KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

PINETOWN DISTRICT

TEACHING AND LEARNING SUPPORT – CURRICULUM FET (GRADES 10-12)

Grade 11 Mathematics Teacher Support Document

TABLE OF CONTENTS:


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TOPIC: EXPONENTS AND SURDS (Lesson 1)	Weighting	12 ± 3	Grade	11
Term	1	Week no.		
Duration	1 hour	Date		
Sub-topics	Revision of Laws of Exponents and Definitions (negative exponents and exponent 0)			
RELATED CONCEPTS/ TERMS/VOCABULARY	Base, exponent, power, Prime numbers, Factorisation			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

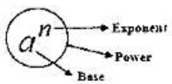
Laws of Exponents

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Product with same bases, multiply exponents instead of adding them.
- Subtracting smaller exponent from the bigger one even if the bigger exponent is on the denominator
- Writing $x^{m+n} as x^m + x^n$

METHODOLOGY



PRODUCT RULE

To multiply when two bases are the same, write the base and ADD the exponents: $x^m \times x^n = x^{m+n}$

1. $x^3.x^8 = x^{3+8} = x^{11}$ 2. $2^4 \times 2^2 = 2^{4+2} = 2^6$ 3. $(x^2y)(x^3y^4) = x^{2+3}y^{1+4} = x^5y^5$

QUOTIENT RULE

To divide when two bases are the same, write the base and SUBTRACT the exponents: $\frac{x^m}{x^n} = x^{m-n}$

1. $\frac{x^5}{x^2} = x^{5-2} = x^3$ 2. $\frac{3^4}{3^7} = 3^{4-7} = 3^{-3}$ 3. $\frac{x^2y^5}{xy^3} = x^{2-1}y^{5-3} = xy^2$

POWER RULE:

To raise a power to another power, write the base and MULTIPLY the exponents: $(x^m)^n = x^{m\times n}$

1. $(x^3)^2 = x^{3\times2} = x^6$ 2. $(3^2)^4 = 3^{2.4} = 3^8$ 3. $(y^3z^5)^2 = y^{3\times2}z^{5\times2} = y^6z^{10}$

EXPANDED POWER RULE: $(xy)^m = x^my^m$ $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

1. $(2a)^3 = 2^3a^3 = 8a^3$ 2. $(6x^3)^2 = 6^2x^{3\times2} = 36x^6$ 3. $\left(\frac{2x^2}{3y}\right)^3 = \frac{2^3x^{2\times3}}{3^3y^3} = \frac{8x^6}{27y^3}$

ZERO EXPONENT RULE

Any base (except 0) raised to the zero power is equal to one: $x^0 = 1$

1. $y^0 = 1$ 2. $6^0 = 1$ $(7a^3b^{-1})^0 = 0$

NEGATIVE EXPONENTS:

If a factor in the numerator or denominator is moved across the fraction bar, the sign of the exponent is changed. $x^{-m} = \frac{1}{x^m}$ $\frac{1}{x^{-m}} = x^m$ $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

1. $x^{-3} = \frac{1}{x^3}$ 2. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ 3. $-4x^5y^{-2} = \frac{-4x^5}{y^2}$ 4. $\left(\frac{x^2}{y}\right)^{-3} = \left(\frac{y}{x^2}\right)^3 = \frac{y^3}{x^6}$

Examples:

Simplify the following and leave the answer in exponential Form:

1. $-4x^2.(-5)^2x$
$= -4.(-5)(-5).x^{2+1}$
$= -100x^3$

2. $\frac{5^7\times3^3\times3^6\times5^8}{3^7\times5^{20}\times3}$
$= 5^{7+8-20} \times 3^{3+6-7-1}$
$= 5^{-5} \times 3^1 = \frac{3}{5^5}$

3. $\frac{3x^2\times4xy^7}{2x^2y\times6x^2y}$
$= \frac{12x^3y^7}{12x^4y^2}$
$= x^{3-4}y^{7-2}$
$= x^{-1}y^5 = \frac{y^5}{x}$

Bases to the power of **<u>more than one term</u>** can be expanded and written as a product.

1. $2^{x+1} = 2^x \times 2$ 2. $3^{2x} = 3^x.3^x$ 3. $5^{1-2x} = 5^1.5^{-2x}$

Examples:

1. $\frac{2^{x+2}+2^{x+3}}{12.2^x}$
$= \frac{2^x.2^2+2^x.2^3}{12.2^x}$ **expand**
$= \frac{2^x(4+8)}{12.2^x}$ **common factor**
$= \frac{12}{12} = 1$

2. $\frac{8^a.2^a+2.16^{a+1}}{11.2^{a+1}}$
$= \frac{(2^3)^a.2^a+2.(2^4)^{a+1}}{11.2^a.2^1}$
$= \frac{2^{3a}.2^a+2^{4a+4+1}}{11.2^a.2}$
$= \frac{2^{4a}+2^{4a}.2^5}{2^a.11\times2}$
$= \frac{2^{4a}(1+32)}{2^a.22} = \frac{3}{2}2^{3a}$

3. $\frac{9^x+3^x-2}{9^x-4}$
$= \frac{3^{2x}+3^x-2}{3^{2x}-4}$ …**prime bases**
$= \frac{(3^x+2)(3^x-1)}{(3^x+2)(3^x-2)}$ … factorise
Trinomial on numerator
Difference of two squares
On denominator
$= \frac{(3^x-1)}{(3^x-2)}$

ACTIVITIES/ ASSESSMENT

Simplify the following:

1. $2x^5 \times (-3x^3y)^2$
2. $\frac{7x^2y^5}{14\ ^3y} \times \frac{36x^3y}{6y}$
3. $\frac{(-2xy^3)^3}{4xy^4\times2x^2y^5}$
4. $\left(\frac{3p^3q^2}{9p^5}\right)^3$
5. $\left(\frac{6x^7}{12x^9}\right)^{-2}$
6. $\frac{2^{x+2}-2^{x=1}}{2^x+2^{x+2}}$
7. $\frac{3^{x+1}+3^{x+2}}{8.3^{x+1}}$
8. $\frac{4^x+3.2^{2x+1}}{7.2^{2x+1}}$
9. $\frac{12^x+4^x.3^{x+1}}{2^{2x+4}.3^x}$
10. $\frac{2.3^x+3^{x-2}}{5.2^{x+1}-7.3^{x-1}}$
11. $\frac{9^x-3^x-6}{3^x-3}$

TOPIC: EXPONENTS AND SURDS (Lesson 2)		Weighting	12 ± 3	Grade	11
Term	I	Week no.			
Duration	1 hour	Date			
Sub-topics	Simplify Expressions with Rational Exponents				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Rational numbers

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Write $x^{m+n} as x^m + x^n$

METHODOLOGY

The laws of exponents can also be extended to include the **rational numbers**. A rational number is any number that **can be written** as a **fraction** with an integer in the numerator and in the denominator.

Definition: $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$

Examples:

Simplify without using a calculator:

1. $8^{\frac{2}{3}}$

$= (2^3)^{\frac{2}{3}}$ ... base as a prime number

$= 2^2$ ...multiply exponents

$= 4$

2. $4^{-\frac{3}{2}}$

$= (2^2)^{-\frac{3}{2}}$

$= 2^{-3}$

$= \frac{1}{2^3} = \frac{1}{8}$

3. $\sqrt[4]{\left(\frac{625}{81}\right)^3}$

$= \sqrt[4]{\left(\frac{5^4}{3^4}\right)^3}$

$= \left(\frac{5^4}{3^4}\right)^{\frac{3}{4}}$

$= \frac{125}{27}$

4. $\left(\frac{81x^{-3}y^4}{16xy^{-4}}\right)^{-\frac{3}{4}}$

$= \left(\frac{81x^{-3-1}y^{4+4}}{16}\right)^{-\frac{3}{4}}$ ... same bases

$= \left(\frac{3^4x^{-4}y^8}{2^4}\right)^{-\frac{3}{4}}$ ... prime numbers

$= \frac{3^{-3}x^3y^{-6}}{2^{-3}}$ ... multiply exponents

$= \frac{2^3x^3}{3^3y^6}$ ...

$= \frac{8x^3}{27y^6}$

5. $\sqrt[3]{\frac{27a^3b^6}{64c^9}}$

$= \left(\frac{3^3a^3b^6}{4^3c^9}\right)^{\frac{1}{3}}$

$= \frac{3ab^2}{4c^3}$

6. $\sqrt{\frac{15^x.3^x}{9^{x+1}.5^{x-2}}}$

$= \sqrt{\frac{(3\times5)^x.3^x}{(3^2)^{x+1}.5^{x-2}}}$

$= \sqrt{\frac{3^x5^x3^x}{3^{2x+2}5^{x-2}}}$

$= \sqrt{3^{x+x-2x-2}5^{x-x+2}}$

$= \sqrt{3^{-2}5^2}$

$= (3^{-2}5^2)^{\frac{1}{2}} = 3^{-1}.5 = \frac{5}{3}$

ACTIVITIES/ ASSESSMENT

Simplify without the use of a calculator

1. $32^{\frac{3}{5}}$

2. $27^{-\frac{2}{3}}$

3. $\sqrt[3]{\left(\frac{25}{8}\right)^2}$

4. $\left(\frac{8a^6b^{-2}}{27\ ^{-3}b}\right)^{-\frac{3}{2}}$

5. $\sqrt[3]{\frac{64x^3y^6}{27z^9}}$

6. $\sqrt[5]{\frac{8^{x+1}.2^{x+2}}{16^x}}$

TOPIC: EXPONENTS AND SURDS (Lesson 3)		Weighting	12 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Simplifying Exponential Expressions Involving Rational Exponents				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Write x^{m+n} as $x^m + x^n$

METHODOLOGY

Expressions with **ONE** term.

- Exponential expressions that involve multiplication and division
- Rewrite all bases in terms of prime numbers
- Apply Laws of Exponents

Examples:

1. $\frac{2^x.12^{x-1}.3^{2x}}{6^{3x-1}}$

$=\frac{2^x.(3\times2^2)^{x-1}.3^{2x}}{(2\times3)^{3x-1}}$ ... prime numbers

$=\frac{2^x.3^{x-1}.2^{2x-2}.3^{3x}}{2^{3x-1}.3^{3x-1}}$

$=2^{x+2x-2-3x+1}.3^{x-1+2x-3x+1}$ ...law of exponents

$=2^{-1}.3^0$

$=\frac{1}{2}.1=\frac{1}{2}$

2. $\frac{2^{4x+1}.9^x.6^{2x-1}}{12^{3x}.3^x}$

$=\frac{2^{4x-1}.(3^2)^x.(2\times3)^{2x-1}}{(3\times2^2)^{3x}.3^x}$

$=\frac{2^{4x-1}.3^{2x}.2^{2x-1}.3^{2x-1}}{3^{3x}.2^{6x}.3^x}$

$=2^{4x-1+2x-1-6x}.3^{2x+2x-1-3x-x}$

$=2^{-2}.3^{-1}$

$=\frac{1}{2^2}\times\frac{1}{3}=\frac{1}{12}$

Expressions with **more than one** terms.

- Expressions that involve addition and subtraction
- Separate exponents
- Take out a common factor

Examples:

Simplify Fully:

1. $3^{x+1}-3^x+3^{x-1}$

$=3^x.3^1-3^x+3^x.3^{-1}$ ... separate exponents

$=3^x\left(3-1+\frac{1}{3}\right)$ ... take out common factor

$=3^x\left(\frac{7}{3}\right)$

2. $2^{2x}+2^x$

$=2^x.2^x+2^x$

$=2^x(2^x+1))$

3. $\frac{2.5^{n+1}+3.5^{n+2}}{4.5^{n+1}-3.5^n}$

$=\frac{2.5^n.5+3.5^n.5^2}{4.5^n.5-3.5^n}$ ... separate exponents

$=\frac{5^n(2.5+3.25)}{5^n(4.5-3)}$ ... take out common factor

$=\frac{85}{17}=5$

4. $\frac{2^{2x}-2^x-6}{2^x+2}$ ... quadratic expression

$=\frac{(2^x+2)(2^x-3)}{2^x+2}$ factors of numerator

$=2^x-3$

5. $\frac{5^{2x}+5^{x+1}-6}{5^x+6}$

$=\frac{5^{2x}+5^x.5-6}{5^x+6}$

$=\frac{(5^x-1)(5^x+6)}{5^x+6}$

$=5^x-1$

6. $\frac{4.3^{2m}-5^{4p}}{2.3^m-5^{2p}}$ ... difference of two squares

$=\frac{(2.3^m-5^{2p})(2.3^m+5^{2p})}{23^m-5^{2p}}$

$=2.3^m+5^{2p}$

ACTIVITIES/ ASSESSMENT

Simplify Fully:

1. $2^{x+2}-2^{x-1}$

2. $\frac{5^{n+1}-5^{n-1}}{3.5^{n-2}}$

3. $\frac{4.3^{2x+2}-3^{x-1}.3^{x+2}}{9.3^{2x-2}}$

4. $\frac{2^{2x}-9}{2^x+3}$

5. $\frac{9.4^x-9^{x+1}}{3^{x+1}-3.2^x}$

6. $\frac{3^{2x}-3^x-6}{3^x-3}$

7. $\frac{4^x+2^{x+2}+3}{2^x+3}$

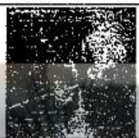
8. $\frac{x^{\frac{2}{5}}+x^{\frac{1}{5}}-2}{x^{\frac{1}{5}}-1}$

TOPIC: EXPONENTS AND SURDS (Lesson 4)	Weighting	12 ± 3	Grade	11
Term	1	Week no.		
Duration	1 hour	Date		
Sub-topics	Equations of Exponents			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Write $x^{m+n} as x^m + x^n$

METHODOLOGY

Most of the time, in an exponential equation, **the variable is in the exponent.**
Also take note that $a^x > 0$for all real values of x where $a > 0$ and $a \neq 1$

Equations with **ONE** term on each side of the equation.
This type of equation is using the basic premise that if $a^x = a^b, then\ x = b, for\ a \neq 0$

- To solve, express **both sides of the equation** with the **same base** so that we can equate the exponents

Examples:

1. $9^{x+1} = 27^x$

$(3^2)^{x+1} = (3^3)^x$ ... prime number bases
$3^{2x+2} = 3^{3x}$
$2x + 2 = 3x$
$x = 2$

2. $\sqrt{4^x} = \frac{1}{8^{x+1}}$

$4^{\frac{x}{2}} = 8^{-x-1}$ ... apply $\sqrt[q]{x^p} = x^{\frac{p}{q}}$ and $x^{-m} = \frac{1}{x^m}$
$(2^2)^{\frac{x}{2}} = (2^3)^{-x-1}$
$2^x = 2^{-3x-3}$
$x = -3x - 3$
$4x = -3$
$x = -\frac{3}{4}$

3. $3.5^x = 0{,}6$

$5^x = 0{,}2$ ... divide by 3 both sides
$5^x = \frac{2}{10} = \frac{1}{5}$
$5^x = 5^{-1}$
$x = -1$

4. $4.3^{7x} = 9.2^{7x}$

$\frac{3^{7x}}{2^{7x}} = \frac{9}{4} = \frac{3^2}{2^2}$
$\left(\frac{3}{2}\right)^{7x} = \left(\frac{3}{2}\right)^2$ ... apply $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
$7x = 2$
$x = \frac{2}{7}$

Equations with **more** than one term on each side of the equation

- Collect the terms with exponents on one side and the constants on the other side.
- Separate exponents (exponents with two terms or more)
- Take out a common factor
- If one of the same bases has an exponent **twice** the other or the **negative** of the other, this

indicates a **quadratic equation.**

Examples:

1. $2^{x+2} - 2^x = 12$

$2^x.2^2 - 2^x = 12$ ... separate exponents
$2^x(4-1) = 12$ ... common factor
$2^x = 4$ ... divide by 3 both sides
$2^x = 2^2$
$x = 2$

2. $3^{2x} + 6.3^x - 27 = 0$

Let $3^x = k$
$k^2 + 6k - 27 = 0$ ... $3^{2x} = (3^x)^2$
$(k+9)(k-3) = 0$ ...factors
$k = -9\ or\ k = 3$
$3^x = 3\ or\ 3^x = -9$... substitute $k = 3^x$
$x = 1$ N/A ($3^x > 0\ for\ all\ values\ of\ x$)

3. $3.3^{2x} - 4.3^x = -1$

$3.3^{2x} - 4.3^x + 1 = 0$ ... standard form
Let $3^x = k$
$3k^2 - 4k + 1 = 0$
$(3k-1)(k-1) = 0$
$3k = 1\ or\ k = 1$
$k = \frac{1}{3} or\ k = 1$
$3^x = \frac{1}{3}\ or\ 3^x = 1$
$3^x = 3^{-1} or\ 3^x = 3^0$
$x = -1 or\ x = 0$

4. $4^x - 32.4^{-x} + 4 = 0$

$4^x - \frac{32}{4^x} + 4 = 0$
Let $4^x = k$
$k - \frac{32}{k} + 4 = 0$
$k^2 - 32 + 4k = 0$ ...multiply terms by LCD
$k^2 + 4k - 32 = 0$ ... standard form
$(k-4)(k+8) = 0$
$k = 4 or\ k = -8$
$4^x = 4\ or\ 4^x = -8$
$x = 1$ N/A ($4^x cannot\ be\ negative$)

ACTIVITIES/ ASSESSMENT

Solve for x

1. $3.2^x = 24$
2. $4^{x-1} = \frac{1}{8}$
3. $9^{x-1} = 27^x.3^{x+2}$
4. $16^x.2^{x+1} = \sqrt[5]{4}$
5. $9^x - 3^{2x-1} = 24$
6. $3^x - 3^{x-2} = 24$
7. $4^{x+1} - 64 = 0$
8. $5.2^x + 3.2^{x+2} = 68$
9. $9^x - 4.3^{x+1} + 27 = 0$
10. $2.2^x - 8.2^{-x} - 15 = 0$

TOPIC: EXPONENTS AND SURDS (Lesson 5)		Weighting	12 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Equations involving Rational Exponents				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Solving for

METHODOLOGY

A rational exponent indicates a power in the numerator and a root in the denominator. There are multiple ways of writing an expression, a variable, or a number with a rational exponent:

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m = \sqrt[n]{a^m}$$

To solve equations with rational exponents:

- isolate the variable that has a fractional exponent
- raise both sides to the reciprocal power
- express the number to the product of its prime numbers
- if the exponent of the variable is even number divided by odd number, the answer will be " ± "

Examples:

Solve for x

1. $x^{\frac{3}{5}} = 27$

$x^{\frac{3}{5}\times\frac{5}{3}} = 3^{3\times\frac{5}{3}}$ ... raise both sides to reciprocal power

$x = 3^5 = 343$

2. $2x^{\frac{2}{3}} = 32$

$x^{\frac{2}{3}} = 16$ ... divide by 2 both sides

$x^{\frac{2}{3}\times\frac{3}{2}} = \pm 2^{4\times\frac{3}{2}}$ ... raise to reciprocal power

$x = \pm 2^6 = \pm 64$

3. $x^{-\frac{3}{2}} = 64$

$x^{-\frac{3}{2}\times-\frac{3}{2}} = 2^{6\times-\frac{3}{2}}$

$x = 2^{-9} = \frac{1}{2^9}$

4. $x^{\frac{5}{4}} = -32$, **No Solution**

It is not possible to have the even root of something equal to a negative value.

5. $-8x^{-\frac{3}{2}} = 1$

$x^{-\frac{3}{2}} = -\frac{1}{8}$

It is impossible to have the square root of

Something equal to a negative value.

NO SOLUTION

6. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 8 = 0$

$\left(x^{\frac{1}{3}}\right)^2 - 2x^{\frac{1}{3}} - 8 = 0$ ... quadratic equation

$\left(x^{\frac{1}{3}} - 4\right)\left(x^{\frac{1}{3}} + 2\right) = 0$

$x^{\frac{1}{3}} = 4 \text{ or } x^{\frac{1}{3}} = -2$

$x^{\frac{1}{3}\times 3} = 4^3 \text{ OR } x^{\frac{1}{3}\times 3} = (-2)^3$

$x = 64 \text{ or } x = -8$

ACTIVITIES/ ASSESSMENT

Solve for x

1. $2x^{\frac{5}{4}} + 10 = 74$

2. $x^{\frac{2}{3}} = 16$

3. $x^{\frac{3}{2}} = -64$

4. $x^{-\frac{5}{3}} = 32$

5. $3x^{\frac{3}{5}} = -24$

6. $-16x^{-\frac{2}{3}} + 4 = 0$

7. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 8 = 0$

8. $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} = 10$

9. $x - 3x^{\frac{1}{2}} + 2 = 0$

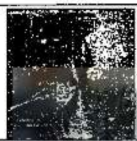
10. $x - 5\sqrt{x} + 4 = 0$

TOPIC: EXPONENTS AND SURDS (Lesson 6)		Weighting	12 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Add, Subtract, Multiply and Divide Simple Surds				

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Write $x^{m+n} as x^m + x^n$

METHODOLOGY

A **surd** is the root of a whole number that produces an irrational number

An **irrational number** is a number that cannot be expressed as an integer or as a fraction

LAWS OF SURDS	EXAMPLES
1. $\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{a.b}$ $for\ n\epsilon N;\ n\geq 2;\ a,b\geq 0$	• $\sqrt{5}\times\sqrt{2}=\sqrt{5.3}=\sqrt{15}$ • $\sqrt{20}=\sqrt{4}\times\sqrt{5}=2\sqrt{5}$
2. $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $for\ n\epsilon N; n\geq 2; a\geq 0; b>0$	• $\sqrt{\frac{16}{25}}=\frac{\sqrt{16}}{\sqrt{25}}=\frac{4}{5}$ • $\frac{\sqrt{3}}{\sqrt{18}}=\sqrt{\frac{3}{18}}=\sqrt{\frac{1}{6}}$
3. $\sqrt[p]{\sqrt[q]{a}}=\sqrt[p\times q]{a}$ $for\ a\geq 0; p,q\geq 2$	• $\sqrt{\sqrt[3]{25}}=\sqrt[6]{5^2}=5^{\frac{2}{6}}=5^{\frac{1}{3}}=\sqrt[3]{5}$ • $\sqrt[6]{36}=\sqrt[6]{6^2}=6^{\frac{2}{6}}=6^{\frac{1}{3}}=\sqrt[3]{6}$
4. $\left(\sqrt[p]{a}\right)^q=\sqrt[p]{a^q}$ $for\ p\epsilon N; p\geq 2; a\geq 0$	• $\left(\sqrt{2}\times\sqrt{3}\times\sqrt{5}\right)^2=\left(\sqrt{2\times 3\times 5}\right)^2=\left(\sqrt{30}\right)^2=30$ • $\left(\sqrt{3b}\right)^3=\sqrt{3^3b^3}=\sqrt{27b^3}$

Examples:

Simplify the following without using a calculator:

1. $\sqrt{48}$
$=\sqrt{16\times 3}$ ...
$=\sqrt{16}\times\sqrt{3}$
$=4\sqrt{3}$

2. $\sqrt{2}+\sqrt{8}$
$=\sqrt{2}+\sqrt{4\times 2}$
$=\sqrt{2}+2\sqrt{2}$
$=3\sqrt{2}$ ... like terms

3. $2\sqrt{8}-4\sqrt{32}+3\sqrt{50}$
$=2\sqrt{4\times 2}-4\sqrt{16\times 2}+3\sqrt{25\times 2}$
$=2\times 2\sqrt{2}-4\times 4\sqrt{2}+3\times 5\sqrt{2}$
$=4\sqrt{2}-16\sqrt{2}+15\sqrt{2}$
$=3\sqrt{2}$ ... like surds

4. $\frac{\sqrt{12}\times\sqrt{24}}{\sqrt{8}}$
$=\frac{\sqrt{4\times 3}\times\sqrt{4\times 6}}{\sqrt{4\times 2}}$
$=\frac{2\sqrt{3}\times 2\sqrt{6}}{2\sqrt{2}}$
$=\frac{4\sqrt{3\times 6}}{2\sqrt{2}}$
$=2\sqrt{\frac{18}{2}}=2\sqrt{9}=2\times 3=6$

5. $\frac{\sqrt{80}-\sqrt{48}}{\sqrt{20}-\sqrt{12}}$
$=\frac{\sqrt{16\times 5}-\sqrt{16\times 3}}{\sqrt{4\times 5}-\sqrt{4\times 3}}$
$=\frac{4\sqrt{5}-4\sqrt{3}}{2\sqrt{5}-2\sqrt{3}}$
$=\frac{4(\sqrt{5}-\sqrt{3})}{2(\sqrt{5}-\sqrt{3})}$ ... common factor
$=2$

6. $\left(2-\sqrt{3}\right)^2+\left(2+\sqrt{3}\right)^2$
$=\left(2-\sqrt{3}\right)\left(2-\sqrt{3}\right)+\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)$
$=4-2\sqrt{3}-2\sqrt{3}+3+4+2\sqrt{3}+2\sqrt{3}+3$
$=7-4\sqrt{3}+7+4\sqrt{3}$
$=14$

ACTIVITIES/ ASSESSMENT

Simplify the following without using a calculator:

1. $\sqrt{54}$
2. $\sqrt{27}-\sqrt{3}$
3. $\sqrt{18}-\sqrt{50}-\sqrt{32}$
4. $\frac{\sqrt{50}+\sqrt{2}}{\sqrt{18}}$
5. $\frac{6\sqrt{8}+4\sqrt{18}}{3\sqrt{32}-\sqrt{72}}$
6. $\left(\sqrt{12}+\sqrt{27}\right)^2$
7. $\frac{\sqrt{32x}-\sqrt{18x}}{\sqrt{8x}}$
8. $3\sqrt{18x^2}+2\sqrt{32x^2}+\sqrt{50x^2}$

TOPIC: EXPONENTS AND SURDS (Lesson 7)		Weighting	12 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Simple Surd Equations				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

- If $x \geq 0$, then the expression $\sqrt{x}$ will be a real number.
- If $x < 0$, then the expression $\sqrt{x}$ will be non-real number.

To solve this equation:

- Isolate the square root
- Square both sides
- Check/test the solutions to ensure that the square root does not equal a negative number.

Examples:

Solve for x:

1. $\sqrt{x-1} = 3$
   $\left(\sqrt{x-1}\right)^2 = (3)^2$
   $x - 1 = 9$
   $x = 10$

2. $\sqrt{x-4} = -5$
   Square root is equal to a negative number
   NO SOLUTION

3. $2x + \sqrt{2-7x} = 0$
   $\sqrt{2-7x} = -2x$ … isolate square root
   $\left(\sqrt{2-7x}\right)^2 = (-2x)^2$
   $2 - 7x = 4x^2$
   $0 = 4x^2 + 7x - 2$ … standard form
   $4x^2 + 7x - 2 = 0$
   $(4x-1)(x+2) = 0$ … factorise
   $x = \frac{1}{4}$ or $x = -2$
   $x = \frac{1}{4}$ is not a solution $x = -2$

4. $\sqrt{x+3} - x = 1$
   $\sqrt{x+3} = x + 1$
   $\left(\sqrt{x+3}\right)^2 = (x+1)^2$
   $x + 3 = x^2 + 2x + 1$
   $0 = x^2 + 2x - x + 1 - 3$
   $x^2 + x - 2 = 0$
   $(x+2)(x-1) = 0$
   $x = -2$ or $x = 1$
   $x = -2$ is not a solution $x = 1$

ACTIVITIES/ ASSESSMENT

Solve for x:

1. $\sqrt{x} = 3$
2. $\sqrt{x-4} = 5$
3. $\sqrt{x+1} = -2$
4. $\sqrt{5x+6} = x$
5. $\sqrt{3x-2} - x = 0$
6. $x + \sqrt{-4x-3} = 0$
7. $\sqrt{x-1} = x - 3$
8. $\sqrt{x+5} + x - 1 = 0$
9. $\sqrt{x+14} - x = 2$
10. $\sqrt{x+8} + 4 = x$
11. $2x = \sqrt{2x+3} + 9$
12. $\sqrt{2x+4} + 5 = x + 3$

TEST: EXPONENTS AND SURDS

FROM PAST PAPERS

MARKS: 25 **Duration: 30 Min**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1 [10 Marks]

Simplify the following expressions **(Write answers with positive exponents)**

1.1 $\frac{2^x \times 8^{x+2}}{4^{3x} \times 2^{-2x}}$ (4)

1.2 $\frac{4.3^{x+1} - 6.3^{x-1}}{3^{x+1} - 3^x}$ (3)

1.3 $\frac{\sqrt{27m^6} - \sqrt{48m^6}}{\sqrt{12m^6}}$ (3)

QUESTION 2 [11 Marks]

Solve for x:

2.1 $x^{\frac{2}{3}} = 4$ (2)

2.2 $x^{-\frac{3}{4}} = 8$ (3)

2.3 $2^{x+2} + 2^x = 20$ (3)

2.4 $\sqrt{2 - x} = x + 4$ (3)

QUESTION 3 [4 Marks]

Show that $\frac{2}{1+\sqrt{2}} - \frac{8}{\sqrt{8}} = 2$ (4)

TOPIC: EQUATIONS AND INEQUALITIES (Lesson 1)		**Weighting**	18 ± 3	Grade	11
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Solving Quadratic Equations by Factorisation**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Zero-factor law, standard form				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Facrorisation: common factor, difference of two squares, quadratic trinomial

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The first step of factorising an expression is to **'take out' any common factors** which the terms have.

Common Factor

Examples:

1. $5m^3 - 10m^2 + 15m = 5m(m^2 - 2m + 3)$

2. $3x(2a - 1) - 5(1 - 2a) = 3x(2a - 1) + 5(2a - 1)$ … $[(1 - 2a) = -(2a - 1)]$
$= (2a - 1)(3x + 5)$

Difference of two squares

To spot a difference of two squares, look for expressions:

- consisting of two terms; made of one square number minus another square number
- to factorise, open two brackets with same terms and different signs between the terms

Examples:

1. $4x^2 - 25 = (2x + 5)(2x - 5)$

5. $32a^2 - 98) = 2(16a^2 - 49)$ .. common factor
$= 2(4a + 7)(4a - 7)$

A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where x is a variable and a, b and c are non-zero constants.

There is **no simple method of factorising a quadratic expression**, but with a little practise it becomes easier.

Examples:

1. $x^2 + 2x - 8$
$= (x + 4)(x - 2$

2. $3x^2 - 21x - 24$
$= 3(x^2 - 7x - 8)$ ... common factor
$= 3(x - 8)(x + 1)$

4. $4x^2 - 19x + 12$
$= 4x^2 - 3x - 16x + 12$ … split up $19x$ into two numbers whose multiple is 48, the product of $-3x \times -16x$
$= x(4x - 3) - 4(4x - 3)$ … common factor first two terms and last two terms
$= (4x - 3)(x - 4)$ ... common factor

A quadratic equation is an equation where the exponent of the variable is at most 2 with at most two real solutions and has a standard form $ax^2 + bx + c = 0,\ where\ a, b\ and\ c\ are\ constants\ and\ a \neq 0$
There are some situations, however, in which a quadratic equation has either **one solution or no solutions.**
The **quadratic expression** on the left-hand side has to be factorised.
Zero-factor law states that if $a \times b = 0\ then\ a = 0\ or\ b = 0$
Solutions of a quadratic equation are called **roots.** The roots of a quadratic equation are the **values of x that satisfy the equation**, i.e., that will make the equation true.

To solve a Quadratic Equation:

- Write the equation in a standard form
- Factorise the equation
- Apply Zer0-factor law

Examples:
Solve for x

1. $x^2 + 2x - 8 = 0$
   $(x + 4)(x - 2) = 0$
   $x + 4 = 0\ or\ x - 2 = 0$
   $x = -4\ or\ x = 2$

2. $2x^2 + x = 3$
   $2x^2 + x - 3 = 0$ ... standard form
   $(2x + 3)(x - 1) = 0$ ... factors
   $2x + 3 = 0\ or\ x - 1 = 0$
   $x = -\frac{3}{2}\ or\ x = 1$

ACTIVITIES/ ASSESSMENT

A. Factorise fully:

1. $12x + 32y$	2. $2xy^2 + xy^2z + 3xy$
3. $4(y - 3) + k(3 - y)$	4. $4x^2 - 1$
5. $49x^4 - 16$	6. $16k^2 - (b - 5)^2$
7. 1. $x^2 + 8x + 15$	8. $2x^2 + 5x - 3$
9. $4x^2 + 10x - 6$	10. $2x^2 - 22x + 20$
11. $6x^2 - 15x - 9$	12. $4p^2 + 7pq - 2p^2$
13. $10m^2 13mn - 3n^3$	14. $a^3 - 6a^2b + 9ab^2$
15. $(a + b)^2 + 8(a + b) - 32$	

B. Solve for x:

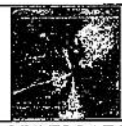
1. $x^2 + 8x + 15$	2. $2x^2 + 5x - 3$
3. $6x^2 - 15x - 9$	4. $12x^2 - 20x + 3$

TOPIC: EQUATIONS AND INEQUALITIES (Lesson 2)		Weighting	18 ± 3	Grade	11
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Solve quadratic equations by completing the Square**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES






ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Completing the square is a method used to

- rewrite a quadratic expression of the form $ax^2 + bx + c$ in the form $\boldsymbol{a(x + p)^2 + q}$
- easily determine the **minimum or maximum value** (q) of a quadratic expression
  - $If\ a > 0, then\ a(x + p)^2 + q\ has\ a\ \boldsymbol{minimum}\ value\ (q)$
  - $If\ a < 0, then\ a(x + p)^2 + q\ has\ a\ \boldsymbol{maximum}\ value\ (q)$

Minimum and Maximum value a Quadratic Expression

Example:
Write $x^2 - 4x + 3$ in the form $a(x + p)^2 + q$ and then write down the minimum or maximum value.

$x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + 3$ ... add and subtract $(half\ the\ coefficient\ of\ x)^2$
$x^2 - 4x + (-2)^2 - 4 + 3$
$(x - 2)(x - 2) - 1$ ... factorise first three terms of the expression and simplify the next 2
$(x - 2)^2 - 1$

Minimum value $(a > 0)$: $q = -1$

When Solving by Completing the Square

- The coefficient of $\boldsymbol{x^2}$ must be 1
- Write the equation $ax^2 + bx + c = 0$ as $\boldsymbol{ax^2 + bx = -c}$
- Add $\boldsymbol{(half\ the\ coefficient\ of\ x)^2}$ to both sides of the equation
- **Factorise** the left-hand side and **simplify** the right-hand side
- Find the square root of both sides

Examples:
Solve by completing the square

1. $x^2 - 4x - 5 = 0$
   $x^2 - 4x = 5$ ... write equation in the as $\boldsymbol{ax^2 + bx = -c}$
   $x^2 - 4x + \left(-\frac{5}{2}\right)^2 = 5 + \left(-\frac{5}{2}\right)^2$ ... Add $\boldsymbol{(half\ coefficient\ of\ x)^2}$ to both sides of the equation
   $\left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = 5 + \frac{25}{4}$ ... **Factorise** the left-hand side and **simplify** the right-hand side

$\left(x-\frac{5}{2}\right)^2=\frac{45}{4}$

$x-\frac{5}{2}=\pm\frac{\sqrt{45}}{2}$ ... square root of both sides

$x-\frac{5}{2}=\frac{\sqrt{45}}{2}$ or $x-\frac{5}{2}=-\frac{\sqrt{45}}{2}$

$x=\frac{5+3\sqrt{5}}{2}$ or $x=\frac{5-3\sqrt{5}}{2}$ ... leaving answer in simple surd form

$x=5{,}85$ or $x=-0{,}85$ ... leaving answer correct to TWO decimal places

2. $-2x^2-12x+14=0$

$\frac{-2x^2}{-2}-\frac{12}{-2}x+\frac{14}{-2}=0$ ... divide by the coefficient of x^2 all the terms (coefficient of x^2 must be 1)

$x^2+6x-7=0$
$x^2+6x=7$

$x^2+6x+\left(\frac{6}{2}\right)^2=7+\left(\frac{6}{2}\right)^2$

$(x+3)(x+3)=7+\frac{36}{4}$

$(x+3)^2=16$
$x+3=\pm4$
$x+3=4$ or $x+3=-4$
$x=1$ or $x=-7$

ACTIVITIES/ ASSESSMENT

Solve the following equations by Completing the Square:

1. $x^2+4x-5=0$
2. $x^2-8x-6=0$ (leave answer in simple surd form)
3. $2x^2+9x-26=0$
4. $-3x^2-11x=9$
5. Show by completing the square that the solutions to equation $ax^2-bx-a=b$ are $x=1$ and $x=\frac{a+b}{a}$
6. Solve for $ax^2+bx+c=0$ by completing the square. Leave the answer in surd form.

TOPIC: EQUATIONS AND INEQUALITIES (Lesson 3)	Weighting	18 ± 3	Grade	11
Term	1	Week no.		
Duration	1 hour	Date		
Sub-topics	Solving Quadratic Equations by using a Quadratic Formula			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES




ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Using the **quadratic formulae** to solve quadratic equations where factorising cannot be used.
The solutions (roots) of any quadratic equation in standard form $ax^2+bx+c=0$ where $a\neq0$ can be determined using the formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\ \text{... QUADRATIC FORMULA}$$

When using the quadratic formula:

- ensure that the equation is in the standard form $(ax^2+bx+c=0)$
- substitute carefully for a, b *and* c

Example

Solve for x using the quadratic formula:

1. $2x^2+9x-6=0$ (correct to 2 decimal places)

$a=2, b=9, c=-6$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(9)\pm\sqrt{(9)^2-4(2)(-6)}}{2(2)}$ ... put brackets when substituting

$x=\frac{-9\pm\sqrt{129}}{4}$

$x=0{,}59$ or $x=-5{,}09$

ACTIVITIES/ ASSESSMENT

Solve for x using the quadratic formula:

1. $x^2+7x-5=0$ (2 decimal places)
2. $x^2-2x-1=0$ (Simplest surd form)
3. $3x^2-27=0$
4. $3x^2+2=9x$ (2 decimals)
5. $(x-2)(x+4)=7$ (2 decimals)
6. $x(x+2)+5=0$ (2 decimals

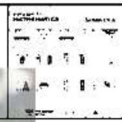
TOPIC: EQUATIONS AND INEQUALITIES (Lesson 4)		Weighting	18 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Solving Quadratic Inequalities				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Linear inequalities, Quadratic equations

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Inequalities are solved using methods similar to those used to solve equations.

The only **difference between inequalities and equations** is the sign ($>, <, \geq, \leq$), and everything you can do to an equation, you can also do to an inequality.

The **inequality sign** tells us whether the graph is **positive or negative**

When **dividing or multiplying** by a **negative number** to solve an inequality, **reverse** (change direction of) the inequality sign.

Solutions to inequalities are represented on a number line, table or graph.

Linear Inequalities

Examples:

Solve for x and represent the answers on a number line:

1. $2x + 3 < x - 5$
   $2x - x < -5 - 3$
   $x < -8$

2. $\frac{2x+4}{7} \geq \frac{3(x-3)}{3}$
   $6x + 12 \geq 21(x-3)$ ... multiply by LCD
   $6x + 12 \geq 21x - 63$
   $-15x \geq -75$
   $x \leq 5$ ... divide by -15 both sides

Quadratic Inequalities

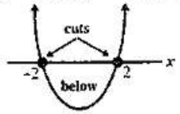
A quadratic inequality involves **determining the values of** x for which the graph of a parabola **lies either above or below the x-axis.**

Represent solutions of a quadratic inequality, by using **the graphical method** (parabola graph).

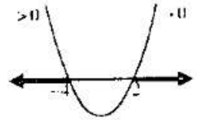
Examples:

Solve the following inequalities and represent the answers graphically:

1. $x^2 \leq 4$
   $x^2 - 4 \leq 0$ ... standard form
   $(x-2)(x+2) \leq 0$ ... factorise
   $x = 2$ or $x = -2$ ... critical values

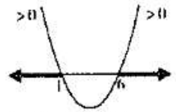


2. $x^2 + 2x - 8 \geq 0$
   $(x-2)(x+4) \geq 0$ ... factorise
   $x = 2$ or $x = -4$ ... critical values



Solution is where the graph cut the x-axis (x-intercepts) .... and **above** the x-axis ($x^2 + 2x - 8 \geq 0$)
And **below** the x-axis ($x^2 - 4 \leq 0$)
$-2 \leq x \leq 2$ $\qquad$ $x \leq -4$ or $x \geq 2$

3. $-x^2 + 5x - 6 < 0$ (the coefficient of x^2 is -1)
   $x^2 - 5x + 6 > 0$ (When multiplying or dividing an inequality by a **negative number, the inequality sign** changes direction)
   $(x+1)(x-6) > 0$ ... factorise
   $x = -1$ or $x = 6$ ... critical values



$x < -1$ or $x > 6$

ACTIVITIES/ ASSESSMENT

A. Solve the following inequalities and represent answers on a number line:

1. $x + 15 \leq 6 - 2x$
2. $4(x-1) > 6(x-1)$
3. $4(x - 3 - 2(x-1) \geq 0$
4. $\frac{x+5}{3} < 1$

B. Solve the following inequalities and represent the answers graphically:

1. $x^1 > 1$
2. $4x^2 \leq 9$
3. $x^2 - 4x + 3 < 0$
4. $x^2 - 2x - 3 \geq 0$
5. $-x^2 + 5x + 3 \leq 6$
6. $-x^2 + 4x < 0$

TOPIC: EQUATIONS AND INEQUALITIES (Lesson 5)		Weighting	18 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Solving Quadratic Inequalities				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Quadratic equations

RESOURCES

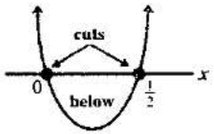


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

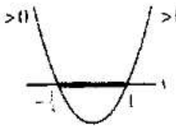
METHODOLOGY

Examples:

1. $x - 2x^2 \geq 0$
   $2x^2 - x \leq 0$ ... standard form
   $x(2x - 1) \leq 0$ ... factorise
   $x = 0$ or $x = \frac{1}{2}$ ... critical values



2. $3x^2 - 2x \leq 1$
   $3x^2 - 2x - 1 \leq 0$ ... standard form
   $(3x + 1)(x - 1) \leq 0$ ... factorise
   $x = -\frac{1}{3}$ or $x = 1$ ... critical values



ACTIVITIES/ ASSESSMENT

Solve the following inequalities and represent the answers graphically:

1. $2x^2 - 5x - 3 < 0$
2. $2x^2 - 3x + 1 > 0$
3. $-3x^2 + 4x \leq -4$
4. $3x + 9 \geq 2x^2$
5. $3x^2 - 2x \leq 0$
6. $(1 - 2x)(x + 3) < 0$
7. $(x - 3)(x - 4) > 12$
8. $9 > -x(x - 6$
10. $x^2 \leq 5$ (surd form)
10. $x^2 - 3x - 2 > 0$ (Surd form)

TEST 1: EQUATIONS AND INEQUALITIES

MARKS: 25 **DURATION: 30 Min**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1 [14 Marks]

Solve for x:

1.1 $(x + 2)(3x - 7) = 0$ (2)

1.2 $x^2 - 7x + 12 = 0$ by completing the square (4)

1.3 $x^2 - 5x = 2$ (Correct to 2 decimal places) (4)

1.4 $6x - 7 = \frac{4}{4}$ (4)

QUESTION 2 [11 Marks]

2.1 Given:

$x^2 - 3x \leq 40$ and $-4x + 3 < -2$

2.1.1 Solve for x if $x^2 - 3x \leq 40$ (4)

2.1.2 Solve for x if $-4x + 3 < -2$ (2)

2.1.3 If it is given that x is a natural number, solve for x if $x^2 - 3x \leq 40$ and $-4x + 3 < -2$ (2)

2.2 Given: $m + \frac{1}{m} = 3$

Determine the value of $m^2 - 1 + \frac{1}{m^2}$ (3)

TOPIC: ALGEBRA PART 3 (Lesson 6)	Weighting	18 ± 3	Grade	11
Term	1	**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Simultaneous equations**			

RELATED CONCEPTS/ TERMS/VOCABULARY: Algebraically, Graphically, System of equations, simultaneously

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Variables, linear equations, quadratic equations, exponential equations, substitution

RESOURCES




ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Forget to solve for the second variable, multiplying the second term in the bracket

METHODOLOGY

When solving for **two** unknown variables, two independent equations are required and these equations are known as **simultaneous equations.**

In this case, one of the equations will be **linear** and the other equation **quadratic**.

The solutions are the values of the unknown variables which **satisfy both equations simultaneously.**

Simultaneous equations can be solved **algebraically** using **substitution method.** System of simultaneous equations can be solved **graphically** (to determine where the two lines intersect).

Substitution Method

- **Use the simplest** of the two given equations (usually **linear equation**) to express one of the variables in terms of the other variable.
- Substitute into the second equation (usually **quadratic equation**). By doing this we reduce the number of equations and the number of variables by one.
- We now have one equation with one unknown variable which can be solved.
- Use the solution to substitute back into the first equation to find the value of the other unknown variable.

Examples:

Solve for x *and* y simultaneously in the following set of equations:

1. $y - 2x = -4$ and $x^2 + y = 4$

$y = 2x - 4$ ...make y the subject of the formula and substitute to the second equation.

$x^2 + (2x - 4) = 4$ ... substitute (y *by* $2x - 4$)

$x^2 + 2x - 4 - 4 = 0$ ... quadratic equation **must** be equal to 0

$x^2 + 2x - 8 = 0$ ... standard form

$(x - 2)(x + 4) = 0$ ... factorise

$\mathbf{x = 2}$ *or* $\mathbf{x = -4}$ (Substitute back into the first equation to find the value(s) of y)

$y = 2(2) - 4$

$\mathbf{y = 0}$

Or $y = 2(\mathbf{-4}) - 4$

$\mathbf{y = -12}$

As coordinates: (2; 0) and (-4; -12)

2. $2x + 3y = 7$ and $y = x^2 - 3x + 1$ (correct to 2 decimal places)

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$3y = 7 - 2x$

$y = \frac{7-2x}{3}$ ... go substitute to 2[nd] equation

$\frac{7-2x}{3} = x^2 - 3x + 1$

$7 - 2x = 3x^2 - 9x + 3$ ... multiply all terms by LCD

$3x^2 - 7x - 4 = 0$ ... standard form

$a = 3, b = -7$ *and* $c = -4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ... quadratic formula

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-4)}}{2(3)}$ ... remember to put brackets when substituting

$\mathbf{x = 2,81}$ *or* $\mathbf{x = -0,47}$ ... go substitute to equation 1

$y = \frac{7-2(2,81)}{3}$

$y = \frac{23}{50} = \mathbf{0,46}$

$y = \frac{7-2x}{3}$

$y = \frac{7-2(-0,47)}{3}$

$y = \frac{397}{150} = \mathbf{2,65}$

ACTIVITIES/ ASSESSMENT

Solve for x and y simultaneously (correct to 2 decimal places where necessary)

1. $y + x = 5$
   $y - x^2 + 3x - 5 = 0$

2. $y = x + 2$
   $x + xy = 4$

3. $2x + y = 1$
   $xy - x^2 + y^2 = 5$

4. $2x - y = 2$
   $4x - 2x^2 = x - 4$

5. $3x = y + 4$
   $y^2 - xy = 9x + 7$

6. $2x = 3y$
   $x^2 - xy + y^2 = 7$

7. $2x - 3y = 1$
   $x^2 - 2x - 2y^2 + 4y = 9$

8. $x - 2y + 3 = 0$
   $(x - y)(x + y) = 0$

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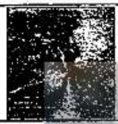
TOPIC: EQUATIONS (Lesson 7)		Weighting	18 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	**Nature of Roots**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Roots, real, non-real, rational, irrational				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Quadratic Equations, quadratic formula

RESOURCES






ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The **roots of an equation** refer to the solutions of that equation or are the values of the variables that satisfy that equation.

