KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NAME & SURNAME OF LEARNER: ______________________________ **TOTAL** $\overline{75}$

GRADE 11

TRIGONOMETRY (Reduction Formulae)

INVESTIGATION

14th & 15th FEBRUARY 2024

MARKS: 75

TIME: 2 hours (1 hour day 1 & 1 hour day 2)

This question paper consists of 13 pages (part 1 pgs. 1-8 and part 2 pgs. 9-13).

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 2 questions, question one in part 1 and question 2 in part 2.
2. Answer the question assigned for each day.
3. Answer the questions in the space(s) provided.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
8. Read the notes supplied for your use.

Investigation aim: Derive and use reduction formulae.

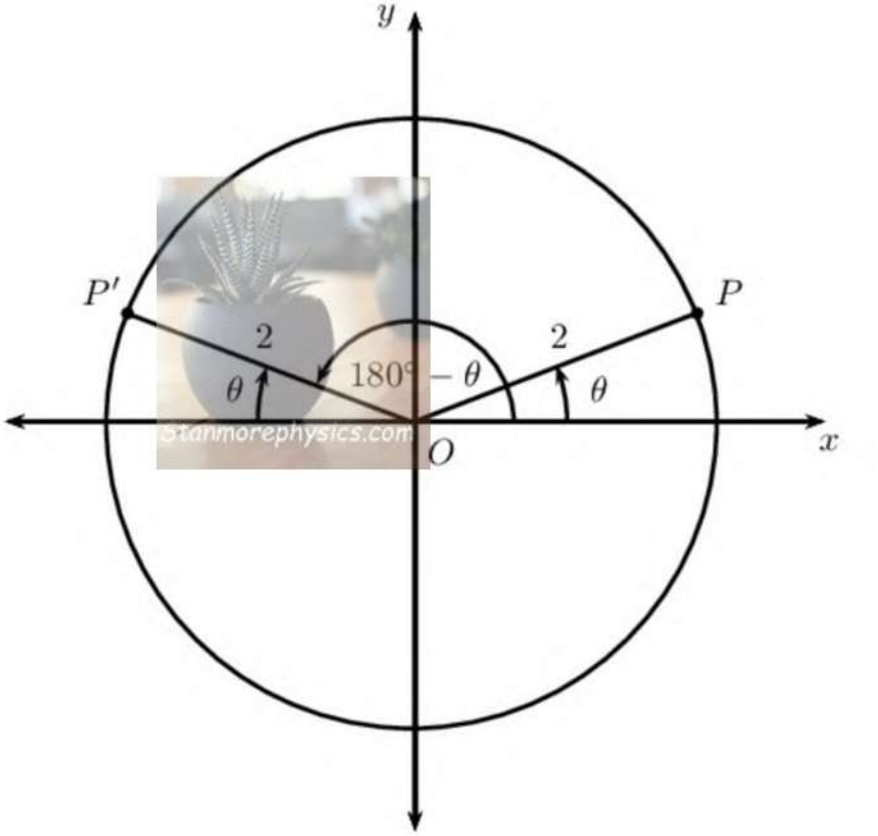
Notes:

- Any trigonometric function whose argument is $90° \pm \theta$; $180° \pm \theta$; $-\theta$; and $360° \pm \theta$ can be written in terms of θ, where θ is an acute angle.
- Reduction formulae are used to reduce the trigonometric ratio of any angle to the trigonometric ratio of an acute angle.
- In the Cartesian plane, we measure angles from the positive x-axis to the terminal arm, which means that an anti-clockwise rotation gives a positive angle.
- We can therefore measure negative angles by rotating in a clockwise direction.
- For an acute angle θ, this implies that $-\theta$ will lie in the fourth quadrant.
- Sine and cosine are known as **co-functions.** Two functions are called co-functions if $f(A) = g(B)$ whenever $A + B = 90°$ (that is, A and B are complementary angles).

