the third quadrant, considering $\theta \in [90°; 270°]$ and the terminal arm will lie in the **third** quadrant.



Use Pythagoras Theorem to calculate the unknown side

$x^2 + y^2 = r^2$ Pythagoras Theorem

$x^2 + (-5)^2 = 13^2$

$x^2 + 25 = 169$

$x^2 = 144$

$x = \pm\sqrt{144}$

$x = \pm 12$

But x is negative in the third quadrant $\therefore x = -12$

cos θ + sin θ

$= \frac{12}{13} + \frac{-5}{12}$

$= \frac{-17}{13} = -1\frac{4}{13}$

ACTIVITIES/ ASSESSMENT

1. If $\sin\theta = \frac{3}{5}$ and $0° \leq \theta \leq 90°$, determine by means of a diagram:

a) $sin^2\theta$ b) $2\tan\theta$

2. If $\tan\theta = \frac{5}{12}$ and $\sin\theta > 0$, determine by means of a diagram:

a) $13\cos\theta$ b) $cos^2\theta + sin^2\theta$

3. If $5\cos A + 3 = 0$ and $180° < A < 360°$, determine by means of a diagram:

a) $tan^2 A$ b) $\frac{sinA}{cosA}$

4. If $8\tan\theta + 15 = 0$ and $\theta \in [90°; 270°]$, determine by means of a diagram:

a) $\sin\theta + \cos\theta$ b) $34\sin\theta - 17\cos\theta$

5. If $13\cos\theta - 5 = 0$ and $180° \leq \theta \leq 360°$, determine by means of a diagram:

a) $sin^2\theta + cos^2\theta$ b) $25tan^2\theta$

SIYAVULA: pg. 136 Exercise 5 – 7 No. 1 - 5

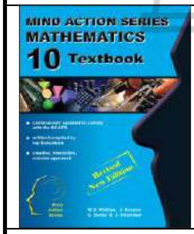
TOPIC: TRIGONOMETRY (Lesson 10)	Weighting	40 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		

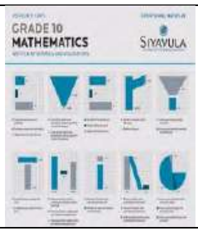
Sub-topics	**Ratios in the Cartesian plane: Use diagrams to determine the numerical values of ratios for angles where $\theta \in [0°; 360°]$**
RELATED CONCEPTS/ TERMS/VOCABULARY	Using quadrants to solve trigonometric ratios

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Quadrants on a cartesian plane, trigonometric ratios

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Choosing the correct/right quadrant

METHODOLOGY

Corrections of previous activities

ACTIVITIES/ ASSESSMENT

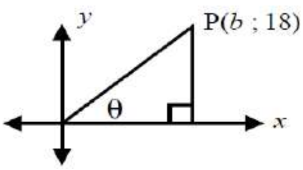
1. If $4\tan B - 3 = 0$ and $\cos B < 0$, determine by means of a diagram:

a) $(\sin B + \cos B)^2$ b) $25(\sin B - \cos B)^2$

2. If $2\sin\theta + 1 = 0$ and $90° < \theta < 270°$, calculate without the use of a calculator and with the aid of a diagram the value of the following:

a) $4cos^2\theta$ b) $81tan^2\theta$

3. In the diagram alongside $\tan\theta = \frac{12}{5}$ and $P(b; 18)$



Determine the value of b without using a calculator.

4. If $\tan\theta = \frac{a}{b}$ where $\theta \in [0°; 90°]$, determine $sin^2\theta$ by means of a diagram.

SIYAVULA: pg. 136 Exercise 5 – 7 No. 6 - 10

TEST 2: TRIGONOMETRY

MARKS: 25 **DURATION: 25 Min**

QUESTION 1 [4 Marks]

Use a calculator to determine θ (correct to ONE decimal place), ($\theta < 90^\circ$) in:

$5\sin(2\theta + 10^\circ) - 4 = 0$ (4)

QUESTION 2 [13 Marks]

2.1 If $\sin\theta = \frac{5}{13}$, determine each of the following without the use of a calculator:

(Hint: Use a sketch) ($\theta < 90^\circ$)

2.1.1 $\tan\theta$ (3)

2.1.2 $\frac{\sin\theta}{\cos\theta}$ (3)

2.1.3 $\sin^2\theta + \cos^2\theta$ (3)

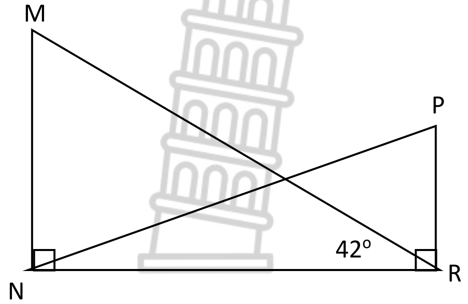
2.2 Make a conjecture about

a) $\frac{\sin\theta}{\cos\theta}$ (2)

b) $\sin^2\theta + \cos^2\theta$ (2)

QUESTION 3 [8 Marks]

In the figure alongside MN ⊥ NR, ∠MRN = 42°, MN = 8 units, PR = 5 units and PR ⊥ NR.



4.1 Calculate NR. (4)

4.3 Calculate PN (4)

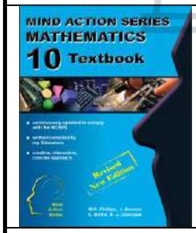
TOPIC: TRIGONOMETRY (Lesson 11)	Weighting	40 ± 3	Grade	10

Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Two-dimensional problems involving right-angled triangles.**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Angle of elevation and angle of depression, rounding off		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trigonometric ratios, right-angled triangle

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing angle of elevation and angle of depression. Do not complete reading the instructions.

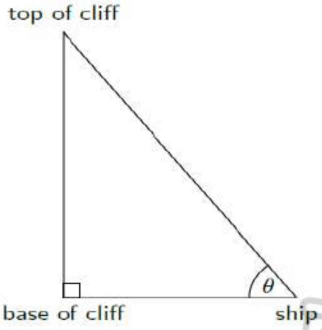
METHODOLOGY

In two-dimensional problems we will often refer to the **angle of elevation** and the **angle of depression**.

The angle of elevation is the angle formed by the line of sight and the horizontal plane for an object above the horizontal plane.

To understand these two angles let us consider a person sailing alongside some cliffs.

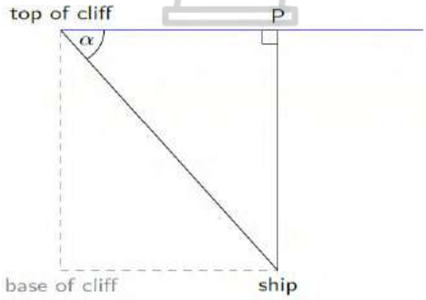
The person looks and sees the top of the cliffs as shown below:



In the above diagram, θ is an angle of elevation

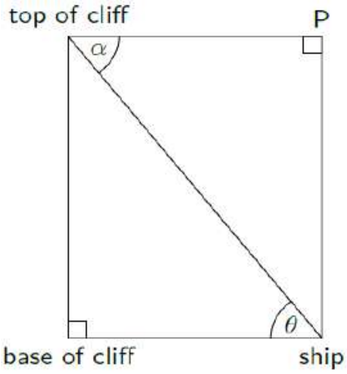
The angle of depression is the angle formed by the line of sight and the horizontal plane for an object below the horizontal plane.

To understand the angle of depression let us now consider the same situation as above but instead our observer is standing on top of the cliffs looking down at the ship.



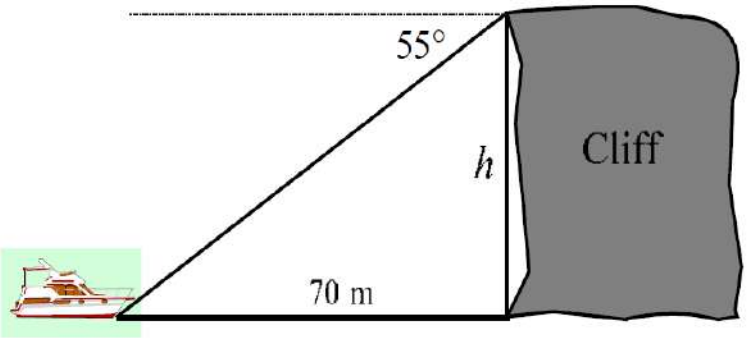
In this diagram α is the angle of depression

In the following diagram the line from the base of the cliffs to the ship is parallel to the line from the top of the cliffs to P. The angle of elevation and the angle of depression are indicated.



Finally, we can compare the angle of elevation and the angle of depression. $\hat{\theta} = \alpha$ alternate angles top of the cliff is parallel to base of the cliff.

Example:
The angle of depression of a boat on the ocean from the top of a cliff is 55°. The boat is 70 metres from the foot of the cliff.



1. What is the angle of elevation of the top of the cliff from the boat?
2. Calculate the height of the cliff.

Solution:

a) The angle of elevation of the top of the cliff from the boat is 55°

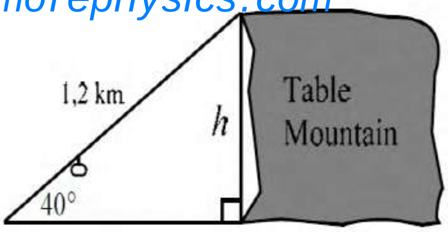
b) $\frac{h}{70m} = \tan 55°$

$h = 70\ m \times \tan 55°$
$h = 100\ m$

ACTIVITIES/ ASSESSMENT

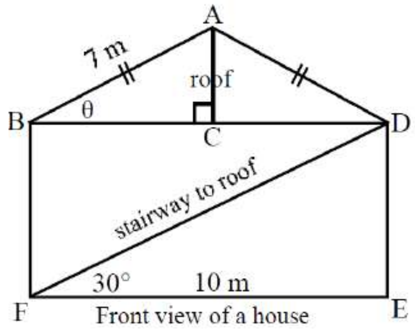
(Round your answers off to one decimal place in this exercise)

1. The Cape Town cable car takes tourists to the top of Table Mountain. The cable is 1,2 kilometres in length and makes an angle of 40° with the ground.



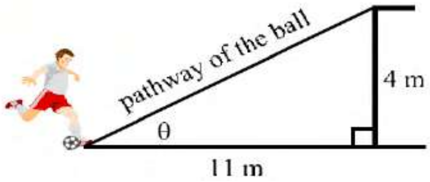
Calculate the height (h) of the mountain.

2. An architectural design of the front of a house is given below. The length of the house is to be 10 metres. An exterior stairway leading to the roof is to form an angle of elevation of 30° with ground level. The slanted part of the roof must be 7 metres in length.



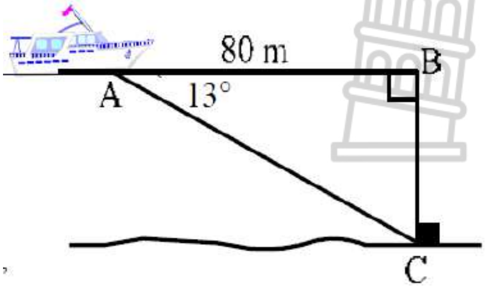
a) Calculate the height of the vertical wall (DE).
b) Calculate the size of θ, the angle of elevation of the top of the roof (A) from the ceiling BCD.
c) Calculate the length of the beam AC.

3. In a soccer World Cup, a player kicked the ball from a distance of 11 metres from the goalposts (4 metres high) in order to score a goal for his team. The shortest distance travelled by the ball is in a straight line. The angle formed by the pathway of the ball and the ground is represented by θ.



a) Calculate the largest angle θ for which the player will possibly score a goal.
b) Will the player score a goal if the angle θ is 22° ? Explain.

4. Treasure hunters in a boat, at point A, detect a treasure chest at the bottom of the ocean (C) at an angle of depression of 13° from the boat to the treasure chest. They then sail for 80 metres so that they are directly above the treasure chest at point B. In order to determine the amount of oxygen they will need when diving for the treasure, they must first calculate the depth of the treasure (BC).



Calculate the depth of the treasure for the treasure hunters.

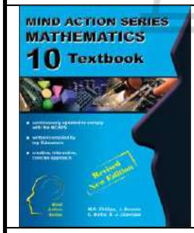
SIYAVULA: pg. 395 – 396 exercise 11 – 1 No. 1 and 2

TOPIC: TRIGONOMETRY (Lesson 12)	Weighting	40 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Two-dimensional problems involving right-angled triangles.**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Angle of elevation and angle of depression, rounding off			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trigonometric ratios, right-angled triangle

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

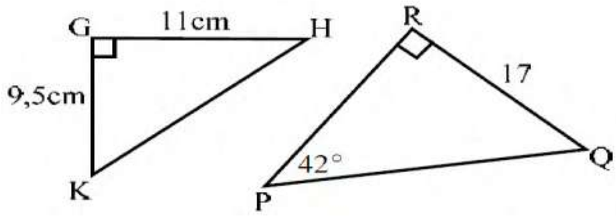
Confusing angle of elevation and angle of depression. Do not complete reading the instructions.

METHODOLOGY

Checking common mistakes on the previous activity and remedial work follows.

ACTIVITIES/ ASSESSMENT

Two right-angled triangles, ΔGHK and ΔPQR are given.
Calculate, rounded off to two decimal places:

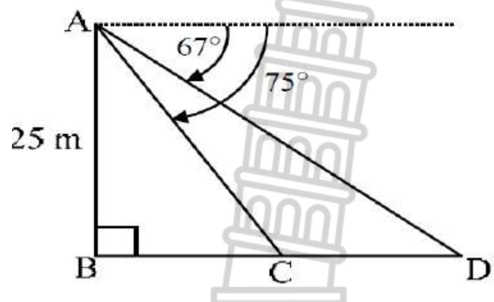


Calculate, rounded off to two decimal places:

a) the length of PQ and PR

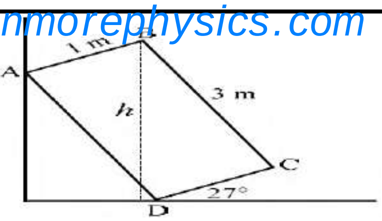
b) the value of $\hat{K}$

2. In the accompanying figure AB represents a lamp pole with height 25 m. Two cables from the top of the pole, are anchored at points C and D. From A, the angles of depression of C and D in the same horizontal line as B are 75° and 67° .



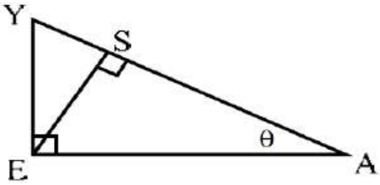
Calculate the distance (CD) between the anchor points.

3. A rectangular slab is placed against a wall as shown in the diagram. It has a length of 3 m and a width of 1 m. It is inclined at an angle of 27° to the ground.



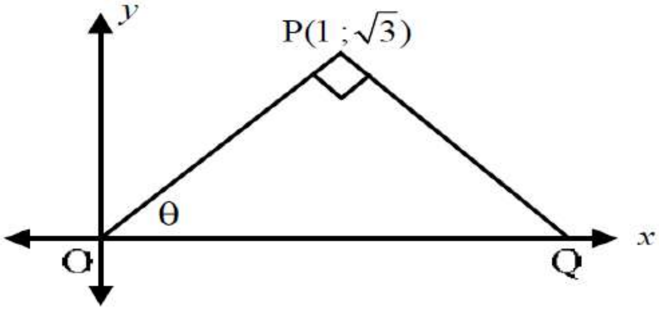
Calculate the distance (h) of the slab's highest point above the ground.

4. In ΔEAY, ES ⊥ YA, $\tan\theta = \frac{5}{12}$ and ES = 7,5 cm.



a) Calculate the lengths of EA and YA without solving θ.
b) Calculate the length of YS by first solving θ.

5. On the Cartesian Plane below O is the origin. Q lies on the x-axis and P is the point (1; 3).



a) Calculate θ
b) Determine the length of OQ.

SIYAVULA: pg. 395 – 400 no. 3,4,9,11,12,13,14 and 19.

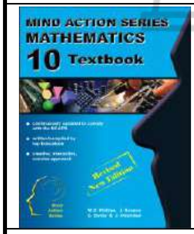
TOPIC: TRIGONOMETRY (Lesson 13)	Weighting	40 ± 3	Grade	10

Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Two-dimensional problems involving right-angled triangles.**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Calculator usage		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trigonometric ratios, Pythagoras Theorem

RESOURCES

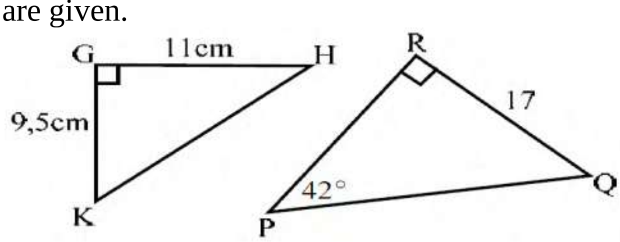


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Rounding off, using the correct trigonometric ratio

METHODOLOGY

Two right-angled triangles, ΔGHK and ΔPQR are given.



Calculate, rounded off to two decimal places:

a) the length of PQ and PR

b) the value of $\widehat{K}$

Solution:

a) $\sin 42° = \frac{17}{PQ}$ and $PR^2 = 25{,}41^2 - 17^2$

$PQ \sin 42° = 17$ $\qquad PR = \sqrt{356.6681} = 18.89cm$

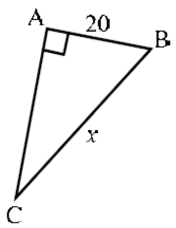
$PQ = \frac{17}{\sin 42°} = 25.41cm$

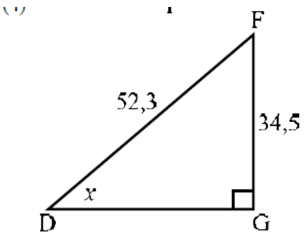
b) $\tan K = \frac{11}{9.5} = 1.15789\ldots \tan K = \frac{opp}{adj}$

$\widehat{K} = 49.18°\ldots$shift sin(1.15789)

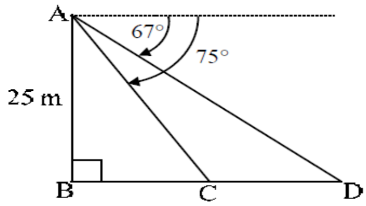
ACTIVITIES/ ASSESSMENT

1. Calculate the value of *x* in each case.





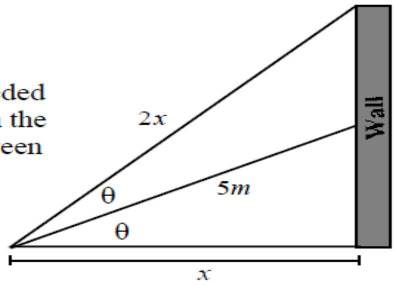
2. In the accompanying figure AB represents a lamp pole with height 25 m. Two cables from the top of the pole, are anchored at points C and D. From A, the angles of depression of C and D in the same horizontal line as B are 75° and 67° .



Calculate the distance (CD) between the two anchor points.

3.

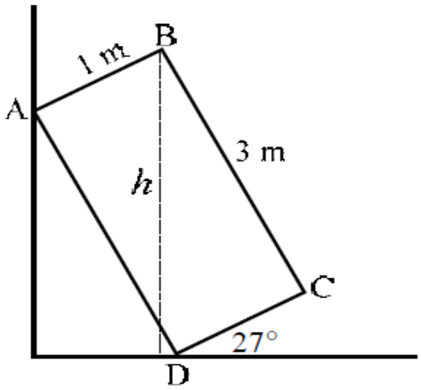
A handy man attempts to reach the roof of a hall with a ladder 5 metres in length. Unfortunately, the ladder is too short and a new ladder will be required. Suppose that the length of the ladder needed to reach the top has to be double the distance from the foot of the ladder to the wall. Also, the angle between his current ladder and the ground will need to be equal to the angle between the two ladders.



(1) Calculate the value of θ

(2) Hence, or otherwise, determine what the length of the ladder should be to get the handy man to the roof.

4. A rectangular slab is placed against a wall as shown in the diagram. It has a length of 3 m and a width of 1 m. It is inclined at an angle of 27° to the ground. Calculate the distance (*h*) of the slab's highest point above the ground.



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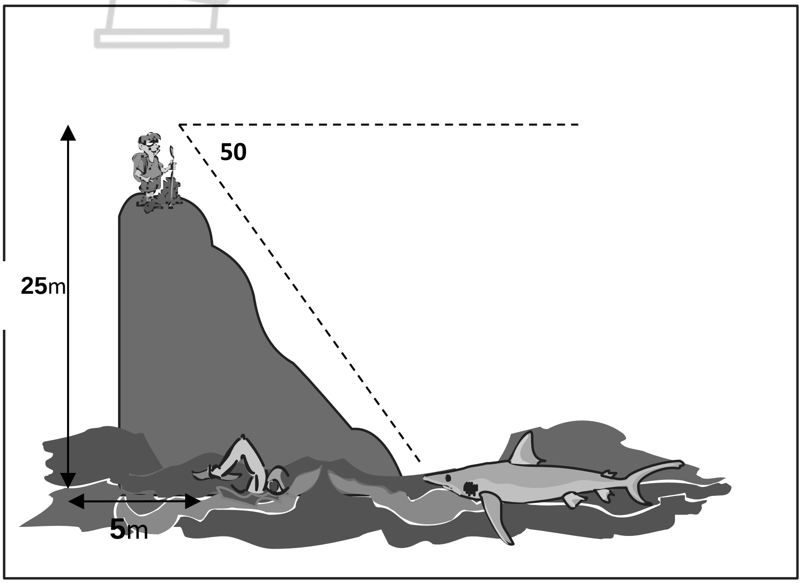
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

TEST 3: TRIGONOMETRY

MARKS: 25 **DURATION: 30 Min**

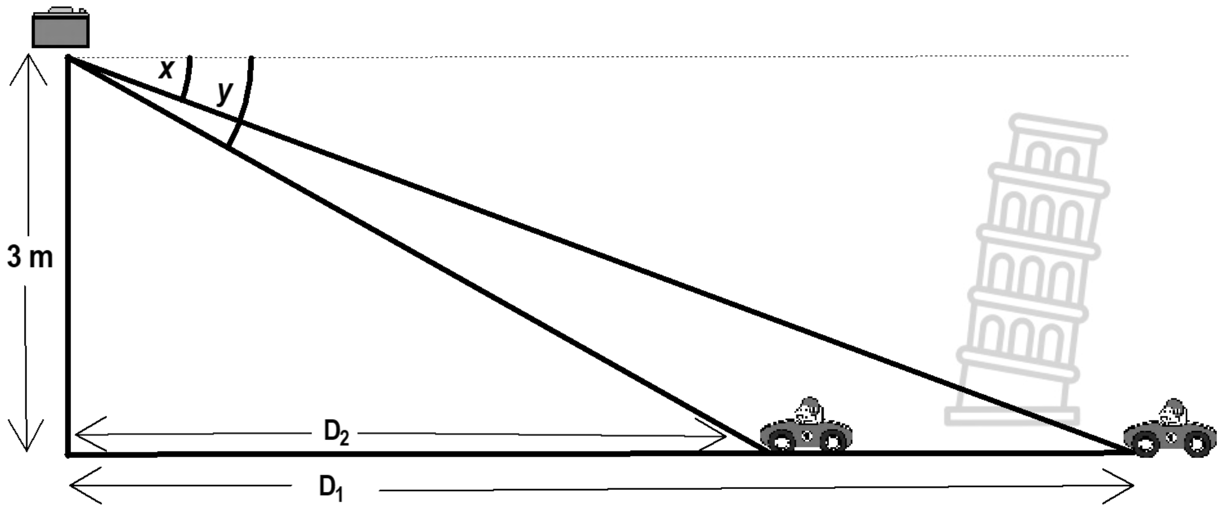
QUESTION 1 [6 Marks]

A shark spotter is standing at lookout point 25m above the water's edge. He spots a shark in the water at an angle of depression of 50°. If the swimmer that is also in the water is 5m from the foot of the lookout spot, how far is the shark from the swimmer? (6)



QUESTION 2 [8 Marks]

A laser speed trapping device is mounted on a pole that is 3m high. The device measures the initial angle of depression of a car (x) and then measures the angle of depression again 1 second later (y).



These measurements are used to work out how much distance the car has covered in 1 second and then to determine whether or not the driver is breaking the speed limit.

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M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

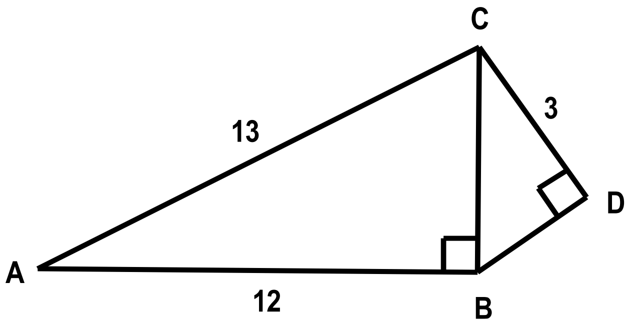
2.1 Show that the initial distance from the camera is given by: $D_1 = \dfrac{3}{\tan x}$ and the distance from the camera, one second later, is given by $D_2 = \dfrac{3}{\tan y}$ (3)

2.2 If $x = 3{,}5°$ and $y = 6°$, calculate the distance (in metres) covered by the car in 1 second. (3)

2.3 If the speed limit is 60 km/hour, determine whether or not the motorist is exceeding the speed limits. Show all calculations. (If you were unable to do 6.1, assume that the car covered 20,51 m in 1 second.) (2)

QUESTION 3 [11 Marks]

In the diagram, AC=13 units, AB = 12 units and BD = 4 units. $C\hat{B}A$ and $BD\hat{C}$ are right-angles.



3.1 Calculate the measurement of CB and BD. (4)

3.2 Now determine the value of:

3.2.1 $\tan A$ (1)

3.2.2 $\sin C\hat{B}D$ (1)

3.2.3 $\tan A\hat{C}B + \cos D\hat{B}C$ (3)

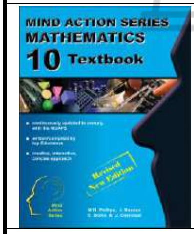
3.3 Use you calculator to determine the size of angle A, correct to one decimal place. (2)

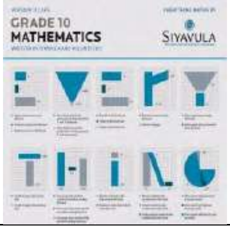
TOPIC: FUNCTIONS AND GRAPHS (Lesson 1)		Weighting	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Functions**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Functional Notation, dependent and independent variables				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Variable, input, output, set-builder notation, interval notation

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Notational error: set-builder notation and interval notation. Learners tend to ignore restrictions on certain functions. Difference between g(0) and g(x)=0

METHODOLOGY

What is a **function**?

A **function** is a mathematical relationship between two **variables**, where every **input** variable has one **output** variable.
A **function** is a rule by means of which each element of a first set, called the domain, is associated with only one element of a second set, called the range. Each element of the range is an image of corresponding elements of the range.
Dependent and Independent variables

In functions, the ***x***-variable is known as the **input** or **independen**t variable, because its value can be chosen freely. The calculated ***y*-variable** is known as the **output** or **dependent** variable, because its value depends on the chosen input value.

Set-Builder Notation

$x \in R, x > 0$: The set of all *x*-values such that *x* is an element of the set of real numbers and is greater than 0.

$3 < y \leq 5$: The set of all *y*-values such that *y*, is greater than 3 and is less than or equal to 5.

Interval Notation

It is important to note that this notation can only be used to represent an interval of real numbers.

(3; 11): Round brackets indicate that the number is not included. This interval includes all real numbers greater than but not equal to 3 and less than but not equal to 11.

$(-\infty; -2)$: Round brackets are always used for positive and negative infinity. This interval includes all real numbers less than, but not equal to -2.

[1; 9): A square bracket indicates that the number is included. This interval includes all real numbers greater than or equal to 1 and less than but not equal to 9.

Functional Notation

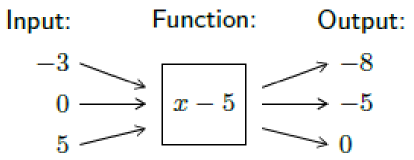
This is a very useful way to express a function. Another way of writing $y = 2x + 1$ is $f(x) = 2x + 1$. We say "*f* of *x* is equal to $2x + 1$". Any letter can be used, for example, $g(x)$, $h(x)$, $p(x)$, etc.

Given: $f(x) = 2x + 1$

Determine f (-3). This means that replace *x* by -3: $f(-3) = 2(-3) + 1 = -5$

Functions can be expressed in many different ways for different purposes.

1. Mapping Diagram:



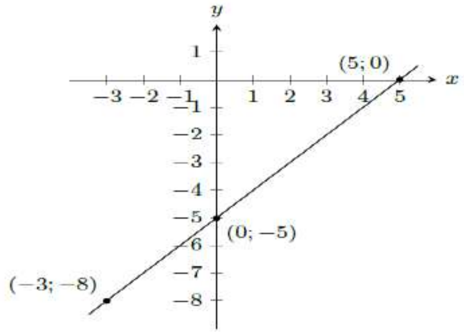
2. Table:

Input variable (x)	-3	0	5
Output variable (y)	-8	-5	0

3. Coordinates or Ordered pairs: (independent variable; dependent variable)
$(x; y)$: (-3; -8), (0; -5), (5; 0)

4. Algebraic formula: $f(x) = x - 5$

5. Graph:



The **domain** of a function is the set of **all independent *x*-values** from which the function produces a single *y*-value for each *x*-value.
The **range** is the set of **all dependent *y*-values** which can be obtained using an independent *x*-value.

ACTIVITIES/ ASSESSMENT

1. Write the following in se-builder notation:

(a) $(-\infty; 7]$

(b) [- 13; 4)

(c) (35;∞)

(d) $[\frac{3}{4}; 21)$

(e) $[-\frac{1}{2}; \frac{1}{2}]$

2. Write the following in interval notation:

(a) $p \leq 6$ (b) $-5 < k < 5$

(c) $x > \frac{1}{5}$ (d) $21 \leq x \leq 41$

3. Complete the following tables and identify the function:

a)

x	1	2	3	4	5	6
y	5	10		20		

b)

x	1	2	3	4	5	6
y	5	5			5	5

(c)

x	2			8	10	12
y	1	2	3			6

4. Plot the following points on a graph:

(a)

x	1	2	3	4	5	6
y	5	9	13	17	21	25

(b)

x	1	2	3	4	5	6
y	0,1	0,2	0,3	0,4	0,5	0,6

5. Given functions $y = \frac{1}{2}x + 2$ and $y = x - 2$,

(a) Create table of values for each function with at least 5 ordered pairs.

(b) Plot ordered pairs in a Cartesian plane and join points.

6. Given Functions: $f(x) = x^2 + 1$, $g(x) = x - 4$, $h(x) = 7 - x^2$ and $k(x) = 3$

Find the value of the following:

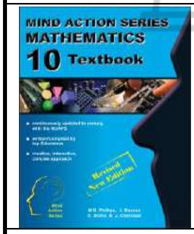
(a) f(-1) (b) g(-7) (c) h(3) (d) k(100) (e) f(-2) + h(2) (f) k(-5) + h(3) (g) f(g(1)) (h) k(f(6))

TOPIC: FUNCTIONS AND GRAPHS (Lesson 2)		**Weighting**	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Linear Function (Straight line)**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Substitution, intercepts, vertical shift, steepness/slope, gradient.				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Variable, input, output, domain, range

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Difference between the domain and the range

METHODOLOGY

Linear functions are functions of the form $y = x$

Examples:

1.

(a) Complete the following table for $y = x$ and plot the points on a set of axes (cartesian plane).

x	-1	0	1
y			

(b) Join the points with a straight line.

(c) Determine the domain and range.

(d) Using the graph, determine the value of x for which $y = 4$. Confirm your answer graphically.

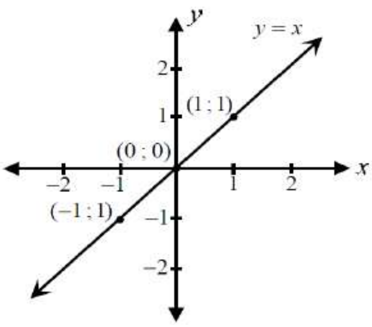
(e) Where does the graph cut the axes?

Solution:

(a) (-1, -1), (0; 0), (1; 1)

x	-1	0	1
y	-1	0	1

(b) The graph of this line is obtained by plotting the points on the Cartesian plane and drawing a solid line through the points.



(c) Domain: $x \in$ R and Range: y∈ R

(d) From the graph we see that when $y = 4$, $x = 4$. This gives the point (4; 4).

(e) The graph cut the x-axis at $x = 0$.

This graph is referred to as the "mother" graph of straight lines

Investigation of the effect of the value of *a*: $y = ax$

On the same set of axes, plot the following graphs by selecting one *x*-value and then determine the corresponding *y*-value. For all four graphs, choose *x* =1.

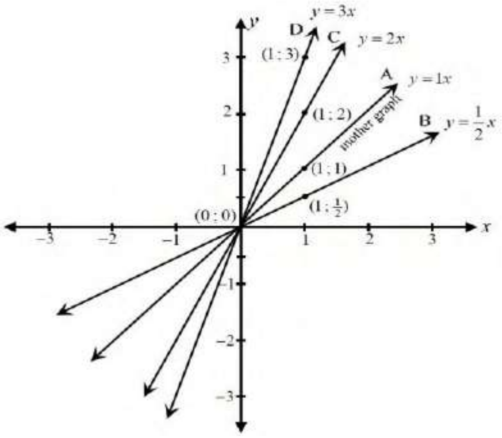
(a) $y = x$ (b) $y = \frac{1}{2}x$ (c) $y = 2x$ (d) $y = 3x$

(a) $y = (1) = 1$
(0; 0) and (1; 1)

(b) $y = \frac{1}{2}(1) = \frac{1}{2}$
(0; 0) and (1; $\frac{1}{2}$)

(c) $y = 2(1) = 2$
(0; 0) and (1; 2)

(d) $y = 3(1) = 3$
(0; 0) and (1; 3)

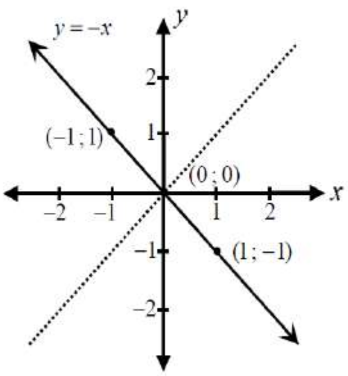


The value of *a* in the equation *y* = *ax* + *q*, determines the **steepness** of the line (closeness to the *y*-axis). The larger the value of a, the closer the graph to the y-axis (the steeper the line).

Let's now consider what happens if the value of a in $y = ax$ is **negative**.

Consider the graph $y = -x$

x	-1	0	1
y	1	0	-1



The graph of *y* = –*x* is the reflection of the mother graph (*y* = *x*) in the *x*-axis.
Therefore, a **negative sign** in the equation $y = ax + q$ causes a reflection in the *x*-axis.

If *a* > 0 then the graph **increases from left to right** (slopes upwards).
If *a* < 0 then the graph **increases from right to left** (slopes downwards). For this reason, ***a*** is referred to as the gradient of a straight-line graph.

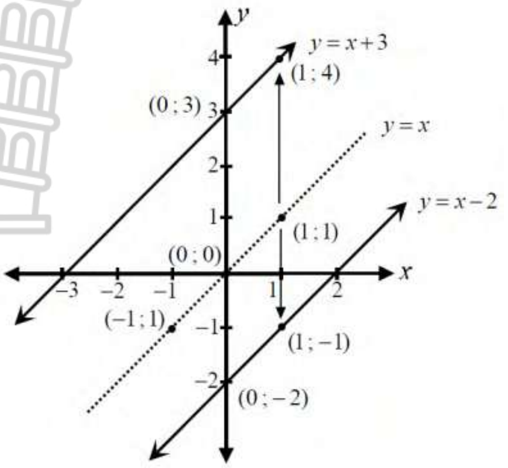
Investigation of the effect of the value of *q* in $y = ax + q$

On the same set of axes, plot the following graphs by selecting one x-value and then determine the corresponding y-value. For both graphs, choose x =1.

(a) $y = x + 3$ (b) $y = x - 2$

x=0: y = 0+3=0…(0; 3) y=0-2=-2…(0; -2)

x=1: y= 1+3=4…(1; 4) y=1-2=-1…(1; -1)



NOTE: $y = x + 3$ is the mother graph $y = x$ shifted 3 units up and $y = x - 2$ is the mother graph $y = x$ shifted 2 units down.

∴ if $q>0$ the graph **shifts** vertically **upwards** and if $q<0$ the graph **shifts** vertically **downwards**.

Also, the y-intercept of $y = x + 3$ is 3 and the y-intercept of $y = x - 2$ is -2.

The value of q in the equation $y = ax + q$ determines the shift of the graph of $y = ax$ up or down. It also represents the y-intercept of the graph of $y = ax + q$.

Intercepts are the points where the graph cuts the axes. There are two types pf intercepts.

x-intercepts (points where the graph cuts the x-axis) and **y-intercepts** (points where the graph cut the y-axis.

ACTIVITIES/ ASSESSMENT

1. Given: $y = x$ $y = \frac{3}{2}x$ $y = \frac{1}{4}x$

(a) Which of the three lines is the steepest? Explain

(b) Which of the three lines is the steepest? Explain

2. Given: $y = -x$ $y = -\frac{3}{2}x$ $y = -\frac{1}{4}x$

(a) Which of the three lines is the steepest? Explain

(b) Which of the three lines is the steepest? Explain

3. Given: $y = -x + 4$ and $y = 3x - 6$

(a) Describe the transformation of $y = x$ into the graph of $y = -x + 4$

(b) Describe the transformation of $y = x$ into the graph of $y = 3x - 6$

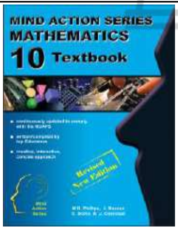
(c) Sketch the graphs of these two functions on the same set of axes using transformations.

TOPIC: FUNCTIONS AND GRAPHS (Lesson 3)		Weighting	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Linear Graph $y = ax + q$: Dual-Intercept Method**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Intercepts,				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Substitution, ordered pairs, points plotting

RESOURCES



https://saylordotorg.github.io/text_elementary-algebra/s06-03-graph-using-intercepts.html

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Forgetting that the sign of *a* determines the direction of the graph

METHODOLOGY

This method involves determining the **intercepts** with the axes algebraically, i.e., x-intercepts and y-intercepts.

In order to sketch graphs of the form, $f(x) = ax + q$, we need to determine three characteristics:

1. Sign of *a* (gradient)
2. x-intercept
3. y-intercept

Only two points are needed to plot a straight-line graph. These points are the *x*-intercept and the *y*-intercept.

The *x*-intercept is the point where the graph of a line **intersects the *x*-axis**. The *y*-intercept is the point where the graph of a line **intersects the *y*-axis**. These points have the form $(x, 0)$ and $(0, y)$, respectively.

To find the *x*- and *y*-intercepts algebraically, **use the fact that all *x*-intercepts have a *y*-value of zero** and all ***y*-intercepts have an *x*-value of zero.** To find the *y*-intercept, set x=0 and determine the corresponding *y*-value. Similarly, to find the *x*-intercept, set y=0 and determine the corresponding *x*-value.

Keep in mind that the **intercepts are ordered pairs** and **not numbers**. In other words, the *x*-intercept is not x=2 but rather (2, 0).

Examples:

1. Sketch the graph of g(x)= $x - 1$

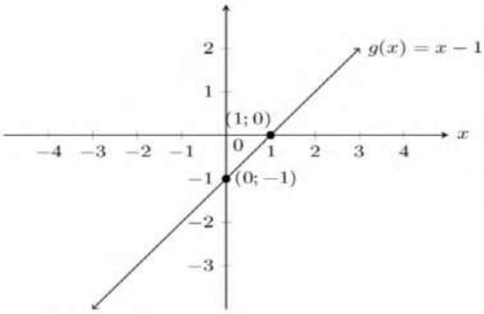
y-intercept: Let $x = 0$

g(0) = 0 -1 = -1…(0; -1)

x-intercept: Let $y = 0$

0=x-1

1 = x…(1; 0)

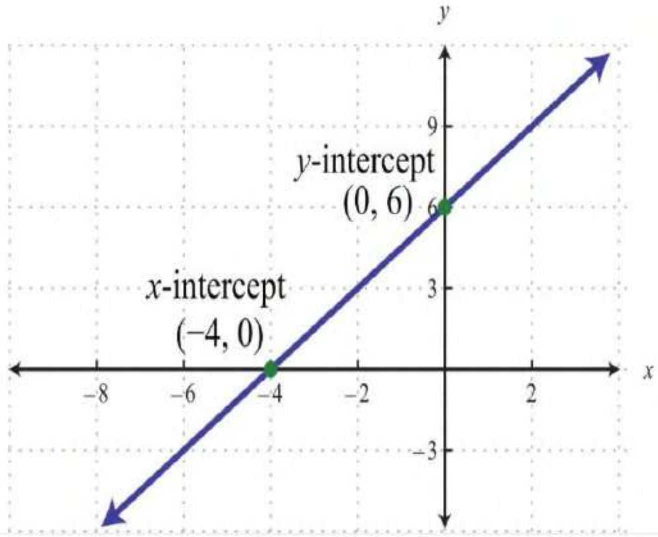


Linear functions can be also in the form $ax + by = c$

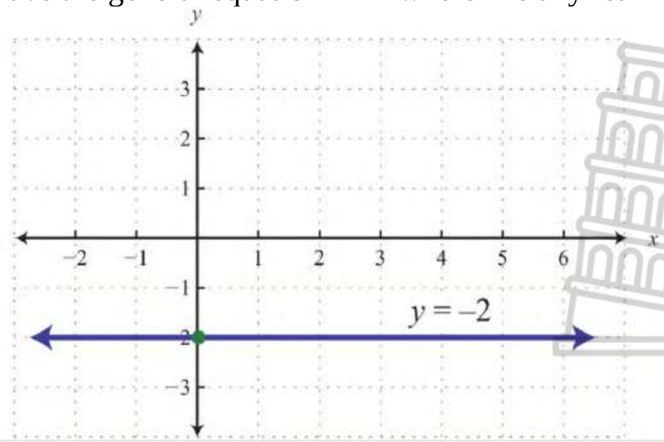
2. Sketch the graph of $-3x + 2y = 12$

y-intercept: Let $x = 0$
$-3(0) + 2y = 12$ …divede by 2 both sides
$y = 6$…(0; 6)

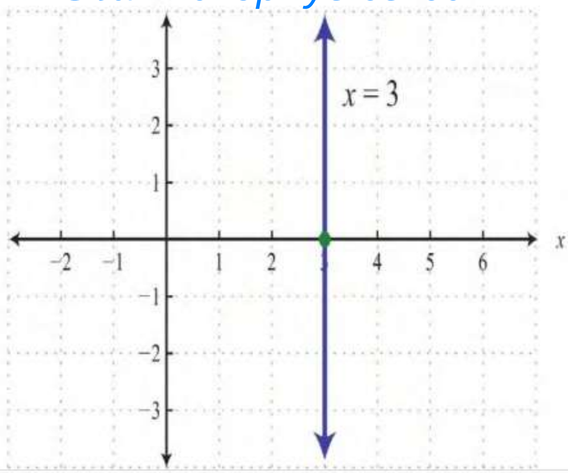
x-intercept: Let $y = 0$
$-3x + 2(0) = 12$…divide by -3 both sides
$x = -4$…(-4; 0)



Not all graphs necessarily have both intercepts. horizontal lines have the general equation $y = n$ where n is any real number. Vertical lines have the general equation $x = n$ where n is any real number.



The horizontal line graphed above has a y-intercept of (0, −2) and no x-intercept.



The vertical line graphed above has an *x*-intercept (3, 0) and no *y*-intercept.

ACTIVITIES/ ASSESSMENT

Use dual intercept method to sketch graphs of the following functions:

1. $f(x) = x - 4$

2. $g(x) = 4x + 8$

3. $y = \frac{3}{5}x - 3$

4. $x = 2$

5. $y = -3$

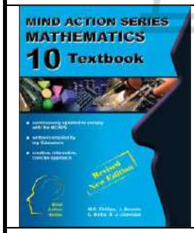
6. $5x - 2y = 10$

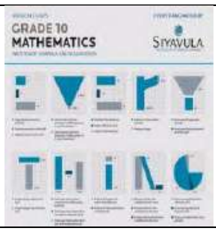
TOPIC: FUNCTIONS AND GRAPHS (Lesson 4)		Weighting	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Finding the equation of a linear function**				
RELATED CONCEPTS/ TERMS/VOCABULARY	x-intercept, y-intercept, substitution to the standard form.				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Standard form of a linear function.

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

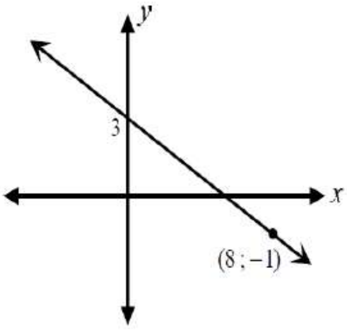
Forgetting that the sign of *a* determines the direction of the graph

METHODOLOGY

Examples:

1. Determine the equation of the following lines in the form $f(x) = ax + q$.

(a)



The y-intercept is 3, therefore q = 3. $y = ax + 3$

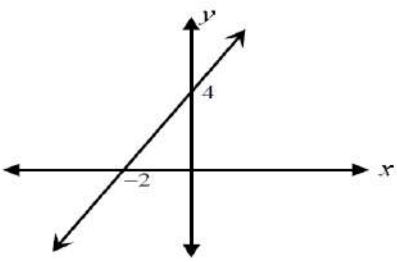
Substitute the point (8; -10) to get *a*: $-1 = a(8) + 3$

$-1 = 8a + 3$

$4 = 8a$

$\frac{1}{2} = a$. Therefore, the equation is $f(x) = \frac{1}{2}x + 3$

(b)



The y- intercept is 4, therefore q = 4. $y = ax + 4$

Substitute point (-2;0) to get *a:* $0 = a(-2) + 4$

$0 = -2a + 4$

$2a = 4$

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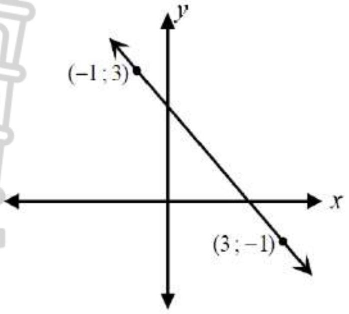
$a = 2$. Therefore, the equation is $g(x) = 2x + 4$

(c)



Substitute two points into $y = ax + q$ and solve simultaneous equations.