Roots are **classified** according to three criteria:

- Real or non-real
- Rational or irrational
- Equal (two equal solutions) or unequal (two different solutions)

The nature of the roots is determined by $b^2 - 4ac$ taken from the quadratic equation $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$b^2 - 4ac$ is called **discriminant** and the Greek letter Δ is used to represent it.

SQUARE ROOT	$\Delta = b^2 - 4ac$	NATURE	EXAMPLE
$\sqrt{negative\ number}$	$\Delta < 0$	**Non-real**	$\sqrt{-25}, \sqrt{-100}$
$\sqrt{perfect\ square}$	$\Delta \geq 0$	Unequal, real and **rational**	$\sqrt{25} = 5, \sqrt{\frac{9}{4}} = \frac{3}{2}$
$\sqrt{not\ a\ perfect\ square}$	$\Delta \geq 0$	Unequal real and **irrational**	$\sqrt{3} = 1{,}732, \sqrt{15} = 3{,}8729$
$\sqrt{zero}$	$\Delta = 0$	**Equal**, real and rational	$\sqrt{0} = 0$

Examples:

Determine the nature of the roots of the following equations:

1\. $x^2 - 5x - 6 = 0$

$a = 1, b = -5\ and\ c = -6$

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4(1)(-6)$

$\Delta = 49$ ... **perfect square**

Roots are Unequal, real and **rational**

2\. $2x^2 + 3x - 7 = 0$

$a = 2, b = 3\ and\ c = -7$

$\Delta = b^2 - 4ac$

$= (2)^2 - 4(2)(-7)$

$\Delta = 65$ ... **not a perfect square**

Roots are unequal, real and **irrational**

3\. $x^2 + 3x + 4 = 0$

$a = 1, b = 3\ and\ c = 4$

$\Delta = b^2 - 4ac$

$= (3)^2 - 4(1)(4)$

$\Delta = -7$ ... negative number

Roots are **non-real/unreal/imaginary**

4\. $4x^2 - 4x + 1 = 0$

$a = 4, b = -4\ and\ c = 1$

$\Delta = b^2 - 4ac$

$= (-4)^2 - 4(4)(1)$

$\Delta = 0$ ... zero

Roots are **equal**, real and rational

ACTIVITIES/ ASSESSMENT

Determine the nature of the roots of the following equations:

1\. $x^2 + x + 1 = 0$

2\. $x^2 - 3x - 1 = 0$

3\. $x^2 - 2x + 1 = 0$

4\. $x^2 - 5x - 7 = 0$

5\. $x^2 - x + 3 = 0$

6\. $4x + x^2 - 1 = 0$

7\. $6x^2 - 6x - 1 = 0$

8\. $x^2 + 3x = -2$

9\. $2x^2 + 6 = -7x$

10\. $-2x^2 - 16x - 32 = 0$

11\. $x^2 = 36$

12\. $x^2 = 4x$

13\. $x^2 = 2(x + 1)$

TOPIC: EQUATIONS (Lesson 8)		Weighting	18 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Nature of Roots				

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES




ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

1. Show that the roots of $x^2 - 2x = 7$ are irrational.

For roots to be rational, calculate discriminant $(b^2 - 4ac)$
$a = 1, b = -2 \text{ and } c = -7$
$\Delta = b^2 - 4ac$
$= (-2)^2 - 4(1)(-7)$
$\Delta = 32$
$\Delta > 0$ and is not a perfect square, $\therefore$ roots are irrational.

2. Prove that $x^2 + px + p = 1$ are rational for all rational values on p.
$x^2 + px + (p - 1) = 0$ … standard form

For roots to be real and equal $\Delta = 0$
$a = 1, b = p \text{ and } c = p - 1$
$\Delta = b^2 - 4ac$
$= (p)^2 - 4(1)(p - 1)$
$= p^2 - 4p + 4$
$= (p - 2)(p - 2)$
$\Delta = (p - 2)^2$ … perfect square
$\therefore$ roots are rational

3. For which value(s) of m will $x^2 + 4x + m = 0$ have unequal real roots?

$a = 1, b = 4 \text{ and } c = m$
$\Delta = b^2 - 4ac$
$= (4)^2 - 4(1)(m)$
$= 16 - 4m$
For unequal roots $\Delta > 0$
$\therefore 16 - 4m > 0$
$-4m > -16$
$m < 4$

4. For which value(s) of k will $2(x + 1) = x^2 + k + x$ have non-real roots?

$2x + 2 = x^2 + x + k$ … simplify
$x^2 + x - 2x + k - 2 = 0$ … transpose
$x^2 - x + (k - 2) = 0$ … standard form

$a = 1, b = -1 \text{ and } c = k - 2$
$\Delta = b^2 - 4ac$
$= (-1)^2 - 4(1)(k - 2)$
$= 1 - 4k + 8$
$= 7 - 4k$
For real roots $\Delta < 0$
$\therefore 7 - 4k < 0$
$-4k < -7$
$k > \frac{7}{4}$

ACTIVITIES/ ASSESSMENT

1. Show that the roots of $x^2 - (m + 2)x + m = 0$ will be real and unequal for all real values of m.
2. Prove that $x^2 + k = -(k - 1)x$ has rational roots for all rational values of k.
3. For which values of k will $3x^1 - 3x + k = 0$ have equal root?
4. Determine the value(s) of r for which $x^2 - 3rx + r = 0$ has real roots.
5. Calculate the value(s) of d for which $5x^2 + 6x - d = 0$ non-real roots.
6. If $f(x) = 0$ has roots $x = \frac{-5\pm\sqrt{3-12k^2}}{4}$, for which values of k will have equal roots.
7. The roots of the equation $f(x) = 0$ are $x = \frac{4\pm\sqrt{16-4m(-m+5)}}{2m}$.
Determine the value(s) for which the roots will be non-real.
8. Given: $P = \sqrt{\frac{5}{x+2}} + \frac{x}{5}$
(a) For which value(s) of x will P be a real number?
(b) Show that P is rational if $x = 3$

TEST: SIMULATIONS EQUATIONS AND NATURE OF ROOTS

MARKS: 19 **DURATION: 25 MIN**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

1. Solve for x and y simultaneously:

   $3y+x=2$ and $y^2+x=xy+y$ (5)

2. Discuss the nature of the roots of $x^2=4x$. (4)

3. For which values of k will the equation $x^2+x+2=3x-k$ have non-real roots? (5)

4. Show that the roots of the equation $mx(x-4)=-4m$ are equal for all real values of m (5)

MEMO

1.	$3y+x=2$ $x=2-3y$	√ A
	$y^2+x=xy+y$ $y^2+(2-3y)=y(2-3y)+y$	√ CA Sub
	$y^2+2-3y=2y-3y^2+y$ $4y^2-6y+2=0$ $2y^2-3y+1=0$	√ Std form
	$(2y-1)(y-1)=0$	√ A
	$y=\frac{1}{2}$ or $y=1$	√ A Both
	$x=\frac{1}{2}$ or $x=-1$	√ A Both

2.	$x^2=4x$ $x^2-4x=0$	√ std form
	$\Delta=(-4)^2-4(1)(0)$ $=16$	√ CA (sub) √ A
	$\therefore$ Roots are real; rational and unqual.	√ CA (reason)
3.	$x^2+x+2=3x-k$ $x^2-3x+2-k=0$	√ std form
	$\Delta=(-4)^2-4(1)(2+k)$ $=4-8-4k$ $-4-4k$	√ CA (sub) √ A
	$\Delta<0$ $-4-4k<0$ $-4k<4$ $k>-1$	√ CA <0 √ A
4.	$mx(x-4)=-4m$ $mx^2-4mx+4m=0$	√ std form
	$\Delta=(-4m)^2-4(m)(4m)$ $=16m^2-16m^2$ $=0$	√ CA (sub) √ A
	$\Delta=0$ $\therefore$ The roots are always equal.	√ CA (reason)

TOPIC: EUCLIDEAN GEOMETRY (Lesson 1)		Weighting	50 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Revise lines and angles, Triangles and Quadrilaterals				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Types and properties of angles, triangles and quadrilaterals

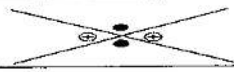
RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

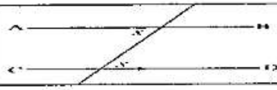
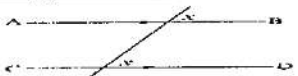
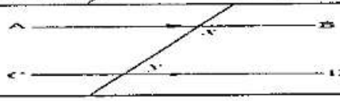
Assuming that lines are parallel. Using congruency conditions without understanding

METHODOLOGY

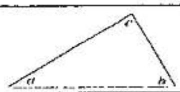
Intersecting Lines

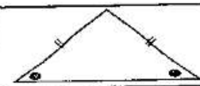
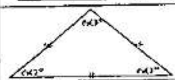
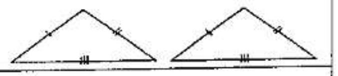
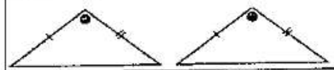
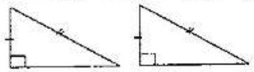
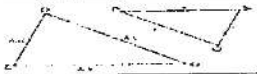
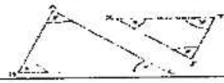
The **sum** of the angles around a point is 360°.	$a+b+c=360°$ 
Adjacent angles at a point on a line segment are **supplementary**.	$\hat{B}_1+\hat{B}_2=180°$ 
When two lines intersect, the **vertically opposite angles** are **equal**.	

Parallel lines

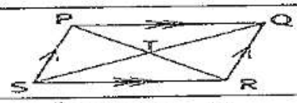
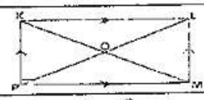
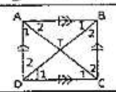
When a transversal cuts **parallel lines**, the **alternate** angles are **equal**.	
When a transversal cuts two **parallel lines**, the **corresponding angles** are **equal**.	
When a transversal cuts **parallel lines**, the **co-interior** angles are **supplementary**.	

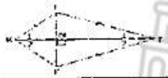
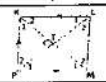
Triangles

The **interior angles** of a triangle are **supplementary**.	$a+b+c=180°$ 

The **angles opposite two equal sides** of an isosceles triangle **are equal**.	
All **interior** angles of an equilateral triangle are **60°**.	
An **exterior angle** of a triangle is equal to the sum of the opposite interior angles	$c=a+b$ 
When the **sides** of one triangle are equal to the to the three sides of the other triangle, the two triangles are **congruent**. [SSS]	
Two triangles are **congruent** when **two sides** and the **included angle** are equal to two sides and the included angle of the other triangle. [SAS]	
When **two angles** and **a side** of one triangle are equal to two angles and the corresponding side of another triangle, the two triangles are **congruent**. [AAS]	
Two triangles are **congruent** when the **hypotenuse** and **one side** of a **right-angled** triangle are equal to the hypotenuse and one side of another right-angled triangle. [90°HS]	
The line segment joining the **midpoints** of two sides of a triangle is **parallel to the third side** and **half the length of the third side**. [DE‖DB]	
When the **corresponding sides** of two triangles are in the same ratio, the two triangles are **similar**. [SSS]	
When the **corresponding angles** of two triangles are equal, the two triangles are **similar**. [AAA]	

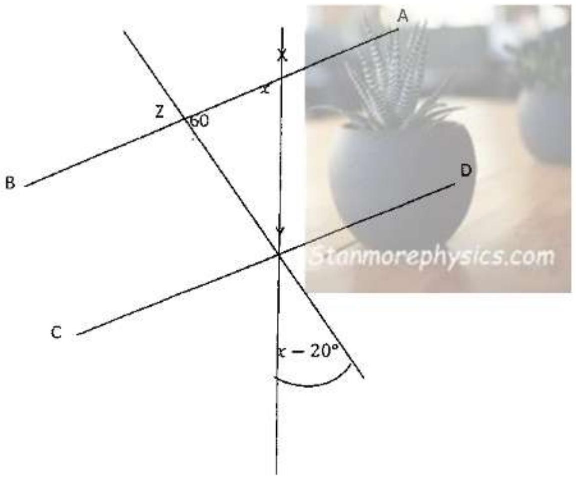
Quadrilaterals

A **parallelogram** is a quadrilateral with opposite sides parallel. [PQ‖SR, PS‖QR]	
A **rectangle** is a parallelogram with a 90° interior angle. [∠KPM = 90°]	
An isosceles **trapezium** is a quadrilateral with one pair opposite sides parallel and the other pair equal. [PQ‖SR, PS = QR]	
A **square** is a rhombus with a 90° interior angle. [AB = BC = CD = DA]	

A **kite** is a quadrilateral with two pairs of adjacent sides equal. [KI = KE, ET = TI]	
A **rhombus** is a parallelogram with all sides equal. [KL = LM = MP = PK]	

ACTIVITIES/ ASSESSMENT

1. In the diagram below, AB and DC are two parallel lines cut by two transversal lines at X, Y and Z respectively.



(a) Determine giving reasons, the value of x in the diagram:

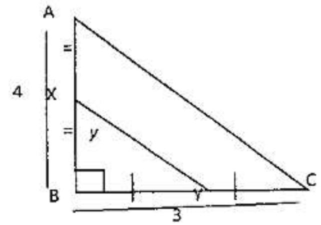
(b) Name one pair of co-interior angles

(c) Name one pair of alternate angles

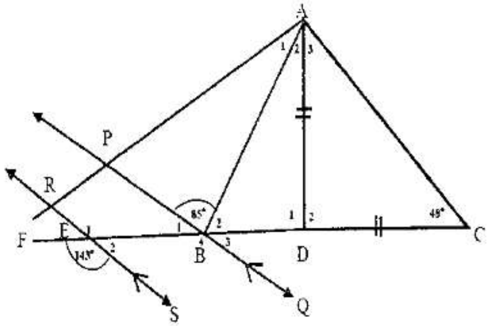
(d) Complete: If two parallel lines are cut by a transversal, then the co-interior angles are ....

(e) Complete: The size of angle XYD = Reason ....

2. Determine with reasons, the value of y (XY) in the diagram below. Given that AB = 4 units and BC = 3 units. X and Y are the midpoints of AB and BC respectively.



3. In the diagram below, AD = CD and PQ||RS. AR and FC are straight lines. RS and FC intersect at E also PQ intersects FC at B.



(a) Determine the sizes of the following angles, giving appropriate reasons:

1) $\hat{D}_1$

2) $\hat{B}_1$

3) $\hat{A}_2$

(b) Show that $R\hat{E}F = \hat{B}_3$

TOPIC: EUCLIDEAN GEOMETRY (Lesson 2)	Weighting	50 ± 3	Grade	11
Term	1	Week no.		
Duration	1 hour	Date		
Sub-topics	Circle Geometry: Line from Centre to chord			
RELATED CONCEPTS/ TERMS/VOCABULARY	Chord, perpendicular bisector			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Circle, diameter, radius, circumference, Congruent, Pythagoras theorem

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

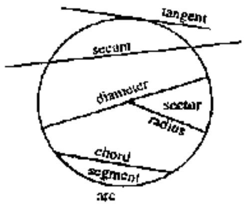
Proving congruency and reasons for congruency

Failing to write reasons correctly/ write wrong reasons, sometimes do not understand reasons

METHODOLOGY

Terminology

- **Chord** — a straight line joining the ends of an arc.
- **Circumference** — the perimeter or boundary line of a circle.
- **Radius** (r) — any line from the centre of the circle to a point on the circumference.
- **Secant** ----- a line that cuts the circle in two places.



A **theorem** is a hypothesis (proposition) that can be shown to be true by accepted mathematical operations and arguments.

A **proof** is the process of showing a theorem to be correct.

The **converse** of a theorem is the reverse of the hypothesis and the conclusion.

Theorem:

The line drawn from the centre of a circle **perpendicular** to a chord **bisects the chord.**

Given: Circle with centre O with OM ⊥ AB. AB is a chord

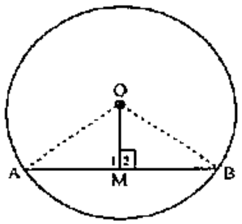
Required to prove (RTP): AM = MB

Proof:

Join OA and OB

In Δ OAM and Δ OBM

1. $OA = OB$	Radii
2. $\hat{M}_1 = \hat{M}_2 = 90°$	Given
3. $OM = OM$	Common
∴ Δ OAM ≡ Δ OBM	HRS
∴ AM = MB	



Reason: **Perpendicular from centre to chord**

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Converse:

1. The line drawn from the centre of a circle to the **midpoint** of a chord **is perpendicular** to the chord.

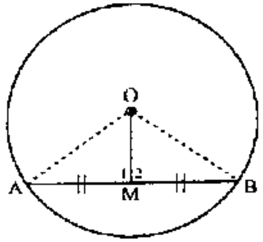
Given: Circle with centre O. M is a point on AB such that AM = MB

Required to prove: OM ⊥ AB

Proof: Join OA and OB

In Δ OAM and Δ OBM

1. OA = OB	radii
2. AM = BM	given
3. OM = OM	common
∴ Δ OAM ≡ Δ OBM	SSS
$\hat{M}_1 = \hat{M}_2 = 90°$	AMB is a straight line
∴ OM ⊥ AB	



Reason: **Line from centre midpoint of chord**

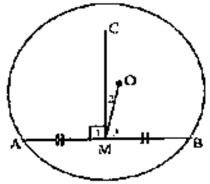
Theorem:

The **perpendicular bisector** of a chord **passes through the centre** of the circle.

CM ⊥ AB cutting AB at M, AM = BM

If $\hat{M}_3 = 90°$, $\hat{M}_2 = 0°$

then O lies on CM



Reason: **Perpendicular bisector of chord**

Examples:

1. Given a circle with centre O with PR = 8 units. Determine the value of x.

PQ = QR = 4 units …. **Perpendicular from centre to chord**

In Δ OPQ

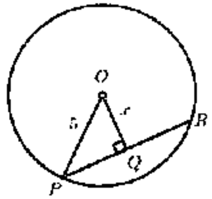
$OP^2 = OQ^2 + PQ^2$ … Pythagoras Theorem

$5^2 = x^2 + 4^2$

$25 - 16 = x^2$

$9 = x^2$

$x = 3$ units



2. O is the centre. AC = 16, AB = BC and OA = 17. Calculate the length of OB.

AB = BC = 8 **Line from centre midpoint of chord**

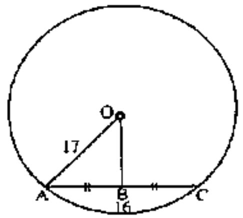
$OB^2 = AO^2 - AB^2$ Pythagoras Theorem

$= 17^2 - 8^2$

$= 289 - 64$

$OB^2 = 225$

$OB = 15$



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3. Given the circle with centre O. AB = 6 cm, CD = 8 cm and the radius is 5 cm.

(a) Determine the length of:

1) OP

AP = 3 cm Line from centre to midpoint of chord

OA = 5 cm Radius

$OP^2 = OA^2 - AP^2$ Pythagoras Theorem

$OP^2 = 5^2 - 3^2 = 16$

$OP = 4cm$

2) PQ

CQ = 4 cm Line from centre to midpoint of chord

OC = 5 cm Radius

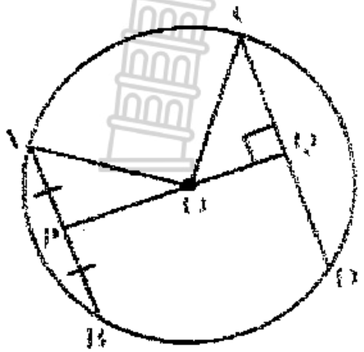
$OQ^2 = OC^2 - CQ^2$ Pythagoras Theorem

$OP^2 = 5^2 - 4^2 = 9$

$OP = 3\ cm$

$PQ = OP + OQ$

$= 4\ cm + 3\ cm = 7\ cm$



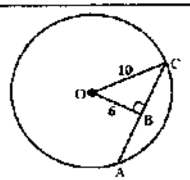
(b) Explain why AB // CD.

$\hat{P}_1 + \hat{Q}_1 = 180°$ ... both angles are equal to 90° each

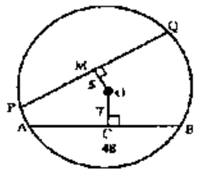
$\therefore$ AB//CD ... co-interior angles supplementary

ACTIVITIES/ ASSESSMENT

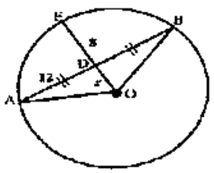
1. O is the centre. OB = 6, OC = 10 and OB ⊥ AC. Calculate the length of AC.



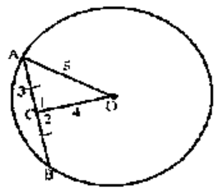
2. O is the centre of the circle. PQ and AB are both chords of the circle with OM ⊥ PQ and OC⊥AB. OM = 5cm, OC = 7cm and AB = 48cm. Calculate the length of:
   (a) the radius of the circle.
   (b) PQ.



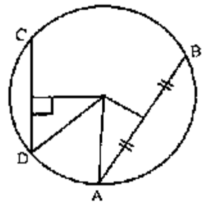
3. AB is a chord of circle centre O. OE bisects AB. AD = 12cm, ED = 8cm and OD = x.
(a) Determine the radius OB in terms of x.
(b) Hence, calculate the length of the radius OB.



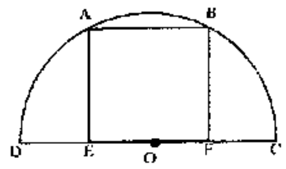
4. AB is a chord of the circle. AC = CB, and the length of the radius is 5 units. AC = 3 units and OC= 4 units. Show that OC ⊥ AB and explain why OC passes through the centre O



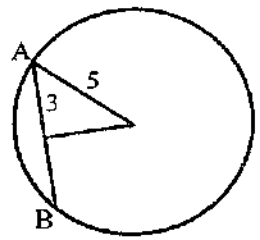
5. O is the centre. AB = 8 cm, OF = 3 cm, OE = 4 cm, AF = FB and CD ⊥ OE. Calculate the length of chord CD.



6. The radius of the semi-circle centre O is 5 cm. A square is fitted into the semi-circle as shown in the diagram. Calculate the area of the square. (Hint: Let the length of the square equal x.)



7. AB is a chord of the circle. AC = CB, and the length of OA is 5 units. AC = 3 units and OC = 4 units. Show that OC ⊥ AB and explain why OC passes through the centre O.



TOPIC: EUCLIDEAN GEOMETRY (Lesson 3)		Weighting	50 ± 3	Grade	11
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Circle Geometry: Angles subtended by a chord/arc				
RELATED CONCEPTS/ TERMS/VOCABULARY	Subtends, arc, segment, chord, semi-circle Converse of a theorem, corollary				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Exterior angle of a triangle

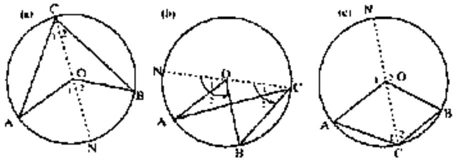
RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

- **Arc** — a portion of the circumference of a circle.
- **Chord** — a straight line joining the ends of an arc.
- **Segment** — part of the circle that is cut off by a chord. A chord divides a circle into two segments.
- **Subtend** — side opposite an angle subtends that angle

➢ **Theorem:**
The angle that an **arc** of a circle **subtends at the centre** of the circle is **twice** the angle it **subtends at any point on the circumference**



Given: Circle with centre O. Arc AB subtends $A\hat{O}B$
At centre and $A\hat{C}B$ at the circumference.

Required to prove (RTP): $A\hat{O}B = 2A\hat{C}B$

Proof: Join CO and produce to N

$\hat{O}_1 = \hat{C}_1 + \hat{A}$ .... Ext ∠ of Δ OAB

But $\hat{C}_1 = \hat{A}$ .... AO = OC, Radii

$\therefore \hat{O}_1 = 2\hat{C}_1$

Similarly, in Δ OCB: $\hat{O}_2 = 2\hat{C}_2$

Diagram (a) and (c)

$\hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$

$= 2(\hat{C}_1 + \hat{C}_2)$

$\therefore A\hat{O}B = 2A\hat{C}B$

Diagram (b)

$\hat{O}_2 - \hat{O}_1 = 2\hat{C}_2 - 2\hat{C}_1$

$= 2(\hat{C}_2 - \hat{C}_1)$

$\therefore A\hat{O}B = 2A\hat{C}B$

Reason: ∠ **at centre** = 2 × ∠ **at circum**

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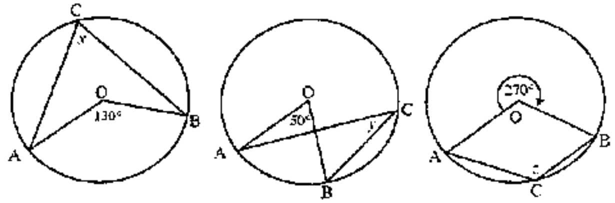
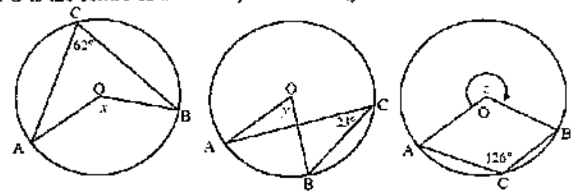
Examples:

1. O is the centre of the circle, determine *x*, *y* and *z*



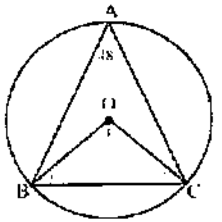
$x = 65°$ ∠ **at centre** = 2 × ∠ **at circum**
$y = 25°$ ∠ **at centre** = 2 × ∠ **at circum**
$z = 135°$ ∠ **at centre** = 2 × ∠ **at circum**

2. O is the centre of the circle, determine *x*, *y* and *z*



$x = 124°$ ∠ **at centre** = 2 × ∠ **at circum**
$y = 46°$ ∠ **at centre** = 2 × ∠ **at circum**
$z = 252°$ ∠ **at centre** = 2 × ∠ **at circum**

3. In the given diagram, O is the centre of the circle. $B\hat{A}C = 48°$
Determine with reasons the size of:



(a) $\hat{O}_1$

$\hat{O}_1 = 2\hat{A} = 96°$ ... ∠ **at centre** = 2 × ∠ **at circum**

(b) $\hat{B}_1$

$\hat{B}_1 = \hat{C}_1$ ... ∠s opposite = sides

$2\hat{B}_1 + 96° = 180°$ ... ∠s of Δ

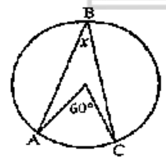
$2\hat{B}_1 = 84°$

$\therefore \hat{B}_1 = 42°$

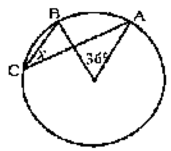
ACTIVITIES/ ASSESSMENT

1. Determine, with reasons, the value of the unknowns. O is the centre of the circle.

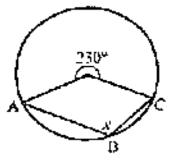
(a)



(b)



(c)



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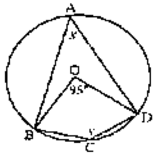
(d)



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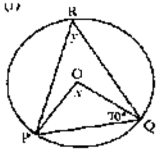

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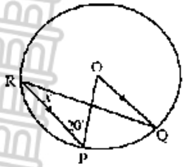
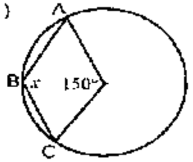
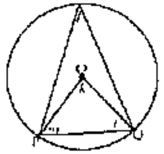


(g)



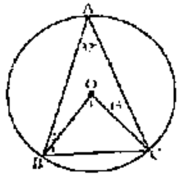
(h)



2. In the diagram alongside, O is the centre of the circle.

Determine, with reasons, the size of:

(a) $\hat{B}_1$ (b) $\hat{B}_2$

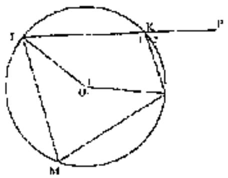


3. O is the centre of the circle, LKP is a straight line and $\hat{O}_1 = 2x$.

(a) Determine $\hat{O}_2$ and $\hat{M}$ in terms of x.

(b) Determine $\hat{R}_1$ and $\hat{R}_2$ in terms of x

(c) Determine $\hat{R}_1 + \hat{M}$. What do you notice? Write down Your observation.



TOPIC: EUCLIDEAN GEOMETRY (Lesson 4)		**Weighting**	50 ± 3	Grade	11
Term	1	**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Circle Geometry: Angles subtended by a chord/arc**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Subtends, arc, segment, chord, semi-circle Converse of a theorem, corollary				
PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE					
Exterior angle of a triangle					
RESOURCES					

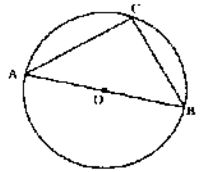
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

- **Diameter** — a **special chord** that passes through the centre of the circle.
- **Segment** — part of the circle that is cut off by a chord. A chord divides a circle into two segments.

➢ **Theorem:**
The angle **subtended by a diameter** at the circumference of a circle is a **right angle.**

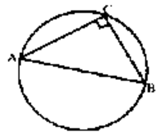
If AB is a diameter, then $\hat{C} = 90°$



Reason: ∠ **in semi-circle**

Converse: Reverse of a theorem
If the angle subtended by a chord at a point on the circle is 90°, then the chord is a diameter

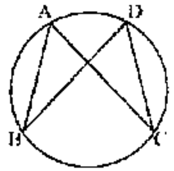
If $\hat{C} = 90°$, then AB is a diameter



Reason: **Chord subtends 90°**

➢ **Theorem:**
Angles subtended by a chord/arc at the circumference of a circle on the same side of the chord are equal; or angles in the same segment of a circle are equal.

If AD subtends $\hat{B}$ *and* $\hat{C}$,
then $\hat{B} = \hat{C}$



Reason: ∠**s in same segment**

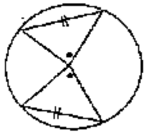
Corollaries

Corollary is a true statement that is a simple deduction from a theorem.

1. Equal chords (arcs) of a circle subtend equal angles at the circumference of a circle.

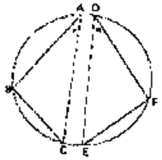
Reason:
Equal chords subt equal ∠s at circum

2. Equal chords (arcs) of a circle subtend equal angles at the centre of the circle.



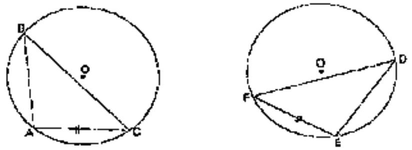
Reason:
Equal chords subt equal ∠ at centre

3. Chords (arcs) of a circle are equal when they subtend equal angles at the circumference or at the centre of the circle.



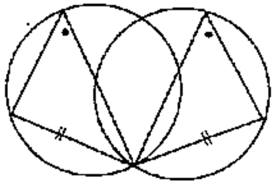
Reason:
Equal ∠s subt by equal chords

4. Equal chords (arcs) in different circles with equal radii (diameter) subtend equal angles on the circles.



Reason:
Equal chord, equal radii, equal ∠s

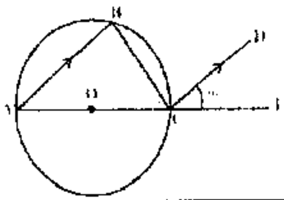
Equal chords of equal circles subtend equal angles at the circumference.



Reason:
Equal chords, equal circles, equal ∠s

Examples:

1. In the diagram alongside, O is the centre of the circle.
AB // CD and $\hat{C}_1 = 36°$
Determine with reasons, the size of:
(a) $\hat{C}_3$ (b) $\hat{A}$



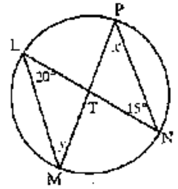
(a) $\hat{B} = 90°$ ... **∠ in semi-circle**
$\hat{C}_2 = \hat{B} = 90°$ ... alternate ∠s, AB // CD

$\hat{C}_3 = 180° - 90° - 36° = \mathbf{54°}$ ... ∠s on a straight line

(b) $\hat{A} = 36°$ ... ∠s of Δ

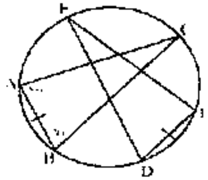
2. Calculate with reasons, the value of the unknowns:

$x = 20°$ ... **∠s in same segment** (subtended by MN)
$y = 15°$ ... **∠s in same segment** (subtended by LP)



3. In the diagram alongside, AB = DE
$\hat{A} = 80°$ and $\hat{B} = 70°$

Determine, with reasons, the size of $\hat{F}$.

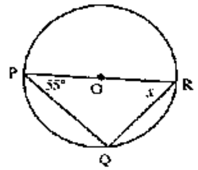


$\hat{C} = 30°$ ∠s of Δ
$\hat{F} = \hat{C} = 30°$ equal chords sub. Equal ∠s

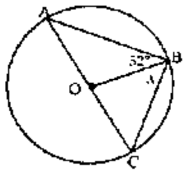
ACTIVITIES/ ASSESSMENT

1. Determine, with reasons, the value of the unknowns. O is the centre of the circle.

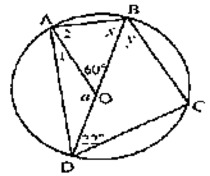
(a)



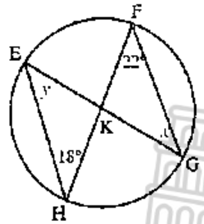
(b)



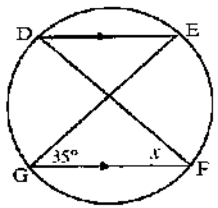
(c)



(d)



(e)



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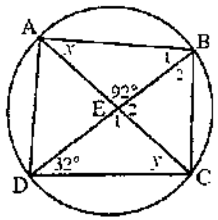
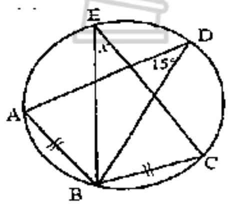
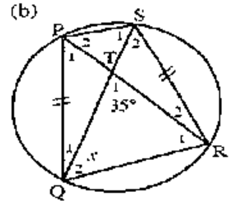


2. Calculate the value of the unknown.

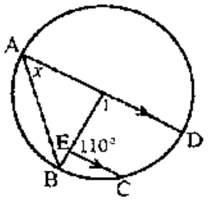
(a)



(b)



(c)



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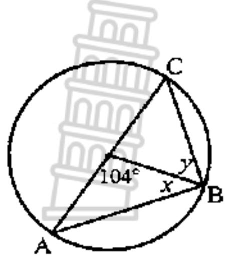

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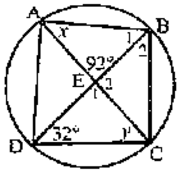

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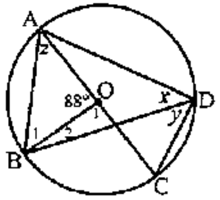


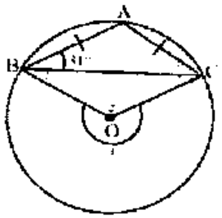
2. In the diagram alongside, O is the centre of the circle.

AB = AC and $\hat{B}_1 = 31°$

Determine, with reasons, the size of:

1) $\hat{O}_1$

2) $\hat{B}_2$



TOPIC: EUCLIDEAN GEOMETRY (Lesson 5)		Weighting	50 ± 3	Grade	10
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Circle Geometry problems and proofs of riders				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

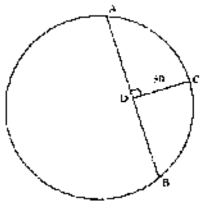



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

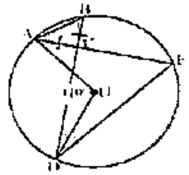
1. D is the midpoint of the chord AB and DC ⊥ AB with C on the circle. If AB = 300mm, and DC =50mm, calculate the radius of the circle.



2. In the given diagram, AC = BC.

   (a) Determine with reasons, the size of $\hat{A}_1$

   (b) Show that AB // DE

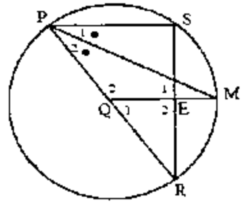


3. PR is a diameter of circle PRMS with centre Q. PS, SR and PM are chords. PM bisects $R\hat{P}S$. Prove that:

   (a) PS // QM

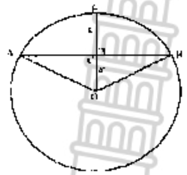
   (b) QM ⊥ SR

   (c) QM bisects SR



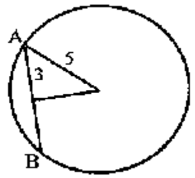
ACTIVITIES/ ASSESSMENT

1. AB is the chord of the circle with centre O and is 24cm long. C is the midpoint of AB. CE ⊥ AB cuts the circle at E. Calculate the value of x if CE = 8cm.

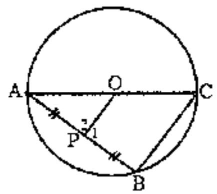


2. AB and CD are two chords of a circle with centre O. M is on AB and N is on CD such that OM ⊥ AB and ON ⊥ CD. Also, AB = 50mm, OM = 40mm and ON = 20mm. Determine the radius of the circle and the length of CD.

3. AB is a chord of the circle. AC = CB, and the length of OA is 5 units. AC = 3 units and OC = 4 units. Show that OC ⊥ AB and explain why OC passes through the centre O.

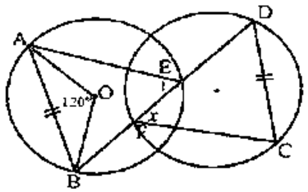


4. AC is a diameter of circle centre O. B is a point on the circle. OP bisects AB. Prove that OP || BC.



5. O is the centre of the circle ABE.

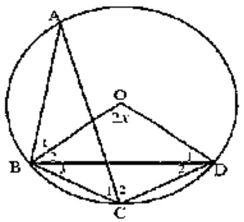
Show that $x = 60°$



6. 1. O is the centre of the circle through A, B, C and D.
BC = CD and $B\hat{O}D = 2x$.
Express the following in terms of x.

(a) $\hat{B}_2$
(b) $B\hat{O}C$

(c) $\hat{A}$



TEST 1: EUCLIDEAN GEOMETRY

Taken from Eastern Cape November 2015

MARKS: 25 DURATION: 30 MIN

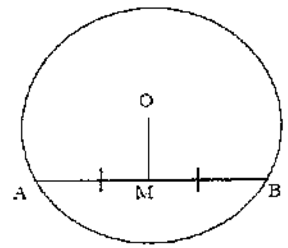
INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1 [12 Marks]

1.1 In the diagram below, AB is a chord of the circle with centre O. M is the midpoint of AB.

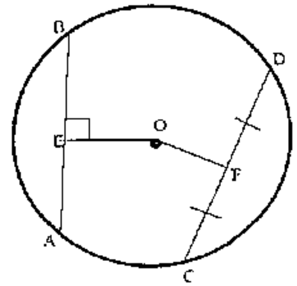
Prove the theorem that states that OM ⊥ AB.



(5)

1.2 In the figure below, AB and CD are chords of the circle with centre O.
OE ⊥ AB. CF = FD. OE = 4 cm, OF = 3 cm and CD = 8 cm.

1.2.1 Calculate the length of OD (3)
1.2.2 Hence, calculate the length of AB (4)



QUESTION 2 [13 Marks]

2.1 COMPLETE: The angle subtended by an arc at the centre of the circle is ____________ (1)

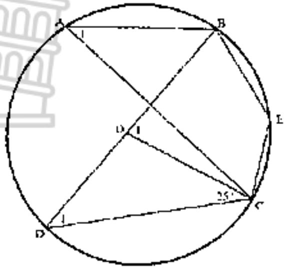
2.2 In the figure alongside, $D\hat{O}C = 25°$ and O is the centre of the circle.
A, B, E C and D are points on the circumference.
Calculate, giving reasons, the sizes of:

2.2.1 $\hat{D}_1$ (2)

2.2.2 $\hat{O}_1$ (2)

2.2.3 $\hat{A}_1$ (2)

2.2.4 $\hat{E}$ (2)



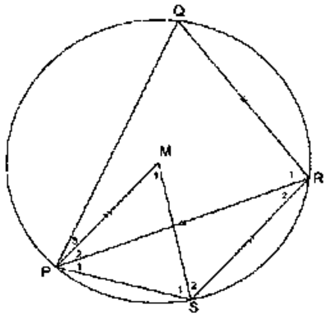
2.3 In the diagram alongside, M is the centre of circle PQRS.

PM ∥ RS, QR = PR and $\hat{R}_1 = 28°$

Determine, giving reasons, the size of the following angles:

2.3.1 $P\hat{M}S$ (2)

2.3.2 $\hat{S}_2$ (2)



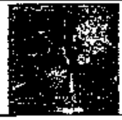
TOPIC: EUCLIDEAN GEOMETRY (Lesson 6)	Weighting	50 ± 3	Grade	11
Term	1	Week no.		
Duration	1 hour	Date		
Sub-topics	Cyclic Quadrilateral			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Quadrilateral, exterior angle, supplementary

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Treating opposite angles of a cyclic quadrilateral as opposite angles of a parallelogram

METHODOLOGY

Cyclic quadrilateral is a quadrilateral with **all vertices** (corner) lying on/touching the circumference

Theorem: The opposite angles of a cyclic quadrilateral are supplementary.

Given: Circle with centre O. ABCD is a cyclic quadrilateral
Required to prove: $\hat{A} + \hat{C} = 180°$ and $A\hat{B}C + A\hat{D}C = 180°$

Proof: Join OB and OD

$\hat{O}_1 = 2\hat{A}$ ... ∠ at centre = 2 × ∠ at circum

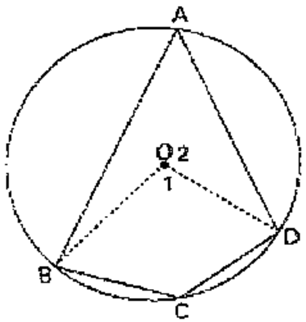
$\hat{O}_2 = 2\hat{C}$ ... ∠ at centre = 2 × ∠ at circum

$\hat{O}_1 + \hat{O}_2 = 360°$ ... ∠s round a point

$2\hat{A} + 2\hat{C} = 360°$

$2(\hat{A} + \hat{C}) = 360°$

$\therefore \hat{A} + \hat{C} = 180°$



Similarly, by joining AO and OC, it can be proven that $A\hat{B}C + A\hat{D}C = 180°$

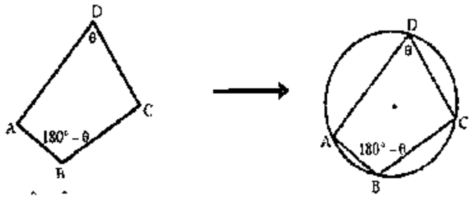
Reason: Opp. ∠s of cyclic quad.

Converse: IF the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

If $\hat{A} + \hat{C} = 180°$ and $\hat{B} + \hat{D} = 180°$, then

ABCD is a cyclic quadrilateral.

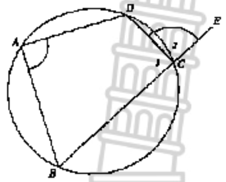
Reason: Opp. ∠s of cyclic quad suppl.



Theorem: An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

If BC is produced to E, then $\hat{C}_2 = \hat{A}$

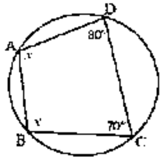
Reason: Ext. ∠ of cyclic = int opp. ∠



Examples:

1. Calculate, with reasons, the value of the unknowns

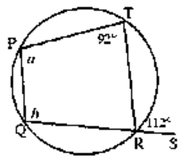
(a)



$x = 110°$ ... Opp. ∠s of cyclic quad

$y = 100°$ ... Opp. ∠s of cyclic quad

(b)



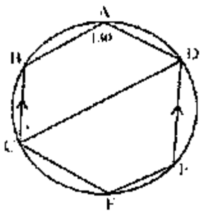
$a = 112°$ ... Ext. ∠ of cyclic = int opp. ∠

$b = 88°$ ... Opp. ∠s of cyclic

2. In the following diagram, BA // ED and $\hat{A} = 130°$

Determine, with reasons, the size of

(a) $\hat{C}_1$ (b) $\hat{F}$



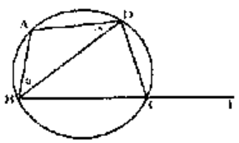
(a) $\hat{C}_1 = 50°$ ... Opp. ∠s of cyclic quad

(b) $\hat{D}_2 = \hat{C}_1 = 50°$

∴ $\hat{F} = 130°$ ... Opp. ∠s of cyclic quad

3. In the given diagram, $A\hat{B}D = 40°$ and $A\hat{D}B = 35°$

Determine, with reasons, the size of $\hat{C}_1$



$\hat{A} = 105°$ ... ∠s of Δ

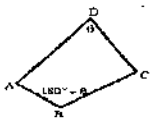
$\hat{C}_1 = \hat{A} = 105°$ ... Ext. ∠ of cyclic = int opp. ∠

PROVING THAT A QUADRILATERAL IS CYCLIC

- Prove that the opposite angles are supplementary

If $\hat{A} + \hat{C} = 180°$ and $\hat{B} + \hat{D} = 180°$, then

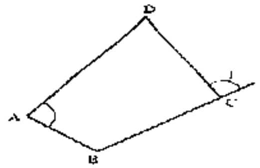
ABCD is a cyclic quadrilateral **Opp. ∠s supp**



- Prove that an exterior angle is equal to the interior opposite angle

If $\hat{C}_1 = \hat{A}$ then ABCD is a cyclic quadrilateral

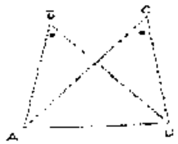
Ext. ∠ = int opp. ∠



- Prove that two points subtend equal angles at two other points on the same side.

If $\hat{B} = \hat{C}$, then A, B, C and D are concyclic i.e., ABCD is a cyclic quadrilateral

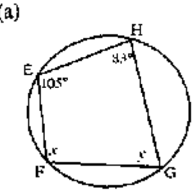
Two points subtend equal ∠s on the same side



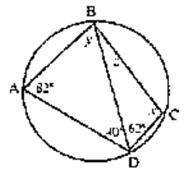
ACTIVITIES/ ASSESSMENT

1. Determine, with reasons, the value of the unknown

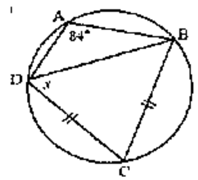
(a)



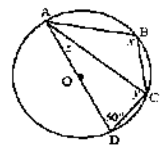
(b)



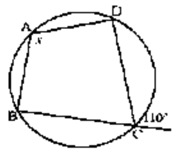
(c)



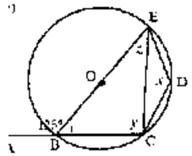
(d)



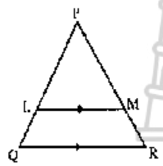
(e)



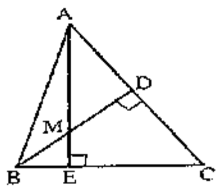
(f)



2. Show that LMRQ is a cyclic quadrilateral if PQ = PR and LM || QR



3. Write down with reasons, two cyclic quadrilateral in the diagram below.



TOPIC: EUCLIDEAN GEOMETRY (Lesson 7)		Weighting	50 ± 3	Grade	10
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Tangents to the circle				
RELATED CONCEPTS/ TERMS/VOCABULARY	Tangent				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Congruency, perpendicular

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

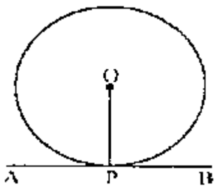
A **tangent** is a straight line that touches the circle at only one point.

Theorem:
The **radius** of a circle is **perpendicular** to the **tangent** at the point of contact.

APB is a tangent to the circle with centre O.

OP is a radius drawn to P.
Then OP ⊥ APB

Reason: rad ⊥ tan



Theorem:

Two tangents drawn to a circle from the **same point** outside the circle are **equal** in length

Given: Circle with centre O and two tangents AB and AC touching the circle at B and C respectively.