QUESTION 1 (DAY 1- PART 1)

1.1 Function values of $180° - \theta$

In the figure $P\left(\sqrt{3};1\right)$ and P' lie on the circle with radius 2. OP makes an angle $\theta = 30°$ with the *x*-axis:



1.1.1 If P and P' are symmetrical about the line $x = 0$ (*y*-axis), write down the coordinates of P' (1)

1.1.2 Use the coordinates of *P*, to write down the values of: (3)

a) $\sin\theta$ __________

b) $\cos\theta$ __________

c) $\tan\theta$ __________

1.1.3 Use the coordinates of P' to determine the values of: (3)

a) $\sin(180°-\theta)$ _________

b) $\cos(180°-\theta)$ _________

c) $\tan(180°-\theta)$ _________

1.1.4 From your results in 1.1.2 and 1.1.3 above, determine a relationship between function values of $(180°-\theta)$ and θ (3)

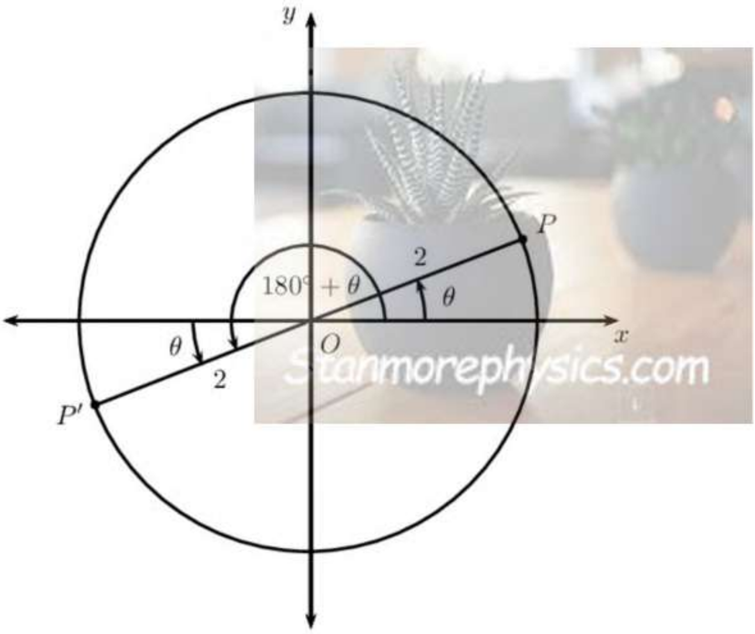
a) $\sin(180°-\theta)=$ _________

b) $\cos(180°-\theta)=$ _________

c) $\tan(180°-\theta)=$ _________

1.2 Function values of $180°+\theta$

In the figure $P(\sqrt{3};1)$ and P' lie on the circle with radius 2. OP makes an angle $\theta=30°$ with the *x*-axis:



1.2.1 If P is rotated through $180°$ to get point P', write down the coordinates of P' (1)

1.2.2 Use the coordinates of P' to determine the values of: (3)

a) $\sin(180° + \theta)$ ________

b) $\cos(180° + \theta)$ ________

c) $\tan(180° + \theta)$ ________

1.2.3 From your results in 1.1.2 and 1.2.2 above, determine a relationship between function values of $(180° + \theta)$ and θ (3)

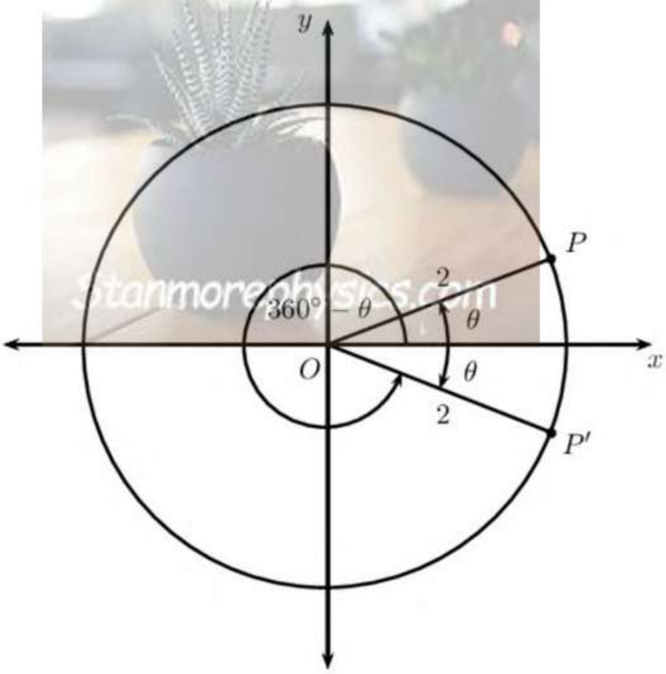
a) $\sin(180° + \theta) =$ ________

b) $\cos(180° + \theta) =$ ________

c) $\tan(180° + \theta) =$ ________

1.3 Function values of $360° - \theta$ and $-\theta$

In the figure $P(\sqrt{3};1)$ and P' lie on the circle with radius 2. OP makes an angle $\theta = 30°$ with the *x*-axis:



1.3.1 If P and P' are symmetrical about the line $y = 0$ (*x*-axis), write down the coordinates of P' (1)

1.3.2 Use the coordinates of P' to determine the values of: (3)

a) $\sin(360°-\theta)$ _________

b) $\cos(360°-\theta)$ _________

c) $\tan(360°-\theta)$ _________

1.3.3 Use the coordinates of P' to determine the values of: (3)

a) $\sin(-\theta)$ _________

b) $\cos(-\theta)$ _________

c) $\tan(-\theta)$ _________

1.3.4 From your results in 1.1.2, 1.3.2, and 1.3.3 above, determine a relationship between function values of $(360°-\theta)$ and $-\theta$ (6)

a) $\sin(360°-\theta)=$ _________

b) $\cos(360°-\theta)=$ _________

c) $\tan(360°-\theta)=$ _________

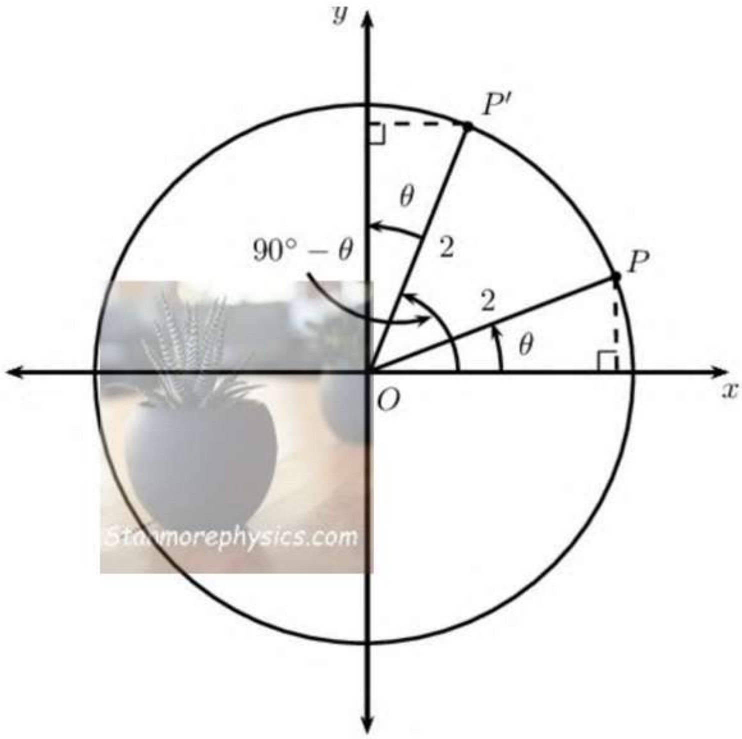
d) $\sin(-\theta)=$ _________

e) $\cos(-\theta)=$ _________

f) $\tan(-\theta)=$ _________

1.4 Function values of $90° - \theta$

In the figure $P\left(\sqrt{3};1\right)$ and P' lie on the circle with radius 2. OP makes an angle $\theta = 30°$ with the x-axis:



1.4.1 If P and P' are symmetrical about the line $y = x$, write down the coordinates of P' (1)

1.4.2 Use the coordinates of P' to determine the values of: (2)

a) $\sin\left(90° - \theta\right)$ _________

b) $\cos\left(90° - \theta\right)$ _________

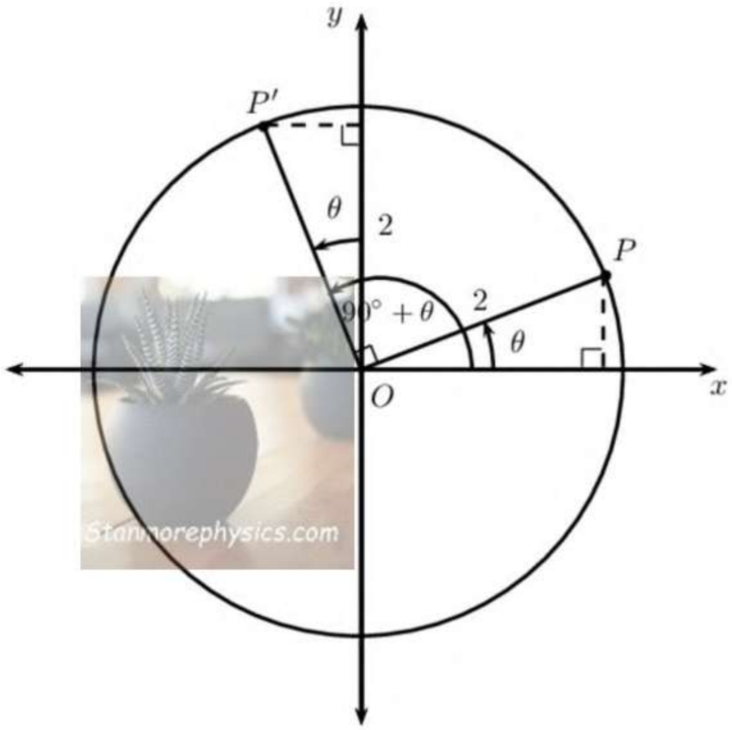
1.4.3 From your results in 1.1.2 and 1.4.2 above, determine a relationship between function values of $\left(90° - \theta\right)$ and θ (2)

a) $\sin\left(90° - \theta\right) =$ _________

b) $\cos\left(90° - \theta\right) =$ _________

1.5 Function values of $90° + \theta$

In the figure $P(\sqrt{3};1)$ and P' lie on the circle with radius 2. OP makes an angle $\theta = 30°$ with the x-axis:



1.5.1 If P is rotated through $90°$ to get point P', write down the coordinates of P' (1)

1.5.2 Use the coordinates of P' to determine the values of: (2)

a) $\sin(90° + \theta)$ _________

b) $\cos(90° + \theta)$ _________

1.5.3 From your results in 1.1.2 and 1.5.2 above, determine a relationship between function values of $(90° + \theta)$ and θ (2)

a) $\sin(90° + \theta) =$ _________

b) $\cos(90° + \theta) =$ _________

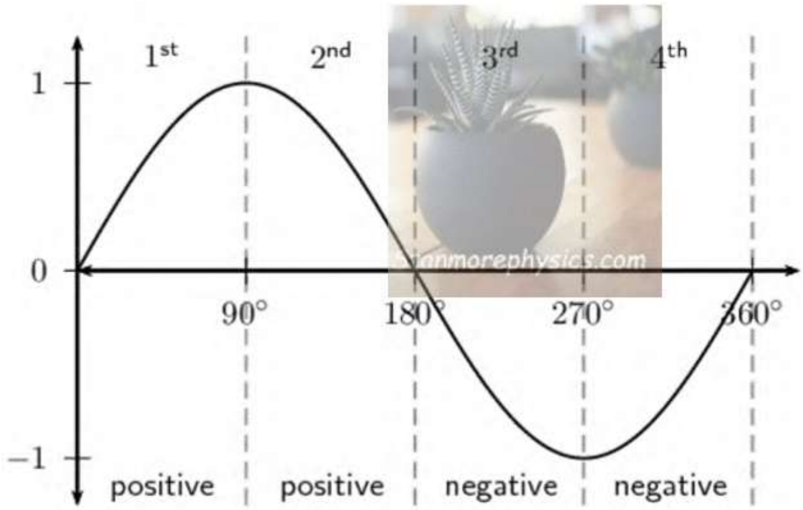
[40]