For (-1;3): $3 = a(-1) + q$

$3 = -a + q$ …(1) OR $3 + a = q$

For (3; -1) $-1 = a(3) + q$

$-1 = 3a + q$ …(2) $-1 = 3a + 3 + a$

(1) – (2): $4 = -4a$ $-4 = 4a$

$a = -1$ $a = -1$

Substitute the value of a to any of the two equations: $3 = -(-1) + q$

$2 = q$

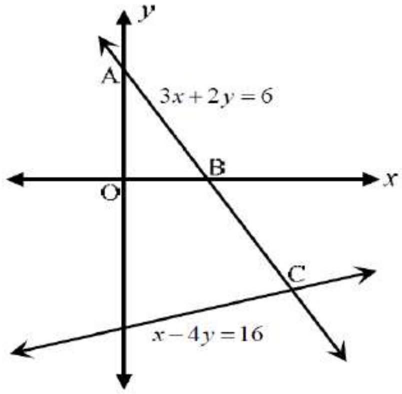
Substitute a to $3 + a = q$

$3 - 1 = q$

$2 = q$

Therefore, the equation is $f(x) = -x + 2$

2. In the diagram, two lines are drawn, $3x + 2y = 6$ and $x - 4y = 16$. The first line cuts the y-axis at A and the x-axis at B. The two lines intersect at C.



Determine:

(a) the coordinates of A and B.
(b) the gradient of $3x + 2y = 6$
(c) the gradient and y-intercept of $x - 4y = 16$
(d) the coordinates of C
(e) the values of x for which the lines are increasing or decreasing.

Solution:

(a) A: y-intercept, let $y=0$

$3x + 2(0) = 6$

$x = 2$

A (2;0)

B: x-intercept, let $x = 0$

$3(0) + 2y = 6$

$y = 3$

B (0;3)

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(b) First write $3x + 2y = 6$ in the form $y = ax + q$

$$2y = -3x + 6$$

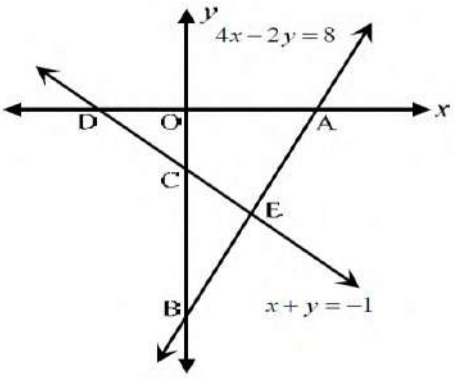
$$y = -\frac{3}{2}x + 6$$ (the coefficient of x is the gradient which is $-\frac{3}{2}$)

(c) $-4y = -x + 16$

$y = \frac{1}{4}x - 4$ Gradient is $\frac{1}{4}$ and the y-intercept is (0; -4)

ACTIVITIES/ ASSESSMENT

1. In the diagram, line $4x - 2y = 8$ cuts the axes at A and B. Line $x + y = -1$ cuts the axes at C and D. The two lines intersect at E.



Determine:

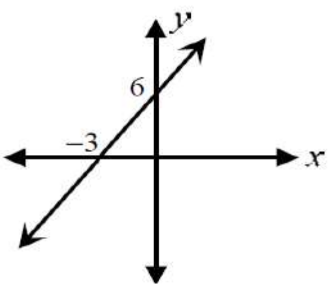
(a) the coordinates of A and B.
(b) the coordinates of C and D.
(c) the gradient of $4x - 2y = 8$
(d) the gradient of $x + y = -1$
(e) the coordinates of E
(f) the values of x for which the lines are increasing or decreasing.

2. Given: $3x - y = 4$ and $2x - y = 5$

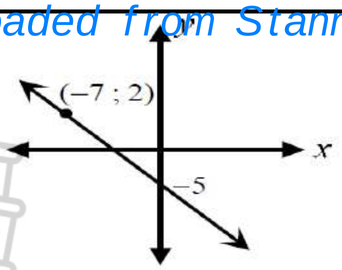
(a) On the same set of axes, draw neat sketch graphs of the two functions.

(b) Determine the coordinates of the point of intersection.

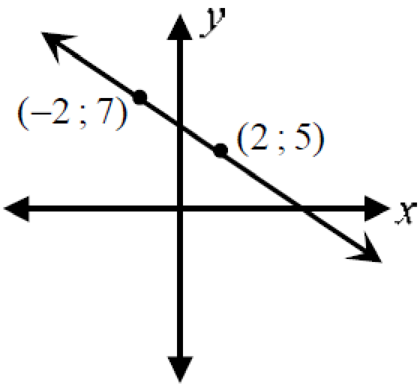
3. Determine the equations of the following lines in the form $f(x) = ax + q$:



(a)



(b)



(c)

4. Determine the equation of the line passing through the point (0; -1) and parallel to the *x*-axis. Do you remember what the gradient of this line is?

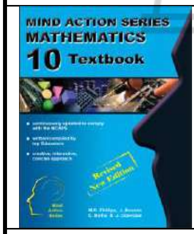
5. Determine the equation of the line passing through the point (-1;0) and parallel to the *y*-axis. Do you remember what the gradient of this line is?

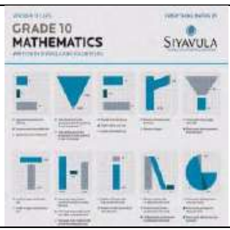
TOPIC: FUNCTIONS AND GRAPHS (Lesson 5)		Weighting	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Quadratic Functions: $y = ax^2 + q$**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Parabola, symmetrical, reflection, vertical				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Substitution, plotting points on the system of axes.

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

If the y-intercept of a graph is 6, learners write the coordinate of the point is (6;0) and if the x-intercepts are -2 and 4, they write the coordinates as (0;-2) and (0;4) sometimes.

METHODOLOGY

Functions of the general form $y = ax^2 + q$ are called parabolic functions. In the equation $y = ax^2 + q$, a and q are constants and have different effects on the parabola.

Consider the graph of $y = x^2$

Complete the table and plot the points on the systems of axes/cartesian plane

x	-3	-2	-1	0	1	2	3
y	9						

(a) Join the points with a smooth curve

(b) The domain of $y = x^2$ is $x \in R$. Determine the range.

(c) About which line is $y = x^2$ symmetrical?

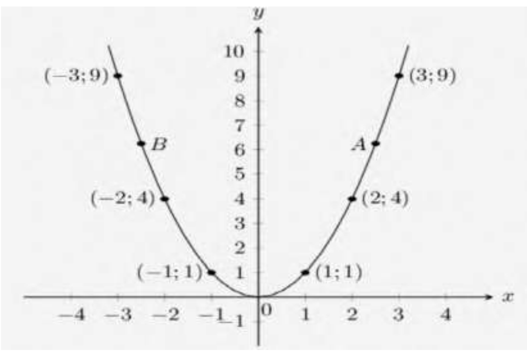
(d) Where does the graph cut the axes?

Solution:

Substitute values of x into the equation $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

All output values are positive.



The graph is not linear but rather a curve referred to as the graph of a **parabola**

(b) $y \geq 0$ $\quad OR \quad$ $y \in [0; \infty)$ $\quad OR \quad$ $0 \leq y < \infty$

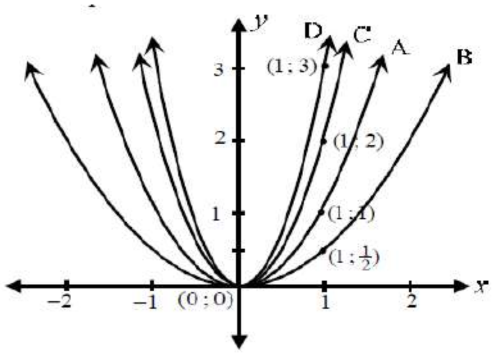
(c) y is symmetrical about the *y-axis*. Therefore, the axis of symmetry of $y = x^2$ is $x = 0$.

(d) Intercepts of $y = x^2$ are at the origin (0;0)

The effect of the value of *a*

Sketch the following graphs on the same set of axes.

A. $y = x^2$ B. $y = \frac{1}{2}x^2$ C. $y = 2x^2$ D. $y = 3x^2$

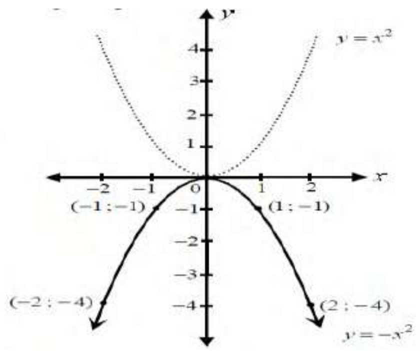


Notice that the arms of the **mother graph parabola** (A) are closer to the *y*-axis than those of B.
The arms of parabola C are closer to the *y*-axis than those of A.
The arms of parabola D are closer to the *y*-axis than those of C.
The value of the coefficient of *x* affects the shape of the parabola (or what is called its **vertical stretch**).
The greater the value of this number, the closer the arms of the parabola will be to the *y*-axis OR
If *a* increases, the graph becomes narrower or stretches. As *a* decreases the graph becomes wider or flatter.
Also note that the coefficient of x^2 for each parabola is positive and the graphs are concave up.

Negative coefficient of *x* (-*a*)

Consider the graph of $y = -x^2$

x	-2	-1	0	1	2
y	-4	-1	0	-1	-2

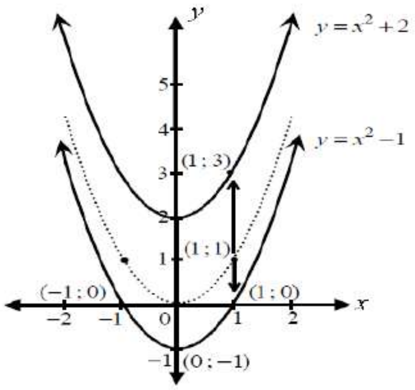


If you now compare the mother graph $y = x^2$ to the graph of $y = -x^2$, it is interesting to note that the graph of $y = -x^2$ is the reflection of the mother graph in the *x*-axis.
The negative sign therefore causes a **reflection in the *x*-axis**.
As *a* decreases the graph becomes narrower or stretches. As *a* increases the graph becomes wider or flatter.

The value of a in the equation $y = ax^2 + q$ (ignoring negative signs), determines the closeness of the arms of the parabola to the y-axis. The larger the value of a, the closer the arms are to the y-axis.
A negative sign will cause a reflection in the x-axis.
If $a > 0$, then the parabola is concave up and has a minimum turning point at $(a; q)$.
The range = $y \geq 0, y \in R$
If $a < 0$, then the parabola is concave down and has a maximum turning point at $(a; q)$.
The range = $y \leq 0, y \in R$.
The value of a is sometimes referred to as the **vertical stretch factor**

The effect of the value of q
Consider the following graphs:

1. $y = x^2 - 1$
2. $y = x^2 + 2$



The graph of $y = x^2 - 1$ is the graph of $y = x^2$ (mother graph) shifted one unit down.
The graph of $y = x^2 + 2$ is the graph of $y = x^2$ (mother graph) shifted 2 units up.

Therefore, the value of q in the equation $y = ax^2 + q$ determines the **shift of the graph up or down**. It also represents the ***y*-intercept** of the graph.

Example
Given: $y = -2x^2 + 8$ and $y = -2x^2 - 2$

(a) Sketch the graphs on the same set of axes.
(b) For these graphs, determine algebraically the coordinates of the intercepts with the axes.

Solution:
Reflect the graph of $y = 2x^2$ in the x-axis to form $y = -2x^2$ and then shift $y = -2x^2$ eight units up to form $y = -2x^2 + 8$. Then shift the graph of $y = -2x^2$ two units down to form $y = -2x^2 - 2$.

(b) $y = -2x^2 + 8$
y-intercept: Let $x = 0$
$y = -2(0)^2 + 8$
$y = 8$
(0;8)
x-intercept: y = 0
$0 = -2x^2 + 8$
$2x^2 = 8$
$x^2 = 4$
$x = \pm 2$

$y = -2x^2 - 2$
y-intercept: Let $x = 0$
$y = -2(0)^2 - 2$
$y = -2$
(0; -2)

$0 = -2x^2 - 2$
$2x^2 = -2$
$x^2 = -1$
$x = \pm\sqrt{-1}$ non-real

ACTIVITIES/ ASSESSMENT

1. Given: $y = 3x^2$ and $y = \frac{1}{4}x^2$

(a) Which parabola has arms that are closest to the y-axis?

(b) Sketch the graphs of these parabolas on the same set of axes.

(c) Are the parabolas concave up or down? Explain

2. Given: $y = -\frac{1}{2}x^2$ and $y = -4x^2$

(a) Which parabola has arms that are closest to the y-axis?

(b) Sketch the graphs of these parabolas on the same set of axes.

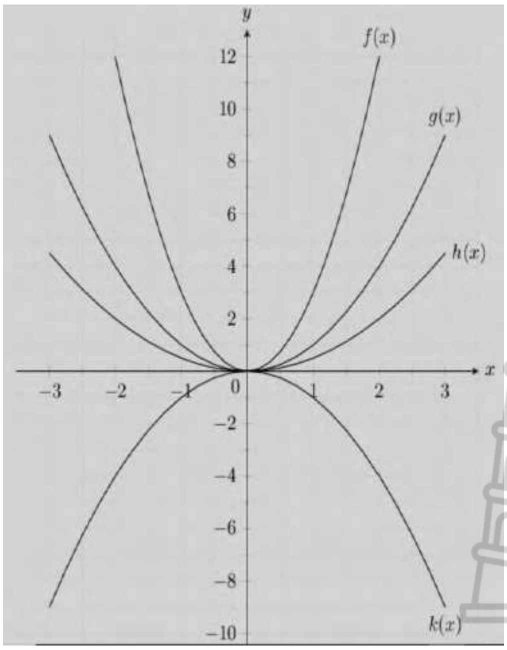
(c) Are the parabolas concave up or down? Explain

3. Given: $y = x^2 - 4$ and $y = -4x^2 - 2$

(a) Sketch the graphs on the same set of axes.

(b) For these graphs, determine algebraically the coordinates of the intercepts with the axes.

4. Given the following graph, identify a function that matches each of the following:



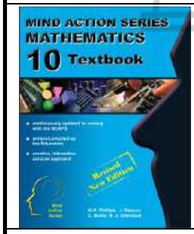
(a) $y = \frac{1}{2}x^2$ (b) $y = x^2$ (c) $y = -x^2$ (d) $y = 3x^2$

TOPIC: FUNCTIONS AND GRAPHS (Lesson 6)		Weighting	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Hyperbolic functions (sketching hyperbolas):** $\boldsymbol{y = \frac{a}{x} + q}$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Asymptotes: vertical and horizontal, line of symmetry				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Intercepts, domain, range

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Meaning of sign of a in the equation $\boldsymbol{y = \frac{a}{x} + q}$

METHODOLOGY

Consider the graph: $y = \frac{1}{x}$

Complete the following table for and plot the points on a system of axes.

x	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2
y									

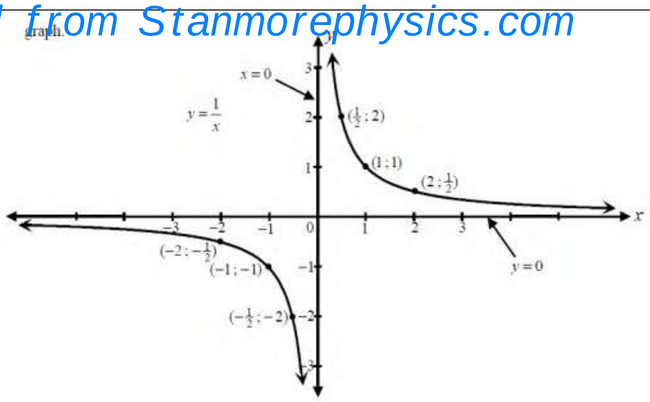
(a) Join the points with smooth curves.
(b) What happens if $x = 0$?
(c) Explain why the graph consists of two separate curves.
(d) What happens to y as the values of x become very small or very large?
(e) Determine the domain and the range.
(f) About which two lines is the graph symmetrical?

Solution:

x	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	-4	$\frac{1}{0}$	4	2	1	$\frac{1}{2}$

$\frac{1}{0}$ is undefined, this is called a discontinuity at $x = 0$.

Since the y-value is undefined at $x = 0$, this means that the graph has no y-intercept.



This graph is referred to as the "mother" graph of hyperbolas and has no intercept with the axes.

As the value of x gets larger, the value of y gets closer to, but does not equal 0.

The same happens in the third quadrant, as x gets smaller y gets closer to, but does not equal 0.

The graph gets closer and closer to the axes but never actually cuts them.

An **asymptote** is a horizontal or vertical line that a graph approaches but never touches.

The vertical line $x = 0$ (lying on the y-axis) is called the **vertical asymptote** of the graph.

The horizontal line $y = 0$ (lying on the x-axis) is called the **horizontal asymptote** of the graph.

Domain: $x \in R, x \neq 0$ Range: $y \in R, y \neq 0$

The graph of y has two lines of symmetry: $y = x \text{ and } y = -x$. About these two lines, one half of the hyperbola is a mirror image of the other half.

The Effect of the value of *a*

Consider the following graphs:

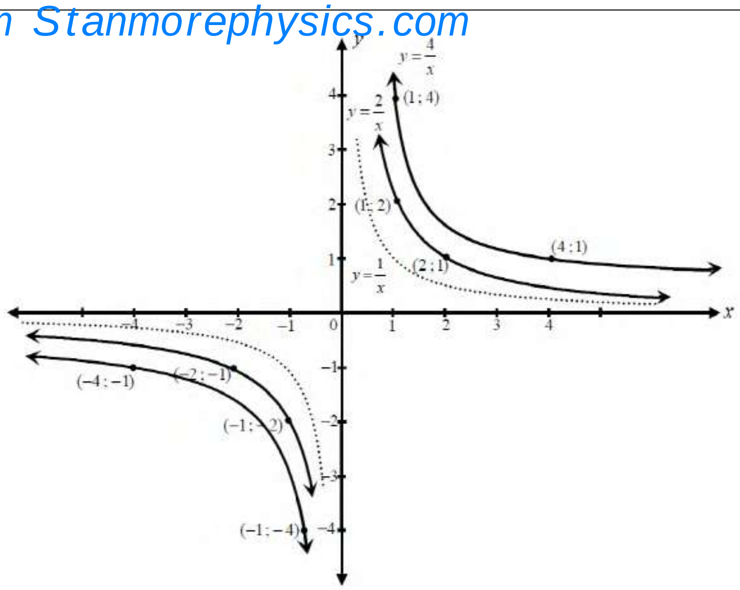
(a) $y = \frac{1}{x}$ (c) $y = \frac{2}{x}$ (c) $y = \frac{4}{x}$

$y = \frac{2}{x}$

x	-2	-1	0	1	2
y	-1	-2	U	2	1

$y = \frac{4}{x}$

x	-4	-1	0	1	4
y	-1	-4	U	4	1

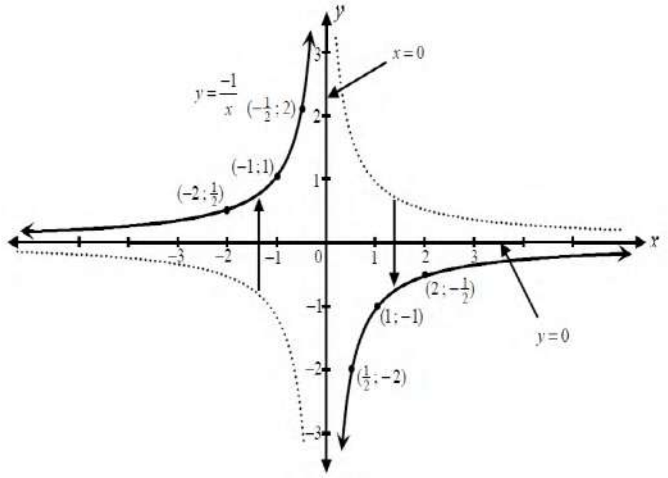


Notice that as the number in the numerator (a) gets larger, the branches of the hyperbolas are stretched vertically away from the x-axis.

Negative value of *a*

Consider $y = \frac{-1}{x}$

As with lines and parabolas, the negative sign indicates a **reflection in the *x*-axis**.



NOTE:

The value of a in the equation y= $\frac{a}{x} + q$ (ignoring negative signs), determines the vertical stretch of the branches of the hyperbola from the x-axis.

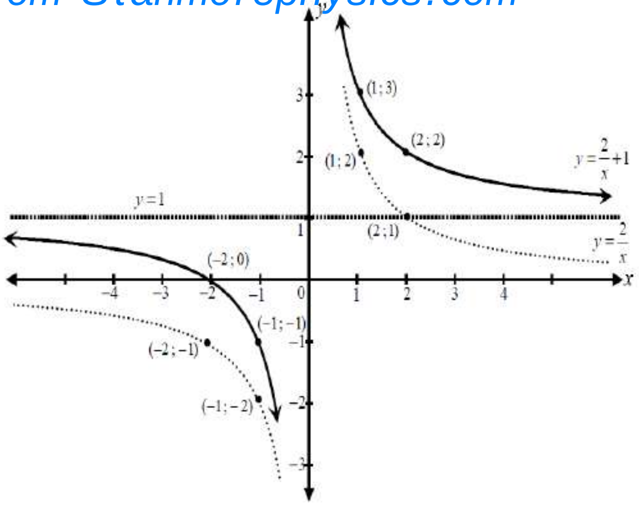
The larger the value of a, the further the stretch away from the axes.

A negative sign will cause a reflection in the x-axis.

The effect of the value of q:

Consider the following graph: $y = \frac{2}{x} + 1$

Let us draw the graph of $y = \frac{2}{x}$ and then shift it up 1 unit and see what effect this shifting has.



Notice that the graph of $y = \frac{2}{x} + 1$ cuts the *x*-axis at (-2; 0).

The horizontal asymptote shifts 1 unit up and has the equation *y* =1.

The constant in the equation $y = \frac{2}{x} + 1$ therefore represents the horizontal asymptote.

The vertical asymptote is still the line *x* = 0 (lying on the *x*-axis).

The effect of *q* is called a **vertical shift** because all points are moved the same distance in the same direction (it slides the entire graph up or down).

It also represents the **horizontal asymptote** of the graph of the equation.

The **horizontal asymptote** is the line *y* = *q* and the **vertical asymptote** is always the *y*-axis, the line *x* = 0.

Example:

Given: $y = -\frac{3}{x} - 1$

(a) Determine algebraically the coordinates of the *x*-intercept for this graph.

(b) Describe the different transformations of $y = \frac{3}{x}$ to $y = -\frac{3}{x} - 1$

(c)Sketch the graph of $y = -\frac{3}{x} - 1$ on a set of axes, clearly showing the asymptotes and *x*-intercept.

(d) Determine the domain and the range of the graph.

(e) Determine the line of symmetry a positive gradient.

Solution:

(a) x-intercepts: Let *y*=0

$0 = -\frac{3}{x} - 1$

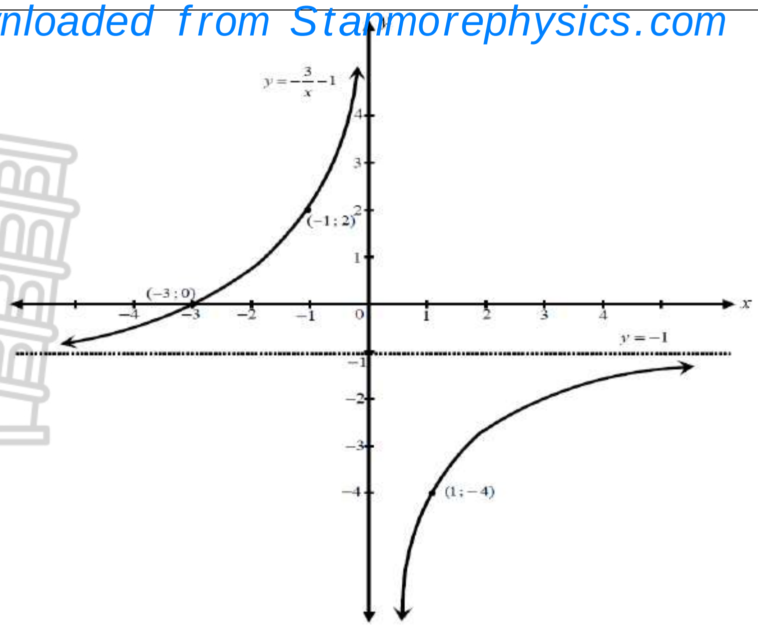
$1 = -\frac{3}{x}$ Transpose 1

$x = -3$ Multiply by *x* n both sides of the equation

(-3;0)

(b) $y = -\frac{3}{x} - 1$ is the graph of $y = \frac{3}{x}$ reflected in the *x*-axis and then shifted 1 unit down.

(c)



(d) Domain: $x \in R, x \neq 0$ Range: $y \in R, y \neq 1$

(e) The line passes through (0; -1): $y = x + c$

$-1 = 0 + c$

c = -1 Therefore, $y = x - 1$ is the line of symmetry.

ACTIVITIES/ ASSESSMENT

1. Given: $y = \frac{5}{x}$ and $y = \frac{1}{x}$

(a) Which graph has branches that have the furthest stretch away from the *x*-axis? Explain.

(b) Sketch the graphs on the same set of axes.

(c) Now sketch the graph of $y = -\frac{5}{x}$ on the same set of axes.

2. Given: $y = -\frac{4}{x} - 1$

(a) Write down the equations of the vertical and horizontal asymptotes.

(b) Determine the coordinates of the *x*-intercept.

(c) Sketch the graph on a set of axes. Indicate the coordinates of the *x*-intercept as well as the asymptotes.

(d) Determine the domain and range of the function.

(e) Describe the different transformations of $y = \frac{4}{x}$ to $y = -\frac{4}{x} - 1$.

(f) Determine the line of symmetry a negative gradient.

TOPIC: FUNCTIONS AND GRAPHS (Lesson 7)	**Weighting**	30 ± 3	Grade	10

Term		**Week no.**	
Duration	1 hour	**Date**	

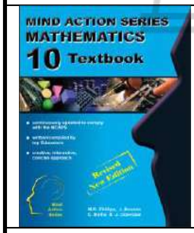
Sub-topics: **Exponential functions (sketching exponential graphs):** $\boldsymbol{y = ab^x + q}$

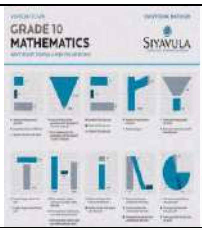
RELATED CONCEPTS/ TERMS/VOCABULARY: Vertical shift, horizontal asymptote

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Intercepts with the axes: x-intercept and y-intercept, asymptote

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Failing to differentiate between decreasing and increasing function. No understanding why $\left(\frac{1}{2}\right)^x = 2^{-x}$

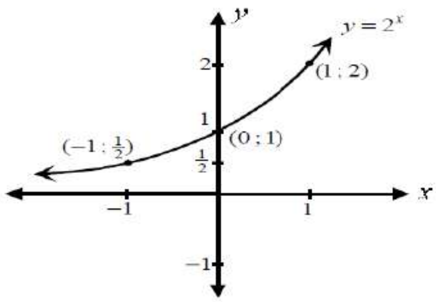
METHODOLOGY

Functions of the general form $y = ab^x + q$ are called exponential functions. In the equation a and q are constants and have different effects on the function.

The function $y = ab^x$ (Mother graph)

Consider: $y = 2^x$

x	-1	0	1
y	$\frac{1}{2}$	1	2

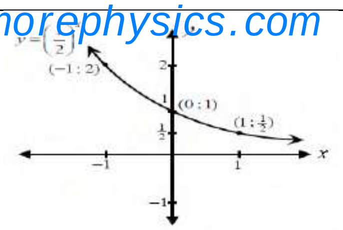


There is no x-intercept. The x-intercept is calculated by letting $y = 0$: $0 = 2^x$. But there are no real values of x for which $2^x = 0$.

The graph will therefore not cut the x-axis. In fact, the graph has a **horizontal asymptote** lying on the x-axis with equation $y = 0$.

Consider: $y = \left(\frac{1}{2}\right)^x$

x	-1	0	1
y	2	1	$\frac{1}{2}$



The graphs of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$ are called the "mother graphs" of the exponential functions and based on these graphs, we can generate different types of exponential graphs, depending on the value of a, b and q in the general equation of an exponential function, which is $y = ab^x + q$ where $0 < b < 1$ Or $b > 1$. These restrictions on *b* will be explained later in the chapter.

NOTE: The exponential function does not cut the x-axis unless the mother graph is translated down.

Investigation of the effect of the value of *b*

1. Exponential graphs where *b* >1.

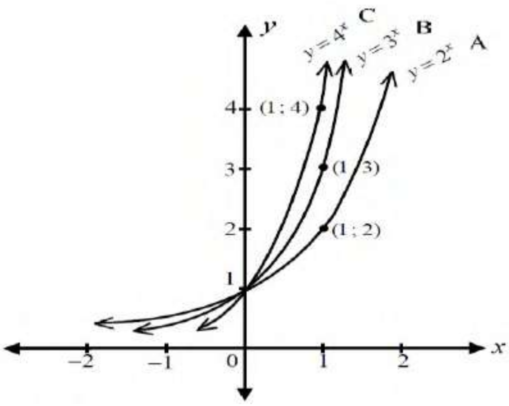
On the same set of axes, plot the following graphs (*a* = 1, *q* = 0 and *b* changes):

(a) $y = 2^x$ (b) $y = 3^x$ (c) $y = 4^x$

For all of these graphs, the *y*-intercept is (0 ;1) since $b^0 = 1$

Select one other *x*-value ($x = 1$) and determine the corresponding *y*-value for all given functions.

(a) $y = 2^1 = 2$: (1;2) (b) $y = 3\;\;1 = 3$: (1;3) (c) $y = 4^1 = 4$: (1;4)



Notice that if *b* >1, all of the exponential graphs move upwards (increasing function) as the *x*-values increase. Also, as the value of *b* **increases** in value, the **steeper** the graph becomes (graph gets closer to the y-axis).

2. Exponential graphs where 0 < *b* < 1.

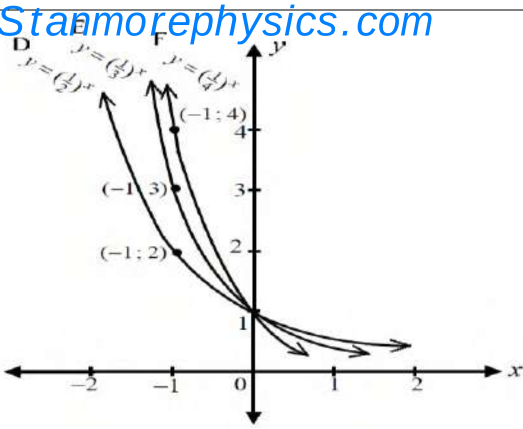
On the same set of axes, plot the following graphs (*a* = 1, *q* = 0 and *b* changes):

(a) $y = \left(\frac{1}{2}\right)^x$ (b) $y = \left(\frac{1}{3}\right)^x$ (c) $y = \left(\frac{1}{4}\right)^x$

For all of these graphs, the *y*-intercept is (0 ;1) since $b^0 = 1$

Select one other *x*-value ($x = -1$) and determine the corresponding *y*-value for all given functions.

(a) $y = \left(\frac{1}{2}\right)^{-1} = 2$: (-1;2) (b) $y = \left(\frac{1}{3}\right)^{-1} = 3$: (-1;3) (c) $y = \left(\frac{1}{4}\right)^{-1} = 4$: (-1;4)



Notice that if 0 < *b* <1, all of the exponential graphs move downwards (decreasing function) as the *x*-values increase. Notice that as the value of the base *b* **decreases** in value, the **steeper** (graph get closer to the y-axis) the graph becomes.

Investigation of the effect of the value of *a*

The **sign of *a*** determines whether the graph curves upwards or downwards.

On the same set of axes, plot the following graphs (*b* = 2, *q* = 0 and *a* changes).

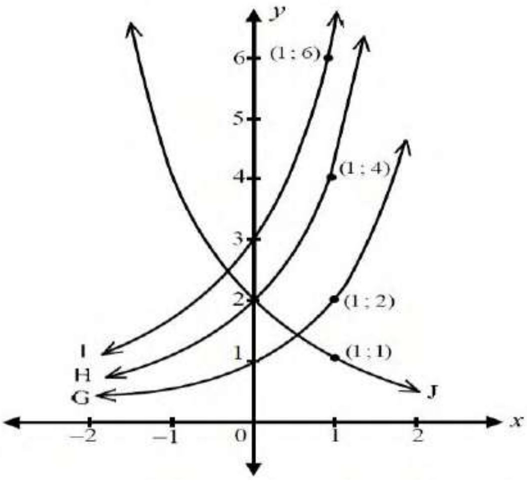
(a) $y = 2^x$ (b) $y = 2.2^x$ (c) $y = 3.2^x$ (d) $y = 2.\left(\frac{1}{2}\right)^x$

Calculate the y-intercept and one other point (use $x = 1$) for each graph.

y-intercept: x= 0

(a) $y = 2^0 = 1$: (0;1) (b) $y = 2.2^0 = 2$: (0;2) (c) $y = 3.2^0 = 3$: (0;3) (d) $y = 2\left(\frac{1}{2}\right)^0 = 2$: (0;2)

$y = 2^1 = 2$: (1;2) $y = 2.2^1 = 4$: (1;4) $y = 3.2^1 = 6$: (1;6) $y = 2.\left(\frac{1}{2}\right)^1 = 1$: (1;1)

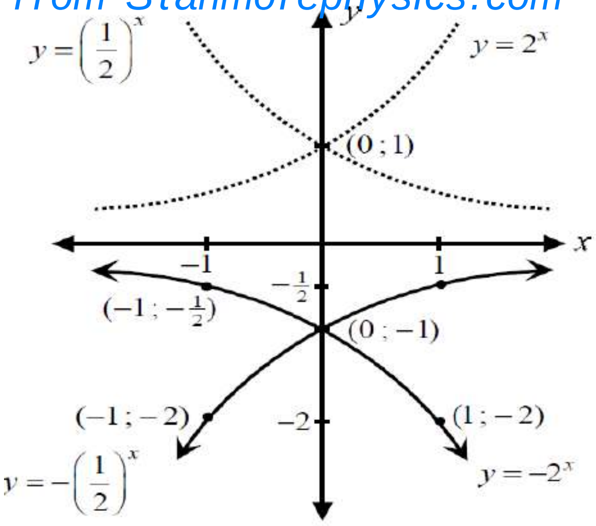


Notice that the value of *a* causes a vertical stretch of the mother graphs. Also note that the graphs have different *y*-intercepts.

For *a* > 0 and *b* > 1, the graph curves upwards [check (a), (b) and (c)].

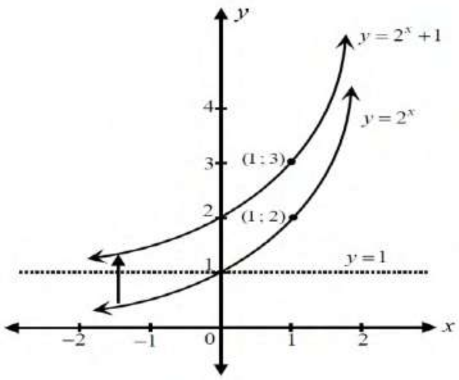
For *a* > 0 and 0 < *b* < 1, the graph curves downwards [check (d)]. It reflects the graph about the horizontal asymptote.

The graphs of $y = -2^x$ and $y = -\left(\frac{1}{2}\right)^x$ are reflections of the two mother graphs in the *x*-axis.



In the expression -2^x , the value of *b* is 2 and not -2. Remember that $-2^x = -(2)^x$
For *a* < 0, the graph curves upwards.

Investigation of the effect of the value of *q*
As with other graphs discussed thus far, the graph of $y = ab^x + q$ is the graph of $y = ab^x$ shifted up or down by *q* units.
The effect of q is called a vertical shift because all points are moved the same distance in the same direction (it slides the entire graph up or down).
Example
$y = 2^x + 1$ is the graph of $y = 2^x$ shifted 1 unit up. The horizontal asymptote is indicated by the constant in the equation $y = 2^x + 1$. The equation of the asymptote is *y* =1.



For *q* > 0, the graph is shifted vertically upwards by *q* units.
For *q* < 0, the graph is shifted vertically downwards by *q* units.

ACTIVITIES/ ASSESSMENT

1. Given graphs below:
   (a) Which graph is the steepest? Explain.
   (b) Determine the coordinates of the *y*-intercept for each graph.
   (c) Write down the equation of the horizontal asymptote for the three graphs.
   (d) Sketch the graphs on the same set of axes indicating the *y*-intercept and one other point.

1.1 $y = 3^x$ and $y = 5^x$

1.2 $y = \left(\frac{1}{3}\right)^x$ and g(x)= $\left(\frac{1}{5}\right)^x$

1.3 h(x)= $4.\,2^x$ and f(x)= $3\left(\frac{1}{2}\right)^x$

2. Given: $y = -\left(\frac{1}{2}\right)^x$ and $y = -3\left(\frac{1}{2}\right)^x$

(1) Determine the coordinates of the *y*-intercept for each graph.
(2) Write down the equation of the horizontal asymptote for both graphs.
(3) Sketch the graphs on the same set of axes indicating the *y*-intercept and one other point.
(4) Explain the transformations of $y = \left(\frac{1}{2}\right)^x$ to $y = -3\left(\frac{1}{2}\right)^x$

3. Consider: f(x)= $2^x - 2$ and $y = 2^x + 1$
(a)Determine the intercepts of f(x)= $2^x - 2$ with the axes.
(b) Write down the equation of the horizontal asymptote of f(x)= $2^x - 2$.
(c) Sketch the graph of $f(x) = 2^x - 2$ on a set of axes.
(d) Explain the transformation of $y = 2^x$ into the graph of f(x)= $2^x - 2$
(e) Sketch the graph of $y = 2^x + 1$ on the same set of axes.
(f) Explain algebraically why the graph of $y = 2^x + 1$ does not cut the *x*-axis.

TEST 1: GRAPH SKETCHING

TOTAL: 25 Marks **Duration: 30 Min**

Answer ALL questions

QUESTION 1[21 Marks]

1. Given: $f(x) = 3\left(\frac{1}{2}\right)^x$, $g(x) = \frac{3}{x}$ and $h(x) = -x^2$

1.1 Write down names of the functions (3)

1.2 Determine the values of;

(a) $h(2)$ (b) $g(-1)$ (c) x if $f(x) = 0$ (6)

1.3 Draw a sketch graph of each of the functions showing all critical points**, asymptotes, axes of symmetry and intercepts with the axes.** Each function must be sketched on a separate set of axes. (6)

1.4 Determine the domain and the range of the given functions. (6)

QUESTION 2 [4 Marks]

Consider the functions: $f(x) = x^2 - 9$ and $g(x) = 2x - 6$

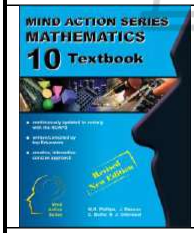
Sketch the graphs of f and g on the same system of axes, showing ALL intercepts with the axes And turning points. (4)

TOPIC: FUNCTIONS AND GRAPHS (Lesson 8)		Weighting	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**The Equation of the quadratic Function: $y = ax^2 + q$** **Finding the equation of a quadratic function(parabola)**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Turning point, decreasing and increasing function, maximum value, minimum value, axis of symmetry				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Shape of a parabola, vertical shift ,how to calculate intercepts with the axes.

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Knowing that q is the y-value of the turning point, q is either the minimum or maximum value of the parabola, q is part of the y-intercept.

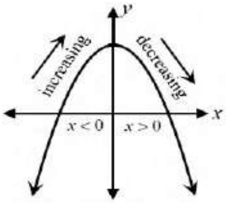
METHODOLOGY

The **turning point** is a point of the graph where the graph changes from increasing to decreasing (rising to falling) or from decreasing to increasing (falling to rising)

The turning point of the function of the form $\boldsymbol{f(x) = ax^2 + q}$ is determined by examining the **range of the function.**

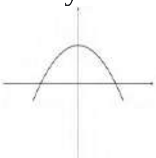
Increasing Function: when the y-value increases as the x-value increases.

Decreasing Function: when the y-values decreases as x-values increases.



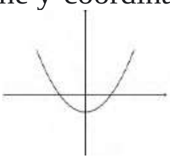
The **maximum value** is the largest y-value on the graph**.**

The **maximum value of a parabola** is the y-coordinate of the turning point that opens down.



The **minimum value** is the smallest y-value on the graph.

The **minimum value of a parabola** is the y-coordinate of the turning point that opens up.



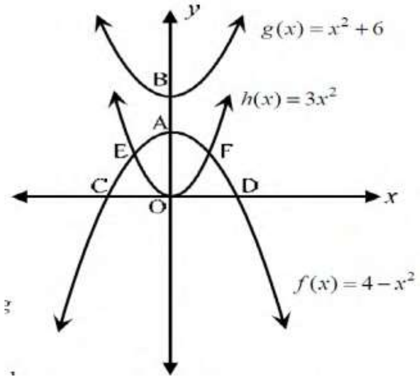
We can identify the minimum or **maximum value of a parabola** by identifying the y-coordinate of the turning point.

Axis of symmetry is a line that divides an object into two equal halves.

Axis of symmetry of a parabola is a vertical line that divides the parabola into two equal halves.

Example:

1.In the diagram, the graphs of $f(x) = 4 - x^2$, $g(x) = x^2 + 6$ and $h(x) = 3x^2$ are shown. The graph of f cuts the axes at A, C and D. The graph of g cuts the y-axis at B.



Determine:

(a) the coordinates of A, B, C and D
(b) the coordinates of E and F
(c) the values of x for which f is increasing
(d) the values of x for which g is decreasing
(e) the maximum or minimum values of f and g
(f) the turning point of f and g
(g) the equation of the axis of symmetry for f and g

Solution:

(a) A: y-intercept of f, y-intercept when x = 0. (0;4)
B: y-intercept of g, (0;6)
C and D are the x-intercepts of f. x-intercepts when y = 0

$$4 - x^2 = 0$$
$$(2 - x)(2 + x) = 0 \ldots \text{ difference of two squares}$$
$$x = 2 \text{ or } x = -2$$
D (2;0) and C (-2;0)

(b) E and F are the points of intersection of $f(x)$ and $g(x)$
At the point of intersection, graphs/functions are equal.
Therefore, at E and F $f(x) = g(x)$

$$4 - x^2 = 3x^2$$
$$4 = 4x^2$$
$$1 = x^2 \ldots \text{divide by 4 on both sides}$$
$$\pm\sqrt{1} = \sqrt{x^2}$$
$$x = 1 \text{ or } x = -1$$

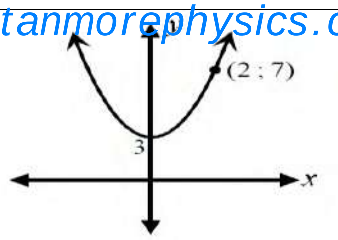
Determine the corresponding y-values by substituting the values of x into either of the two equations
In $f(x) = 4 - x^2$: $f(1) = 4 - (1)^2 = 3$ and $f(-1) = 4 - (1)^2 = 3$
Therefore, E (-1;3) and F (1;3)

(c) f is increasing for all $x < 0$
(d) g is decreasing for all $x < 0$
(e) The maximum value of f is 4 and the minimum value of g is 6
(f) f: (0;4) and g: (0;6)
(g) For both graphs: y-axis and the equation is $x = 0$

Finding the equation of a quadratic function (parabola)

Example:

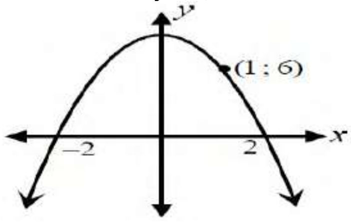
2. (a)Determine the equation of the below graph in the form $f(x) = ax^2 + q$.



Solution:
The y- intercept is 3. Therefore, $q = 3$. Then $y = ax^2 + 3$
Now substitute (2;7) into $y = ax^2 + 3$ to get the value of a

$7 = a(2)^2 + 3$
$4 = 4a$
$1 = a$ Therefore $f(x) = 1x^2 + 3$

(b) Determine the equation of the below graph in the form $f(x) = ax^2 + q$.

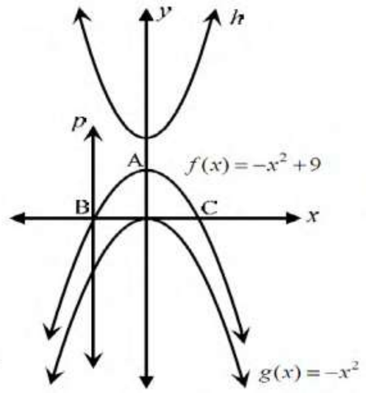


Solution:
The x-intercepts are (0; -2) and (0; 2)
The x-intercept formula of a parabola is $y = a(x - x_1)(x - x_2$, where $x_1 and\ x_2$ are x- intercepts.

$y = a(x - (-2))(x - 2)$
$y = a(x + 2)(x - 2)$
$y = a(x^2 - 4)$
Now substitute (1; 6): $6 = a(1^2 - 4)$
$6 = -3a$
$a = -2$ Therefore, $y = -2(x^2 - 4)$
$f(x) = -2x^2 + 8$ … multiply by -2

ACTIVITIES/ ASSESSMENT

1. In the diagram, the graphs of $f(x) = -x^2 + 9$, $g(x) = -x^2$,*h* and *p* are shown. The graph of *f* cuts the axes at A, B and C. Line *p* cuts the *x*-axis at B and is parallel to the *y*-axis.

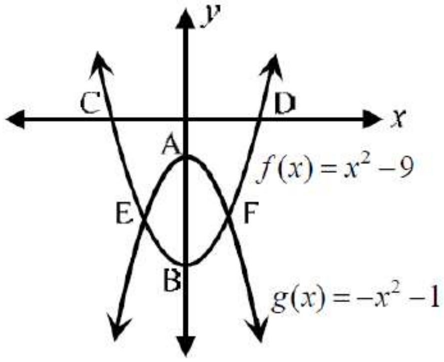


Determine:

(a) the coordinates of A, B, and C.
(b) the values of *x* for which *f* is increasing
(c) the values of *x* for which *g* is decreasing
(d) the maximum value of *f* and *g*
(e) the turning points of *f* and *g*
(f) the equation of the axis of symmetry for *f*
(g) the domain and range of *f* and *g*

(h) the equation of h if $h(x) = -g(x) + 12$. Describe the transformations.
(i) the equation of p and the value of its gradient.
(j) the domain and range of p

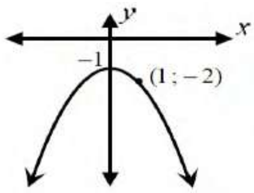
2. Given: $f(x) = x^2 - 9$ and $g(x) = -x^2 - 1$



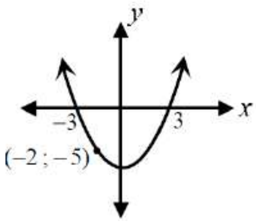
(a) Determine the coordinates of A, B, C and D
(b) Determine the coordinates of E and F.