Required to prove: AB = AC

Proof: Draw radii OB and OC. Join OA

In Δ OBA and Δ OCA

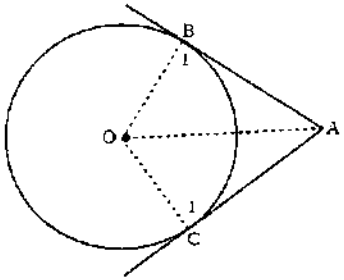
OB = OC ... Radii

$\hat{B}_1 = \hat{C}_1$ ... rad ⊥ tan

OA = OA ... Common

∴ Δ OBA ≡ Δ OCA ... RHS

∴ AB = AC

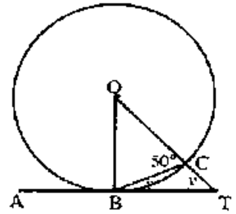


COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

Examples:

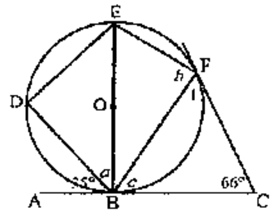
Calculate, with reasons, the value of the unknown angles.

1.



$O\hat{B}C = 50°$	Radii
$x = 40°$	tan ⊥ rad
$x + y = 50°$	ext. ∠s of Δ
$y = 10°$	

2.

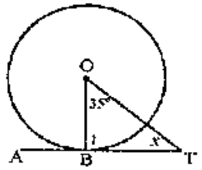


$a = 55°$	tan ⊥ rad
$b = 90°$	∠ in semi-circle
$c = \hat{F}_1$	∠s opp. Equal sides
$c = 57°$	∠s of Δ

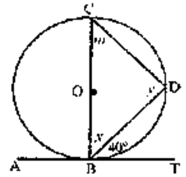
ACTIVITIES/ ASSESSMENT

Calculate, with reasons, the value of the unknown angles.

1.



2.



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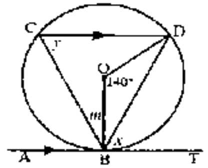

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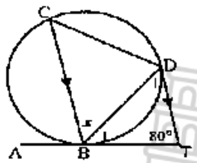

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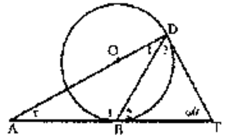
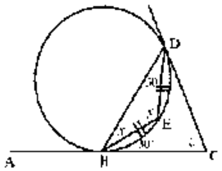
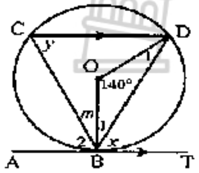


6.



7.



COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

TOPIC: EUCLIDEAN GEOMETRY (Lesson 8)		Weighting	50 ± 3	Grade	10
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Tangents to the circle				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Theorem:

The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.

Method 1

Given: Circle with centre O. Tangent ABC, chord BD

Required to prove: $C\hat{B}D = B\hat{E}D$ or $A\hat{B}E = B\hat{D}E$

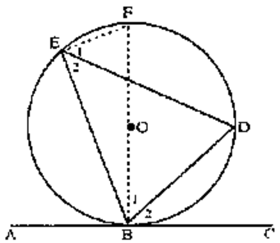
Proof: Draw diameter BOF and join EF

$\hat{E}_1 + \hat{E}_2 = 90°$ ... $\angle$ in semi-circle

$\hat{B}_1 + \hat{B}_2 = 90°$ ... rad $\perp$ tan

$\hat{E}_1 = \hat{B}_1$ ... FD subt = $\angle s$

$\therefore \hat{B}_2 = \hat{E}_2$



Method 2: **Required to prove:** $A\hat{B}E = B\hat{D}E$

Proof: Draw diameter BOF and join FD

$\hat{B}_1 = 90°$ tan $\perp$ rad

$\hat{D}_1 = 90°$ $\angle$ in semi-circle

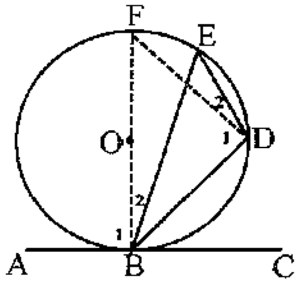
$\therefore \hat{B}_1 = \hat{D}_1$

But $\hat{B}_2 = \hat{D}_2$ $\angle$ s in the same segment

$\therefore \hat{B}_1 + \hat{B}_2 = \hat{D}_1 + \hat{D}_2$

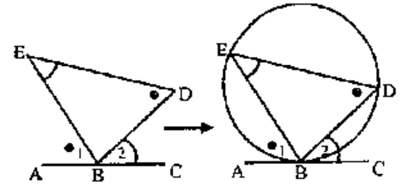
$\therefore$; $A\hat{B}E = B\hat{D}E$

Reason: tan-chord



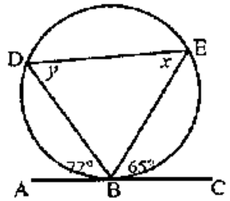
Converse

If a line is drawn through the endpoint of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. **(Between line and chord)** or **(converse of tan-chord theorem)**



Examples:

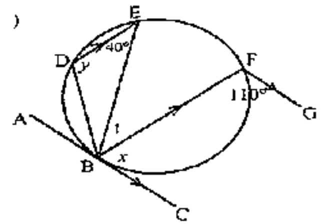
1.



$y = 65°$ tan-chord theorem

$x = 72°$ tan-chord theorem

2.



$x = 70°$ co-int. $\angle s$ FG // AC

$\hat{B}_1 = 40°$ alt. $\angle s$ FG // AC

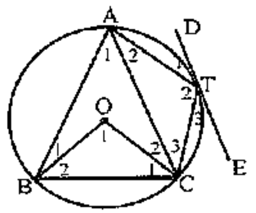
$y = E\hat{B}C = 110°$ tan-chord

3. O is the centre of the circle through A, B, C and T. DTE is a tangent at T and chord BC = chord CT

(a) Why is $\hat{A}_1 = \hat{A}_2$?

(b) Prove that $\hat{O}_1 = 2\hat{T}_3$

(c) Prove that $\hat{T}_2 + \hat{B}_1 + \hat{C}_1 = 180°$



(a)

(b) $\hat{O}_1 = 2\hat{A}_1$ $\angle$ at centre = 2 x $\angle$ at circumf.

$\hat{A}_1 = \hat{A}_2$ Proven in (a)

$\hat{A}_1 = \hat{T}_3$ tan-chord theorem

$\therefore \hat{O}_1 = 2\hat{T}_3$

(c) $\hat{T}_2 + (\hat{B}_1 + \hat{B}_2) = 180$ opp. $\angle s$ of cyclic quad.

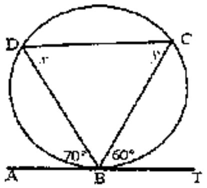
$\hat{B}_2 = \hat{C}_1$ Radii

$\therefore \hat{T}_2 + \hat{B}_1 + \hat{C}_1 = 180°$

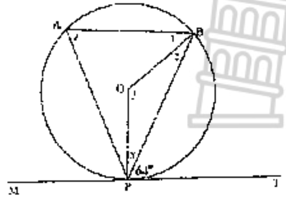
ACTIVITIES/ ASSESSMENT

1. Determine, with reasons, the value of the unknown angles.

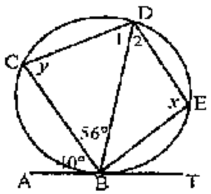
(a)



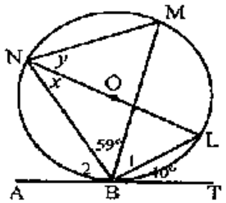
(b)



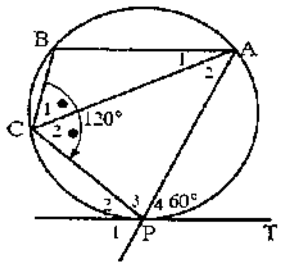
(c)



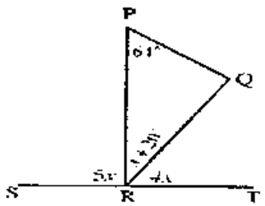
(d)



2. (a) Prove that PT is a tangent to the circle if $\hat{P}_4 = 60°, B\hat{C}P = 120°$ *and* $\hat{C}_1 = \hat{C}_2$



(b) Prove that SRT is a tangent to the circle through PQR.



COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

TOPIC: EUCLIDEAN GEOMETRY (Lesson 9)		Weighting	50 ± 3	Grade	10
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Circle Geometry problems and proofs of riders				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

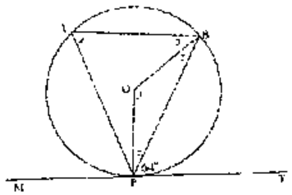
RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

1. O is the centre of the circle below. MPT is the tangent and OP ⊥ MT. $B\hat{P}T = 64°$
Determine, with reasons, x, y and z



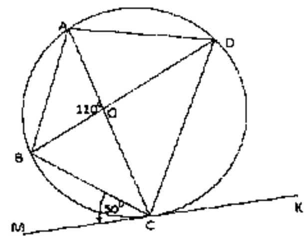
$x = 26°$ tan⊥ rad

$y = 128°$ $\angle s\ of\ \Delta$

$z = 64°$ tan-chord theorem

2. ABCD is a cyclic quadrilateral. MK is a tangent touching the circle at C. CA bisects $B\hat{A}D$. If AC and BD intersect at O and $B\hat{C}M = 50°$ and $B\hat{O}A = 110°$
Determine, with reasons,

(a) $B\hat{A}D$
(b) $A\hat{C}D$
(c) $D\hat{C}K$



(a) $B\hat{A}C = 50°$ tan-chord theorem
$D\hat{A}C = B\hat{A}C$ given
$B\hat{A}D = B\hat{A}C + D\hat{A}C = 100°$

(b) $A\hat{B}O = 20°$ $\angle s\ of\ \Delta$
$A\hat{C}D = A\hat{B}O = 20°$ $\angle s$ in same segment

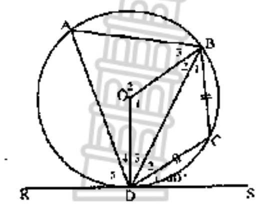
(d) $D\hat{C}K = D\hat{A}B = 50°$ tan-chord theorem

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

ACTIVITIES/ ASSESSMENT

1. In the figure below, RDS is a tangent to the circle centre O at D.
   BC = DC and $C\hat{D}S = 40°$
   Determine, with reasons, the size of:

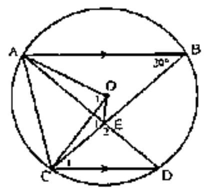
   (a) $\hat{C}_1$ (b) $\hat{D}_2$ (c) $\hat{C}$ (d) $\hat{O}_2$
   (e) $\hat{O}_1$ (f) $\hat{D}_3$ (h) $\hat{A}$



2. In the diagram, O is the centre of the circle passing through A, B, C and D. AB//CD and $\hat{A} = 20°$

   Calculate, with reasons, the size of:

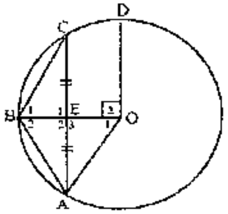
   (a) $\hat{C}_1$ (b) $\hat{O}_1$ (c) $\hat{D}$ (d) $\hat{E}_1$



3. In the circle centre O, BO ⊥ OD, AE = EC and $A\hat{O}D = 116°$.

   Calculate, with reasons, the size of:

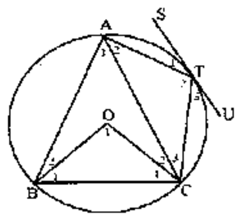
   (a) $\hat{C}$ (b) $A\hat{B}C$



4. O is the centre of the circle. STU is a tangent at T. Chord BC = chord CT, $A\hat{T}C = 105°$ and $C\hat{T}U = 40°$.

   Determine, with reasons, the size of:

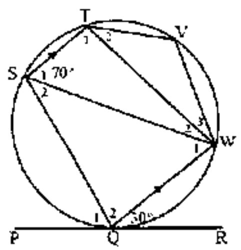
   (a) $\hat{A}_2$ (b) $\hat{A}_1$ (c) $\hat{B}_1 + \hat{B}_2$ (d) $\hat{C}_2$



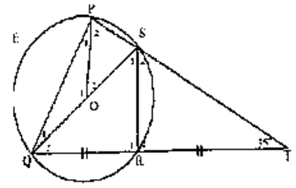
5. PQR is a tangent at Q. ST || QW. $W\hat{Q}R = 30°$ and $T\hat{S}W = 70°$

   Determine, with reasons, the size of:

   (a) $\hat{V}$ (b) $\hat{Q}_1$ (c) $\hat{T}_1$ (d) $\hat{W}_2$

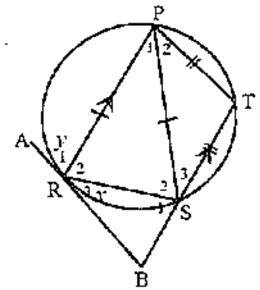


6. QOS is a diameter of the circle centre O. QR = RT and $\hat{T} = 35°$
   Calculate, with reasons, the size of:



7. In the diagram, chord PR is parallel to chord TS. PR = PS and PT = TS. ARB is a tangent to the circle at R and RST is a straight line.
   $\hat{R}_3 = x$ and $\hat{R}_1 = y$.

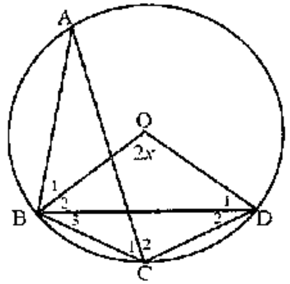
   (a) Write down, with reasons, three other angles equal to x And three other angles equal to y.
   (b) Express $\hat{T}$ in terms of x
   (c) Express $\hat{T}$ in terms of y



8. O is the centre of the circle through A, B, C and D. BC = CD and $B\hat{O}D = 2x$.

   Express the following in terms of x.

   (a) $\hat{B}_2$ (b) $B\hat{C}D$ (c) $\hat{A}$



TOPIC: EUCLIDEAN GEOMETRY (Lesson 10)		Weighting	50 ± 3	Grade	10
Term	1	Week no.			
Duration	1 hour	Date			
Sub-topics	Circle Geometry problems and proofs of riders				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

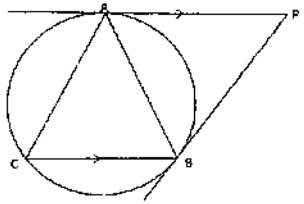
RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

1. PA and PB are tangents to the circle ABC at A and B respectively. PA is parallel to BC.



(a) Prove that:

1) AB = AC

$P\hat{A}B = A\hat{B}C$	alt. $\angle s$, PA//BC
$P\hat{A}B = A\hat{C}B$	tan-chord theorem
$\therefore A\hat{B}C = A\hat{C}B$	
∴ AB = AC	sides opp. Equal $\angle s$

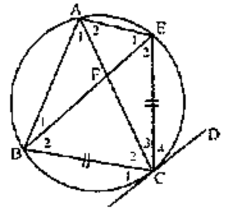
2) AB bisects $P\hat{B}C$

$P\hat{A}B = A\hat{B}C$	alt. $\angle s$, PA//BC
$P\hat{A}B = A\hat{B}P$	tans from same point
$\therefore A\hat{B}C = A\hat{B}P$	
∴ AB bisects $P\hat{B}C$	

(b) If $A\hat{P}B = 40°$, determine, with reasons:

1) $A\hat{C}B$		2) $B\hat{A}C$	
$P\hat{A}B = A\hat{B}P = 70°$	$\angle s$ opposite equal sides	$A\hat{B}C = A\hat{C}B = 70°$	proven
$P\hat{A}B = A\hat{C}B = 70°$	tan-chord theorem	$B\hat{A}C = 40°$	$\angle s$ of Δ

2. CD is a tangent at C to the circle through A, B, C and E. BC = CE and chords AC and BE intersect at F.



Prove that:

(a) $\hat{C}_1 = \hat{A}_2$

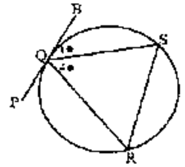
(b) $\hat{C}_4 = \hat{E}_2$

(a) $\hat{C}_1 = \hat{A}_1$	tan-chord theorem
$\hat{A}_1 = \hat{A}_2$	equal chords, equal $\angle s$
$\therefore \hat{C}_1 = \hat{A}_2$	
(b) $\hat{C}_4 = \hat{B}_2$	tan-chord theorem
$\hat{B}_2 = \hat{E}_2$	$\angle s$ opposite equal sides
$\therefore \hat{C}_4 = \hat{E}_2$	

ACTIVITIES/ ASSESSMENT

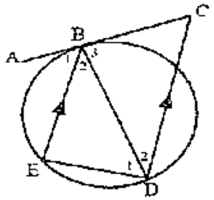
1. PQB is a tangent to the circle QRS at Q. QS bisects $B\hat{Q}R$.

Prove that QS = RS



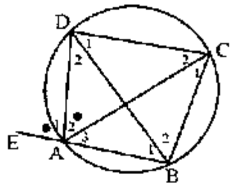
2. ABC is a tangent to the circle BED. BE || CD.

Prove that $\hat{D}_1 = \hat{C}$



3. BCD is a cyclic quadrilateral. BA is produced to E. AD bisects $E\hat{A}C$.

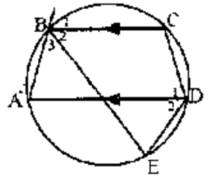
Prove that DC = DB.



4. In circle ABCDE, BC and AD are parallel chords. AB is produced to F.

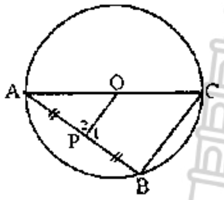
(a) Name 2 cyclic quadrilaterals

(b) Prove that:

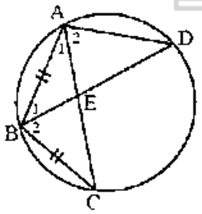


5. AC is a diameter of circle centre O.
   B is a point on the circle.
   OP bisects AB.

   Prove that OP || BC.

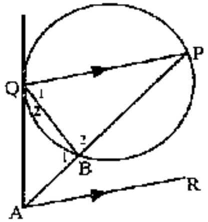


6. In circle ABCD, AB = BC.
   Prove that AB is a tangent to circle AED in A.



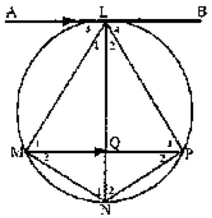
7. AQ is a tangent to the circle in Q and QP || AR.

   Prove that RA is a tangent to circle ABQ at A.

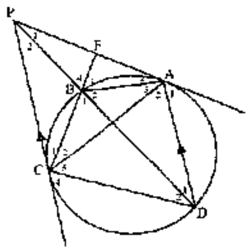


8. ALB is a tangent to circle LMNP.
   ALB || MP.

Prove that:
   (a) LM = LP
   (b) LN bisects $M\hat{N}P$
   (c) LM is a tangent to circle MNQ



9. PA and PC are tangents to the circle at A and C. AD || PC, and PD cuts the B.
   CB is produced to meet AP at F.
   AB, AC and DC are drawn.
   Prove:
   (a) AC bisects $P\hat{A}D$
   (b) $\hat{B}_1 = \hat{B}_3$
   (c) $A\hat{P}C = A\hat{B}D$



TEST 2: EUCLIDEAN GEOMETRY

TOTAL: 26 **DURATION: 30 Min**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1 [11 Marks]

EC November 2015

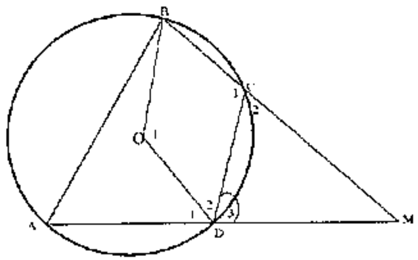
1.1 COMPLETE: Opposite angles of a cyclic quadrilateral ____________. (1)

1.2 In the figure, ABCD is a cyclic quadrilateral.
   AB || DC in circle with centre O.
   BC and AD produced meet at M. $\hat{D}_3 = x$

1.2.1 Show that MC = MD. (5)

1.2.2 If $\hat{D}_3 = x$, determine the value of $\hat{M}$ in terms of x. (2)

1.2.3 Hence, show that BODM is a cyclic quadrilateral. (3)



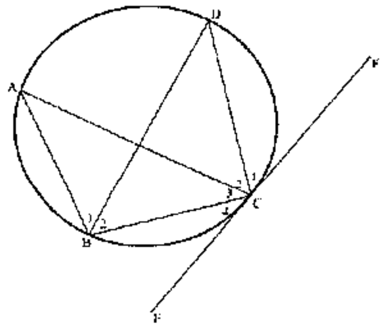
QUESTION 2 [15 Marks]

EC November 2015

2.1 A, B, C and D are points on the circumference of the circle in the diagram below. ECF is a tangent at C, $\hat{B}_1 = \hat{B}_2$

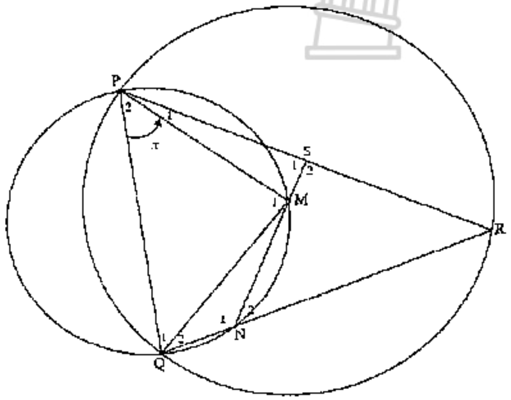
2.1.1 If $\hat{B}_1 = x$, find, with reasons, TWO other angles equal to x. (4)

2.1.2 Hence, show that DC bisects $A\hat{C}F$ (2)



DBE November 2015

2.2 In the diagram, PQ is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S. $\hat{P}_2 = x$.



2.2.1 Give a reason why $\hat{N}_2 = x$. (1)

2.2.2 Write down another angle equal in size to x. Give a reason. (2)

2.2.3 Determine the size of $\hat{R}$ in terms of x. (3)

2.3.4 Prove that PS = SR. (3)

TOPIC: TRIGONOMETRY (Lesson 1)		Weighting	50 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Grade 10 Revision				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trigonometric Ratios, Pythagoras Theorem, quadrants

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Failing to identify the correct quadrant

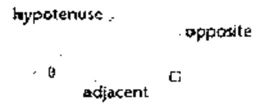
METHODOLOGY

Definition of the three **trigonometric ratios** for the angle θ in a right-angled.

$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan\theta = \frac{Opposite}{Adjacent}$$

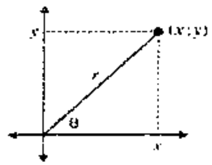


When **angles on a Cartesian plane** are formed then:

$$\sin\theta = \frac{y}{r}$$

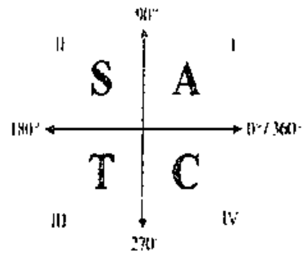
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



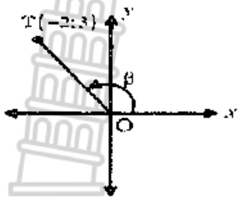
The Cartesian plane is divided into **four quadrants**. The angle formed will determine the sign of each trigonometric function.
The positive angle θ always rotates **anti-clockwise** from the positive x-axis about the origin.
If angle θ rotates clockwise from the positive x-axis about the origin, it is negative.

- Quadrant I: All Ratios are positive
- Quadrant II: Sin is positive, Cos and Tan are negative
- Quadrant III: Tan is positive, Sin and Cos are negative
- Quadrant IV: Cos is positive Sin and Tan are negative



Examples:

1. Consider the diagram alongside. Calculate the following without the use of a calculator.
   (a) $\tan\beta$ (b) $sin^2\beta + cos^2\beta$ (c) $13\sin\beta.\cos\beta$



- Complete the triangle and form a right-angled triangle
- Calculate the third side (Pythagoras theorem)

$x^2 + y^2 = r^2$ Pythagoras Theorem
$x = -2$ *and* $y = 3$
$(-2)^2 + (3)^2 = r^2$
$4 + 9 = r^2$
$r = \sqrt{13}$ (hypotenuse)… square root on both sides

(a) (a) $\tan\beta = \frac{3}{-2}$

(b) $sin^2\beta + cos^2\beta$
$= \left(\frac{3}{\sqrt{13}}\right)^2 + \left(\frac{-2}{\sqrt{13}}\right)^2$
$= \frac{9}{13} + \frac{4}{13} = \frac{13}{13} = 1$

(c) $13\sin\beta.\cos\beta$
$= 13.\left(\frac{3}{\sqrt{13}}\right).\left(\frac{-2}{\sqrt{13}}\right)$
$= 13.\frac{-6}{13} = -6$

2. If $\cos\theta = \frac{8}{17}$ and $\sin\theta < 0$, determine the value of:
   (a) $\tan\theta$ (b) $\frac{\sin\theta}{\cos\theta}$ (c) $sin^2\theta + cos^2\theta$

- Identify the quadrant in which the angle lies and draw a triangle

$\cos\theta = \frac{8}{17}$ …$(8 = x)$, x is positive in quadrants I and IV
$\sin\theta < 0$ … Sin is negative in quadrants III and IV

Quadrant IV is common, therefore, the right-angled triangle will be in quadrant IV

NB: DRAW DIAGRAM

- Calculate the third side by using Pythagoras Theorem

$x = 8$ *and* $r = 17$
$x^2 + y^2 = r^2$ Pythagoras Theorem
$(8)^2 + y^2 = (17)^2$
$y^2 = 289 - 64$
$y^2 = 225$
$y = \pm 15$, therefore, $y = -15$ … quadrant IV

(a) $\tan\theta = \frac{-15}{8}$

(b) $\frac{\sin\theta}{\cos\theta} = \frac{-15}{17} \div \frac{8}{17}$
$= \frac{-15}{17} \times \frac{17}{8} = \frac{-15}{8}$

(c) $sin^2\theta + cos^2\theta$
$= \left(\frac{-15}{17}\right)^2 + \left(\frac{8}{17}\right)^2$
$= \frac{225}{289} + \frac{64}{289} = \frac{289}{289} = 1$

ACTIVITIES/ ASSESSMENT

1. If $3\tan\theta - 4 = 0$ and $\cos\theta < 0$, with the aid of a diagram, calculate without the use of a calculator the value of
   (a) $\frac{\sin\theta}{\cos\theta}$ (b) $10\sin\theta - 25cos^2\theta$

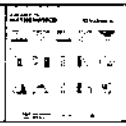
2. If $\sin A = \frac{12}{13}$ and $A \in (90°; 270°)$. With the aid of a diagram and without the use of a calculator, calculate the value of:
   (a) $\tan A$ (b) $cos^2A - sin^2A$ (c) $13\cos A - 5\tan A$

TOPIC: TRIGONOMETRY (Lesson 2)		**Weighting**	50 ± 3	**Grade**	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Trigonometric Identities**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Identity				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trigonometric ratios, factorisation, addition and subtraction of fractions

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Forgetting that $1 - cos^2\theta = sin^2\theta$ and $1 - sin^2\theta = cos^2\theta$
Challenge with addition and subtraction in expressions that are in fraction form

METHODOLOGY

An **identity** is a mathematical statement that is true for all values of the variable excluding the values for which the statement is not defined.

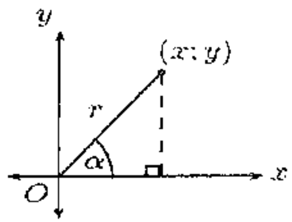
A **trigonometric identity** is an expression that is always true.

The two basic identities are:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ **quotient identity**
And $sin^2\theta + cos^2\theta = 1$ **square identity**

Proofs of identities:

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$ and $\tan\theta = \frac{y}{x}$



1. $\frac{\sin\theta}{\cos\theta} = \sin\theta \div \cos\theta$
$= \frac{y}{r} \div \frac{x}{r}$
$= \frac{y}{r} \times \frac{r}{x} = \frac{y}{x} = \tan\theta$
$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}, \theta \neq 90° + k.180°$

2. $sin^2\theta + cos^2\theta$
$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$
$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$
$= \frac{y^2+x^2}{r^2}$ … common denominator
$= \frac{r^2}{r^2} = 1$ … $x^2 + y^2 = r^2$ (Pythagoras Theorem)
$\therefore sin^2\theta + cos^2\theta = 1$ for all values of θ

The square identity can also be expressed as:

$sin^2\theta = 1 - cos^2\theta$

$cos^2\theta = 1 - sin^2\theta$

Examples:

1. Use the identities to simplify the following into a single trigonometric ratio:

(a) $tan^2\theta \times cos^2\theta$

$= \frac{sin^2\theta}{cos^2\theta}.cos^2\theta$

$= sin^2\theta$

(b) $\frac{1}{cos^2\theta} - tan^2\theta$

$= \frac{1}{cos^2\theta} - \frac{sin^2\theta}{cos^2\theta}$

$= \frac{1-sin^2\theta}{cos^2\theta}$ ... common denominator

$= \frac{cos^2\theta}{cos^2\theta} = 1$ ...$1 - sin^2\theta = cos^2\theta$

(c) $(1 + tan^2\theta)(1 - sin^2\theta)$

$= (1 + \frac{sin^2\theta}{cos^2\theta})(cos^2\theta)$

$= \left(\frac{cos^2\theta + sin^2\theta}{cos^2\theta}\right)(cos^2\theta)$ ... LCD when adding/ subtracting fractions

$= cos^2\theta + sin^2\theta = 1$

ACTIVITIES/ ASSESSMENT

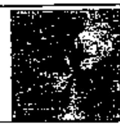
Use the identities to simplify the following into a single trigonometric ratio:

1. $\frac{\sin\theta}{\tan\theta}$
2. $\frac{\sin x}{\cos x \times \tan x}$
3. $tan^2\theta(1 - sin^2\theta)$
4. $\frac{1-cos^2 x}{\sin x}$
5. $8sin^2\theta + 8cos^2\theta$
6. $(3 - 3\sin\theta)(3 + 3\sin\theta)$
7. $\frac{1}{\sin\theta} - \frac{\cos\theta}{\tan\theta}$
8. $\frac{\sin x.\cos x}{1+cos^2 x - sin^2 x}$

TOPIC: TRIGONOMETRY (Lesson 3)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Trigonometric Identities			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ **quotient identity**

And $sin^2\theta + cos^2\theta = 1$ **square identity**

$sin^2\theta = 1 - cos^2\theta$

$cos^2\theta = 1 - sin^2\theta$

When proving Identities:

- Work with one side at a time
- Write $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$
- If terms are in fraction form, find the LCD
- Factorise where necessary.

Examples:

Prove that:

1. $cos\theta.tan\theta + sin\theta = 2sin\theta$

LHS: $cos\theta.tan\theta + sin\theta$

$= cos\theta.\frac{sin\theta}{cos\theta} + sin\theta$

$= sin\theta + sin\theta$

$= 2sin\theta = RHS$

2. $sinx.cosx.tanx + cos^2 x = 1$

LHS: $sinx.cosx.tanx + cos^2 x$

$= sinx.cosx.\frac{sinx}{cosx} + cos^2 x$

$= sin^2 x + cos^2 x$

$= 1 = RHS$

3. $\frac{\sin\theta - \sin\theta\cos\theta}{\cos\theta - (1 - sin^2\theta)} = \tan\theta$

LHS: $\frac{\sin\theta - \sin\theta\cos}{\cos\theta - (1 - sin^2\theta)}$

$= \frac{sin\theta(1-\cos\ )}{\cos\theta - cos^2\theta}$

$= \frac{sin\theta(1-\cos\ )}{cos\theta(1-\cos\ )}$

$= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$

4. $\tan x + \frac{cosx}{1+\sin x} = \frac{1}{\cos x}$

LHS: $\tan x + \frac{cosx}{1+\sin x}$

$= \frac{sinx}{cosx} + \frac{six}{1+six}$

$= \frac{sinx(1+sinx)+cosx(cosx)}{cosx(1+sinx)}$ ... LCD

$= \frac{sinx+sin^2x+cos^2x}{cosx(1+sinx)}$

$= \frac{sinx+1}{cosx(1+sinx)}$

$= \frac{1}{cosx} = RHS$

ACTIVITIES/ ASSESSMENT

Prove the following Identities:

1. $sinx.cosx.tanx = 1 - cos^2x$

2. $cos^3\theta + cos\theta.sin^2\theta = cos\theta$

3. $\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

4. $\frac{tanx.cosx}{sinx} = 1$

5. $\frac{sinx+cosx}{1+t} = cosx$

6. $\frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta}$

7. $\frac{cosx}{1+sinx} + tanx = \frac{1}{cosx}$

8. $tanx + \frac{cosx}{sinx} = \frac{1}{cosx.sinx}$

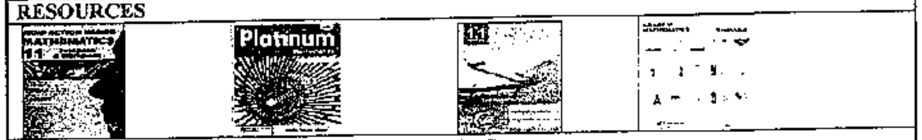
9. $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = \frac{2}{cos^2\theta}$

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

TOPIC: TRIGONOMETRY (Lesson 4)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Reduction Formulae: $(180° - \theta)$, $(180° + \theta)$ and $(360° - \theta)$			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

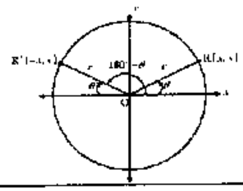
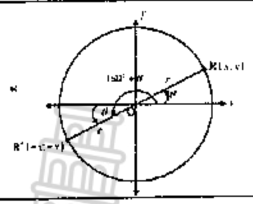
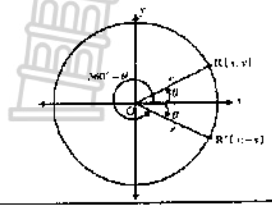


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Reduction formulae – trigonometric identities that express the trigonometric ratios of an angle of any size in terms of the trigonometric ratios of an acute angle, i.e., reduction formulae are used to **reduce** the trigonometric ratio of any angle to the trigonometric ratio of an **acute angle**.

REDUCTION FORMULAE:

	**QUADRANT 2** $\sin(180° - \theta) = \sin\theta$ $\cos(180° - \theta) = -cos\theta$ $\tan(180° - \theta) = -tan\theta$
	**QUADRANT 3** $\sin(180° + \theta) = -\sin\theta$ $\cos(180° + \theta) = -cos\theta$ $\tan(180° + \theta) = tan\theta$
	**QUADRANT 4** $\sin(360° - \theta) = \sin\theta$ $\cos(360° - \theta) = cos\theta$ $\tan(360° - \theta) = -tan\theta$

NOTICE THAT:

- θ is an acute angle
- the **ratios stay the same** but the **signs change** according to the CAST diagram

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

Examples:

Simplify the following:

1. $\frac{\sin(180°+x)}{\sin(180°-x)}$
$= \frac{-\sin}{\sin x}$
$= -1$

2. $\frac{\cos(180°+\theta).\tan(360°-\theta)}{\tan(180-\theta)}$
$= \frac{-\cos\theta \times -\tan\theta}{-\tan\theta}$
$= -\cos\theta$

3. $\frac{3\cos(180°+\theta)\sin(360°-\theta)}{\sin\theta\cos(180°-\theta)}$
$= \frac{3\times -\cos \quad \times -\sin\theta}{\sin\theta \times -\cos\theta}$
$= -3$

4. $\frac{\sin^2(180°+x)+\cos^2(180°-x)}{\tan(180°-x).\cos(360°-x)}$
$= \frac{\sin(180°+x)\sin(180°+x)+\cos(180°-x)\cos(180°-x)}{-ta \quad \times \cos x}$
$= \frac{-\sin x \times -\sin x + -\cos x \times -\cos x}{-\frac{\sin x}{\cos x}\times \cos x}$
$= \frac{\sin^2 x+\cos^2 x}{-\sin}$
$= \frac{1}{-\sin x} = -\frac{1}{\sin x}$

ACTIVITIES/ ASSESSMENT

Simplify the following:

1. $\sin(180° - \theta) + \sin(360° - \theta)$
2. $\frac{\tan(180°-A)}{\tan(180°+A)}$
3. $\tan(180° - \theta) \times \sin(180° + \theta)$
4. $\frac{\tan(360°-x)\cos(180°+x)}{\sin(180°-x)}$
5. $\frac{\sin(180°-B).\tan(360°-B)}{\sin(180°+B)}$
6. $\frac{\tan(360°-x).\tan(180°+x)}{\tan^2(180°-x)}$
7. $\frac{2\sin(180°-\theta)\cos(360°-\theta)}{\sin(180°+\theta)\sin(180°-\theta)}$
8. $\frac{\sin^2(360°-\theta).\cos^2(360°-\theta)}{\sin(180°+\theta).\sin(180°-\theta)}$
9. $1 - \frac{\sin^2(180°+x)}{1-\mathrm{co}\ (180°+x)}$

TOPIC: TRIGONOMETRY (Lesson 5)		**Weighting**	50 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Reduction Formulae:** $(90° - \theta)$ *and* $(90° + \theta)$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Co-ratio or co-function				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Forgetting to change the name of the ratio

METHODOLOGY

Co-ratios are also called complementary ratios. Co-ratios are $(90° - \theta)$ *and* $(90° + \theta)$

When using 90° the **name of the ratio changes** to its co-ratios or co-function.

- Co-ratio of $\sin\theta$ is $\cos\theta$
- Co-ratio of $\cos\theta$ is $\sin\theta$

$(90° - \theta)$: Quadrant 1

$\sin(90° - \theta) = \cos\theta$
$\cos(90° - \theta) = \sin\theta$

$(90° + \theta)$: Quadrant 2

$\sin(90° + \theta) = \cos\theta$
$\cos(90° + \theta) = -\sin\theta$

$\cos 80° = \cos(90° - 10°)$
$\cos 80° = \sin 10°$ ... angles add up to 90° (80° + 10°)

$\therefore \cos(angle) = \sin(\textbf{complementary angle})$

Examples:

1. Write cos 50° in terms of the ratio **sin**.

$\cos 50° = \sin 40°$

2. Simplify the following into a single ratio:

(a) $\frac{\cos(90°+A)}{\sin(360°-A)}$
$= \frac{-\sin A}{-\sin A}$
$= 1$

(b) $\frac{\cos(180°-x)\cos(90°-x).\tan(360°-x)}{\sin(180°+x).\sin(90°+x)}$
$= \frac{-\cos x.\sin x.-\tan x}{-\sin x.\cos x}$
$= -\tan x$

ACTIVITIES/ ASSESSMENT

Simplify the following:

1. $\frac{\cos(90°+\theta)}{\sin(360°-\theta)}$

2. $\frac{\sin(90°-A).\sin A}{\cos A.\cos(90°+A)}$

3. $\frac{\sin^2(360°-x)}{\sin(180°-\theta).\cos(90°+x)}$

4. $\frac{\cos(90°-B).\cos B.\tan(360°-B}{\sin^2(180°-B).COS(90°+B)}$

5. $\frac{2\sin(180°-x).\cos(360°-x)}{\cos(90°-x).\cos(180°+x)}$

6. $\frac{\sin(360°-\theta).\sin(90°+\theta)}{\cos(90°-\theta).\cos(360°-\theta)}$

7. $\frac{\sin(360°-\theta)+\cos(90°+\theta}{\sin(360°-\theta)}$

8. $\frac{\cos^2(90°-x)+s\ \ ^2(90°+x)}{\tan(180°+x).\sin(90°+x)}$

9. $\frac{\tan(180°-x)\cos(360°-x)+co\ \ (90°+x)}{\cos(90°-x).\sin(90°+x)}$

10. $\frac{2\cos(90°-\theta)-si\ \ (360°-\theta)}{2\sin(90°-\theta)-\cos(180°+\theta)}$

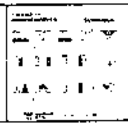
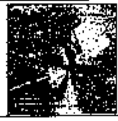
11. $\frac{2\,si\ \ (90°-\theta)+\cos(180°-\theta}{\sin(90°-\theta)-\cos(180°+\theta)}$

12. $\frac{\tan(180°+x).\sin(90°-x)}{\cos(90°-x)} - \frac{\cos(180°-x)}{\sin(90°+x)}$

TOPIC: TRIGONOMETRY (Lesson 6)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Reduction Formulae: Negative angles and angles greater than 360°			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

An **angle is negative** if the rotation is in a **clockwise** direction.

$(-\theta)$ ... FOURTH QUADRANT
$(-180° + \theta)$ ... THIRD QUADRANT
$(-180° - \theta)$ ... SECOND QUADRANT

Note:
We don't work with **negative angles**, so if negative, add 360° to the angle to make positive.

$(-\theta)$ Fourth Quadrant	$(-180° + \theta)$ Third Quadrant	$(-180° - \theta))$ Second Quadrant
$\sin(-\theta) = -\sin\theta$	$\sin(-180° + \theta) = -\sin\theta$	$\sin(-180° - \theta) = \sin\theta$
$\cos(-\theta) = \cos\theta$	$\cos(-180\theta + \theta) = -\cos\theta$	$\cos(-180\theta - \theta) = -\cos\theta$
$\tan(-\theta) = -\tan\theta$	$\tan(-180° + \theta) = \tan\theta$	$\tan(-180° + \theta) = -\tan\theta$

Note:
We don't work with **angle greater than 360°**, so if angle is greater than 360, **subtract 360°**, until the angle falls into the cast diagram **(0° to 360°)** to be able to use it.

$(360° + \theta)$ ... FIRST QUADRANT

$\sin(360° + \theta) = \sin\theta$ $\cos(360° + \theta) = \cos\theta$ $\tan(360° + \theta) = \tan\theta$

Examples:
Simplify the following:

1. $\tan(\theta - 180°)$
$= \tan(\theta - 180° + 360°)$
$= \tan(\theta + 180°)$
$= \tan\theta$ ... 3rd Quadrant

2. $\sin(-180° - x)$
$= \sin(-180° + 360° - x)$
$= \sin(180° - x)$
$= \sin x$ ... 2nd Quadrant

3. $\cos(540° + \theta)$
$= \cos(540° - 360° + \theta)$
$= \cos(180° + \theta)$
$= -\cos\theta$ ... 3rd Quadr.

4. $\frac{\cos(360°+x).\sin(360°-x)}{\cos(-x).\sin(x+360°)}$

$= \frac{\cos x.-\sin}{\cos x.\sin x}$

$= -1$

5. $\frac{\cos(90°-\theta).\cos(720+\theta).\tan(\theta-360°)}{\sin^2(\theta+360°).\cos(\theta+90°)}$

$= \frac{\sin\theta.\cos(720°-720°+\theta).\tan(\theta-360°+360°)}{\sin^2\theta.-\sin\theta}$

$= \frac{\sin\theta.\cos\theta.\tan\theta}{\sin^2\theta.\sin\theta}$

$= \frac{\cos\theta.\frac{\sin\theta}{\cos\theta}}{\sin^2\theta}$

$= \frac{1}{\sin\theta}$

ACTIVITIES/ ASSESSMENT

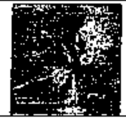
Simplify the following:

1. $\cos(-\theta)$
2. $\sin(\theta - 180°)$
3. $\tan(-\theta - 180°)$
4. $\cos(540° - x)$
5. $\frac{\tan(360°+x).\cos(-x)}{\cos(360°-x)}$
6. $\frac{\tan(180°-\theta).\sin(\theta-360°)}{\sin(360°-\theta)}$
7. $\frac{\cos(360°+\theta).\cos(-\theta).\sin(-\theta)}{\cos(90°+\theta)}$
8. $\frac{\sin(180°-x)-\sin(-x)}{\sin(369°+x)}$
9. $\frac{\sin(-x).\tan(180°+x0.\cos180°-x)}{\sin(180°-x0.\tan9360°-x).\cos(-x)}$
10. $\frac{\sin(\theta+180°0.\tan^2(180°-\theta)}{\tan(\theta-180).\sin720°-\theta)}$
11. $\frac{\cos^2(x-180°).\sin(540°-x)}{\sin(180°-x).\tan(-x-360°)}$
12. $\frac{\sin(\theta-180°.\cos(540+\ ).\sin(-\theta)}{\cos(180°-\theta).\sin^2(180°+\theta)}$

TOPIC: TRIGONOMETRY (Lesson 7)		Weighting	50 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Reduction Formulae: Numerical angles				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Reduction Formulae:

QUADRANT 1	QUADRANT 2	QUADRANT 3	QUADRANT 4
θ $(90° - \theta)$ $(360° + \theta)$	$(90° + \theta)$ $(180 - \theta)$ $(-180° - \theta)$	$(180° + \theta)$ $(-180° + \theta)$	$(360° - \theta)$ $(-\theta)$
$\cos\theta$ is positive $\cos\theta$ is positive $\tan\theta$ is positive	$\sin\theta$ is positive	$\tan\theta$ is positive	$\cos\theta$ is positive
ALL ratios are positive	$\cos\theta$ is positive $\tan\theta$ is positive	$\sin\theta$ is positive $\cos\theta$ is positive	$\cos\theta$ is positive $\tan\theta$ is positive

Special Angles:

30°	60°	45°
$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 45° = \frac{1}{\sqrt{2}}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\cos 45° = \frac{1}{\sqrt{2}}$
$\tan 30° = \frac{1}{\sqrt{3}}$	$\tan 60° = \sqrt{3}$	$\tan 45° = 1$

Examples:

Calculate without the use of a calculator:

1. $\frac{\cos 300°}{\tan 135°}$

$= \frac{\cos(360°-60°)}{\tan(180°-45°)}$

$= \frac{\sin 60°}{-\tan 45°}$

$= \frac{\frac{\sqrt{3}}{2}}{-1} = -\frac{\sqrt{3}}{2}$

2. $\frac{\tan 240°.\sin(-280°)}{\tan(-30°).\cos 370°}$

$= \frac{\cos(180°+60°).-\sin(360°-80°)}{-\tan 30°.\cos 360°+10°)}$

$= \frac{-\cos60°.\sin80°}{-\tan3\ °.\cos10°}$

$= \frac{1}{2} \div \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

5. If $\cos 20° = p$, determine the value(s) of the following in terms of p:

(a) $\cos 160°$ (b) $\tan(-200°)$ (c) $\cos 340°$

(d) $\sin 20°$ (e) $\tan 20°$ (f) $\sin(-70°)$

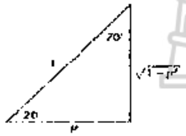
$\cos 20° = \frac{p}{1} = \frac{x}{r}$

$x^2 + y^2 = r^2$ Pythagoras Theorem

$p^2 + y^2 = 1^2$

$y^2 = 1 - p^2$

$y = \sqrt{1 - p^2}$



(a) $\cos 160° = \cos(180° - 20°)$

$= -\cos 20°$

$= -p$

(b) $\tan(-200°) = -\tan(180° + 20°)$

$= -\tan 20°$

$= -p$

(c) $\cos 340° = \cos(360° - 20°)$

$= \cos 20°$

$= p$

(d) $\sin 20° = \frac{\sqrt{1-p^2}}{1}$

(e) $\tan 20° = \frac{\sqrt{1-p^2}}{p}$

(f) $\sin(-70°) = -\sin 70°$

$= -\frac{p}{1} = -p$

ACTIVITIES/ ASSESSMENT

1. Calculate the following, without the use of a calculator:

(a) $\frac{\sin 160°}{\cos 250°}$

(b) $\tan 225° + \cos(-60°) - \sin^2 510°$

(c) $\frac{\tan 240°.\sin 115°}{\sin 330°.\cos 205°}$

(d) $\frac{\tan(-60°).\sin 158° + \sin 1\quad°.\cos 248°}{\cos 570°.\cos 2\quad°}$

(e) $\frac{\tan 160°.\cos 200°}{\sin 340°}$

(f) $\sin 193°.\cos(-77°) - \frac{\sin^2 13°}{\tan^2 347°}$

(g) $\frac{\cos^2 213°}{\tan 135° - s\quad°.\sin 327°}$

(h) $\frac{\sin^2 10° + \sin^2 100° - \cos^2 200°}{\sin(-20°).\cos 250°}$

2. If $\sin 10° = m$, write down the following in terms of m, with the aid of a diagram:

(a) $\sin 350°$ (b) $\sin(-10°)$ (c) $\cos 260°$ (d) $\cos 10°$ (e) $\tan 170°$

3. If $\tan 22° = k$, write down the following in terms of k, with the aid of a diagram:

(a) $\tan 202°$ (b) $\tan 518°$ (c) $\tan(-22°)$ (d) $\sin 338°$ (e) $\cos 68°$

4. If $\cos 38° = t$, express the following in terms of t. Draw the diagram.

(a) $\cos 322°$ (b) $\sin 52°$ (c) $\sin 232°$ (d) $\tan 38°$ (e) $\cos 142°$

TEST 1: TRIGONOMETRY

MARK: 25 **DURATION: 30 MIN**

DBE/NOV. 2014

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1 [10 Marks]

1.1 Simplify the following expression to a single trigonometric ratio:

$\frac{\sin(360° - x).\tan(-x)}{\cos(180° + x).(\sin^2 x + \cos^2 x)}$ (5)

1.2 Prove the identity:

$\frac{\cos\theta}{1 = SIN\theta} + \frac{1 + \sin x}{\cos x} = \frac{2}{\cos x}$ (5)

QUESTTION 2 [15 Marks]

2.1 If $\cos 23° = p$, express, without the use of a calculator, the following in terms of p:

2.1.1 $\cos 203°$ (2)

2.1.2 $\sin \sin 293°$ (3)

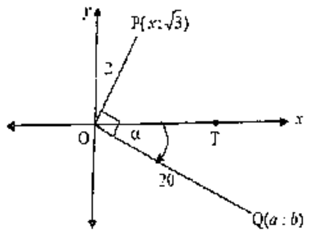
2.2 In the diagram below, $(x; \sqrt{3})$ is a point on the Cartesian plane such that OP = 2.