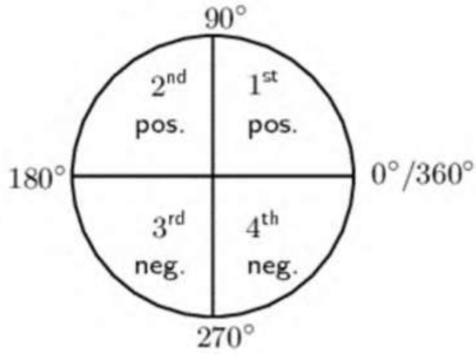
END OF PART 1

NAME & SURNAME OF LEARNER: ____

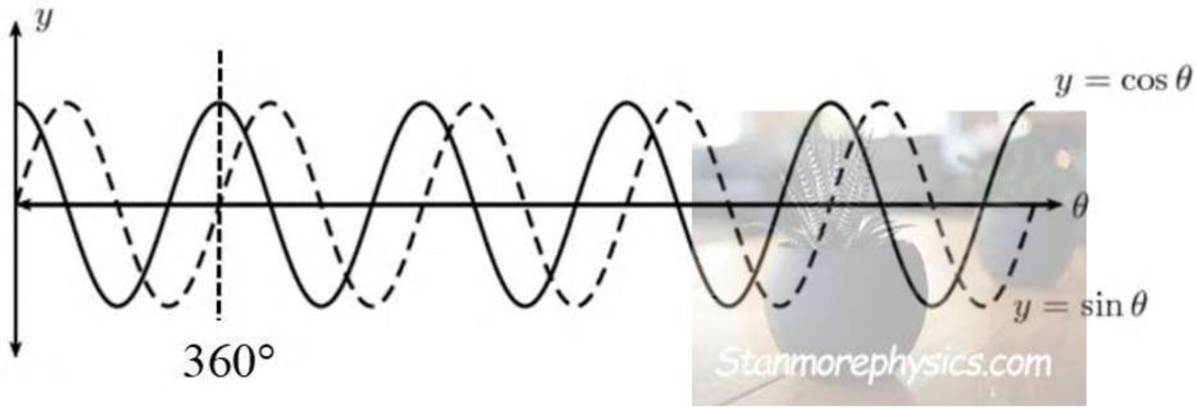
(DAY 2 - PART 2)

Notes: We can also have angles that are larger than $360°$. The angle completes a revolution of $360°$ and then continues to give an angle of θ. From working with functions, we know that the graph of $y = \sin\theta$ has a period of $360°$, therefore, one complete wave of a sine graph is the same as one complete revolution for $\sin\theta$ in the cartesian plane.





We can also have multiple revolutions. A complete sine or cosine graph is completed in $360°$.



If k is an integer, then

$$\sin(k.360° + \theta) = \sin\theta$$
$$\cos(k.360° + \theta) = \cos\theta$$
$$\tan(k.360° + \theta) = \tan\theta$$

QUESTION 2

2.1 Function values of $360°+\theta$

Complete the following reduction formulae:

(3)

2.1.1 $\sin(360°+\theta) =$ _________

2.1.2 $\cos(360°+\theta)=$ _________

2.1.3 $\tan(360°+\theta)=$ _________

2.2 Function values in quadrants

Notes & Examples

To simplify expressions using reduction formulae, determine:

- quadrant
- the sign of the function value and reduce.

Example

Simplify the following as far as possible:

$$\frac{\tan(180°+\theta).\sin(360°-\theta)}{\tan(360°-\theta)}$$

Solution

$$\frac{\overset{3^{rd}}{\tan(180°+\theta)}.\overset{4^{th}}{\sin(360°-\theta)}}{\underset{4^{th}}{\tan(360°-\theta)}}$$ **Determine the quadrant**

$$=\frac{(+\tan\theta)(-\sin\theta)}{-\tan\theta}$$ **Determine the sign of the function value and reduce**

$$=\sin\theta$$

Simplifying expressions with negative angles:

Example

$\sin(-\theta-90°)$

$=\sin\left(-(\theta+90°)\right)$ Take out a negative sign

$=-\sin(\theta+90°)$ $\sin(-angle)=-\sin(angle)$

$=-\sin(90°+\theta)$

$=-\cos\theta$ $\sin(90°+\theta)=\cos\theta$

Simplifying angles greater than $360°$:

Example

$$\begin{aligned}&\cos(540° + \beta)\\&= \cos(360° + 180° + \beta) \qquad 540° = 360° + 180°\\&= \cos\left(360° + (180° + \beta)\right)\\&= \cos(180° + \beta) \qquad \cos\left((1)360 + \theta\right) = \cos\theta\\&= -\cos\theta\end{aligned}$$

2.2.1 From what you have discovered, determine which trigonometric function(s) are positive after reduction. (4)

1st Quadrant $(90° - \theta)$	**2nd Quadrant** $(90° + \theta)$ or $(180° - \theta)$	**3rd Quadrant** $(180° + \theta)$	**4th Quadrant** $(360° - \theta)$
a) ______________	b) ______________	c) ______________	d) ______________