3. Determine the equation of the following graphs in the form $f(x) = ax^2 + q$.

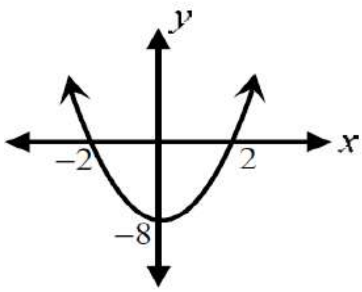
(a)



(b)



(c)

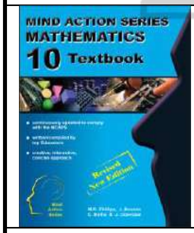


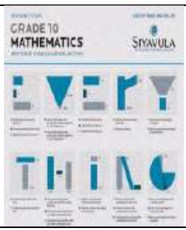
TOPIC: FUNCTIONS AND GRAPHS (Lesson 9)	Weighting	30 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**The hyperbolic function: $y = \frac{a}{x} + q$.** **Finding the equation of a hyperbola**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Equations of asymptotes, axes of symmetry			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Shape of a parabola, effects of *a* and *q*, intercepts with the axes, decreasing and increasing functions

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

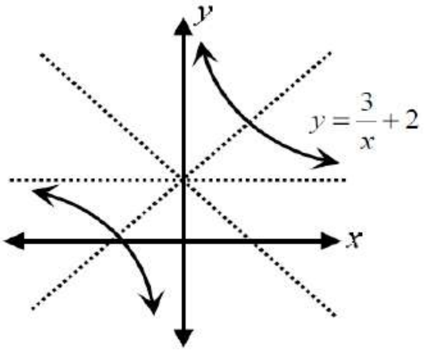
Writing down equations of the asymptotes, point to substitute when determining the line of symmetry.

METHODOLOGY

The hyperbola has **two asymptotes**, vertical asymptote and horizontal asymptote.
The **horizontal asymptote** is the line ***y* = *q*** and the **vertical asymptote** is always the ***y*-axis**, the line $x = 0$.
The **hyperbola** has two axes of symmetry, one has a gradient of -1 and the other has a gradient of 1. These two lines about which a hyperbola is symmetrical are: ***y* = *x* + *q*** and ***y* = -*x* + *q***.

Example:

1. Given: $f(x) = \frac{3}{x} + 2$



Determine:

(a) the equations of the asymptotes
(b) the equations of the axes of symmetry
(c) the values of *x* for which *g* is decreasing
(d) the domain and range of *g*

Solution:

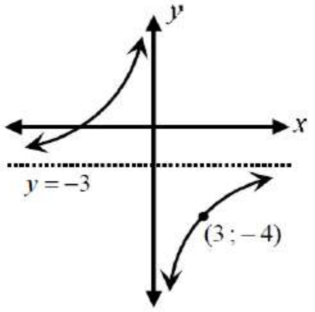
(a) Horizontal asymptote is $y = 2$ Vertical asymptote is $x = 0$

(b) $y = -x + 2$ and $y = x + 2$

(c) The graph of *g* is decreasing on the interval $x \in (-\infty; 0)$ as well as the interval $x \in (0; \infty)$

(d) Domain of *g*: $x \in R \quad x \neq 0$ and Range of *g*: $y \in R \quad y \neq 2$

2. Determine the equation of the below graph in the form $f(x) = \frac{a}{x} + q$.



Solution:

The horizontal asymptote is $y = -3$. Therefore, $q = -3$ and $f(x) = \frac{a}{x} - 3$

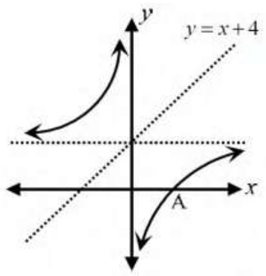
Substitute (3; -4): $-4 = \frac{a}{3} - 3$

$-1 = \frac{a}{3}$

$a = -3 \quad \therefore f(x) = \frac{-3}{x} - 3$

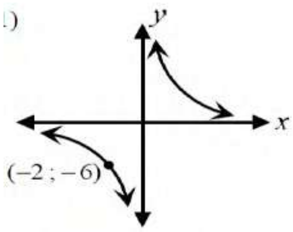
ACTIVITIES/ ASSESSMENT

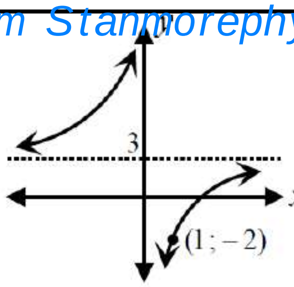
1. The line $y = x + 4$ is an axis of symmetry of the graph of $f(x) = \frac{-2}{x} + q$. The graph of f cuts the x-axis at A.

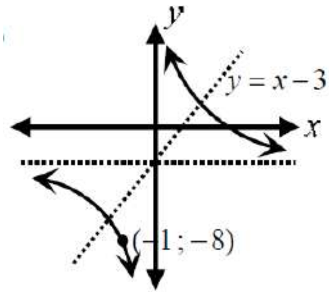


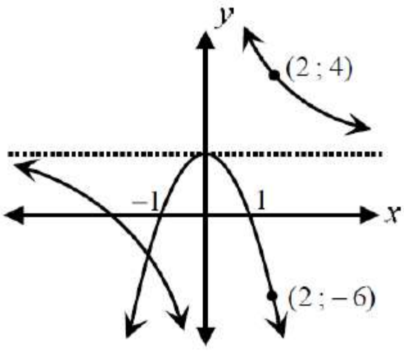
(a) Write down the value of q
(b) Write down the equation of f
(c) State the domain and range of f
(d) For which values of x is f increasing?
(e) Write down the equations of the asymptotes
(f) Write down the equation of the other axis of symmetry.

2. Determine the equation of the following graphs in the form $f(x) = \frac{a}{x} + q$.



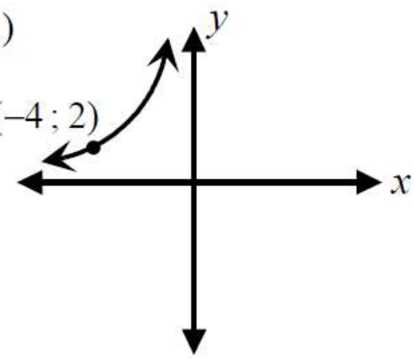




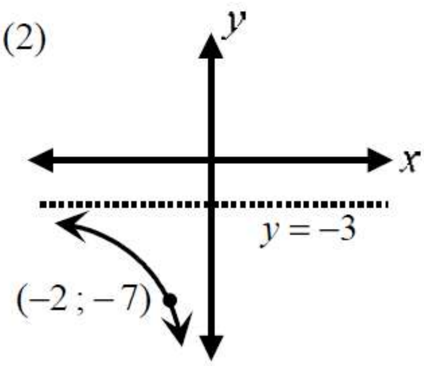


3. For each function below, state the domain and range and determine the equation:

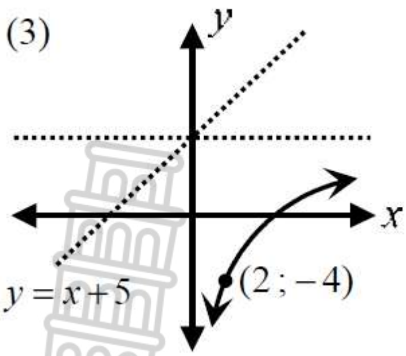
(1)



(2)



(3)

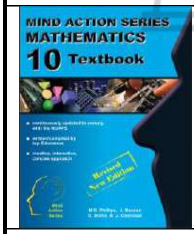


TOPIC: FUNCTIONS AND GRAPHS (Lesson 16)		Weighting	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Exponential function: $y = ab^2 = q$** **Finding the equation of an exponential function**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Horizontal asymptote, equation of the asymptote				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Substitution,

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

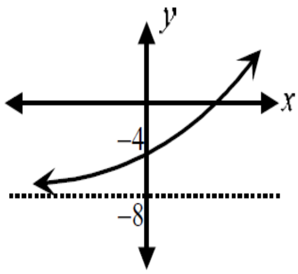
Writing the equation of the asymptote, recognizing the asymptote

METHODOLOGY

Exponential functions have one asymptote, horizontal asymptote the line *y* = *q*.

Examples:

1. Determine the equation of the given graph in the form $f(x) = a.2^x + q$



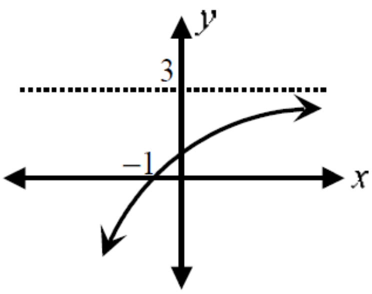
The horizontal asymptote is *y* = –8. Therefore, $q = -8$

Then $f(x) = a.2^x - 8$

Substitute (0; -4): $-4 = a.2^0 - 8$

$4 = a \quad \therefore f(x) = 4.2^x - 8$

2. Determine the value of *b* and *q* if the equation of the given graph is $g(x) = -b^x + q$.



Solution:

Horizontal asymptote is $y = 3$ *making* $q = 3$. $\therefore g(x) = -b^x + 3$

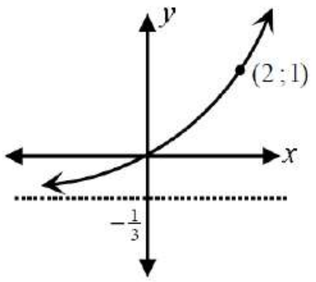
Substitute (-1;0): $0 = -b^{-1} + 3$

$-3 = -\frac{1}{b}$

$-3b = -1$

$b = \frac{1}{3}$ $\therefore g(x) = -\left(\frac{1}{3}\right)^x + 3$

3. Determine the equation of the given graph in the form $y = ab^x + q$



Solution:

$y = ab^x - \frac{1}{3}$

Substitute (2; 1): $1 = \frac{1}{3}b^2 - \frac{1}{3}$ and Substitute (0; 0): $0 = ab^0 - \frac{1}{3}$

$3 = b^2 - 1$ $a = \frac{1}{3}$

$0 = b^2 - 4$

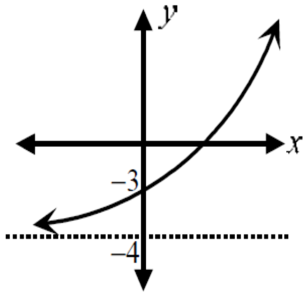
$0 = (b-2)(b+2)$ difference of two squares

$b = 2$ *or* $b = -2$

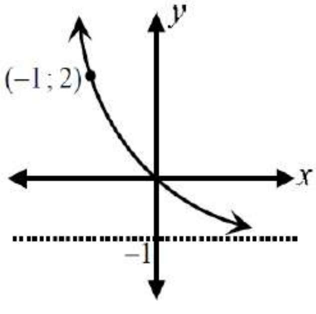
$b = 2$ N/A Therefore, $y = \frac{1}{3}.2^x - \frac{1}{3}$

ACTIVITIES/ ASSESSMENT

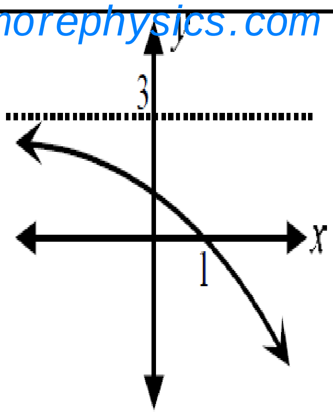
1. Determine the equation of the given graph in the form $f(x) = a.2^x + q$



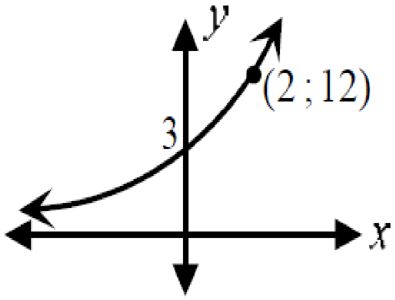
2. Determine the equation of the given graph in the form $f(x) = b^x + q$



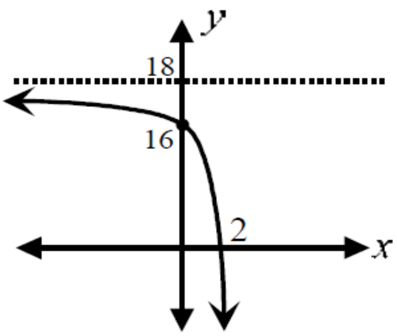
3. Determine the equation of the given graph in the form $f(x) = -b^x + q$



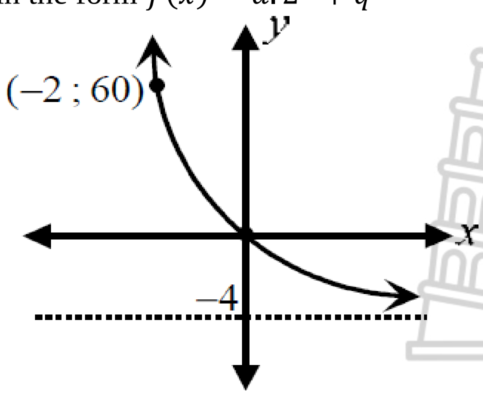
4. Determine the equation of the given graph in the form $f(x) = a.2^x + q$



5. Determine the equation of the given graph in the form $f(x) = a.2^x + q$



6. Determine the equation of the given graph in the form $f(x) = a.2^x + q$

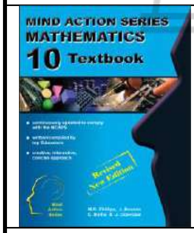


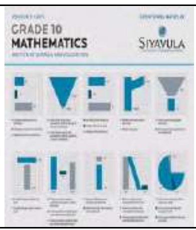
TOPIC: FUNCTIONS AND GRAPHS (Lesson 11)	Weighting	30 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Graph interpretation: Length of line segment**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Point of intersection, vertical length between TWO graphs and horizontal length between TWO graphs.			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Point of intersection, vertical line, horizontal line

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Giving the length of a line as a negative number,

METHODOLOGY

To interpret a graph:

- Study the graph to understand what it shows
- Distinguish between the different types of graphs
- Types of graphs studied in this grade are linear function (straight line), quadratic function (parabola), hyperbolic function and exponential function.

This topic involves determining the **lengths of line segments** using the different functions you have studied thus far and also discusses the **graphical interpretation of inequalities.**

<u>**Lengths of line segment**</u>

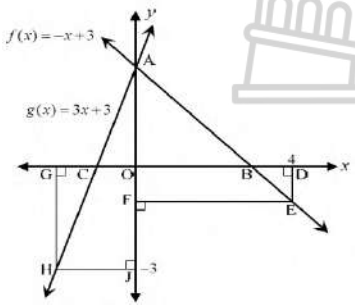
- The **length** of a line segment is always **positive.**
- To determine a length **along the *y*-axis,** let $x = 0$ and to determine a length **along the *x*-axis**, let $y = 0$
- The **point of intersection** between two graphs is obtained by **equating the equations of the functions** e.g. ($f(x) = g(x)$) and solving for x and hence for y. The value of x will give the horizontal length and the value of y will give the vertical length.
- A **vertical length between two graphs** can be calculated using the formula: ***y* top graph - *y* bottom** graph (substitute the x-value into this formula to get the required length).
- A **horizontal length between two graphs** can be calculated using the formula: x right end point - x left end point.

Examples:

1. Two lines cut the y-axis at A.

Determine:

(a) the length of OA, OB, OC and BC
(b) the length of OD, FE, OF and DE
(c) the length of OJ, GH, OG and JH



Solution:

(a) A (0; 3)

∴ OA = 3 units

For x_B

$0 = -x + 3$

$x = 3$ 0B = 3units

For x_C

$0 = 3x + 3$

$x = -1$ OC = 1 unit

$BC = y_B - y_C$

BC = 3-(-1)

BC = 4 units

(b) OD = 4 units

4FE = 4 units (opp sides rectangle)

Substitute $x = 4$ into $y = -x + 3$

$y = -4 + 3 = -1$

$y_E = y_F = -1$

OF =1 unit and DE =1 unit

(c) OJ = 3 units

GH = 3units (opp sides rectangle)

Substitute $y =$ 3 into $y = 3x + 3$

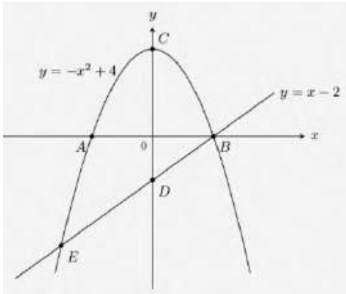
$-3 = 3x + 3$

$-3x = 6$

$x = -2$

OG = 2 units and JH = 2 units

2. The graph of $y = -x^2 + 4$ and $y = x - 2$



Calculate:

(a) the coordinates of A, B, C and D
(b) the coordinates of E
(c) distance CD

Solution:

(a) A and B are the x-intercepts of the parabola.

To calculate x-intercepts, y = 0

$0 = -x^2 + 4$

$x^2 - 4 = 0$

$(x - 2)(x + 2) = 0$... difference of two squares

$x = 2 \text{ or } x = -2$

A (-2; 0) and B (2; 0)

C is the y- intercept

C (0; 4)

D is the y-intercept of line

D (0; -2)

(b) E is the point of intersection of the two graphs.

At the point of intersection graphs are equal

$x - 2 = -x^2 + 4$

$x^2 + x - 6 = 0$

$(x - 2)(x + 3) = 0$

$x = 2 \text{ or } x = -3$

At E, $x = -3$, the substitute it for a corresponding y-value. $y = -3 - 2 = -5$. ∴ $E(-3; -5)$

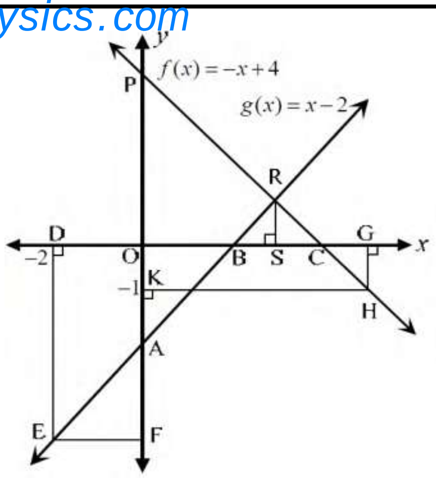
(c) CD = CO + OD

CD = 4 – (-2) = 6 units

ACTIVITIES/ ASSESSMENT

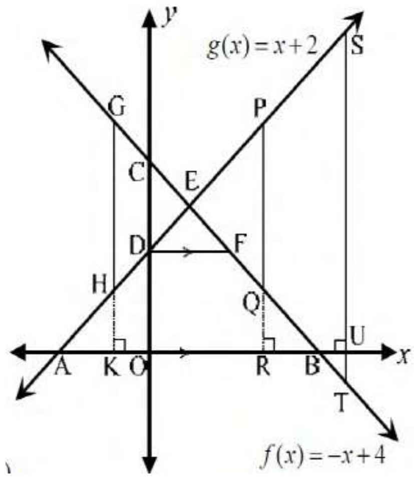
1. Two lines $f(x) = -x + 4$ and $g(x) = x - 2$ intersect at R.

By using the information on the diagram, determine:
(a) the length of OA, OP, OB and OC
(b) the length of AP and BC
(c) the length of OD, EF, OF and DE
(d) the length of OK, GH, OG and KH
(e) the length of RS and OS



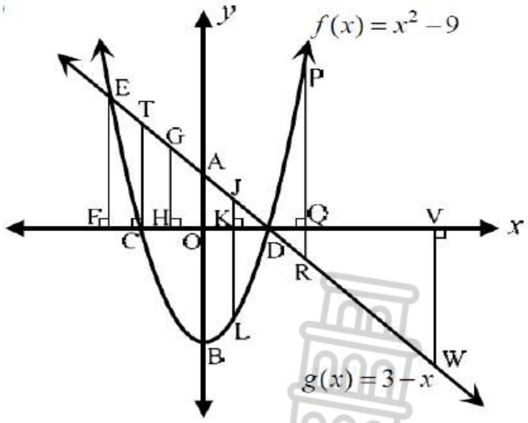
2. Two lines $f(x) = -x + 4$ and $g(x) = x + 2$ intersect at E.

Determine:
(a) the length of AB, CD and DF
(b) the length of PQ if OR = 3 units
(c) the length of OU if ST = 8 units
(d) the length of GH if HK $=1\frac{1}{2}$



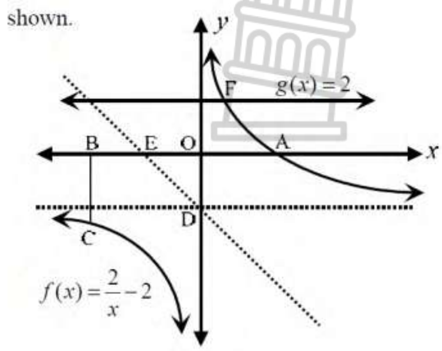
3. The diagram shows the graphs of $f(x) = x^2 - 9$ and $g(x) = 3 - x$ intersecting at E and D.
The graph of f cuts the axes at B, C and D.

Determine:
(a) the length of AB, CT and CD
(b) the length of OF and EF
(c) the length of GH if OH =1 unit
(d) the length of OV if VW = 8 units
(e) the length of JL if OK =1 unit
(f) the length of OQ if PR = 8 units



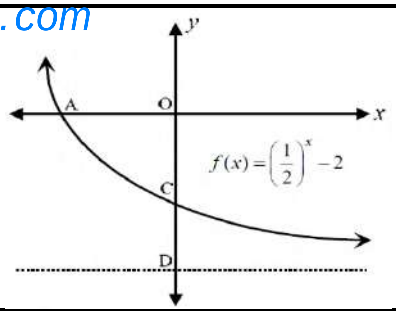
4. The graph of $f(x) = \frac{2}{x} - 2$ and $g(x) = 2$ is shown.

Determine:
(a) the length of OA.
(b) the length of BC if OB = 4 units
(c) the length of OD and OE
(d) the coordinates of F



5. The graph of $f(x) = \left(\frac{1}{2}\right)^x - 2$ is shown.

Determine the length of OA and CD.

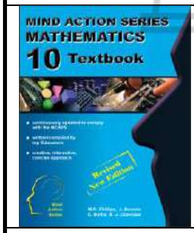


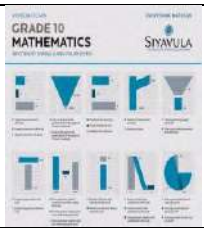
TOPIC: FUNCTIONS AND GRAPHS (Lesson 12)		**Weighting**	30 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Graph interpretation: Using graphs to solve inequalities**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Solving inequalities graphically				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Inequalities, interval notation, set-builder notation.

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Lack of understanding or the meaning of $f(x) \lesssim 0, f(x) \lesseqqgtr 0$

METHODOLOGY

We can **use the graphs** to **solve the inequalities graphically**:

If questions on graphs are as follows:

- For which values of x is $f(x) > 0$?
  We are required to determine the x-values for which the y-values of f are positive.
  Where the parabola lies above the x-axis will be where the y-values are positive.

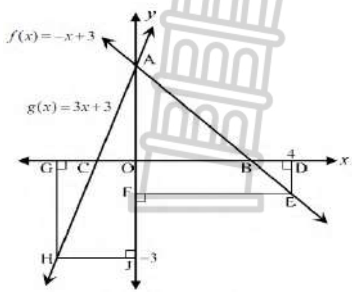
- For which values of x is $f(x) \leq 0$?
  We are required to determine the x-values for which the y-values of f are negative.
  Where the parabola lies below the x-axis will be where the y-values are negative.

- For which values of x is $f(x) \geq g(x)$?
  We are required to determine the values of x for which the y-values of f are greater than or equal to the y-values of g. This is where the graph of f is above the graph of g.

- For which values of x is $f(x) < g(x)$?
  We are required to determine the values of x for which the y-values of f are smaller than the y-values of g. This is where the graph of f is below the graph of g.

Examples:

Two lines cut the y-axis at A.



Determine:

(a) the values of x for which $f(x) \geq 0$
(b) the values of x for which $g(x) < 0$
(c) the values of x for which $f(x) \leq g(x)$

Solution:

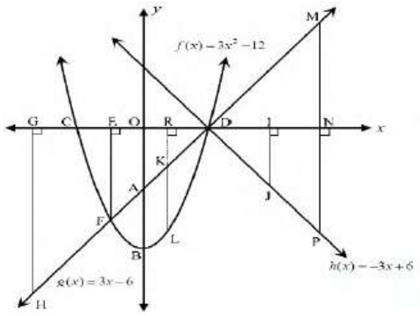
(a) $x \leq 3$ OR $x \in (-\infty; 3]$

(b) $x < -1)$OR $x \in -\infty; -1)$

(c) $x \geq 0$ OR $x \in [0; \infty)$

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

2. The diagram below shows the graphs of $f(x) = 3x^2 - 12$, $g(x) = 3x - 6$ and $h(x) = -3x + 6$.
The graphs of f, g and h share a common x-intercept (D). The graph of f and g intersect at D and F.
The diagram is not drawn to scale.



Determine:

(a) the values of x for which $f(x) \geq 0$
(b) the values of x for which $f(x) < g(x)$

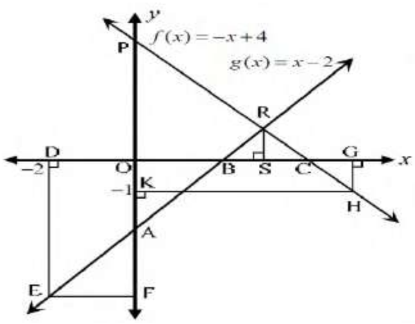
Solution:

(a) C (-2; 0) and D (2; 0)
Therefore, $x < -2$ or $x > 2$, also write as $x \in (-\infty; -2) \cup (2; \infty)$

(b) E (-1; 0) and D (2; 0)
Therefore, $-1 < x < 2$ OR $x \in (-1; 2)$

ACTIVITIES/ ASSESSMENT

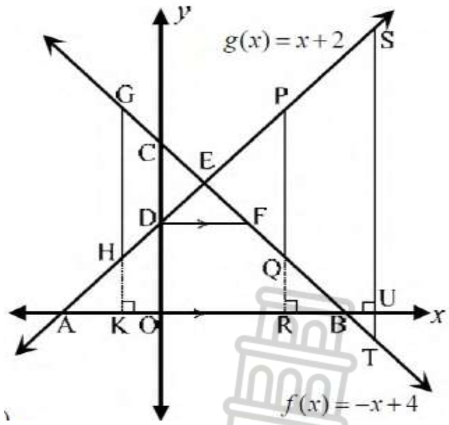
1. Two lines $f(x) = -x + 4$ and $g(x) = x - 2$ intersect at R.



Determine:

(a) the values of x for which $f(x)$ ε 0
(b) he values of x for which $g(x) < 0$
(c) the values of x for which $f(x) < g(x)$

2. Two lines $f(x) = -x + 4$ and $g(x) = x + 2$ intersect at E.

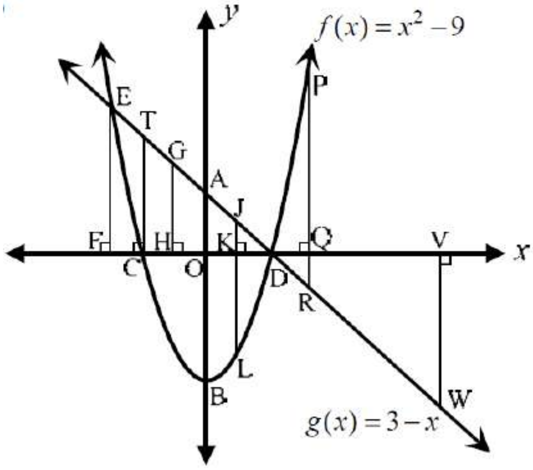


Determine:

(a) the values of x for which $f(x) > 0$

(b) the values of x for which $g(x)$ δ 0
(c) the values of x for which $g(x) < f(x)$

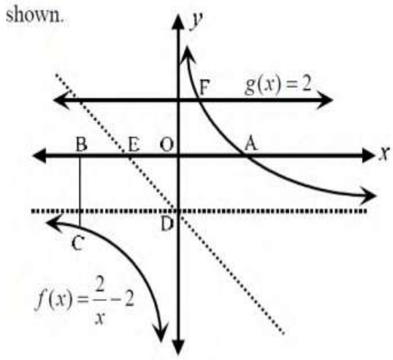
3. The diagram shows the graphs of $f(x) = x^2 - 9$ and $g(x) = 3 - x$ intersecting at E and D.
The graph of f cuts the axes at B, C and D.



Determine:

(a) the values of x for which $f(x) > 0$
(b) the values of x for which $f(x) \geq g(x)$

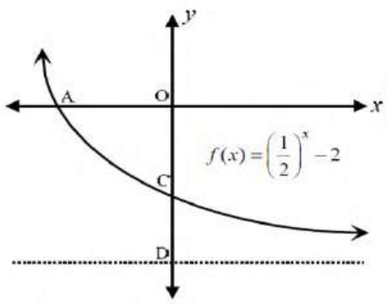
4. The graph of $f(x) = \frac{2}{x} - 2$ and $g(x) = 2$ is shown.



Determine;

(a) the values of x for which $f(x) \geq 0$
(b) the values of x for which $f(x) < 0$
(c) the values of x for which $f(x) > -2$
(d) the values of x for which $f(x) > g(x)$

5. The graph of $f(x) = \left(\frac{1}{2}\right)^x - 2$ is shown.



Determine:

(a) the values of x for which $f(x) > 0$
(b) the values of x for which $f(x) \leq 0$
(c) the values of x for which $f(x) > -2$
(d) the values of x for which $f(x) > -1$
(e) the values of x for which $f(x) \leq -1$

TEST 2: GRAPH INTERPRETATION

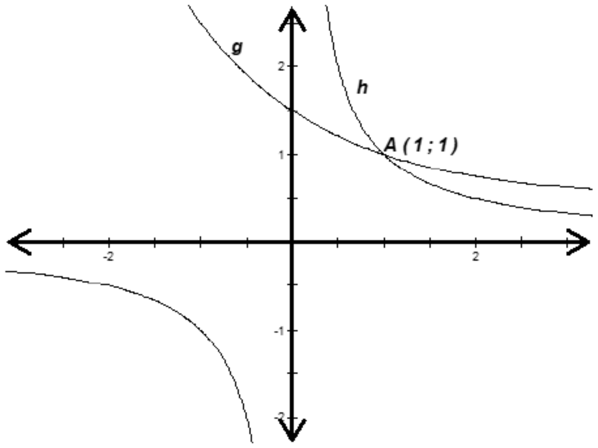
MARKS: 25 **Duration: 30 Min**

Instructions:

Answer ALL questions

QUESTION 1 [10 Marks]

Sketched below are the functions $g(x) = b^x + q$ and $h(x) = \frac{a}{x}$, A (1; 1) is the point of intersection of the two graphs.



1.1Determine the values of a, b and q (3)

1.2 Write down the equation of the asymptote of *g* and *h* (3)

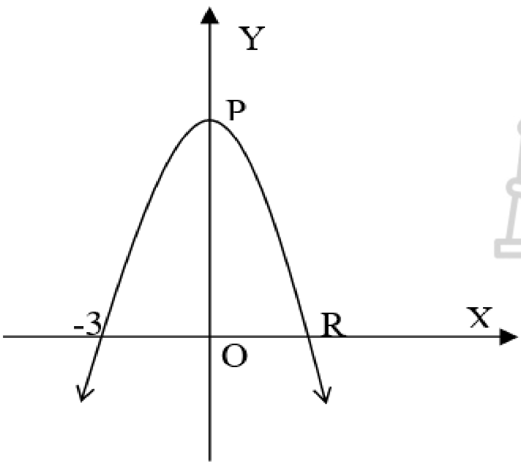
1.3 Determine the range of *g* and *h* (2)

1.2 *f*(*x*) is the reflection of *g*(*x*) in the y-axis. Write down the equation of *f*(*x*). (2)

QUESTION 2 [7 Marks]

2.1 Determine the equation of a linear function $f(x) = ax + q$ if $f(0) = -7$ and $f(2) = 0$ (3)

2.2 A sketch graph of $f(x) = -x^2 + 9$ is shown.

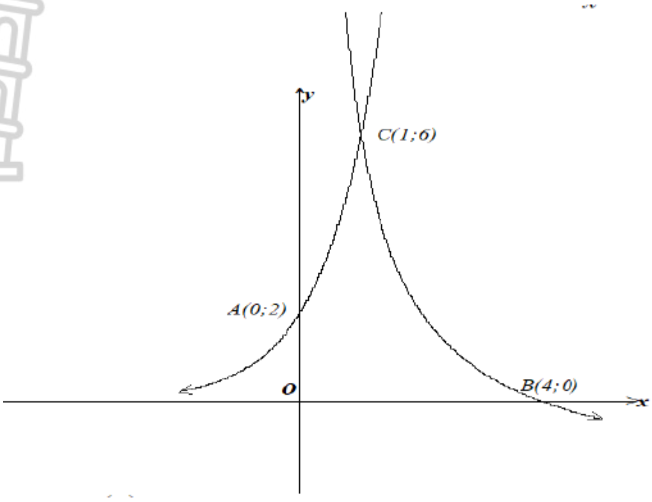


2.2.1For which values of *x* is $f(x) > 0$? (2)

2.2.2 Write down the equation of $y = 3.f(x)$ (2)

QUESTION 3 [8 Marks]

3.1 Sketched below are graphs of $f(x) = a.b^x$ and $g(x) = \frac{a}{x} + q$.



3.1.1 Given that $f(x)$ cuts the y-axis at A (0; 2) and that C (1; 6) lies on both graphs.

Calculate the values of a and b. (4)

3.1.2 It is further given that $g(x)$cuts the x-axis at the point B (4; 0).

Calculate the values of a and q. (3)

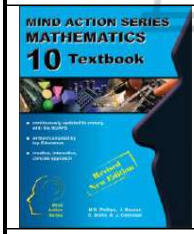
3.1.3 Write down the equation of the asymptote of $f(x)$. (1)

TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 1)		Weighting	40 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Point by point plotting of basic trigonometric graphs $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$, where $\theta \in [0°; 360°]$**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Amplitude, period, asymptote				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Minimum, Maximum, turning point, coordinates, domain, range, x-intercepts, y-intercept

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Determining the amplitude of the graph by inspection, understanding period in trig. graphs

METHODOLOGY

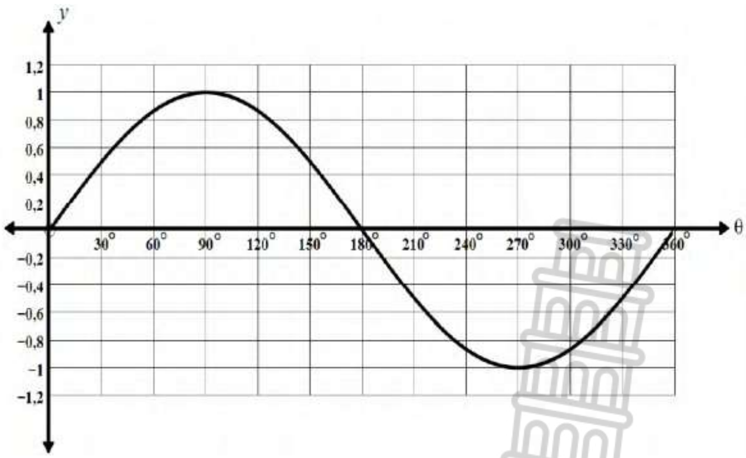
Basic trigonometric graphs:

1.Consider $y = \sin\theta$, where $\theta \in [0°; 360°]$

Use your calculator to complete the table below and round answers to 1 decimal place.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin\theta$	0	0,5	0,9	1	0,9	0,5	0	-0,5	-0,9	-1	-0,9	-0,5	0

We can represent the values of θ on the horizontal axis and the values of $\sin\theta$ on the vertical axis and then draw the graph of $y = \sin\theta$.



Notice the wave shape of the graph. Each complete wave takes 360° to complete. This is called the **period**.

The **maximum value** of $y = \sin\theta$ is 1 and the **minimum value** is -1.

The height of the wave above and below the x-axis is called the **amplitude** of the graph.

Amplitude is defined to be $\frac{1}{2}$ [distance between maximum and minimum value]

Amplitude of $y = \sin\theta$ is $\frac{1}{2}$ [1-(-1)] = 1

Domain: $\theta \in [0; 360°]$

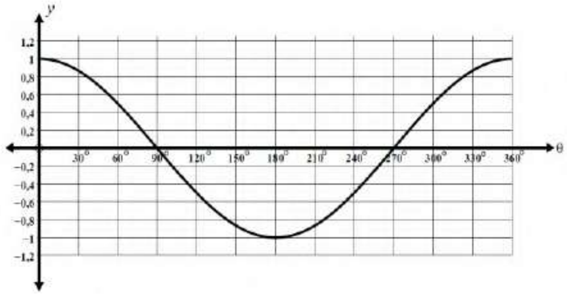
Range: $y \in [-1; 1]$

x- intercepts: (0°; 0), (180°; 0), 360°; 0)
y- intercept: (0°; 0)
Maximum turning point: (90°; 1)
Minimum turning point: (270°; −1)

2. Consider $y = \cos\theta$, where $y \in [0°; 360°]$
Use your calculator to complete the table below and round answers to 1 decimal place.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\cos\theta$	0	0,5	0,9	1	0,9	0,5	0	-0,5	-0,9	-1	-0,9	-0,5	0

We can represent the values of θ on the horizontal axis and the values of $\sin\theta$ on the vertical axis and then draw the graph of $y = \cos\theta$.

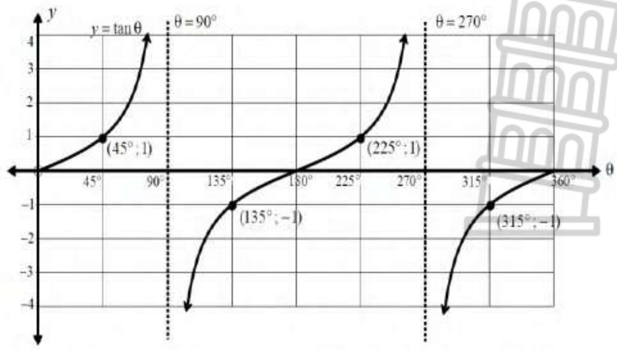


Notice the similar wave shape of the graph. The **period** is also 360° and the **amplitude** is 1. The **maximum value** of $y = \cos\theta$ is 1 and the **minimum value** is -1.
Domain: $\theta \in [0; 360°]$
Range: $y \in [-1; 1]$
x- intercepts: (90°; 0), (270°; 0)
y- intercept: (0°; 1)
Maximum turning point: (0°; 1); (360°; 1)
Minimum turning point: (180°; −1)

3. Consider $y = \tan\theta$, $0° \leq \theta \leq 360°$
Use your calculator to complete the table below and round answers to 1 decimal place.

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$\tan\theta$	0	0,6	1	1.7	error	-1,7	-1	-0,6	0	0,6	1	1.7	error	-1.7	-1	-0,6	0

We can represent the values of θ on the horizontal axis and the values of $\sin\theta$ on the vertical axis and then draw the graph of $y = \tan\theta$.



Consider our definitions of $\sin\theta$ and $\cos\theta$ for right-angled triangles:

$$\sin\theta = \frac{opposite}{hypotenuse} \text{ and } \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{opposite}{hypotenuse} \times \frac{hypotenuse}{adjaceni}$$

$$= \frac{opposite}{adjacent} = \tan\theta$$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$$

So, we know that for values of θ for which $\sin\theta = 0$, we must also have $\tan\theta = 0$. Also, if $\cos\theta = 0$ the value of $\tan\theta$ is undefined as we cannot divide by 0.

As the values of θ approach $90°$ from the left, the values of y tend towards $+\infty$. At $90°$, the y-value is **undefined**. This means that the curve moves upwards and never cuts or touches the line $\theta = 90°$. As the values of θ approach $90°$ from the right, the values of y tend towards $-\infty$. The curve moves downwards and never cuts or touches the line $\theta = 90°$.

The **dotted vertical lines** are at the values of θ where $\tan\theta$ is not defined and are called the **asymptotes**.

An **asymptote** is a vertical line that a graph approaches but never touches. Therefore, the line $\theta = 90°$ is an asymptote of the graph of $y = \tan\theta$. All of this applies to $270°$. The line $\theta = 270°$ is therefore also an asymptote.

The points (45° ;1), (135° ; -1), (225° ;1) and (315° ; -1) may be referred to as the **critical points** on the basic graph $y = \tan\theta$.The points are useful when sketching tan graphs involving **vertical stretches**, **reflections** in the *x*-axis and vertical shifts.

Tan graph has **no maximum** and **no minimum** values, hence, **no amplitude.**

The period of the graph of $y = \tan\theta$ is $180°$

Domain: $\theta \in [0; 360°]$, $\theta \neq 90°; 270°$

Range: $y \in R$

x- intercepts: $(0°; 0), (180°; 0), 360°; 0)$

y- intercept: $(0°; 0)$

ACTIVITIES/ ASSESSMENT

1. Sketch the following graphs where $0° \leq x \leq 360°$

$y = tanx\ and\ f(x) = \sin x$ on the same set of axes.

2. With the aid of a graph, write down the following about $f(x) = \cos x$, where $x \in [0°; 360°]$:

a) Maximum value

b) Minimum value

c) Amplitude

d) Period

e) Domain

e) Range

f) x-intercepts

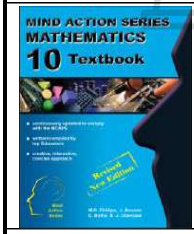
g) y-intercepts

TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 2)		Weighting	40 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**The effect of "a" on the basic trigonometric graphs: $y = a\sin\theta$, $y = a\cos\theta$ and $y = a\tan\theta$, where $\theta \in [0°; 360°]$**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Reflection about the x-axis, vertical stretch				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Amplitude, period, asymptote

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Determining the amplitude of the graph by inspection.

METHODOLOGY

The value of a affects the amplitude of the graph. Note that **amplitude** is **always positive.**

For $a > 0$, there is a **vertical stretch** and the **amplitude increases**.
For $0 < a < 1$, the **amplitude decreases**.

For $a < 0$, there is a **reflection about the *x*-axis**.
For $-1 < a < 0$, there is a **reflection about the *x*-axis** and the **amplitude decreases**.
For $a < -1$, there is a **reflection about the *x*-axis** and the **amplitude increases**.

Examples:

1. Sketch the graphs of:

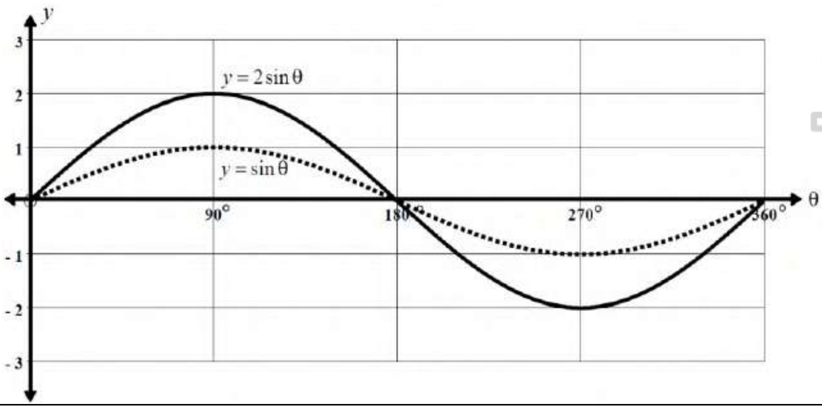
(a) $y = 2\sin\theta$
(b) $y = -3\sin\theta$

2. Sketch the graph of $y = 4\cos\theta$

3. Sketch the graph of $y = -2\tan\theta$ for the interval $\theta \in [0°\ ; 270°)$.

Solution:

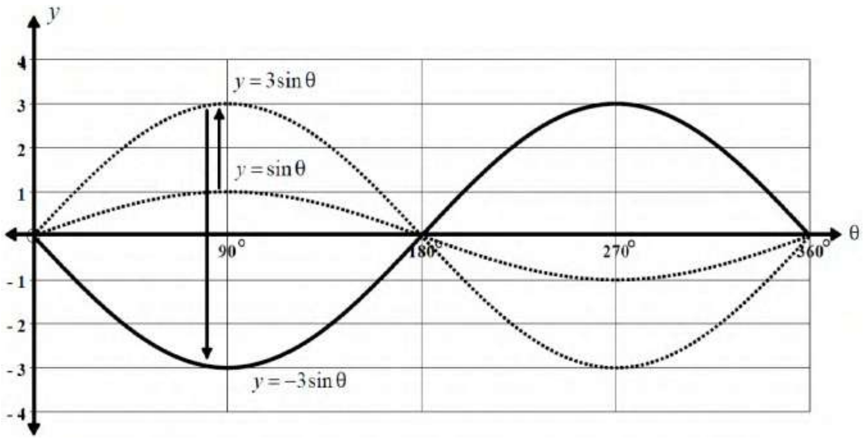
(a)



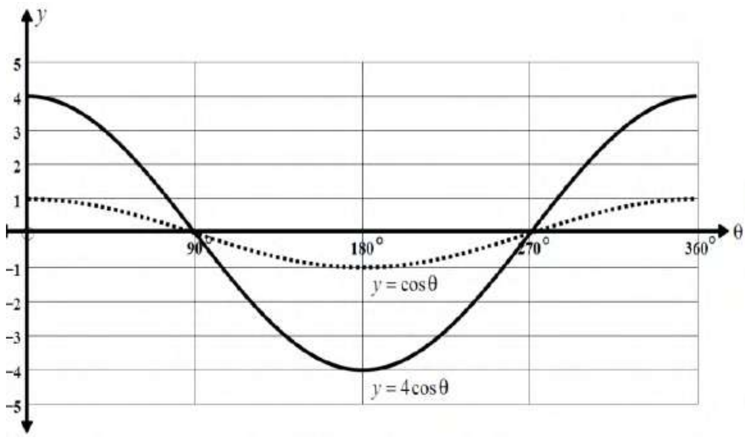
COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

Notice that the graph of $y = 2\sin\theta$ is a vertical stretch of the basic graph $y = \sin\theta$ by a factor of 2.
The **maximum value** is 2 and the **minimum value** is -2
Range is $y \in [-2; 2]$.
The **amplitude** of the graph of $y = 2\sin\theta$ is $\frac{2-(-2)}{2} = 2$
The **period** is 360° .
This **vertical stretch** of the graph of $y = \sin\theta$ is called an **amplitude shift**.
The number **2** in the equation $y = 2\sin\theta$ tells us what the amplitude of the graph is.