$Q(a; b)$ is a point such that $T\hat{O}Q = \alpha$ and OQ = 20. $P\hat{O}Q = 90°$

2.2.1 Calculate the value of x (2)

2.2.2 Hence, calculate α (3)

2.2.3 Determine the coordinates of Q. (5)



TOPIC: TRIGONOMETRY (Lesson 8)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Trigonometric Equations			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The **trigonometric equations** are similar to **algebraic equations** and can be linear equations, quadratic equations, or polynomial equations.

In trigonometric equations, the trigonometric ratios of $\sin\theta, \cos\theta$ *and* $\tan\theta$ are represented in place of the variables, as in a normal polynomial equation.

Solving trigonometric equations requires that we **find the value of the angles** that satisfy the equation by using both the **reference angles** and **trigonometric identities** that you've learned, together with a lot **of the algebra** you've learned.

STEPS TO SOLVE TRIGONOMETRIC EQUATIONS

- Simplify the equation
- Determine the **reference angle** (acute angle) … shift → ratio → number on the right-hand side
- Use quadrants to determine where the function is positive or negative
- Find angles in the interval $(0°; 360°)$

SIMPLE TRIGONOMETRIC EQUATIONS

Examples:

Solve for x where $x \in (0°; 360°)$

1. $2\sin x = 1$

$\sin x = \frac{1}{2}$

ref. $\angle = sin^{-1}(\frac{1}{2})$

ref. $\angle = 30°$

Sin is **positive** in the **first**: $\boldsymbol{x = 30°}$

and the **second** quadrant: $x = 180° - Ref.\angle$

$\boldsymbol{x = 180° - 30}$

$x = 150°$

2. $3\tan x + 4 = 0$

$\tan x = -\frac{4}{3}$

ref. $\angle = tan^{-1}(\frac{4}{3})$ … ignore negative sign

ref. $\angle = 53{,}13°$

tan is **negative** in the **second**: $\boldsymbol{x = 180° - 53{,}13°}$

$x = 126{,}87°$

3. $\sin(x - 24°) = -0{,}7$

$(x - 24°) = sin^{-1}(0{,}7)$

$(x - 24°) = 44{,}43°$ … $Ref.\angle$

II: $(x - 24°) = 180° + 44{,}43°$

$x - 24° = 224{,}43$

$x = 248{,}43°$

IV: $(x - 24°) = 360° - 44{,}43$

$x - 24° = 315{,}57°$

$x = 339{,}57$

4. $\cos 2x = 0{,}867$ *and* $2x \in (0°; 360°)$

$2x = cos^{-1}(0{,}867)$

$2x = 29{,}89°$ … $Ref.\angle$

I: $2x = 29{,}89°$

$x = 14{,}94°$

IV: $2x = 360° - 29{,}89°$

$2x = 330{,}11$

$x = 165{,}05°$

ACTIVITIES/ ASSESSMENT

Solve for x correct to ONE decimal place in the interval $(0°; 360°)$

1. (a) $\sin x = 0{,}3$ (b) $\cos x = -0{,}7$ (c) $\tan x = 5$

2. (a) $5\sin x = 3$ (b) $\cos x + 0{,}939 = 0$ (c) $3\tan x + 4 = 0$

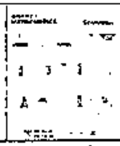
3. (a) $\sin(x - 22°) = -\frac{1}{2}$ (b) $5\cos(x + 15°) - 2 = 0$ (c) $\tan(x - 10°) + 10 = 1$

4. (a) $\sin 2x = 0{,}522$ (b) $\cos 3x - 2 = 0$ (c) $2\tan 2x = -4$

TOPIC: TRIGONOMETRY (Lesson 9)		Weighting	50 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	General Solution				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The solutions of trigonometric equations beyond 360° are all consolidated and expressed as a **general solution** of the trigonometric equations.

The trigonometric functions **sin θ and cos θ** have a **period of 360°** and **tan θ** has a **period of 180°**. The **period** is the number of degrees needed for a trigonometric function to complete one cycle.

This means for **sin θ and cos θ** the same numerical value will be obtained when **adding or subtracting 360°** to the specific angle. For **tan θ**, the same numerical value will be obtained when 180° is added or subtracted to the specific angle.

NOTE: For,

$\sin\theta = a$ ... where a is real number

$\text{ref.}\angle = sin^{-1}(a)$

General Solution: $\theta = sin^{-1}(a) + k.360°, k \in Z$ or $\theta = 180° - sin^{-1}(a) + k.360°, k \in Z$

$\cos\theta = a$ ... where a is real number

$\text{ref.}\angle = cos^{-1}(a)$ ... Reference Angle (ref. ∠)

General Solution: $\theta = cos^{-1}(a) + k.360°, k \in Z$ and $\theta = 360° - cos^{-1}(a) + k.360°, k \in Z$

OR $\theta = \pm\, cos^{-1}(a) + k.360°, k \in Z$

$\tan\theta = a$ ... where a is real number

$\text{ref.}\angle = tan^{-1}(a)$

General Solution: $\theta = tan^{-1}(a) + k.180°$

STEPS FOR DETERMINING THE GENERAL SOLUTION

- Determine the reference angle (use a positive value).
- Use the CAST diagram to determine where the function is positive or negative (depending on the given equation).

- Find the angles in the interval [0°; 360°] that satisfy the equation and add multiples of the period to each answer.

Remember that Trigonometric Equations can be **linear equations, quadratic equations**, or polynomial equations.

LINEAR EQUATIONS

The linear equation $ax + b = 0$ can be written as a trigonometry equation as $a\sin\theta + b = 0$, which is also sometimes written as $\sin\theta = \sin\alpha$.

Examples:
Determine the general solution of the following equations:

1. $5\sin\theta + 2 = 0$

$\sin\theta = -\frac{2}{5}$ ... equation in a simplified form

$\text{ref.}\angle = sin^{-1}\left(\frac{2}{5}\right)$ ... you do not include the negative sign

$\text{ref.}\angle = 23{,}58°$

$\theta = 180° + 23{,}87° + k.360°, k \in Z$ ... sin is negative in quadrant 2

$\boldsymbol{\theta = 203{,}58° + k.360°}$

Or $\theta = 360° - 23{,}58° + k.360°, k. \in Z$ ... and sin is negative in quadrant 4

$\boldsymbol{\theta = 336{,}42° + k.360°}$

2. $\tan 2x = 4$

$\text{ref.}\angle = tan^{-1}(4) = 75{,}96°$

$2x = 75{,}96° + k.180°$ ... tan is positive in quadrant 1 and the period is 180°

$\boldsymbol{x = 37{,}98° + k.90°}$ ... divide by 2 all the terms

3. $2\cos(A - 20°) = -\sqrt{3}$

$\cos(A - 20°) = -\frac{\sqrt{3}}{2}$

$\text{ref.}\angle = COS^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$

$A - 20° = 180° - 30° + k.360°, k \in Z$ ... cos is negative in quadrant 2

$A = 170° + k.360°$

$A - 20° = 180° + 30° + k.360°, k \in Z$ ... cos is negative in quadrant 3

$A = 230° + k.360°$

4. $\cos 4x = -\cos(2x - 30°)$

$\text{ref.}\angle = 2x - 30°$ (negative sign indicated quadrants where **cos is negative**)

$4x = 180 - (2x - 30°) + k.360°, k \in Z$ or $4x = 180 + (2x - 30°) + k.360°, k \in Z$

$4x = 180 - 2x + 30° + k.360°$ | $4x = 180 + 2x - 30° + k.360°$

$6x = 210° + k.360°$ | $2x = 150° + k.360°$

$x = 35° + k.60°$ | $x = 75° + k.180°$

ACTIVITIES/ ASSESSMENT

Determine the general solution of the following equations:

1. (a) $\sin(x-16^\circ)=0{,}616$

   (b) $3\cos(x-15^\circ)+1=-0{,}456$

   (c) $2\tan(2x-10^\circ)=10{,}67$

2. (a) $\sin 2x=-\frac{2}{3}$

   (b) $2\cos 3\theta=-1$

   (c) $\tan 2x-2=0$

3. (a) $\sin 2\theta=-\sin(\theta-30^\circ)$

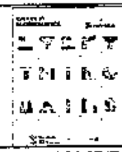
   (b) $\tan 3x=-\tan(2x+45^\circ)$

   (c) $\cos(x-10^\circ)=\cos(2x+15^\circ)$

TOPIC: TRIGONOMETRY (Lesson 10)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	General Solution			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Equations with more than one ratio.

USE OF IDENTITY $\frac{\sin\theta}{\cos\theta}=\tan\theta$ TO SOLVE: $a\sin x=b\cos x$

Example:

Determine the general solution of the equation:

$3\sin x+4\cos x=0$
$3\sin x=-4\cos x$
$3\tan x=-4$ ... divide by $\cos x$ on both sides
$\tan x=-\frac{4}{3}$
ref. $\angle=53{,}13^\circ$
$x=180^\circ-53{,}13+k.180^\circ, k\in Z$ ... quadrant 2
$x=126{,}87^\circ+k.180^\circ$

CO-FUNCTIONS/CO-RATIOS

Example:

Determine the general solution of the equation:

$\sin(\theta-20^\circ)=\cos 2\theta$
$\sin(\theta-20^\circ)=\sin(90^\circ-2\theta)$ ... $\sin\theta=\cos(90^\circ-\theta)$
ref. $\angle=90^\circ-2\theta$

$\theta-20^\circ=90^\circ-2\theta+k.360^\circ, k\in Z$
$3\theta=110^\circ+k.360^\circ$
$\theta=36{,}67^\circ+k.120^\circ$

or $\theta-20^\circ=180^\circ-(90^\circ-2\theta)+k.360^\circ, k\in Z$
$-\theta=110^\circ+k.360^\circ$
$\theta=-110^\circ-k.360^\circ$

EQUATIONS REQUIRING FACTORISATION/QUADRATIC FORM

The quadratic equation $ax^2 + bx + c = 0$ is as an example of trigonometric equation is written as $acos^2x + b\cos x + c = 0$.

But unlike normal solutions of equations with the number of solutions based on the **degree** of the variable, in trigonometric equations, the same value of solution exists for different values of θ.

Examples:

Determine the General Solution:

1. $2sin^2\theta - \sin\theta\cos\theta = 0$

$\sin\theta\,(2\sin\theta - \cos\theta) = 0$

$\sin\theta = 0$ or $2\sin\theta - \cos\theta = 0$

ref.$\angle = 0°$ $\quad$ $2\sin\theta = \cos\theta$

$\tan\theta = \frac{1}{2}$

$\theta = 0° + k.360°, k \in Z$ $\quad$ **ref.**$\angle = 26{,}57°$

$or\ \theta = 180° + k.360°, k \in Z$ $\quad$ $or\ \theta = 26{,}57° + k.180°, k \in Z$

2. $5tan^2x + 7\tan x = 6$

$5tan^2x + 7\tan x - 6 = 0$ ... standard form

$(5\tan x - 3)(\tan x + 2) = 0$ ... can use the **quadratic formula**

$5tanx - 3 = 0$ or $tanx + 2 = 0$

$\tan x = \frac{3}{5}$ $\quad$ $\tan x = -2$

ref.$\angle = 30{,}96°$ $\quad$ **ref.**$\angle = 63{,}43°$

$x = 30{,}96° + k.180°, k \in Z$ $\quad$ $x = 180° - 63{,}43° + k.180°, k \in Z$

$x = 116{,}57°$

Quadratic Formula:

$5tan^2x + 7\tan x - 6 = 0$ ... standard form

$a = 5, b = 7\ and\ c = -6$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\tan x = \frac{-(7)\pm\sqrt{(7^2)-4(5)(-6)}}{2(5)}$

$\tan x = \frac{3}{5}$ or $\tan x = -2$

3. $4cos^2x.\sin x - 3\sin x = 0$

$\sin x\,(4cos^2x - 3) = 0$ ... take out common factor

$\sin x = 0$ $\quad$ $or\ \ 4cos^2x - 3 = 0$

ref.$\angle = 0°$ $\quad$ $cos^2x = \frac{3}{4}$

$x = 0° + k.360°, k \in Z$ $\quad$ $\cos x = \pm\frac{\sqrt{3}}{2}$ ... (all quadrants)

$or\ x = 180° + k.360°, k \in Z$ $\quad$ **ref.**$\angle = 30°$

$x = 30° + k.360°, k \in Z$ $\quad$ $x = 180° - 30° + k.360°, k \in Z$

$x = 360° - 30° + k.360°, k \in Z$ $\quad$ $x = 150° + k.360°$

$x = 330° + k.360°$ $\quad$ $x = 180° + 30° + k.360°, k \in Z$

$x = 210° + k.360°$

BOUNDARY ANGLES

Angles that lie at the boundary of quadrants. Trigonometric equations that have boundary angles are:

$\begin{Bmatrix}\sin\theta = 0\\ \sin\theta = 1\\ \sin\theta = -1\end{Bmatrix}$ $\quad$ $\begin{Bmatrix}\cos\theta = 0\\ \cos\theta = 1\\ \cos\theta = -1\end{Bmatrix}$ $\quad$ $\{\tan\theta = 0\}$

$\begin{bmatrix}\sin\theta = 0\\ \theta = 0° + k.360°\ or\\ \theta = 180° + k.360°, k\epsilon Z\end{bmatrix}$ $\quad$ $\begin{bmatrix}\cos\theta = 0\\ \theta = 90° + k.360°, k\epsilon Z\end{bmatrix}$ $\quad$ $\begin{bmatrix}\tan\theta = 0\\ \theta = 0° + k.180°, k\epsilon Z\end{bmatrix}$

$\begin{bmatrix}\sin\theta = 1\\ \theta = 90° + k.360°, k\epsilon Z\end{bmatrix}$ $\quad$ $\begin{bmatrix}\cos\theta = 1\\ \theta = 0° + k.360°, k\epsilon Z\end{bmatrix}$

$\begin{bmatrix}\cos\theta = -1\\ \theta = 180° + k.360, k\epsilon Z\end{bmatrix}$

ACTIVITIES/ ASSESSMENT

Determine the General Solution of the following equations:

1. (a) $\sin\theta = -\cos\theta$ $\quad$ (b) $3\cos x = \sin x$

(c) $-4\sin A = 5\cos A$ $\quad$ (d) $\sqrt{3}\cos\theta - 3\sin\theta$

(e) $\frac{5}{9}\cos\alpha = -\sin\alpha$

2. (a) $\sin(x + 30°) = \cos 2x$ $\quad$ (b) $\cos(2x - 10°) = \sin(x - 40)$

(c) $\sin(2x + 5°) = \cos(x - 35°)$ $\quad$ (d) $\cos(x - 20°) = -\sin(x + 30°)$

3. (a) $3\sin\theta = 2\sin\theta\cos\theta$ $\quad$ (b) $2sin^2\theta = \sin\theta$

(c) $2\tan x.cos^2x = cos\,x$ $\quad$ (d) $3cos^2x + 2\cos x - 1 = 0$

(e) $sin^2x = 2\cos x + 2$ $\quad$ (f) $tan^2\theta = 1$

(g) $3cos^2x + 11\cos x + 1 = 0$ $\quad$ (h) $2\tan x.cos^2x = \cos x$

TOPIC: TRIGONOMETRY (Lesson 11)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	General Equations: Finding Solutions in a given Interval			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Solutions of a trigonometric Equation that lie in a specific Interval.

Example:

1. Solve for $1+\sin\theta = cos^2\theta, -360° < \theta < 360°$

$1+\sin\theta = 1 - sin^2\theta$ ... $(sin^2\theta + cos^2\theta = 1)$

$sin^2\theta + \sin\theta = 0$

$\sin\theta(\sin\theta + 1) = 0$ ... common factor

$\sin\theta = 0$ or $\sin\theta = -1$

$\theta = 0° + k.360°$ $\quad\theta = 270° + k.360°, k\epsilon Z$

Or $\theta = 180° + k.360°, k\epsilon Z$

ACTIVITIES/ ASSESSMENT

1. Solve each of the following equations in the given interval: $(-180°; 180°)$
   (a) $5\sin x = 2$ (b) $\cos(\theta + 25°) = 0{,}231$
   (c) $\sin 2\alpha = -0{,}327$ (d) $\tan\frac{\theta}{2} = 0{,}9$

2. Solve each of the following equations in the given interval: $(0°; 360°)$
   (a) $\cos x = \sin 32°$ (b) $3\sin\theta - 4\cos\theta = 0$
   (d) $2sin^2x = 3\cos x$ (d) $sin^2x + \frac{1}{2}\sin x = 0$

3. Solve each of the following equations in the given interval: $(-360°; 360°)$
   (a) $2sin^2\theta = \sin\theta\cos\theta$ (b) $2cos^2x - \cos x - 1 = 0$
   (c) $4cos^2A.\sin A = 3\sin A$ (d) $1 + \sin\beta\cos\beta = 7cos^2\beta$

TEST 2: TRIGONOMETRY

MARKS: 25 **DURATION: 25 Min**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1 [17 Marks]

Determine the General Solution

1.1 $\sin(x - 30°) = \cos 2x$ (5)

1.2 $2\sin\theta\cos\theta = \cos\theta$ (6)

1.3 $6sin^2x + \cos x = 4$ (6)

QUESTION 2 [8 Marks]

Solve for x

2.1 $\tan(x - 45°) = 3,$ $\quad 0° < x < 270°$ (4)

2.2 $\sin 2x = 4\cos 2x,$ $\quad x\epsilon(-180°; 180°)$ (4)

TOPIC: TRIGONOMETRY (Lesson 12)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Solution of triangles			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

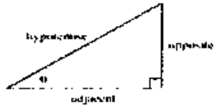
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

All triangles have three angles and three sides. If you are given three of any of them, as long as one is a length, you can solve the triangle (find all the other sides and angles).

For a right-angled triangle use the three trigonometric definitions and Pythagoras Theorem.

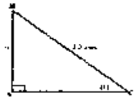
$\sin\theta = \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{adjacent}{hypotenus}$ $\tan\theta = \frac{opposite}{adjacent}$



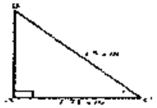
Examples:

1. ΔABC is given with $\hat{A} = 90°$, $\hat{C} = 40°$ and BC = $12cm$. Determine the length of the side AB.

2. ΔABC is given with $\hat{A} = 90°$, AC = $7{,}71cm$ and BC = $15cm$. Determine the size of $\hat{C}$.



Sketching diagrams is very helpful.



$\sin 40° = \frac{AB}{12} \cdots \frac{Opp}{Hyp}$

$12 \times \sin 40 = AB$

$AB = 7{,}71\ cm$

$\cos C = \frac{7{,}71}{15}$

$\hat{C} = \cos^{-1}\left(\frac{7{,}71}{15}\right)$

$\hat{C} = 59{,}07°$

ACTIVITIES/ ASSESSMENT

1. It would be helpful if you sketch the diagrams
   (a) If $\hat{A} = 90°$, $\hat{C} = 33°$ and BC = $13{,}5cm$ determine the length of AC.
   (b) If $\hat{A} = 90°$, $\hat{B} = 33°$ and BC = $7{,}35cm$ determine the length of AC.
   (c) If $\hat{A} = 90$, AB = $10{,}5cm$ and AC = $13{,}5cm$, determine $\hat{C}$.
   (d) If $\hat{C} = 90$, BC = $17{,}1cm$ and AB = $22{,}5cm$, determine $\hat{B}$

2. Find the size of the unknown sides and angles of the triangles.
   (a) (b) (c) (d)

COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
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TOPIC: TRIGONOMETRY (Lesson 13)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Sine Rule			
RELATED CONCEPTS/ TERMS/VOCABULARY	Perpendicular height			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Trigonometric ratios

RESOURCES

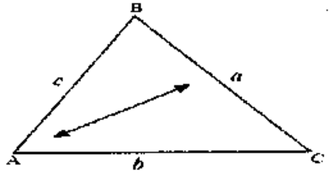
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

For triangles that have no right angle you cannot use the trigonometric definitions. You therefore need to know and prove two new rules called the sine rule, cosine rules and area rule

LABELLING TRIANGLES

- Upper letters are used to represent vertices
- Lower letters are used to represent sides
- Aside opposite a vertex is labelled with the lower case of the letter used to label the vertex.



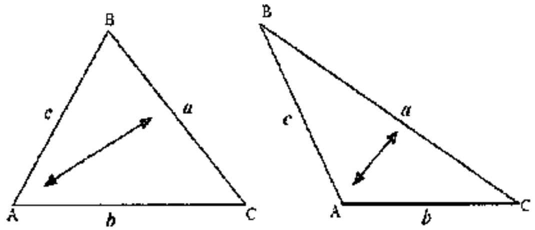
SINE RULE

In any triangle ABC,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

OR

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Acute-angled triangle Obtuse-angled triangle

For proofs: Construct perpendicular heights on triangles:

- Acute angled-triangle: Perpendicular height should be inside a triangle from a vertex to the opposite side.
- Obtuse-angled triangle: Perpendicular height should be outside a triangle, from a vertex to the outside of an obtuse angle

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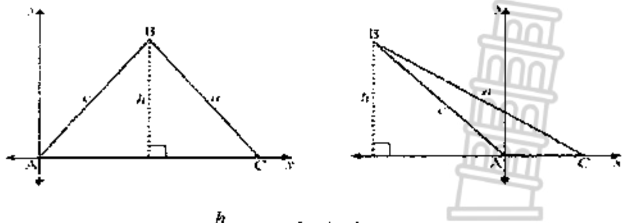
PROOF:



$\sin A=\frac{h}{c}, \quad \therefore c\sin A=h$

$\sin C=\frac{h}{a}, \quad \therefore a\sin C=h$

$\therefore c\sin A=a\sin C$

$\therefore \frac{\sin A}{a}=\frac{\sin C}{c}$ ... divide both by ac

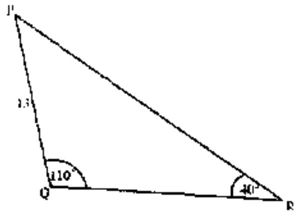
Similarly, when the perpendicular height is from A or from C, it can be proven that $\frac{\sin B}{b}=\frac{\sin C}{c}$ OR $\frac{\sin A}{a}=\frac{\sin B}{b}$.

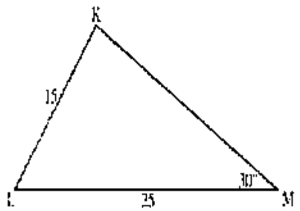
$\therefore \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ OR $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

NB: sine-rule can be used when **two sides** and **one angle** (not included) are known, or when **two angles** and **one side** are known.

Examples:

1.





Determine QR

$\hat{P}=30°$

$\frac{QR}{\sin 30}=\frac{13}{\sin 40}$

$QR=\frac{13\sin 30}{\sin 40}$

$QR=10,11$ units

Determine $\hat{L}$

$\frac{\sin 30}{15}=\frac{\sin K}{25}$

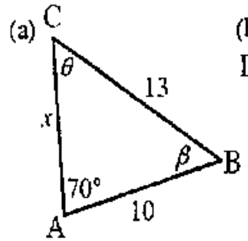
$\sin K=\frac{25\sin 30}{15}$

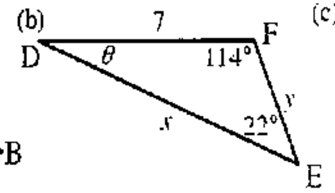
$\sin K=0,8333$

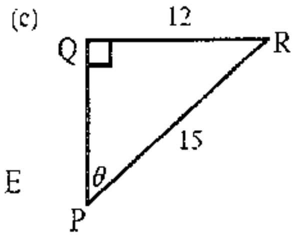
$\hat{K}=56,44°$ $\quad \therefore \hat{L}=93,56°$

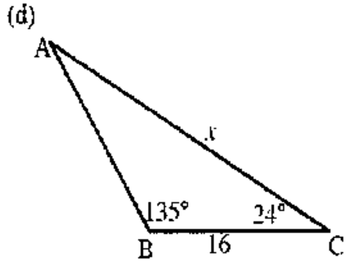
ACTIVITIES/ ASSESSMENT

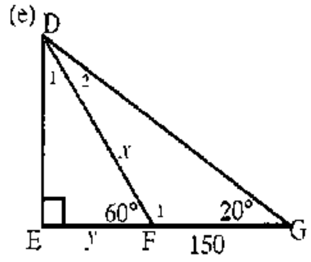
1. Determine the value of the unknown in each case

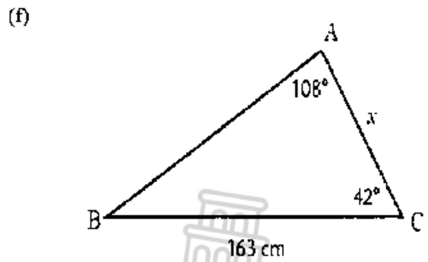


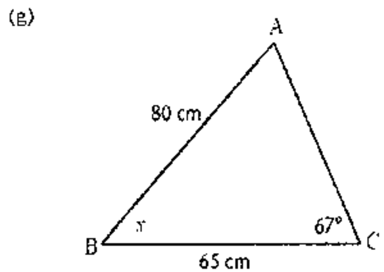












2. In ΔKMS, $\hat{K}=20°$, $\hat{M}=100°$ and $s=23cm$. Determine the length of the side m.

TOPIC: TRIGONOMETRY (Lesson 14)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Cosine Rule			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Labelling triangle sides with small letters, solving trigonometric equations, distance formula

RESOURCES

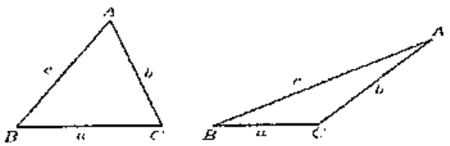
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Signs when solving for an angle in the cosine rule

METHODOLOGY

In any triangle ABC:

$a^2 = b^2 + c^2 - 2bcCosA$
$b^2 = a^2 + c^2 - 2acCosB$
$c^2 = a^2 + b^2 - 2abCosC$



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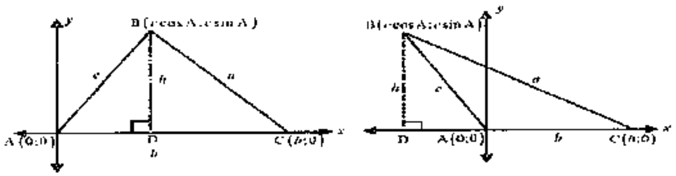

$B(x;y)$, $AC = b = x$ and $BD = h = y$

$\sin A = \frac{h}{c}$, $\therefore c\sin A = h = y$

$\cos A = \frac{b}{c}$, $\therefore c\cos A = b = x$

$\therefore A(0;0)$, $B(c\cos A; c\sin A)$ and $\therefore C(b;0)$

$a^2 = (c\cos A - b)^2 + (c\sin A - 0)^2$ ... distance formula

$= c^2\cos^2 A - 2bc\cos A + b^2 + c^2\sin^2 A$

$= b^2 + c^2(\cos^2 A + \sin^2 A) - 2bc\cos A$

$\therefore a^2 = b^2 + c^2 - 2bc\cos A$ ... $\cos^2 A + \sin^2 A = 1$

Similarly, when:

- B is put at the origin: $b^2 = a^2 + c^2 - 2acCosB$
- C is put at the origin: $c^2 = a^2 + b^2 - 2abCosC$

Cosine Rule can be used when

- Two sides and included angle are given/known
- All three sides are given/known

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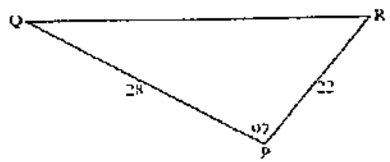
Examples:

1. Calculate the length of QR.

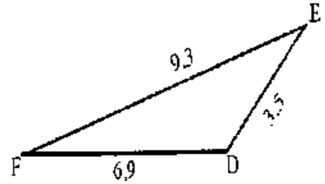


$QR^2 = QP^2 + PR^2 - 2QP.PRCosP$

$QR^2 = 28^2 + 22^2 - 2.28.22Cos97° = 1343,071515$

$QR = 36,65$ units

2. Calculate the size of $\hat{D}$



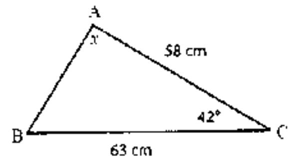
$EF^2 = FD^2 + DE^2 - 2.EF.DE.CosD$

$9,3^2 = 6,9^2 + 3,5^2 - 2\times 6,9\times 3,5\times CosD$

$\cos D = \frac{6,9^2 + 3,5^2 - 9,3^2}{2\times 6,9\times 3,5} = -0,551346$

$\hat{D} = 123,46°$

3. Determine the value of x



$AB^2 = 63^2 + 58^2 - 2.63.58Cos42$

$AB^2 = 1902,0976$

$AB = 43,61$ cm

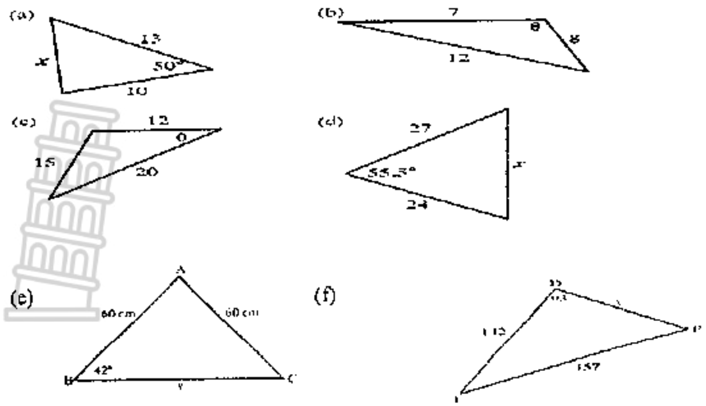
$\frac{\sin x}{63} = \frac{\sin 42}{43,61}$

$\sin x = \frac{63\sin 42}{43,61} = 0,96664$

$x = 75,16°$

ACTIVITIES/ ASSESSMENT

1. Calculate the value of x and θ



2. Determine the length of the third side of triangle XYZ where $\hat{X} = 71,4°$, $y = 3,42$, $z = 4,03$.

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TOPIC: TRIGONOMETRY (Lesson 15)	Weighting	50 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Area Rule			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Formula for the area of a triangle, trigonometric ratio sin

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

It was learnt that the area of a triangle is $A=\frac{1}{2}\times base\times height$.

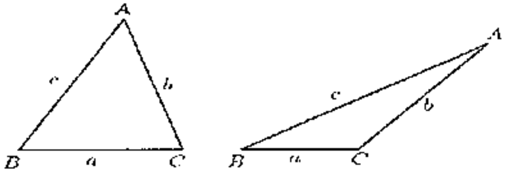
If there the triangle has no height, the **area rule** is used to calculate the area of a triangle.

In any triangle ABC,

$Area=\frac{1}{2}bc\sin A$

$Area=\frac{1}{2}ac\sin B$

$Area=\frac{1}{2}ab\sin C$



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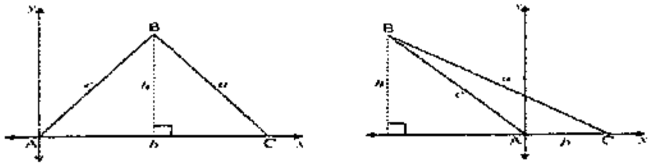


$Area=\frac{1}{2}\times b\times h$ and $\sin A=\frac{h}{c}$, $h=c\sin A$

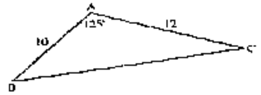
$Area=\frac{1}{2}\times b\times c\times\sin A$

Similarly, when

- B at the origin, $Area=\frac{1}{2}\times a\times c\times\sin B$
- C at the origin, $Area=\frac{1}{2}\times a\times b\times\sin C$

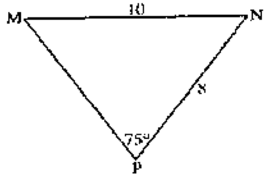
Examples:

1. Calculate the area of triangle ABC



$Area\Delta ABC=\frac{1}{2}\times 10\times 12\times\sin 125°=49,15$ square units

2. Calculate the area of $Area\Delta$ MNP



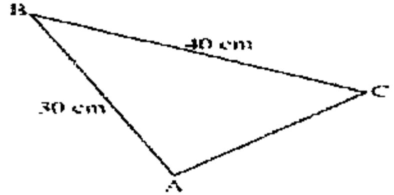
$\frac{\sin M}{8}=\frac{\sin 75°}{10}$

$\sin M=\frac{8\times\sin 75°}{10}=0,77274066$

$\hat{M}=50,60$ and $\hat{N}=180°-(75°+50,60°)=54,4°$

$Area\Delta MNP=\frac{1}{2}\times 10\times 8\times\sin 54,4=33,52$ square units

3. ΔABC has an area of 400 cm^2. Determine the size of $\hat{B}$.



$Area\Delta ABC=\frac{1}{2}\times AB\times BC\times\sin B$

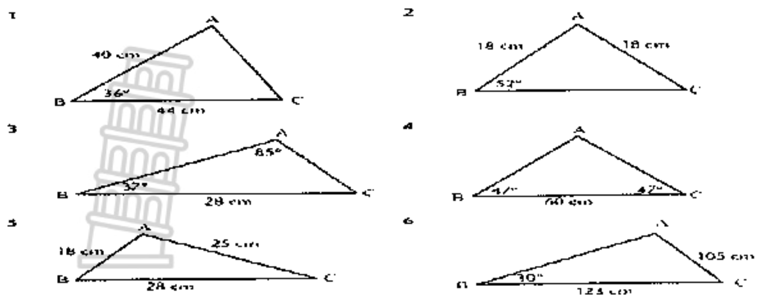
$400=\frac{1}{2}\times 30\times 40\times\sin B$

$\sin B=\frac{400}{15\times 40}=0,6666$

$\hat{B}=41,81°$

ACTIVITIES/ ASSESSMENT

Calculate the area of each triangle. Give answers correct to two decimal places.



7. Determine the area of a parallelogram in which two adjacent sides are 10 cm and 13 cm and the angle between them is 55°

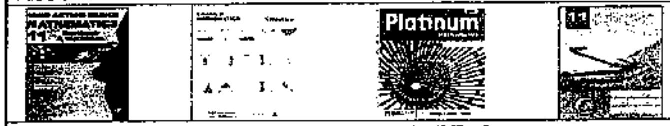
8. If the area of ΔABC is 5000 m^2 with $a=150m$ and $b=70m$, what are the two possible sizes of $\hat{C}$?

TOPIC: TRIGONOMETRY (Lesson 16)		Weighting	50 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Problems in TWO Dimensions using Sine, Area and Cosine Rule				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Sin rule, Cosine rule and area rule

RESOURCES



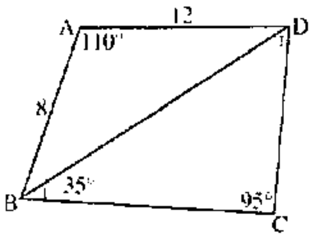
ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing the area, sin and cosine rules

METHODOLOGY

Examples:

1. ABCD is a quadrilateral. Calculate:



(a) the length of CD

Start by calculating BD

$BD^2 = 8^2 + 12^2 - 2 \times 8 \times 12 \times \cos 110° = 273,6678675$

$BD = 16,54$ units

$\frac{CD}{\sin 35°} = \frac{BD}{\sin 95°}$ $\quad \therefore CD = \frac{16,54 \sin 35°}{\sin 95°} = 9,52$ units

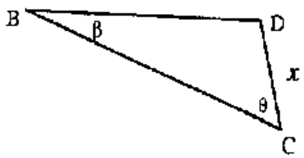
(b) the area of ABCD

$AreaABCD = Area\Delta ABD + Area\Delta BCD$ $\quad \hat{D}_1 = 180° - (95° + 35°) = 50°$

$AreaABCD = \frac{1}{2} \times 8 \times 12 \times \sin 110° + \frac{1}{2} \times 9,52 \times 16,54 \times \sin 50°$

$= 45,1052 + 60,31 = 105,42$ square units

2. (a) Determine $\hat{D}$ in terms of β and θ



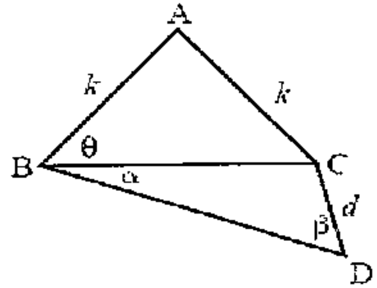
$\hat{D} = 180° - (\beta + \theta)$

(b) Show that $BC = \frac{x \sin(\beta + \theta)}{\sin \beta}$

$\frac{BC}{\sin(180° - (\beta + \theta))} = \frac{x}{\sin \beta}$ ... sine rule

$\therefore BC = \frac{x \sin(\beta + \theta)}{\sin \beta}$

3. Use the given information to show that $d = \frac{k \sin 2\theta \sin \alpha}{\sin \theta \sin \beta}$



$\hat{C} = \theta$ ... angles opposite equal sides

$\hat{A} = 180° - 2\theta$ ... sum angles of a triangle

$\frac{BC}{\sin 180° - 2\theta} = \frac{k}{\sin \theta}$ $\quad \therefore BC = \frac{k \sin 2\theta}{\sin \theta}$

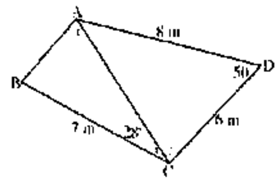
$\frac{d}{\sin \alpha} = \frac{BC}{\sin \beta}$ $\quad \therefore d = \frac{BC \sin \alpha}{\sin \beta}$

$\therefore d = \frac{\frac{k \sin 2\theta}{\sin \theta} \sin \alpha}{\sin \beta} = \frac{k \sin 2\theta \sin \alpha}{\sin \theta \sin \beta}$

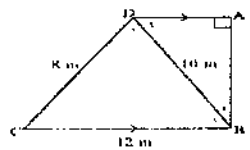
ACTIVITIES/ ASSESSMENT

Give your answers correct to two decimal places

1. Calculate:
   (a) the length of AC
   (b) the length of AB
   (c) the area of ABCD

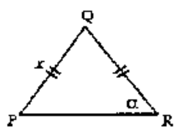


2. Calculate the area of ABCD

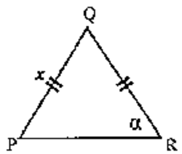


3. Show that:

(a) $QR = \frac{x \sin 2\alpha}{\sin \alpha}$



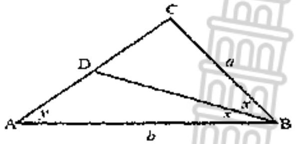
(b) $QR^2 = 2x^2(1 + \cos 2\alpha)$



4. (a) Prove that $DC = \dfrac{x\sin x}{\sin(x+y)}$

(b) Show that $AD = \dfrac{b\sin x}{\sin(x+y)}$

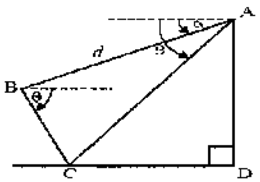
(c) Determine the ratio DC: AD



5. A cableway AB connects the top of two mountains across an intervening valley. The length of AB is d. From A the angle of depression of B is α and the angle of depression of the bottom of the valley at C is β. From B the angle of depression of the bottom of the valley at C is θ.

(a) Show that $A\hat{C}B = 180° - (\theta + \beta)$

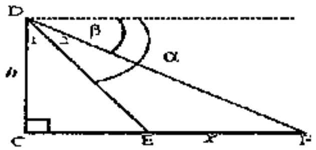
(b) Prove that $AC = \dfrac{d\sin(\theta+\alpha)}{\sin(\theta+\beta)}$



6. An observer on a cliff wishes to determine the height of the cliff h. He notices two boats on the sea. From his position (D), the angle of depression of the two boats E and F on the sea directly east of him are α and β respectively.

(a) Determine the length of DE in terms of h and α

(b) Determine $\hat{D}_2$ in terms of α and β.



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TEST 3: TRIGONOMETRY

MARKS: 25 **DURATION: 30 Min**

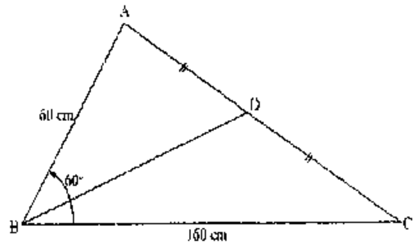
QUESTION 1 [9 Marks]

In ΔABC, AB = 60 cm, BC = 160 cm and $C\hat{B}A = 60°$.
BD is the bisector of AC with D a point on AC.

1.1 Calculate the length of AC. (3)

1.2 Determine the value of $\sin A$. (3)

1.3 Calculate the area of ΔABD. (3)

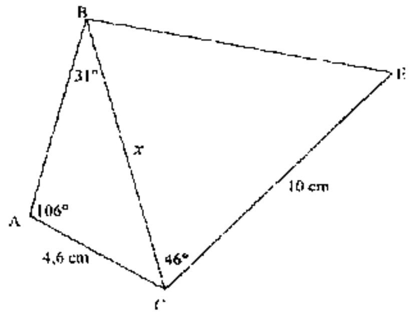


QUESTION 2 [16 Marks]

2.1 In the diagram A, C, E and B are the vertices of a quadrilateral.

2.1.1 Calculate the length of BC. (3)

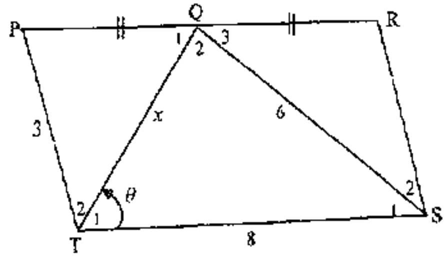
2.1.2 Calculate the area of quadrilateral ACEB. (4)



2.2 In the diagram below, PTSR is a parallelogram. Q is the midpoint of PR.

2.2.1 Show that $\cos\theta = \dfrac{x^2+18}{16x}$ (3)

2.2.2 Hence, determine the length of QT. (6)



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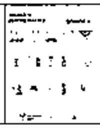
TOPIC: ANALYTICAL GEOMETRY (Lesson1)	Weighting	30 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Distance Formula, Midpoint Formula and Gradient Formula			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Distance between any two points, Midpoint of any two given point and gradient in any two given points

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Confusing formulas
- Failing to put correct signs in formulas, forgetting where to put a minus or a plus
- Making x a numerator in the gradient formula.

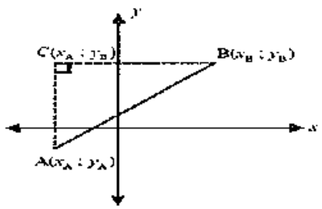
METHODOLOGY

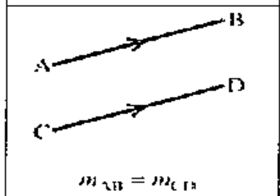
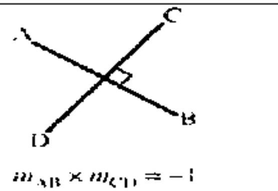
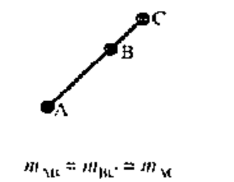
For two points: $A(x_A; y_A)$ and B $(x_B; y_B)$

Distance Formula: $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

Midpoint Formula: M $\left(\frac{x_A+x_B}{2}; \frac{y_A+y_B}{2}\right)$

Gradient Formula: $m = \frac{y_B-y_A}{x_B-x_A}$



PARALLEL LINES	PERPENDICULAR LINES	COLLINEAR POINTS
If two lines AB and DC are parallel, then their gradients are equal	If two lines AB and DC are perpendicular, then the product of their gradients equal −1	Points that are collinear lie on the same line. The gradient between each pair of points is the same.
		

Examples:

1. Given the points $P(-5; -4)$ and $Q(0; 6)$

(a) Calculate the length of the line PQ

(b) Determine the coordinates of M, the midpoint of PQ.

$PQ = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

$= \sqrt{(0-(-5))^2 + (6-(-4))^2}$

$= \sqrt{5^2 + 10^2}$

$= \sqrt{125}$

$= 5\sqrt{5}$

M $\left(\frac{x_P+x_Q}{2}; \frac{y_P+y_Q}{2}\right)$

M $\left(\frac{-5+0}{2}; \frac{-4+6}{2}\right)$

M $\left(-\frac{5}{2}; 1\right)$

(c) If Q is the midpoint of PR, calculate the coordinates of R.

$P(-5; -4)$ $Q(0; 6)$ $R(x; y)$

Midpoint

$\frac{x_R+x_P}{2} = 0$ $\frac{y_R+y_P}{2} = 6$

$\frac{x-5}{2} = 0$ $\frac{y-4}{2} = 6$

$x - 5 = 0$ $y - 4 = 12$

$x = 4$ $y = 16$

$R(4; 16)$

(d) Show that P, Q and Z are collinear if Z is the point $(-2; 2)$

$m_{PQ} = \frac{y_Q-y_P}{x_Q-x_P}$ $m_{QZ} = \frac{y_Z-y_Q}{x_Z-x_Q}$

$= \frac{6-(-4)}{0-(-5)}$ $= \frac{2-6}{-2-0}$

$= 2$ $= 2$

P, Q and Z are collinear, $m_{PQ} = m_{QZ}$

2. Given the points A $(-2; 1)$, B $(1; 3)$, C $(5; -3)$ and D $(-1; k)$. Determine the values of k if:

(a) AB // CD

$m_{AB} = m_{CD}$

$\frac{y_B-y_A}{x_B-x_A} = \frac{y_D-y_C}{x_D-x_C}$

$\frac{3-1}{1-(-2)} = \frac{k-(-3)}{-1-5}$

$\frac{2}{3} = \frac{k+3}{-6}$

$3k + 9 = -12$

$3k = -21$

$k = -7$

(b) AB ⊥ CD

$m_{AB} \times m_{CD} = -1$

$\frac{y_B-y_A}{x_B-x_A} \times \frac{y_D-y_C}{x_D-x_C} = -1$

$\frac{3-1}{1-(-2)} \times \frac{k-(-3)}{-1-5} = -1$

$\frac{2}{3} \times \frac{k+3}{-6} = -1$

$\frac{2k+6}{-18} = -1$

$2k + 6 = 18$

$k = 6$

ACTIVITIES/ ASSESSMENT

1. If P $(8; 4)$ is the midpoint between A and B $(13; 6)$, determine:

(a) The coordinates of A
(b) The length of AB
(c) The gradient of AB

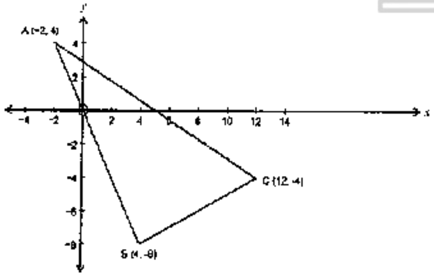
2. Given A $(-3; -5)$, B $(0; -3)$ and C $(6; 1)$, show that A, B and C are collinear.