2.2.2 Use reduction formulae to simplify the following expressions to a single trigonometric ratio of θ

a) $\dfrac{\sin(180° + \theta).\cos(180° - \theta)}{\cos(180° + \theta).\sin(180° - \theta)}$ (5)

b) $\dfrac{\tan(360^\circ-\theta).\cos(180^\circ+\theta)}{\cos(360^\circ-\theta)-\cos(180^\circ+\theta)}$ (5)

c) $\dfrac{\sin(360^\circ-\theta).\sin(90^\circ+\theta)}{\cos(90^\circ-\theta).\cos(360^\circ-\theta)}$ (5)

2.2.3 Write each of the following as a function value of θ

a) $\sin(\theta-180)$ (2)

b) $\tan(-\theta-180^\circ)$ (2)

c) $\cos(\theta-90^\circ)$ (2)

d) $\cos(360^\circ+\theta)$ (1)

e) $\tan(720^\circ+\theta)$ (3)

f) $\cos(540^\circ-\theta)$ (3)

[35]

END OF PART 2

TOTAL: 75 marks

KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

REDUCTION FORMULAE

INVESTIGATION

14 - 15 FEBRUARY 2024

MARKING GUIDELINE

MARKS: 75

This marking guideline consists of 5 pages.

GRADE 11
Marking Guideline

PART 1
QUESTION 1

1.1.1		$P'\left(-\sqrt{3};1\right)$	A✓ both values correct	(1)
1.1.2	a)	$\frac{1}{2}$	AAA ✓✓✓	
	b)	$\frac{\sqrt{3}}{2}$		
	c)	$\frac{1}{\sqrt{3}}$		(3)
1.1.3	a)	$\frac{1}{2}$	AAA ✓✓✓	
	b)	$\frac{-\sqrt{3}}{2}$		
	c)	$\frac{1}{-\sqrt{3}}$		(3)
1.1.4	a)	$\sin\theta$	AAA ✓✓✓	
	b)	$-\cos\theta$		
	c)	$-\tan\theta$		(3)
1.2.1		$P'\left(-\sqrt{3};-1\right)$	A✓ both values correct	(1)
1.2.2	a)	$\frac{-1}{2}$	AAA ✓✓✓	
	b)	$\frac{-\sqrt{3}}{2}$		
	c)	$\frac{1}{\sqrt{3}}$		(3)
1.2.3	a)	$-\sin\theta$	AAA ✓✓✓	
	b)	$-\cos\theta$		
	c)	$\tan\theta$		(3)
1.3.1		$P'\left(\sqrt{3};-1\right)$	A✓ both values correct	(1)
1.3.2	a)	$\frac{-1}{2}$	AAA ✓✓✓	
	b)	$\frac{\sqrt{3}}{2}$		
	c)	$\frac{-1}{\sqrt{3}}$		(3)

1.3.3	a)	$\frac{-1}{2}$	AAA ✓✓✓	
	b)	$\frac{\sqrt{3}}{2}$		
	c)	$\frac{-1}{\sqrt{3}}$		(3)
1.3.4	a)	$-\sin\theta$	AAA AAA ✓✓✓ ✓✓✓	
	b)	$\cos\theta$		
	c)	$-\tan\theta$		
	d)	$-\sin\theta$		
	e)	$\cos\theta$		
	f)	$-\tan\theta$		(6)
1.4.1	$P'\left(1;\sqrt{3}\right)$		A✓ both values correct	(1)
1.4.2	a)	$\frac{\sqrt{3}}{2}$	AA ✓✓	
	b)	$\frac{1}{2}$		(2)
1.4.3	a)	$\cos\theta$	AA ✓✓	
	b)	$\sin\theta$		(2)
1.5.1	$P'\left(-1;\sqrt{3}\right)$		A✓ both values correct	(1)
1.5.2	a)	$\frac{\sqrt{3}}{2}$	AA ✓✓	
	b)	$-\frac{1}{2}$		(2)
1.5.3	a)	$\cos\theta$	AA ✓✓	
	b)	$-\sin\theta$		(2)
END OF PART 1				[40]