(b) The negative sign indicates a **reflection in the horizontal axis**.
All you need to do is first draw the basic graph of $y = \sin\theta$, stretch this graph vertically by a factor of 3 and then reflect this graph in the horizontal axis to obtain the graph of $y = -3\sin\theta$.



The amplitude of the graph of $y = -3\sin\theta$ is: $\frac{3-(-3)}{2} = 3$
Therefore, in the equation $y = -3\sin\theta$, the number 3 tells us that the amplitude is 3 and the negative sign indicates a **reflection** in the horizontal axis.
2.



Notice that the graph of $y = 4\cos$ (is a **vertical stretch** of the basic graph $y = \cos\theta$ by a factor of 4.
The **maximum value** is 4 and the **minimum value** is -4.
Range is $y \in [-4; 4]$.
The amplitude of the graph of $y = 4\cos\theta$ is $\frac{4-(-4)}{2} = 4$.
The **period** is 360°.
This **vertical stretch** of the graph of $y = \cos\theta$ is called an **amplitude shift**.
The number 4 in the equation $y = 4\cos\theta$ tells us what the amplitude of the graph is.

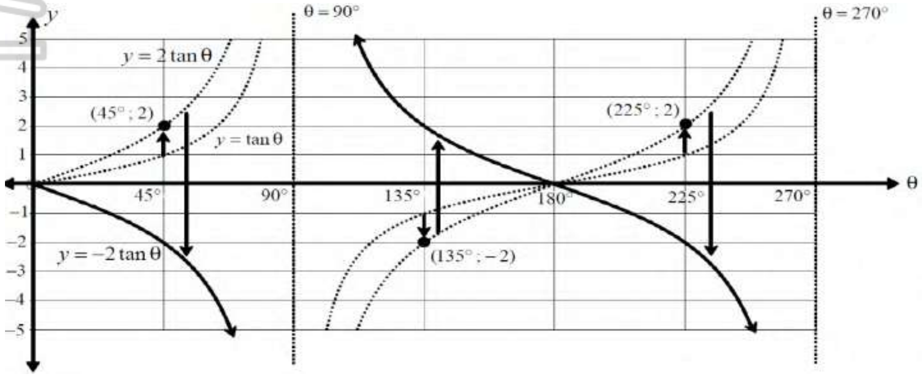
3. First draw the graph of $y = 2\tan\theta$ by vertically stretching the graph of $y = \tan\theta$ by a factor of 2. The y-values of the critical points of $y = \tan\ \theta$ are multiplied by 2 to form the following new points on the graph of $y = 2\tan\ \theta$: (45° ; 2), (135° ; - 2), (225° ; 2)

The graph of $y = -2\tan\ \theta$ is then formed by **reflecting** $y = 2\tan\ \theta$ in the x-axis.

The signs of the y-values of the critical points of $y = 2\tan\ \theta$ now become: (45° ; - 2), (135° ; 2), (225° ; - 2)

The **asymptotes** of this graph remain the same as well as the x-intercepts.

Now plot the **critical points**, ***x*-intercepts and asymptotes** for $y = -2\tan\theta$ and restrict the graph in the interval $\theta \in [0° ; 270°)$. The graph is shown below.



The **period** of $y = -2\tan\theta$ is 180° and the **asymptotes** are $\theta = 90°$ and $\theta = 270°$.

ACTIVITIES/ ASSESSMENT

1. Given: $y = 3\sin\theta$ and $y = -2\sin\theta$
   (a) Sketch the graphs on the same set of axes for $\theta \in [0° ; 360°]$.
   (b) Write down the maximum and minimum values for each graph.
   (c) Write down the range, amplitude and period for each graph.

2. Given: $y = 2\cos\theta$ and $y = -3\cos\theta$
   (a) Sketch the graphs on the same set of axes for $\theta \in [0° ; 360°]$.
   (b) Write down the maximum and minimum values for each graph.
   (c) Write down the range, amplitude and period for each graph.

3. Given: $y = -\tan\theta$ and $y = \frac{1}{2}\tan\theta$
   (a) Sketch the graph of $y = -\tan\theta$ and $y = \frac{1}{2}\tan\theta$ for the interval $\theta \in [0° ; 270°]$
   (b) Write down the range and the period of each graph.

4. Given: $y = \frac{1}{2}\cos\theta$ and $y = -\sin\theta$
   (a) Sketch the graphs on the same set of axes for $\theta \in [0° ; 360°]$.
   (b) Write down the maximum and minimum values for each graph.
   (c) Write down the range, amplitude and period for each graph.

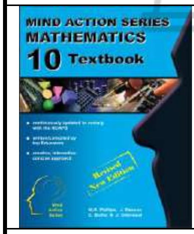
5. Given: $y = 2\tan\ \theta$ and $y = -3\sin\ \theta$
   (a) Sketch the graphs on the same set of axes for $\theta \in [0° ; 270°]$.
   (b) Write down the period for each graph.

TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 3)		Weighting	40 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**The effect of "q" on the basic trigonometric graphs; $y = a\sin\theta + q$, $y = a\cos\theta + q$ and $y = a\tan\theta + q$, where $0° \leq \theta \leq 360°$**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Vertical shift, translation, move graph, reflection about the axis				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Domain, range, amplitude, period, maximum and minimum value

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Negative amplitude, if the domain is not up to 360° for sin and cos, learners write wrong period of the graph

METHODOLOGY

The effect of q is called a **vertical shift** because the whole graph shifts up or down by q units.

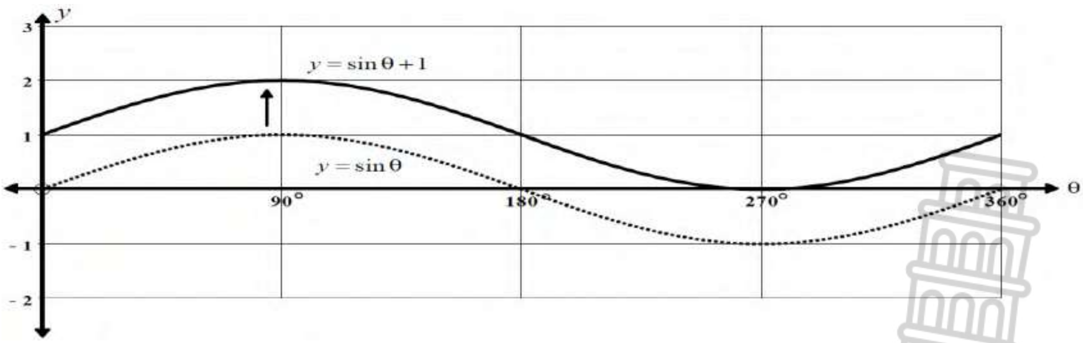
For $q > 0$, the graph is shifted vertically upwards by q units.
For $q < 0$, the graph is shifted vertically downwards by q units.

Examples:

1. Sketch the graph of $y = \sin\theta + 1$ and $y = -\cos\theta - 1$ for $\theta \in [0°\ 360°]$.

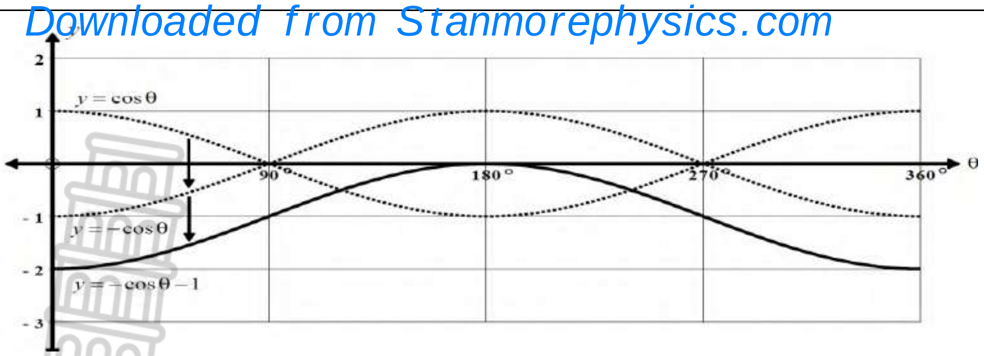
Solution:

The graph of $y = \sin\theta + 1$ is the graph of $y = \sin\theta$ shifted 1 unit up.



The **maximum** value is 2 and the **minimum** value is 0.
The **range** is $y \in [0; 2]$.
The **amplitude** is $\frac{2-0}{2} = 1$
The **period** is 360°.

The graph of $y = -\cos\theta - 1$ is the graph of $y = \cos\theta$ **reflected in the x-axis** and then shifted 1 unit down.

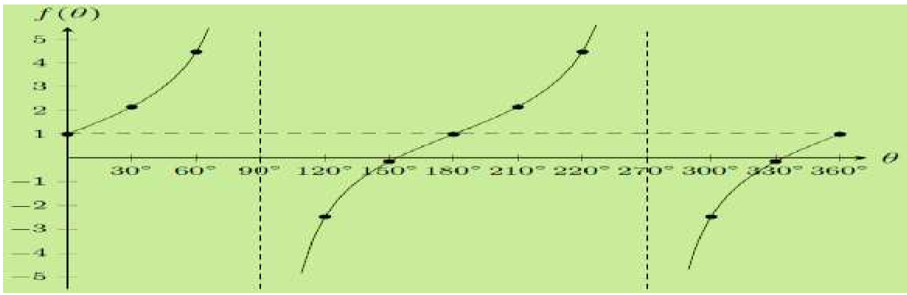


The **maximum** value is 0 and the **minimum** value is -2.
The **range** is y θ [-2; 0].
The **amplitude** is $\frac{0-(-2)}{2} = 1$
The **period** is 360°.

2. Sketch the graph of $y = 2\tan\theta + 1$ for $\theta \in [0°; 360°]$.

Solution:

We see that $a > 1$ so the branches of the curve will be steeper. We also see that $q > 0$ so the graph is shifted vertically upwards by 1 unit.



Domain: $0° \leq \theta \leq 360°$ but $\theta \neq 90°, 270°$.
Range: $y = f(\theta) \in R$

ACTIVITIES/ ASSESSMENT

1. Given: $y = \sin x + 2$ and $y = \cos x - 1$
(a) Sketch the graphs on the same set of axes for $x \in [0° ; 360°]$.
(b) Write down the maximum and minimum values for each graph.
(c) Write down the range, amplitude and period for each graph.

2. Given: $y = -\cos x + 3$ and $y = -\sin x - 2$
(a) Sketch the graphs on the same set of axes for $x \in [0° ; 360°]$.
(b) Write down the maximum and minimum values for each graph.
(c) Write down the range, amplitude and period for each graph.

3. Given: $y = 2\sin +x\ 4$ and $y = -3\cos -x1$
(a) Sketch the graphs on the same set of axes for $x \in [0° ; 360°]$.
(b) Write down the maximum and minimum values for each graph.
(c) Write down the range, amplitude and period for each graph.

4. Given: $f(x) = \tan x - 2$

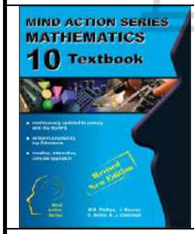
(a) Sketch the graph f(x) for $x \in [0° ; 360°]$.
(b) Write down the domain, range and period for the graph.

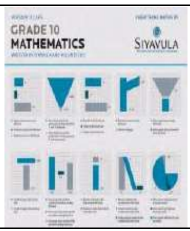
TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 4)		**Weighting**	40 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Sketching graphs of $y = a\sin\theta = q, y = a\cos\theta + q$ *and* $y = a\tan\theta + q$**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Intersection points, intercepts, interval notation, inequality signs				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Range, Period, asymptote

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Taking note of the value of a and the value of q and use these values correctly.

METHODOLOGY

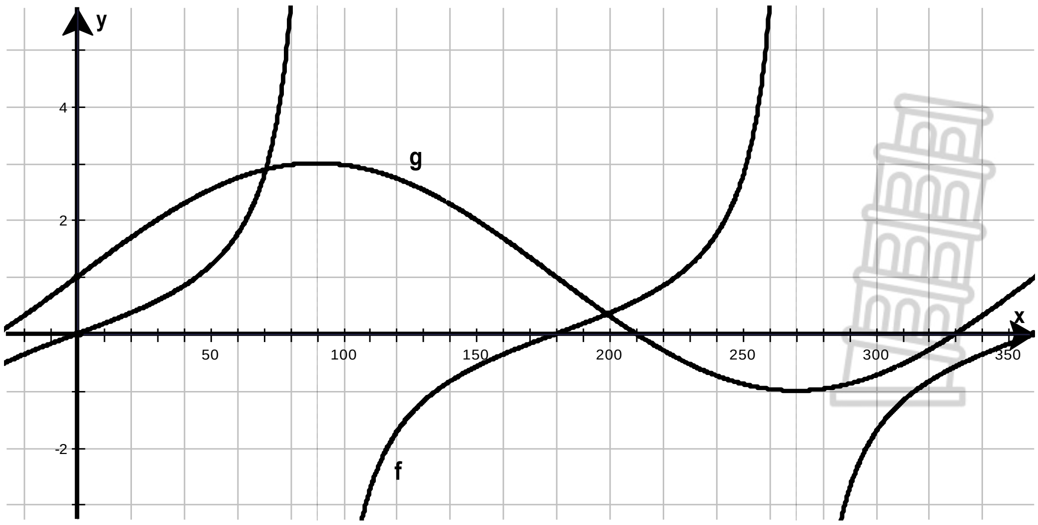
Example:

Given: $f(x) = \tan x$ and $g(x) = 2\sin x + 1$, where $-30° \leq x \leq 360°$

(a) Sketch graphs on the same set of axes.
(b) Write down the range of g
(c) Write down the period of f
(d) Write down the equations of the asymptotes of f
(e) Write down the values of x for which $g(x) - f(x) = 0$
(f) For which Values of x is $f(x) = 0$?
(g) Write down the values of x for which $f(x) \geq g(x)$

Solution:

(a)



(b) Range of g: $y \in [-1; 3]$

(c) Period of f is 180°

(d) $x = 90°, x = 270°$

(e) $x = 70°, x = 200°$ where the graphs intersect

(f) $x = 0°, x = 180°$ where the graph cut the x-axis (x-intercepts)

(g) $x \in [70°; 90°) and\ x \in [200°; 270°)$ where f(x) is above g(x)

ACTIVITIES/ ASSESSMENT

1. Given: $f(x) = 2 \tan x$ and $g(x) = -3\sin x - 3$

(a) Sketch the graphs on the same set of axes for $x \in [0° ; 270°]$.
(b) Write down the period for each graph.
(c) Write down the range of g
(d) For which values of x is $f(x > g(x)$
(e) Write down the equations of the asymptotes od f.

2. 2. Sketch the graphs of $y = f(x) = \sin x$ and $y = g(x) = \cos x + 1$ for $x \in [0°; 360°]$ on the same set of axes and then answer the questions that follow.

(a) For which values of x is $f(x) = g(x)$?
(b) For which values of x is $g(x) \geq f(x)$?
(c) For which values of x is $f(x) \leq 0$?
(d) For which values of x is $g(x) > 0$?
(e) For which values of x is $g(x) - f(x) = 2$?
(f) For which values of x is $f(x).\ g(x) \geq 0$?

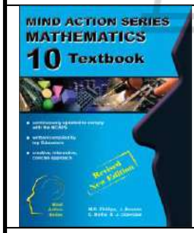
TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 3)	Weighting	40 ± 3	Grade	10

Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Finding the Equation of given Trigonometric graphs**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Amplitude, Minimum, Maximum, Range, Turning Point		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Substitution, simultaneous equations

RESOURCES



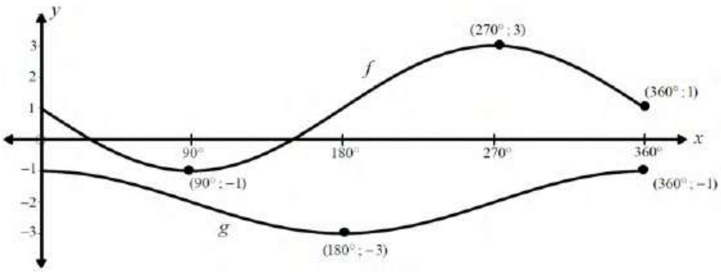
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Taking amplitude as the value of ‘a’ whether negative or positive.

METHODOLOGY

Examples:

In the diagram below, the graphs of $y = f(x) = a \sin x + q$ and $y = g(x) = m \cos x + n$ are shown for the domain $x \in [0°\ ;\ 360°]$.



Note: The variable x in the equations may also be used to represent the angles.

(a) Write down the amplitude and range of f.
(b) Write down the amplitude and range of g.
(c) Determine the values of a and q.
(d) Determine the values of m and n.

Solutions:

(a) For f: Amplitude: 2 Range: $y \in [-1; 3]$

(b) For g: Amplitude: 1 Range: $y \in [-3; -1]$

(c) Substitute two points on the graph into the equation $y = a \sin x + q$ and solve simultaneously.

$(90°; -1)$: $-1 = a \sin 90° + q$
$-1 = a(1) + q$
$-1 = a + q$..........(1)

$(270°; 3)$: $3 = a \sin 270° + q$
$3 = a(-1) + q$
$3 = -a + q$...........(2)

(1) + (2): $2 = 2q$
$q = 1$

$3 = -a + 1$
$a = -2$

(d) Substitute two points on the graph into the equation $y = m \cos x + n$ and solve simultaneously:

$(180°; -3)$: $-3 = m\cos 180° + n$
$-3 = m(-1) + n$
$-3 = -m + n$..........(1)

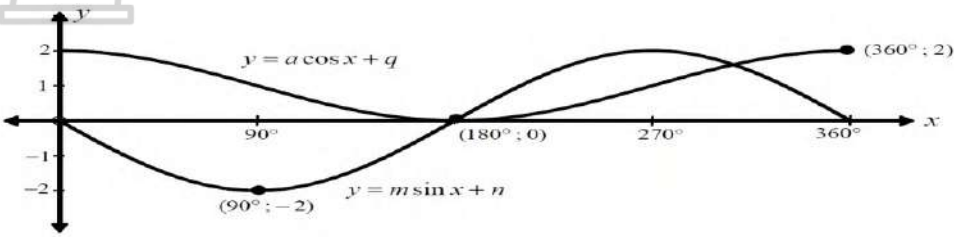
$(360°; -1)$: $-1 = m\cos 360° + n$
$-1 = m(1) + n$
$-1 = m + n$.............(2)

(1) + (2): $-4 = 2n$
$n = -2$

$-1 = m - 2$
$m = 1$

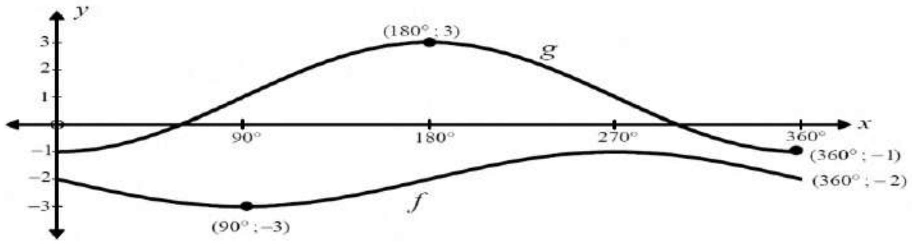
ACTIVITIES/ ASSESSMENT

1. In the diagram below, the graphs of $y = f(x) = a\cos x + q$ and $y = g(x) = m\sin x + n$ are shown for the domain $x \in [0° ; 360°]$.



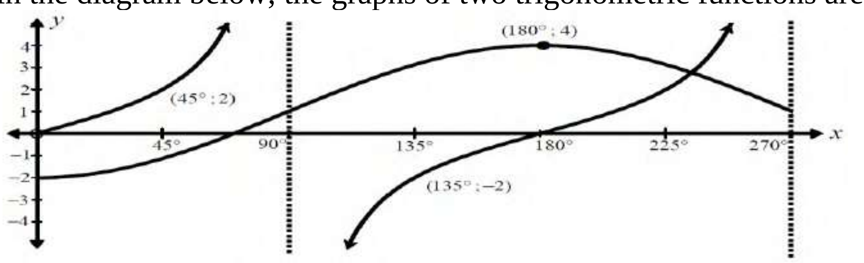
(a) Write down the amplitude and range of *f*.
(b) Write down the amplitude and range of *g*.
(c) Determine the values of *a* and *q*.
(d) Determine the values of *m* and *n*.

2. In the diagram below, the graphs of $y = f(x) = a\sin x + q$ and $y = g(x) = m\cos x + n$ are shown for the domain $x \in [0° ; 360°]$.



(a) Write down the range, amplitude and period of *f*.
(b) Write down the range, amplitude and period of *g*.
(c) Determine the values of *a* and *q*
(d) Determine the values of *m* and *n*.

3. In the diagram below, the graphs of two trigonometric functions are shown.



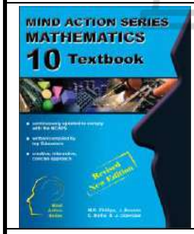
(a) Determine the equations of the two graphs.
(b) Write down the range, amplitude and period of each graph, where possible.

TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 6)		Weighting	40 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Interpreting graphs of $y = a\sin\theta = q, y = a\cos\theta + q$ and $y = a\tan\theta + q$**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Distance between two points, Substitution				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Amplitude, Reflection on x-axis, period, substitution

RESOURCES

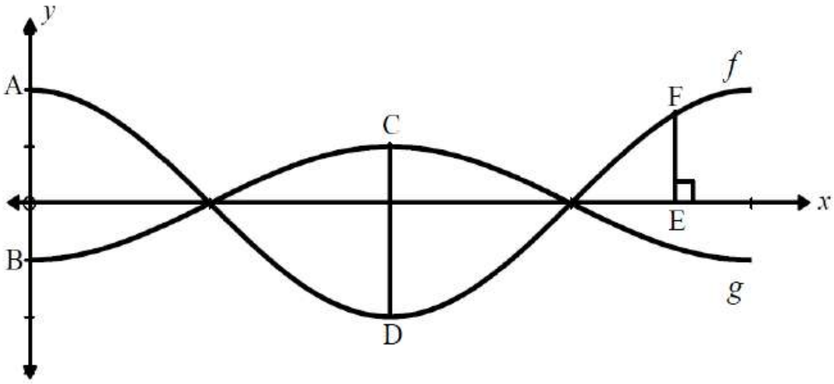


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Calculating the length of a line between TWOgraphs

METHODOLOGY

1. The diagram below represents the graphs of $f(x) = 2\cos x$ and $g(x) = -\cos x$ for the interval $x \in [0°; 360°]$.



(a) Determine the lengths of OA and OB.
(b) Determine the length of CD.
(c) Determine the length of EF if OE = 315°.
(d) Determine graphically the values of $x \in [0° ; 360°]$ for which:

(i) $f(x) = 0$	(ii) $g(x) = 0$
(iii) $f(x) = 2$	(iv) $g(x) = -1$
(v) $f(x) < 0$	(vi) $g(x) \leq 0$
(vii) $f(x) \geq g(x)$	(viiii) $g(x) > f(x)$

Solution:

(a) OA = 2 units and OB = 1 unit (Maximum and Minimum values of both graphs)

(b) CD = 1-(-2) = 3 units

(c) OE = 315° $and\ EF\ is\ y = 2\cos(315°)$ = 1.41 units……F is on $y = 2\cos x$

(d) (i) $x = 90°, x = 270°$ (ii) $x = 90°, x = 270°$

(iii) $x = 0°, x = 360°$ (iv) $x = 0°, x = 360°$

(v) $90° < x < 270°$ / $x \in (90°; 270°)$ ….. where f(x) is below x-axis

(vi) $0° \leq x \leq 90°$ and $270° \leq x \leq 360°$ …where g(x) is below and equal to the x-axis

OR $x \in [0°; 90°]$ and $x \in [270°; 360°]$

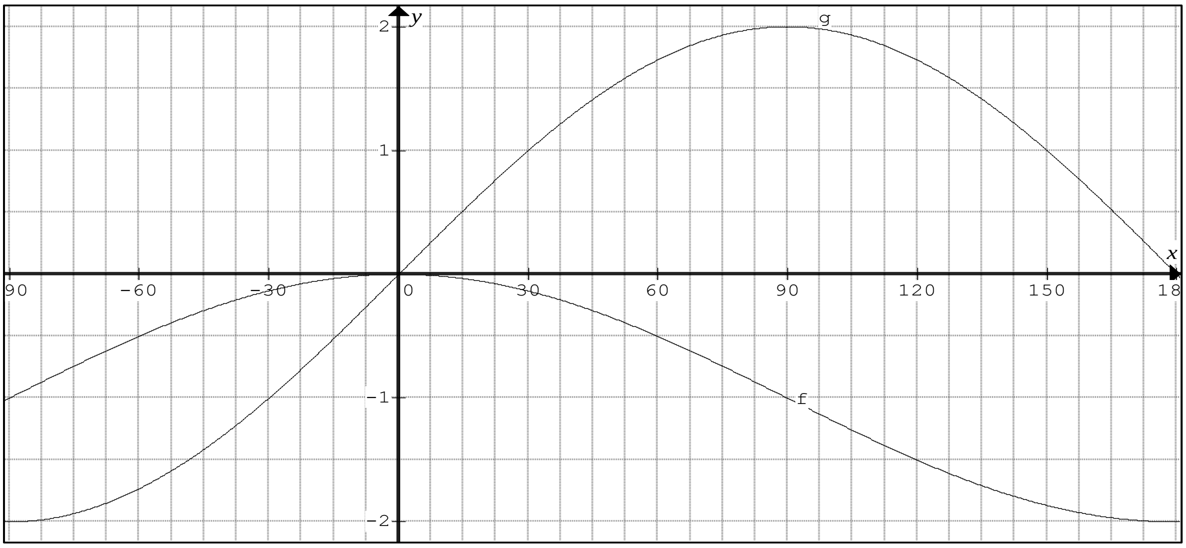
(vii) $0° \leq x \leq 90°$ and $270° \leq x \leq 360°$…where f(x) is above and equal to g(x)

OR $x \in [0°; 90°]$ and $x \in [270°; 360°]$

(viii) $90° < x < 270°$ OR $x \in (90°; 270°)$ …where g(x) is above f(x)

ACTIVITIES/ ASSESSMENT

1. Below is a sketch of $f(x) = \cos x + q$ and $g(x) = a\sin x$



(a) Write down the amplitude of f and g

(b) What is the range of f

(c) What is the period of f

(d) Determine the values of a and q

2. Sketch the graphs of $y = f(x) = -\tan x$ and $y = g(x) = \tan x + 2$ for $x\ [0°;270°]$ on the same set of axes and then answer the questions that follow.

(a) Calculate $f(45°)$ - $g(45°)$

(b) For which values of x is $g(x) = f(x)$?

(c) For which values of x is $g(x) \geq f(x)$?

(d) For which values of x $g(x)$ - $f(x) = 2$?

TEST: TRIGONOMETRIC FUNCTIONS

MARKS: 25 **DURATION: 30 Min.**

Instructions:

Answer ALL the questions.

QUESTION 1 [13 Marks]

1.1 Given $f(x) = \sin x - 1$ and $g(x) = 2\cos x$ for $0° \leq x \leq 270°$

Sketch on the same set of axes the graph of *f* and *g*. (6)

1.2 Write down the following:

1.2.1 Amplitude of g (1)

1.2.2 Range of *f* (2)

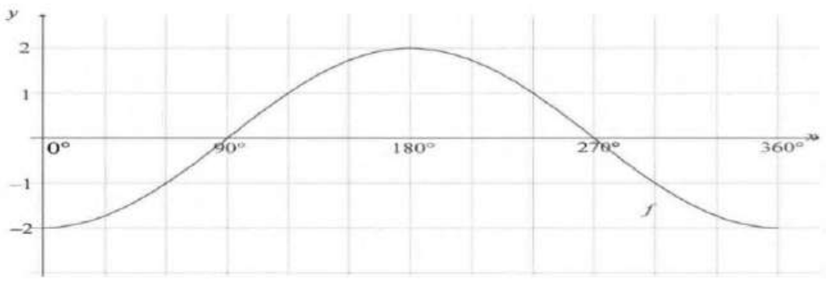
1.3 Use your graph to determine the following:

1.3.1 Number of solutions to $f(x) = g(x)$ in the interval $0° \leq x \leq 270°$ (1)

1.3.2 Values of *x* in the interval for which $\sin x = 2 + 2\cos x$ (3)

QUESTION 2 [12 Marks]

In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for the interval $0° \leq x \leq 360°$



2.1 Write down the amplitude of *f* (1)

2.2 Write down the minimum value of $f(x) + 3$ (1)

2.3 On the same set of axes, draw the graph of *g*, where $g(x0 = \sin x + 1$ for $0° \leq x \leq 360°$. (3)

2.4 Use the graphs to determine the following:

2.4.1 The value of $f(180°) - g(180°)$ (1)

2.4.2 For which values of *x* will $f(x).g(x) > 0$ (2)

2.5 The graph of *f* is reflected about the x-axis and then moved 3 units downwards to form the Graph of *h*. Determine:

2.5.1 The equation of *h* (2)

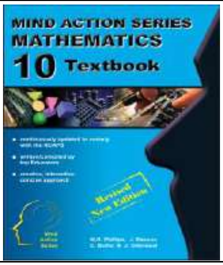
2.5.2 The range of *h* for the interval $0° \leq x \leq 360°$. (2)

TOPIC: NUMBER PATTERNS (Lesson 1)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Investigate number patterns leading to those where there is a constant difference, and the general term is therefore linear.** **Linear number Patterns: Number Sequence**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Constant difference, linear pattern, sequence, general term/rule				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Difference between numbers, substitution

RESOURCES

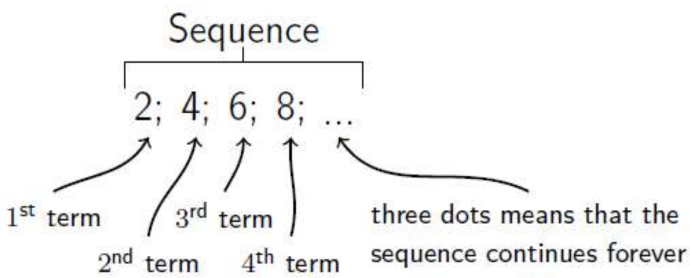


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Common difference as $T_1 - T_2$, confusing the difference between the term (T) and its position (n).

METHODOLOGY

A **sequence** is an ordered list of items, usually numbers. Each item which makes up a sequence is called a **term**.



Number pattern is a pattern in which a series/list of numbers follows a certain sequence. This pattern generally establishes a **common relationship** between all numbers.

Number patterns could be ascending, descending, multiples of a certain number, or series of even numbers, odd numbers, etc.

Linear pattern is a sequence of numbers in which there is a **common difference (d)** between any term and the term before.

The **common difference** is the difference between any term and the term before it.

Consider the number pattern 3; 5; 7; 9; 11; …

The pattern is formed by adding 2 to each new term. We say that the **constant difference** between the terms is 2.
NOTE: A number pattern with a constant difference is called a **linear number pattern**.

To solve the problems of number pattern, **the general term/rule** being followed in the pattern must be understood.

To determine the general term of linear number patterns, find the **difference between consecutive terms/numbers.**

Examples:

1. Given sequence: 6; 10; 14; 18; …
   (a) Calculate the common difference.
   (b) Determine the general term.
   (c) Calculate the 12th term.
   (d) Which term of the sequence is 242?

Solution:

(a) $d = T_2 - T_1 OR\ T_3 - T_2 OR\ T_4 - T_3$
$= 10 - 4\ OR\ 14 - 10\ OR\ 18 - 14$
$d = 4$, common difference.

(b) Draw a table.

The position of the term	T_1	T_2	T_3	T_4	T_n
The common difference multiplies by the position of the term	4(1)	4(2)	4(3)	4(4)	4(n)
What to do to get the actual term?	+ 2	+ 2	+ 2	+ 2	+ 2
The actual term in the sequence	6	10	14	16	**4n + 2**

Therefore, $T_n = 4n + 2$

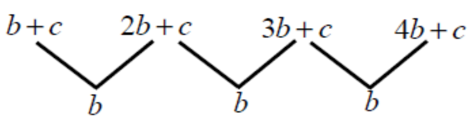
(c) $T_{12} = 4(12) + 2 = 50$…substitute into the nth term

(d) The actual **term** in the sequence **is 242** and we want to find its **position**.
Let T*n* = 242 where ***n* represents the position** to be determined.
$T_n = 4n + 2$
$242 = 4n + 2$
$240 = 4n$…transpose 2
$60 = n$…divide by 4 on both sides of the equation
Therefore, $T_{60} = 242$ (242 is the 60th term in the sequence.

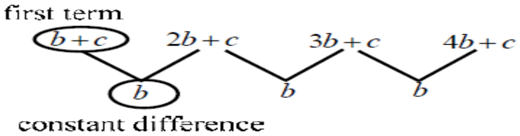
AN ALTERNATIVE METHOD FOR LINEAR NUMBER PATTERNS

For linear pattern the general term or **nth term** will always be of the form: $\boldsymbol{T_n = bn + c}$ where b is the common difference between each successive terms and c is some constant.

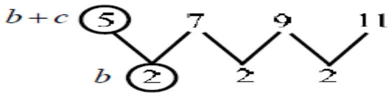
We can determine the first few terms using this rule: $T_n = bn + c$
$T_1 = b(1) + c = b + c$
$T_2 = b(2) + c = 2b + c$
$T_3 = b(3) + c = 3b + c$
$T_4 = b(4) + c = 4b + c$



The common diffeence is b and the first term is $b = c$



Now let's determine the general term of the linear pattern 5; 7; 9 ;11; ... using these findings.



$b = 2$

$b + c = 5$
$2 + c = 5$…substitute $b = 2$
$\therefore c = 3$
$T_n = bn + c$
$\therefore T_n = 2n + 3$

ACTIVITIES/ ASSESSMENT

1. For each of the following sequences determine the common difference. If the sequence is not linear, write "no common difference".

(a) 5; 12; 19; 26; 32; … (b) 9; -7; -8; -25; -34; … (c) 9; -7; -8; -25; -34; …

(d) 9; 13; 17; 21; … (e) 9; 13; 17; 21; … (f) 0; -3; -6; …

(g) 5; 1; -3; -7; … (h) $3\frac{1}{2}$; 4; $4\frac{1}{2}$; … (i) $\frac{1}{4}$; 1; $\frac{7}{4}$; …

(j) -13; -7; -1; …

2. Write down the next three terms in each of the following sequences:

(a) 5; 15; 25; … (b) -8; -3; 2; … (c) 30; 27; 24; …

(d) -13,1; -18,1; -23,1; … (e) 9*x*; 19*x*; 29*x*; … (f) -1,8; 4,2; 24,2; …

(g) 30*b*; 34*b*; 38*b*; … (h) C; D; E; F; …

3. Given a pattern which starts with the numbers: 3; 8; 13; 18; …determine the values of T_6 *and* T_9.

4. The general term is given for each sequence below. Calculate the missing terms.

(a) 1; 10;19; …; 37 $T_n = 9n - 8$ (b) 3; 2; 1; 0; …; -2 $T_n = -n + 4$

(c) 111; …; -7; …; -3 $T_n = 2n - 13$ (d) 9; …; 21; …; 33 $T_n = 6n + 3$

5. For each of the following sequences, determine the general rule (*n*th term) and hence calculate the 25th term.

(a) 6; 9 ;12 ;15; ... (b) 3; 8 ;13 ;18; ... (c) 10 ;16; 22; 28; ...

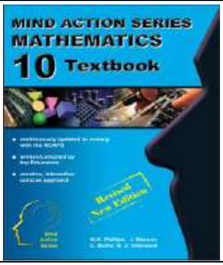
(d) 5; 0; – 5; –10; ... (e) –6; –11; –16; ... (f) –5; –11; –17; ...

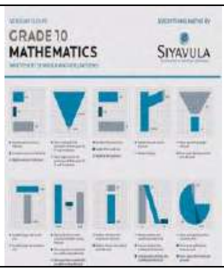
TOPIC: NUMBER PATTERNS (Lesson 2)	Weighting	15 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Investigate number patterns leading to those where there is a constant difference, and the general term is therefore linear.** **Linear Number Pattern: Shapes**			
RELATED CONCEPTS/ TERMS/VOCABULARY	General term,			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Addition and subtraction, calculating the next term and the n^{th} term

RESOURCES





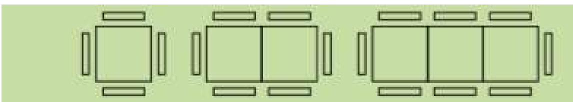
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Common difference as $T_1 - T_2$, confusing the difference between the term (T) and its position (n).

METHODOLOGY

Patterns are repetitive sequences and can be found in nature, shapes, events, sets of numbers and almost everywhere you care to look. For example, **geometric designs** on quilts or tiles.

Example:
You and 3 friends are studying for Mathematics and are sitting together at a square table. A few minutes later 2 other friends arrive so you move another table next to yours. Now 6 people can sit at the table. Another 2 friends also join your group, so you take a third table and add it to the existing tables. Now 8 people can sit together as shown below.



Solution:
Sequence is 4; 6; 8; …

Common difference is 2

Number of tables	T_1	T_2	T_3	T_4	T_n
The common difference multiplies by the number of the tables	2(1)	2(2)	2(3)	2(4)	2(n)
What to do to get the actual term?	+2	+2	+2	+2	+2
The actual term in the sequence	4	6	8	10	**2n +2**

(a) Find an expression for the number of people seated at n tables.

$$T_n = 2n + 2$$

(b) Use the general formula to determine how many people can sit around 12 tables.

$$T_n = 2n + 2$$

$T_{12} = 2(12) + 2 = 26$…12 is the position (12^{th} term)

Therefore, 12 tables can have 26 people.

(c) How many tables can sit 20 people?

$T_n = 2n + 2$
$20 = 2n + 2 \ldots T_n = 20$, calculate $\boldsymbol{n}$
$18 = 2n$
$9 = n$, therefore, 20 people can be seated on 9 tables

AN ALTERNATIVE METHOD FOR LINEAR NUMBER PATTERNS: $\boldsymbol{T_n = bn + c}$

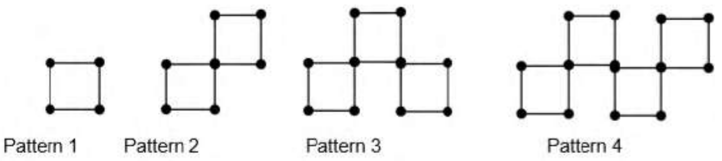
(a) $b = 2\ldots$common difference

$b + c = 4\ldots$first term (T_1)
$2 + c = 4\ldots$substitute the common difference
$c = 2$

$\boldsymbol{T_n = bn + c}$
$T_n = 2n + 2$

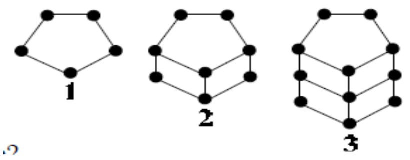
ACTIVITIES/ ASSESSMENT

1. The following shapes were formed with matchsticks.



(a) Write down the sequence
(b) Determine the common difference.
(c) Determine the general term of the pattern.
(d) How many sticks will be in the 50th pattern?

2. Consider the diagram made up of black dots joined by thin black lines.



(a) How many dots are there in figure 4?
(b) How many lines are there in figure 4?
(c) How many dots are there in figure 8?
(d) How many lines are there in figure 8?
(e) Determine the general rule to find the number of dots in the *n*th figure.
(f) How many dots are there in the 186th figure?
(g) Which figure will contain 272 dots?
(h) Determine the general rule to find the number of lines in the *n*th figure.
(i) How many lines are there in the 900th figure?
(j) Which figure will contain 650 lines?

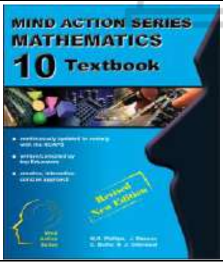
TOPIC: NUMBER PATTERNS (Lesson 3)	Weighting	15 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		

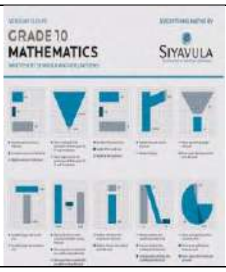
Sub-topics	**Linear Number Pattern: Number Sequence**
RELATED CONCEPTS/ TERMS/VOCABULARY	General term ($\boldsymbol{T_n = bn + c}$), Common difference

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

General term of a linear number pattern

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

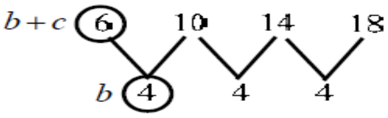
Common difference as $T_1 - T_2$, confusing the difference between the term (T) and its position (n).

METHODOLOGY

Example

1. Determine the general term (nth term) and hence the 20th term of the following number patterns:
   6; 10; 14; 18; ...

Solution:



$b = 4$...common difference

$b + c = 6$...first term (T_1)
$4 + c = 6$
$c = 2$

$\boldsymbol{T_n = bn + c}$
$T_n = 4n + 2$ AND $T_{20} = 4(20) + 2 = 82$

2. Consider the sequence: 3; 8; 13; 18; …
   (a) Calculate the next term.
   (b) Determine the nth term of the pattern.
   (c) Which term is equal to 748?

Solution:

(a) common difference is 5. The next term is 18+ 5 = 23

(b) $b = 5$

$b + c = 3$
$5 + c = 3$
$c = -2$

Therefore, $T_n = 5n - 2$

(c) $\boldsymbol{T_n = bn + c}$
$748 = 5n - 2$
$750 = 5n$
$150 = n$, therefore, $T_{50} = 748$

ACTIVITIES/ ASSESSMENT

1. Given a sequence: 19; 16; 13; 10; …

(a) Determine the common difference

(b) Determine the nth term.

(c) Determine the 45th term.

(d) Which term of the sequence is -113?

2. $T_n = 9n - 4$ is the nth term of a linear number pattern.

(a) Determine the first four terms of the sequence.

(b) Which term is equal to 987?

3. Consider the pattern: $4 \times 7; 7 \times 15; 10 \times 23; 13 \times 31; ...$

(a) Determine the nth term of the sequence

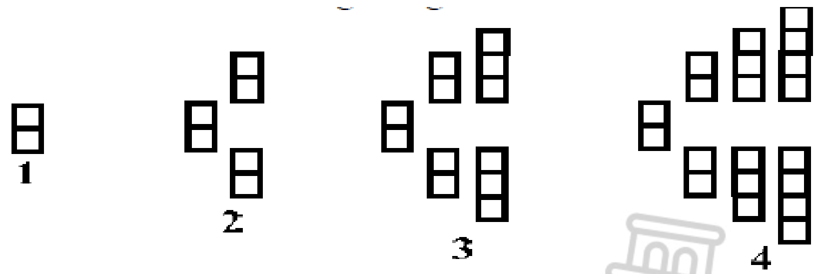
(b) Determine the 50th term.

4. Consider the number pattern: -2; -5; -8; -11; …

(a) Determine the nth term and hence the 145th term.

(b) Determine which term of the number pattern equals -389.

5. Consider the following designs:



(a) Write down the number of squares in design 1, 2, 3, 4, and 5.
(b) Determine the number of squares in design *n*.
(c) How many squares are there in design 20?

TEST: NUMBER PATTERNS

MARKS: 25 **DURATION: 30 Min.**

Instructions:

Answer ALL the questions.

QUESTION 1[9 Marks]

Write down the next three terms and the general (or nth term) of each pattern:

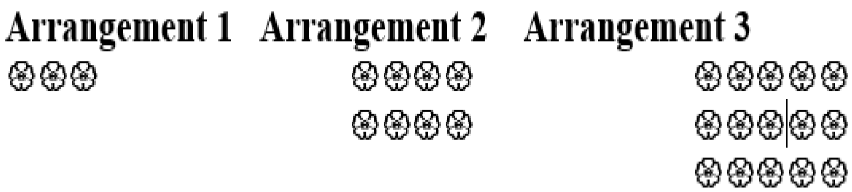
1.1 1; 7; 13; 19; … (3)

1.2 $\frac{1}{2}$; 1; $\frac{3}{2}$; 2; … (3)

1.3 $x - 1; 2x - 2; 3x - 3; 4x - 4; ...$ (3)

QUESTION 2 [15 Marks]

2.1 Consider the following pattern:

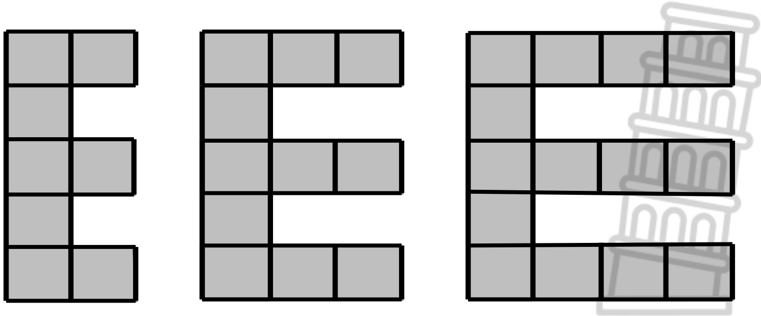


2.1.1 How many flowers will be used in the 4^{th} arrangement? (1)

2.1.2 How many flowers will be used in the n^{th} arrangement? (3)

2.1.3 Which arrangement will have 99 flowers? (2)

2.2 Consider the following sequence of Es:



2.2.1 How many blocks will be needed to build the 10^{th} E? (2)

2.2.2 How many blocks will be needed for the *n*th E? (3)

2.2.3 116 blocks are needed for the *n*th E. Calculate the value of *n*. (2)

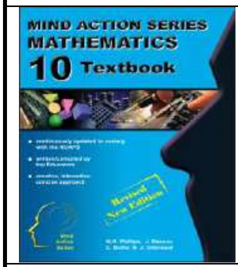
2.2.4 Can the total number of blocks ever be a multiple of 10? Explain. (3)

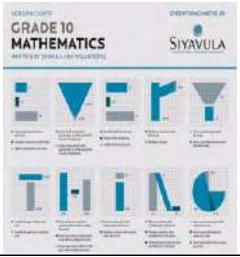
TOPIC: ANALYTICAL GEOMETRY(Lesson1)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Derivation of the Distance Formula for any two points.**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Point, Distance				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Calculating the distance between two points using the formula.

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Learners not knowing where to put a plus or a mins in a distance formula

METHODOLOGY

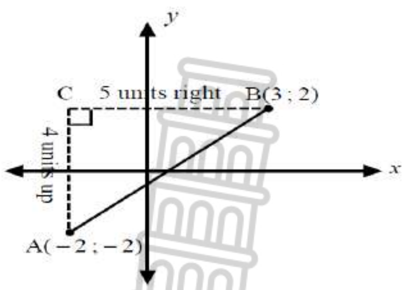
A **point** is a simple geometric object having location as its only property.
A **point** is an ordered pair of numbers written as a coordinate (x; y).