3. M $(p; 4)$ is the midpoint of CD, where C is the point $(-3; 1)$ and D is the point $(5; q)$. Calculate the values of p and q.

4. A (8; 4), B (8; 4), (8; 4) and D (8; 4) are points on the Cartesian plane. For which values of t is

(a) AB // CD (b) AB ⊥ CD

5. Consider the diagram below:



(a)Show that is right-angled at B. (3) ABC∆

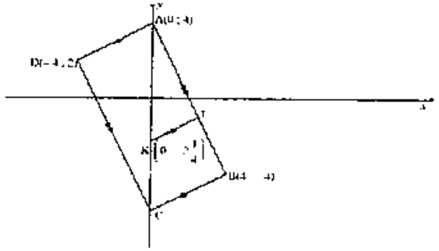
(b) Determine the coordinates of P and Q, the mid-points of AB and AC respectively.

(c) Use analytical methods to show that the line joining P and Q is parallel to BC.

(d) Use analytical methods to prove that. $PQ = \frac{1}{2}BC$

(e) Determine the coordinates of D if ABCD is a rectangle.

6. In the diagram, C is point on the y-axis such that A (0; 4), B (4; -40, C and D (-4; 2) are vertices of a parallelogram ABCD. K is a point $\left(0; -2\frac{1}{4}\right)$ and L is a point on AB such that KL // CB.



(a) Calculate the length of diagonal DB.
(b) Calculate the coordinates of M, the midpoint of DB
(c) Calculate the gradient of AD.
(d) Prove that AD ⊥ AB
(e) Write down with reasons, the coordinates of C.

TOPIC: ANALYTICAL GEOMETRY (Lesson2)		Weighting	30 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Equation of a line**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Gradient of any two given points, Gradients of parallel lines, Gradients of perpendicular lines

RESOURCES




ERRORS/MISCONCEPTIONS/PROBLEM AREAS

- Making x a numerator in the gradient formula.
- Confusing gradients of parallel lines and perpendicular lines

METHODOLOGY

The different forms of a **straight line** are used depending on the information provided.

- The **gradient–point** form of the straight line equation: $y - y_1 = m(x - x_1)$, where m is the gradient and $(x_1; y_1)$ a point on the line
- The **gradient–intercept** form of the straight line equation: $y = mx + c$, where m is the gradient and c is the y-intercept

THE EQUATION OF A LINE THROUGH **TWO POINTS**

- Calculate the **gradient** (m) between two points and substitute to m in the equation
- Calculate the **y-intercept**: Substitute any **one of the two points** to x **and** y or to x_1 **and** y_1 in the equation

Example:

1. Determine the equation of a line through A (-1; -5) and B (5; 4).

$$m_{AB} = \frac{4-(-5)}{5-(-1)} = \frac{9}{6} = \frac{3}{2}$$

$y = mx + c$ or $y - y_1 = m(x - x_1)$

$y = \frac{3}{2}x + c$ ... substitute m_{AB}

$-5 = \frac{3}{2}(-1) + c$ ... A (-1; -5)

$-5 + \frac{3}{2} = c$

$-\frac{7}{2} = c$

$\therefore y = \frac{3}{2}x - \frac{7}{2}$

$y - y_1 = \frac{3}{2}(x - x_1)$

$y - (-5) = \frac{3}{2}(x - (-1))$

$y + 5 = \frac{3}{2}(x + 1)$

$y = \frac{3}{2}x + \frac{3}{2} - 5$

$y = \frac{3}{2}x - \frac{7}{2}$

EQUATION OF A LINE THROUGH ONE POINT AND **PARALLEL OR PERPENDICULAR** TO A GIVEN LINE

- Write the equation of the given line in standard form to find m and c.
- Determine the gradient of the line with the unknown equation by using the rules of parallel and perpendicular lines
- Substitute the gradient (m) and the coordinates of the given **point** into the standard equation for a straight line to find the equation.

Examples:

2. Determine the equation of a straight line passing through the point (1; - 4) and parallel to the line $y = 2x - 3$.

Gradient of line $y = 2x - 3$: $m = 2$
Parallel lines have same/equal gradients
$\therefore m = 2$ $\quad y = mx + c$
$-4 = 2(1) + c$ ... substitute $m = 2$ and point (1; -4)
$-4 - 2 = c$
$\therefore y = 2x - 6$

3. Determine the equation of the line perpendicular to $3y - 2x = 6$ and passing through the point (-6;2)

$3y = 2x + 6$
$y = \frac{2}{3}x + 2$ ... equation in standard form
$m = \frac{2}{3}$ and the gradient of the perpendicular line is $m = -\frac{3}{2}$ (products of gradients must be $= -1$)
$y = mx + c$
$2 = -\frac{3}{2}(-6) + c$ ... substitute m and a point
$2 - 9 = c$
$\therefore y = -\frac{3}{2}x - 7$

ACTIVITIES/ ASSESSMENT

1. Determine the equations of straight line passing through the following **two given points**:

(a) $(-3; 1)$ and $(-5; 5)$ (b) $(3; 2)$ and $(-9; -2)$ (c) $(3; -7)$ and $(5; -4)$

(d) $(-5; 1)$ and $(-5; 7)$ (e) $(0; 4)$ and $(10; 4)$

2. Determine the equation of a line:

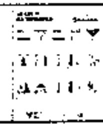
(a) passing through the point $(-4;3)$ and perpendicular to $2y = 3x + 6$.
(b) passing through the point $(-8;-1)$ and parallel to $x - 2y + 2 = 0$.
(c) parallel to $2y - x = 4$ and passing through $(-1;-2)$.
(d) perpendicular to $3x - y = 4$ and passing through $(6;4)$.

TOPIC: ANALYTICAL GEOMETRY (Lesson3)	Weighting	30 ± 3	Grade 11
Term		Week no.	
Duration	1 hour	Date	
Sub-topics	Angle of Inclination		
RELATED CONCEPTS/ TERMS/VOCABULARY	Angle of inclination		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Gradient of a line, reference angle, period of $y = \tan\theta$, obtuse angle, acute angle

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The **angle of inclination** of a line is the **angle between that line and the positive** x-axis and it is measured in an anticlockwise direction.

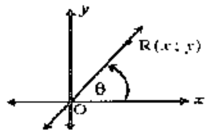
The angle of inclination of a line is **related to the gradient of a line gradient**. If the gradient changes, the value of the angle also changes.

Therefore, the **gradient** of a **straight line** is **equal to the tangent** of the angle formed between the line and the positive direction of the x-axis.

$m = \tan\theta$ for $0° < \theta < 180°$

- **Lines with positive gradients**

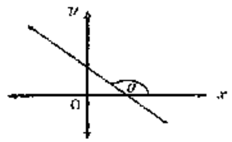
If a straight line has a negative gradient then the **angle formed** between the line and the positive direction of the x-axis is **acute** ($0° < \theta < 90°$)



$\theta = ref.\angle$

- **Lines with negative gradients**

If a straight line has a negative gradient then the **angle formed** between the line and the positive direction of the x-axis is **obtuse** ($90° < \theta < 180°$).



$\theta = 180° - ref.\angle$

Examples:

1. Calculate the angle of inclination of a line passing through P (2; 1) and Q (-3; -3)

$m_{PQ} = \frac{-3-2}{-3-1} = \frac{-5}{-4} = \frac{5}{4}$
$\tan\theta = m_{PQ} = \frac{5}{4}$
$\tan\theta = \frac{5}{4}$
ref. $\angle = 51{,}34°$ ... acute angle, positive gradient

2. Determine the inclination of a straight line $2y + 3x = 5$, correct to two decimal places.

$$2y = -3x + 5$$
$$y = -\frac{3}{2}x + \frac{5}{2}$$
$$\tan\theta = m = -\frac{3}{2}$$
$$\text{ref.}\angle = 56{,}31^\circ$$
$$\theta = 180^\circ - 56{,}31^\circ = 123{,}69^\circ$$ … negative gradient, obtuse angle

3. Determine the equation of the straight line passing through the point (3; 1) and with an angle of inclination of 135°.

$m = \tan\theta = \tan 135^\circ = -1$ … negative gradient, θ is an obtuse angle

$y = mx + c$
$y = -1x + c$
$1 = -1(3) + c$
$4 = c$
$\therefore y = -x + 4$

ACTIVITIES/ ASSESSMENT

1. Determine the inclination of a line passing through

(a) (3; -7) and (5; -4) (b) (-3; 1) and (-5; 5) (c) (3; 2) and (-9; -2)

2. Calculate the inclination of a straight line, correct to two decimal places.

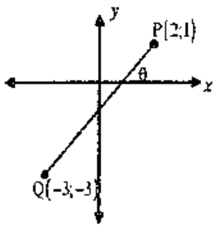
(a) $5x + 3y = 10$ (b) $2y - x = 6$ (c) $x = 3y + \frac{1}{2}$

3. Determine the gradient of a straight line with inclination

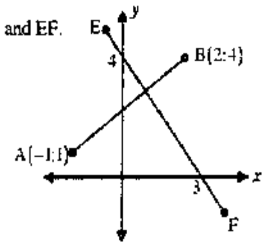
(a) 75° (b) 60° (c) 140°

4. Determine the inclination:

(a) of line PQ



(b) of line AB and EF.



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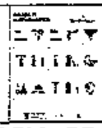
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MQADI

TOPIC: ANALYTICAL GEOMETRY (Lesson4)		Weighting	30 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Angle of Inclination between two lines				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

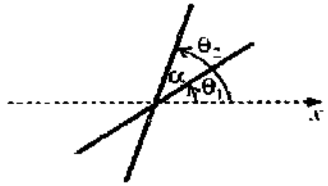
Gradient of a line, reference angle, period of $y = \tan\theta$

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY



α is between two lines
- Calculate the size of θ_1
- Calculate the size of θ_2

$\therefore \alpha = \theta_2 - \theta_1$

Examples:

1. Given the diagram alongside, with line AC and line BD.

Determine the size of α

$\tan\beta = m_{BD} = \frac{-3-4}{6-(-1)} = \frac{-7}{7} = -1$

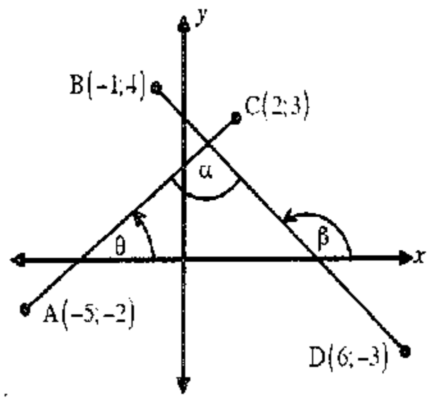
ref.$\angle = 45^\circ$

$\beta = 180^\circ - 45^\circ = 135^\circ$ … obtuse angle

$\tan\theta = m_{AC} = \frac{3-(-2)}{2-(-5)} = \frac{5}{7}$

ref.$\angle = 35{,}54^\circ$ …acute angle

$\therefore \alpha = \beta - \theta$
$= 135^\circ - 35{,}54^\circ = 99{,}46^\circ$



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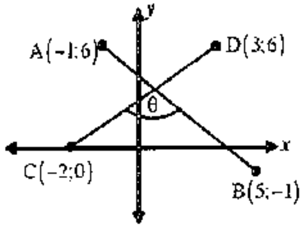
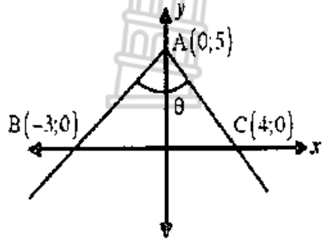
ACTIVITIES/ ASSESSMENT

1. Calculate the size of θ. Round off your answer correct to two decimal places.

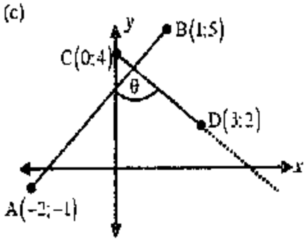
(a)



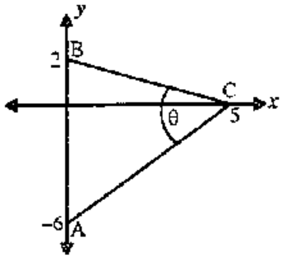
(b)



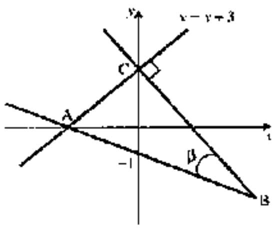
(c)



(d)



2. Determine the size of β in the following diagram:



3. Given points P $(-1;-1)$, Q $(4;2)$, R $(7;-5)$ and S $(-3;-7)$. Determine, correct to 2 decimal places:

(a) the acute angle between QR and the x-axis.

(b) $P\hat{Q}R$

(c) the angle of inclination of PQ

(d) $S\hat{P}R$

(e) the angle of inclination of SP

(f) $P\hat{S}R$

TEST : ANALYTICAL GEOMETRY

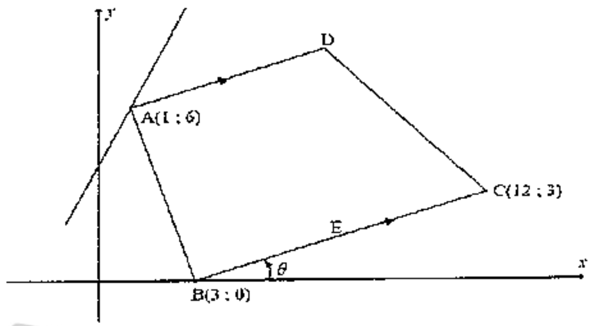
MARKS: 25 **DURATION: 25 Min**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1[14 Marks]

A (1; 6), B (3; 0), C (12; 3) and D are the vertices of a trapezium with AD || BC. E is the midpoint of BC. The angle of inclination of the straight line BC is θ, as shown in the diagram.



1.1 Calculate the coordinates of E. (2)

1.2 Determine the gradient of the line BC. (2)

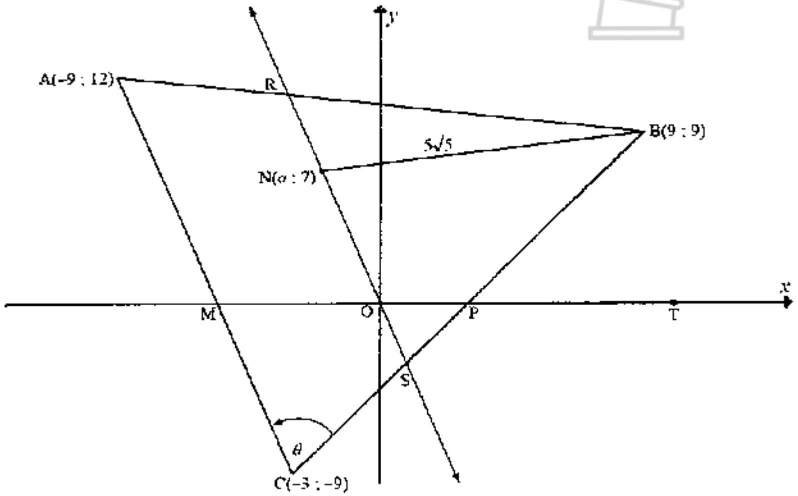
1.3 Calculate the magnitude of θ. (2)

1.4 Prove that AD is perpendicular to AB. (3)

1.5 A straight line passing through vertex A does not pass through any of the sides of the trapezium. This line makes an angle of 45° with side AD of the trapezium. Determine the equation of this straight line. (5)

QUESTION 2 [11 Marks]

In the diagram A (–9; 12), B (9; 9) and C (–3; –9) are the vertices of ΔABC. N $(a; 7)$ is a point such that BN $= 5\sqrt{5}$. R is a point on AB and S is a point on BC such that RNS is parallel to AC and RNS passes through the origin. T lies on the x-axis to the right of point P.
$A\hat{C}B = \theta$, $A\hat{M}O = \alpha$ and $B\hat{P}T = \beta$

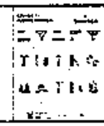


2.1 Calculate the gradient of the line AC. (2)

2.2 Determine the equation of line RNS in the form $y = mx + c$. (2)

2.3 Calculate the value of a. (3)

2.4 Calculate the size of θ. (4)

TOPIC: NUMBER PATTERNS (Lesson 1)		Weighting	25 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Linear Number Pattern				
RELATED CONCEPTS/ TERMS/VOCABULARY					
PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE					
RESOURCES					
   					
ERRORS/MISCONCEPTIONS/PROBLEM AREAS					

METHODOLOGY

A **pattern** is an ordered set of numbers or variables. In grade 10, a linear number pattern was done.

A **linear number pattern** is a sequence of terms/numbers in which there is a **common difference between** any term and the term before. It has a constant first difference.

NOTE: $b = T_2 - T_1$ and not $T_1 - T_2$

The general term (n^{th}) of a linear number pattern is: $T_n = bn + c$ $\left(\begin{array}{c} n = position\ of\ the\ term \\ b = common\ difference \\ c = T_1 - b \end{array}\right)$

$T_1 = b + c$

Examples:

1. Given the linear number pattern: 4; 7; 10; 13; ...

(a) Write down the next 2 terms

$b = 7 - 4\ or 10 - 7 or 13 - 10$
$b = 3$
Next two terms: 16; 19 ... add the value of d

(b) Determine the n^{th} term of the sequence.

$T_n = bn + c$
$= 3n + c$ and $c = T_1 - b = 4 - 3 = 1$
$T_1 = 3n + 1$

OR $b = 3$ and $T_1 = b + c$
$4 = 3 + c$

(c) Calculate the 20^{th} term of the number pattern.

$T_n = 3n + 1$
$T_{20} = 3(20) + 1 = 61$ ... substitute 20 to n

(d) **Which term** of the number pattern is 301? position

$T_n = 3n + 1$ ... T_n is the unknown term
$301 = 3n + 1$
$n = 100$ ... transpose 1 and divide by 3

ACTIVITIES/ ASSESSMENT

1. Given the sequence: 3; 0; -3; …

   (a) Write down the next term of the sequence.

   (b) Determine the general term of the sequence.

   (c) calculate the 15^{th} term of the sequence.

   (d) Determine which term of the sequence is equal to – 60?

2. Given linear number pattern: 8; 3; -2; …

   (a) Write down the next two terms of the pattern.

   (b) Determine the n^{th} term of the pattern

   (c) Determine T_{30}, the thirtieth term of the pattern

   (d) Which term of the pattern is equal to – 492?

3. The first four terms of Pattern A and Pattern B are shown in the table below:

Position of term (n)	1	2	3	4
Pattern A	1	3	5	71
Pattern B	1	9	25	49

   (a) Determine the general term for the n^{th} term of Pattern A

   (b) Hence, determine the general formula n^{th} term of Pattern B

4. $3x + 1$; $2x$; $3x - 7$; … are the first three terms of a linear number pattern.

   (a) If the value of x is three, write down the first three terms

   (b) Determine the formula for T_n, the general term of the sequence

   (c) Which term in the sequence is first to be less than – 31?

TOPIC: NUMBER PATTERNS (Lesson 2)		Weighting	25 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Quadratic Number Pattern				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

a

METHODOLOGY

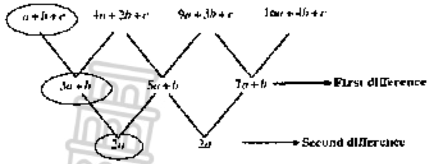
A **quadratic number pattern** is a sequence of numbers in which the **second difference** between any two consecutive terms is constant. Quadratic patterns are referred as **second difference patterns** that have a general formula:

$$T_n = an^2 + bn + c \qquad \begin{pmatrix} a = \frac{2^{nd} difference}{2} \\ c = T_0 \\ b = T_1 - a - c \end{pmatrix} \ldots T_0 \text{ method}$$

$$T_1 = a(1)^2 = b(1) + c$$
$$\therefore T_1 = a + b + c$$

OR from the general term: $T_n = an^2 + bn + c$

$T_1 = a(1)^2 = b(1) + c$, $T_2 = a(2)^2 = b(2) + c$, $T_3 = a(3)^2 = b(3) + c$, $T_4 = a(4)^2 = b(4) + c$, ..
$= a + b + c$ $\quad = 4a + 2b + c$ $\quad = 9a + 3b + c$ $\quad = 16a + 4b + c$



The **first differences** of a quadratic number pattern form a linear number pattern.

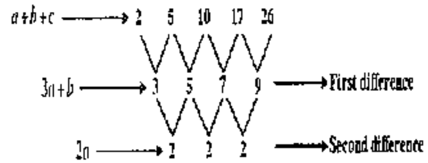
Examples:

1. Consider the number pattern: 2; 5; 10; 17; 26; …

   (a) Write down the next two terms of the number pattern.

   (b) Determine the n^{th} term of the number pattern.

   (c) Determine the 23^{rd} term of the number pattern.



(a) 37 … (26 + 11) and 50 … (37+13)

(b) $2a = 2$ $\quad a = 1$ $\qquad 3a + b = 3$ $\quad 3(1) + b = 3$ $\quad b = 0$ $\qquad a + b + c = 2$ $\quad 1 + 0 + c = 2$ $\quad c = 1$

$\therefore T_n = 1n^2 = 0n + 1$

$T_n = n^2 + 1$

(c) $T_{23} = (23)^2 + 1 = 530$

ZERO METHOD

Before 3 (first term in the first differences) there is 1 leading to T_0 in the sequence.

$\therefore T_0 = 2 - 1 = 1$

$c = T_0 = 1$

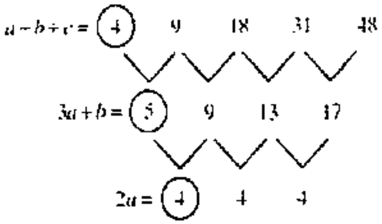
$b = T_1 - a - c$

$= 2 - 1 - 1 = 0$

2. Given the quadratic number pattern: 4; 9; 18; 31; …

(a) Determine the next term

48 … (31+17)



(b) Determine the n[th] term

$2a = 4$ $\quad a = 2$ $\qquad 3a + b = 5$ $\quad 3(2) + b = 5$ $\quad b = -1$ $\qquad a + b + c = 4$ $\quad 2 - 1 + c = 4$ $\quad c = 3$

$\therefore T_n = 2n^2 - n + 3$

(c) Which term of the quadratic number pattern is equal to 234?

$T_n = 234$

$2n^2 - n + 3 = 234$

$2n^2 - n + 231 = 0$

$(2n + 21)(n - 11) = 0$ … (you can use a quadratic formula)

$n = -\frac{21}{2}$ or $n = 11$ … (n must be a natural number)

$n = 11 \quad \therefore T_{11} = 234$

ACTIVITIES/ ASSESSMENT

1. Determine the first five terms of a quadratic sequence defined by $T_n = 5n^2 + 3n + 4$

2. Given the quadratic pattern: -3; 1; 7; 15; 25; …

(a) Write down the 6[th] and the 7[th] terms. (b) Determine the T_n of the pattern

3. Consider the quadratic number pattern: 2; 3; 6; 11; …

(a) Write down the next two terms of the given number pattern

(b) Determine the general term (T_n) of the number pattern

(c) Determine the T_{50}

(d) Which term of the quadratic number pattern is 443?

4. Given $T_n = 2n^2$. For which value(s) of n is $T_n = 32$?

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TOPIC: NUMBER PATTERNS (Lesson 3)		Weighting	25 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Quadratic Number Pattern				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

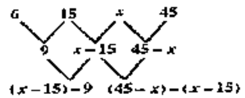
RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

1. 6; 15; x; 45; … is a quadratic number pattern.



(a) Determine the value of x

$(x - 15) - 9 = (45 - x) - (x - 15)$

$x - 15 - 9 = 45 - x - x + 15$

$x + x + x = 45 + 15 + 15 + 9$

$3x = 84$

$x = 28$

(b) Determine the general term.

$2a = (x - 15) - 9$

$2a = (28 - 15) - 9 = 4$

$a = 2$

$3a + b = 9$

$3(2) + b = 9$

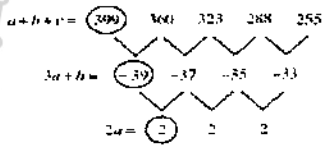
$b = 3$

$a + b + c = 6$

$2 + 3 + c = 6$

$c = 1 \quad \therefore T_n = 2n^2 + 3n + 1$

2. Consider the quadratic number pattern: 399; 360; 328; 288; 255; …



(a) Determine the formula for the n[th] term.

$2a = 2$ $\quad a = 1$ $\qquad 3a + b = -39$ $\quad 3(1) + b = -39$ $\quad b = -42$ $\qquad a + b + c = 399$ $\quad 1 - 42 + c = 399$ $\quad c = 440$

$\therefore T_n = n^2 - 42n + 440$

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(b) Which term of the pattern will have the lowest value?

Lowest value is the minimum value of a quadratic sequence.

To determine the minimum value, complete the square.

$$T_n = n^2 - 42n + 440$$
$$T_n = n^2 - 42n + \left(\frac{-42}{2}\right)^2 - \left(\frac{-42}{2}\right)^2 + 440$$
$$T_n = n^2 - 42n + (-21)^2 - 441 + 440$$
$$T_n = (n - 21)^2 - 1$$

For minimum $n = 21$ $\therefore T_{21}$ will have the lowest value

3. Given a quadratic pattern: 4; 4; 8; 16; ...

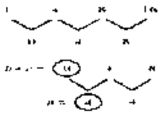
(a) What is the difference between the 50th and the 51th term?

Difference between T_1 and T_2 of a quadratic gives T_1 of the first differences

i.e., $(T_2 - T_1) quadratic = T_1$ (first difference)

$\therefore (T_{51} - T_{50}) quadratic = T_{50}$ (first difference)

Calculate first differences



$b = 4$ $\quad b + c = 0$

$4 + c = 0$

$c = -4$

$T_n = 4n - 4$

$T_{50} = 4(50) - 4 = 196$

$\therefore$ the difference between the 50th and the 51th terms of a quadratic sequence is 196.

(b) Between which two consecutive terms will difference be 28 088?

Determine which term of the linear pattern (first differences) is equal to 28 088

$T_n = 28\ 088$ $\qquad T_n = 4n - 4$

$$28\ 088 = 4n - 4$$
$$28\ 092 = 4n$$
$$7\ 023 = n$$

$T_{7\ 023} = 28\ 088$ ... from linear pattern (first differences)

$\therefore$ difference (28 088) is between $T_{7\ 023}$ and $T_{7\ 024}$ ... $((T_{7\ 024} - T_{7\ 023}) quadratic = T_{7\ 023}$ (first difference)

ACTIVITIES/ ASSESSMENT

1. 1; 7; x; 31; ... is a quadratic number pattern.

(a) Determine the value of x (b) Determine the general term.

2. Given the quadratic number pattern: -4; x; -28; -46; ...

(a) Determine the value of x (b) Determine the general term.

3. The constant second difference of the quadratic number pattern:

4; x; 8; y; 20; ... is 2.

(a) Determine the values of x and y. (b) Determine which term equals 125.

4. x, 50; 32; y; 8; ... is a quadratic number pattern. Determine the values of x an x and y.

5. Consider a quadratic number pattern: -4; -1; -2; -7; ...

(a) Determine the general term of the first differences of the sequence.

(b) Calculate the difference between the 25th and 26th terms of the quadratic sequence.

(c) Determine the general term of a quadratic number pattern

(d) Explain why the quadratic number pattern will never have a positive term.

6. -20; -9; 0; 7; ... is a quadratic number pattern.

(a) Determine T_n (b) Determine which term of the pattern will have the highest value.

7. Given the quadratic sequence: 846; 789; 734; 681; ...

Determine the position and the value of the term with the lowest value.

TEST : NUMBER PATTERNS

MARKS: 25 **DURATION: 25 Min**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUEASTION 1 [14 Marks]

1.1 Given the linear pattern: 5; -2; -9; …; -289

1.1.1 Write down the constant difference. (1)

1.1.2 Write down the value of T_4 (1)

1.1.3 Determine the number of terms in the number patter. (4)

1.2. $x-1; 2x-3; x+6;$ ... are the first three terms of a linear number pattern

Determine

1.2.1 the value of x (2)

1.2.1 the general term. (2)

1.3. A linear pattern has a difference of 3 between consecutive terms and its $T_{20}=64$.

1.3.1 Determine the value of the 22nd term. (1)

1.3.2 Which term in the pattern will be equal to $3T_5-2$? (4)

QUESTION 2 [11 Marks]

2.1 Consider the quadratic pattern: 5 ; 12 ; 29 ; 56 ; ...

2.1.1 Write down the next two terms of the pattern (2)

2.1.2 Prove that the first differences of this pattern will always be odd. (3)

2.2 A certain quadratic pattern has the following characteristics:

- $T_1=p$
- $T_2=18$
- $T_4=4T_1$
- $T_3-T_2=10$

Determine the value of p (6)

TOPIC: FUNCTIONS AND GRAPHS (Lesson 1)	**Weighting**	25 ± 3	Grade	11
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Quadratic Function:** $f(x)=ax^2+q$			
RELATED CONCEPTS/ TERMS/VOCABULARY	Function, graph, domain, range			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Mother graph, parameters "a" and "q"(vertical shift)

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

A **function** is a mathematical relationship between two **variables**, usually x and y are usually represented on graphs.
A **graph** is a is a visual representation of a function.

The set of all x-values that are the inputs in a function is called the **domain.**
The set of all y-values that are the outputs in a function is called **range**.

1. Mother graph $f(x)=ax^2$ and Parameter "a"

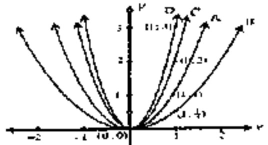
- The value of "a" stretches the graph away or towards the axes

A. $f(x)=x^2$ B. $f(x)=\frac{1}{2}x^2$ C. $f(x)=2x^2$ D. $f(x)=3x^2$

Domain: $x\in R$ or $x\in(-\infty;\infty)$ or $-\infty<x<\infty$

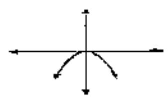
Range: $y\geq 0$ or $y\in[0;\infty)$ or $0\leq y<\infty$

Turning Point $(0;0)$

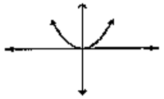


If "a" **increases**, the graph becomes **narrower** or stretches and as "a" **decreases** the graph becomes wider or flatter.

- The sign of "a" gives the shape of the graph, whether opens upwards or opens downwards.
  - ✓ $a<0$(negative), the graph opens downwards and has a **maximum turning point** the graph **reflects** on the x-axis



✓ $a > 0$(positive), the graph opens upwards and has a **minimum turning point**

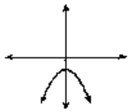


2. **The graph** $f(x) = ax^2 + q$ **and Parameter "q"** (vertical shift)

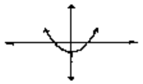
Vertical shift: Moves the graph up or down only and **do not change the x-coordinates.** Increases or decreases the **y-coordinate.**

- $q < 0$ (negative), the **graph shifts downwards** (decreases y-coordinate0

$a < 0$(negative)
Maximum turning Point

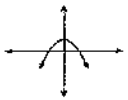


$a > 0$(positive)
Minimum Turning Point

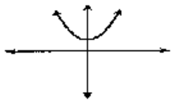


- $q > 0$ (positive), the **graph shifts upwards** (increases y-coordinate)

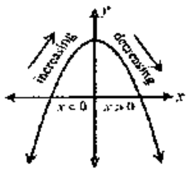
$a < 0$(negative)
Maximum turning point



$a > 0$(positive)
Minimum turning point



If $a < 0$, then the graph of the parabola **increases** for all $x < 0$ and **decreases** for all $x > 0$.



If $a > 0$, then the graph of the parabola **decreases** for all $x < 0$ and **increases** for all $x > 0$.

Example:

Given $f(x) = -2x^2 + 8$

Determine:
1. the x- and y- intercepts

x-intercepts: $y = 0$ AND
$0 = -2x^2 + 8$ ... $f(x) = y$
$2x^2 = 8$
$x^2 = 4$ ... divide by 2 both sides
$x = \pm 2$ ... square root sign on both sides

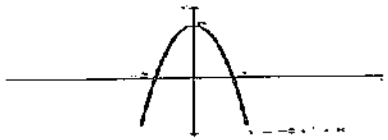
y-intercept: $x = 0$
$f(x) = -2(0)^2 + 8$
$y = 8$

2. Domain and Range of $f(x)$

Domain: $x \in R$

Range: $y < 8$

3. Sketch the graph of $f(x)$, showing all the intercepts and the turning point.



4. For which value (s) of x is
   (a) $f(x)$ decreasing

   $x \in (-\infty; 0]$ or $-\infty < x \leq 0$

   (b) $f(x) > 0$

   graph above the x-axis at $x \in (-2; 2)$ or $-2 < x < 2$

5. the turning point (state whether minimum or maximum)

   T.P. $(0;8)$, Maximum turning point

ACTIVITIES/ ASSESSMENT

1. Given: $f(x) = 3x^2$ and $g(x) = \frac{1}{4}x^2$

(a) Which parabola has arms that are closest to the y-axis?

(b) Sketch the graphs of these parabolas on the same set of axes.

(c) Are the parabolas concave up or down? Explain

2. Given: $f(x) = -\frac{1}{2}x^2$ and $g(x) = -4x^2$

(a) Which parabola has arms that are closest to the y-axis?

(b) Sketch the graphs of these parabolas on the same set of axes.

(c) Write down the domain and the range of each function.

3. Given: $f(x) = x^2 - 4$ and $g(x) = -4x^2 - 2$

(a) Sketch the graphs on the same set of axes.

(b) For these graphs, determine algebraically the coordinates of the intercepts with the axes.

(c) Write down the domain and the range of each function.

(d) For which value(s) of x is
1) $f(x)$ increasing 2) $g(x)$ decreasing 3) $g(x) < 0$ 4) $f(x) \geq 0$

TOPIC: FUNCTIONS AND GRAPHS (Lesson 2)		Weighting	30 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Quadratic Function: $f(x)=a(x+p)^2+q$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Horizontal shift, axis of symmetry				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Domain, Range, Turning point

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The effect of the parameter "p"

The effect of "p" is a **horizontal shift** because all points are moved the same distance in the same direction (the entire graph slides to the left or to the right). $f(x)=ax^2 \to f(x)=a(x+p)^2$

The **axis of symmetry** is the **vertical line** that divides the graph into two identical halves. It passes through the turning point $x=-p \ldots \ (x+p=0)$

If $p<0$, the graph will shift p units to the right.

Consider the following functions:

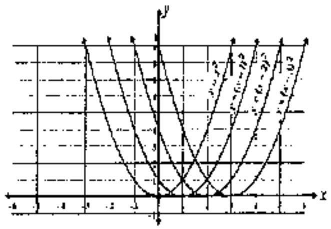
1. $y=x^2$ 2. $y=(x-1)^2$ 3. $y=(x-2)^2$ 4. $y=(x-3)^2$

Equation of the Axis of symmetry

1. $x=0$ 2. $x=1$ 3. $x=2$ 4. $x=3$

Turning Point of the graphs

1. $(0;0)$ 2. $(1;0)$ 3. $(2;0)$ 4. $(3;0)$



If $p>0$, the graph will shift p units to the left.

Consider the following functions:

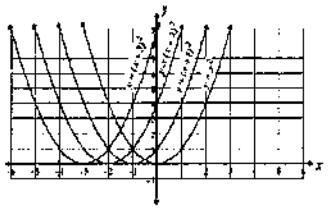
1. $y=x^2$ 2. $y=(x+1)^2$ 3. $y=(x+2)^2$ 4. $y=(x+3)^2$

Equation of the Axis of symmetry

1. $x=0$ 2. $x=-1$ 3. $x=-2$ 4. $x=-3$

Turning Point of the graphs

1. $(0;0)$ 2. $(-1;0)$ 3. $(-2;0)$ 4. $(-3;0)$



NOTE:

- The **x-value** of the turning point also **shifts** but the **y-value remains the same**.
- The **shape** of the graph does not change.

The graph of the form $f(x)=a(x+p)^2+q$

- Equation of the axis of symmetry: $x=-p$
- Turning point of the graph: $(-p;q)$
- Domain has no restrictions, therefore, $x\in R$
- Range: $a>0$, $y\geq$ y-value of the turning point ($y\geq q$) … **Minimum** value of the function is q
  $a<0$, $y\leq$ y-value of the turning point ($y\leq q$) … **Maximum** value of the function is q

Examples:

1. Given: $f(x)=2x^2+2$

Determine the equation of the new graph if $f(x)$ is shifted:

(a) 2 units to the right
$f(x)=2(x-2)^2+2$

(b) 1 unit to the left
$f(x)=2(x+1)^2+2$

(c) 4 units up
$f(x)=2x^2+2+4$
$f(x)=2x^2+6$

(d) 3 units down
$f(x)=2x^2+2-3$
$f(x)=2x^2-1$

(e) 2 units to the left and 3 units up.
$f(x)=2(x+2)^2+2+3$
$f(x)=2(x+2)^2+5$

(f) 3 units to the right and 2units down
$f(x)=2(x-2)^2+2-2$
$f(x)=2(x-2)^2$

To sketch the graph of a parabola $f(x)=a(x+p)^2+q$**, you need:**

- $x-$ and $y-$intercepts
- Axis of symmetry
- Turning point

2. Consider the function: $f(x)=-2(x+1)^2+8$

(a) Write down the equation of the axis of symmetry.
$x+1=0$
$x=-1$

(b) Determine the turning point of $f(x)$.
$(-p;q)=(-1;8)$

(c) Determine the x- and the y-intercepts.

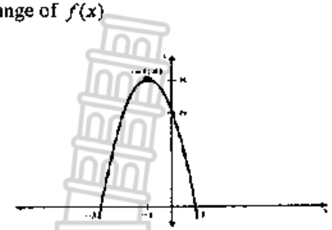
y-intercept: $x=0$
$f(x)=-2(x+1)^2+8$
$y=-2(0+1)^2+8$
$y=10$
$(0;10)$

x-intercepts: $y=0$
$0=-2(x+1)^2+8$
$2(x+1)^2=8$
$(x+1)^2=4$
$x+1=\pm 2$
$x=2-1=1$ or $x=-2-1=-3$
$(1;0)$ and $(-3;0)$

(d) Sketch the graph of $f(x)$. Determine the domain and the range of $f(x)$

Domain: $x \in R$

Range: $y \leq 8$



(e) Determine the x value for which the graph increases.

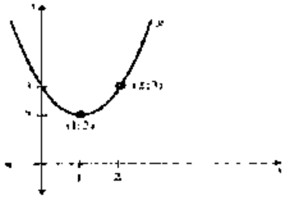
$x \in (-\infty;-1]$ or $-\infty < x \leq -1$

ACTIVITIES/ ASSESSMENT

1. Given: $f(x)=x^2-4$. Determine the equation of the new graph formed if the graph of $f(x)=x^2-4$ is:
   (a) shifted 2 units to the left.
   (b) shifted 2 units to the right.
   (c) shifted 4 units upwards.
   (d) shifted 1 unit downwards.
   (e) shifted 2 units to the left and 3 units upwards.
   (f) shifted 2 units to the right and 2 units downwards.
   (g) reflected about the x-axis.

2. Consider the function $g(x)=(x-2)^2-9$ and $f(x)=-(x+1)^2+4$
   (a) Draw a neat sketch graph of each function indicating the coordinates of the **intercepts** with the axes, the coordinates of the **turning point** and the equation of the **axis of symmetry**.
   (b) Determine the domain and the range in each function
   (c) Determine the x-values for which the graph of $g(x)$ increases.
   (d) Determine the values of x for which the graph of $f(x)$ decreases.
   (e) Determine the maximum or minimum value of the graphs.

3. Use the given diagram to determine:
   (a) the intercepts of the graph
   (b) the axis of symmetry of the graph
   (c) the turning point of the graph
   (d) the domain and the grange of the graph
   (e) the maximum or minimum value of the graph
   (f) the values of x for which the graph decreases
   (g) *the equation of the given diagram.

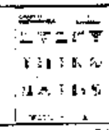


TOPIC: FUNCTIONS AND GRAPHS (Lesson 3)		Weighting	30 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Quadratic Function: $f(x)=a(x+p)^2+q$, $f(x)=ax^2+bx+c$				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Quadratic Equation, Factorisation, Quadratic Formula, Completing the square, axis of symmetry

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

A quadratic function is also given in the form $f(x)=ax^2+bx+c$, where c is the y-intercept

To sketch the graph of the function in the form $f(x)=ax^2+bx+c$, Calculate:

- the x- and the y-intercepts
- the axis of symmetry: $x=\frac{-b}{2a}$
- y-value by substituting the value of the axis of symmetry to the given equation
- turning point (axis of symmetry, y value)

Examples:

1. Determine the turning point of the following:

(a) $f(x)=x^2-5x-6$

$x=\frac{-b}{2a}=\frac{-(-5)}{2(1)}=\frac{5}{2}$

$f(x)=x^2-5x-6$

$y=\left(\frac{5}{2}\right)^2-5\left(\frac{5}{2}\right)-6=-\frac{49}{4}$

T.P. $\left(\frac{5}{2};-\frac{49}{4}\right)$

(b) $g(x)=-x^2+2x-3$

$x=\frac{-b}{2a}=\frac{-(2)}{2(-1)}=1$

$f(x)=x^2-5x-6$

$y=-(1)^2+2(1)-3=-2$

T.P. $(1;-2)$

$f(x)=ax^2+bx+c$ can be converted into the form $f(x)=a(x+p)^2+q$ by **completing the square.**

(a) $f(x)=x^2-5x+\left(\frac{5}{2}\right)-\left(\frac{5}{2}\right)-6$

$=x^2-5x+\left(-\frac{5}{2}\right)^2-\frac{25}{4}-6$

(b) $g(x)=-[x^2-2x+3]$

$=-[x^2-2x+\left(-\frac{2}{2}\right)^2-\left(-\frac{2}{2}\right)^2+3]$

$=\left(x-\frac{5}{2}\right)^2-\frac{49}{4}$

T.P. $\left(\frac{5}{2};-\frac{49}{4}\right)$

$=-[(x-1)^2+2]$

$=-(x-1)^2-2$ T.P $(1;-2)$

2. Sketch the graph of $f(x)=3x^2+6x-9$

y-intercept: $y=-9$

Axis of symmetry: $x=\frac{-b}{2a}=\frac{-(6)}{2(3)}=-1$

x-intercept: $0=3x^2+6x-9$

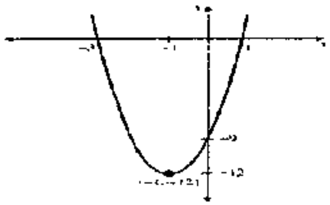
$0=x^2+2x-3$ ... common factor

$0=(x+3)(x-1)$

$x=-3$ or $x=1$

$f(x)=3(-1)^2+6(-1)-9=-12$

T.P $(-1;-12)$



Another form of the quadratic function is $f(x)=a(x-x_1)(x-x_2)$, where x_1 and x_2 are x-intercepts

ACTIVITIES/ ASSESSMENT

1. Determine the turning point of the following:

(a) $f(x)=x^2-6x+5$

(b) $g(x)=-x^2+4x+12$

(c) $f(x)=2x^2-3x-4$

(d) $h(x)=-2x^2+8x+10$

2. Sketch the graphs of the following functions:

(a) $f(x)=x^2-2x-3$

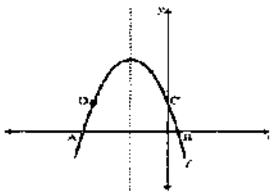
(b) $g(x)=-x^2+4x-3$

(c) $h(x)=2x^2+8x-10$

(d) $f(x)=8-2x-x^2$

3. The diagram shows the graph of $f(x)=-x^2-6x+7$. D is the mirror image of C in the axis of symmetry of $f(x)$

(a) Determine the equation of the axis of symmetry
(b) Determine the domain and the range
(c) Determine the turning point
(d) Determine the coordinates of A, B, C and D



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TOPIC: FUNCTIONS AND GRAPHS (Lesson 4)	**Weighting** 30 ± 3	**Grade**	11
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Determining the Equations of a Parabola**		
RELATED CONCEPTS/ TERMS/VOCABULARY			

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The quadratic Function (Parabola) is defined by:

$$\left\{\begin{array}{l} y=a(x+p)^2+q \\ y=ax^2+bx+c \\ y=a(x-x_1)(x-x_2) \end{array}\right\}$$

Examples:

1. Determine the equation of a parabola which has points (2;0), (– 4;0) and (4;8).

(2;0), (– 4;0) are x-intercepts, therefore, $y=a(x-x_1)(x-x_2)$

$y=a(x-2)(x+4)$

$8=a(4-2)(4+4)$ ... substitute (4;8)

$8=16a$

$a=\frac{1}{2}$, $\therefore y=\frac{1}{2}(x-2)(x+4)$

2. Determine the equation of the parabola if it has a turning point (1;2) and passes through (2; –1).

(1;2) is the turning point $(-p;q)$, therefore, $y=a(x+p)^2+q$

$y=a(x-1)^2+2$

$-1=a(2-1)^2+2$ ... substitute (2; –1)

$a=-1$, $\therefore y=-3(x-1)^2+2$

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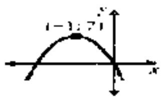
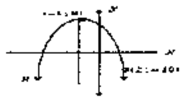
3. Determine the equation of a parabola on the diagram below.

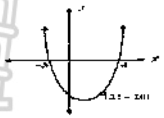
(a)



(b)



(c)



(a)	(b)	(c)
T.P (-1;7)	T.P. (-1;8)	x-intercepts (-3;0) and (4;0)
$y=a(x+p)^2+q$	$y=a(x+p)^2+q$	$y=a(x-x_1)(x-x_2)$
$y=a(x+1)^2+7$	$y=a(x+1)^2+8$	$y=a(x+3)(x-4)$
$0=a(0+1)^2+7$	$-10=a(2+1)^2+8$	$-20=a(2+3)(2-4)$
$a=-7$	$-18=9a$	$-20=-10a$
	$a=-2$	$a=2$
$y=-7(x+1)^2+7$	$y=-2(x+1)^2+8$	$y=2(x+3)(x-4)$

ACTIVITIES/ ASSESSMENT

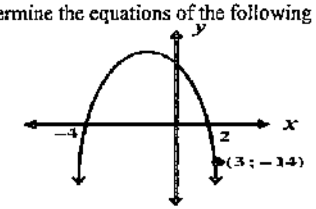
1. Determine the equation of the parabola which passes through the points $(-4;0)$, $(1;0)$ and $(0;12)$

2. Determine the equation of the parabola which has a turning point $(1;-2)$ and passes through the point $(2;1)$.

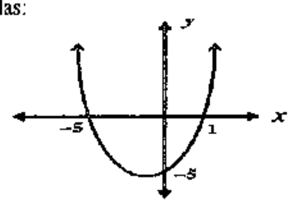
3. Determine the equation of the parabola with a turning point $(-2;4)$ and a y-intercept of 2.