PART 2
QUESTION 2

2.1.1	$\sin\theta$	A ✓	(1)
2.1.2	$\cos\theta$	A ✓	(1)
2.1.3	$\tan\theta$	A ✓	(1)
2.2.1	a) All b) Sin c) Tan d) Cos	AAAA ✓ ✓ ✓ ✓	(4)
2.2.2	a) 3rd quad, 2nd quad $=\frac{\sin(180°+\theta).\cos(180°-\theta)}{\cos(180°+\theta).\sin(180°-\theta)}$ 3rd quad, 2nd quad $=\frac{-\sin\theta.-\cos\theta}{-\cos\theta.\sin\theta}$ $=-1$	A ✓ $-\sin\theta$ A ✓ $-\cos\theta$ A ✓ $-\cos\theta$ A ✓ $+\sin\theta$ CA ✓ Answer	(5)
	b) 4th quad, 3rd quad $=\frac{\tan(360°-\theta).\cos(180°+\theta)}{\cos(360°-\theta)-\cos(180°-\theta)}$ 4th quad, 2nd quad $=\frac{-\tan\theta.\cos\theta}{\cos\theta-(-\cos\theta)}$ $=\frac{-\tan\theta.\cos\theta}{\cos\theta+\cos\theta}$ $=\frac{-\tan\theta.\cos\theta}{2\cos\theta}$ $=-\frac{\tan\theta}{2}$	Check Errata Below A ✓ $-\tan\theta$ A ✓ $+\cos\theta$ A ✓ $+\cos\theta$ A ✓ $-\cos\theta$ CA ✓ Answer	(5)
	c) 4th quad, 2nd quad $=\frac{\sin(360°-\theta).\sin(90°+\theta)}{\cos(90°-\theta).\cos(360°-\theta)}$ 1st quad, 4th quad		(5)

		$=\dfrac{-\sin\theta.\cos\theta}{\sin\theta.\cos\theta}$ $=-1$	**A** ✓ $-\sin\theta$ **A** ✓ $+\cos\theta$ **A** ✓ $+\sin\theta$ **A** ✓ $+\cos\theta$ **CA** ✓ Answer	
2.2.3	a)	$=-\sin(180°-\theta)$ $=-\cos\theta$	**A** ✓ taking out a negative sign. **CA** ✓ Answer	(2)
	b)	$=-\tan(180°+\theta)$ $=-\sin\theta$	**A** ✓ taking out a negative sign. **CA** ✓ Answer	(2)
	c)	$=-\cos(90°-\theta)$ $=-\cos\theta$	**A** ✓ taking out a negative sign. **CA** ✓ Answer	(2)
	d)	$=\cos\theta$	**A** ✓ Answer	(1)
	e)	$=\tan(360°+360°+\theta)$ $=\tan(360°+\theta)$ $=\tan\theta$	**A** ✓ $720°=360°+360°$ **A** ✓ $\cos(180°-\theta)$ **CA** ✓ Answer	(3)
	f)	$=\cos(360°+180°-\theta)$ $=\cos(180°-\theta)$ $=-\cos\theta$	**A** ✓ $540°=360°+180°$ **A** ✓ $\cos(180°-\theta)$ **CA** ✓ Answer	(3)
		END OF PART 2		**[35]**

TOTAL: 75

Errata

Marking guideline Grade 11

2.2.2

	b)	4th quad 3rd quad $=\dfrac{\tan(360°-\theta).\cos(180°+\theta)}{\cos(360°-\theta)-\cos(180°-\theta)}$ 4th quad 2nd quad $=\dfrac{-\tan\theta.-\cos\theta}{\cos\theta-(-\cos\theta)}$ $=\dfrac{-\tan\theta.-\cos\theta}{\cos\theta+\cos\theta}$ $=\dfrac{\tan\theta.\cos\theta}{2\cos\theta}$ $=\dfrac{\tan\theta}{2}$	A ✓ $-\tan\theta$ A ✓ $-\cos\theta$ A ✓ $+\cos\theta$ A ✓ $-\cos\theta$ CA ✓ Answer	(5)

2.2.3	a)	$=-\sin(180°-\theta)$ $=-\sin\theta$	A ✓ taking out a negative sign. CA ✓ Answer	(2)
	b)	$=-\tan(180°+\theta)$ $=-\tan\theta$	A ✓ taking out a negative sign. CA ✓ Answer	(2)
	c)	$=-\cos(90°-\theta)$ $=-\sin\theta$	A ✓ taking out a negative sign. CA ✓ Answer	(2)