Distance is a measure of the length between **two points** (length of the line segment).

Given points A (-2; -2); B (3; 2) and C (-2; 2)

Suppose that we wish to calculate the length of line segment AB, with endpoints A (–2; – 2) and B (3; 2).

NOTE:
Drawing a sketch helps with your calculation and makes it easier to check if your answer is correct.



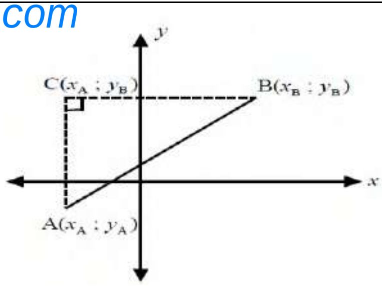
Length of $CB = 3 - (-2) = 5\ units$ ... (maximum x-value – minimum x-value)

Length of $AC = 2 - (-2) = 4\ units$ ... (maximum y-value – minimum y-value)

Length of AB, use Pythagoras Theorem

$$AB^2 = BC^2 + AC^2 \text{ ... Pythagoras Theorem}$$
$$AB^2 = (3 - (-2))^2 + (2 - (-2))^2$$
$$AB^2 = 5^2 + 4^2 = 41$$
$$AB = \sqrt{41} \approx 6{,}40$$

We can generalise this concept to create what is known as the "*Distance Formula*".



Distance between C and B (length CB) = $x_B - x_A$
Distance between A and C (length AC) = $y_B - y_A$

To derive a general formula for the distance between two points $A(x_A; y_A)$ and $B(x_B; y_B)$ we use the theorem of Pythagoras.

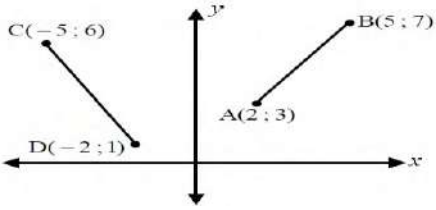
$$AB^2 = CB^2 + AC^2 \dots \text{ Pythagoras Theorem}$$
$$\therefore AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$
$$\therefore AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

Therefore, the formula to calculate the distance or length of the line segment between any two points, $(x_1; y_1)$ and $(x_2; y_2)$, we use:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Examples:

1. Calculate the lengths of line segments AB and CD in the given diagram.



Solution:

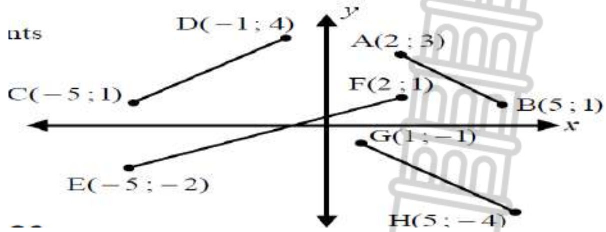
$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ units}$$

and

$$CD = \sqrt{(x_D - x_C)^2 + (y_D - y_C)^2}$$
$$CD = \sqrt{(-2 - (-5))^2 + (1-6)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{34} \approx 5{,}83 \text{ units}$$

ACTIVITIES/ ASSESSMENT

1. Calculate the lengths of the line segments in the given diagram.



2. Siyavula: Exercise 8 – 2. Page 292

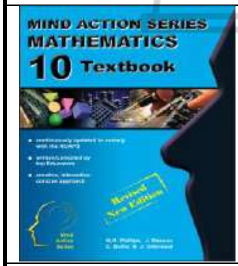
Number: 1, 2, 3 and 4

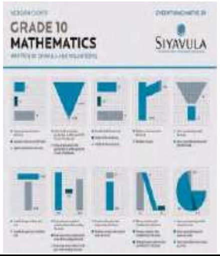
TOPIC: ANALYTICAL GEOMETRY (Lesson 2)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Applications of the Distance Formula:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Equidistant, perimeter				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Isosceles triangle, perimeter of a triangle, distance formula

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

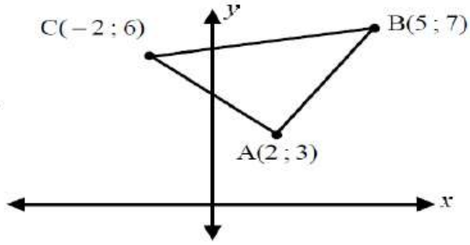
Confusing the perimeter and the area, understanding of equidistant points.

METHODOLOGY

The **perimeter** of a polygon (or any other closed curve such as a circle) is the distance around the outside.
Perimeter is the distance around a two-dimensional shape.

Equidistant
Equidistant comes from two words, equal and distant. **Equidistant** means equal distance from every point

1. In the diagram, the vertices of ΔABC are A (2; 3), B (5; 7) and C (–2; 6).



(a) Show that ΔABC is an isosceles triangle.

(b) Calculate the perimeter of ΔABC correct to one decimal place.

Solution:

(a) We can show that ΔABC is an isosceles triangle by proving two sides equal. From the diagram above, the obvious choice is to prove that AB = AC.

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$= \sqrt{(5-2)^2 + (7-3)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{25} = 5 \text{ units}$$

and

$$AC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}$$
$$AC = \sqrt{(-2-2)^2 + (6-3)^2}$$
$$= \sqrt{(-4)^2 + (3)^2}$$
$$= \sqrt{25} = 5 \text{ units}$$

$$\therefore AB = AC$$

(b) The perimeter of ΔABC is the sum of its three sides:

$$BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}$$
$$= \sqrt{(-2-5)^2 + (6-7)^2}$$
$$= \sqrt{(-7)^2 + (1)^2} = \sqrt{50}$$

Perimeter = AB+AC+ BC

$= 5 + 5 + \sqrt{50}$ = 17,071…≈ 17,1

2. Calculate the possible values of k, if the distance between A and B is 5 units, where A (2; 5) and B (–1; k).

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$5 = \sqrt{(-1-2)^2 + (k-5)^2}$… AB = 5 units
$$5 = \sqrt{9 + k^2 - 10k + 25}$$
$25 = k^2 - 10k + 34$... square on both sides to remove the square root
$0 = k^2 - 10k + 9$... standard form of quadratic equation
$0 = (k-9)(k-1)$... factors
$$k = 9 \text{ or } k = 1$$

3. (a) Show that the point Q (- 6 ;1) is equidistant from the points P (- 4; 5) and R (- 2; 3).

We are required to show that QP = QR.

$$QP = \sqrt{\left(x_P - x_Q\right)^2 + \left(y_P - y_Q\right)^2}$$
$$= \sqrt{(-4-(-6))^2 + (5-1)^2}$$
$$= \sqrt{2^2 + 4^2}$$
$$= \sqrt{20}$$

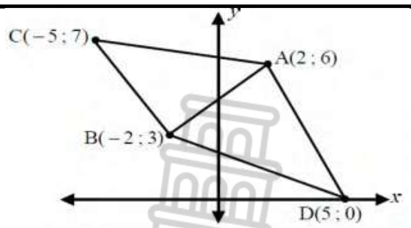
$$QR = \sqrt{\left(x_R - x_Q\right)^2 + \left(y_R - y_Q\right)^2}$$
$$= \sqrt{(-2-(-6))^2 + (3-1)^2}$$
$$= \sqrt{4^2 + 2^2}$$
$$= \sqrt{20}$$

∴ QP = QR and Q is therefore equidistant from P and R.

(b) The point T (x ;1) is equidistant from the points A (- 2; –1) and N (1; 2). Determine the value of x.

ACTIVITIES/ ASSESSMENT

1. In the given diagram, two triangles have been drawn.



(a) Show that ΔABC is an isosceles triangle.

(b) Determine the perimeter of ΔABD.

2. Show that C (2; 3) is equidistant from the points A (3; 6) and B (–1; 4).

3. C is the point (1; – 2). The point D lies in the second quadrant and has coordinates (x; 5). If the length of CD is $\sqrt{53}$ units, determine the value of x.

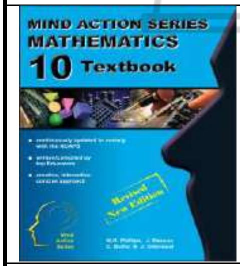
4. Given the points P (–3; 2), Q (2; 7) and R (–10; y). Determine the values of y if P is equidistant from Q and R.

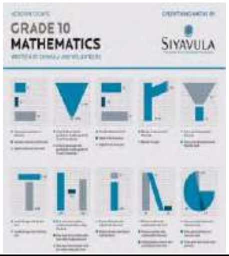
TOPIC: ANALYTICAL GEOMETRY (Lesson 3)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Derive for the formulae for calculating the midpoint of the line segment connecting the two points.**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Midpoint, line segment				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Addition and Division, fractions

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Midpoint of a line: Subtracting the values of *x* or the values of *y* instead of adding.

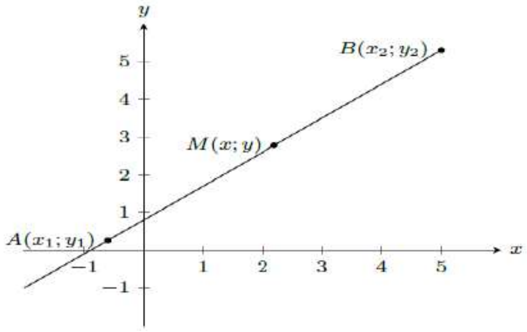
METHODOLOGY

The **midpoint** of a line segment is the halfway mark on the line segment.

Investigation: Finding the **mid-point** of a line

Consider the numbers 1 and 7 for example. Halfway between 1 and 7 is 4.
How do we get to 4? Add the two values and then divide the answer by 2, that is $\frac{7+1}{2} = \frac{8}{2} = 4$

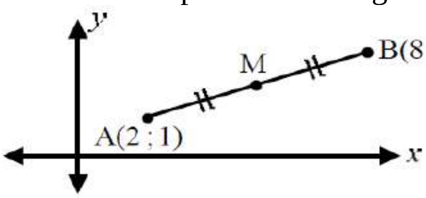
This concept can be applied to a **line segment joining two points** on the Cartesian plane.
Line segment is a line with two endpoints



Midpoint $(x; y) = M\left(\frac{x_1+x_2}{2}\right); \left(\frac{y_1+y_2}{2}\right)$

Examples:

1. Determine the coordinates of M, if M is the midpoint of line segment AB, where A (2;1) and B (8; 3).

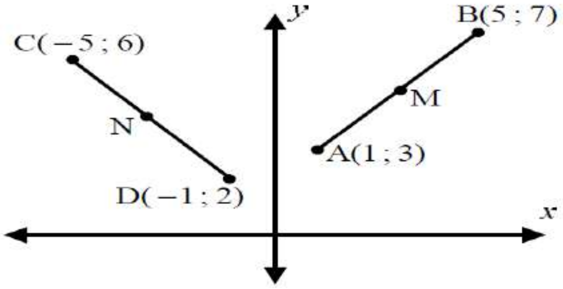


$M(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

$= M(\frac{2+8}{2}; \frac{1+3}{2})$
$= M(5; 2)$

2. Calculate the midpoints of line segments AB and CD in the given sketch.



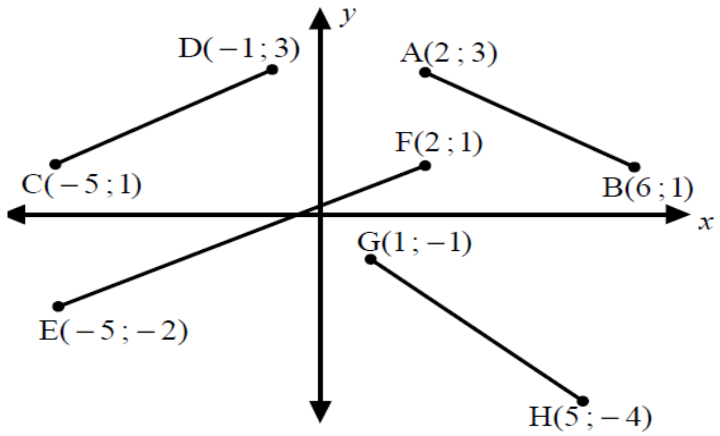
Midpoint AB $= M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ and Midpoint CD $= M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$\therefore M\left(\frac{1+5}{2};\frac{3+7}{2}\right)$ $\therefore M\left(\frac{-1+-5}{2};\frac{2+6}{2}\right)$

$\therefore M(3;5)$ $\therefore M(-3;4)$

ACTIVITIES/ ASSESSMENT

1. Determine the midpoints of the given line segments. Use the midpoint formula.



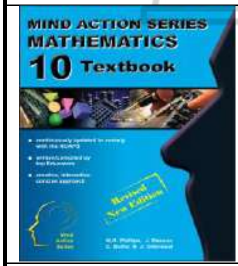
2. Siyavula: Page 313 – 314, Exercise 8 - 5
Number 1, 2, 3 and 4

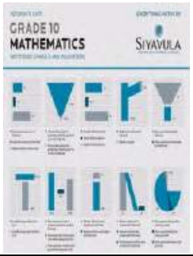
TOPIC: ANALYTICAL GEOMETRY (Lesson 4)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Applications of the Midpoint Formula:** $M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Diagonals of a parallelogram				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Midpoint formula, line segment, properties of quadrilateral, parts of a circle.

RESOURCES



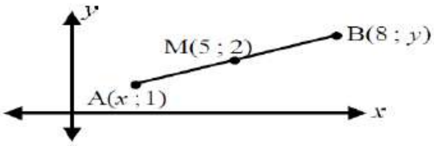


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Midpoint of a line: Subtracting the values of *x* or the values of *y* instead of adding.

METHODOLOGY

1. Determine the values of *x* and *y* if M (5; 2) is the midpoint of the line segment joining the points A (*x* ;1) and B (8; *y*).



$x_M = \frac{x_A+x_B}{2}$

$5 = \frac{x+8}{2}$

$x + 8 = 10$

$x = 2$

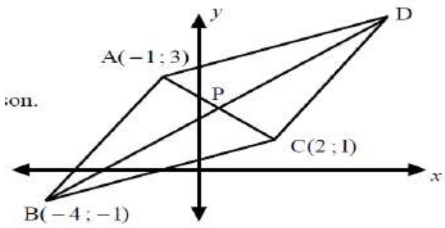
$y_M = \frac{y_A+y_B}{2}$

$2 = \frac{1+y}{2}$

$1 + y = 4$

$y = 3$

2. The following sketch shows parallelogram ABCD, with P the point where the diagonals intersect.



(a) Determine the coordinates of P, giving a reason.

P is the midpoint of both AC and BD, because the **diagonals** of a parallelogram **bisect** each other.

$P\left(\frac{-1+2}{2};\frac{3+1}{2}\right)$

$\therefore P\left(\frac{1}{2};2\right)$

(b) Determine the coordinates of D.

$x_P = \frac{x_B+x_D}{2}$

$\frac{1}{2} = \frac{-4+x_D}{2}$

$-8 + 2x_D = 2$

$y_P = \frac{y_B+y_D}{2}$

$2 = \frac{-1+y_D}{2}$

$-1 + y_D = 4$

$2x_D = 10$ $y_D = 5$

$x_D = 5$

$\therefore$ D (5; 5)

ACTIVITIES/ ASSESSMENT

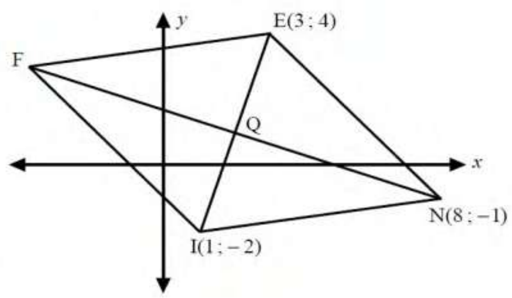
1. A circle has diameter AB with endpoints A (–1; 4) and B (5; – 2).

(a) Determine the centre of the circle.
(b) Determine the radius of the circle.

2. Answer the following questions. You may want to draw a diagram to help you visualise the scenario.

(a) If M (–3; 2) is the midpoint of the line segment joining the points A (*x* ;1) and B (–1; *y*), calculate the values of *x* and *y*.
(b) If M (–1; 7) is the midpoint of the line segment joining the points A (*x;* 6) and B (2; *y*), calculate the values of *x* and *y*.
(c) If M (–1; – 5) is the midpoint of the line segment joining the points A (*x; y*) and B (–6; – 3), calculate the values of *x* and *y*.

3. Given below is rhombus FINE. Q is the point where the diagonals intersect.



Determine the coordinates of F.

4. A (- 2; 3), B (*x; y*), C (1; 4) and D (-1; 2) are the vertices of a quadrilateral. Find B (*x; y*) if ABCD is a parallelogram.

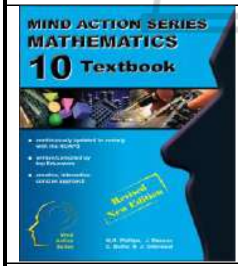
5. Siyavula: Page 314, Exercise 8 – 5
Number 5

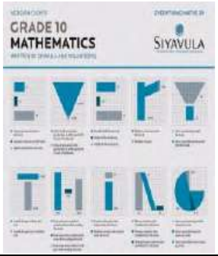
TOPIC: ANALYTICAL GEOMETRY (Lesson 5)		**Weighting**	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Derivation for the formulae for calculating the gradient of the line segment joining the two points**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Gradient, vertical movement/change, horizontal movement/change				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Relationship between change in *y* and change in *x*, gradient as rise over run

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Gradient as change in *x* divided by change in *y* or $m = \frac{y_1 - y_2}{x_2 - x_1}$

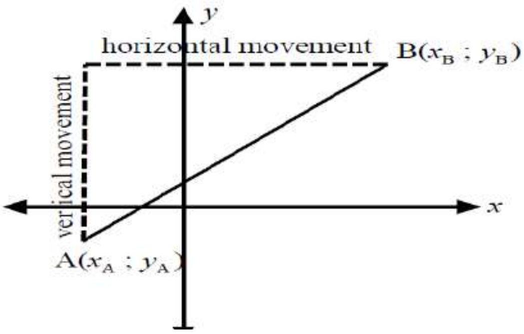
METHODOLOGY

Gradient describes the slope or steepness and direction of the line joining two points.

A line can either slant up (**gradient is positive**), slant down (**gradient is negative**), be horizontal (**gradient is zero**) or be vertical (**gradient is undefined**). The symbol used for gradient is *m*.

Gradient is determined by the **ratio** of **vertical change to horizontal change**.

To **derive the formula for gradient**, we consider any two points $A(x_A; y_A)$ and $B(x_B; y_B)$ as shown in the diagram alongside.



The **gradient is determined** by the ratio of the length of the vertical movement (vertical side) of the triangle to the length of the horizontal movement (horizontal side) of the triangle.

$$m = \frac{vertical\ movement}{horizontal\ movement}$$

The length of the vertical side of the triangle is the difference in *y*-values of points *A* and *B*.

$$vertical\ side = y_B - y_A$$

The length of the horizontal side of the triangle is the difference in *x*-values of points *A* and *B*.

$$horizontal\ dise = x_B - x_A$$

Therefore

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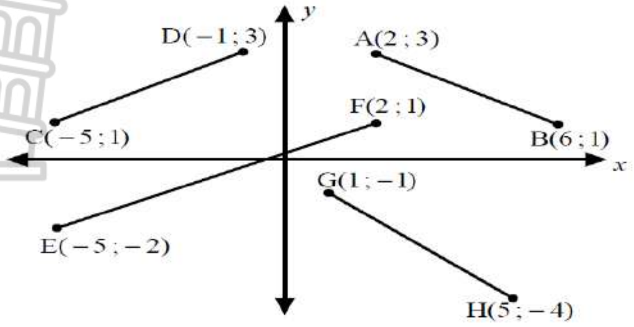
A formula to calculate the gradient of a line joining two points A $(x_A; y_A)$ and B $(x_B; y_B)$, is:

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

Examples:

Calculate the gradients of the following lines using the formula for gradient.



$m_{AB} = \frac{1-3}{6-2} = \frac{-2}{4} = -\frac{1}{2}$... slopes down from left to right

$m_{CD} = \frac{3-1}{-1-(-5)} = \frac{2}{4} = \frac{1}{2}$... slopes up from left to right

$m_{EF} = \frac{1-(-2)}{2-(-5)} = \frac{3}{7}$... slopes up from left to right

$m_{GH} = \frac{-4-(-1)}{5-1} = \frac{-3}{4} = -\frac{3}{4}$... slopes down from left to right

ACTIVITIES/ ASSESSMENT

1. Calculate the gradients of the lines joining the following points.

(a) A (1; 3) and B (5; 7)

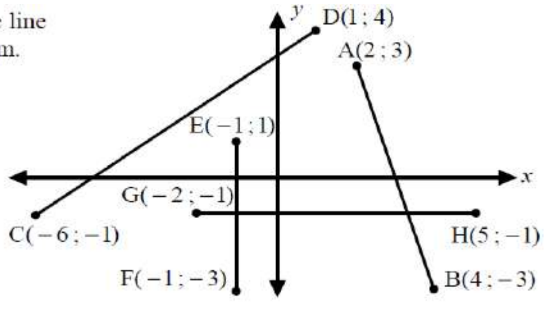
(b) A (1; 3) and B (–5 ; – 7)

(c) A (–1; – 3) and B (–5; – 7)

(d) A (–1; 3) and B (5 ; –7)

2.

Calculate the gradients of the line segments in the given diagram.



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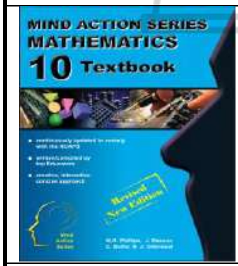
3. Siyavula: Page 296, Exercise 8 - 3
Number 1, 2 and 3

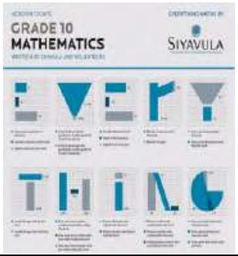
TOPIC: ANALYTICAL GEOMETRY (Lesson 6)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Applications of gradient: Parallel and Perpendicular Lines**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Parallel lines, collinear points, perpendicular lines.				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Formula for calculating gradient. Properties of quadrilaterals, area of polygons.

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

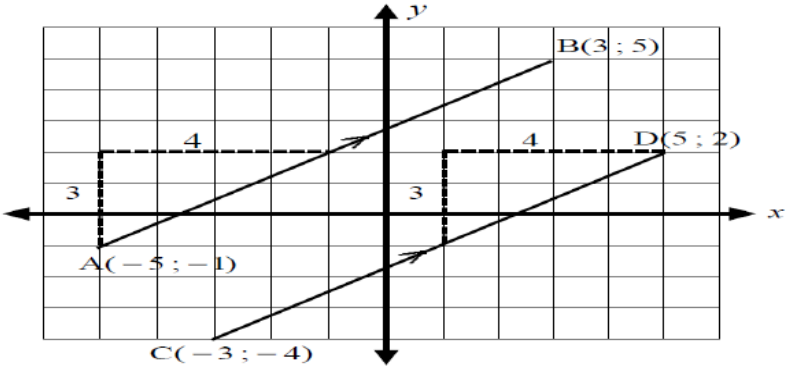
Gradient as change in *x* divided by change in *y* or $m = \frac{y_1 - y_2}{x_2 - x_1}$

METHODOLOGY

<u>Parallel Lines</u>

Two lines that run **parallel** to each other are always the **same distance** apart and **have equal gradients.**

Parallel lines slope in the exactly the same direction and will therefore never intersect.



For any pair of parallel lines AB and CD: $m_{AB} = m_{CD}$

Example 1

Given are the points A (–1; 5), B (–2; 3), C (9 ;10) and D (5; 2). Show that AB||CD.

$m_{AB} = \frac{3-5}{-2-(-1)} = \frac{-2}{-1} = 2$ $\qquad$ $m_{CD} = \frac{2-10}{5-9} = \frac{-8}{-4} = 2$

$\therefore m_{AB} = m_{CD}$

$\therefore AB \parallel CD$

<u>Collinear points</u>

Points are said to be **collinear** when they lie on the same line. Refer to the diagram. A, B and C lie on the same line and are therefore collinear. This implies that the **gradients between each pair of points are the same.**

When points A, B and C are collinear: $m_{AB} = m_{AC} = m_{BC}$

Example 2

Show that the points A, B and C are collinear if the coordinates of the points are:
A (2; –2), B (1;1) and C (–1; 7).

$m_{AB} = \frac{1-(-2)}{1-2} = \frac{3}{-1} = -3$ $\quad$ $m_{BC} = \frac{7-1}{-1-1} = \frac{6}{-2} = -3$

$\therefore m_{AB} = m_{BC}$

Therefore, A, B and C are collinear.

Perpendicular lines

Lines that meet or intersect each other at right angles (90°)

Perpendicular lines have gradients that are the negative inverses of each other and the **product of their gradients is equal to -1**.

∴ **For any pair of *perpendicular lines*:** $m_1 \times m_2 = -1$

Examples:

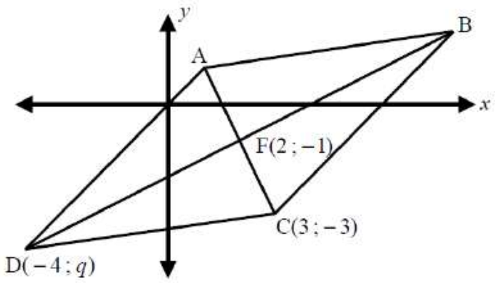
1. Given are the points A (3; – 3), B (6; – 7), C (–5; 0) and D (–1; 3). Show that AB is perpendicular to CD.

$m_{AB} = \frac{-7-(-3)}{6-3} = \frac{-4}{3} = -\frac{4}{3}$ $\quad$ $m_{CD} = \frac{3-0}{-1-(-5)} = \frac{3}{4}$

$m_{AB} \times m_{CD} = -\frac{4}{3} \times \frac{3}{4} = -1$

∴ AB⊥CD

2. In the diagram, ABCD is a rhombus. F is the point where the diagonals intersect.



(a) Determine the value of q.

AC ⊥ BD because the diagonals of a rhombus are perpendicular to each other.

$m_{FC} = \frac{-3-(-1)}{3-2} = -2$ $\quad$ $m_{FD} = \frac{q-(-1)}{-4-2} = \frac{q+1}{-6}$

$m_{FC} \times m_{FD} = -1$

$-2 \times \frac{q+1}{-6} = -1$... substitute gradients

$\frac{-2q-2}{-6} = -1$... multiply numerators

$-2q - 2 = 6$... cross multiply

$-2q = 8$

$q = -4$

(b) Calculate the area of ΔCFD.

Area of ΔCFD= $\frac{1}{2} b \times h = \frac{1}{2} FC \times FD$

$= \frac{1}{2}\sqrt{45} \times \sqrt{5}$

$=$ 7,5 square units

$FD = \sqrt{(-4-2)^2 + (-1-(-1))^2}$

$= \sqrt{45}$

$FC = \sqrt{(3-2)^2 + (-3-(-1))^2}$

$= \sqrt{5}$

(c) What is the area of the rhombus?

Area of rhombus = 4 (7,5) = 30 square units

ACTIVITIES/ ASSESSMENT

1. Determine whether line segments AB and CD are parallel, perpendicular or neither, in each of the following cases.

(a) A (–1; –3), B (2 ;1) and C (4; –1), D (7; 3).
(b) A (1; – 3), B (2 ;1) and C (4; –1), D (7; 3)
(c) A (1; – 3), B (2 ;1) and C (–3 ;1), D (1; 0)

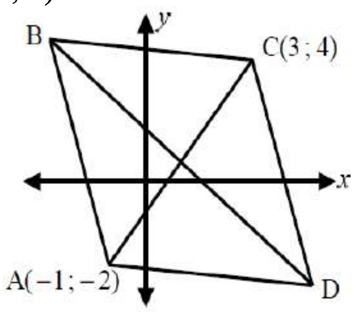
2. (a) Line segment AB is parallel to line segment CD. A (–5; –1) and B (–3; *a*) are points on AB. C (–4; – 3) and D (–1; 3) are points on CD. Calculate the value of *a*.
   (b) Line segment AB is perpendicular to line segment CD. A (–5; 2) and B (*b;* –1) are points on AB. C (–4; – 3) and D (–1; 3) are points on CD. Calculate the value of *b*.

3. Show that points F, R and N are collinear if F (3; 2), R (4; – 2) and N (7; –14).

4. A (3; 4), B (-1; 7), C (*x;* –1) and D (1; 8) are points on the Cartesian plane.
Calculate the value of *x* in each case if:

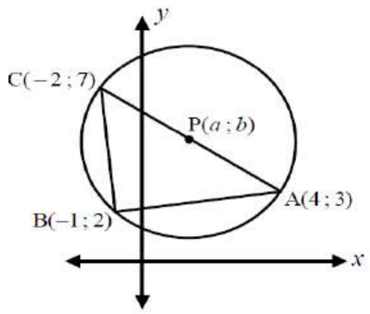
(a) AB||CD
(b) AB ⊥ CD
(c) B, C and D are collinear

5. Rhombus ABCD is drawn with A (–1; – 2) and C (3; 4).



Determine the gradient of BD.

6. In the diagram below, a circle with centre P is drawn. A, B and C are points on the circle, with AC the diameter.



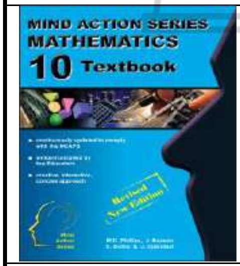
(1) Determine the length of the radius.
(2) Determine the coordinates of P.
(3) Show that AB ⊥ BC.
(4) Hence determine the area of ΔABC.

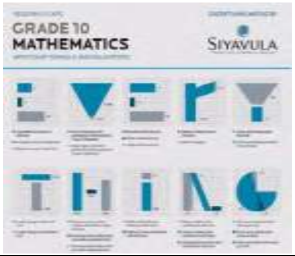
TOPIC: ANALYTICAL GEOMETRY (Lesson 7)		**Weighting**	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Proving Quadrilaterals using Analytical Geometry**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Parallelogram, rhombus, rectangle				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Properties od quadrilaterals, distance formula, midpoint formula, gradient formula

RESOURCES





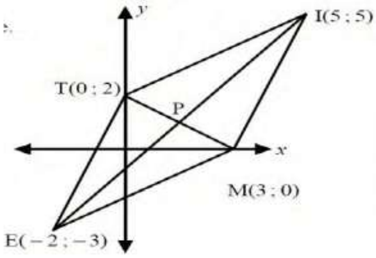
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing properties of quadrilaterals, confusing midpoint and gradient formula

METHODOLOGY

Examples:

1. Quadrilateral TIME is drawn on the Cartesian plane. Prove that it is a parallelogram.



Midpoint of TE $= \left(\frac{-2+5}{2};\frac{-3+5}{2}\right) = \left(\frac{3}{2};1\right)$ AND Midpoint TM$= \left(\frac{3+0}{2};\frac{0+2}{2}\right) = \left(\frac{3}{2};1\right)$

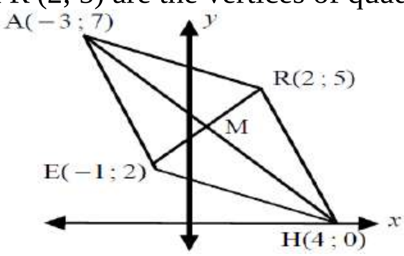
Therefore, P$\left(\frac{3}{2};1\right)$is the midpoint of both diagonals

Quadrilateral TIME is a parallelogram, because its diagonals bisect each other.

NOTE:

Also, can use that: (a) both pairs of opposite sides are parallel
(b) both pairs of opposite sides are equal

2. H (4; 0), E (–1; 2), A (–3; 7) and R (2; 5) are the vertices of quadrilateral HEAR.



Prove that quadrilateral HEAR is a rhombus.

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We are required to prove that the **diagonals bisect at a 90°** angle. i.e. $m_{HA} \times m_{ER} = -1$

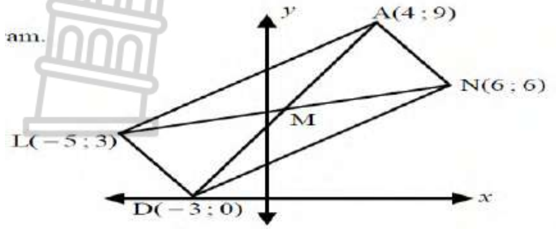
$m_{HA} = \frac{7-0}{-3-4} = \frac{7}{-7} = -1$ AND $m_{ER} = \frac{5-2}{2-(-1\_} = \frac{3}{3} = 1$

$\therefore m_{HA} \times m_{ER} = -1 \times 1 = -1$

$\therefore$ HA⊥ER

Quadrilateral HEAR is a rhombus because its diagonals bisect at 90°.

3. Quadrilateral LAND is shown in the given diagram.



Prove that quadrilateral LAND is a rectangle.

We are required to prove that one interior angle of parallelogram LAND is 90°.

We need to prove that LA ⊥ LD

$m_{LA} = \frac{9-3}{4-(-5)} = \frac{6}{9} = \frac{2}{3}$ AND $m_{LD} = \frac{0-3}{-3-(-5)} = \frac{-3}{2} = -\frac{3}{2}$

$\therefore \ m_{LA} \times m_{LD} = \frac{2}{3} \times -\frac{3}{2} = -1$

$\therefore$ LA⊥LD

Quadrilateral LAND is a rectangle because it is a parallelogram with one interior angle equal to 90°.

ACTIVITIES/ ASSESSMENT

1. Quadrilateral DEFG is formed by the points D (– 5; 3), E (3; 5), F (2;1) and G (–6; –1). Show that DEFG is a parallelogram.

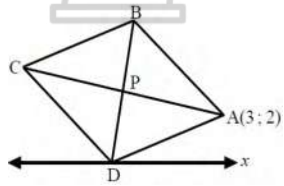
2. Given: A (– 4; 3), B (3; 4), C (8; –1) and D (1; – 2). Show that ABCD is a rhombus. (Hint: First show that it is a parallelogram)

3. Given: A (0; – 3), B (4; 0), C (- 2; 8) and D (– 6; 5). Show that ABCD is a rectangle.

4. M (– 3; 2), N (3; 6), O (9; – 2) and P (3; – 6) are the points of quadrilateral MNOP.
Show that:
   (a) MNOP is a parallelogram.
   (b) MNOP is not a rectangle.

5. A (– 2; 3), B (*x*; *y*), C (1; 4) and D (–1; 2) are the vertices of a quadrilateral. Determine B (*x*; *y*) if ACDB is a parallelogram. Drawing a diagram will be useful.

6. In the following sketch ABCD is a parallelogram. The diagonals AC and BD intersect in P (–1,5 ; 3). D is a point on the *x*-axis. The gradient of BD is 6.



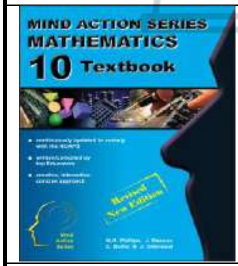
   (a) Determine the coordinates of C and D.
   (b) Use Analytical techniques to determine whether or not ABCD is a rhombus.

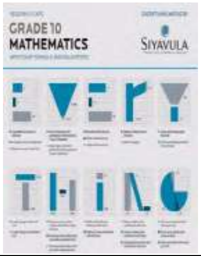
TOPIC: ANALYTICAL GEOMETRY (Lesson 8)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Application of Analytical Geometry: Consolidation Exercise**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Properties of triangles, properties of quadrilaterals

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing properties of quadrilaterals, confusing midpoint and gradient formula

METHODOLOGY

Revision of:
Distance formula, Midpoint formula, Gradient formula, parallel and perpendicular lines, collinear points

ACTIVITIES/ ASSESSMENT

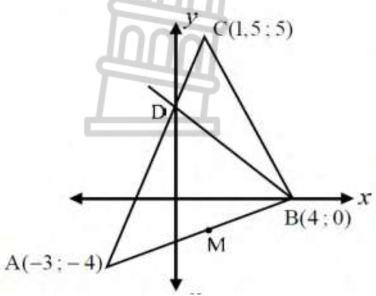
1. A (4; 3) and B (10; 5) are two points on a Cartesian plane.

   (a) Calculate the length of AB. Round off your answer to one decimal place.
   (b) Determine the coordinates of M, the midpoint of AB.
   (c) Determine the coordinates of P if B is the midpoint of AP.

2. ΔABC has coordinates A (–4; 2), B (1; 2) and C (–1; 6)
   (a) Determine the perimeter of Δ ABC
   (b) What kind of triangle is ΔABC?
   (c) Explain why ΔABC cannot be right-angled. Show all workings.

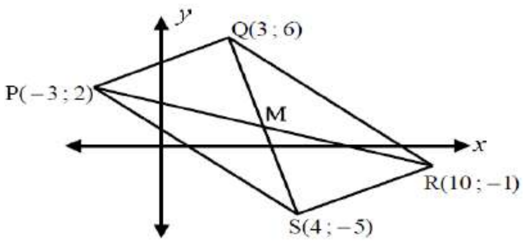
3. Points F (–3; – 4), A (1; *b*) and N (3; 5) are collinear. Determine the value of *b*.

4. Refer to the diagram alongside.

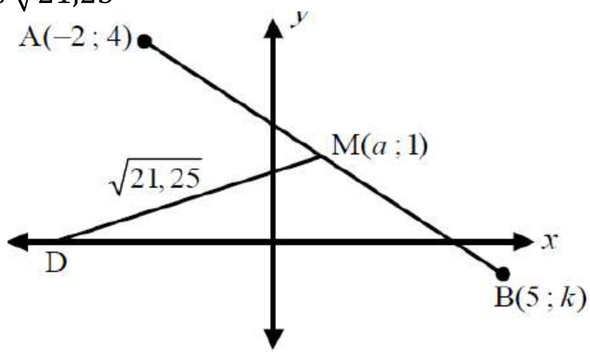


   (a) Determine the gradient of AC.
   (b) Determine the length of BC (one decimal place).
   (c) Determine the coordinates of M, the midpoint of line AB.
   (d) Determine the coordinates of D.
   (e) Show that BD ⊥ AC.

5. Given the diagram below, use analytical methods to show that PQRS is a parallelogram

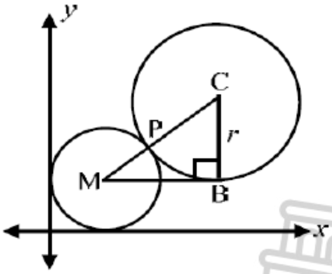


6. On the Cartesian plane below M (a; 1) is the midpoint of line AB with A (–2; 4) and B (5; k) . Point D lies on the x-axis. The length of MD is $\sqrt{21,25}$



(a) Determine the values of a and k.
(b) Determine the coordinates of D. Show all calculations.

7. In the diagram below, two circles with centres M (2; 2) and C respectively are drawn such that they are touching at P. B is a point on the larger circle such that MBC is a right-angled triangle. If the radius of the smaller circle is 2 with MB = 4 units and BC = r.



Determine:
(a) the co-ordinates of B.
(b) the co-ordinates of C in terms of r.
(c) the length of the radius of the larger circle.

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TEST: ANALYTICAL GEOMETRY

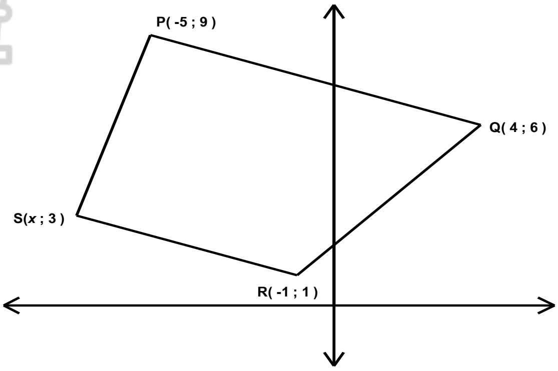
MARKS: 25 **DURATION: 30 Min**

INSTRUCTIONS:

Answer ALL the questions

QUESTION 1 [13 Marks]

Refer to diagram below:



1.2 If it is given that $PQ \perp PS$, find the value of x in the point S (5)

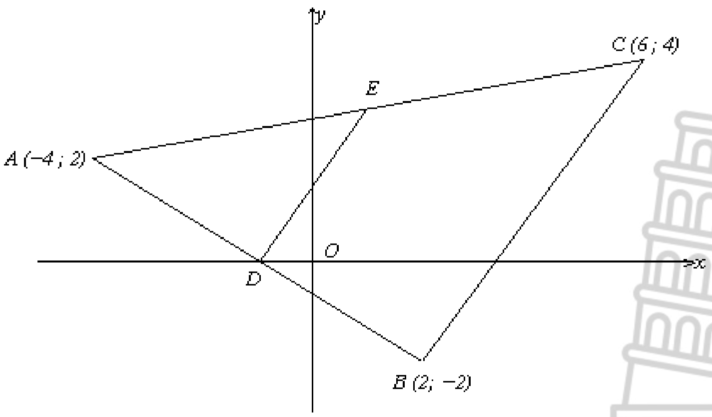
1.3 Join PR

1.3.1 What type of triangle is ΔPRS ? Show all working. (5)

1.3.2 Find the area of ΔPRS (3)

QUESTION 2 [12 Marks]

Sketched below is ΔABC . The co-ordinates of the vertices are as indicated on the sketch.



2.1 Calculate the co-ordinates of the mid-points D and E of AB and AC respectively. (4)

2.2 Show that DE // DC (4)

2.4 The vertices of a rhombus $R(-4;2)$, $H(-4;-3)$, $O(0;0)$, $M(0;5)$.

Prove that: the diagonals RO and HM bisect each other. (4)

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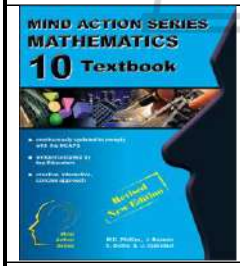
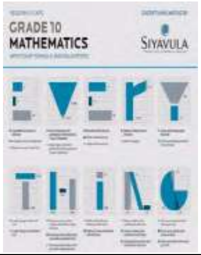
TOPIC: FINANCE AND GROWTH (Lesson 1)		Weighting	10 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Simple Interest**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Interest, Interest rate Simple interest				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Simple interest and compound interest

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Using the value of A as the value for P.

METHODOLOGY

Simple interest is the interest that is calculated on the original amount for the length of time for which it is saved or borrowed. Simple interest is **due at the end of the term**.

Interest can be seen as:

1. **Interest earned** – the reward that a bank or company pay their clients for investing money with them
2. **Interest owed** – the fee or charge that a person pays for borrowing money.

Interest **depends on three factors**:

1.The **amount of money** invested or borrowed, denoted by P

2.The **duration** of the investment or loan

3.The **interest rate per period**, denoted by r.

Interest Rate is the rate at which a person is **rewarded** for money that has been invested charged for money that has been borrowed.

Interest rate is **usually expressed as a percentage**. In Financial Mathematics, we say that

$i = \frac{r}{100}$ (the rate divided by 100 i.e., a decimal)

Example:

Suppose that R2500 is invested at an interest rate of 10% per annum (p.a.) simple interest. Calculate the accumulated amount after 3 years.

After year 1: $A = 2\,500 + 10\%\ of\ 2\,500$

$= 2\,500 + 0{,}10 \times 2\,500 = R2\,750$

NOTE that the interest made in the first year is R250

After year 2: $A = 2\,750 + 0{,}10 \times 2\,500 = R3\,000$

NOTE that the interest made in the second year is also R250 and R500 over the first two years

After year 3: $A = 3\,000 + 0{,}10 \times 2\,500$

$= 3\,250$

NOTE that the interest made in the third year is also R250 and R750 over the three-year period.

The general formula for calculating simple interest is $\boldsymbol{A = P(1 + i.n)}$.

A = accumulated amount/final amount

P = original amount borrowed or invested

$i =$ interest

n = number of years/periods

Examples:

1. Carine deposits R 1000 into a special bank account which pays a simple interest rate of 7% p.a. for 3 years.
How much will be in her account at the end of the investment term?

P = R1 000, $i = \frac{7}{100} = 0{,}07$, n = 3 years and A =?

$$\begin{aligned} \boldsymbol{A} &= \boldsymbol{P(1 + i.n)} \\ &= 1\,000(1 + 0{,}07 \times 3) \\ &= R1\,210 \end{aligned}$$

2. Sarah borrows R 5000 from her neighbour at an agreed simple interest rate of 12,5% p.a. She will pay back the loan in one lump sum at the end of 2 years. How much will she have to pay her neighbour?

P = R5 000, $i = \frac{12{,}5}{100} = 0{,}125$, n = 2 years, A =?

$$\begin{aligned} \boldsymbol{A} &= \boldsymbol{P(1 + i.n)} \\ &= 5\,000(1 + 0{,}125 \times 2) \\ &= R6\,250 \end{aligned}$$

3. Themba deposits R 30 000 into a bank account that pays a simple interest rate of 7,5% p.a. How many years must he invest for to generate R 45 000?

A = R45 000, P = R30 000, $i = \frac{7{,}5}{100} = 0{,}075$, n =?

$$\boldsymbol{A = P(1 + i.n)}$$

$$45\,000 = 30\,000(1 + 0{,}075 \times n)$$

$$\frac{45\,000}{30\,000} = 1 + 0{,}075n$$

$$\frac{3}{2} - 1 = 0{,}075n$$

$$\frac{1}{2} \div 0{,}075 = n$$

$$6{,}6666666667 = n$$

It will take 6 years and 8 months… $(0{,}6666666667 \times 12 \approx 8)$

4. Five years ago, a certain amount of money was invested in a bank. The value of the investment is currently R200 000. Calculate the original amount invested (P) if the interest rate was 5% per annum

simple interest.

A = R200 000 P =? $i = \frac{5}{100} = 0,05$ n = 5

$\boldsymbol{A = P(1 + i.n)}$

$200\ 000 = P(1 + 0,05 \times 5)$

$P = \frac{200\ 000}{(1+0,05\times5)} = R160\ 000$ was invested.

ACTIVITIES/ ASSESSMENT

1. An amount of R 3500 is invested in a savings account which pays simple interest at a rate of 7,5% per annum. Calculate the balance accumulated by the end of 2 years.

2. A bank offers a savings account which pays simple interest at a rate of 6% per annum. If you want to accumulate R 15 000 in 5 years, how much should you invest now?

3. Noma wanted to calculate the number of years she needed to invest R 1000 for in order to accumulate R 2500. She has been offered a simple interest rate of 8,2% p.a. How many years will it take for the money to grow to R 2500?

4. Joseph deposited R 5000 into a savings account on his son's fifth birthday. When his son turned 21, The balance in the account had grown to R 18 000. If simple interest was used, calculate the rate at Which the money was invested.

5. Andrew wants to invest R 3010 at a simple interest rate of 11,9% p.a. How many years will it take for the money to grow to R 14 448? Round **up** your answer to the nearest year.