4. Determine the equations of the following parabolas:

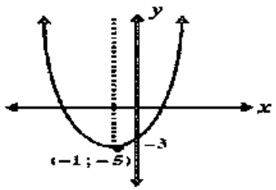
(a)



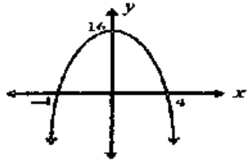
(b)



(c)



(d)



5. The parabola with the equation $y=-2(x+p)^2+q$ has axis of symmetry $x=1$ and range $(-\infty;5]$. Determine the x-intercepts of the parabola.

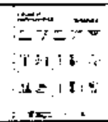
TOPIC: FUNCTIONS AND GRAPHS (Lesson 5)		Weighting	30 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Hyperbola: $f(x)=\frac{a}{x}+q$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Asymptotes, line of symmetry				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

x- and y- intercepts, vertical shift

RESOURCES






ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Writing equation of asymptotes as $p=...$ and $q=...$

Failing to identify correct quadrants for the graph

METHODOLOGY

Mother graph: $y=\frac{a}{x}$

The graph does not touch the x-axis and the y-axis. These lines are called **asymptotes**.

Asymptote is a line that a graph gets closer to but never touches. Hyperbola has TWO asymptotes

- Horizontal asymptote: $y=0$
- Vertical asymptote: $x=0$

The effect of "a" on the hyperbola is the same as the effect of "a" on the parabola.

- If "a" **increases**, the graph becomes **narrower** or stretches and as "a" **decreases** the graph becomes **wider** or flatter.
- If $a>0$, the graph lies on the first and the third quadrants.
- If $a<0$, the graph lies in the second and the fourth quadrants and the graph reflects on the x-axis.

Line of symmetry of a hyperbola passes through the point of **intersection of the asymptotes**.

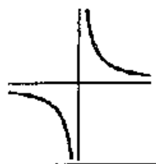
Example 1

Consider the function: $y=\frac{6}{x}$

Equations of Asymptotes: $x=0$ (vertical asymptote)

$y=0$ (horizontal asymptote)

Lines of symmetry: $y=x$



$y = -x$

Domain: $x \in R, x \neq 0$ Range: $y \in R, y \neq 0$

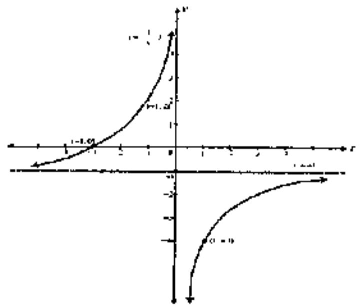
The graph of $f(x) = \frac{a}{x} + q$

Like in the parabola, q **shifts the graph** up or down (vertical shift) and the asymptote

- q gives the equation of the asymptote, $y = q$

Example 2.

Given the function: $f(x) = \frac{-3}{x} - 1$



Equations of asymptotes: $x = 0$ (vertical0
$y = -1$ (horizontal)

Lines of symmetry: $y = x - 1$
$y = -x - 1$

Domain: $x \in R, x \neq 0$ Range: $y \in R, y \neq -1$

ACTIVITIES/ ASSESSMENT

1. Consider the function $f(x) = \frac{2}{x}$.

(a) Determine the equation of the new graph formed if the graph of $f(x) = \frac{2}{x}$ is:

1) shifted 2 units upwards.
2) shifted 2 units downwards.
3) reflected about the y-axis.
4) reflected about the x-axis.

(b) Determine the equations of asymptotes of 1)
(c) Determine the domain and the range of $f(x)$
(d) Determine the line of symmetry of 2)

2. Sketch the following graphs on different sets of axes. In each function;

- Determine the equations of the asymptotes.
- Determine the equations of the axes of symmetry.
- Determine the coordinates of the x-intercept.
- Determine the coordinates of the y-intercept.
- Clearly indicate the intercepts with the axes, the asymptotes and the symmetry lines on the graphs.
- Write down the domain and the range.

(a) $f(x) = \frac{4}{x} - 2$ (b) $g(x) = -\frac{1}{x} + 3$

TOPIC: FUNCTIONS AND GRAPHS (Lesson 6)		Weighting	30 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Hyperbola: $f(x) = \frac{a}{x+p} + q$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Parameter "p"				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Equations of asymptotes, equations of lines of symmetry, domain and the range

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing vertical and horizontal asymptotes
Confusing p with x and q with y in equations of asymptotes

METHODOLOGY

Functions of the form $y = \frac{a}{x+p} + q$ and the effects of the parameter "p"

"p" shifts the graph horizontally (to the left or to the right)

If $p > 0$, the graph shifts p units to the left (remember the asymptote also shifts p units to the left).
If $p < 0$, the graph shifts p units to the right (remember the asymptote also shifts p units to the right).

To sketch the graph of a hyperbola $y = \frac{a}{x+p} + q$

- Identify the quadrants by checking the sign of a
- Indicate asymptotes on the axes
- Determine the x- and the y-intercepts
- Determine the lines of symmetry/axes of symmetry: $y = x + c$ and $y = -x + c$ (lines of symmetry passes through the point of intersection of the asymptotes)

Example:

Consider the function: $f(x) = \frac{2}{x+1} + 2$

(a) Write down the equations of the asymptotes
(b) Determine the x- and the y-intercepts
(c) Sketch the graph of $f(x)$
(d) Write down the domain and the range
(e) Determine the lines of symmetry

(a) $y = 2$ and $x = -1$ ... $(x + 1 = 0)$

(b) x-intercept $(y = 0)$ y-intercept $(x = 0)$

$0=\frac{2}{x+1}+2$

$-2=\frac{2}{x+1}$

$-2(x+1)=2$

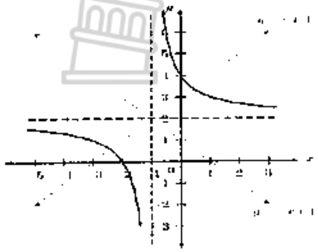
$x+1=-1$

$x=-2 \ldots (-2;0)$

$f(0)=\frac{2}{0+1}+2$

$y=2+2$

$y=4 \ldots (0;4)$



(c) Domain: $x\in R, x\neq -1$
Range: $y\in R, y\neq 2$

(d) $y=x+c$
$2=(-1)+c$
$3=c$
$y=x+3$

$y=-x+c$
$2=-(-1)+c$
$1=c$
$y=-x+1$

DETERMINING THE EQUATION OF A HYPERBOLA

2. A hyperbola has two asymptotes, $x=2$ and $y=-1$, and it passes through the point $(4;1)$. Determine the equation of the hyperbola and state the equations of its symmetry lines.

$y=\frac{a}{x+p}+q$

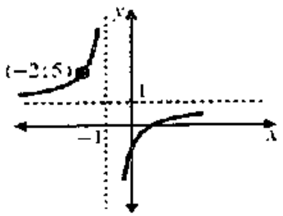
$y=\frac{a}{x-2}-1$ ... substitute asymptotes

$2=\frac{a}{4-2}$ ... substitute (4;1) into x and y

$a=4$, therefore, $y=\frac{4}{x-2}-1$

3. Determine the equation of:

Equations of asymptotes: $x=-1$ and $y=1$



$y=\frac{a}{x+1}+1$

$5=\frac{a}{-2+1}+1$

$a=-4$, therefore, $y=\frac{-4}{x+1}+1$ or $y=-\frac{4}{x+1}+1$

ACTIVITIES/ ASSESSMENT

1. For each hyperbolic function below, sketch the graph showing the **asymptotes**, the **intercepts** with the axes and the **lines of symmetry**. Hence determine the **domain and range** for each function.

(a) $f(x)=\frac{1}{x+2}-3$

(b) $y=\frac{3}{x-1}-3$

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(c) $g(x)=-\frac{4}{x-3}-1$

(d) $f(x)=-\frac{3}{x-1}+2$

(e) $(y-2)(x+2)=3$

(f) $x=\frac{2}{3-y}+5$

(g) $y=\frac{x+5}{x+2}$

(h) $f(x)=\frac{-2x-7}{x+4}$

2. A hyperbola has two asymptotes, $x=-1$ and $y=4$, and it passes through the point $(1;3)$. Determine the equation of the hyperbola and state the equations of its symmetry lines.

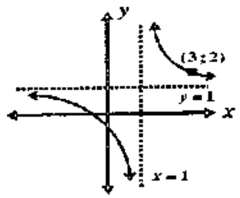
3. A hyperbola has two asymptotes, $x=2$ and $y=1$, and it passes through the point $(5;2)$. Determine the equation of the hyperbola and state the equations of its symmetry lines.

4. A hyperbola has two asymptotes, $x=-3$ and $y=-2$, and it passes through the point $(-2;-3)$. Determine the equation of the hyperbola and state the equations of its symmetry lines.

5. A hyperbola has two asymptotes, $x=4$ and $y=-3$, and it passes through the point $(-1;-4)$. Determine the equation of the hyperbola and state the equations of its symmetry lines.

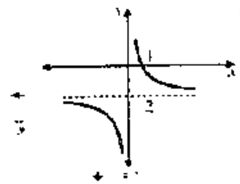
6. Determine the equation of the following hyperbolas:

(a)



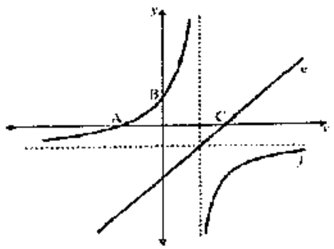
(b)

(c)



7. The graph of $f(x)=-\frac{4}{x-2}-1$ and one of its axes of symmetry g are shown.

Determine:
(a) The coordinates of A and B
(b) The equation of the vertical asymptotes of f
(c) The range of f
(d) The coordinates of C



*8. Draw a rough sketch of $y=\frac{a}{x+b}+c$ if $a<0, b<0$ and $c=-\frac{a}{b}$

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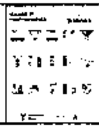
TOPIC: FUNCTIONS AND GRAPHS (Lesson 7)		Weighting	30 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Exponential Function: $y = ab^{x+p} + q$ for $b > 0$ and $b \neq 1$				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Failing to differentiate between decreasing and increasing function. No understanding why $\left(\frac{1}{2}\right)^x = 2^{-x}$

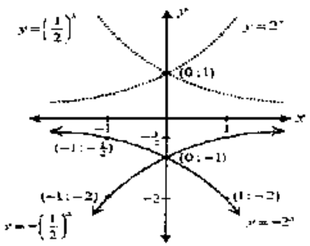
Failing to differentiate between an exponential function from a hyperbola

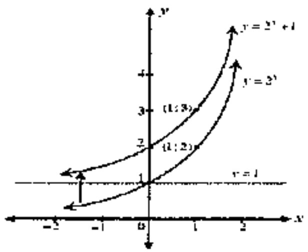
METHODOLOGY

The graph of the function: $y = ab^x + q$ for $b > 0$ and $b \neq 1$

The effect of "a"	The effect of "b"	The effect of "q"
$a > 0$: Graph above asymptote	$b > 0$: Graph increasing	$q > 0$: shifts graph upwards
$a < 0$: Graph below asymptote	$0 < b < 1$: Graph decreasing	$q < 0$: shifts graph downwards
	$b \leq 0$: Graph undefined	$y = q$: horizontal asymptote

Examples:





The graph of the function: $y = ab^{x+p} + q$ and the effect of "p"

"p" shifts the graph horizontally, left or right

"p" does not shift the horizontal asymptote ($y = p$)

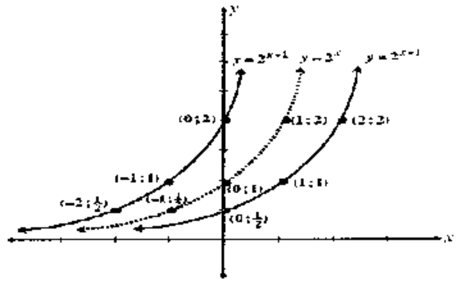
"p" disturbs the y-intercept of the graph.

NOTE: Exponential function has ONE asymptote

Consider the functions: $y = 2^x$ ... ($p = 0$) $y = 2^{x+1}$ ... ($p > 0$) $y = 2^{x-1}$ ... ($p < 0$)

$a > 0$

$b > 0, b \neq 1$



Example

1. Consider the function: $f(x) = 2^{x+2} - 2$
   (a) Determine the x- and the y-intercepts
   (b) Determine the equation of the asymptote
   (c) Write down the domain and the range.

(a) x-intercept ($y = 0$)

$0 = 2^{x+2} - 2$

$2 = 2^{x+2}$

$x + 2 = 1$

$x = -1$ ... $(-1; 0)$

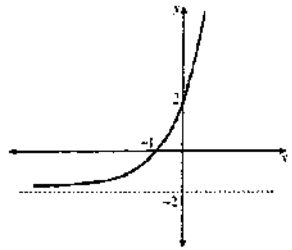
y-intercept ($x = 0$)

$f(x) = 2^{0+2} - 2$

$y = 2^2 - 2$

$y = 4 - 2$

$y = 2$ ... $(0; 2)$



(b) $y = -2$

(c) Domain: $x \in R$ Range: $y > -2$

2. (a) Sketch the graph of the function $y = 2.2^{x-1} - 4$.
   Indicate the intercepts with the axes and the asymptote.
   (b) Determine the domain and the range.

(a) x-intercept ($y = 0$)

$0 = 2.2^{x-1} - 4$

$4 = 2.2^{x-1}$

$2 = 2^{x-1}$

$x - 1 = 1$

$x = 2$ ... $(2; 0)$

y-intercept ($x = 0$)

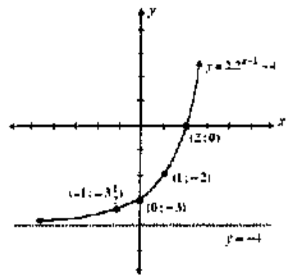
$y = 2.2^{0-1} - 4$

$y = 2.2^{-1} - 4$

$y = 2 \times \frac{1}{2} - 4$

$y = 1 - 4$

$y = -3$ ... $(0; -3)$



Equation of asymptote: $y = -4$

(b) Domain: $x \in R$ Range: $y > -4$

DETERMINING THE EQUATION OF AN EXPONENTIAL FUNCTION

3. $h(x) = ab^{x+p} + q$ and passes through C(3;4) and D(2;8). The asymptote of h is given by $y = 2$. If $a = q$,
determine the values of b, p and q.

$q = 2$ ... asymptote

$a = q \therefore a = 2$

$4 = 2.b^{3+p} + 2$ ... substitute (3;4)

$2 = 2.b^{3+p}$

$1 = b^{3+p}$

$b^0 = b^{3+p}$

$3 + p = 0$

$p = -3$

$8 = 2.b^{2+p} + 2$ ... substitute (2;8)

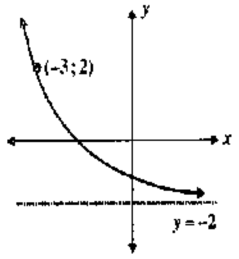
$6 = 2.b^{2+p}$

$3 = b^{2+p}$

$3 = b^{2-3}$

$3 = \frac{1}{b}$, therefore, $b = \frac{1}{3}$

4. Given the diagram below, determine the equation of



$g(x) = b^{x+1} + q$

$2 = b^{-3+1} - 2$

$4 = b^{-2}$

$2^{2x-\frac{1}{2}} = b^{-2x-\frac{1}{2}}$ or $4 = \frac{1}{b^2}$

$b = 2^{-1}$ $\quad 4b^2 = 1$

$b = \frac{1}{2}$ $\quad b^2 = \frac{1}{4}$, $b > 0$

$g(x) = \left(\frac{1}{2}\right)^{x+1} - 2$

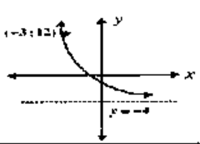
ACTIVITIES/ ASSESSMENT

1. For each exponential function below:
   (a) sketch the graph showing the asymptotes and any intercepts with the axes.
   (b) Hence determine the domain and range for each function.
   (c) State the values of x for which the function increases or decreases.

1) $f(x) = 2^{x+2} - 1$ 2) $y = -5^{x-2} + 1$ 3) $g(x) = 3.2^{x+1} - 6$ 4) $y = 2\left(\frac{1}{3}\right)^{x+1} - 2$

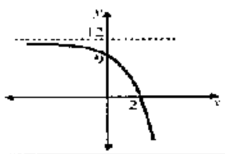
2. Determine the equation of the graphs shown below:

(a)



$y = b^{x+1} + q$

(b)



TEST 1: FUNCTIONS AND GRAPHS

MARKS: 25 **DURATION: 30 Min**

INSTRUCTIONS

1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations
4. Write neatly and legibly

QUESTION 1 [15 Marks]

Given $f(x) = -x^2 + 2x + 3$ and $g(x) = -2^x + 1$

1.1 Write down the equation of the asymptote of g (1)

1.1 Sketch the graphs of f and g on the same set of axes. (9) (6+3)

1.2 Determine the domain and range of g (2)

1.3 Determine the value of x for which the graph of $f(x)$ decreases. (2)

1.4 Write $f(x) = -x^2 + 2x + 3$ in the form $f(x) = a(x+p)^2 + q$ (1)

QUESTION 2 [10 Marks]

Given: $f(x) = \frac{8}{x-2} + 3$

2.1 Write down the equations of the asymptotes. (2)

2.2 Calculate the $x-$ and the $y-$intercepts of f (3)

2.3 Sketch the graph of f. Show clearly the intercepts with the axes and the asymptotes. (3)

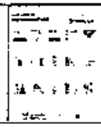
2.4 If $y = x + k$ is an equation of the line of symmetry of f, calculate the value of k. (2)

TOPIC: FUNCTIONS AND GRAPHS (Lesson 8)		**Weighting**	30 + 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Average Gradient between two points on a curve**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Distance between any two points, Midpoint of any two given point and gradient in any two given points

RESOURCES

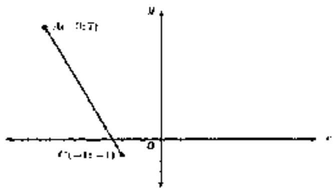

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

AVERAGE GRADIENT

The **gradient of a curve changes** at every point on the curve, therefore we need to work with the average gradient.

The **average gradient** of a function/curve between any two points is defined to be the **gradient of the straight line** joining the two points.



A linear function (straight line) has a **fixed gradient**, defined by: $m=\frac{y_2-y_1}{x_2-x_1}$

In a quadratic (parabola) function, hyperbola function and an exponential function, the **gradient changes** from point to point.

Therefore, the **average gradient** between any two points can be determined, by calculating the gradient of a line joining those two points.

The formula for calculating the average gradient of the curve, $f(x)$ between $x=a$ and $x=a+h$ is given by:

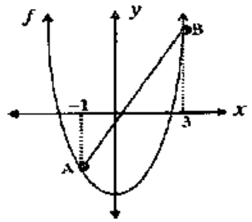
$$\text{Average gradient}=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$
$$=\frac{f(a+h)-f(a)}{a+h-a}$$
$$=\frac{f(a+h)-f(a)}{h} \text{ ... average gradient of a curve.}$$

Examples:

1. Determine the average gradient of the graph of $f(x)=x^2-4$ between $x=-1$ and $x=3$.

$f(-1)=(-1)^2-4=-3$ and $f(3)=(3)^2-4=5$

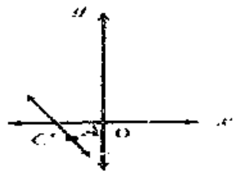
$$\text{Average gradient}=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$
$$=\frac{f(3)-f(-1)}{3-(-1)}$$
$$=\frac{5-(-3)}{4}=2$$



GRADIENT AT A SINGLE POINT ON A CURVE

At the point where **two points overlap**, the straight line only passes through one point on the curve. This line is known as a **tangent** to the curve.

Therefore, the idea of gradient at a single point on a curve is then introduced.



The **gradient at a point** on a curve is the **gradient of the tangent** to the curve at the given point.

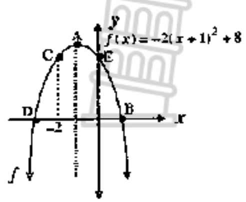
NOTE: This concept will be explored further in Grade 12.

ACTIVITIES/ ASSESSMENT

1. Determine the average gradient of the curve $f(x)=x(x+3)$ between $x=5$ and $x=3$.
2. Given $f(x)=x^2-4x+3$. Determine the average gradient between $x=-3$ and $x=-1$
3. Determine the average gradient of f between:

(a) A and B (b) B and C

(c) D and E

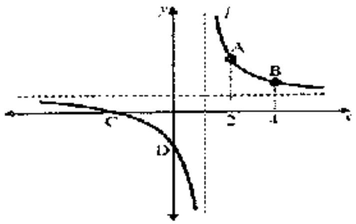


4. The sketch shows the graph of $f(x)=\frac{4}{x-1}+2$

Determine the average gradient between;

(a) A and B

(b) C and D



TOPIC: FUNCTIONS AND GRAPHS (Lesson 9)	Weighting	30 ± 3	Grade	11
Term		**Week no.**		
Duration	1 hour	**Date**		
Sub-topics	**Interpretation, Application and practical problems**			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

NOTE:

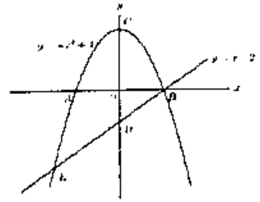
BASIC PRINCIPLES	INEQUALITIES
x- intercept(s) make $y=0$ and solve for x	$f(x)>0$ Values of x for which the graph f is above the $x-axis$
y- intercept make $x=0$ and solve for y	$f(x)<0$ Values of x for which the graph of f is below the $x-axis$
Point of intersection between 2 graphs Equate 2 functions, e.g., $f(x)=g(x)$ Solve for x Substitute x to any of the 2 functions to get y	$f(x)>g(x)$ Values of x for which the graph of f is above the graph of g
Horizontal distance/length between 2 points $=x_{Right}-x_{Left}$	$f(x)<g(x)$ Values of x for which the graph of f is below the graph of g.
Vertical distance/length between 2 points $=y_{Top}-y_{Bottom}$	$f(x).g(x)>0$ f and g have same sign (Both graphs above the x-axis OR both graphs are below the x-axis)
Average gradient $=\frac{f(x_2)-f(x_1)}{x_2-x_1}$	$f(x).g(x)<0$ f and g have opposite signs (f above x-axis and g below x-axis OR g above x-axis and f below axis.
BE ABLE TO DETERMINE • Domain, Range • Equations of symmetry • Equations of asymptotes • Turning point • Minimum and Maximum Value	Sometimes ≥ or ≤ are used. Means, the equal sign is included with the inequality sign. **TRANSFORMATIONS** in the graph. • $g(x)=f(-x)$: reflection about $y-axis$ • $g(x)=-f(x)$: reflection about $x-axis$

Examples

1. The graphs of $y=-x^2+4$ and $y=x-2$ are given.

Determine:

(a) the coordinates of A, B, C, D
(b) the coordinates of E
(c) the length of CD



(a) A and B are x-intercepts of the parabola and B is also the x-intercept of the straight line.

x-intercepts: $y=0$

$0=-x^2+4$

$x^2=4$

$x=\pm2$

$A(-2;0)$ and $B(2;0)$

C is the y intercept the parabola

$y=4$ $\quad\therefore C(0;4)$

D is the y-intercept of the straight line

$y=-2$ $\quad\therefore D(0;-2)$

(b) E and B are points of intersection of both graphs

At the point of intersection, graphs are equal. $-x^2+4=x-2$

$x^2+x-6=0$

$(x+3)(x-2)=0$ … can use quadratic formula

$x=-3$ or $x=2$

At E x is negative, $\therefore x=-3$

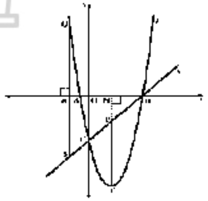
Substitute $x=-3$ into any of the two equations: $y=-3-2=-5$ $\therefore E(-3;-5)$

(c) y-value at C is 4 and y-value at D is -2

$\therefore CD = 4-(-2) = 6$ units

2. The following sketch show the graphs of $f(x)=x^2-6x-7$ and $g(x)=ax+b$

T is the turning point of f. TPN and QRS are perpendicular to the x-axis



(a) Determine:

1) The length of AB
2) The coordinates of C
3) The values of A and B
4) The coordinates of T
5) The length of PT
6) The length of PR if QS = 18 units

(b) Write down the range of f

(c) For which values of x is: 1) $f(x)\geq g(x)$

2) $f(x).g(x)<0$

(d) Write down the coordinates of the turning point of: 1) $y=f(x-2)$

2) $y=f(-x)$

(a) 1) $f(x)=x^2-6x-7$, A and B are x-intercepts of parabola

$x^2-6x-7=0$ ... calculate x-intercepts of parabola

$(x-7)(x+1)=0$ ... can use quadratic formula

$x=7$ or $x=-1$ $\therefore A(-1;0)$ $B(7;0)$

AB = 7-(-1) = 8 units

2) C is y-intercept of both f and g, $y=-7$. $\therefore C(0;-7)$

3) $B(7;0)$ and $C(0;-7)$, $b=-7$ ... y-intercept of g

$a=m=\frac{-7-0}{0-7}=1$

4) $x=\frac{-b}{2a}=\frac{-(-6)}{2(1)}=3$ $f(3)=(3)^2-6(3)-7=-16$ $T(3;-16)$

5) PT = Above graph – Below graph

$PT=g(x)-f(x)$

$=x-7-(x^2-6x-7)$

$=x-7-x^2+6x+7$

$=-x^2+7x$

$PT=-(3)^2+7(3)=12$ units

6) $QS=x^2-6x-7-(x-7)$

$18=x^2-6x-7-x+7$

$18=x^2-7x$

$x^2-7x-18=0$ ... standard form

$(x-9)(x+2)=0$ ... or quadratic formula

$x=9$ or $x=-2$

At R $x=-2$, $\therefore OR=2$ units (length is positive)

(b) $y\geq-16$

(c) 1) Where $f(x)$ is above $g(x)$: $x\in(-\infty;0]$ and $x\in[7;\infty)$ OR $x\leq0$ and $x\geq7$

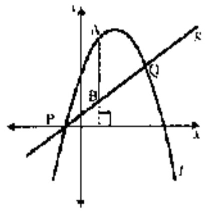
2) Where $f(x)$ is above x-axis and $g(x)$ is below x-axis OR where $f(x)$ is below x-axis and $g(x)$ is above x-axis: $x\in(-\infty;-1)$ OR $x<1$

(d) 1) Shift x-value of turning point 2 units to the right: $(5;-16)$

2) Reflection about the y-axis: x-value changes the sign. $(-3;-16)$

ACTIVITIES/ ASSESSMENT

1. The following sketch show the graphs of $f(x)=-x^2+10x+24$ and $g(x)=2x+4$



(a) Determine:

1) The value of P
2) The coordinates of Q
3) The coordinates of the turning point of f
4) The length of PQ
5) The length of AB if the x-value on A is 1.

(b) Write down the range of f

(c) For which values of x is:

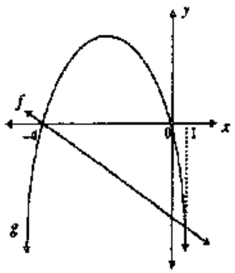
1) $f(x)\geq g(x)$
2) $f(x).g(x)<0$

(d) Write down the coordinates of the turning point of:

1) $y=f(x-2)$
2) $y=f(-x)$

2. The graphs of $f(x)=-2x-8$ and $g(x)=-2x^2-8x$ are represented in the diagram below.

Determine the value(s) of x for which:

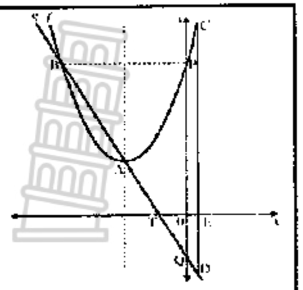


(a) $g(x)>0$
(b) $f(x)\leq0$
(c) $f(x)=g(x)$
(d) $f(x)<g(x)$
(e) $f(x).g(x)\geq0$
(f) $g(x)-f(x)=8$

2. The graphs of $f(x)=x^2+6x+14$ and $g(x)=ax+b$ are shown.

B is the mirror image of P in the axis of symmetry of f

(a) Determine the coordinates of A
(b) Write down the equation of the axis of symmetry
(c) Determine the coordinates of B
(d) Show that $a=-3$ and $b=-4$
(e) Calculate the length of 1) PQ
2) OE if CD = 28 units
(f) For which value(s) of x is 1) $f(x) < g(x)$ and
2) $f(x).g(x) \geq 0$
(g) If $y = f(x+c)$ has the same y-intercept as $y = f(x)$, determine the value of c



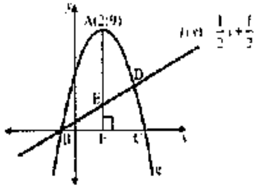
3. In the following sketch, the graphs $f(x)=\frac{1}{2}x+\frac{1}{2}$ of and $g(x)=ax^2+bx+c$ are shown.

(a) Write down the range of g (b) Determine a, b and c

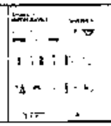
(c) Determine the length of 1) BC 2) AE

(d) Determine the coordinates of D

(e) Write the coordinates of the turning point of h if 1) $h(x) = g(-x)$ 2) $h(x) = -g(x+1)$



TOPIC: FUNCTIONS AND GRAPHS (Lesson 10)	Weighting	30 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Interpretation, Application and practical problems			
RELATED CONCEPTS/ TERMS/VOCABULARY				
PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE				
RESOURCES				

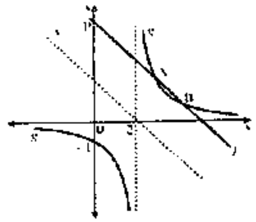




ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

1. The sketch shows the graph of $f(x)=-x+5$ and g(hyperbola)
(a) Determine the equation of g
(b) Write down the domain of g
(c) Determine the equation of s, an axis of symmetry of g
(d) Determine the average gradient of g between A and B.



(a) $y=\frac{a}{x+p}+q$ Equations of asymptotes: $x=2$ and $y=0$

$y=\frac{a}{x-2}$

$-1=\frac{a}{0-2}$ ... substitute (0; -1)

$a=2$ $\therefore g(x)=\frac{2}{x-2}$

(c) $y=-x+c$

$0=-(2)+c$

$c=2$ $\therefore y=-x+2$

(e) At A and B: $\frac{2}{x-2}=-x+5$

$(x-2)(-x+5)=2$

$-x^2+7x-12=0$

$x^2-7x+12=0$

$(x-3)(x-4)=0$

$x=3$ or $x=4$

Av. gradient $=\frac{f(4)-f(3)}{4-3}$

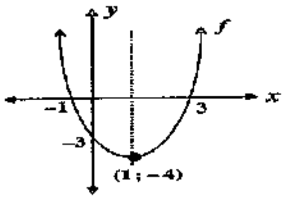
$=\frac{1-2}{1}=-1$

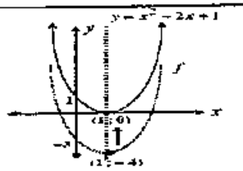
(b) $x \in \mathbb{R}, x \neq 2$

NATURE OF ROOTS

2. Consider the graph $f(x)=x^2-2x-3$

Determine for which value(s) of k for which
(a) $x^2-2x+k=0$ has equal roots
(b) $x^2-2x+k=0$ has non-real roots
(c) $x^2-2x+k=0$ has unequal roots
(d) $x^2-2x-3=k$ has equal roots
(e) $x^2-2x-3=k$ has non-real roots
(f) $x^2-2x-3=k$ has unequal roots

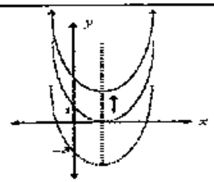




(a) If the graph is shifted 4 units up, the turning point will be on the x-axis, and roots will be equal. The y-intercept will also shift 4 units and will be $y=1$.

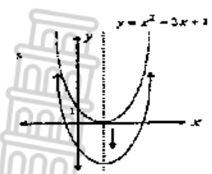
$\therefore y=x^2-2x+1$

Since k represents the y-intercept, $k=1$.

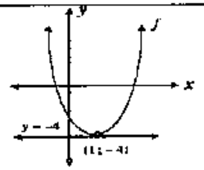


(b) If $y=x^2-2x+1$ is shifted upwards, the newly formed parabolas will have no x-intercepts.
The y-intercepts of will be greater than 1.
$\therefore k>1$

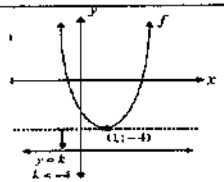
(c) if



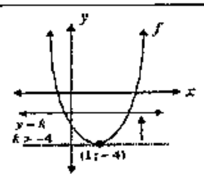
$y=x^2-2x+1$ is shifted downwards, the new parabola will have two unequal x-intercepts.
The y-intercepts of will be less than 1.
$\therefore k<1$



The horizontal line $y=k$ will pass through the turning point
$\therefore k=-4$



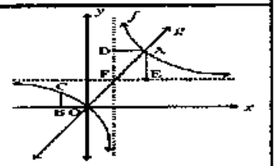
The horizontal line $y=k$ doesn't intersect with the parabola
$y=k$ is below $y=-4$
$\therefore k>-4$



The horizontal line $y=k$ will intersect with the parabola in two distinct points.
$y=k$ is above $y=-4$
$\therefore k>-4$

ACTIVITIES/ ASSESSMENT

1. The graph of $f(x)=\dfrac{2}{x-1}+2$ and $g(x)=2x$ are shown.

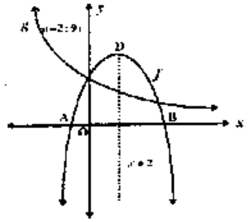


(a) Determine the equations of the asymptotes of f.
(b) Determine the coordinates of A.
(c) Determine the length of BC if OB =1 unit.
(d) Determine the area of rectangle ADFE.

2. The sketch represents the graphs of $f(x)=ax^2+bx+c$ and $g(x)=k^x$. The equation of the axis of symmetry of f is $x=2$ and the point (−2; 9) lies on g. The length of AB is 6 units. A and B are the x-intercepts of f and D is the turning point.

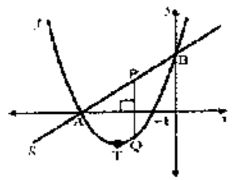
3. Sketch the graph of $f(x)=-x^2-2x+8$. Determine the values of k for which
(a) $-x^2-2x+k=0$ has real and equal roots
(b) $-x^2-2x+k=0$ has non-real roots
(c) $-x^2-2x+k=0$ has real and unequal roots
(d) $-x^2-2x+k=0$ has two unequal, negative roots

(e) $-x^2-2x+8=k$ has non-real solutions
(f) $-x^2-2x+8=k$ has equal roots
(g) $-x^2-2x+8=k$ has two distinct real solutions
(h) $-x^2-2x+8=k$ has two unequal roots with different signs.



(a) Determine the value of k.
(b) Determine the coordinates of A and B.
(c) Show that $a=-\dfrac{1}{5}$ and $b=\dfrac{4}{5}$
(d) Determine the coordinates of D.
(e) Use the graph to determine the values of p for which
$-\dfrac{1}{5}x^2+\dfrac{4}{5}x+p<0$

4. The following sketch shows the graphs of $f(x)=ax^2+bx+c$ and $g(x)=x+5$



(a) Determine the values of a, b and c
(b) Determine the coordinates of T, the turning point of f
(c) Determine the maximum length of PQ
(d) If $f(x-t)$ has one positive and one negative roots, determine the possible values of t
(e) For which values of k will $f(x)=k$ have
1) two distinct roots
2) non-real roots
3) two negative real roots

TEST 2: FUNCTIONS AND GRAPHS

MARKS: 25 **DURATION: 30 Min**

INSTRUCTIONS

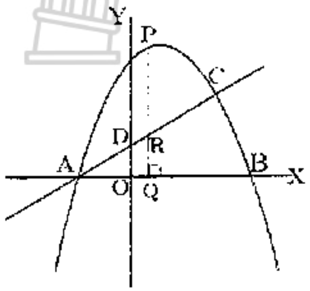
1. Answer ALL the questions
2. Round off correct to TWO decimal places, unless stated otherwise
3. Clearly show ALL Calculations

4. Write neatly and legibly

QUESTION 1 [18 Marks]

The diagram shows the graphs of $f(x)=-x^2+10x+24$ and $g(x)=2x+4$.

1.1 Calculate the length of AB (3)
1.2 Determine the coordinates of C (3)
1.3 Determine the coordinates of the turning point of f in the form $y=a(x+p)^2+q$ (4)
1.4 Determine the length of PR if OQ = 3 (3)
1.5 For which values of x will $f(x)>g(x)$
1.6 Write down the equation of $h(x)$ if $f(x)$ is shifted 2 units to the right and 3 units down (3)
1.7 For which value(s) of p $-x^2+10x+p=0$ have non-real roots? (2)



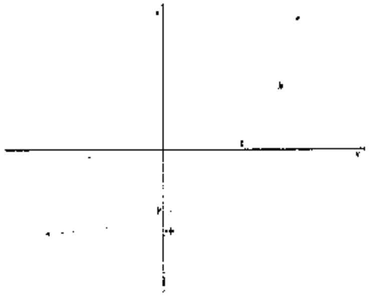
QUESTION 2 [7 Marks]

Given $h(x)=a.2^{x-1}+q$

2.1 Write down the value of q (1)

2.2 If the graph of h passes through the point $\left(-1;-5\frac{1}{4}\right)$, calculate the value of a (4)

2.3 Determine the equation of p if $p(x)=h(x-2)$ in the form $p(x)=a.2^{x-1}+q$ (2)

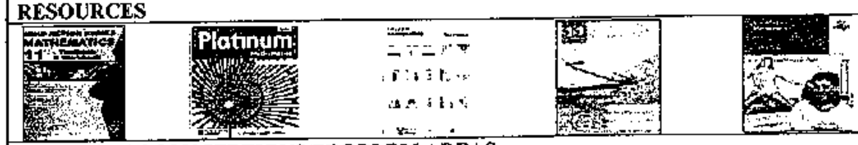


TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 1)	Weighting	15 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	The Effect of a and q on Graphs defined by $y=a\sin x+q$, $y=a\cos x+q$ and $y=a\tan x+q$ for $x\in[-360°;360°]$			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Amplitude, period, asymptote, Minimum, Maximum, turning point, coordinates, domain, range, x-intercepts, y-intercept, Vertical shift, translation, reflection about the axis

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

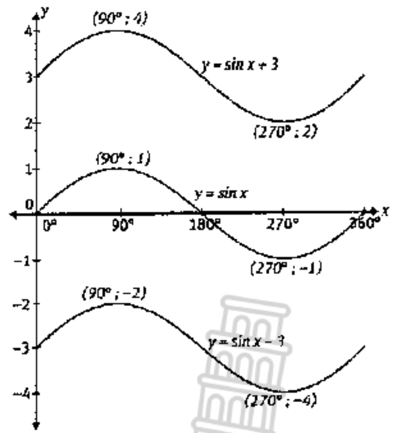
The Effect of a and q

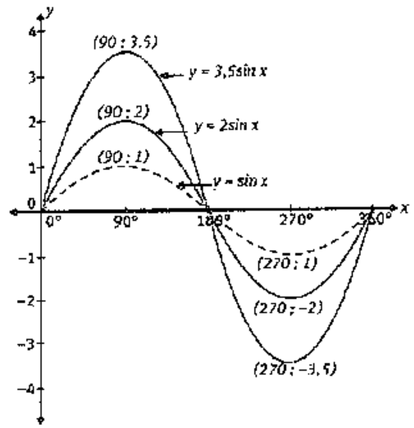
Amplitude is the positive value of a.

$a>1$: amplitude increases
$0<a<1$: amplitude decreases
$a<1$: Reflection about the $x-axis$

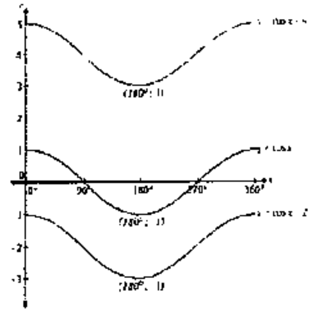
q shifts the graph up (q *is positive*) or down (q *is negative*): Vertical shift

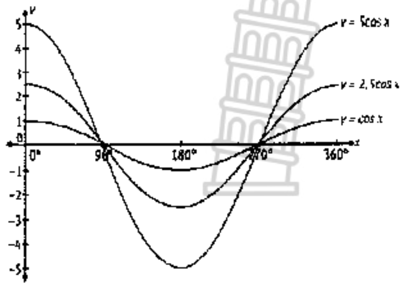
1. $y=a\sin x+q$ for $x\in[0°;360°]$



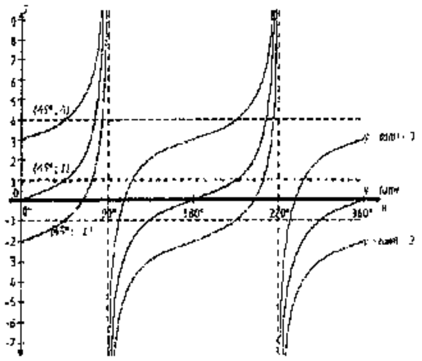


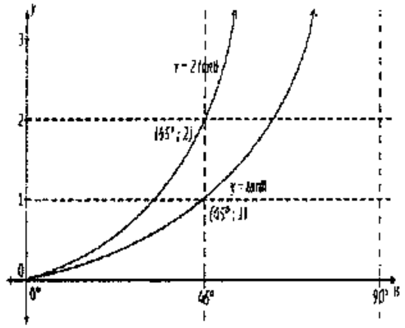
2. $y=a\cos x+q$ $x\in[0°;360°]$





3. $y = a\tan x + q$ $x \in [0°;360°]$





The graph $y = a\tan x + q$ has no amplitude because it has no maximum and minimum value.

Important points of Basic Trigonometric Graphs

$x \in [-360°; 360°]$	$y = a\sin x$	$y = a\cos x$	$y = a\tan x$
Amplitude	1	1	None
Period	360°	360°	180°
Asymptote	None	None	$x = \pm 90°$ $x = \pm 270°$
Range	$y \in [-1; 1]$	$y \in [-1; 1]$	$y \in [-\infty; \infty]$
Interval Spacing	90°	90°	45°

ACTIVITIES/ ASSESSMENT

1. Given: $f(x) = -2\sin x,\ x\epsilon[0°; 360°]$

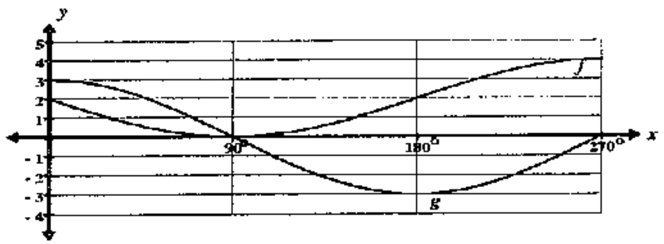
$g(x) = \frac{1}{2}\cos x - 1, x\epsilon[-180°; 180°]$
$h(x) = \tan x + 1, x\epsilon[-45°; 135°]$

(a) Sketch the graphs of the given functions on separate axes
(b) Determine the amplitude of g
(c) Write down the period of h
(d) Determine the range of f

2. Given the functions: $f(x) = -2\sin x + 1$ and $g(x) = \frac{1}{2}\cos x$ for $x\epsilon[-90°; 270°]$

(a) Sketch the graphs on the same set of axes
(b) For which value(s) of x is $g(x) \geq 0$?
(c) If f is reflected in the $x - axis$, write down the new equation in the form $y = \cdots$
(d) Determine $f(180°) - g(180°)$

3. The diagram below represents the graphs of two functions, f and g.



(a) Write down the equation of f and g.
(b) Write down the minimum and maximum values for f and g.
(c) Write down the amplitude for f and g.
(d) Determine graphically the value of $f(180°) - g(180°)$
(e) Determine graphically the value(s) of x for which:

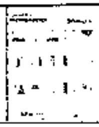
(1) $g(x) \geq 0$ (2) $g(x) < 0$ (3) $g(x) > 0$
(4) $f(x) = g(x)$ (5) $f(x) > g(x)$ (6) $f(x) = 0$
(7) $g(x) = -3$ (8) $f(x) - g(x) = 4$

TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 2)		Weighting	30 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	The effect of parameter k defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Interval Spacing				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Amplitude, period, asymptote, Minimum, Maximum, turning point, coordinates, domain, range, x-intercepts, y-intercept

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

k changes the period of the graph.

Period of $y = \sin(kx)$ and $y = \cos(kx)$ is $\frac{360°}{k}$ $(k > 0)$ and Period of $y = \tan(kx)$ is $\frac{180°}{k}$

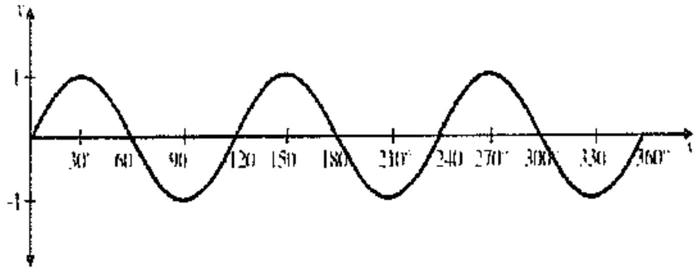
$k > 1$: the period of the graph decreases
$0 < k < 1$: the period of the graph increases

NOTE: Interval Spacing $= \frac{\text{period of the graph}}{4}$

Examples:

1. Sketch the graphs of each of the following functions on the separate axes:

(a) $f(x) = \sin 3x$ for $x \in [0°; 360°]$



Period$= \frac{360°}{3} = 120°$

Interval Spacing $= \frac{120°}{4} = 30°$

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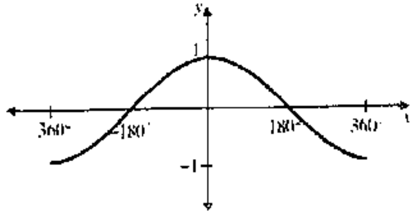
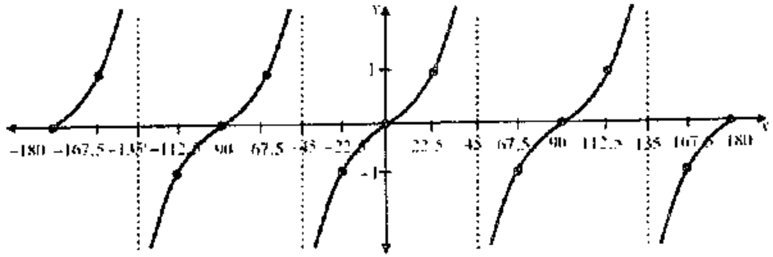
(b) $g(x) = \cos\frac{1}{2}x$ for $x \in [0°; 360°]$

Period $= \frac{360°}{\frac{1}{2}} = 720°$

Interval Spacing $= \frac{720°}{4} = 180°$



(c) $f(x) = \tan 2x$ for $x \in [-180°; 180°]$



Period $= \frac{180°}{2} = 90°$

Interval Spacing $= \frac{90°}{4} = 22{,}5°$

ACTIVITIES/ ASSESSMENT

Sketch the graphs of each of the following functions and write down the

(a) period, (b)amplitude and (c) range.

1. $f(x) = \tan\frac{1}{2}x$ for $x \in [0°; 360°]$

2. $f(x) = -\sin 3x$ for $x \in [-180°; 180°]$

3. $g(x) = \cos 4x$ for $x \in [-90°; 90°]$

4. $f(x) = \frac{1}{2}\cos 2x$ for $x \in [-90°; 180°]$

5. $g(x) = -2\sin 2x$ for $x \in [-90°; 180°]$

6. $f(x) = \tan\frac{1}{2}x$ for $x \in [-60°; 120°]$

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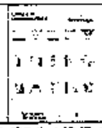
TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 3)		Weighting	15 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	The Effect of Parameter p on graphs defined by $y = a\sin(x+p)$, $y = a\cos(x+p)$ and $y = a\tan(x+p)$				
RELATED CONCEPTS/ TERMS/VOCABULARY	Horizontal shift				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Amplitude, period, asymptote Minimum, Maximum, turning point, coordinates, domain, range, x-intercepts, y-intercept,

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

p shifts the graph **horizontally**, $p°$ to the left ($p > 0$) or $p°$ to the right ($p < 0$)

Examples:

Sketch the graphs of the following functions:

1. $f(x) = \sin(x + 30°)$ for $-30° \leq x \leq 330°$

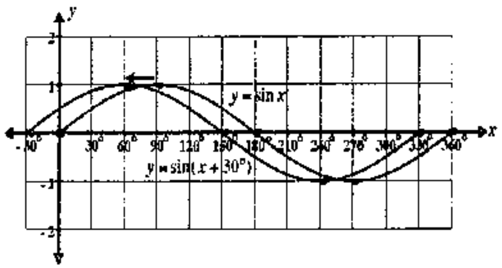
$p = 30°$

The graph is shifted 30° to the left

Range: $y \in [-1; 1]$

Amplitude is 1

Period is 360°

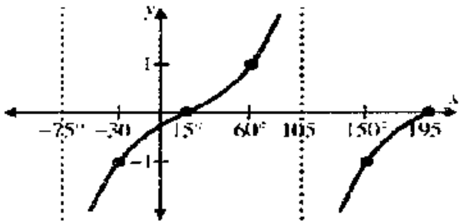


2. $g(x) = \tan(x - 15°)$ where $x \in [-75°; 195°]$

$p = -15°$

The graph is shifted 15° to the right

Period is 180°



3. $h(x) = -2\cos(x - 20°)$ for $-160° \leq x \leq 200°$

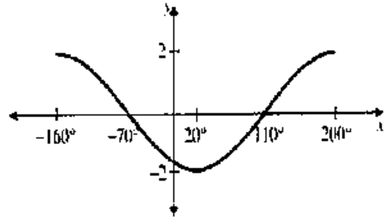
$p = -20°$

The graph is shifted 20° to the right

Range: $y \in [-2; 2]$

Amplitude is 2

Period is 360°



ACTIVITIES/ ASSESSMENT

Sketch the graphs of each of the following functions on separate axes and write down the:

(a) maximum and minimum values,
(b) range
(c) amplitude
(d) period of each function

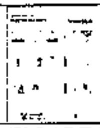
1. $f(x) = \sin(x + 45°)$ for $-135° \leq x \leq 405°$
2. $g(x) = \cos(x - 30°)$ for $x \in [-90°; 360°]$
3. $f(x) = -\tan(x - 20°)$ for $-70° \leq x \leq 200°$
4. $g(x) = 2\sin(x + 60°)$ for $x \in [-180°; 270°]$
5. $f(x) = \frac{1}{2}\tan(x - 45°)$, $x \in [-90°; 360°]$
6. $h(x) = \cos(x - 60°)$, $-150° \leq x \leq 360°$
7. $g(x) = -2\cos(x + 30°)$, $-120° \leq x \leq 330°$

TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 4)		Weighting	15 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Graphs defined by $y = a\sin k(x+p)+q$, $y = a\cos k(x+p)+q$ and $y = a\tan k(x+p)+q$				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Amplitude, period, asymptote Minimum, Maximum, turning point, coordinates, domain, range, x-intercepts, y-intercept

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

1. Sketch the graphs of the following functions:

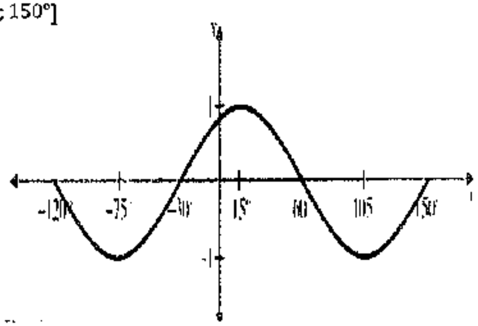
(a) $f(x) = \cos 2(x - 15°)$ for $x \in [-120°; 150°]$

$\text{Period} = \frac{360°}{2} = 180°$

$\text{Steps/interval} = \frac{180°}{4} = 45°$

Amplitude = 1

Shift to right



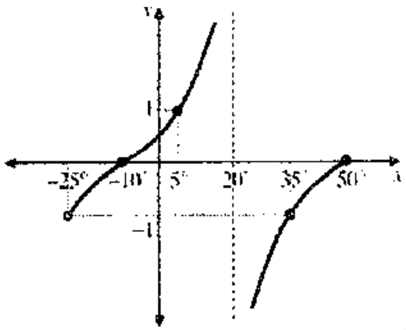
(b) $g(x) = \tan(3x + 30°)$ for $x \in [-25°; 50°]$

$g(x) = \tan 3(x + 10°)$

$\text{Period} = \frac{180°}{3} = 60°$

$\text{Steps/interval} = \frac{60°}{4} = 15°$

Shift to left



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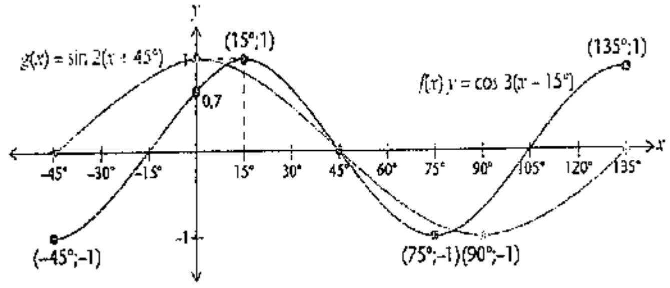
2. Sketch the following graphs on the same set of axes:

$f(x) = \sin(2x - 90°)$ and $g(x) = \cos 3(x - 15°)$ for $x \in [-45°; 135°]$

$f(x) = \sin 2(x - 45°)$

Function	Amplitude	Period	Steps	Shift
$f(x) = \sin 2(x - 45°)$	1	$= \frac{360°}{2} = 180°$	$= \frac{180°}{4} = 45°$	To the right
$g(x) = \cos 3(x - 15°)$	1	$= \frac{360°}{3} = 120°$	$= \frac{120°}{4} = 30°$	To the right



ACTIVITIES/ ASSESSMENT

Sketch the graphs of each of the following functions on separate axes and for each, determine the:

(a) Period
(b) Range
(c) x- and y-intercepts
(d) Maximum and minimum turning point(s)
(e) Function undefined
(f) Function increasing and decreasing

1. $g(x) = \sin 2(x - 30°)$ $x \in [-60°; 240°]$

2. $f(x) = \cos 3(x + 20°)$ $x \in [-80°; 100°]$

3. $f(x) = \tan 2(x - 45°)$ $-45° \leq x \leq 135°$

4. $g(x) = \cos(3x - 180°)$ $[0° \leq x \leq 360°]$

5. $f(x) = \cos(x - 60°)$ and $g(x) = \tan(x - 15°)$ on the same set of axes

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TOPIC: TRIGONOMETRIC FUNCTIONS (Lesson 5)		Weighting	15 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Interpretation of Trigonometric Graphs				

RELATED CONCEPTS/ TERMS/VOCABULARY

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Amplitude, period, asymptote Minimum, Maximum, turning point, coordinates, domain, range, x-intercepts, y-intercept,

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

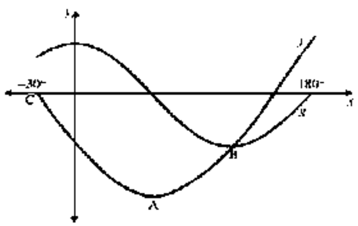
METHODOLOGY

SUMMARY

1. Complete the table.

FUNCTION	AMPLITUDE	PERIOD	STEPS	ASYMPTOTES	VERTICAL SHIFT	HORIZONTAL SHIFT
$y=-2\sin 3x$						
$y=-3\cos x+2$						
$y=\tan 2x-1$						
$y=\sin(x-30°)$						
$y=2\cos(x+60°)$						
$y=2\tan\frac{1}{2}x$						
$y=\tan(x-45°)$						
$y=\sin 2(x+45°)$						
$y=\cos 3(x+15°)$						
$y=\sin(x-75°)+1$						
$y=\cos(x-90°)-2$						

2. The functions $f(x)=-2\sin(x+p)$ and $g(x)=\cos\frac{3}{2}x$, $-30°\le x\le 180°$ are sketches alongside. A is a turning point of f and B is a turning point of g and also a point of intersection of f and g.



(a) Write down the value of p. $p=30°$

(b) What is the minimum value of f. -2

(c) Write down the period of g. $240°$

(d) Write down the amplitude of f. 2

(e) Determine the coordinates of A and B $A(60°;-2)$ and $B(120°;-1)$

(f) Determine the range of h if $h(x)=\frac{1}{2}f(x)-1$ $-2\le y\le 0$

(g) Write down the equation of m if m is obtained by shifting g $15°$ to the right and 2 units up.

$m(x)=\cos(\frac{3}{2}(x-15°))+2$

(h) For which values of x is:

1) $f(x)=g(x)$ — $x=120°$

2) $f(x)>g(x)$ — $120°<x\le 180°$

3) $f(x).g(x)\le 0$ — $-30°\le x\le 60°\cup 150°\le x\le 180°$

(i) For which value(s) of k will $g(x)=k$ have one solution. $k=1$ or $k=-1$

ACTIVITIES/ ASSESSMENT

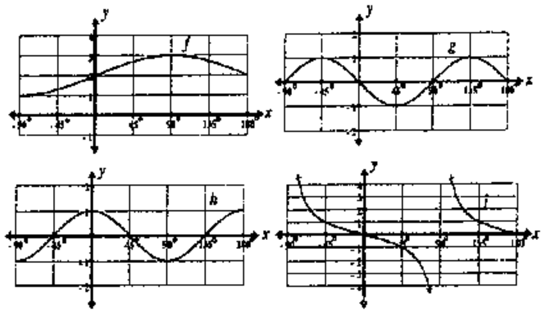
1. Sketch the graphs of $f(x)=3\cos x$ and $g(x)=\tan\frac{1}{2}x$ on the same set of axes for $x\in[-90°;270°]$

(a) Write down the 1) period of g and the minimum value of f

(b) For which values of x will $f(x)-g(x)>0$

(c) Determine the graph of $h(x)$ if $h(x)=g(2x)$

2. Determine the equations of graphs shown below:



(a) Determine the equations of each graph.

(b) Write down the amplitude of f.

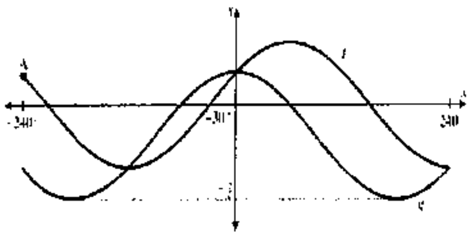
(c) Write down the period of f and g

(d) Determine graphically the value(s) of x for which the graph of:

1) h increases

2) j decreases

3. The functions $f(x)=\sin(x+p)$ and $g(x)=\cos x+q$ are sketched below. For both functions $-240°\le x\le 240°$



(a) Determine the values of p and q

(b) What is the maximum value of g ?

(c) What is the range of $y=-2g(x)+1$?

(d) Determine the equation of h, if h is obtained by shifting f f $60°$ to the right and 2 units down.

(e) Determine the coordinates of A.

(f) For which values of x is: 1) f increasing? 2) $g(x)=0$? 3) $f(x)<0$? 4) $f(x).g(x)\ge 0$?

TEST: TRIGONOMETRIC FUNCTIONS

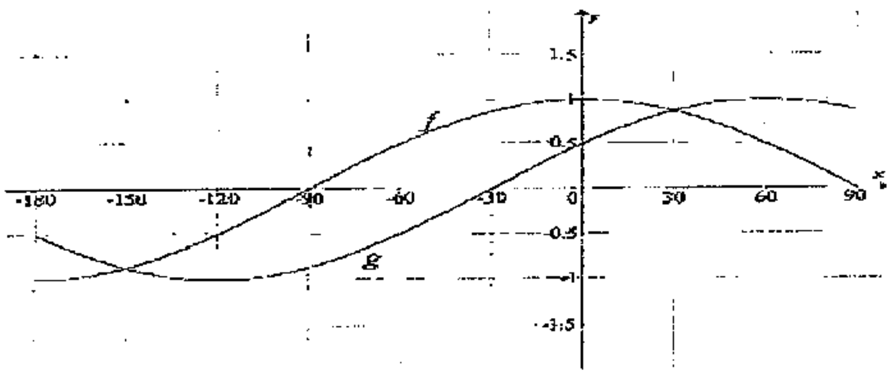
MARKS: 26 DURATION: 30 MIN

INSTRUCTIONS

1. Answer ALL questions
2. Unless stated or otherwise, round off answers correct to TWO decimal places
3. You may use an approved scientific calculator

QUESTION 1 [9 Marks]

In the diagram the graphs of $f(x)=\cos x$ and $g(x)=\sin(x+p)$ are drawn for the interval $-180° \le x \le 90°$



1.1 Write down the value of b (1)

1.2 Write down the period of g (1)

1.3 Write down the value(s) of x in the interval $-180° \le x \le 90°$ for which $f(x)-g(x)=0$ (2)

1.4 For which values of x in the interval $-180° \le x \le 90°$ is $\sin(90°-x) > g(x)$ (3)

1.5 The graph of h is obtained by shifting f 3 units upwards. Determine the range of h (2)

QUESTION 2 [13 Marks]

Given $f(x)=\frac{1}{2}\tan x$ and $g(x)=\sin 2x$

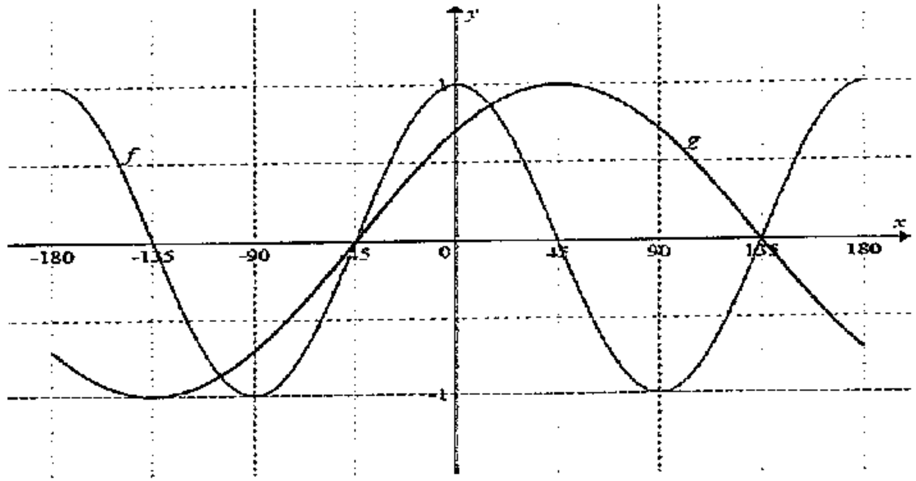
2.1 Draw the graph of f and g for $x \in [-90°;180°]$. Show all the turning points and intercepts with the axes. Clearly show the asymptotes using dotted lines. (6)

2.2 Determine the values of x, for $x \in [-90°;180°]$, for which $f(x) > g(x)$ (6)

2.3 Write down the period of $g(2x)$ (1)

QUESTION 3 [11 Marks]

In the diagram below the graphs of $f(x)=a\cos bx$ and $g(x)=\sin(x+p)$ are drawn for $x \in [-180°;180°]$



1.1 Write down the values of a, b and p (3)

1.2 For which of x in the given interval does the graph of f increase as the graph of g increases. (2)

1.3 Write down the period of $f(2x)$ (2)

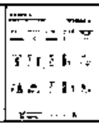
1.4 Determine the minimum value of h if $h(x)=3f(x)-1$ (2)

1.5 Describe how the graph of g must be transformed to form the graph of k, where $k(x)=-\cos x$ (2)

TOPIC: FINANCE, GROWTH AND DECAY (Lesson 1)	Weighting	15 ± 3	Grade	11
Term		Week no.		
Duration	1 hour	Date		
Sub-topics	Simple and Compound Interest Simple and Compound Decay			
RELATED CONCEPTS/ TERMS/VOCABULARY				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Simple interest: Interest is calculated as the percentage of the original amount invested or borrowed. $A = P(1+i.n)$

Compound interest: Interest is added after every period and the interest for the next period is calculated as a percentage of the new total. $A = P(1+i)^n$

Certain items **depreciate** in value over time. Depreciation is calculated exactly like interest but with a negative interest rate. There are **two types** of depreciation, or decay:

- Simple decay, or the straight-line method of depreciation: $A = P(1-i.n)$
- **Compound decay**, or the reducing balance method of depreciation: $A = P(1-i)^n$

A = Accumulated amount/final amount
P = Principal value/Present value
i = interest rate
n = time period in years/number of years

Examples:

1. Sam wants to invest R 3450 for 5 years. Wise Bank offers a savings account which pays simple interest at a rate of 12,5% per annum, and Grand Bank offers a savings account paying compound interest at a rate of 10,4% per annum. Which bank account would give Sam the greatest accumulated balance at the end of the 5-year period?

Simple Interest: P = R3 450, $n = 5$ years, i = 12,5% and A = ?
Compound interest: P = R3 450, $n = 5$ years, i = 10,4% and A = ?

A = P (1+i.n) $\qquad A = P(1+i)^n$

= 3 450 (1+ 12,5%.5) = R5 606,25 $\qquad = 3450(1+10,4\%)^5 = $ R5 658,02

Grand Bank

2. A new smartphone costs R 6000 and depreciates at 22% p.a. on a straight-line basis. Determine the value of the smartphone over a 4-year period.

P = R6 000, i = 22%, $n = 4$ years and A = ?
$A = P(1-i.n)$

A = 6 000 (1 – 22%. 4) = R720

3. A laptop cost R9 000 and, after four years, has a scrap value of R2 000. Find the annual depreciation rate if it is calculated using on a reducing balance.

P = R9 000, A = R2 000, $n = 4$ years
$A = P(1-i)^n$
$2000 = 9000(1-i)^4$
$\sqrt[4]{\frac{2000}{9000}} = 1-i \quad \therefore i = 0{,}3134 \times 100 = 31{,}34\%$

ACTIVITIES/ ASSESSMENT

1. Stephanie invests R5 000 for 6 years. Find the future value of her investment if the interest she receives is:
(a) 12% p.a. simple interest
(b) 12% p.a. compound interest

2. Marc saved an amount of money and it grew to R15 000 over a period of seven years. Calculate the amount of money originally invested if the interest received was:
(a) 13% p.a. simple interest
(b) 13% p.a. compound interest

3. The value of an investment grows from R 2200 to R 3850 in 8 years. Determine the simple interest rate at which it was invested.

4. James had R 12 000 and invested it for 5 years. If the value of his investment is R 15 600, what compound interest rate did it earn?

5. In five years' time Peter wants to have saved R50 000 in order to visit his friend who lives in Ireland. He manages to receive an interest rate of 14% per annum simple interest. How much must he invest now in order to achieve this goal?

DEPRECIATION

6. Mavis bought a new car for R280 000. The value of the car depreciates at 18% p.a. using the straight-line method. What will the value of the car be after 3 years?

7. A farmer bought a tractor. Five years later it had a book value of R168 345,22. Determine the original value of the tractor if the annual rate of depreciation was 14% p.a. on a reducing balance.

8. How long will it take an item bought for R12 800 to depreciate to R8 600 if depreciation is calculated at 11,5% p.a. on the straight-line method? Give your answer to the nearest year.

9. Rhino poaching is a serious problem, and has resulted in rhinos becoming an endangered species, particularly the black rhino. Statistics suggest that there were 60 000 black rhinos in 1970, but that only 4 200 remained in 2011.
(a) Determine the annual rate of depreciation that these statistics represent if the rate is:
1) Simple decay $\qquad$ 2) Compound decay

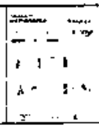
(b) Using your answer to 2), determine the number of black rhinos there would be in 2050 if this rate of compound decay continues.

TOPIC: FINANCE, GROWTH AND DECAY (Lesson 2)		Weighting	15 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Different Compounding Period				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES






ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Compound interest is often compounded more often than once a year. Interest can be quoted per annum but calculated over different time periods during a year.

Interest can be calculated:

- **Yearly**: once per year (usually at the end of the year
- **Semi-annually/half-yearly**: twice per year (every 6 months)
- **Quarterly**: 4 times per year (every 3 months)
- **Monthly**: 12 times per year (every month)

Examples:

1. Khabela deposits R7 000 into an account offering an interest rate of 8,5% p.a. compounded monthly. Calculate how much he will have in his account at the end of 4 years.

$P = R7\,000,\ i = \frac{18,5\%}{12},\ n = 4\times 12 = 48$ and $A = ?$

$$A = P(1+i)^n$$

$$A = 7000\left(1+\frac{18,5\%}{12}\right)^{48} = R\ 14\ 566,94$$

2. What amount must be invested for 2 years at an interest rate of 10% per annum compounded half-yearly in order to receive R10 000?

$P = ?,\ n = 2\times 2 = 4,\ i = \frac{10\%}{2}$ and $A = R10\,000$

$$A = P(1+i)^n$$

$$10000 = P\left(1+\frac{10\%}{2}\right)^4$$

$$P = \frac{10000}{\left(1+\frac{10\%}{2}\right)^4} = R8\ 227,02$$

3. Thulani borrows R2 000 with an agreement that he will pay back R3 000 exactly 2 years from now. Calculate the interest rate charged per annum, compounded quarterly.

$P = R2\,000,\ A = R3\,000,\ n = 2\times 4 = 8$ and $i = ?$

$$A = P(1+i)^n$$

$$3000 = 2000\left(1+\frac{i}{4}\right)^8$$

$$\sqrt[8]{\frac{3000}{2000}} = 1+\frac{i}{4}$$

$$\frac{i}{4} = 1,051989 - 1$$

$$i = 0,051989\times 4\times 100 = 20,80\%$$

ACTIVITIES/ ASSESSMENT

1. Khadija invested R25 000 into an account offering interest at 10,3% p.a. compounded quarterly.
   (a) Determine how much she has in her account after 6 years.
   (b) How much interest has she received after 6 years?

2. David invests R8 000 in a savings account which pays 8% per annum compounded monthly. Calculate the value of his investment in ten years' time.

3. Joseph borrows R15 000 at 21% p.a. compounded semi-annually. How much does he has to pay back at the end of 3 years?

4. Joseph invested an amount of money six years ago. Now, after six years, it is worth R1 200 000. The interest rate for the savings period was 18% per compounded monthly. What was the amount that was originally invested six years ago?

5. Wouter has R8 000 to invest over a period of 10 years. If he requires R15 000 at the end of the ten-year period, what annual interest rate, compounded monthly, will he need? Give your answer correct to one decimal place.

6. Cyril has inherited money and wants to invest a part of it in order to have R1 000 000 in 10 years' time. How much must he invest if interest is calculated at 6,8% p.a. compounded monthly?

7. At what interest rate, compounded quarterly, must R25 000 be invested in order to grow to R40 000 in 5 years' time?

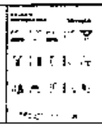
8. Thabo and Patrick each invest R8 000 for 8 years. Thabo invest his money at 7% p.a. simple interest and Patrick invest his money at 6% p.a. compounded semi-annually.
   Which one of them will have more money in 8 years' time?

TOPIC: FINANCE, GROWTH AND DECAY (Lesson 3)		Weighting	15 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Effective and Nominal Interest Rates**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES






ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

The **nominal interest rate** is the annual interest rate that is quoted and the **effective interest rate** is the actual rate achieved per annum

When interest is compounded more often than once a year, the effective interest rate will be higher than the nominal interest rate due to the effect of compound interest which adds interest to a growing total.

The formula that can be used to convert between nominal and effective interest rates for any compounding period is:

$$1 + i_{effective} = \left(1 + \frac{i_{nominal}}{m}\right)^m$$ where m is the number of compounding per year

Whenever we use this formula, the value that the interest is divided by will always be the same as the value of the exponent.

Examples:

1. Convert an annual effective rate of 13,5% per annum, to a nominal rate per annum compounded semi-annually.

$$1 + i_{effective} = \left(1 + \frac{i_{nominal}}{m}\right)^m$$

$$1 + 13,5\% = \left(1 + \frac{i_{nominal}}{2}\right)^2$$

$$\sqrt{1,135} = 1 + \frac{i_{nominal}}{2}$$

$$\frac{i_{nominal}}{2} = \sqrt{1,135} - 1 = 0,0653637$$

$$i_{nominal} = 0,0653637 \times 2 \times 100 = 13,07\%$$

2. Determine the effective interest rate, which results in a nominal interest rate of 11,5% p.a. compounded quarterly (correct to one decimal place).

$$1 + i_{effective} = \left(1 + \frac{i_{nominal}}{m}\right)^m$$

$$i_{effective} = \left(1 + \frac{11,5\%}{4}\right)^4 - 1$$

$$i_{effective} = 0,120055 \times 100 = 12,01\%$$

ACTIVITIES/ ASSESSMENT

1. Determine the effective annual interest rate, to one decimal place, for these interest rates 10,5% compounded quarterly.

2. If an effective annual interest rate is 8%, determine the nominal interest rate p.a. (correct to two decimal places) if interest was compounded every 6 months

3. What nominal annual interest rate, compounded monthly, would give the same return on your investment as 9% p.a. effective? Give the answer to one decimal place.

4. Tamara invests R24 000 at 14% per annum compounded quarterly for a period of twelve years.

   (a) Calculate the future value of the investment using the nominal rate.
   (b) Convert the nominal rate of 14% per annum compounded quarterly to the equivalent effective rate
   (c) Use the annual effective rate to show that the same accumulated amount will be obtained as when using the nominal rate.

5. Michelle inherited R500 000 and deposited the money into a savings account for a period of six years. The accumulated amount at the end of the six-year period was R650 000. Calculate the interest rate paid in each of the following cases:

   (a) The annual effective rate.
   (b) The nominal rate per annum compounded monthly.

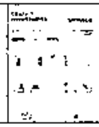
6. Sarah will need R800 000 to buy a flat in 5 years' time.

   (a) Calculate how much she must deposit now into an account offering 10% p.a. compounded monthly, to have the necessary funds in 5 years' time.
   (b) What was the effective interest rate, to 2 decimals, that Sarah received each year?

TOPIC: FINANCE, GROWTH AND DECAY (Lesson 4)		Weighting	15 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Timeline: Changing Interest Rates and Additional Payments and Withdrawals				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

When more than one transaction occurs which are linked to each other, it is useful to picture the process over time with the help of a **timeline**.

Examples:

Changing Interest Rates

1. R 5500 is invested for a period of 4 years in a savings account. For the first year, the investment grows at a simple interest rate of 11% p.a. and then at a rate of 12,5% p.a. compounded quarterly for the rest of the period. Determine the value of the investment at the end of the 4 years.

$$A = P(1+i.n)\,(1+i)^n$$
$$= 5500(1+11\%)\left(1+\frac{12,5\%}{4}\right)^{12} = \text{R}8\,831{,}88$$

2. Henry deposited R4 000 into an account. The interest rate for the first 2 years was 6,5% p.a. compounded quarterly, 7% p.a. compounded semi-annually for the next 3 years and 8,5% p.a. *effective* thereafter. Calculate how much he will have saved after 9 years.

$$A = P(1+i)^n$$
$$= 4000\left(1+\frac{6,5\%}{4}\right)^{8}\left(1+\frac{7\%}{2}\right)^{6}(1+8,8\%) = \text{R}6\,069{,}27$$

Additional Payments and Withdrawals

3. Sindisiwe wants to buy a motorcycle. The cost of the motorcycle is R 55 000. In 1998 Sindisiwe opened an account at Sutherland Bank with R 16 000. Then in 2003 she added R 2000 more into the account. In 2007 Sindisiwe made another change: she took R 3500 from the account. If the account pays 6% p.a. compounded half-yearly, will Sindisiwe have enough money in the account at the end of 2012 to buy the motorcycle?

$$A = P(1+i)^n$$
$$= 16000\left(1+\frac{6\%}{2}\right)^{28} + 2000\left(1+\frac{6\%}{2}\right)^{18} - 3500\left(1+\frac{6\%}{2}\right)^{10}$$
$$= \text{R}35\,308{,}00$$

OR $$A = \left\{\left[16000\left(1+\frac{6\%}{2}\right)^{10} + 2000\right]\left(1+\frac{6\%}{2}\right)^{8} - 3500\right\}\left(1+\frac{6\%}{2}\right)^{10}$$
$$= \text{R}35\,308{,}00$$

ACTIVITIES/ ASSESSMENT

1. Sikhumbuzo deposits R30 000in a bank account and leaves the money in the account for 4 years. For the first 2 years interest is calculated at 4,3% p.a. compounded quarterly. For the remaining 2 years, interest is calculated at 5% p.a. compounded monthly. How much money will be in the account at the end of 4 years?

2. Vicky invests R10 000 for a period of 10 years. During the first 3 years, the interest rate is 9% p.a. compounded monthly. Thereafter, interest changes to 12% p.a. compounded semi-annually. Calculate the future value of the investment after 10 years.

3. After a 20-year period Josh's lump sum investment matures to an amount of R 313 550. How much did he invest if his money earned interest at a rate of 13,65% p.a. compounded half yearly for the first 10 years, 8,4% p.a. compounded quarterly for the next five years and 7,2% p.a. compounded monthly for the remaining period?

4. Mr Jacobs invested R60 000. Four years later he withdrew R5 000 from his account. After a further two years he deposited R8 000. Interest was 10% p.a. compounded half-yearly. Use a time line to determine how much he had in his account after a total of 10 years.

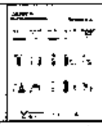
5. Bradley deposits a birthday gift of R14 000 into a savings account in order to save up for an overseas trip in six years' time. At the end of the second year, he withdraws R2 000 from the account. How much money will he have saved at the end of the six-year period, assuming that the interest rate for the whole savings period is 8% per annum compounded monthly?

6. Johannes opened a savings account 20 years ago. On opening the account, he immediately deposit R4 00. He added a further R10 000 ten years ago. Five years ago, he withdrew R5 000 to contribute towards his uncle's funeral. The effective annual interest rate on the account is 7,2% p.a. How much money is in the account today?

TOPIC: FINANCE, GROWTH AND DECAY (Lesson 5)		Weighting	15 + 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Timeline: Changing Interest Rates and Additional Payments and Withdrawals				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Examples:

1. R 150 000 is deposited in an investment account for a period of 6 years at an interest rate of 12% p.a. compounded half-yearly for the first 4 years and then 8,5% p.a. compounded yearly for the rest of the period. A deposit of R 8000 is made into the account after the first year and then another deposit of R 2000 is made 5 years after the initial investment. Calculate the value of the investment at the end of the 6 years.

$$A = P(1+i)^n$$

$$A = 150000\left(1+\frac{12\%}{2}\right)^8(1+8,5\%)^2 = 8000\left(1+\frac{12\%}{2}\right)^6(1+8,5\%)^2 + 2000(1+8,5\%)$$

$$= \text{R } 296\ 977,00$$

2. Mrs Naidoo opened a savings account and the following transactions occurred:
   - She deposited R15 000 immediately.
   - Three years later she withdrew Rx.
   - After a further four years she deposited R21 500.

   Seven years later (fourteen years after the initial investment), she had R25 735,50 in her account. Interest is calculated at 6% p.a. compounded annually for the first five years, and 10% p.a. compounded quarterly for the next nine years. Draw a time line to represent the above and calculate the value of x.

$$A = P(1+i)^n$$

$$25733,50 = 15000(1+6\%)^5\left(1+\frac{10\%}{4}\right)^{36} - x(1+6\%)^2\left(1+\frac{10\%}{4}\right)^{36} + 21500\left(1+\frac{10\%}{4}\right)^{28}$$

$$x(1+6\%)^2\left(1+\frac{10\%}{4}\right)^{36} = 15000(1+6\%)^5\left(1+\frac{10\%}{4}\right)^{36} + 21500\left(1+\frac{10\%}{4}\right)^{28} - 25733,50$$

$$x = \frac{15000(1+6\%)^5\left(1+\frac{10\%}{4}\right)^{36} + 21500\left(1+\frac{10\%}{4}\right)^{28} - 25733,50}{(1+6\%)^2\left(1+\frac{10\%}{4}\right)^{36}} = \text{R}24\ 155,00$$

ACTIVITIES/ ASSESSMENT

3. Theresa wants to save for an overseas trip in three years' time. She will need to have saved R50 000 for the trip. The interest rate during the first year will be 14% per annum compounded quarterly. For the remaining two years, the interest rate will be 11% per annum compounded monthly. What must she invest now in order to receive R50 000 in three years' time?

2. Mrs Mohamed opens a savings account and these transactions take place:
   - She deposits R8 000 immediately, and a further R6 000 five years later.
   - Two years after the deposit of R6 000 she withdraws R10 000.
   - Interest is calculated at 10% p.a. compounded annually for the first two years, and 9,5% p.a. compounded quarterly thereafter.

   Draw a time line to represent the above and calculate the amount of money that she will have saved after 10 years.

3. R 60 000 is invested in an account which offers interest at 7% p.a. compounded quarterly for the first 18 months. Thereafter the interest rate changes to 5% p.a. compounded monthly. Three years after the initial investment, R 5000 is withdrawn from the account. How much will be in the account at the end of 5 years?

4. R 75 000 is invested in an account which offers interest at 11% p.a. compounded monthly for the first 24 months. Then the interest rate changes to 7,7% p.a. compounded half-yearly. If R 9000 is withdrawn from the account after one year and then a deposit of R 3000 is made three years after the initial investment, how much will be in the account at the end of 6 years?

5. Christopher wants to buy a computer, but right now he doesn't have enough money. A friend told Christopher that in 5 years the computer will cost R 9150. He decides to start saving money today at Durban United Bank. Christopher deposits R 5000 into a savings account with an interest rate of 7,95% p.a. compounded monthly. Then after 18 months the bank changes the interest rate to 6,95% p.a. compounded weekly. After another 6 months, the interest rate changes again to 7,92% p.a. compounded two times per year. How much money will Christopher have in the account after 5 years, and will he then have enough money to buy the computer?

6. Uthmaan purchased a car five years ago. After paying a deposit, he took out a loan for the balance that he owed. He paid off the loan with two payments: R30 000 after 2 years and a final payment of R113 582,40 which he made 5 years after taking out the loan. Interest on the loan was 10% p.a. compounded monthly during the first 3 years, and 11,5% p.a. effective for the remaining 2 years. Draw a time line and determine the original price of the car.

7. Mandy has just finished reading a book on the importance of saving for the future. She immediately opens a savings account and deposits R5 000 into the account. Two years later, she deposits a further R6 000 into the account. Thirty six months later, she withdraws R3 000 to buy a birthday gift for her husband. The interest rate during the first three years of the investment is 8% per annum compounded monthly. The interest rate then changes to 9% per annum compounded quarterly. Calculate the value of Mandy's investment two years after her withdrawal of R3 000.

8. Jordan deposited Rx into a savings account six years ago. He added R$2x$ three years ago and R50 000 a year ago. For the first three years, interest has been calculated at 5% p.a. effective. Before that, interest was calculated at 4% p.a. effective. There is currently R88 674,20 in the account. Calculate the value of x.

TEST: FINANCE, GROWTH AND DECAY

MARKS: 25 DURATION: 30 MIN

INSTRUCTIONS

2 Answer ALL questions
3 Unless stated or otherwise, round off answers correct to TWO decimal places
4 You may use an approved scientific calculator

QUESTION 1 [13 Marks]

1.1 A new cell phone was purchased for R 7 200. Determine the depreciation value after 3 years if the cell phone depreciates at 25% per annum on reducing balance method. (3)

1.2 An amount of R 500 is invested at x % per annum compounded half yearly. After 6 years it has grown to R 1 126,10. Calculate the value of x, correct to two decimal places. (4)

1.3 John invest R 120 000. He is quoted a nominal interest rate of 7,2 % per annum compounded monthly.

1.3.1 Calculate the effective interest rate p.a. correct to three decimal places. (3)
1.3.2 Use the effective interest rate to calculate the value of John's investment if the invested the money for 3 years. (3)

QUESTION 2 [12 Marks]

2.1 James invests a certain amount for 5 years. The investment earns interest at per annum, compounded monthly, for the full term. James withdraws R2 000 from the account after 18 months. After 5 years the value of the investment is R23 564.

What amount did James initially invest? (5)

2.2 Susan made an initial payment of R28 000 into an investment account. Three years later, she made another deposit of R12 000. She withdrew R6 500 from the account 5 years after the initial deposit was made. The interest rate for the first 4 years was 12% p.a. compounded monthly. Thereafter, the interest rate changed to 12,95 p.a. compounded half-yearly.

2.2.1 Calculate how much Susan had in this investment account 2 years after the initial deposit was made? (2)

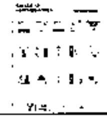
2.2.2 How much will the investment be worth 8 years after the initial deposit was made? (5)

TOPIC: STATISTICS (Lesson 1)		Weighting	20 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Measure of Central Tendency and Measure of Dispersion in Ungrouped Data and Grouped Data				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Mean, Median, Mode, Range, Interquartile range

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Ungrouped data is data with individual data point listed as part of the set.

Measure of central tendency is a single value used to summarise data set. Measure of central tendency are:

- **Mean** $(\bar{x})$ which is the average (addition of all data values and divide by the number of data values) of the data set. $\bar{x} = \frac{\sum x}{n}$
- **Median** which is the value in the **middle** of an ordered (ascending order) data set
- **Mode** which is the value that appears most frequently in the data set.

Measures of dispersion tell us how spread out a data set is.

- If a measure of dispersion is small, the data are clustered in a small region.
- If a measure of dispersion is large, the data are spread out over a large region.

Measure of dispersion are:

- **Range** = Maximum value – Minimum value
- **Inter-quartile range** = third quartile (Q_3) - first quartile (Q_1)
- **Semi-interquartile range** = $\frac{1}{2}(Q_3 - Q_1)$

ACTIVITIES/ ASSESSMENT

The results for Argentina for the past 14 World Cup tournaments are recorded in the table below.

WC Tournament	Matches played	Wins	Draws	Losses	Goals for	Goals against
2006 Germany	5	3	2	0	11	3
2002 Japan	3	1	1	1	2	2
1998 France	5	3	1	1	10	4
1994 USA	4	2	0	2	8	6
1990 Italy	7	2	3	2	5	4
1986 Mexico	7	6	1	0	14	5
1982 Spain	5	2	0	3	8	7
1978 Argentina	7	5	1	1	15	4
1974 Germany	6	1	2	3	9	11
1966 England	4	2	1	1	4	2
1962 Chile	3	1	1	1	2	3
1958 Sweden	3	1	0	2	5	10
1934 Italy	1	0	0	1	2	3
1930 Uruguay	5	4	0	1	18	9

Source: www.2010 Fifa World Cup:Statistics – MediaClubSouthAfica.com

(a) Calculate the mean for the number of matches played over the 14 tournaments.

(b) Calculate the mean for the number of goals scored for Argentina played over the 14 tournaments.

(c) What is the mode for the number of matches played?

(d) What is the mode for the number of losses?

(e) Determine the quartiles for the number of matches played.

(f) Determine the quartiles for the number of goals scored for Argentina.

(g) Calculate the interquartile range for the number of matches played.

(h) Calculate the interquartile range for the number of goals scored for Argentina.

(i) What is the mode for the goals scored for Argentina?

(j) What is the mode for the games won by Argentina?

(k) What is the mode for the goals scored against Argentina?

(l) What is the mode for the games in which Argentina scored a draw?

(m) Draw box and whisker plots for the matches played by Argentina, the wins and the goals scored against Argentina.

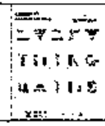
(n) Comment on the performance of Argentina over the 14 tournaments (refer to 1(m)).

TOPIC: STATISTICS (Lesson 2)	**Weighting** 20 ± 3	**Grade**	11
Term		**Week no.**	
Duration	1 hour	**Date**	
Sub-topics	**Grouped data: Histogram and Frequency Polygon**		
RELATED CONCEPTS/ TERMS/VOCABULARY	Grouped data, Estimated mean, Histogram and Frequency polygon		

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Measure of Central tendency

RESOURCES


ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Grouped data is the data has been grouped into intervals.

When given a grouped data, **mean is estimated**. To calculate an estimated mean, find **midpoints** of class intervals.

Example:

1. The table shows the monthly cellphone expenses of a group of 30 grade 11 learners. Estimated the mean cellphone expenses of these learners.

Class Interval (in Rand)	Frequency (f)	Midpoint (x)	Frequency × Midpoint ($f \times x$)
$0 \le x < 100$	3	50	150
$100 \le x < 200$	5	150	750
$200 \le x < 300$	12	250	3000
$300 \le x < 400$	6	350	2100
$400 \le x < 500$	4	450	1800
TOTAL	**30**		**7800**

$$\bar{x} = \frac{\sum (f \times x)}{\sum f} = \frac{7800}{30} = R260$$

The groped data has the **modal class** and not the mode.

Histogram, Frequency polygon and cumulative frequency curve are used to represent grouped data.

HISTOGRAM

Histograms are similar to bar graphs. The difference is that in histograms the **bars are adjacent** to each other **with no gaps** between the rectangles whereas in bar graphs the bars are sometimes separate rectangles.

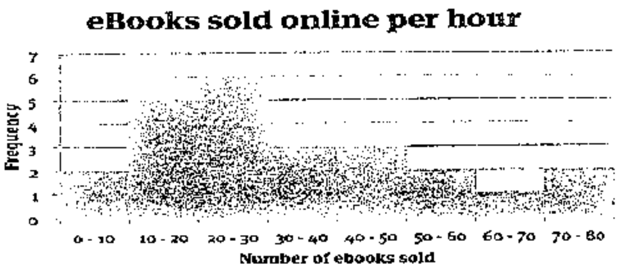
Example 2:

The following frequency table represents this the numbers of eBooks sold on an online store per hour over a 1-day period.

eBooks sold per hour	Frequency
$0 \le x < 10$	2
$10 \le x < 20$	5
$20 \le x < 30$	6
$30 \le x < 40$	3
$40 \le x < 50$	3
$50 \le x < 60$	2
$60 \le x < 70$	1
$70 \le x < 80$	2

(a) Draw a histogram to represent this data.
(b) Determine the modal class.
(c) Determine the class containing the median

(a)



(b) The modal class is represented by the highest bar. $20 \le x < 30$
(c) There are 24 data points in total. Therefore, the median is the average of the 12th and 13th values. $20 \le x < 30$

FREQUEMCY POLYGON

A frequency polygon enables us to represent the information in a **frequency table** by means of line graphs.

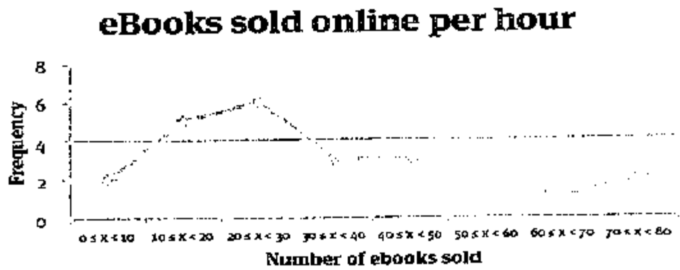
To draw a frequency polygon:

- Use the frequency table to calculate the midpoints of every class interval
- Draw a set of axes for the data and mark each of the frequencies corresponding to the class interval of it
- Join the marks to create the frequency polygon.
- Remember to **ground your frequency polygon** at each side of the histogram.

Example 3

Draw the frequency polygon of example 2.

eBooks sold per hour	Frequency	Midpoint
$0 \le x < 10$	2	5
$10 \le x < 20$	5	15
$20 \le x < 30$	6	25
$30 \le x < 40$	3	35
$40 \le x < 50$	3	45
$50 \le x < 60$	2	55
$60 \le x < 70$	1	65
$70 \le x < 80$	2	75



ACTIVITIES/ ASSESSMENT

1. The table below represents the ages of final 23 players selected by coach to play the tournament:

Class Interval (ages)	Frequency	Midpoint of Class Intervals
$16 \le x < 20$		
$20 \le x < 24$		
$24 \le x < 28$		
$28 \le x < 32$		
$32 \le x < 36$		
$36 \le x < 40$		

(a) Redraw and complete the table
(b) Draw a histogram and a frequency polygon for the data
(c) Calculate the estimated mean age of the players.

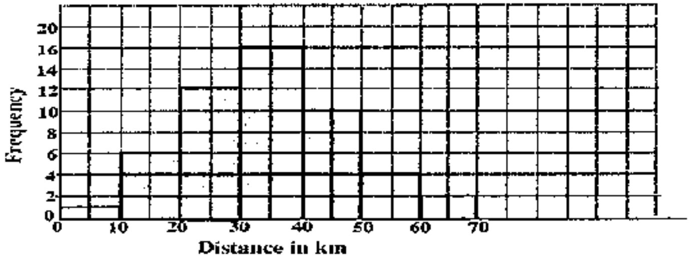
(d) Determine the modal age.
(e) Determine the class interval containing the median

2. The annual earnings (in pounds) of the top 20 soccer players during 2011 are represented as grouped data in the following table.

Class Interval (in millions of pounds)	Frequency (Number of players)
$5 \leq x < 10$	9
$10 \leq x < 15$	5
$15 \leq x < 20$	2
$20 \leq x < 25$	1
$25 \leq x < 30$	3

(a) Calculate the estimated mean for this data.
(b) Calculate the estimated median for this data.
(c) Write down the modal class interval for this data.

3. The following histogram represents the distance run by marathon runners.



(a) Redraw and complete the following table:

(b) Calculate the estimated mean

Class interval	Frequency
$0 < x \leq 10$	
$10 < x \leq 20$	
$20 < x \leq 30$	
$30 < x \leq 40$	
$40 < x \leq 50$	
$50 < x \leq 60$	
$60 \leq x \leq 70$	

4. In a traffic survey, a random sample of 50 motorists were asked the distance (d) they drove to work daily. The results of the survey are shown in the table below.

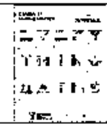
Distance (d)	$5 \leq x < 10$	$5 \leq x < 10$	$5 \leq x < 10$	$5 \leq x < 10$	$5 \leq x < 10$
Frequency	9	19	15	5	4

Draw a frequency polygon to represent the data.

TOPIC: STATISTICS (Lesson 3)		Weighting	20 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Grouped data: Cumulative Frequency Curve/Ogive				
RELATED CONCEPTS/ TERMS/VOCABULARY	Cumulative frequency				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

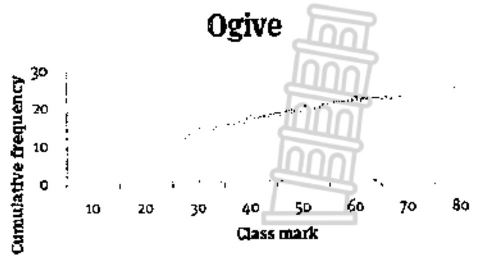
An **ogive** is a graph representing cumulative frequency (a running total of the frequencies) and not individual frequencies, and as such are also known as **cumulative frequency curve**.
The **last value** for the cumulative frequency will always **be equal to the total number of data values**, since all frequencies will already have been added to the previous total.

Example:

The following frequency table represents this the numbers of eBooks sold on an online store per hour over a 1-day period.
(a) Redraw the table and complete the last two columns

eBooks sold per hour	Frequency	Cumulative frequency	Graph Points
$0 \leq x < 10$	2	2	(10; 2)
$10 \leq x < 20$	5	2+5= 7	(20; 7)
$20 \leq x < 30$	6	7+6= 13	(30; 13)
$30 \leq x < 40$	3	13+3= 16	(40; 16)
$40 \leq x < 50$	3	16+3= 19	(50; 19)
$50 \leq x < 60$	2	19+2= 21	(60; 21)
$60 \leq x < 70$	1	21+1= 22	(70; 22)
$70 \leq x < 80$	2	22+2= 24	(80; 24)

(b) Draw an ogive for the data set.