6. Refilwe wants to purchase a stove costing R12 000. She wants to pay back this amount with interest in two years', time. The interest rate is 24% per annum simple interest.
   (a) Calculate the amount that she will repay in two years' time.
   (b) If she wants to pay the loan off in monthly payments over the two-year period, what will her monthly payments be?

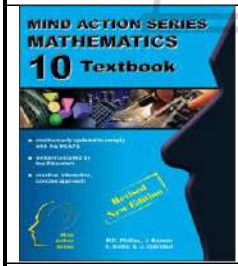
7. Calculate how long it would take for an investment of R9 000 to double if the simple interest rate is 11% per annum.

TOPIC: FINANCE AND GROWTH (Lesson 2)		Weighting	10 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Compound Interest**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Compound interest				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Interest rate,

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Not expressing interest rate as a percentage. Using the value of A as the value for P.

METHODOLOGY

Compound interest allows interest to be earned on interest, i.e., the original investment and the interest earned on it, both earn interest. Compound interest is **advantageous for investing** money but **not for taking out a loan.**

Compounding can be your friend if you are saving money over a long-term It can also be your worst enemy if you are paying back a bank loan over a long-term period.

Compound interest is the interest earned on the principal amount and on its accumulated interest. Compound interest is used with **long-term loans and investments**

Example

1. An amount of R5000 invested at 6% per annum compound interest for a period of 3 years.

After Year 1: $A = 5\,000 \times 0{,}06 \times 5\,000 = R5\,300$

NOTE that the interest made in the first year is R 300

After year 2: $A = 5\,000 \times 0{,}06 \times 5\,300 = R5\,618$

NOTE that the interest made in the second year is R318, which is more than the interest made in the first year.

After year 3: $A = 5\,000 \times 0{,}06 \times 5\,518 = R5\,955{,}08$

NOTE that the interest made in the third year is R337,08 which is more than the interest made in the second year.

The general formula for calculating simple interest is $\boldsymbol{A = P(1 + i)^n}$

Example

2. Blessing invests R18000 for 6 years at 15% per annum compounded annually. Find the future value of his investment after 6 years and the interest he receives.

$A = ?$ $P = R18\ 000$ $i = \frac{15}{100} = 0{,}15$ $n = 6$

$\boldsymbol{A = P(1+i)^n}$

$= 18\ 000(1 + o, 15)^6 = R41\ 635{,}09$

3. Charlie has been given R 5000 for his sixteenth birthday. Rather than spending it, he has decided to invest it so that he can put down a deposit of R 10 000 on a car on his eighteenth birthday. What compound **interest rate** does he need to achieve this growth? Comment on your answer.
   $A = R10\ 000$ $P = R5\ 000$ $i = ?$ $n = 2$

   $\boldsymbol{A = P(1+i)^n}$
   $10\ 000 = 5\ 000(1+i)^2$
   $\frac{10\ 000}{5\ 000} = (1+i)^2$
   $\sqrt{2} = 1 + i$… square root on both sides
   $\sqrt{2} - 1 = i$
   $0{,}4142 = i$
   $\therefore\ r = 0{,}4142 \times 100$… Interest rate is **usually expressed as a percentage**

4. Five years ago, a certain amount of money was invested in a bank. The value of the investment is currently R200 000. Calculate the original amount invested (P) if the interest rate was 5% per annum compound interest.
   $A = R200\ 00$ $P = ?$ $i = \frac{5}{100} = 0{,}05$ $n = 5$

   $\boldsymbol{A = P(1+i)^n}$
   $200\ 000 = P(1 + 0{,}05)^5$
   $200\ 000 = P(1{,}27628)$
   $\frac{200\ 000}{1{,}27628} = P$
   $P = R156\ 705{,}43$

ACTIVITIES/ ASSESSMENT

1. An amount of R 3070 is invested in a savings account which pays a compound interest rate of 11,6% p.a. Calculate the balance accumulated by the end of 6 years. As usual with financial calculations, round your answer to two decimal places, but do not round off until you have reached the solution.

2. Nceba invested R500 000 in the share market. He managed to secure an average compound interest rate of 14% per annum during the first two years.
   (a) Calculate the value of his investment at the end of the two-year period.
   (b) During the next three years, he managed to secure an average compound interest rate of 12% per annum. What was his investment then worth at the end of the next three years?

3. Thobeka wants to invest some money at a compound interest rate of 11,8% p.a. How much money should be invested if she wants to reach a sum of R 30 000 in 2 years' time?

4. Morgan invests R 5000 into an account which pays out a lump sum at the end of 5 years. If he gets R 7500 at the end of the period, what compound interest rate did the bank offer him?

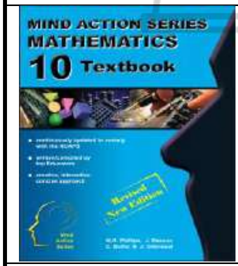
5. Bongani invests R 6110 into an account which pays out a lump sum at the end of 7 years. If he gets R 6904,30 at the end of the period, what compound interest rate did the bank offer him? Give the answer correct to one decimal place.

TOPIC: FINANCE AND GROWTH (Lesson 3)		Weighting	10 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Hire-Purchase**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Hire purchase agreement, deposit, insurance				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Simple interest formula, interest rate, interest

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Not subtracting the deposit from the principal amount.

METHODOLOGY

A **hire purchase agreement** is a financial agreement between the shop and the customer about how the customer will pay for the desired product. The **interest on a hire purchase** loan is always charged at a **simple interest rate** and only charged on the amount owing.

NOTE: when you are asked a hire purchase question, don't forget to always use the simple interest Formula ($\boldsymbol{A = P(1 + i.n)}$)

A hire purchase agreement (HP) is a **short - term loan**. Household appliances and furniture are often bought on HP.

The buyer signs an agreement with the seller to pay a specified amount per month. Most agreements **require that a deposit is paid** initially before the product can be taken by the customer and the balance is paid over a short time period.

The **principal amount of the loan** is therefore the cash price minus the deposit. The **total loan amount** is then divided into monthly payments over the period of the loan.

The buyer will be required to **pay the total interest charged** on the loan even if the loan can be paid off in a shorter time period.

Examples:

1. Vanessa buys a laptop costing R16 000. She pays a 10% deposit and then takes out a twenty four month hire-purchase loan on the balance. The interest rate charged on the loan is 22% per annum simple interest. Calculate her monthly payments and what she will actually pay for the computer.

$$Deposit = 10\%\ of\ 16\,000$$

$$= 0{,}1 \times 16\,000 = R1\,600$$

$$Balance = 16\,000 - 1\,600 = R14\,400 \text{ (P)}$$

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

Interest is charged on R14 400 for a period of 24 months or 2 years (n).

HP loan amount (A):

A = P(1+ in)

A = 14 400(1+ 0, 22 $\times$ 2)

A = R20 736

Monthly payments:

$\frac{R20\ 736}{24} = R864$ (The monthly payment is also called the monthly instalment)

She will pay R20 736 plus the deposit of R1 600 for the laptop, which is a total amount of R22 336.

2. Cassidy wants to buy a TV and decides to buy one on a hire purchase agreement. The TV's cash price is R 5500. She will pay it off over 54 months at an interest rate of 21% p.a. An insurance premium of R 12,50 is added to every monthly payment. How much are her monthly payments?

$P = R5\ 500 \qquad n = \frac{54}{12} = 4{,}5\ years \qquad i = \frac{21}{100} = 0{,}21$

$$\boldsymbol{A = P(1 + i.n)}$$
$$= 5\ 500(1 + 0{,}21 \times 4{,}5)$$
$$= R10\ 697{,}50$$

Monthly payment $= \frac{R10697{,}50}{54} = R198{,}10$

Add the insurance: $198{,}10 + 12{,}50 = R210{,}60$

Cassidy will pay R 210,60 per month for 54 months until her TV is paid off.

3. Mike buys a mobile phone costing R8 000 on HP, pays a deposit of R800, and then pays 36 monthly payments of R344. Calculate the simple interest rate.

$n = \frac{36}{12} = 3 \qquad i = ? \qquad monthly\ payments = R344 \qquad deposit = R800$

$P = Balance = 8\ 000 - 800 = R7\ 200$

$A = amount\ repaid\ over\ 3\ years = 344 \times 36 = R12\ 384$

$$\boldsymbol{A = P(1 + i.n)}$$
$$12\ 384 = 7\ 200(1 + 3.i)$$
$$\frac{12\ 384}{7\ 200} = 1 + 3i$$
$$1{,}72 - 1 = 3i$$
$$0{,}24 = i$$
$$r = 100 \times 0{,}24 = 24\%$$

ACTIVITIES/ ASSESSMENT

1. Patricia wants to buy a furniture suite for R38 000. She decides to take out a hire purchase loan

involving equal monthly payments over five years. The deposit is 20% and the simple interest rate charged per annum is 15%. Calculate:

(a) how much must be paid each month
(b) the amount of interest paid
(c) the actual amount paid for the furniture suite

2. Two stores are offering a fridge and washing machine combo package. Store A offers a monthly payment of R 350 over 24 months. Store B offers a monthly payment of R 175 over 48 months. If both stores offer 7,5% interest, which store should you purchase the fridge and washing machine from if you want to pay the least amount of interest?

3. Richard is planning to buy a new stove on hire purchase. The cash price of the stove is R 6420. He has to pay a 10% deposit and then pay the remaining amount off over 36 months at an interest rate of 8% p.a. An insurance premium of R 11,20 is added to every monthly payment. Calculate Richard's monthly payments.

4. Shaun buys a smartphone on HP which costs R11 799,90. He will have to pay R639 per month for 24 months. No deposit will be required. Calculate the simple interest rate.

5. A hire purchase contract for a sound system requires James to pay a deposit of R2 000 and to then make six monthly payments of R3 375. If the price of the sound system is R20 000, calculate the total simple interest paid and the rate of simple interest.

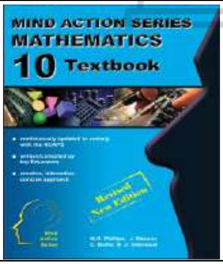
6. Ayanda wants to buy a new car and can afford to pay R4 899 per month. A car dealership offers her a payment plan over 72 months at a simple interest rate of 10,5% per annum. What is the price of the car she can afford to buy?

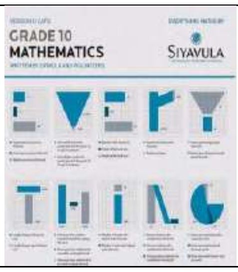
TOPIC: FINANCE AND GROWTH (Lesson 4)		**Weighting**	10 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Inflation**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Inflation, rate of inflation				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Compound interest formula

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Substitute the interest rate wrongly e.g. when the interest rate is 9%, learners substitute 9

METHODOLOGY

There are many factors that influence the **change in price of an item**, one of them is **inflation**.
Inflation is the average increase (steady compounded increase) in the price of goods each year and is **given as a percentage**.
Since the **rate of inflation** increases year on year, it is calculated using the **compound interest formula** ($\boldsymbol{A = P(1+i)^n}$)
The rate of inflation is therefore the percentage of money you'll need more every year to buy the same things you were able to buy the previous year.

Examples:

1. Milk costs R14 for two litres. How much will it cost in 4 years' time if the inflation rate is 9% p.a.?

$P = R14 \qquad n = 4\ years \qquad i = 0{,}09 \qquad A = ?$

$$\boldsymbol{A = P(1+i)^n}$$
$$= 14(1+0{,}09)^4 = R19{,}76$$

∴In 4 years 'time, two litres of milk will cost R19,76.

2. The average salary of a computer programmer in South Africa in 1995 was R4 500. Assuming an annual average rate of inflation of 6,1%, would a salary of R9 000 have the same buying power in 2015?

$P = R4\,500 \qquad i = 0{,}061 \qquad n = 20\ years \qquad A = ?$

$$\boldsymbol{A = P(1+i)^n}$$
$$= 4\,500(1+0{,}061)^{20} = R14\,706{,}87$$

A salary of R9 000 would be way below the buying power of a salary of R14 706,87.

3. A box of chocolates costs R55 today. How much did it cost 3 years ago if the average rate inflation

was 11% p.a.?

$A = R55$ $n = 3\ years$ $i = 0{,}11$ $P = ?$

$$\boldsymbol{A = P(1+i)^n}$$
$$55 = P(1+0{,}11)^3$$
$$\frac{55}{(1+0{,}11)^3} = P$$
$$R40{,}22 = P$$

∴Three years ago, the box of chocolates would have cost R40,22.

ACTIVITIES/ ASSESSMENT

1. The price of a bag of apples is R 12. How much will it cost in 9 years time if the inflation rate is 12% p.a.?

2. A box of biscuits costs R 24 today. How much did it cost 5 years ago if the average rate of inflation was 11% p.a.? Round your answer to 2 decimal places.

2. Arnold paid R2 599,99 for a car sound system in 1997. Assuming an average inflation rate of 7% per annum, what did he pay for a sound system with the equivalent value in 2014?

3. If the average rate of inflation for the past few years was 7,3% p.a. and your water and electricity account is R 1425 on average, what would you expect to pay in 6 years time?

4. The average salary of a domestic worker in South Africa in the year 2000 was R2 000. Assuming an annual average rate of inflation of 5,7%, would a salary of R3 000 have the same buying power in 2015?

5. Forty years ago, John deposited R5 000 in a bank paying him 3% per annum compound interest. The average inflation rate over the forty years was 6%.
   (a) How much money will he have saved after forty years?
   (b) Calculate the buying power of R5 000 after forty years.
   (c) Comment on the value of John's savings after forty years.

6. If salaries double every seven years, what will the rate of inflation be?

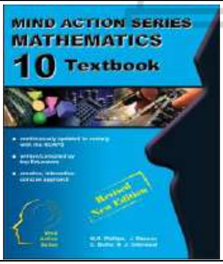
7. Suppose that a cold drink costs R4,50 now but will cost double in eight years' time. What will the average inflation rate be?

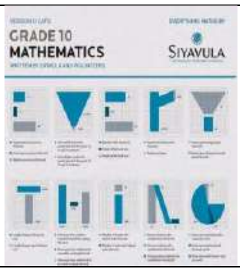
TOPIC: FINANCE AND GROWTH (Lesson 5)		Weighting	10 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Foreign exchange rate**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Exchange rate, currency				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Conversions

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Conversions: multiply instead of dividing or divide instead of multiplying.

METHODOLOGY

There are different money systems in different countries. **Currency** is the term used to describe the particular money system of a country. Different countries have their own currencies.

Here are the currencies of a few countries.

Country	Currency used	Currency symbol
South Africa	Rand	R
United States of America	US dollar	$
United Kingdom	British Pound	£
Several European countries	Euro	€

An **exchange rate** is the relative price of two monies. Exchange rates affect a lot more. The price of oil increases when the South African rand weakens. This is because when the rand is weaker, we can buy less of other currencies with the same amount of money.

A **currency** gets **stronger** when money is **invested in the country**. When we buy products that are made in South Africa, we are investing in South African business and keeping the money in the country. When we buy products imported from other countries, we are investing money in those countries and as a result, the rand will weaken. The more South African products we buy, the greater the demand for them will be and more jobs will become available for South Africans.

Examples:

1. Sean wants to buy the latest DJ equipment, which has been advertised in a US catalogue for $4 000. He wants to order and pay for the equipment online. The current rand/dollar exchange rate is R12,56 to the US dollar. Calculate the cost in rands of the DJ equipment.

$\$1 = R12,56$

$\therefore \$1 \times 4\,000 = R12,56 \times 4\,000$

$\$4\,000 = R50\,240$

The DJ equipment will cost R50 240.

2. Saba wants to travel to see her family in Spain. She has been given R10 000 spending money. How many euros can she buy if the exchange rate is currently € 1 = R10,68

$1€ = R10,68$

$\frac{1€}{R10,68} = R1$

$\frac{1€}{R10,68} \times 10\ 00 = R1 \times 10\ 000$

$936,33€ = R10\ 000$

∴Saba can buy € 963,33 with R10 000.

3. Simone is on a trip to the UK to visit her mom. The current rand/pound exchange rate is R18,50 to the British pound. She has R40 000 to spend in the UK. How many pounds does she have to spend?

£1 = R18,50

$\frac{£1}{18,50} = R1$

$\frac{£1}{18,50} \times 40\ 000 = R1 \times 40\ 000$

$£2\ 162,16 = R40\ 000$

Judy has £2 162,16 to spend in London.

ACTIVITIES/ ASSESSMENT

1. The latest Playstation game costs $645 in New York. What would it cost in South Africa if the rand/dollar exchange rate is R12,25 to the US dollar?

2. A South African teacher works in England for 2 years. H saves £400 every month. How much money in rand would he save in this time if the average exchange rate during the two years is R12,46 to the pound.

Jill is visiting a friend in California for a week. She has R3 000 to spend and will exchange the money for US dollars. How many dollars will she have to spend if the rand/dollar exchange rate is R11,28 to the US dollar?

3. Nthabiseng wants to buy an iPad that costs £ 120, with the exchange rate currently at £ 1 = R 14. She estimates that the exchange rate will drop to R 9 in a month.

(a) How much will the television cost in rands, if he buys it now?
(b) How much will he save if the exchange rate drops to R 9?
(c) How much will he lose if the exchange rate moves to R 19?

4. A certain watch costs €350 in Germany or £245 in England. Which price is better for the South African buyer if the exchange rates are R11,24 to the euro and R18,06 to the pound?

5. Mathe is saving up to go visit her friend in Germany. She estimates the total cost of her trip to be R50 000. The exchange rate is currently € 1 = R 13,22. Her friend decides to help Mathe out by giving her € 1000. How much (in rand) does Mathe now need to save up?

6. Brenda won a competition where she can fly to three international destinations free of charge with spending money. The destinations she chose were Germany, Japan and England. For Germany, she was allocated €9 000. For Japan, she was allocated ¥30 000. For England, she was allocated £2 500.

Use the exchange rates in the table on the next page to calculate the total amount she had been allocated in rands.

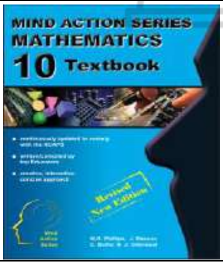
Exchange Rates		
Germany (€)	Japan (¥)	England (£)
R10,46	R0,29	R17,12

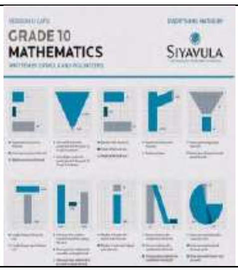
TOPIC: FINANCE AND GROWTH (Lesson 6)		Weighting	10 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Population Growth**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Population growth rate, exponential growth				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Compound interest formula

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Manipulation of the formula when calculating $P, i \text{ or } n$

METHODOLOGY

Population growth is the increase in the number of individuals in a population.

The **population growth rate** is the rate at which the number of individuals in a population increases in a given time period as a fraction of the initial population. It measures how the size of the population is changing over time.

In the first few years, a **population grows exponentially** in the same way that money grows through compounding. For example, family trees increase exponentially as every person born has the ability to start another family.
The formula for calculating exponential growth of a population is similar to the compound interest formula: $\boldsymbol{A = P(1+i)^n}$

Examples:

1. If the current population of Johannesburg is 3 888 180, and the average rate of population growth in South Africa is 2,1% p.a., what can city planners expect the population of Johannesburg to be in 10 years?

$P = 3\ 888\ 180 \qquad i = 0{,}021 \qquad n = 10\ years \qquad A = ?$

$$\boldsymbol{A = P(1+i)^n}$$
$$A = 3\ 888\ 180(1+0{,}021)^{10} = 4\ 786\ 343$$

City planners can expect Johannesburg's population to be 4 786 343 in ten years time.

2. The mid-year population in South Africa in 2014 was 54 002 200. Calculate the size of the population in five years' time if the average population growth rate is 1,56%.

$P = 54\ 002\ 200 \qquad n = 5\ years \qquad i = 0{,}0156 \qquad A = ?$

$$\boldsymbol{A = P(1+i)^n}$$
$$\boldsymbol{A} = 54\ 002\ 200(1+0{,}0156)^5 = 58\ 347\ 857{,}54$$

The population size will be approximately 58 347 857 people in five years' time.

ACTIVITIES/ ASSESSMENT

1. The current population of Durban is 3 879 090 and the average rate of population growth in South Africa is 1,1% p.a.
   What can city planners expect the population of Durban to be in 6 years' time? Round your answer To the nearest integer.

2. The current population of Polokwane is 3 878 970 and the average rate of population growth in South Africa is 0,7% p.a.
   What can city planners expect the population of Polokwane to be in 12 years' time? Round your answer to the nearest integer.

3. The number of people in the USA as at June 2014 was estimated at 318 857 056. If the average population growth rate was 1,42%, calculate the estimated population size of the USA in June 2017.

4. The number of black rhinos in Africa during 2012 was estimated at 5 487. If the average population growth rate of black rhinos is 4,9% per annum, calculate how many rhinos were there in Africa in 2007.

5. The world population during the year 2015 was estimated to be 7 320 248 940. If the average annual exponential growth rate was 1,14%, what was the population in the year 2000?

6. A small town in Ohio, USA is experiencing a huge increase in births. If the average growth rate of the population is 16% p.a., how many babies will be born to the 1600 residents in the next 2 years?

7. A family of 6 mice can multiply into a family of 60 mice in 3 months.
   (a) Calculate the estimated monthly growth rate for the mice population.
   (b) If not controlled, how many mice will there possibly be in one year if the initial population is 6 mice?

8. You are studying the population growth of a species of frog. In a pond constructed for the frogs, you start off with 50 frogs and notice that after 10 months, the number of frogs has increased to 61. What is the average monthly growth rate?

TEST: FINANCE AND GROWTH

MARKS: 25 **DURATION: 30 Min**

INSTRUCTIONS:

Answer ALL the questions

QUESTION 1 [14 Marks]

1.1 Determine through calculation which of the following investments will be more profitable:

1.1.1 R7 000 at 10% p.a compound interest for 5 years. (4)

1.1.2 R7 000 at 12% p.a simple interest for 5 years. (3)

1.2 Simphiwe has just received a gift of R 500 from her grandmother and she promises another R 500 in a year's time. She decides to invest this money in an account which pays 8,5% p.a. compounded annually. How much additional money will she have to save to add to her investment after two years if she withdraws all the money for her trip. (7)

QUESTION 2 [11 Marks]

2.1 Inflation is set at 5,5% for the next three years. A small scooter currently costs R7999,00. I want to buy this scooter on my birthday in 3 years' time.

2.1.1 What will it cost in three years' time? (3)

2.1.2 The dealer offers me a hire purchase on the scooter on the following terms: 15% deposit and the balance payable over 36 months. The current interest rate on a hire purchase deal is 23% per annum. Calculate my monthly payments. (5)

2.2 You want to import a personal computer from Japan at a total cost of 96180 Yen. The equivalent computer cost R8 500 in South Africa. Will you import or buy locally? Show all calculations to justify your answer. (3)

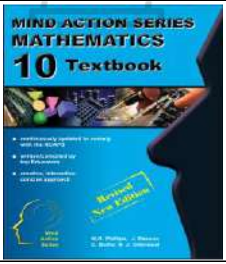
TOPIC: DATA HANDLING (Lesson 1)	Weighting	15 ± 3	Grade	10

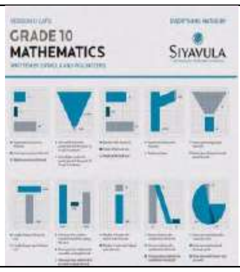
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Representing Ungrouped Data**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Data, discreet data, ungrouped data		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Bar graph, frequency table, stem and leaf plot

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Not separating bars when sketching a bar graph

METHODOLOGY

Data refers to the pieces of information that have been observed and recorded, from an experiment or a survey.
There are two types of data, quantitative data and qualitative data.
Quantitative data are data that can be written as numbers and can be discrete or continuous.

Discrete quantitative data can be represented by integers and usually occur when we count things, for example, the number of learners in a class, or the number of SMS messages sent in one day.

Continuous quantitative data can be represented by real numbers, for example, the height or mass of a person, the distance travelled by a car, or the duration of a phone call.

Qualitative data are data that cannot be written as numbers.
In this chapter we are going to deal with **quantitative data.**

Ungrouped data is a set of individual values or observations. The data is **discrete** since the values are distinct values.

Ungrouped data can be **represented in different ways**. The **three most common ways** are: frequency tables, bar graphs and stem-and-leaf plots.

Example:

The American company Deloitte made a prediction that in 2014, the increase in the number of smartphones being used world-wide would be the greatest for people over 55 years. In a survey conducted in a shopping mall during 2014, different people were approached and asked what type of phone they were using. The following table shows the ages of thirty people between the ages of 15 and 60 using smartphones.

16	17	17	17	17	18	18	25	25	27	28	28	28	28	28
28	32	34	34	34	46	46	48	54	55	56	56	56	56	56

(a) Draw a frequency table for this ungrouped, discrete data.

(b) Represent the data in a stem-and-leaf plot.
(c) Draw a frequency bar graph for this data.

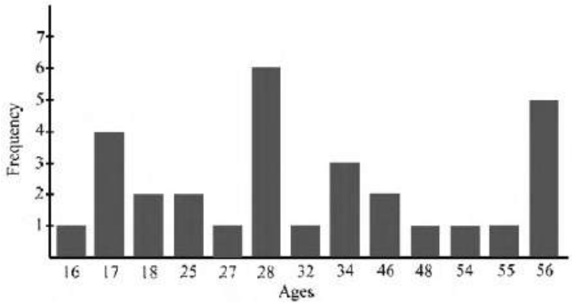
(a) A **frequency table** shows the different observations and how many times they occur.

Age	Tally	Frequency
16	|	1
17	||||	4
18	||	2
25	||	2
27	|	1
28	~~||||~~ |	6
32	|	1
34	|||	3
46	||	2
48	|	1
54	|	1
55	|	1
56	~~||||~~	5
		$n = 30$

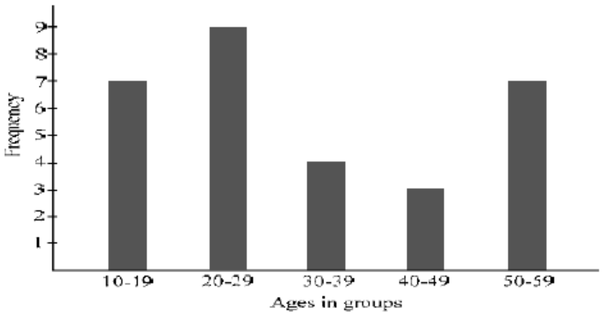
(b) In a stem-and-leaf plot, the tens digit is used as a "stem" and the units as a "leaf". Ensure that the "leaves" are equally spaced.

1	6	7	7	7	7	8	8		
2	5	5	7	8	8	8	8	8	8
3	2	4	4	4					
4	6	6	8						
5	4	5	6	6	6	6	6		

(c) The individual ages are on the horizontal axis and the frequencies on the vertical axis.



It might be useful to place the ages into age groups rather than individual ages.



(d) Although the bar graphs indicate that there were a high number of over-55's using smartphones, there is not enough data to prove this prediction. The sample was too small. It would be far more feasible to increase the number of people surveyed to well in the millions to get a better idea. Also, one would have needed to compare sales in previous years.

ACTIVITIES/ ASSESSMENT

1. The manager of a computer store assessed the quality of service at his store based on the feedback from thirty customers. The rating scale was as follows:
   Extremely poor (0) Poor (1) Average (2) Good (3) Very good (4) Outstanding (5)

   The scores of the thirty customers are provided below.

0	3	4	3	4	1	4	2	4	5	4	2	4	4	2
3	4	4	5	3	4	2	1	1	3	4	2	5	2	4

   (a) Draw a frequency table for this ungrouped, discreet data.
   (b) Draw a frequency bar graph for this data.

2. Donating blood can help to save the life of someone and even yours should you be in an accident one day and require blood. The ages of forty people who donated blood on a particular day are provided below.

18	42	17	35	19	35	20	39	35	26
41	53	42	50	57	43	35	24	55	54
64	22	35	17	65	18	47	19	48	54
27	35	63	24	66	34	39	27	66	35

   (a) Draw a stem-and-leaf plot for this data.
   (b) Draw a frequency bar graph for this data. Group the data into appropriate age groups (10-19 year olds, 20-29 year olds, etc).
   (c) Which age group donated the most blood?

3. The number of air conditioners sold by fifty sales representatives in the year 2015 are recorded below:

25	22	19	27	27	19	23	21	14	12
13	13	9	4	21	18	30	31	28	21
20	3	7	14	14	9	7	27	21	39
18	22	27	30	23	14	14	14	8	1
3	14	4	18	5	24	20	8	10	8

   (a) Draw a stem-and-leaf plot for this data.
   (b) Draw a frequency bar graph for this data.
   (c) How many agents sold twenty or more air conditioners?
   (d) What percentage of the agents sold less than 20 air conditioners?

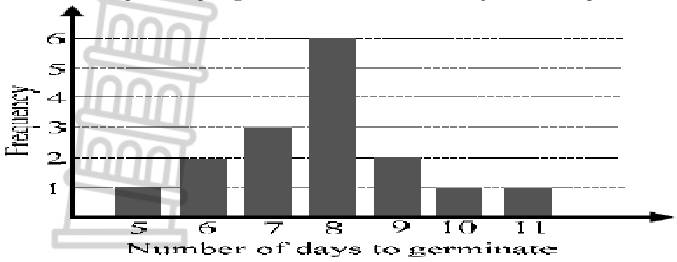
4. An app is a type of software that allows you to perform specific tasks on your laptop or mobile phone. The word "app" is an abbreviation for "application". By the year 2016, there will be almost 310 billion downloaded apps generating close to $74 billion in revenue. With nearly two million apps already developed, competition for someone wanting to develop a successful app is fierce. Having an app in the top 25 will require at least 33 downloads per hour. Two companies recorded the number of downloads per hour for one of their new apps over a period of 15 hours.

Company A	23	30	32	11	33	13	42	41	14	22	33	22	22	33	44
Company B	10	20	11	23	44	24	34	35	43	33	29	32	33	43	43

   (a) Using today's exchange rates, convert $74 billion to rands, pounds and euros.
   (b) Draw a back-to-back stem-and-leaf diagram for the two companies.
   (c) What percentage of Company A's downloads were more than 33 per hour?
   (d) What percentage of Company B's downloads were less than 33 per hour?

(e) In your opinion, which company has the better chance of success with their new app? State a reason for your answer.

5. A number of seeds of a particular variety of flower were sown. All germinated, but not all at the same time.
The following bar graph shows how many seeds germinated after various number of days.



(a) How many seeds were sown?
(b) After how many days did the first seed germinate?
(c) What percentage of seeds germinated within the first 8 days?

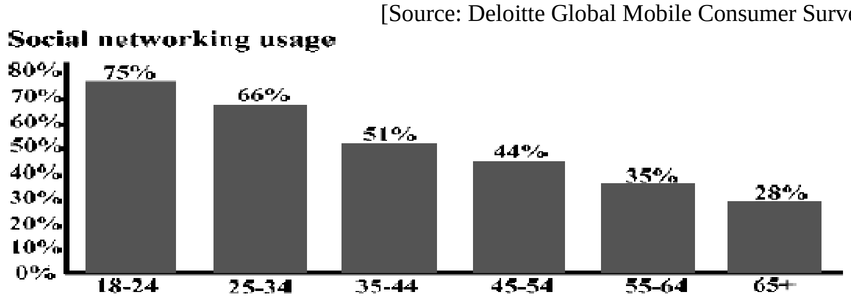
6. The following back-to-back stem-and-leaf diagram shows the average number of hours spent per week on social networking websites by learners from two different classes.

8 7 7 7 5 5 1 1 0	**0**	**1 2 3 3 4 6 6 7 8 9**
1 1 0 0 0	**1**	**0 0 0 1**
9	**2**	**1**
	3	**0**

(a) How many learners are there in Class A?
(b) How many learners are there in Class B?
(c) How many learners in Class A spend exactly one hour per week on a social networking website?
(d) How many learners in Class B spend more than five hours per week on a social networking website?
(e) Which class spends more time, in total, on a social networking website?

7. The following bar graph shows the weekly social networking usage on smartphones by different age groups in developed countries.

[Source: Deloitte Global Mobile Consumer Survey, Developed countries, May-July 2014]



(a) In which age group does 44% use smartphones for social networking?
(b) What percentage of the over 65-year olds do not use smartphones for social networking?
(c) If there are 200 000 people in the 25-34 year age group, how many will be using smartphones for social networking?
(d) If there are 375 000 people in the 18-24 age group using smartphones for social networking, how many people in this age group are not using smartphones for social networking?

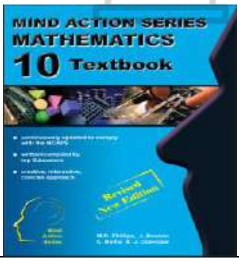
TOPIC: DATA HANDLING (Lesson 2)	Weighting	15 ± 3	Grade	10

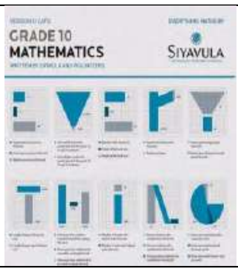
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Measure of Central Tendency**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Central tendency, Mean, Median, Mode		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Decimals, ascending order, even number, odd number, stem and leaf plot

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Miscounting values on the data set, determining the median in a stem and leaf plot

METHODOLOGY

The **Central tendency** is the methods of finding out the central value of a given data. It identifies a single value as a representative of an entire distribution.
A **measure of central tendency** is a single value used to represent the centre or middle of a set of data values.
As such, measures of central tendency are sometimes called **measures of central location.** They are also classed as **summary statistics**.
The **mean** (often called the average) is most likely the measure of central tendency that you are most familiar with, but there are others, such as the **median** and the **mode**.

The **Mean** or average of the data, is the **sum** of the **data values** divided by the **total number of data** values (*n*). We refer to the data values as the *x*-values. The symbol for the mean is $\bar{x}$.

$$\bar{x} = \frac{\text{sum of all } -\text{values}}{\text{total number of } -\text{values}} = \frac{\sum x}{n}$$

Example:

Calculate the **mean** of the following data set:
9 14 9 14 8 8 9 8 9 9. Round your answer to 1 decimal place.

$$\text{Mean} = \frac{9+14+9+14+8+8+9+8+9+9}{10} = \frac{97}{10} = 9{,}7$$

The **Median** is the **middle-most number** when the data values are written in ascending order.

Example:
1. Determine the median for the following data set: 2 9 5 12 10

First arrange the values in ascending order:
2 5 9 10 12
There is an **odd** number of values and therefore the median will be part of the data set.
9 is in the middle of the data set arranged in ascending order. The median is therefore **9** and it divides the data into two equal halves.

2 5 9 10 12
lower half upper half

2. Determine the median for the following data set: 3 9 10 12 15 18

The data is arranged in ascending order.
There is an **even** number of values and therefore the median will be **not** part of the data set.
Since there are an even number of values in this data set (6) the median lies between the 3rd and 4th place. This means that the median is the **average** between the 3rd and 4th data values.

Median$\frac{10+12}{2} = \frac{22}{2} = 11$ The median is **11** and it divides the data into two equal halves.

3 9 10 | 12 15 18
↑
11

It is not a value in the data set.

The median may be a better indicator of the most typical value if a data set has an **outlier**, which is an extreme value that differs greatly from the other values.
An **outlier** is a value that lies outside (much smaller or larger) than most of the other values in the data set, an extreme value compared to the other values.

3. Consider the following data set: 5 12 7 36 8 9 7

(a) Determine the mean and the median.
(b) Which value is an outlier?
(c) Which measure of central tendency is more representative of the data set?

$$\bar{x} = \frac{5+12+7+36+8+9+7}{7} = \frac{84}{7} = 12$$

Arrange the data in ascending order: 5 7 7 **8** 9 12 36

There is an odd number of values and therefore the median will be part of the data set.
The median is **8**

(c) 36 is an extreme value compared to the other values and is an outlier.

(d) The outlier has inflated the mean. It is therefore not a good value to use as a measure of central tendency. The median is not affected by the outlier and is therefore a much better measure than the mean.

The **Mode** of a data set is the value that **occurs most frequently**.
If two numbers tie for the most frequent occurrence, the data set has two modes and is called **bimodal**.
If three numbers tie for the most frequent occurrence, the data set has three modes and is called **trimodal**.
If a data set has an **outlier**, the mode, like the median, may also be a better indicator of the most typical value.

4. Determine the mode for the following data:
4 11 3 15 11 13 25 17 2 11

The mode is the value that occurs the most. In this data set the mode is 11.

5. Determine the mode in the following data:

1 2 2 2 2 3 5 6 6 7 8 8 8 8 9 10 12

There are two modes: 2 and 8 because these values occur four times each in the data set.
The data set is therefore **bimodal**.

6. The ages of thirty people using smartphones between the ages of 15 and 60 were recorded.
The stem-and-leaf diagram of this data is provided below:

1	6	7	7	7	7	8	8		
2	5	5	7	8	8	8	8	8	8
3	2	4	4	4					
4	6	6	8						
5	4	5	6	6	6	6	6		

(a) Calculate the mean for this data.

$$\bar{x} = \frac{16+3(17)+2(18)+2(25)+27+6(28)+32+3(34)+2(46)+48+54+55+5(56)}{30} = 34{,}27$$

(b) Determine the median.

$$\text{Median} = \frac{28+28}{2} = 28$$

(c) Determine the mode.

The most frequently-occurring value is 28.
Mode is therefore 28

ACTIVITIES/ ASSESSMENT

1. Find the mean, median and mode of the following sets of data values:

(a) 4 13 5 7 9 6 5
(b) 13 2 11 2 10 4 5 10 8 10

2. The monthly salaries of nine employees in a small business are:

R15 400 R16 800 R86 300 R13 200 R16 900 R11 900 R17 100 R16 200 R16 900

(a) Calculate the mean, median and mode for this data.
(b) Which measure of central tendency is a sensible measure of the "typical" monthly salary of an employee in this business? Explain.

3. A teacher records the following results for an examination out of 100:

98	63	79	76	58	71	86	78	91	87
89	41	19	88	41	99	97	83	78	90

Which measure of central tendency best describes these results?

4. A dairy farmer has 32 cows for sale. The weights of these cows in kilograms are recorded below. The total weight of the cows is 5 060 kg.

80 82 83 83 84 85 85 86

86	87	87	88	88	89	90	92
92	93	94	95	97	153	153	154
155	321	371	376	377	381	382	391

(a) Calculate the mean and the median.

(b) The farmer describes the cows to a buyer and states that the average weight is over 158 kg. Which measure of central tendency did the farmer use to describe the cows and does this measure describe the cows fairly?

5. The following stem-and-leaf diagram represents the number of air conditioners sold by fifty sales representatives. The total number of air conditioners is 843.

0	1 3 3 4 4 5 7 7 8 8 8 9 9
1	0 2 3 3 4 4 4 4 4 4 4 8 8 8 9 9
2	0 0 1 1 1 1 2 2 3 3 4 5 7 7 7 7 8
3	0 0 1 9

(a) Calculate the mean, median and mode for this data.

(b) Comment on the usefulness of these measures of central tendency.

6. Research was done on families having six children. The table below shows the number of families in the study with the indicated numbers of boys.

Number of boys	0	1	2	3	4	5	6
Frequency	1	24	45	54	50	19	7

(a) Calculate the mean, median and mode for this data.

(b) Comment on the usefulness of these measures of central tendency.

7. (a) The mean of 3; 4; 8; 9; x is 7. Determine x.

(b) The median of five consecutive natural numbers is 12. What is the mean?

(c) The numbers 4; 6; 8; 9; x are arranged from smallest to biggest. If the mean and the median are equal, determine x.

(d) The mean of five numbers is 27. The numbers are in the ratio 1: 2: 3: 4: 5. Determine the five numbers.

(e) Write down three possible sets of five numbers such that the median is 4, the mean is 5 and the mode is 3.

(f) The mean of six numbers is 44 and the mean of five of these numbers is 46. What is the sixth number?

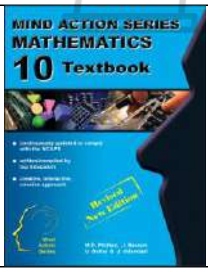
TOPIC: DATA HANDLING (Lesson 3)	Weighting	15 ± 3	Grade	10

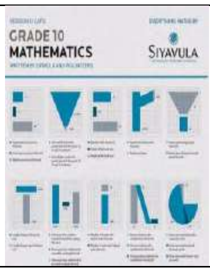
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Measure of dispersion**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Range, quartiles, interquartile, semi-interquartile, percentiles		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Arranging data in ascending order, difference, median

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Not dividing by 2 for semi-interquartile range, difference between quartiles and percentiles

METHODOLOGY

Measure of dispersion is a measure of spread of data about the mean. It is a nonnegative real number. Measures of dispersion help us to determine how data is spread around the mean or median.
There are five commonly used measures, range, quartiles, percentiles, interquartile range and semi-interquartile range.

Range is the difference between the maximum value and the minimum value and is the simplest measure of spread.
The **disadvantage** of the range is that a great deal of information is ignored when calculating the range, since only the largest and smallest data values are considered. The range is greatly influenced by the presence of just one unusually large or small value (outlier).

Example:
1. A group of **15** learners count the number of sweets they each have. This is the data they collect:

4 11 14 7 14 5 8 7 12 12 5 13 10 6 7

4 5 5 6 7 7 7 8 10 11 12 12 13 14 **14**… ascending order

Minimum value is 4 and maximum value is 14.
Range = Maximum value – minimum value
= 14 – 4 = 10.

Quartiles divide the data set into four equal parts.
Consider an ordered set of numbers with the MEDIAN.
The **lower quartile (Q_1)** is the median of the numbers that occur before MEADIAN.
The **upper quartile (Q_3)** is the median of the numbers that occur after the MEADIAN.

The following steps may be used to determine the lower and upper quartiles:

1. Write the data in ascending order and find the median (Q_2/M)
2. Find the median of the lower half of the data (exclude the median of the entire set). This is the lower quartile (Q_1).

3. Find the median of the upper half of the data (exclude the median of the entire set). This is the upper quartile (Q_3).

Example:

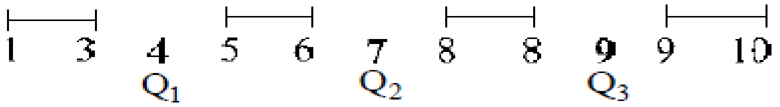
2. Calculate the quartiles for the following sets of data:

(a) 1 3 4 5 6 7 8 8 9 9 10... (11 values)

The **median** is 7 (the 6th value). It is a value in the data set.

The lower half of the data set is: 1 3 4 5 6
The lower quartile is the median of the lower half (consisting of 5 values).
The **lower quartile** is 4 (the 3rd value). It is a value in the lower half.

The upper half of the data set is: 8 8 9 9 10
The upper quartile is the median of the upper half (consisting of 5 values).
The **upper quartile** is 9 (the 3rd value). It is a value in the upper half.

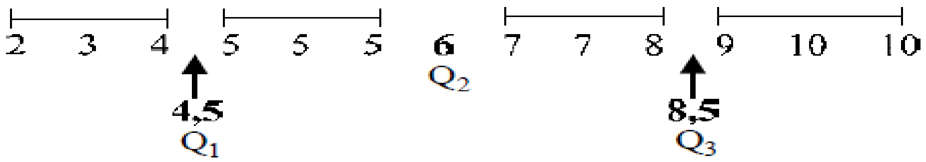


(b) 2 3 4 5 5 5 **6** 7 7 8 9 10 10... (13 values)

The **median** is 6 (the 7th value). It is a value in the data set.

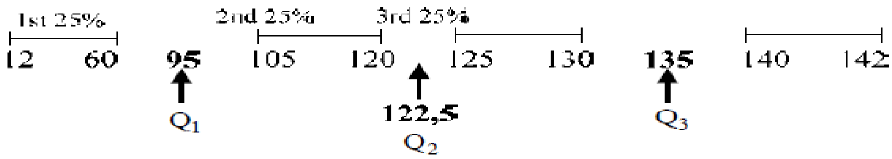
$Q_1 = \frac{4+5}{2} = 4{,}5$ and is not the value in the lower half.

$Q_3 = \frac{8+9}{2} = 8{,}5$ and is not the value in the upper half.



(c) 95 120 125 140 105 12 142 60 135 130... (10 values)

12 60 95 105 120 125 130 135 140 142... ascending order
Median$= \frac{120+125}{2} = 122{,}5\ldots$ between 120 and 125
Lower half: 12 60 **95** 105 120 AND upper half: 125 130 **135** 140 142
$Q_1 = 95$ $Q_3 = 135$



The **interquartile range** is the difference between the upper and lower quartiles. $\boldsymbol{IQR = Q_3 - Q_1}$
It spans 50% of a data set and eliminates the influence of outliers because, in effect, the highest and lowest quarters are removed. It is a **good measure of the spread** of the data either side of the median.

The **semi-interquartile range** is the half the difference between the upper and lower quartiles **($Semi - IQR = \frac{1}{2}(Q_3 - Q_1)$**) and is also **not affected by extreme scores**. It is a **good measure of spread for skewed** distributions.

Example:

3. Consider the following data: 1 2 2 2 3 3 4 4 **4** 6 7 8 8 9 10 10 30

(a) Determine the range, interquartile range and semi-interquartile range.

Range = 30 – 1 = 29

$Q_1 = \frac{2+3}{2} = 2,5$ AND $Q_3 = \frac{8+9}{2} = 8,5$

Interquartile range = Q_3 - Q_1

= 8,5 – 2,5 = 6

$Semi - IQR = \frac{1}{2}(Q_3 - Q_1)$

$= \frac{1}{2}(6) = 3$

(b) Which measure of dispersion is more suitable for this data, the range or interquartile range?

The range is too inflated due to the influence of the maximum value 30. The interquartile range is a more realistic measure of dispersion.

Percentiles are more appropriate with data sets containing a large number of values. Note that the **lower quartile** (Q_1) is also the **25th percentile**, the **median** is the **50th percentile** and the **upper quartile** (Q_3) is the **75th percentile**.

We will use the following method to determine percentiles:

1. Arrange the data in ascending order.
2. Compute index i, the position of the pth percentile using the formula: $100\, i = p\,(n)$
3. If i is not an integer, round up. The pth percentile is the value in the i th position.
4. If i is an integer, the pth percentile is the average of the values in positions i and i +1.

Example:

4. A Maths professor at a university posted a list of marks, without names, on the notice board outside his office. The students were informed as to the percentile they were in. There are 45 students in his class and the marks are as follows:

66	86	65	78	32	52	69	85	87	28	90	98	73	64	56
58	78	65	50	36	67	55	72	57	64	70	92	95	33	32
24	42	54	55	54	68	65	88	80	84	68	61	75	76	82

(a) Jaco scored in the 70th percentile. What is his mark?

Arrange the marks in ascending order:

24	28	32	32	33	36	42	50	52	54	54	55	55	56	57
58	61	64	64	65	65	65	**66**	67	68	68	69	70	72	73
75	76	78	78	80	82	84	85	86	87	88	90	92	95	98

$i = \frac{70}{100} \times 45 = 31,5$

All we now do is round this number up to 32 and the **70th percentile is the 32nd mark** which is 76%. Jaco therefore obtained 76% and scored better than 70% of all students.

(b) Michael scored in the 20th percentile. What is his mark?

$i = \frac{20}{100} \times 45 = 9$

The 20th percentile is the average between the 9th and 10th mark: $\frac{52+54}{2} = 53$

There is no score of 53 in the data and therefore Michael will have obtained 54% and will have scored better than 20% of all students.

(c) Dimpho scored in the 50th percentile. What is her mark?

$i = \frac{50}{100} \times 45 = 22{,}5$ round off this number to 23

The 50th percentile is the 23rd mark which is 66. Dimpho obtained 66% and scored better than 50% of all students.

ACTIVITIES/ ASSESSMENT

1. Determine the range, quartiles, interquartile range and semi-interquartile range for the following sets of data.
   (a) 1 1 2 2 4 4 6 6 8 8 10
   (b) 1 2 6 8 8 8 8 8 8 10 10 10
   (c) 2 5 7 9 12 13 15
   (d) 2 2 2 4 4 6 6 8 8 10 10 10

2. Six data values are represented as follows: $3x$; $x + 4$; $2x + 2$; $5x$; $4x + 1$; $6x + 2$

   (a) Calculate the value of x if the mean is 12.
   (b) Determine the interquartile range.

3. In a data set made up of five numbers, the mean, median, mode and range are all equal. Determine this data set.

4. The table below contains the mean, median and range of the Mathematics final exam for a large group of students.

Mean	Median	Range
56	51	86

The Mathematics teacher added 3 marks to each of the students' marks. Write down the mean, median and range for the new set of Mathematics.

5. Five data values are represented as follows: $2x$; $x + 1$; $x + 2$; $x - 3$; $2x - 2$

   (a) Determine the value of x if the mean of the data set is 15.
   (b) Calculate the quartiles.

6. Tobacco use is a leading cause of death in the United States. Nicotine found in tobacco is rapidly metabolised in the liver to a substance called cotinine. The levels of cotinine in the body are measured in nanograms per millilitre (ng/ml). A nanogram is one billionth of a gram. Consider the following cotinine levels of 50 smokers:

5	6	6	8	22	40	43	44	48	86
88	103	113	122	123	130	131	149	165	168
174	174	198	208	210	223	224	227	233	245
249	250	253	265	267	277	280	284	286	289
290	313	313	314	350	360	401	460	476	490

(a) Calculate the 25th, 50th and 75th percentiles.
(b) Calculate the 30th percentile.
(c) Calculate the 65th percentile.
(d) Calculate the 80th percentile.

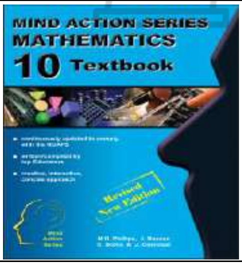
TOPIC: DATA HANDLING (Lesson 4)	Weighting 15 ± 3	Grade	10

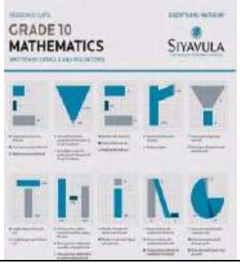
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Five-Number Summary and Box-and-Whisker Diagram**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Five-number summary, box-and-whisker		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Minimum value, maximum value, median, quartiles

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

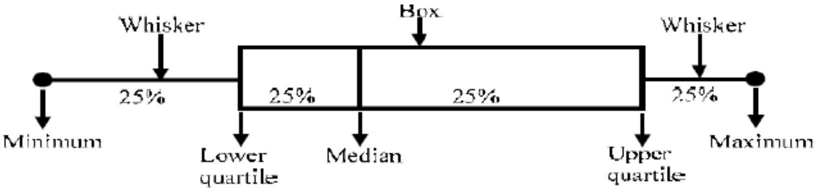
Not using a scale when drawing a box-and-whisker diagram, failing to comment on box-and-whisker

METHODOLOGY

A **five-number summary** is a method for summarising a distribution of data. The five numbers are the minimum, the first quartile, the median, the third quartile, and the maximum. This makes the summary a useful measure of spread.

An efficient method for visually displaying a five-number summary referred to as the **box-and-whisker plot**.

A **box-and-whisker plot** is a visually effective way of viewing a clear summary of one or more sets of data. It is particularly useful for quickly comparing different data sets.



Examples:

1. Given the data: 1 2 2 2 3 3 4 4 **4** 6 7 8 8 9 10 10 10

   Determine the five-number summary and draw a box-and-whisker plot for the data.

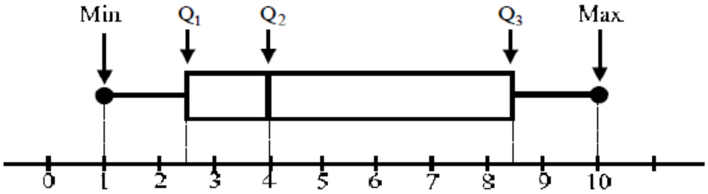
   Minimum value = 1

   Maximum value = 10

   Median = 8

   $Q_1 = \frac{2+3}{2} = 2,5$

   $Q_3 = \frac{8+9}{2} = 8,5$



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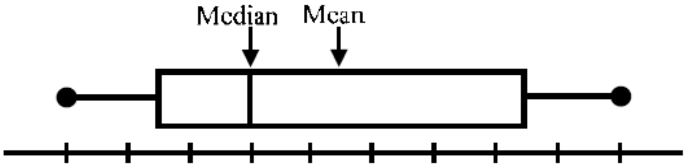
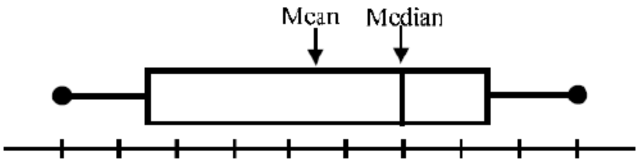
Box-and-whisker plots provide useful information about how data is distributed. Take note of the following three **types of distributions** that are frequently encountered in Statistics.

1. Box-and-whisker plots where the data is **positively skewed.**



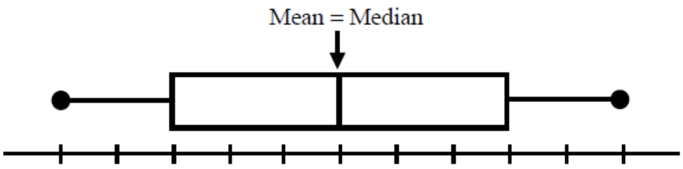
Data is positively skewed if there are some very high values which cause the mean to be greater than the median.

2. Box-and-whisker plots where the data is **negatively skewed.**



Data is negatively skewed if there are some very low values which cause the mean to be less than the median.

3. Box-and-whisker plots where the data is **symmetrical.**



Data is symmetrical if the mean and median are equal.

ACTIVITIES/ ASSESSMENT

1. Consider the following data: 1 1 2 2 4 4 6 6 8 8 10

   (a) Determine the five-number summary.
   (b) Draw a box-and-whisker plot.
   (c) Calculate the mean.
   (d) How is the data distributed? Explain.

2. Consider the following data: 1 2 6 8 8 8 8 8 8 10 10 10

   (a) Determine the five-number summary.
   (b) Draw a box-and-whisker plot.
   (c) Calculate the mean.
   (d) How is the data distributed? Explain.

3. Consider the following data: 2 5 7 9 12 13 15

   (a) Determine the five-number summary.
   (b) Draw a box-and-whisker plot.
   (c) Calculate the mean.
   (d) How is the data distributed? Explain.

4. Consider the following data: 2 2 2 4 4 6 6 8 8 10 10 10

(a) Determine the five-number summary.
(b) Draw a box-and-whisker plot.
(c) Calculate the mean.
(d) How is the data distributed? Explain.

5. A teacher records the following results for an examination out of 100:

98	63	79	76	58	71	86	78	91	87
89	41	19	88	41	99	97	83	78	90

Draw a box-and-whisker plot for this data.

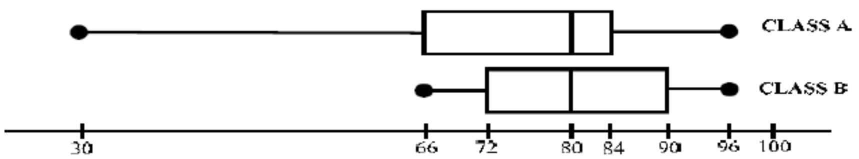
6. The prize money allocated to the first ten positions in the 2015 Comrades Marathon were as follows:

Position 1	Position 2	Position 3	Position 4	Position 5
R350 000	R175 000	R130 000	R65 000	R50 000
Position 6	**Position 7**	**Position 8**	**Position 9**	**Position 10**
R30 000	R25 000	R22 000	R18 500	R16 500

[http://www.comrades.com/marathoncentre/faq/2-race-info/322-medals-and-prizes]

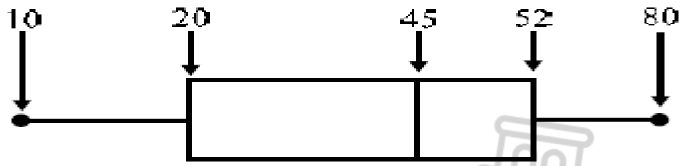
(a) Draw a box-and-whisker plot.
(b) How is the data distributed in the box-and-whisker plot? Explain.

7. The box and-whisker plots below summarise the final test scores for two Mathematics classes from the same grade.



(a) Describe the features in the scores that are the same for both classes.
(b) The Head of Department considers the median of each class and reports that there is no significant difference in the performance between them. Is this conclusion valid? Support your answer with reasons.

8. Consider the following box-and-whisker plot:



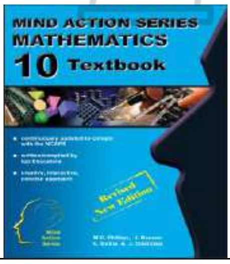
The data set contains a total of nine numbers. The second and third number of the data set are the same. The seventh and eighth numbers are different. The mean for the data set is 40. Write down a possible list of nine numbers which will result in the above box-and- whisker plot.

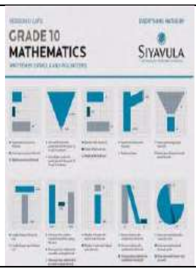
TOPIC: DATA HANDLING (Lesson 5)	Weighting	15 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Grouped Data**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Continuous data, class intervals, class boundary, estimated mean, modal class, median class			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Mean, median, mode

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Estimated mean: add frequency and divide by number of frequencies

METHODOLOGY

In a **grouped data** the data is **continuous** and take on an infinite number of real values within a range. For example, data containing the heights of people is continuous since heights are not restricted to integer values.

Large sets of continuous data are grouped into **class intervals**. A class interval has a given range and consists of class boundaries. The **upper class boundary** is the maximum possible value which could be in the class interval and the **lower class boundary** is the minimum value which could be in the class interval.

In a **continuous grouped data,** the following concepts are discussed:

1. The estimated mean,
2. The median class interval
3. The modal class interval.

Example:

Medical science has always recognised human growth and height as an important measure of the health and wellness of individuals. Research into the average height of people in different countries revealed that the tallest race of humans is the Nilotic peoples of Sudan having an average height of 1,83 m. The tallest man currently living is Sultan Kösen from Turkey who measures 2,51 m. The average heights (ranging from 150-185 cm) of people in 135 countries have been grouped into class intervals.

[Source: https://en.wikipedia.org/wiki/Template:Average_height_around_the_world]

Class intervals (average heights in cm)	Frequency (number of countries)
$150 \leq x < 155$	12
$155 \leq x < 160$	15
$160 \leq x < 165$	19
$165 \leq x < 170$	25
$170 \leq x < 175$	33
$175 \leq x < 180$	22
$180 \leq x < 185$	9

(a) Calculate an estimated value for the mean.

The data is continuous and the actual average heights per class interval are not known. It is therefore, impossible to calculate the actual mean for this data. We can, however, calculate an estimated value for this mean using the following method.

1. Calculate the **midpoint** of each class interval, which represents the average of all heights in that class interval by simply calculating the average of the lower and upper class boundaries.
2. Then multiply the frequencies with the corresponding midpoint.
3. Add up the results, divide by the total frequencies and calculate the **estimated mean** for the data.

Class intervals	Frequency	Midpoint	Freq×Midpt
$150 \le x < 155$	12	$\frac{150+155}{2} = 152,5$	$12 \times 152,5 = 1\ 830$
$155 \le x < 160$	15	$\frac{155+160}{2} = 157,5$	$15 \times 157,5 = 2\ 362,5$
$160 \le x < 165$	19	$\frac{160+165}{2} = 162,5$	$19 \times 162,5 = 3\ 087,5$
$165 \le x < 170$	25	$\frac{165+170}{2} = 167,5$	$25 \times 167,5 = 4\ 187,5$
$170 \le x < 175$	33	$\frac{170+175}{2} = 172,5$	$33 \times 172,5 = 5\ 692,5$
$175 \le x < 180$	22	$\frac{175+180}{2} = 177,5$	$22 \times 177,5 = 3\ 905$
$180 \le x < 185$	9	$\frac{180+185}{2} = 182,5$	$9 \times 182,5 = 1\ 642,5$
Totals	135		22 707,5

Estimated Mean$= \frac{22\ 707,5}{135} = 168,2037037\ cm$

(b) What is the modal class?

Since $170 \le x < 175$ contains the highest frequency of heights, this class interval will be the **modal class**.

(c) In which class interval does the median lie?

It is not possible to determine the actual median. There are 135 values and therefore the position of the median is $\frac{135}{2} = 67,5$... round this number up to 68

The 68th value lies in the class interval $165 \le x < 170$. This class interval is called the **median class**.

ACTIVITIES/ ASSESSMENT

1. A stopwatch was used to find the times that it took a group of athletes to run 100 m.

Class intervals (Time in seconds)	Frequency (number of athletes)
$10 \le x < 15$	6
$15 \le x < 20$	16
$20 \le x < 25$	21
$25 \le x < 30$	8

(a) Calculate the estimated mean.
(b) What is the modal class?
(c) In which class interval does the median lie?

2. In a research survey, a gym measured the weights (in kg) of a number of members.

Class intervals (weights in kg)	Frequency (number of members)
$30 \le x < 35$	11
$35 \le x < 40$	13
$40 \le x < 45$	15
$45 \le x < 50$	17
$50 \le x < 55$	19
$55 \le x < 60$	26
$60 \le x \le 65$	36

(a) Calculate the estimated mean.
(b) What is the modal class?
(c) In which class interval does the median lie?

3. The raw data below shows an athlete's different times in seconds in the 400 m.

43,0	43,1	45,3	44,8	44,9	46,3	44,8	46,3	46,1
45,4	44,7	43,1	44,9	45,3	45,2	45,5	45,6	45,0
45,1	46,2	45,9	43,2	43,3	43,8	43,9	43,7	45,3
45,7	44,7	46,2	45,7	44,9	45,0	45,5	46,0	46,9

(a) Draw a stem-and-leaf diagram for this data.
(b) Complete the following table:

Class interval	Frequency
$43,0 \le x < 44,0$	
$44,0 \le x < 45,0$	
$45,0 \le x < 46,0$	
$46,0 \le x < 47,0$	

(c) Calculate the actual mean, median and mode for this data.
(d) Calculate the range and interquartile range.
(e) Draw a box-and whisker plot for the data.

4. The number of litres of diesel purchased by 30 truck drivers at a petrol station is presented below (litres are rounded off to the nearest whole number).

82	64	55	50	49	44	52	59	68	74
71	78	88	98	96	77	75	54	57	56
64	66	80	84	88	72	71	65	68	97

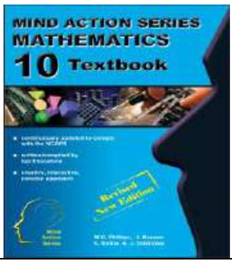
(a) Draw a stem-and-leaf display for this data.
(b) Organise the data into class intervals of your choice.
(c) Calculate the actual mean and median for this data.
(d) Calculate the interquartile range.

TOPIC: DATA HANDLING (Lesson 6)	Weighting	15 ± 3	Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Consolidation and Extension exercise**			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Measure of central tendency, measure of dispersion, five-number summary, ungrouped and grouped data.

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Not separating bars when sketching a bar graph.
Miscounting values on the data set, determining the median in a stem and leaf plot
Arranging data in ascending order, difference, median
Not using a scale when drawing a box-and-whisker diagram, failing to comment on box-and-whisker
Estimated mean: add frequency and divide by number of frequencies

METHODOLOGY

ACTIVITIES/ ASSESSMENT

1. A small company pays their employees hourly rates. The hourly rates of eight employees are as follows: R36 R270 R90 R72 R54 R90 R54 R54

(a) Calculate the mean, median, mode and range for this data.
(b) Which measure would the employer use to claim that the staff were well-paid?
(c) Which measure would the employee use to claim that the staff were badly-paid?

2. A gardener buys ten packets of seeds from two different companies. Each pack contains 25 seeds. The gardener records the number of plants which grow from each pack.

Company A:	25	25	10	25	11	25	25	25	13	25
Company B:	22	23	20	21	23	23	22	20	22	23

(a) Which company does the mode suggest is best?
(b) Which company does the mean suggest is best?

3. A fisherman records the number of fish caught over a number of fishing trips:

3 0 0 5 0 0 13 0 2 0 0 4 16 0 2 0 1

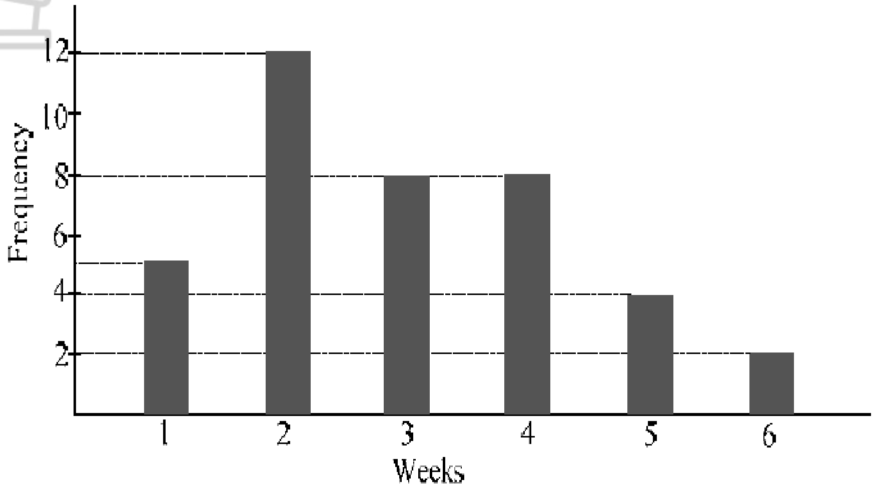
Why does the fisherman object to the mode and median of the number of fishes caught?

4. A school has to select one learner to take part in a Mathematics Quiz. Sandy and Paul took part in six trial quizzes and the following are their scores:

Sandy:	29	25	22	28	25	27
Paul:	34	20	17	33	35	19

By using the mean and range, which learner qualifies to represent the school in your opinion?

5. The following frequency bar graph shows the number of laptops sold at a computer store per week for six weeks.



Calculate the mean number of laptops sold per week.

6. A traffic officer is trying to work out the mean number of parking tickets she has issued per day. She produced the table below, but some of the data has been erased.

Tickets per day	Frequency	No of tickets × frequency
0	1	
1		1
2	10	
3	7	
4		20
5	2	
6		
Totals	26	72

Complete the table for her and then calculate the mean, median and mode.

7. In a certain school, 60 learners wrote examinations in Maths and Science. Information for each subject is provided below.

Maths	**Science**
Minimum 30	Minimum 30
Maximum 85	Range 55
Median 45	Upper quartile 70
Lower quartile 40	Interquartile range 30
Upper quartile 50	Median 55

(a) Draw a box-and-whisker plot for both subjects.

(b) The teacher argues that the number of learners who scored between 30 and 45 in Maths is smaller than the number who scored between 30 and 55 in Science.
Does she have a valid argument? Explain.

8. The following table represents the percentage of monthly income spent on petrol and car expenses by fifty people.

Percentage	Frequency
$12 \leq p < 18$	8
$18 \leq p < 24$	20
$24 \leq p < 30$	12
$30 \leq p < 36$	8
$36 \leq p \leq 42$	2

Calculate the estimated mean, modal class and the interval containing the median.

9. The mean height of a class of 30 learners is 164 cm. A new boy of height 148 cm joins the class. Calculate the mean height of the class now.

10 After five matches, the mean number of goals scored by Orlando Pirates per match is 1,8. If Pirates scores three goals in their next match, what is the mean then?

11. The mean weight of 27 learners in a class is 62 kg. The mean weight of a second class of 30 learners is 59 kg. Calculate the mean weight of all the learners.

12. The mean monthly salary of the eight people who work for a small company is R18 000. When an extra employee is hired, this mean drops to R17 000. How much does the new employee earn?

13. Consider the following set of data values: x; $2x$ -1; $2x$; $2x + 2$; $3x$ -1
The inter-quartile range is 6. Determine the value of x.

TEST: DATA HANDLING

MARKS: 25 **DURATION: 30 Min**

INSTRUCTIONS

Answer ALL the questions

QUESTION 1 [11 Marks]

1.1 At a recent interschool athletics event, the following distances were recorded for all the participants in the girls' long jump:

3,1m 3,2m 5,0m 3,6m 4,1m 2,9m
3,2m 4,3m 4,9m 3,9m 2,8m 4,6m.

Calculate the:

1.1.1 Mean (3)

1.1.2 Median and (2)

1.1.3 Mode (1)

1.2 The marks in a class test of 15 girls in Mrs Mbusi's Science class s given below. The test is out of 50 marks:

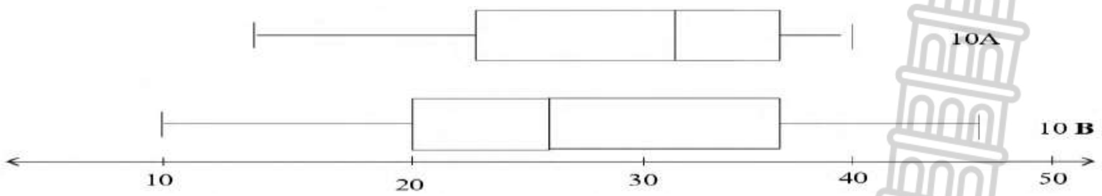
43	42	31	32	22	13	44	38	25
50	9	15	25	35	41			

1.2.1 Calculate interquartile range for the class (3)

1.2.2 Draw a box and whisker plot to represent the marks (2)

QUESTION 2 [14 Marks]

2.1 Consider the box-and-whisker diagrams below representing the marks of two Grade 10 classes in a test out of 50



2.1.1 Write down the five-number summary for 10B (3)

2.1.2 What is the inter quartile rage for 10A (1)

2.1.3 What is the median mark for 10B (1)

2.1.4 What percentage of marks in 10A lies between the highest mark and the median (1)

2.1.5 Which class in your opinion did the best? Give a reason (2)

2.2 A music shop records the sales of CDs over a two-year period. The monthly sales figures are given below:

204	255	310	283	288	393	282	359	364	172	158
407	458	299	109	307	272	283	285	367	479	280
382	258	111								

2.2.1 Complete the frequency table below (3)

Class interval	Tally marks	Frequency	Class midpoint	Frequency x class midpoint
$100 \leq x < 200$	////	4	150	600
				2450
				3150
	Total		Total	

Use the frequency table to determine:

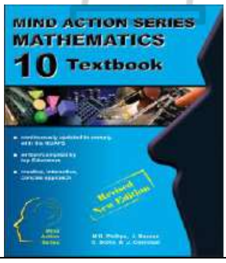
2.2.2 the mean monthly sales to the nearest whole number (2)

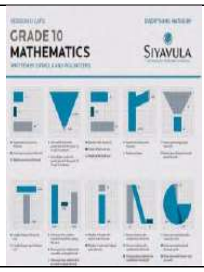
2.2.3 the modal class (1)

TOPIC: MEASUREMENT (Lesson 1)	Weighting	Grade	10
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Revision of the perimeter and the area of 2D Shapes**		
RELATED CONCEPTS/ TERMS/VOCABULARY	2D shape, perimeter, area,		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

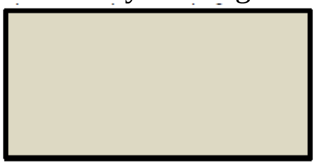
Differentiating between the area and the perimeter

METHODOLOGY

A **2D shape** is a flat shape. We will deal with the following 2D shapes -circle, square, rectangle, triangle.

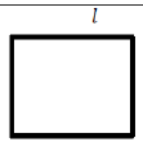
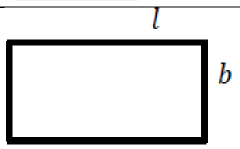
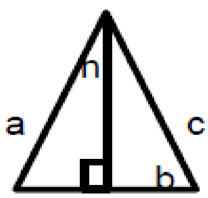
Perimeter of a shape is the distance around the shape.

Area of a diagram is the size of the flat surface enclosed by the diagram.



The black line around the rectangle is the perimeter, while the grey inside the rectangle is the area.

You need to know all the formulas for the area and perimeter of different shapes by heart.
The summary of area and perimeter of different shapes is shown below:

NAME/SHAPE	DIAGRAM	PERIMETER	AREA
Square		$P = 4l$	$A = l^2$
Rectangle		$P = 2(l + b)$	$A = l \times b$
Triangle		$P = a + b + c$	$A = \frac{1}{2} b \times h$

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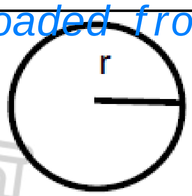
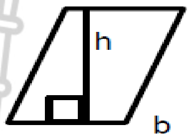
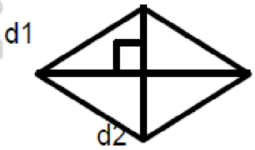
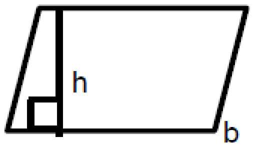
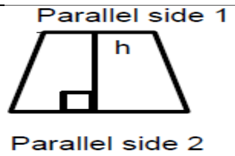
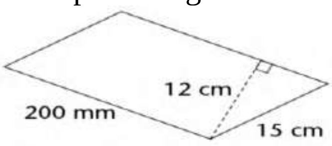
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

Circle		$P = 2\pi r$ also known as circumference.	$A = \pi r^2$
NAME/SHAPE	DIAGRAM	PERIMETER	AREA
Rhombus		P = sum of all four sides P = 4b	$A = b \times h$
Kite		P = sum of all four sides	$A = \frac{1}{2}(diagonal_1 \times diagonal_2)$
Parallelogram		P = sum of all four sides	$A = b \times h$
Trapezium		P = sum of all four sides	$A = \frac{1}{2}(sum\ of\ \parallel sides) \times h$

Examples:

1. Calculate the area parallelograms:



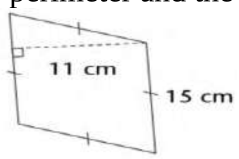
$A = b \times h = 20 \times 12 = 240cm^2$

$h^2 = 5^2 - 3^2$ Pythagoras Theorem

$h = 4cm$

$Area = b \times h = 11 \times 4 = 44cm^2$

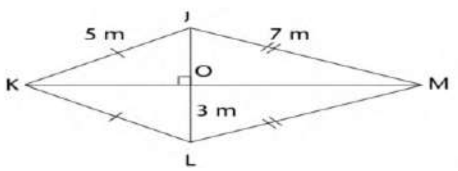
2. Determine the perimeter and the area of rhombus below.



$P = 15 \times 4 = 60cm$

$A = 15 \times 11 = 165cm^2$

3. Calculate the area of kite JKLM correct to 2 decimal places.



$A = \frac{1}{2}(diagonal_1 \times diagonal_2)$

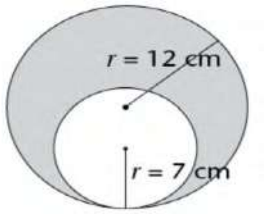
$MO^2 = 7^2 - 3^2$ Pythagoras Theorem

$MO = \sqrt{40} = 6,32$

$KO^2 = 5^2 - 3^2$ Pythagoras Theorem

$KO = 4m$ Therefore,diagonals KM = 10,32m and JL = 6m

$A = \frac{1}{2}(6 \times 10,32) = 30,96m^2$



4. Calculate the area of the shaded part correct to TWO decimal places.

$A_{shaded\ part} = A_{outsice} - A_{inside\ circle}$

$= \pi.12^2 - \pi.7^2 = 95\pi = 298,45cm^2$

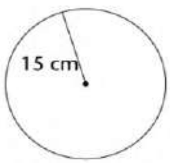
ACTIVITIES/ ASSESSMENT

1. Write down the formulae for the following:

Perimeter of a square	
Perimeter of a rectangle	
Area of a squsreArea of a rectangle	
Area of a triangle	
Area of a rhombus	
Area of a parallelogram	
Area of trapezium	
Circumference of a circle	
Area of a circle	

2. Determine the area of the following

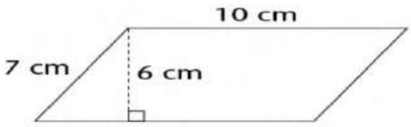
(a)



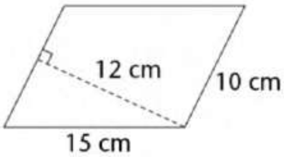
(b)

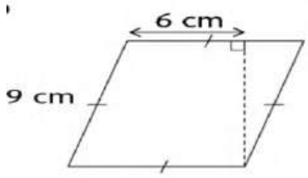
3. Determine the area and the perimeter of the parallelograms below

(a)

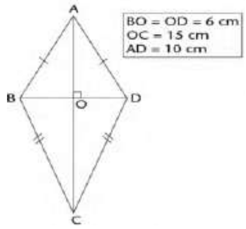


(b)



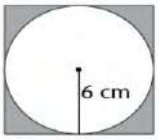


4. Calculate the area of the rhombus.



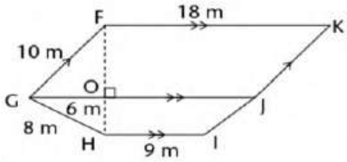
5. use the information in the box to calculate the area.

6. Calculate the perimeter of the square and the area of the shaded parts of the square.

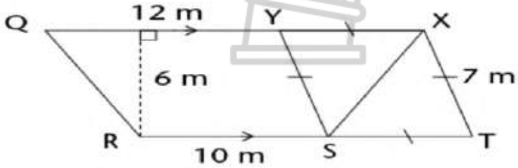


7. Calculate the areas of the following 2D shapes. Round off your answers to two decimal places where necessary

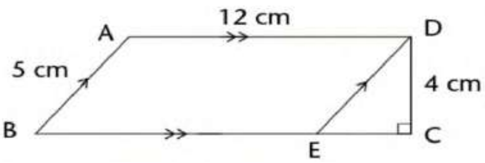
(a)



c)



(b)



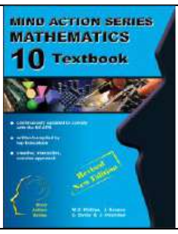
TOPIC: MEASUREMENT (Lesson 2)	Weighting		Grade	10

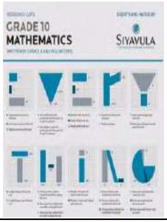
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Revision of the volume and surface areas of right-prisms and cylinders**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Surface area, volume		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Conversions

RESOURCES



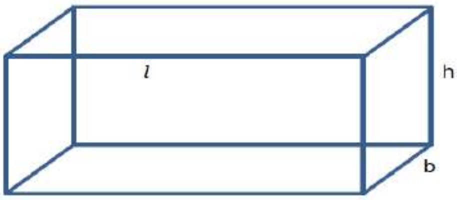


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Mixing formulars, failing to differentiate between surface area and volume, units conversions

METHODOLOGY

Surface area is the area of the surface (or outside faces) of the prism and is measured in **square units**. Given the rectangular Box with length (*l*), breadth (*b*) and height (*h*):



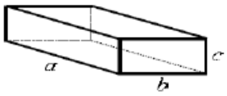
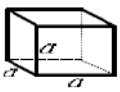
The rectangular box has **6 sides** (faces) and opposite sides are **identical** to each other. Each side has a particular area. For example, the **top side** has an area of $\boldsymbol{l \times b}$, while the **side area** is $\boldsymbol{b \times h}$. The area in the **front** of the box is $\boldsymbol{l \times h}$. Remember that we have 2 of each type of face so the formula for the Surface area of the rectangular prism is:

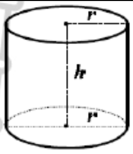
$$\boldsymbol{SA = 2(l \times b) + 2(b \times h) + 2(l \times h)}$$

Volume is the amount of space a 3D shape takes up or occupies and is measured in **cubic units**.

$$Volume = area\ of\ the\ base \times height$$

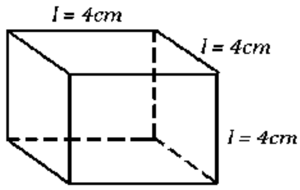
Therefore, volume of a rectangular prism is: $\boldsymbol{V = (l \times b) \times h}$

NAME/SHAPE	DIAGRAM	SURFACE AREA (SA)	VOLUME (V)
Rectangular Prism		Sum of the areas of the six rectangles: $\boldsymbol{SA = 2(ab) + 2(bc) + 2(ac)}$	Area of the base multiplied by the height. $\boldsymbol{V = (a \times b) \times h}$
Cube		Sum of the areas of the six squares: $\boldsymbol{SA = 6(a \times a) = 6a^2}$	Area of a chosen base multiplied by the height. $\boldsymbol{V = (a \times a) \times a = a^3}$

Triangular Prism		Sum of the areas of two triangles and three rectangles. There are three different sized rectangles that make up the sides of this triangular prism. $\boldsymbol{SA = 2\left(\frac{1}{2}bh\right) + db + ad + cd}$	Area of a chosen base multiplied by the height. $\boldsymbol{V = (\frac{1}{2}bh) \times d}$
Cylinder		Sum of the areas of two circles and a curved surface. If the cylinder is closed: $\boldsymbol{SA = 2\pi r^2 + 2\pi rh}$ If open on top (or bottom): $SA = \pi r^2 + 2\pi rh$ If open on top and bottom: $SA = 2\pi rh$	Area of a chosen base multiplied by the height. $\boldsymbol{V = \pi r^2 \times h}$

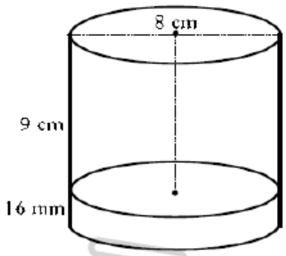
Examples:

1. Calculate the surface area and the volume of the diagram below in metres (m):



10mm = 1cm
100cm = 1m Therefore, $4cm \div 100cm = 0{,}04m$
1000m = 1km
All sides are equal, therefore, $SA = 6(0{,}04m \times 0{,}04m) = 0{,}0096m^2$
$V = 0{,}04m \times 0{,}04m \times 0{,}04m = 0{,}000064m^3$

2. A cylindrical drinking glass is made up of a solid glass base and a top curved part made of glass and which is hollow and open on top.



Calculate:

(a) the total volume of the drinking glass

The **volume** is the amount of space occupied by the prism whereas the **capacity** is the amount of substance that the prism can hold.

Convert 16mm to cm: $16mm \div 10 = 1{,}6cm$

$V = \pi r^2 \times h$

$= \pi . 4^2 \times (9 + 1{,}6) = 532{,}81cm^3$

(b) the capacity of the drinking glass in *l*

Capacity (amount of liquid that the glass can contain)

$= \pi . 4^2 \times 9 = 452{,}39cm^2$

(c) the internal surface area of the glass

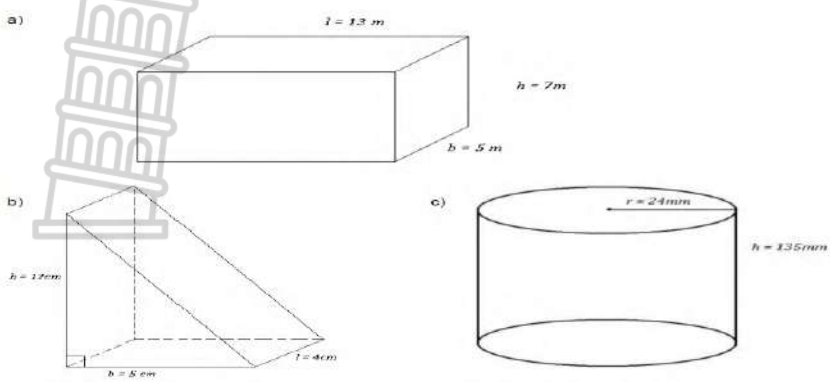
Internal surface area (open on top and therefore excludes the top circle)

$= \pi . 4^2 + 2\pi . 4.9 = 276{,}46cm^2$

ACTIVITIES/ ASSESSMENT

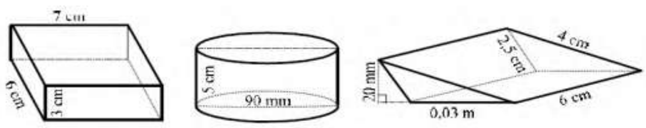
In this exercise, answers must be rounded off to two decimal places where appropriate.

1. Find the volume and surface area of the following prisms:



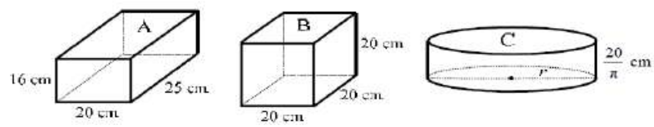
2. (a) The surface area of a rectangular prism is 136 cm^2. If the length is 80 mm and the width is 4 cm, calculate the height of the prism.
   (b) If the volume of a triangular prism is 1 400 cm^3 and the height is 20 cm, then calculate the area of the base.
   (c) The surface area of a cube is 384 cm^2. Calculate the length of a side.
   (d) The surface area of a closed cylinder is (120□) cm^2. If the height is 7 cm, calculate the radius.

3. Consider the following three closed hollow prisms:



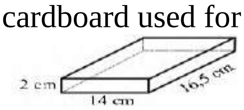
(a) Calculate the volume (cm^3) of each of the three prisms.
(b) Calculate the surface area (cm^2) of each of the three prisms.
(c) If the cylinder and rectangular prism are open on top, calculate the surface area of these prisms.

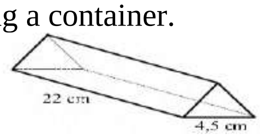
4. Three solid wooden objects, a rectangular prism (A), a cube (B) and a cylinder (C), are shown below.



(a) Show that A and B have the same volume.
(b) Calculate the value of *r* for which C has the same volume as A and B.
(c) Assuming that the radius of C is the same as the value calculated in (b), which prism will have the largest surface area?

5. A company manufacturing solid chocolate bars has two new packaging containers that will have same amount of chocolate inside. The one container is a triangular prism with an equilateral triangle as a base. The other is a rectangular prism. The company wants to cut down on the cost of the cardboard used for making a container.



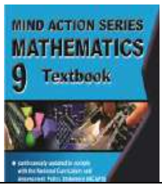


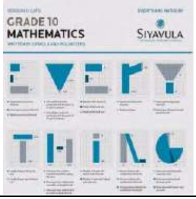
Determine which container will be the least expensive to wrap.

TOPIC: MEASUREMENT (Lesson 3)	Weighting		Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**The effect on volume and surface area when multiplying any dimension by a scale factor**			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

When the **dimensions** of a prism are **multiplied** by a number ***k* (called the scale factor**), then :

- The **surface area** of the new prism formed after multiplying the dimensions of the original prism by a scale factor k is equal to $k^2 \times$ surface area of original prism.
- The **volume** of the new prism formed after multiplying the dimensions of the original prism by a scale factor k is equal to $k^3 \times$ volume of original prism.
- If $k > 1$, then the new prism formed is an **enlargement** of the original prism.
- If $0 < k < 1$ then the new prism formed is a **reduction** of the original prism.

NOTE: The original prism and the enlarged (or reduced) prism are **similar** to each other.

Consider a rectangular prism with length = x, breadth = y and height = z
The volume will be: $V = xyz$ and the surface area will be: $SA = 2xy + 2yz + 2xz$

If each dimension is multiplied by k

$$SA = 2k^2xy + 2k^2yz + 2k^2xz \qquad V = k^3xyz$$

Examples:

1. If a cylinder has a radius of x cm and a height of y cm what will happen to the volume if:

(a) the radius is doubled?

$V = \pi.x^2y$

$V = \pi(2x)^2y = 4\pi x^2y$… radius doubled; the volume is 4 time its original size.

(b) the height is tripled?

$V = \pi.rx^2y$

$V = \pi.x^2 3y = 3\pi x^2y$… height tripled; the volume is 3 time its original size.

(c) both the radius and the height are halved?

$V = \pi.x^2y$

$V = \pi\left(\frac{x}{2}\right)^2.\frac{y}{2} = \frac{1}{8}\pi x^2y$ The volume is one eights of its original size.

2. The surface area of a cube is 2 400 cm² and its volume is 8 000 cm³. Determine the surface area and

volume of the cube formed if the dimensions of the original cube are multiplied by a scale factor of 3.

$SA_{cube} = 2\ 400 \times 3^2 = 21\ 600cm^2$ $V_{cube} = 8\ 000 \times 3^3 = 216\ 000cm^3$

ACTIVITIES/ ASSESSMENT

1. A cylinder has a height of 6 cm and a diameter of 8 cm. The dimensions are doubled.
   (a) What is the scale factor?
   (b) Show that the surface area of the enlarged cylinder is $k^2 \times$ surface area of original.
   (c) Show that the volume of the enlarged cylinder is $k^3 \times$ surface area of original.

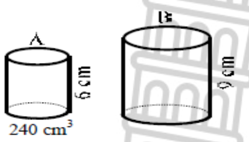
2. The surface area of a cuboid is 0,0292 m^2 and its volume is 24 cm^3.
   (a) Determine the surface area (in cm^2) and volume (in cm^3) of the rectangular prism formed if the dimensions of the original cuboid are multiplied by 5.
   (b) Suppose that the volume of the original cuboid is increased by 8 times its value. What is the surface area of the enlarged cuboid?

3. The surface area of a cube is $96x^2$ and its volume is $64x^3$. Determine, in terms of x:
   (a) the surface area and volume of the cube formed if the dimensions of the original cube are halved.
   (b) the length of a side of the reduced cube.

4. A cylinder has a height of 8 cm and a radius of 7 cm. The height remains constant but the radius is doubled.
   (a) What is the volume of the enlarged cylinder?
   (b) How does the volume of the larger cylinder relate to the volume of the original cylinder?
   (c) What is the surface area of the enlarged cylinder?
   (d) How does the surface area of the larger cylinder relate to the surface area of the original cylinder?

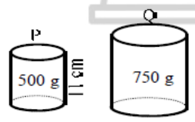
5. A cylinder has a height of b units and a diameter of $2a$ units.
   (a) Determine the volume and surface area in terms of a and b.
   (b) If you want to double the volume but keep the radius the same, by what scale factor will the height increase?
   (c) If the radius is doubled but the height stays the same, by what number will the area of the base of the cylinder increase?
   (d) If the radius is doubled but the height stays the same, by what number will the area of the side surface of the cylinder increase?

6. Cylinder A and B are similar. The volume of A is 240 cm^3.



   (a) Calculate the scale factor and hence the volume of B.
   (b) Calculate the ratio of the surface areas of A and B.

7. Two soup tins are similar. Tin P can hold 500 grams of soup while tin Q can hold 750 grams of soup. The height of tin P is 11 cm.



   (a) Calculate the height of tin Q.
   (b) Calculate the ratio of the areas of the circular bases.

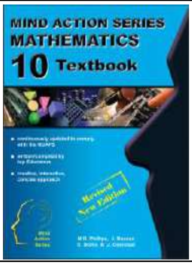
8. The heights of two similar rectangular prisms are in the ratio 4:5.
   (a) Calculate the ratio of the surface areas of the rectangular prisms.
   (b) Calculate the ratio of the volumes of the rectangular prisms

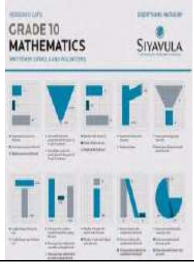
TOPIC: MEASUREMENT (Lesson 4)	Weighting		Grade	10
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Volume and Surface Area of Pyramids, Spheres and Cones**			
RELATED CONCEPTS/ TERMS/VOCABULARY	Right pyramid, sphere, cone			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Area formulas: rectangle, square, circle

RESOURCES



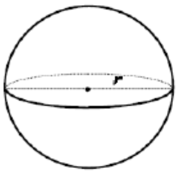


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Not choosing the correct and relevant formula. Formula.
Unable to separate between surface area and volume.

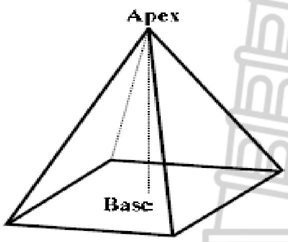
METHODOLOGY

A **sphere** is a perfectly round object in three-dimensional space that resembles the shape of a completely round ball. The points on the surface of the sphere are the same distance from the centre. The radius is the straight line from any point on the sphere to its centre.



The volume of a sphere is given by the formula: $V = \frac{4}{3}\pi r^3$ AND $SA = 4\pi r^2$

A pyramid is a polyhedron in which three or more triangles are based on the sides of the polygonal base and meet in one point called the apex of the pyramid. **Right pyramids** are such that the apex is perpendicularly above the centre of the regular base. The right pyramid shown has a square base and four congruent triangles meeting at the apex of the pyramid.



The volume of a right pyramid is given by the formula: $V = \frac{1}{3}(area\ of\ base) \times height$

SA = Sum of the area of the base (square) and four congruent triangles.

A **right circular cone** is similar to a pyramid in that it has an apex. The difference is that it has a circular base.

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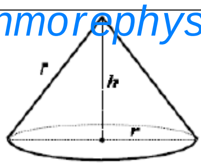
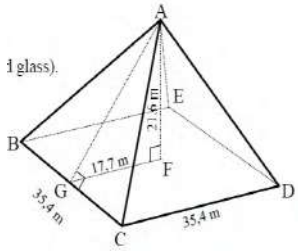


The volume of a right cone is given by formula: $V = \frac{1}{3}\pi r^2 h$ AND $SA = \pi r^2 + \pi rs$, where, l is the slant height.

Examples:

1. Given a large right square pyramid made of metal and glass. The length of one side of the base is 35,4 m and the height is 21,6 m. The base of the pyramid is open.