(c) Use the cumulative frequency curve to estimate the median of the number of eBooks sold per hour.

The median is to be found at the $12\frac{1}{2}$ position on the y-axis of the ogive.

You can read off the approximate median eBooks sold per hour by using the graph. Draw a horizontal line from $12\frac{1}{2}$ on the vertical axis. Then draw a vertical line down to the x-axis and determine an approximate value for the median, which is between 28 and 29 eBooks.

(d) Use the cumulative frequency curve to estimate:
1) lower quartile 2) upper quartile

(e) 80% of hours were used to sell a certain number of eBooks. Determine the number of eBooks.
(f) Use the graph to determine the number of hours used to sell
1) Less than 25 eBooks 2) more than 58 eBooks

ACTIVITIES/ ASSESSMENT

1. The table shows the heights of a group of learners.

Height (in cm)	Frequency	Cumulative frequency	Graph points
$140 \le x < 150$	15		
$150 \le x < 160$	27		
$160 \le x < 170$	18		
$170 \le x < 180$	10		
TOTAL			

(a) Redraw and complete the table
(b) Draw an ogive on a set of axes.
(c) Use your ogive to determine an approximate value of:
1) Median 2) lower quartile 3) upper quartile
(d) Draw a box-and-whisker to represent the spread.

2. Fifty motorists were asked to record the number of kilometres travelled in one week. The following table shows the results:

Number of kilometres	Number of motorists	Cumulative frequency
$10 < x \le 20$	2	2
$10 < x \le 20$		9
$10 < x \le 20$		13
$10 < x \le 20$		26
$10 < x \le 20$		42
$60 < x \le 70$		50

(a) Redraw and complete the table.
(b) Draw the cumulative frequency curve for the data.
(c) Use your graph to estimate the median number of kilometres per week.

3. The table below shows the amount of time it took a group of Grade 11's to get ready in the morning.

Time (in Minutes)	Frequency	Cumulative Frequency
$0 \le x < 10$	1	
$10 \le x < 20$	2	
$20 \le x < 30$	12	
$30 \le x < 40$	15	
$40 \le x < 50$	5	
$50 \le x < 60$	2	

(a) Estimate the mean time these Grade 11's took to get ready in the morning
(b) What is the modal class?
(c) Complete the table and sketch a cumulative frequency graph (ogive) to represent this data
(d) Use your graph to estimate the
1) Median 2) lower quartile
(e) 65% of learners took less than a certain amount of time. What is this amount?
(f) How many learners took more than 35 minutes to get ready for school?

4. The table below shows information about the speeds travelled on the N1 BETWEEN Johannesburg and Pretoria:

Speed (km/h)	Frequency	Cumulative Frequency
$30 \le x < 50$	2	
$50 \le x < 70$		11
$70 \le x < 90$		34
$90 \le x < 110$	31	
$110 \le x < 130$	27	
$130 \le x < 150$		100

(a) Complete the table
(b) Sketch the cumulative frequency graph.
(c) Use the graph to estimate the median speed travelled
(d) Calculate the estimated mean
(e) What percentage of vehicles travelled faster than 120 km/h?

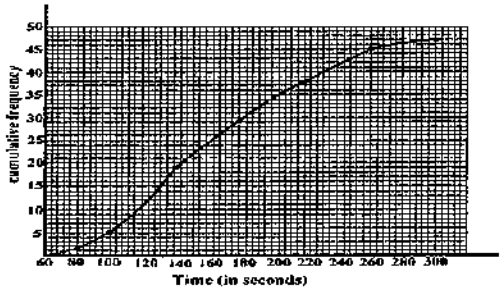
5. The following data set lists the ages of 24 Grade 11 Mathematics learners.

2; 5; 1; 76; 34; 23; 65; 22; 63; 45; 53; 38
4; 28; 5; 73; 79; 17; 15; 5; 34; 37; 45; 56

Use the given data to answer the following questions:

(a) Using an interval width of 10 construct a cumulative frequency plot.
(b) How many are below 30?
(c) How many are below 60?
(d) Giving an explanation state below what value the bottom 50% of the ages fall.
(e) Below what value do the bottom 40% fall?
(f) Construct a frequency polygon.

6. The cumulative frequency graph alongside shows the times between planes landing at an airport.

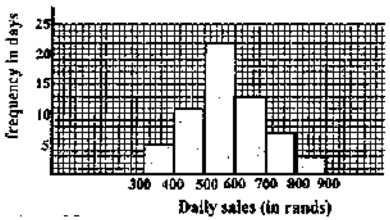


(a) Redraw and complete the following table:

Speed (km/h)	Frequency	Cumulative Frequency
$60 \le t < 100$		5
$100 \le t < 140$		
$140 \le t < 180$		
$180 \le t < 220$		
$220 \le t < 260$		
$260 \le t < 300$		

(b) Determine the estimated median
(c) How many planes had a time between them of 220 seconds or more?
(d) Calculate the estimated mean of the times between the planes.

7. A small coffee shop has kept a record of sales for the past two months. The daily sales, in rands (not in cents), is shown in the following histogram.



(a) Redraw and complete the table below

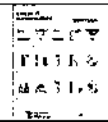
Speed (km/h)	Frequency	Cumulative Frequency
$300 < x \le 400$		5
$400 < x \le 500$		
$500 < x \le 600$		
$600 < x \le 700$		
$700 < x \le 800$		
$800 < x \le 900$		

(b) Draw an ogive curve for the sales over the past 2 months.
(c) Determine the estimated median value of the daily sales
(d) Estimate the interval of the upper 25% of the daily sales

TOPIC: STATISTICS (Lesson 4)		Weighting	20 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Variance and Standard Deviation of Ungrouped Data**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Variance, standard deviation				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES




ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Measures of central tendency (mean, median and mode) provide information on the data values **at the centre** of the data set.
Measures of dispersion (quartiles, percentiles, ranges) provide information on the spread of the data **around the centre**.

In this section we will look at two more **measures of dispersion** called the **variance** and the **standard deviation.** These measures of dispersion use all data values.

The **variance** of the data is the average squared distance between the mean and each data value.

$$\text{Variance} = \frac{\sum\left(x-\bar{x}\right)^2}{n}$$

The variance is **never negative** since every term in the variance sum is squared and therefore either positive or zero.

The variance is a **squared quantity**, it cannot be directly compared to the data values or the mean value of a data set.

Standard deviation is the square root of the variance. Since it is a square root of the variance, it can be compared to the mean value of the data set.

$$\sigma=\sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}}$$

Standard deviation **measures how spread out** the values in a data set are **around the mean**.

The **mean** and the **standard deviation** of a set of data are usually reported together.

If the **data values are all similar**, then the **standard deviation will be low** (closer to zero).
If the data values are **highly variable**, then the **standard variation is high** (further from zero).

The standard deviation is **always a positive number** and is always measured in the same units as the original data.

Examples:

1. A game ranger measured the heights of different buck and recorded the results. The heights at the shoulders are: kudu 150 cm; waterbuck 130 cm; duiker 100 cm; steenbok 50 cm and dik-dik 35 cm.

(a) Determine the mean rounded off to one decimal place.

$$\bar{x}=\frac{150+130+100+50+35}{5}=93,0$$

(b) Use the table to calculate the standard deviation rounded off to one decimal place.

Heights in cm (x)	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
150	57	3249
130	37	1369
100	7	49
50	- 43	1849
35	- 58	3364
TOTAL		9880

$$\sigma=\sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}}=\sqrt{\frac{9880}{n}}=44,5$$

(c) Determine the **standard deviation intervals** for the data.

1) One standard deviation interval: $\left(\bar{x}-\sigma;\bar{x}+\sigma\right)$

$$(93-44,5;93+44,5)=(48,5;137,5)$$

2) Two standard deviation interval: $\left(\bar{x}-2\sigma;\bar{x}+2\sigma\right)$

$$(93-2(44,5);93+2(44,5))=(4;182)$$

3) Three standard deviation interval: $\left(\bar{x}-3\sigma;\bar{x}+3\sigma\right)$

$$(93-3(44,5);93+3(44,5))=(-40,5;226,5)$$

(d) Make **conclusions** about the spread of the data about the mean by establishing how many of the data values lie within or outside of the first standard deviation interval.

One standard deviation interval is $(48;137,5)$
3 of the 5 data values lie within one standard deviation of the mean (50, 100 and 130)
This means that 60% of the data values lie within one standard deviation of the data values

NB: The standard deviation intervals are useful when the data set is reasonably large.
You can use a calculator to determine the mean and standard deviation.

ACTIVITIES/ ASSESSMENT

1. A teacher asked a group of learners how long in minutes it took them to complete their mathematics homework. They gave these answers: 12; 19; 33; 40; 24; 25; 15; 38

(a) Determine the mean number of minutes taken by the learners to complete their homework.
(b) Determine the variance and standard deviation to two decimal places by completing the table below using the formula: $\sigma=\sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}}$

Time taken in minutes (x)	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
12		
19		
33		
40		
24		
25		
15		
38		

(c) How many data values fall within one standard deviation of the mean?

2. The times for 8 athletes who ran a 100 m sprint on the same track are shown below. All times are in seconds. 10,2; 10,8; 10,9; 10,3; 10,2; 10,4; 10,1; 10,4

a) Calculate the mean time.
b) Calculate the standard deviation for the data.
c) How many of the athletes' times are more than one standard deviation away from the mean?

3. A group of people were surveyed about the number of WhatsApp messages they send per day. The following responses were received:
48; 40; 65; 35; 42; 12; 18; 102; 63; 55; 43; 38; 27; 50; 68; 39; 43; 80; 74; 35; 33; 45; 72; 16; 49; 5; 58

(a) Calculate the mean and standard deviation
(b) What percentage of data values are within two standard deviation of the mean?

4. The following data set has a mean of 14,7 and a variance of 10,01.

18; 11; 12; a; 16; 11; 19; 14; b; 13

Compute the values of a and b.

5. Five data values are represented as follows:

$2x;\ x+1;\ x+2;\ x-3;\ 2x-2$

(a) Determine the value of x if the mean of the data set is 15.
(b) Draw a box and whisker plot for the data values.
(c) Calculate the inter-quartile range.
(d) Calculate the standard deviation for this data, rounded off to one decimal place.

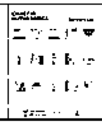
TOPIC: STATISTICS (Lesson 5)		Weighting	20 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Symmetric and Skewed Data**				
RELATED CONCEPTS/ TERMS/VOCABULARY	Skewed data, Symmetric distribution				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Mean, median

RESOURCES






ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing the skewness of the data (positive and negative skewness)

METHODOLOGY

It is important to describe **how the data values are distributed** throughout the range, **relative to the median.**

There are **3 categories** that describe the shape of the data distribution:

- A **symmetric distribution** is one where the left- and right-hand sides of the **distribution** are roughly **equally balanced** around the mean.
  This means that **Mean = Median** or median is **halfway** between the first and third quartile

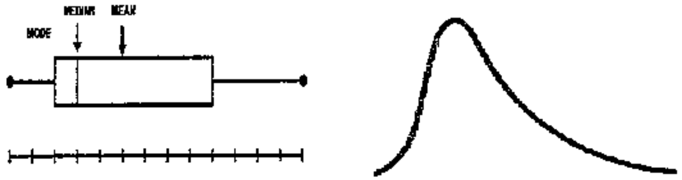
NB: Minimum and maximum values do not have to be equally far away from the median

Skewed data values are more spread out on one side than on the other.

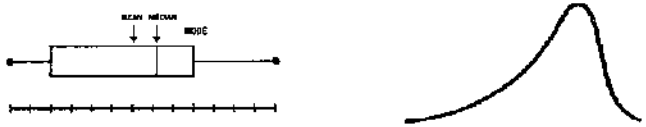
COMPILED BY PINETOWN DISTRICT MATHEMATICS ADVISORS:
M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MOADI

Skewness has an **effect on the mean.**
The more skewed the data, the less reliable the mean becomes as a measure of central tendency

- Skewed to the right **or positively skewed** data is a mass of data values predominantly on the left side of the distribution with fewer higher values on the right.
  NB: For a right skewed distribution, the **mean** is **greater** than the **median.**



- Skewed to the left or **negatively skewed** data is a mass of data values predominantly on the right side of the distribution with fewer higher values on the left.
  NB: For a right skewed distribution, the **mean is less** than the **median.**



The median **remains reliable** as a measure of central tendency, regardless of the skewness

Examples:

1. A small grade 11 class achieved the following marks for Mathematics test:

40 41 41 42 44 48 54 62 72 90

(a) Calculate the mean and the median mark
(b) Comment on the distribution of the data
(c) Which measure of central of tendency best represents the marks achieved by the class for this test?

(a) Mean = 53,4 Median = 46

(b) Positively skewed (Mean > Median)

(c) Median (The mean is too high because the data is positively skewed).

2. The data set below represents the speed (km/h) at which motorists travelled past a school on a specific morning.

46 65 82 68 74 53 18 63 64 76 81 50 40

(a) Draw a Box-and-Whisker diagram to represent the data.

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M.B. MPISI, Z.I. SHANGASE, P.T.C. ZUNGU & S.M. MOADI

(b) Comment on the distribution of the data.
(c) Which measure of central tendency best represents the speed at which the motorists travelled?

(a) 18 40 48 50 53 63 (64) 65 68 74 76 81 82

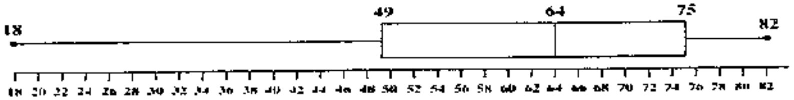
Lower half: 18 40 48 50 53 63 — Q_2 = 64 — Upper half: 65 68 74 76 81 82

The lower (first) quartile (Q_1) is the median of the lower half of the data:

18 40 48 | 50 53 63 $\quad Q_1 = \frac{48+50}{2} = 49$

The upper (third) quartile (Q_3) is the median of the upper half of the data:

65 68 74 | 76 81 82 $\quad Q_3 = \frac{74+76}{2} = 75$



(b) Negatively skewed
(c) The median (the mean is too low because the data is negatively skewed)

ACTIVITIES/ ASSESSMENT

1. Consider the data below:
   27 28 30 32 34 38 41 42 43 44 46 53 56 62

   (a) Draw a box-and-Whisker diagram to represent the data below.
   (b) Is the data set symmetric, skewed right or skewed left? Motivate your answer.

2. The Mathematics Paper 1 June results for Mr Mogodi's class of learners are recorded below. The exam was out of 150.

   101 90 85 97 89 85 84 88 83 96 93 81 88 92 88 96
   92 77 85 91 81 80 87 88

   (a) Draw a Box -and – Whisker diagram
   (b) Comment on the distribution of data.

3. Fifteen households were surveyed in suburb A to find out how much each one spent on electricity for a ten-day period. The results in rand are:

   90 102 50 125 141 220 196 78 137 142 123 157 118 165 121

   (a) Determine:
   1) the median 2) the lower quartile 3) the upper quartile.
   (b) Draw a box-and-whisker diagram to illustrate this data.
   (c) Calculate the mean expenditure on electricity for the 15 households.
   (d) Determine the standard deviation for the data to two decimal places.
   (e) Calculate the percentage of households whose expenditure on electricity falls within one standard deviation of the mean.
   (f) Comment on the distribution of data.
   (g) Which measure of central tendency best represent the amount spent on electricity for a ten-day period?

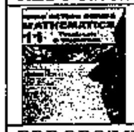
4.

TOPIC: STATISTICS (Lesson 6)		Weighting	20 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Scatterplot and outliers				
RELATED CONCEPTS/ TERMS/VOCABULARY	Scatterplot, line of best fit				

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

Box-and-whisker diagram, set of axes, variables

RESOURCES





ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

An **outlier** is a data value that does not follow the pattern or trend of the rest of the data but a value that is far away from the rest of the values in the data set.

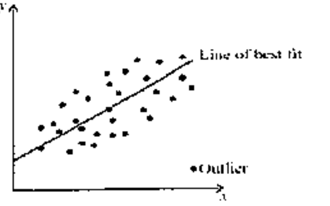
Outliers can be identified in a:
- Box-and-whisker diagram

In a box and whisker diagram, **outliers are usually close to the whiskers** because the whiskers represent the extremes — the **minimum and maximum** — of the data.

- Scatterplot

A **scatter plot** is a graph that shows the relationship (**correlation**) between two random variables. We call these data **bivariate** (literally meaning two variables) and we plot the data for two different variables on one set of axes.

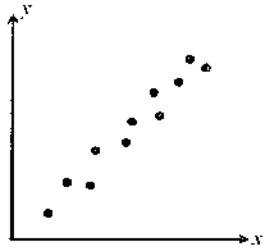
Data could follow a **linear, quadratic or exponential** trend.



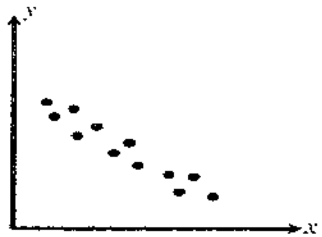
The **strength of the linear relationship** between the two variables in a scatter plot depends on how close the data points are to the **line of best fit.**

Line of best fit is a line that goes through the centre of points of a scatter plot.

- The **closer the points** are to this line of best fit, the **stronger** the relationship.
- If the points are **further away** from the line of best fit, the **weaker** the relationship.
- If the line of best fit slopes to the right and has a positive gradient, then the linear relationship is **positive.**
- If the line of best fit slopes to the left and has a negative gradient, then the linear relationship is **negative.**



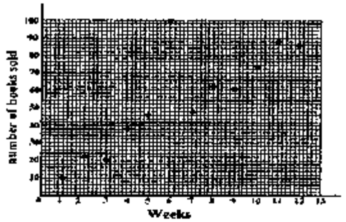
Strong positive association



Strong negative association

Examples:

1. Consider the following scatterplot of information obtained by a publishing company which recorded the number of books sold per week.

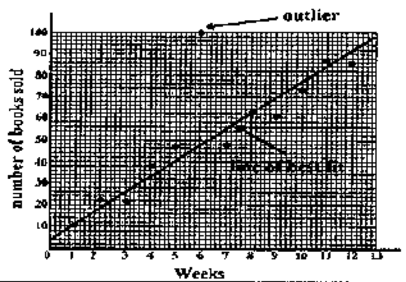


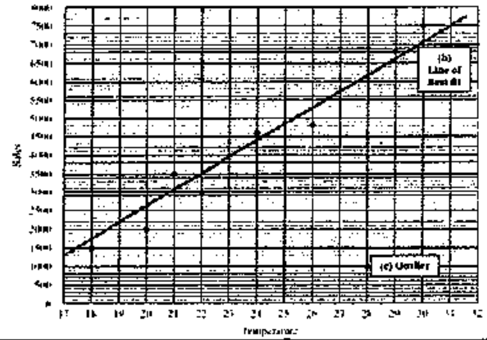
(a) Draw a line of best fit
(b) Indicate an outlier on the scatterplot.

2. The table below represents the temperature on certain days and the sales on an ice cream shop on each of those days.

Temperature (^oC)	18	20	21	22	24	26	28	29	30
Sales (in Rands)	1 500	2 000	3 500	3 000	4 600	4 800	1 000	6 800	7 500

(a) Draw a scatterplot to represent this data.





(b) Draw the line of best fit

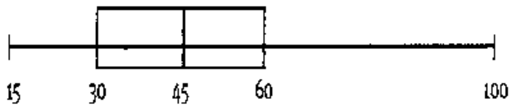
(c) Identify any outlier(s). Give a possible reason for the outlier(s).

ACTIVITIES/ ASSESSMENT

1. For the following data sets, draw a box and whisker diagram and determine whether there are any outliers in the data.
198 166 175 147 125 194 119 170 142 148

2. Draw a box and whisker diagram of the following data set and explain whether it is symmetric, skewed right or skewed left. 15 12 5 3 18 23 11 4

3. Consider the box-and-whisker plot below.



(a) Write down the five-number summary.
(b) Determine the semi-quartile range.
(c) Comment on the spread of the data. What kind of data, do you think, might this represent?
(d) Comment on the skewness of the data.

4. Draw scatter plots for the following sets of pairs. Indicate any outliers.

(a)

x	3	2	5	1	4	6	8	5	4	5	4	5
y	1	2	3	2	1	2	3	2	1	2	3	2

(b)

x	4	2	5	8	1	2.5	5	6	8.5	2	9	4
y	1	1	1	0	0	0	3	7	2	9	5	0

(c)

x	1	5	2	3	6	4	5	6	2	3	1	2
y	3	6	9	5	6	9	3	5	6	9	6	5

5. The table below shows the acidity of eight lakes near an industrial plant and their distance from it.

Distance (in km)	4	34	17	60	6	52	42	31
Acidity (in pH)	3.0	4.4	3.2	7.0	3.2	6.8	5.2	4.8

(a) Draw a scatter plot to illustrate this data.
(b) Draw a line of best fit on the diagram.
(c) Use your line of best fit to predict the acidity of the lake at a distance of 22 kilometres.
(d) Describe the correlation between the distance and acidity.

TEST: STATISTICS

MARKS: 25 DURATION: 30 MIN

INSTRUCTIONS

5 Answer ALL questions
6 Unless stated or otherwise, round off answers correct to TWO decimal places
7 You may use an approved scientific calculator

QUESTION 1 [16 Marks]

1. Mr Ngwane is the sales manager for a furniture shop. Every month his 15 staff members report on the number of customers who visited during the previous month.
The results were given as follows:

12 15 15 19 22 23 26 26 32 33 33 33 33 35 35

1.1 Determine the:
1.1.1 Draw a box-and whisker diagram (3)
1.1.2 interquartile range (3)
1.1.3 mean of the data (2)
1.1.4 standard deviation of the data. (2)
1.1.5 Comment on the skewness of the data. (1)
1.1.6 Are there any outliers in the data set? Explain. (2)
1.2 Determine the percentage of customers who visited the furniture shop that are outside one standard deviation of the mean. (3)
[16]

QUESTION 2 [9 Marks]

Mary wants to buy a car and visits a popular website. She finds a number of advertisements

for the make of the car that she would like to buy. She summarised the selling prices (in thousands of rands) of the cars on sale in the cumulative frequency table.

Selling Price (in thousands of rands	Frequency	Cumulative Frequency
$50 \le x < 60$	3	3
$60 \le x < 70$	4	7
$70 \le x < 80$	a	14
$80 \le x < 90$	19	33
$90 \le x < 100$	12	b
$100 \le x < 110$	5	50

2.1 Write down the values of a and b (2)

2.2 Draw a cumulative frequency graph (ogive). (3)

2.3 Mary wants to spend a maximum of R95 000. Use the cumulative frequency graph to estimate the number of cars that are on sale in the price range that Mary can afford. (2)

2.4 Use the graph to estimate the median selling price. (2)

[9]

TOPIC: PROBABILITY (Lesson 1)		**Weighting**	20 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Grade 10 Revision**				
RELATED CONCEPTS/ TERMS/VOCABULARY					
PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE					
RESOURCES					



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Sample space: the set of all possible outcomes of the experiment.

Event: a subset of a sample space/ set of outcomes of an experiment.

Outcome: the different ways an experiment can turn out.

Probability of an event: a real number between 0 and 1 that describes how likely it is that the event will occur.

- A probability of 0 means the outcome of the experiment will never be in the event set.
- A probability of 1 means the outcome of the experiment will always be in the event set.

$$Probability\ of\ an\ event = \frac{number\ of\ outcomes\ in\ the\ event}{number\ of\ outcomes\ in\ the\ sample\ space}$$

$$P(E) = \frac{n(E)}{n(S)}$$

Mutually exclusive events: events with no outcomes in common, that is (A and B) = ϕ. For example, the event that a number is even and the event that the same number is odd are mutually exclusive, since a number can never be both even and odd.

Complementary events: two mutually exclusive events that together contain all the outcomes in the sample space. For an event called A, we write the complement as "not A". Another way of writing the complement is as A'.

Complementary Rule: $P(notA) = 1 - P(A)$

Addition rule: Also called sum rule:

P(A or B) = P(A) + P(B) - P(A and B)

P(A or B) = P(A) + P(B) (mutually exclusive events)

Examples:

1. Two events, A and B are such that P(A) = 0,3, P(B) = 0,4 and P(A and B) = 0,1
   Determine P(A or B)
   P(A or B) = P(A) + P(B) - P(A and B)
   = 0,3 + 0,4 – 0,1 = 0,6

2. At a restaurant, 40% of customers order a starter and 50% order dessert. 60% of customers order at least one of the two. What percentage of customers order both a starter and dessert?

   Let starter be S and dessert be D.
   P(S or D) = P(S) + P(D) - P(S and D)
   0,6 = 0,4 + 0,5 – P(S and D)
   P(S and D) = 0,9 – 0,6 = 0,3

3. Two mutually exclusive events, A and B, are such that P(A) = 0,3 and P(not B) = 0,4.
   Determine P(A or B)
   P(A or B) = P(A) + P(B) - P(A and B)
   = 0,3 + P(B) – 0
   = 0,3 + 0,6 = 0,9
   P(not B) = 1 – P(B)
   P(B) = 1 – P(not B) = 0,6

Exhaustive events: events which use up the full sample space, that is, all possible outcomes

ACTIVITIES/ ASSESSMENT

1. In a random experiment it was found that: P(A) 0,25; P(B) 0,5 and = = P(A or B) = 0,625

   (a) Calculate P(A and B)

(b) Determine, giving reasons, if events A and B are:
1) mutually exclusive
2) inclusive
3) complementary

2. The probability that Joe will today see a movie is 0,7. The probability that he will go to a restaurant is 0,8. The probability of him seeing a movie and going to a restaurant is 0,6. Determine the probability that:
   (a) he doesn't go to a movie or a restaurant.
   (b) he only goes to a movie.
   (c) he only goes to a restaurant.
   (d) doesn't go to a movie.
   (e) doesn't go to a restaurant.
   (f) he goes to either one or the other.

3. If D and F are mutually exclusive events, with P(not D) = 0,3 and P(D or F) = 0,94, find P(F).

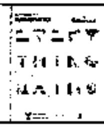
4. The probability of event X is 0,43 and the probability of event Y is 0,24. The probability of both occurring together is 0,10. What is the probability that X or Y will occur?

5. P(H) = 0,62; P(J) = 0,39 and P(H and J) = 0,31. Calculate:
   (a) P(H′) (b) P(H or J) (c) P(H′ or J′) (d) P(H′ or J) (e) P(H′ and J′)

TOPIC: PROBABILITY (Lesson 2)		Weighting	20 ± 3	Grade	11
Term		Week no.			
Duration	1 hour	Date			
Sub-topics	Independent Events				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES



ERRORS/MISCONCEPTIONS/PROBLEM AREAS

Confusing mutually exclusive events and independent events

METHODOLOGY

Two successive events A and B are said to be **independent** if the outcomes/results of the first event do not influence the outcomes/results of the second event.

Example

If a card is **drawn** from one pack and **replaced**, and then a second card is drawn, the possibilities of whichever card is drawn second will not be affected by the card that was drawn first, so these two events are independent.

The product rule for independent events

Two events, A and B are independent if and only if **P(A and B)** $= P(A)\times P(B)$

Examples:

1. A box contains three blue smarties and two green smarties. A smartie is drawn at random and then replaced in the box. Another smartie is then drawn at random and replaced in the box. Determine the probability of:

(a) first drawing a blue smartie and then a green smartie.
(b) first drawing a green smartie and then a blue smartie.
(c) drawing a blue smartie and then another blue smartie.
(d) not drawing a blue smartie on the first or second draw.
(e) drawing a blue and then a green or a green and then a blue.

(a) **P(B and G)** $= P(B)\times P(G)$

$$=\frac{3}{5}\times\frac{2}{5}=\frac{6}{25}$$

(b) **P(G and B)** $= P(G)\times P(B)$

$$=\frac{2}{5}\times\frac{3}{5}=\frac{6}{25}$$

(c) **P(B and B)** $= P(B)\times P(B)$

$$=\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$$

(d) **P(not B and not B)** $= P(B')\times P(B')$

$$=\frac{2}{5}\times\frac{2}{5}=\frac{4}{25}$$

(d) **P(B and G)** or **P(G and B)** $= P(B)\times P(G) + P(G)\times P(B)$

$$=\frac{3}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{3}{5}=\frac{6}{25}+\frac{6}{25}=\frac{12}{25}$$

2. Events A and B are such that P(A) = 0,5, P(not B) = 0,3 and P(A or B) = 0,8.
Are A and P independent events?

NB: Check if the rule **P(A and B)** $= P(A)\times P(B)$ applies

P(B) = 1 – P(not B)
= 1 - 0,3 = 0,7

P(A or B) = P(A) + P(B) – P(A and B)
0,8 = 0,5 + 0,7 – P(A and B)
P(A and B) = 0,5 + 0,7 – 0,8 = 0,4

$P(A)\times P(B) = 0{,}5 \times 0{,}7 = 0{,}35$
P(A and B) $\neq P(A)\times P(B)$
Therefore, events A and B are not independent

NOTE: Mutually exclusive events and **independent** events are two different concepts.

MUTUALLY EXCLUSIVE EVENTS A and B: **P(A and B) = 0**

INDEPENDENT EVENTS A and B: **P(A and B)** $= P(A)\times P(B)$

ACTIVITIES/ ASSESSMENT

1. A coin is tossed and a die is rolled. Determine the probability that the outcome will be:

(a) a head on the coin and a 5 on the die
(b) a tail on the coin or a prime number on the die
(c) a head on the coin and not a 6 on the die.

2. A bag consists of five green marbles and eight blue marbles. If one marble is drawn, then replaced, and a second marble is drawn, determine the probability that:
(a) both marbles are blue
(b) the first marble is blue and the second marble is green
(c) the first marble is green or the second marble is green.

3. Two events, A and B are independent. P(A) = 0,3 and P(not B) = 0,4. Determine P(A or B)

4. A and B are two independent events such that P(A) = 0,6 and P(A or B) = 76. Determine P(B)

5. P(not A) = 0,16 and P(A or B) = 0,92. Determine P(B) if:

(a) A and B are mutually exclusive (b) A and B are independent

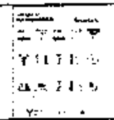
6. P(M) = 0,45; P(N) = 0,3 and P(M or N) = 0,615.

Are the events M and N mutually exclusive or independent?

TOPIC: PROBABILITY (Lesson 3)		**Weighting**	20 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Venn Diagram**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

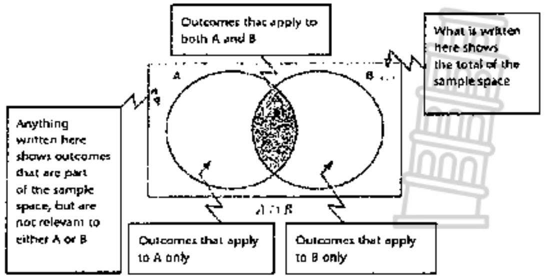




ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

A **Venn diagram** is used to show how events are related to one another and can be very helpful when doing calculations with probabilities.

A Venn diagram representing a **sample space** (the full set of the data), **S**, as a **square** and **events** as **circles**. The intersection of the two circles contains outcomes that are in both A and B.



NB: When using a Venn diagram, always start at the intersection.

Examples:

1. The manager of a hotel in Pretoria recorded the number of guests sitting down for breakfast, lunch and supper on a particular day. Of the 300 guests,

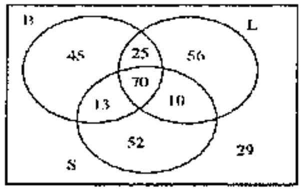
29 did not arrive for any of the three meals.
153 were at breakfast
161 were at lunch
145 were at supper
95 were at breakfast and lunch
80 were at lunch and supper
52 were at supper but did not arrive for the other two meals.
70 were at all three meals

(a) Draw a Venn diagram.
(b) Calculate the probability that a guest chosen at random will have:
  1) been at both breakfast and lunch, but not supper.
  2) been at both breakfast and lunch.
  3) been at breakfast only.
  4) been at one or more of the meals.

(a) The best way to do this is to start with the intersection, i.e., 70 guests and then develop from there.

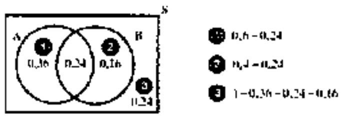
(b) 1) $\frac{25}{300} = \frac{1}{12}$ 2) $\frac{96}{300} = \frac{19}{60}$

3) $\frac{45}{300} = \frac{3}{20}$ 4) $1 - \frac{29}{300} = \frac{271}{300}$



2. Two independent events, A and B, are such that P(A) = 0,6 and P(B) = 0,4.
Determine P(not A) or B)
P(A and B) = 0,6 x 0,4 = 0,24

P(not A) or B) = 0,4 +0,24 = 0,64… P(B) + 0,24



ACTIVITIES/ ASSESSMENT

1. We know the following facts about a group of 32 learners:
   - 12 of them like hamburgers
   - 16 of them like hotdogs
   - 8 of them like chips, but not hamburgers and hotdogs
   - 7 of them like chips and hamburgers
   - 13 of them like chips and hotdogs
   - 2 of the learners who like hotdogs and hamburgers also like chips
   - All the learners like either hotdogs, hamburgers or chips

   (a) Draw a Venn diagram to represent this information, using A for hamburgers, B for hotdogs and C for chips.
   (b) Explain with reasons why:
   1) A, B and C are exhaustive events
   2) A, B and C are not complementary events
   3) B and C are independent events
   4) A and B are not independent events.

2. A group of 70 learners were asked about their subject choice. Their responses showed that: 32 take Physical Sciences, 43 take Mathematics, 25 take Life Sciences and 6 take none of these three subjects. Also, 18 take Physical Sciences and Mathematics but not Life Sciences; 12 take Life Sciences only and 5 take Physical Science and Life Sciences.

   (a) Draw a Venn diagram to represent this information. Use the letters *S*, *M* and *L* to represent Physical Sciences, Mathematics and Life Sciences.
   (b) Determine:
   1) the number of learners who take Physical Sciences, Mathematics and Life Sciences
   2) the probability that a learner who takes Life Sciences does not take Mathematics
   3) the probability that a learner taking Life Sciences and Mathematics does not take Physical Sciences.

3. A group of 80 athletes entered the 100m, 200m and 400m sprints as follows:
   6 entered all three events.
   21 entered none of these events.
   10 entered the 100m and 200m
   11 entered the 200m and 400m
   Of the 21 who entered the 100m, 10 entered nothing else.
   27 entered the 400m

(a) Represent the above situation using a Venn Diagram.
(b) How many athletes entered the 200m event?
(c) What is the probability of an athlete, selected at random, running in at least two of the sprint events?

4. There are 200 registered delegates for an upcoming seminar on Financial Management to be held at the Sandton Convention Centre. The most popular courses in the seminar are usually Share Market Basics, Retirement Planning and Debt Management. There are other less popular courses, which some of the delegates attend. The following information was extracted from the registration forms:
   107 delegates registered for Share Market Basics (S)
   90 delegates registered for Retirement Planning (R)
   63 delegates registered for Debt Management (D)
   35 delegates registered for Retirement Planning and Share Market Basics

23 delegates registered for Share Market Basics and Debt Management
15 delegates registered for Share Market Basics and Retirement Planning and Debt Management.
190 delegates registered for Share Market Basics or Retirement Planning or Debt Management.
x delegates registered for Debt Management and Retirement Planning, but not Share Market Basics

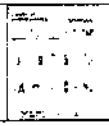
(a) Draw a Venn diagram.
(b) Calculate the value of x.
(c) How many of the delegates have not registered for any of Share Market Basics, Retirement Planning and Debt Management?
(d) How many of the delegates have registered for Retirement Planning and Debt Management, but not Share Market Basics?
(e) What is the probability that a delegate selected at random registered for at least two of the following courses: Share Market Basics, Retirement Planning and Debt Management?

5. Two events, A and B, such that P(A) = 0,3; P(B and A) = 0,2 and P(B) = 0,7
(a) First draw a Venn diagram to represent this information.
(b) Determine the value of P(B and (not A)).

6. Two events, A and B, are such that P(A or B) = 0,59, P(B) = 0,3 and P(B and not A) = 0,19.

(a) Draw a Venn diagram (b) Determine P(A)
(c) Are the events A and B independent? Motivate.

7. R and T are two independent events with P® = 0,3 and P(R or T) = 0,72. Determine:
(a) P(T) (b) P(not R or not T) (c) P(R and not T)

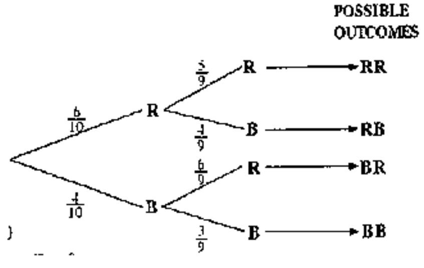
TOPIC: PROBABILITY (Lesson 4)			Weighting	20 ± 3	Grade	11
Term		Week no.				
Duration	1 hour	Date				
Sub-topics	Tree Diagram					
RELATED CONCEPTS/ TERMS/VOCABULARY						
PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE						
RESOURCES						
   						
ERRORS/MISCONCEPTIONS/PROBLEM AREAS						
METHODOLOGY						
When **more than one event** takes place consecutively or simultaneously, it is useful to represent them as a **tree diagram**. We represent each event by a column of branches, and the number of branches is determined by the **number of possible outcomes** for that event.						

Tree diagrams are very helpful for analysing dependent events. A tree diagram allows you to show how each possible outcome of one event affects the probabilities of the other events.

Examples:

1. A bag has 6 red and 4 blue marbles. A marble is drawn at random but not replaced. A second marble is then drawn and not replaced. Calculate the following probabilities:

(a) P(first marble drawn is red)
(b) P(both marbles are blue)
(c) P(one marble is red and the other is blue)



(a) P(first red): $\frac{6}{10}=\frac{3}{5}$

(b) P(both marbles are blue): $\frac{4}{10}\times\frac{3}{9}=\frac{12}{90}=\frac{2}{15}$

(c) P((R and B) or (B and R))
P(R) x P(B) + P(B) x P(R)
$=\frac{6}{10}\times\frac{4}{9}+\frac{4}{10}\times\frac{6}{9}$
$=\frac{24}{90}+\frac{24}{90}=\frac{48}{90}=\frac{8}{15}$

ACTIVITIES/ ASSESSMENT

1. A bag contains 6 blue, 5 red and 9 white marbles. A marble is drawn and not replaced, and another marble is then drawn.
Draw a tree diagram to represent this information, and use it to answer the questions.
(a) Explain why drawing a second blue marble is not independent on drawing a blue marble first.
(b) What are the probabilities that:
1) both marbles are white
2) both marbles are blue
3) one blue and one red marble is chosen
4) neither of the marbles chosen is red?

2. There are eight houses in a school, five for day scholars and three for boarders. Each house has chosen a Head of House. From these Heads of House, the learners must select a Head prefect and a Deputy Head prefect. Assume that each Head of House has an equal chance of being elected. Draw a tree diagram showing the probabilities of each position being filled by a boarder or a day scholar. Then answer the questions.
(a) Determine the probability that the Head prefect of the school will be a day scholar.
(b) Is it true to say that the election of the Deputy is independent of the election of the Head prefect? Give a reason.
(c) What is the probability that both positions are filled by boarders?
(d) What is the probability that at *least* one of the positions is filled by a boarder?

3. In a box of smarties, there are 8 blue smarties and 6 red smarties. A smartie is randomly drawn from the box, the colour is written down and the smartie is replaced. Another smartie is randomly drawn and the colour is recorded
(a) Draw a tree diagram, showing all the possible outcomes.
(b) What is the probability of drawing
1) Two blue marbles
2) a red smartie followed by a blue smartie
3) two smarties of the same colour
4) two smarties of different colours

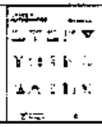
4. Calculators are manufactured in a factory, two machines, A and B, are used. 40% of the calculators are produced by machine A. 5% of all the calculators produced by machine A are faulty and 10% of all the calculators produced by machine B are faulty. What is the percentage of calculators manufactured in this factory will be faulty?

5. In a town, if it rains on a particular day, the probability that it will rain on the next day is 40%. If it doesn't rain on a particular day, the probability that it will rain on the next day is only 10%. If it rains on Monday, what is the probability that it will also rain on Wednesday.

TOPIC: PROBABILITY (Lesson 5)		Weighting	20 ± 3	Grade	11
Term		**Week no.**			
Duration	1 hour	**Date**			
Sub-topics	**Contingency Table**				
RELATED CONCEPTS/ TERMS/VOCABULARY					

PRIOR-KNOWLEDGE/ BACKGROUND KNOWLEDGE

RESOURCES

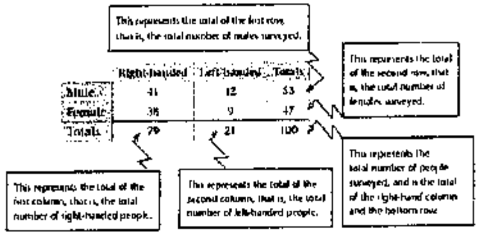

ERRORS/MISCONCEPTIONS/PROBLEM AREAS

METHODOLOGY

Contingency tables are statistical tables that represent the relationships between two or more variables. It is another tool for keeping a record of the counts or percentages in a probability problem.

Examples:

1. A survey of 100 people was done to analyse the relationship between gender and right- or left-handedness. The results were represented in a contingency table be:



(a) Determine the probability of a male chosen at random being lefthanded.

$\frac{12}{53}$

2. A Durban gym recorded the number of members (men and women) who use or don't use the gym on a regular basis. The results are recorded in the following contingency table:

	Men	Women	Total
Use the gym regularly	70	150	220
Don't use the gym regularly	110	40	150
Total	180	190	370

Calculate the probability that a member selected at random from the sample of 370 members:

(a) uses the gym regularly.

$\frac{220}{37} = \frac{22}{37}$

(b) uses the gym regularly given that the member is a woman.

$\frac{150}{190} = \frac{15}{19}$

(c) doesn't use the gym regularly given that the member is a man.

$\frac{110}{180} = \frac{11}{18}$

ACTIVITIES/ ASSESSMENT

1. A school investigated the number of learners who arrived at school on time, arrived late or were absent during a particular week. The results are recorded in the following table. There were 100 learners.

	Boys	Girls	Total
On time	40	25	65
Late	18	7	25
Absent	7	3	10
Total	65	35	100

(a) Calculate the probability of a learner arriving late for school.
(b) Calculate the probability of a learner arriving on time.
(c) Calculate the probability of a learner being absent if the learner is a boy.
(d) Calculate the probability of a learner arriving on time if the learner is a girl.

(e) Calculate the probability of a learner arriving late or being absent.
(f) Calculate the probability of a learner arriving late or being absent if the learner is a boy.

2. Consider the following table and answer questions that follow:

	B	Not B	Total
A	1	3	4
Not A	12	36	48
Total	13	39	52

Determine:

(a) P(A)
(b) P(B)
(c) P(A and B)
(d) P(A or B)
(e) P(A and not B)
(f) P(A or not B)
(g) P(not A and not B)

3. Survey was conducted amongst learners at a school. The number of learners coming late to school and the number of learners using public transport are recorded below:

	Use public transport	Don't use public transport	Total
Late for school	a	100	250
On time	b	d	e
Total	600	c	1 000

A learner is selected randomly from the school.

(a) Determine the values of a to e
(b) What is the probability that the selected learner was late for school?
(c) What is the probability that the learner was on time and doesn't use public transport?
(d) What is the probability that the learner was on time, if he/she is one of the learners using public transport
(e) Are events 'late for school' and 'use public transport' independent? Motivate.

4. The hair colour of thirty learners was recorded in the following contingency table:

	Boys	Girls	Total
Black	6	2	8
Brown	8	4	12
Blonde	4	6	10
Total	18	12	30

Calculate the probability that a learner, chosen at random:
(a) will have black hair.
(b) will have brown hair given that a girl is chosen.
(c) will have blonde hair given that a boy is chosen.
(d) will be a girl given that the hair colour chosen is blonde.

5. A study of speeding fines issued recently and drivers who use car phones was conducted by the Johannesburg Traffic Department. The following data was recorded.

	Speeding fines	No speeding fines	Total
Car phone user	25	280	305
Not car phone user	45	405	450
Total	70	685	755

Calculate the probability that a person selected at random:
(a) is a car phone user.
(b) had no speeding fines.
(c) is a car phone user given that the person had a speeding fine.
(d) had no speeding fine given that the person did not use a car phone.

6. Mathematics teacher suspect that participating in Drama may put learners at a higher risk of failing Mathematics, due to the long hours of rehearsing. The following data was gathered;

	Passed Maths	Failed Maths	Total
Participating in Drama	27	3	30
Does not participate in Drama	108	12	120
Total	135	15	150

Does the data support the claim that participating in Drama has an effect on Mathematics performance? Motivate.

TEST 1: PROBABILITY

MARKS: 26 DURATION: 30 MIN

INSTRUCTIONS

(e) Answer ALL questions
(f) Unless stated or otherwise, round off answers correct to TWO decimal places
(g) You may use an approved scientific calculator

QUESTION 1 [20 Marks]

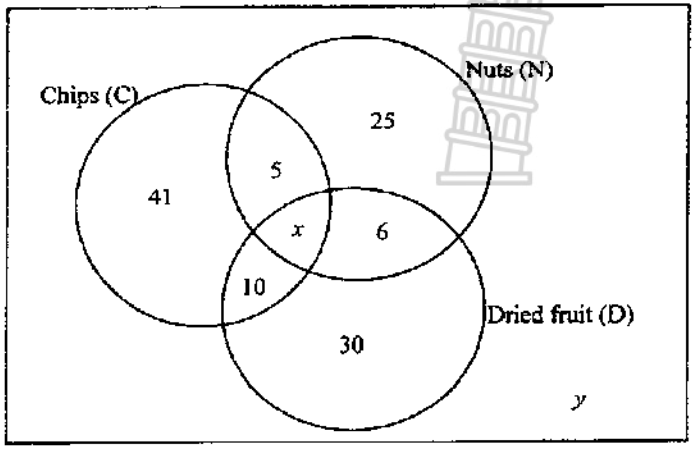
1.1 For any two events, A and B, it is given that P(A) = 0,48 and P(B) = 0,26.

Determine:

1.1.1 P(A and B) if A and B are independent events. (2)

1.1.2 P(A or B0 if A and B are mutually exclusive events. (2)

1.2 A survey was conducted among 130 Grade 11 learners to establish which snack They prefer to eat while they watch television. The results were summarised in the Venn Diagram below. However, some information is missing.



1.2.1 If 29 learners prefer at least two types of snacks, calculate the value of x and y. (4)

1.2.2 Determine the probability that a learner who does not eat nuts will either have another snack or no snack while watching television. (3)

1.3 A group of 200 tourists visited the same restaurant on two consecutive evenings. On both evenings, the tourists could either choose beef (B) or chicken (C) for their Main meal. The manager observed that 35% of the tourists chose beef on the first evening and 70% of them chose chicken on the second evening.

1.3.1 Draw a tree diagram to represent the different choices of main meals Made on the two evenings. Show on your diagram the probabilities Associated with each branch as well as all the possible outcomes of The choices. (4)

1.3.2 Calculate the number of tourists who chose the same main meal on both evenings (3)

1.3.3 Show that more tourists opted not to change their choice on main meal during Their two visits to the restaurant. (2)

QUESTION 2 [6 Marks]

A survey was conducted amongst 60 boys and 60 girls in grade 8 relating to their participation in sport. 20 girls did not participate in any sport and 50 boys did participate in a sport.

2.1 Complete a two-way contingency table for the above survey. (5)

2.2 What is the probability that if a grade 8 pupil is chosen at random that:

2.2.1 It is a girl and participates in sport? (1)

2.2.2 The pupil does not participate in sport and is not female? (1)