Calculate:

(a) the exterior surface area of the pyramid (metal and glass).

The exterior surface area consists of the sum of the areas of the four congruent triangles: ΔABC, ΔACD, ΔAED and ΔABE

$SA = 4(\frac{1}{2}b \times h)$

$A = \frac{1}{2}BC \times AG$

$= \frac{1}{2}(35,4)(27,92579453\ )$

$= 494,2865631\ m^2$

In ΔAGF, GF = $\frac{1}{2}$ CD = 17,7m

Therefore, AG = $\sqrt{21,6^2 + 17,7^2} = 27,92579453$ m

(b) the volume of the pyramid.

$V = \frac{1}{3}(area\ of\ base \times height)$

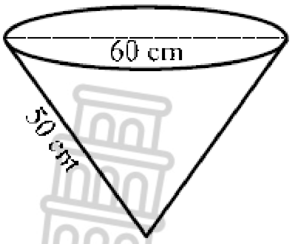
$= \frac{1}{3}(35,4 \times 35,4) \times 21,6 = 9\ 022,75\ cm^3$

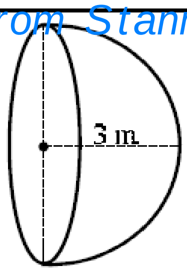
ACTIVITIES/ ASSESSMENT

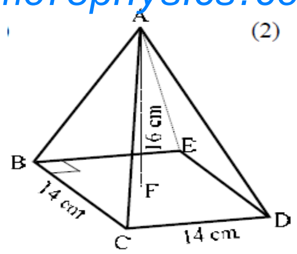
1. Find the volume and surface area of the following shapes:

(a) A sphere with a radius of 2cm.
(b) A right square pyramid, with the length of one side of the square being 4cm and the height from the base to the apex is 5cm.
(c) A right cone with a radius of 55mm and a height of 70mm.
(d) A sphere with a radius of 16mm.
(e) A right equilateral triangle pyramid with the length of one side of the equilateral equal to 15cm and the height equal to 8cm. The distance from the centre of the pyramid (where the height is measured) to the midpoint of one side of the triangle is 6cm.
(f) A right cone with a radius of 3.4cm and a height of 11cm.
(g) A right circular pyramid with a radius of 14mm and a height of 17mm

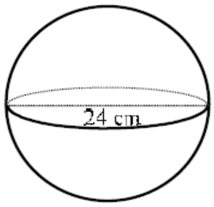
2. Calculate the surface area and volume of the following closed solids. Round your answers off to two decimal places.



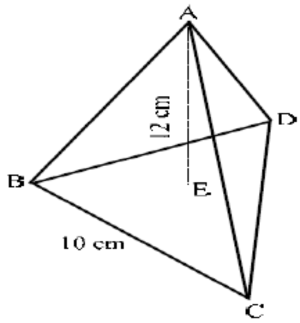




(2)



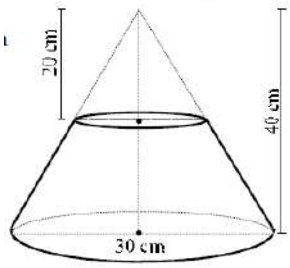
3. The base of the given triangular prism is an equilateral triangle. The height of the prism is 12 cm and the length of a side of the triangular base is 10 cm.



Calculate:

(a) the area of the triangular base (ΔBCD)
(b) the area of ΔABC
(c) the surface area of the prism
(d) the volume of the prism

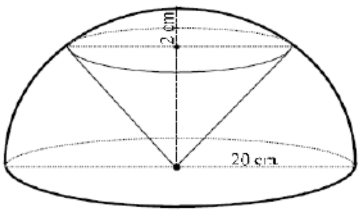
4. A frustum is formed by removing a small cone from a similar larger cone. The height of the small cone is 20 cm and the height of the larger cone is 40 cm. The diameter of the larger cone is 30 cm.



Calculate:

(a) the radius of the small cone
(b) the volume of the frustum

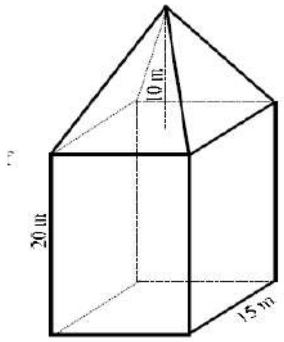
5. A solid right circular cone is placed centrally within a container in the shape of a hemisphere. The radius of the hemisphere is 20 cm and the distance from the circular base of the cone to the top of the hemisphere is 2 cm.



Calculate the volume of the right circular cone. Round off your answer to the nearest whole number.

6. A barn is constructed as follows:
The room space is constructed as a right rectangular prism with a square base. The length of one side of the base of the prism is equal to 15 metres. The height of the wall of the room is 20 metres. The

roof is constructed in the form of a right triangular pyramid with a height of 10 metres. The base of the roof is open.



TEST: MEASUREMENT

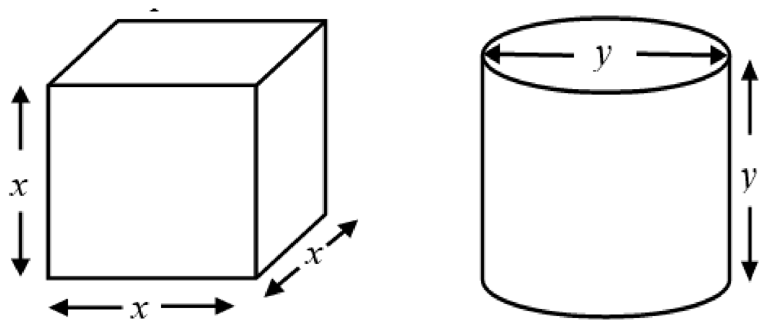
MARKS: 25 **DURATION: 30 Min**

INSTRUCTIONS

Answer ALL the questions

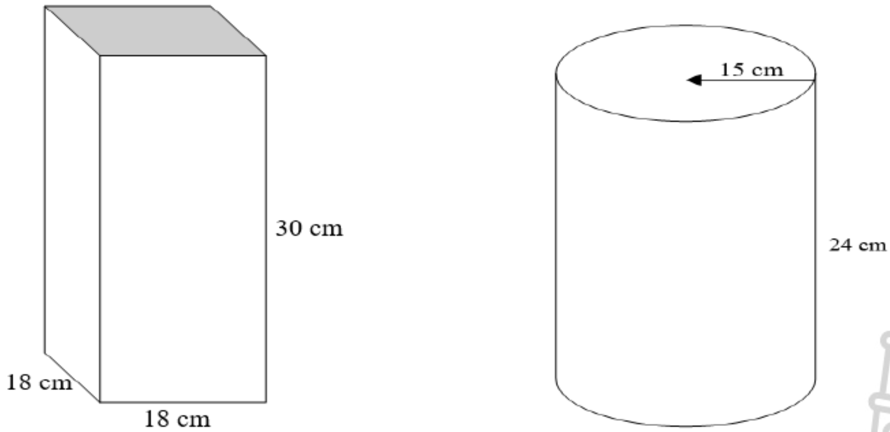
QUESTION 1[15 Marks]

1.1 In the figure, the cube and the cylinder have equal volume. If two sides of the cube are doubled, and the diameter of the cylinder is doubled, will the volumes of the resulting prisms still be equal? Show all calculations.



(5)

1.2 Consider the figures below and determine:



1.2.1 the surface area and the volume of the prism (3)

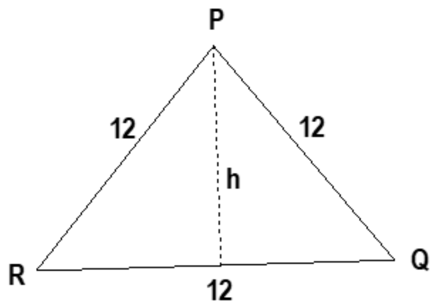
1.2.2 the surface area and the volume of the cylinder (3)

1.2.3 the surface area if the edge of the base of the prism is doubled. (2)

1.2.4 the surface area if the edge of the base of the prism is doubled. (2)

QUESTION 2 [10 Marks]

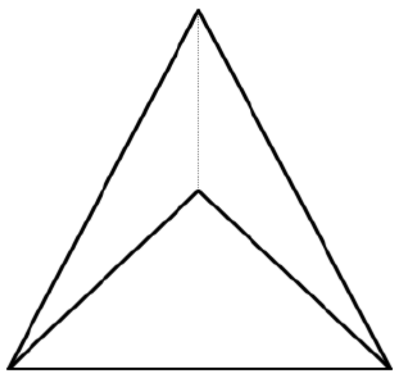
2.1 ΔPQR is an equilateral triangle, with the measurement of each side equal to 12cm



2.1.1 Use any appropriate method to show that the perpendicular height of the triangle is 10,39cm. (3)

2.1.2 Hence, calculate the area of the triangle. (2)

2.2 A triangular pyramid is constructed using four triangles that are the same as the triangle in 2.1 (i.e. an equilateral triangle with sides measuring 12cm).



2.2.1 Calculate the perimeter of the base of the pyramid (1)

2.2.2 Explain how you know that the height of the slanted triangles is 10,39cm. (1)

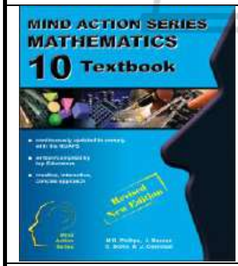
2.2.3 Calculate the total surface area of the pyramid (3)

TOPIC: PROBABILITY (Lesson 1)		**Weighting**	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Theoretical probability and Relative frequency**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Probability, sample space, event, outcome				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Probability, probability scale, likely, impossible, certain

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Not reading the given statement with understanding

METHODOLOGY

Probability is a measure of likelihood of an event to occur. It is a ratio comparing how many times an outcome can occur compare to all possible outcomes.

That is, $Probability = \frac{number\ of\ expected\ outcomes}{number\ of\ possible\ outcomes}$

Possible outcomes of an experiment is called a **sample space(S)** and the number of expected outcomes is called an **event**.
An **event(E)** is a specific set of outcomes of an experiment that you are interested in. It is the subset of the sample space.

The number of ways an event can occur (favourable outcome) divided by the number of total outcomes is called **theoretical probability**. Theoretical probability does not **require an experiment**, it is **what is expected.**

$$\boldsymbol{P(event) = \frac{Number\ of\ favourable\ outcomes}{Number\ of\ total\ outcomes}}$$

$$\boldsymbol{P(E) = \frac{N(E)}{N(S)}}$$

Remember that the probability of an event happening is a number in the interval [0 ;1].
Probability can be expressed as a **fraction**, **decima**l, or **percentage.**

Examples:

1. A bag contains 10 red marbles, 8 blue marbles and 2 yellow marbles. Determine the theoretical probability of getting a blue marble.

There are 8 blue marbles. Therefore, the number of favorable outcomes = 8
There are a total of 20 marbles. Therefore, the number of total outcomes = 20

$$P(event) = \frac{Number\ of\ favourableoutcomes}{Number\ of\ total\ outcomes} = \frac{8}{20} = \frac{2}{5}$$

2. Find the probability of rolling an even number when rolling a die containing the numbers 1 – 6. Express the probability as a fraction, decimal, ratio and percent

The possible even number are 2, 4, 6
Number of favorable outcomes = 3
Total number of outcomes = 6

$$P(event) = \frac{Number\ of\ favourable outcomes}{Number\ of\ total\ outcomes} = \frac{3}{6} = \frac{1}{2}$$

The probability = $\frac{1}{2}$ (fraction) = 0,5 (decimal) = 1:2 (ratio) = 50% (percent)

Comparing theoretical and experimental (relative frequency) Probability

The **relative frequency** of an event is the number of times that the event occurs during an experimental trial, divided by the total number of trials conducted.

$$Relative\ frequency = \frac{Number\ of\ positive\ trials}{Total\ number\ of\ trials}$$

Example:

3. A die is tossed 44 times and lands 5 times on the number 3. What is the relative frequency of observing the die land on the number 3? Write your answer correct to 2 decimal places.

Total number of trials = 44
Number of positive trials = 5

$$Relative\ frequency = \frac{5}{44} = 0{,}11$$

Therefore, the relative frequency of observing the die on the number 3 is 0,11.

4. A die is rolled 12 000 times. Approximately how many times do you expect the die to land on a factor of 6.

Outcome: {1; 2; 3; 4; 5; 6} Factors of 6: {1; 2; 3; 6}

Theoretical probability $= \frac{4}{6} = \frac{2}{3}$

Relative frequency $= \frac{Number\ of\ possible\ trial}{Total\ number\ of\ trials} = \frac{f}{12\ 000} \approx \frac{2}{3}$

Number of possible trials $\approx \frac{12\ 000 \times 2}{4} \approx 8000$ times.

ACTIVITIES/ ASSESSMENT

1. 70 tickets were sold in a competition. The prize is a smartphone. Mpho decided to buy 12 tickets. What is the probability that he will:
(a) win the prize? (b) not win the prize?

2. A die is tossed 27 times and lands 6 times on the number 6. What is the relative frequency of observing the die land on the number 6? Write your answer correct to 2 decimal places.

3. A coin is tossed 30 times and lands 17 times on heads.
What is the relative frequency of observing the coin land on heads? Write your answer correct to 2 decimal places.

4. A six-sided die is thrown. Determine the probability of:
(a) throwing a 6
(b) throwing a 3 or a 4
(c) throwing an even number
(d) not throwing a 2

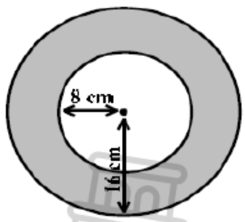
5. A container is filled with 5 blue blocks, 8 red blocks, 6 green blocks and 9 white blocks. A block is taken out of the container at random. Find the probability of taking out:
(a) a blue block
(b) a white block
(c) a red or green block
(d) a brown block
(e) any block that is not white

6, A cupboard contains 5 shirts, 3 pairs of jeans and 8 pairs of socks.
(a) What is the probability of taking out, at random, one pair of socks?
(b) What is the probability of taking out, at random, a shirt or pair of jeans?
(c) What is the probability of not taking out a pair of jeans?
(d) Assuming that you have already taken out a pair of socks from the cupboard and put them on your feet. What will the probability now be of taking out a shirt?

7. There are 52 cards in a standard deck of cards of which 13 are hearts. A card is drawn at random, returned, and then the deck is reshuffled. This is repeated 50 000 times.
Which of the following do you consider to be the most reasonable answer for the number of times that a card of hearts was chosen? Motivate your answer.
A. 31 210 B. 2003 C. 12 685 D. 25 443

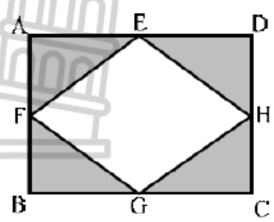
8. A card is drawn from a pack of 52 cards. Determine the probability of drawing:
(a) a heart
(b) a jack of clubs
(c) an ace
(d) a king or queen
(e) neither a heart or a spade

9. A dart is thrown at random onto a board that has the shape of a circle as shown in the figure.
The two circles are concentric (have the same centre).



Calculate the probability that the dart will hit the shaded region.

10. An arrow is shot at random onto rectangle ABCD. E, F, G and H are the midpoints of the sides of rectangle ABCD. Let ED = x and DH = y.



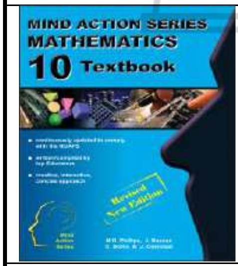
Calculate the probability of the arrow:
(a) landing in ΔAEF.
(b) landing in the shaded region.
(c) landing in either in ΔCGH or the unshaded area.

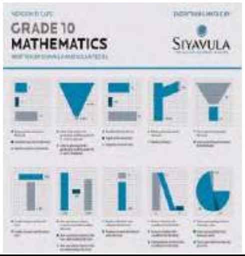
TOPIC: PROBABILITY (Lesson 2)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Venn Diagram**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Sample space, event				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Probability, outcome,

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Difference between P(E) and n(E), not putting remaining items of a sample space in a Venn diagram

METHODOLOGY

The set of all possible outcomes of an experiment is called the **sample space** and is denoted by the symbol S. When rolling a die, the sample space is given by the set **S = {1; 2; 3; 4; 5; 6} .**

An **event** is a subset of the sample space and is denoted by a given capital letter.
When rolling a die, if A is the event in which the die lands on an even number, then **A = {2; 4; 6} .**

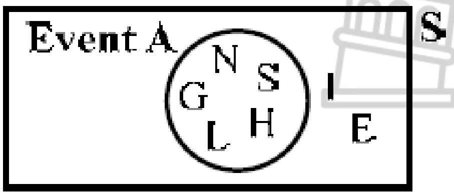
We use a special diagram, called a **Venn diagram** to represent events in a sample space. The sample space is represented by a rectangle and the events by circles inside the rectangle.

The probability of event E occurring is given by the formula: $P(E) = \frac{n(E)}{n(S)}$, where P(E) is the probability of event E occurring, *n*(E) is the number of outcomes in E and *n*(S) is the number of outcomes in the sample space.

Examples:

1. The letters of the word ENGLISH are written on cards and placed in a hat. One card is drawn randomly. Let A be the event in which a consonant is drawn.

   (a) Draw a Venn diagram showing all outcomes.



   (b) Write down the outcomes of the sample space (S) and event A in set form.

   Sample Space: {E; N; G; L; I; S; H)
   Event A: {N; G; L; S; H}

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(c) Write down n(A) and n(S)

n(A) = 5 and n(S) = 7

(d) Calculate P(A)

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{7}$$

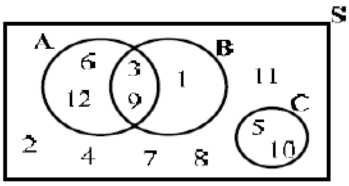
2. A 12-sided dodecahedral die is rolled. The following events are defined:

A = {multiples of 3} B = {factors of 9} C = {multiples of 5}

(a) List the outcomes of events in set form.

A = {3; 6; 9; 12} B = {1; 3; 9} C = {5, 10}

(b) Draw a Venn diagram to represent these events.



(c) Determine P(A), P(B) and P(C).

$P(A) = \frac{4}{12} = \frac{1}{3}$ $P(B) = \frac{3}{12} = \frac{1}{4}$ $P(C) = \frac{2}{12} = \frac{1}{6}$

ACTIVITIES/ ASSESSMENT

1. Pieces of paper labelled with the numbers 1 to 12 are placed in a box and the box is shaken. One piece of paper is taken out and then replaced.

   a) What is the sample space, *S*?
   b) Write down the set *A*, representing the event of taking a piece of paper labelled with a factor of 12.
   c) Write down the set *B*, representing the event of taking a piece of paper labelled with a prime number.
   d) Represent *A*, *B* and *S* by means of a Venn diagram.
   e) Determine: n(S), n(A) and n(B)

2. The letters of the word RANDOMLY are written on cards and placed in a hat. One card is drawn randomly. Let A be the event in which a vowel is drawn.

   (a) Draw a Venn diagram showing all outcomes.
   (b) Write down the outcomes of the sample space (S) and event A in set form.
   (c) Write down n(A) and n(S)
   (d) Calculate P(A)

3. A twelve-sided die is rolled. The following events are defined:

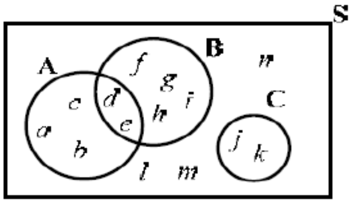
   A = {the first five natural numbers} B = {the first two multiples of 5}

(a) Write down the set A, n(A) and P(A).
(b) Write down the set B, n(B) and P(B).
(c) Draw a Venn diagram to represent the actual outcomes.

4. A twelve-sided die is rolled. The following events are defined:
A = {the first five prime numbers} B = {the first two multiples of 6}

(a) Write down the set A, n(A) and P(A).
(b) Write down the set B, n(B) and P(B).
(c) Draw a Venn diagram to represent the actual outcomes.

5. Consider the given Venn diagram.



(a) Write down the sets S, A, B and C.
(b) Write down n(A), n(B) and n(C).
(c) Calculate P(A), P(B) and P(C).

6. In a survey conducted at a music store in Johannesburg, it was found that 120 people bought only Trance music, 150 bought only Deep House music and 100 people bought both. Twenty people did not buy either Trance or Deep House music.
Let T = {Trance music} and D = {Deep House music}

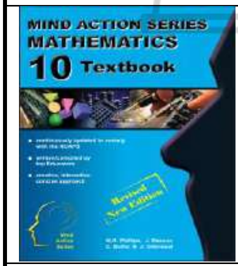
(a) Draw a Venn diagram to represent events T and D.
(b) Determine n(T), n(D) and n(S).
(c) Calculate the probability of selecting, at random, a person who likes only Deep House music.
(d) Calculate the probability of selecting, at random, a person who likes only Trance music.
(e) Calculate the probability of selecting, at random, a person who likes neither types of music.
(f) Calculate the probability of selecting, at random, a person who likes both types of music.

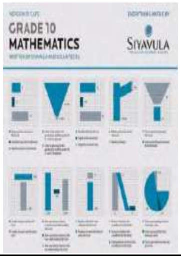
TOPIC: PROBABILITY (Lesson 3)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Complement of events, Union of events and Intersection of events**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Complement of events, union, intersection				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Probability formula, sample space, events, Venn diagram

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Difference between P(E) and n(E), application of terms like union, intersection, complement of events

METHODOLOGY

The **complement** of an event A is the event consisting of all outcomes that are in the sample space, but not in A. We write the complement of A as **not A** or A'.

For example, if you roll a die and A is the event in which the die lands on an even number, then:

$S = \{1; 2; 3; 4; 5; 6\}$ $\quad$ $A = \{2; 4; 6\}$ $\quad$ not $A = \{1; 3; 5\}$

The **intersection** of two events, event A and event B, is the collection of all outcomes that are elements of both A and B simultaneously. It corresponds to combining descriptions of the two events using the word "and". We write this intersection of event A and event B as **A∩B** or **A and B**.

For example, if you roll a die and A is the event in which the die lands on an even number and B is the event that the die lands on a prime number, then:

$A = \{2; 4; 6\}$ $\quad$ $B = \{2; 3; 5\}$ $\quad$ A and $B = \{2\}$

The **union** of two events, event A and event B, is the event consisting of all outcomes that are in at least one of these events. The union consists of outcomes that are either in A, or in B, or in both. This basically means that we put all of the outcomes of A and B together by uniting them into one big set. We write the union of event A and event B as **A∪B** or **A or B**.

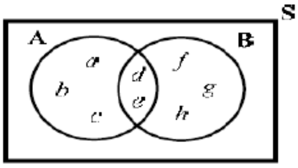
For example, if you roll a die and A is the event in which the die lands on an even number and B is the event that the die lands on a prime number, then:

$A = \{2; 4; 6\}$ $\quad$ $B = \{2; 3; 5\}$ $\quad$ A or $B = \{2; 3; 4; 5; 6\}$

Examples:

1. In a certain experiment, the sample space is $S = \{a; b; c; d; e; f; g; h\}$.
   Events A and B are defined as follows:
   $A = \{a; b; c; d; e\}$ $\quad$ $B = \{d; e; f; g; h\}$

(a) Represent all of the outcomes in a Venn diagram.



(b) List the outcomes in:

(1) not A = {f; g; h}

(2) A and B = {d; e}

(3) A or B = {a; b; c; d; e; f; g; h}

(c) What is the value of:

(1) n(not A) = 3

(2) n(A and B) = 2

(3) n(A or B) = 8

(d) Determine:

(1) n(not A and B)

not A = {**f; g; h**}
B = {d; e; **f; g; h**}
not A and B = {f; g; h}

(2) P(not A and B)

$= \frac{3}{8}$

(3) n(not A or B)

not A or B = {d; e; f; g; h}
= 5

(4) P(not A or B)

$= \frac{5}{8}$

(5) P(not A or not B)

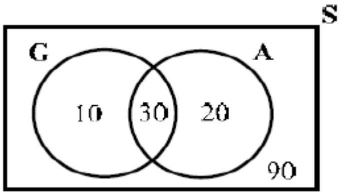
not A = {f; g; h}
not B = {a; b; c}
not A or not B = {a; b; c; f; g; h}
$= \frac{6}{8} = \frac{3}{4}$

(6) P(not A and not B)

not A and not B = { }

$= = \frac{0}{8} = 0$

2. The diagram shows the subjects taken by the matric group in Kwazamakuhle FET school with respect to Geography (G) and Accounting (A).



(a) How many learners take:

(1) both Geography and Accounting? 30 learners

(2) neither Geography nor Accounting? 90 learners

(3) at least one of the subjects? 60 learners (10 + 30 + 20)

(4) Geography but not Accounting? 10 learners

(b) If a learner is chosen at random from this group, determine:

(1) P(not A)

not A = 10 +90 = 100

(2) P(G or (not A))

G = 10 + 30 not A = 10 + 90

P(not A) $= \frac{100}{150} = \frac{2}{3}$

G or not A = 10 + 30 + 90

P(G or (not A)) $= \frac{130}{150} = \frac{13}{15}$

(3) P((not G) **and** A)

not G = **20** + 90 = 110, A = **20** + 30 = 50

P((not G) and A) $= \frac{20}{150} = \frac{2}{15}$

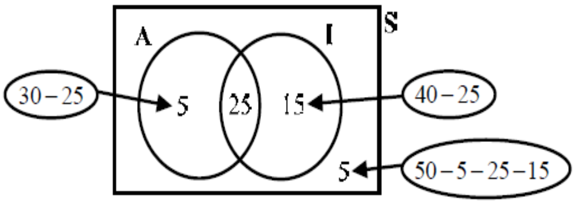
(4) P(not(G and A)).

not (G and A) = 10 +20 + 90 = 120

P(not(G and A)) $\frac{120}{150} = \frac{4}{5}$

3. In a group of 50 people, 30 can speak Afrikaans (A) and 40 can speak IsiZulu (I). There are 25 of these people that can speak both languages.
   (a) Draw a Venn diagram showing this information.

   Start in the middle with the intersection and then work your way outwards.



   (b) How many of these 50 people cannot speak either of the two languages? 5 people

   (c) If a person is chosen from this group at random, find the probability that this person can speak:
   (1) at least one of the two languages
   (2) IsiZulu but not Afrikaans

$= \frac{5+25+15}{50} = \frac{45}{50} = \frac{9}{10}$

Isizulu = 25 + **15**; not Afrikaans = **15** + 5

$= \frac{15}{50} = \frac{3}{10}$

ACTIVITIES/ ASSESSMENT

1. A twelve-sided die is rolled.
   (a) Write down the sample space in set form.
   (b) Determine the probability that the die will land on an even number.
   (c) Determine the probability that the die will land on a prime number.
   (d) Determine the probability that the die will land on an even and prime number.
   (e) Determine the probability that the die will land on an even or prime number.
   (f) Determine the probability that the die will land on a prime number, given that the number is greater than 8.

2. You are given the sample space S = $\{a; b; c; d; e; f; g\}$ with the following events:
   A = $\{a; b; c\}$ B = $\{e; f\}$ C = $\{c; d; e\}$
   Determine:
   (a) P(A) (b) P(A and C) (c) P(A or C)
   (d) P(B or C) (e) P(not A) (f) P(not B)

3. A = $\{a ; b ; c ; d ; e\}$, not A = $\{f ; g ; h\}$ and B = $\{c ; d ; e ; f\}$ are given.
   Determine:
   (a) the sample space S (b) P(A) (c) P(A and B)

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(d) P(A or B) (e) P(not A) (f) P(not B)

4. A twelve-sided die is rolled. Suppose that the following events are given:

A = {multiples of 3} B = {factors of 12}

Determine:

(a) *n*(A and B) (b) *n*(A or B) (c) *n*(not A)
(d) *n*(not B) (e) P(A and B) (f) P(A or B)
(g) P(not A) (h) P(not B) (i) *n*((not A) and B)
(j) P((not A) and B) (k) *n*(A and (not B)) (l) P(A and (not B))
(m) *n*((not A) or B) (n) P((not A) or B) (o) *n*(A or (not B))
(p) P(A or (not B)) (q) *n*(not(A and B)) (r) P(not(A and B))
(s) *n*(not(A or B)) (t) P(not(A or B)) (u) P((not A) and (not B))

5. In a recent sports survey, it was found that 120 people enjoy watching cricket only, 95 people enjoy watching rugby only and 45 people enjoy watching both sports. There were 40 people who don’t watch either sport.

(a) Draw a Venn diagram to illustrate this information.
(b) How many people watch cricket in total?
(c) How many people watch rugby in total?
(d) How many people were there in the survey?
(e) Determine the probability that a person selected watches both sports.
(f) Determine the probability that a person selected watches none of the sports.
(g) Determine:
(i) P(C or (not R)) (ii) P(R and (not C))

6. In a survey on brain diseases conducted by medical researchers, it was found that of a total of 1 520 genes, 454 are associated with Alzheimer’s disease, 1 091 are associated with Multiple Sclerosis and 40 genes are associated with both diseases.

(a) Draw a venn diagram to illustrate this information.
(b) How many genes are not associated with either of the diseases?
(c) Determine the probability that a gene, selected at random, will be associated with Alzheimer’s disease and Multiple Sclerosis.
(d) Determine the probability that a gene, selected at random, will be associated with at least one of the diseases.
(e) Determine the probability that a gene, selected at random, will be associated with only one of the diseases.
(f) Determine:
(i) P((not A) or (not M)) (ii) P((not A) and (not M))

7. The probability that Tumi will not see a movie today is 0,3. The probability that he will go to a restaurant today is 0,6. The probability of him seeing a movie and going to a restaurant today is 0,4. Determine the probability that he:

(a) doesn’t go to a movie or a restaurant. (b) only goes to a movie.
(c) only goes to a restaurant. (d) doesn’t go to a movie.
(e) doesn’t go to a restaurant. (f) goes to either one or the other.

8. 100 boys – 60 from school P and 40 from school K – were included in a survey in which they were asked whether or not they liked Mathematics. The results were as follows:

	Like Mathematics	Don't like Mathematics
School K	16	24
School P	40	20

A learner is chosen at random from this group. Suppose that the following events are defined:

A = {learners from school P} and B = {learners who don't like Maths}

(a) Describe the event (not A) and B in words.

(b) Calculate:

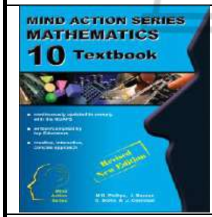
(i) P(A) (ii) P(A or B) (iii) P(A and (not B))

TOPIC: PROBABILITY (Lesson 4)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**The Fundamental Laws of Probability**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Probability formula, sample space, events, Venn diagram

RESOURCES

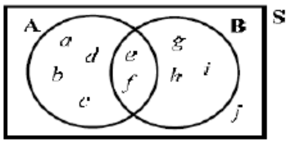



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The following **two laws** are always **valid** and form the basis of probability theory. When used correctly, they can make many probability problems much easier to solve.

Consider the following Venn diagram:



$P(A) = \frac{n(A)}{n(S)} = \frac{6}{10}$ $\qquad$ $P(B) = \frac{n(A)}{n(S)} = \frac{5}{10}$

$P(AandB) = \frac{n(A)}{n(S)} = \frac{2}{10}$ $\qquad$ $P(AorB) = \frac{n(A)}{n(S)} = \frac{9}{10}$

$$P(A) + P(B) - P(AandB)$$
$$= \frac{6}{10} + \frac{5}{10} - \frac{2}{10} = \frac{9}{10}$$

Therefore, $\boldsymbol{P(AorB) = P(A) + P(B) - P(AandB)}$

$P(notA) = \frac{n(notA)}{10} = \frac{4}{10}$ $\qquad$ $1 - P(A) = 1 - \frac{6}{10} = \frac{4}{10}$

Therefore, $\boldsymbol{P(notA) = 1 - P(A)}$

Example:

1. If P(A) = 0,7; P(B) = 0,5 and P(A and B) = 0,4, determine:
   (a) P(A or B) $\qquad$ (b) P(not A)

$\boldsymbol{P(AorB) = P(A) + P(B) - P(AandB)}$
$= 0{,}7 + 0{,}5 - 0{,}4 = 0{,}8$

$\boldsymbol{P(notA) = 1 - P(A)}$
$= 1 - 0{,}7 = 0{,}3$

(c) (c) P(not (A or B))

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$\boldsymbol{P(not(AandB)) = 1 - P(AorB)}$
$= 1 - 0{,}8 = 0{,}2$

2. If P(not A) = 0,25 ; P(A or B) = 0,8 and P(A and B) = 0,15 , determine:

(a) P(A)

$\boldsymbol{P(notA) = 1 - P(A)}$
$0{,}25 = 1 - P(A)$
$0{,}25 - 1 = -P(A)$
$-0{,}75 = -P(A)$
$P(A) = 0{,}75$

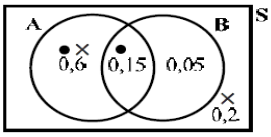
(b) P(B)

$\boldsymbol{P(AorB) = P(A) + P(B) - P(AandB)}$
$0{,}8 = 0{,}75 + P(B) - 0{,}15$
$0{,}8 - 0{,}75 + 0{,}15 = P(B)$
$P(B) = 0{,}2$

(c) P(A and (not B))

P(A and B) = 0,15
$P(A) = 0{,}75 = 0{,}15 + \boldsymbol{0{,}6}$
$P(notB) = 1 - 0{,}2 = 0{,}8 = 0{,}2 + \boldsymbol{0{,}6}$
$\therefore P(A)and(notB) = 0{,}6$

Venn diagram:

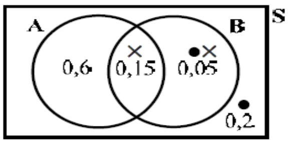


$P(A)and(notB) = 0{,}6$

(d) P((not A) or B)

$P(B) = 0{,}15 + 0{,}05$
$P(notA) = 0{,}25 = 0{,}2 + 0{,}05$
$\therefore P\big((notA)orB\big) = 0{,}15 + 0{,}05 + 0{,}2$
$= 0{,}4$

Venn diagram:



$P\big((notA)orB\big) = 0{,}15 + 0{,}05 + 0{,}2 = 0{,}4$

ACTIVITIES/ ASSESSMENT

1. If P(A) = 0,5 P(B) = 0,7 P(A and B) = 0,3, determine:

(a) P(A or B) (b) P(not A) (c) P(not B)
(d) P(not (A or B)) (e) P(not (A and B)) (f) P((not A) and B)

2. $P(A) = \frac{4}{7}$ $P(AorB) = \frac{6}{7}$ $P(AandB) = \frac{1}{7}$, determine:

(a) P(B) (b) P(not A) (c) P(not (A or B))
(d) P((not A) or B) (e) P(A or (not B)) (f) P((not A) and B)

3. If P(not A) = 0, 4 P(A or B) = 0,9 P(A and B) = 0, 2 , determine:

(a) P(A) (b) P(B) (c) P(A and (not B))
(d) P((not A) or B) (e) P(A or (not B)) (f) P((not A) and B)

4. If P(A) = 0,38 ; P(B) = 0,45 and P(not(A or B)) = 0, 4 , determine:

(a) P(A or B) (b) P(A and B) (c) P(A and (not B))

5. If P((not B) and A) = 0,3 P(A and B) = 0,1 P(not (A or B)) = 0, 2 , determine:

(a) P(A or B) (b) P(B or (not A)) (c) P(B and (not A))

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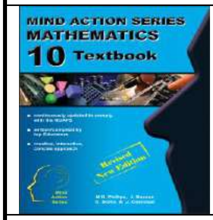
TOPIC: PROBABILITY (Lesson 5)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Mutually exclusive events, Exhaustive and complementary events**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Mutually exclusive, exhaustive, complementary events				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Probability formula, sample space, events, Venn diagram, probability laws

RESOURCES



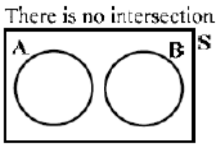
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Using formulars correctly, understanding and applying probability terms correctly

METHODOLOGY

Two events are **mutually exclusive** if they do not share any common outcomes and can therefore never both take place at the same time i.e., there is no intersection. If A and B are mutually exclusive, $P(AandB) = 0$.

$$\boldsymbol{P(AorB) = P(A) + P(B) - P(AandB)}$$
$$P(AorB) = P(A) + P(B) - 0$$
$$\therefore \boldsymbol{P(AorB) = P(A) + P(B)}$$



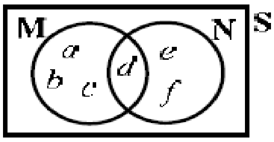
1. If P(A) = 0,3 and P(B) = 0,4 where A and B are mutually exclusive events, determine:
   (a) P(A and B) (b) P(A or B) (c) P(not (A or B))

$P(AandB) = 0$

$P(AorB) = P(A) + P(B) = 0{,}3 + 0{,}4 = 0{,}7$

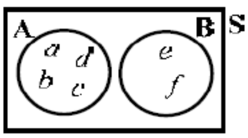
$P(notAorB) = 1 - P(AorB) = 1 - 0{,}7 = 0{,}3$

Two events A and B are said to be **exhaustive** if, together, they cover all elements of the sample space, i.e. P(A or B) =1



$P(MorN) = \frac{6}{6} = 1$

Two events, A and B, are said to be **complementary** if they are both exhaustive and mutually exclusive.



$P(AorB) = \frac{6}{6} = 1$ $P(Aand\ B) = 0$

$$P(AorB) = P(A) + P(B) - P(AandB)$$
$$1 = P(A) + P(B) - 0$$
$$\therefore \boldsymbol{P(A) + P(B) = 1}$$

NOTE: **For complementary events:**
P(A and B) = 0 (mutually exclusive) and **P(A or B) =1** (exhaustive) and P(A) + P(B) =1

2. A and B are mutually exclusive events with P(not A) = 0,3 and P(A or B) = 0,8
   (a) Determine P(B).

$$P(A) = 1 - P(notA) = 1 - 0{,}3 = 0{,}7$$
$$P(AorB) = P(A) + P(B) - P(AandB)$$
$$0{,}8 = 0{,}7 + P(B) - 0$$
$$P(B) = 0{,}1$$

   (b) Explain why A and B are not complementary.

$\therefore \boldsymbol{P(AandB) = 0}$ mutually exclusive
P(A or B) = 0,8 and for exhaustive events **P(A or B) =1**

For complementary events, events must be mutually exclusive and exhaustive.
Event A and Event B are mutually exclusive but not exhaustive
Therefore, event A and event B are not complementary

3. If P(A) = 0,6 and P(B) = 0,5, prove that A and B cannot be mutually exclusive.
For mutually exclusive events: $P(AorB) = P(A) + P(B) = 0{,}6 + 0{,}5 = 1{,}1$
This is impossible since probabilities can never exceed 1. Therefore, the events can never be mutually exclusive.

ACTIVITIES/ ASSESSMENT

1. A and B are mutually exclusive events such that P(A) = 0,6 and P(B) = 0,3 .
   Determine P(A and B) and P(A or B) .

2. A and B are mutually exclusive events such that P(A) = 0, 25 and P(not B) = 0,58.
   Determine P(A or B) .

3. The events C and D are mutually exclusive with P(not C) = 0,3 and P(C or D) = 0,8 .
   Determine P(D) .

4. If P(A) = 0, 4 and P(A or B) = 0,5, determine P(B) if:
   (a) A and B are mutually exclusive (b) P(A and B) = 0,3

5. Determine whether the following events are complementary if
(a) A= $\{a; b; c; d\}$ and B= $d; e; f; g\}$ (b) A = $\{a; b; c; d\}$ and (c) A = $\{a; b; \ ; d\}$ and B = $\{e; f; g\}$

6. If P(A) = 0, 25 ; P(B) = 0,5 and P(A or B) = 0,625
   (a) Calculate P(A and B)
   (b) Are events A and B are complementary? Give reasons.

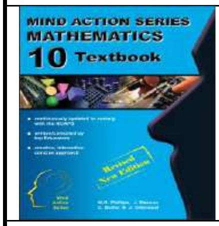
7. A smoke detector system in a large warehouse uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0,95. The probability that it will be detected by device B is 0,98 and the probability that it will be detected by both devices simultaneously is 0,94. What is the probability that the smoke will not be detected

TOPIC: PROBABILITY (Lesson 6)		Weighting	15 ± 3	Grade	10
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Consolidation and Extension Exercise**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Vocabulary: Probability, sample space, event, outcome, Complement of events, union, intersection, Mutually exclusive, exhaustive, complementary events Probability Fundamental Laws: $P(AorB) = P(A) + P(B) - P(AandB)$ $P(notA) = 1 - P(A)$				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Using formulars correctly, understanding and applying probability terms correctly

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Using formulars correctly, understanding and applying probability terms correctly

METHODOLOGY

ACTIVITIES/ ASSESSMENT

1. A survey was conducted at Mbusi High School to establish how many of the 650 learners buy vetkoek and how many buy sweets during break. The following was found:
   - 50 learners bought nothing
   - 400 learners bought vetkoek
   - 300 learners bought sweets

   (a) Represent this information with a Venn diagram.
   (b) If a learner is chosen randomly, calculate the probability that this learner buys:
   i. sweets only
   ii. vetkoek only
   iii. neither vetkoek nor sweets
   iv. vetkoek and sweets
   v. vetkoek or sweets

2. In a class there are
   - 8 learners who play football and hockey
   - 7 learners who do not play football or hockey
   - 13 learners who play hockey
   - 19 learners who play football

   How many learners are there in the class?

3. In a survey on internet usage, it was found that out of a total of 27 people, 16 use ADSL lines, 15 use wireless internet and 3 use neither of the two. What is the probability that a person chosen at random uses both internet connections?

4. Simon and his girlfriend decide to go out one evening. They can see a movie, go to a restaurant or do

both. The probability of them seeing a movie is 0,6. The probability of them going to a restaurant is 0,7. The probability of them seeing a movie without going to a restaurant is 0,2. What is the probability of them not seeing a movie and not going to a restaurant?

5. If $n(S) = 46$, $n(A) = n(B) = 19$ and $n(\text{not } (A \text{ or } B)) = 11$, determine:
(a) P(A and B) (b) P(A or B) (c) P((not A) and B)

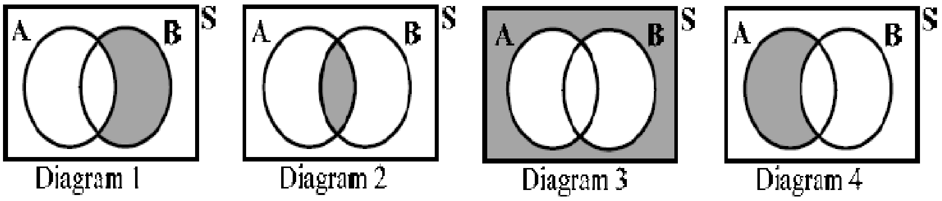
6. Determine whether the following events are complementary:
(a) A and B, if $P(A) = P(notB) = \frac{2}{5}$
(b) C and D, if $P(C) = \frac{1}{2}, P(D) = \frac{2}{3}$ $and\ P\big(not(CorD)\big) = 0$
(c) F and G, if $P(notF) = \frac{9}{11}, P(G) = \frac{8}{11}$ and F and G are mutually exclusive

7. G and H are inclusive events in a sample space S. If it is given that
$P(GorH) = \frac{3}{4}, P(G) = \frac{2}{5}$ $and\ P(H) = \frac{1}{2}$, determine $P(GandH)$ and the value of $P(notG)$.

8. Two events, K and L, are such that P(K) = 0,7; P(L) = 0, 4 and P(K or L) = 0,8 . Determine:
(a) P(K and L) (b) P(K and (not L))
(c) n(L) if K = $\{a; b; c; d; e; f; g\}$

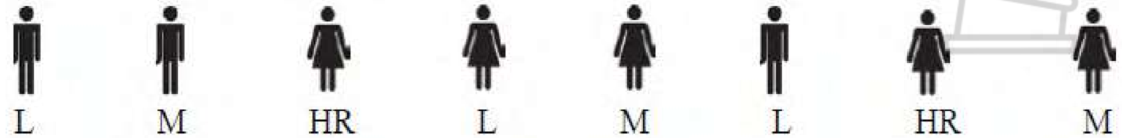
9. Given event A = {1; 2; 3; 4} = , event B = {5; 6} and P(A or B) = $\frac{2}{3}$.
(a) Determine the values of P(A) and P(B) .
(b) A third event C is mutually exclusive with A as well as B and P(B or C) = $\frac{1}{3}$. Determine P(A or C)

10. The diagrams below represent a class of learners. A is the set of girls and B is the set of learners that like rugby.



Indicate the diagram representing:
(a) the girls who like rugby (b) the boys who like rugby
(c) the girls who dislike rugby (d) the boys who dislike rugby
(e) P(A and B) (g) P((not A) and B)
(g) P(A and (not B)) (h) P((not A) and (not B))

11. You are given a group of eight students studying different degrees in Management. Consider the gender and degree in Logistics (L), Marketing (M) or Human Resources (HR)) that a student from this group is registered for:



Calculate the probability that a student selected at random from this group:
(a) is male (b) is female and studies Marketing
(c) studies Logistics or is male (d) must be female and studies Logistics

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TEST: PROBABILITY

MARKS: 30 **DURATION: 36 Min**

INSTRUCTIONS: Answer ALL the questions

QUESTION 1 [16 Marks]

1.1 A fair die is rolled once.

1.1.1 Write down the sample space (2)

1.1.2 what is the probability of getting the following:

(a) The number 6 (1)

(b) A number less than 3 (2)

(c) the number 3 or 6 (2)

1.2 Two dice are rolled simultaneously.

1.2.1Write down the sample space (2)

1.2.2 Determine the probability that:
(a) the number 2 is obtained (1)
(b) the sum of the numbers equals 8 (2)
(c) one of the numbers is an odd number (2)
(d) both of the numbers are factors of 6 (2)

QUESTION 2 [14 Marks]

2.1 There are 120 grade 10 learners at a school. 55 learners offer Mathematics and 80 offer Life Sciences. There are 25 learners who does not offer Mathematics or Life Sciences

2.1.1 Represent the information in a Venn diagram (3)

2.1.2 Use the Venn Diagram to calculate the probability that a randomly chosen learner:
(a) offer Mathematics only (1)
(b) offer Life Science only (1)
(c) offer Mathematics and Life Sciences (2)

2.2 In a staff of 20 teachers a survey was conducted to establish how many drink coffee and how many drink tea.

The following was found: 3 staff members did not drink either coffee or tea; 11 drank coffee and 8 drank tea

2.2.1 Represent this information in a Venn diagram. (3)

C= {staff members who drink coffee} and
T= {staff members who drink tea}.
Let the number of staff members who drink both coffee and tea = x

2.2.2 Calculate the value of x (2)
2.2.3 If a staff member is chosen randomly, calculate the probability that s/he drinks

(a) Coffee or tea (1)
(b) Coffee and tea (1)