KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

INVESTIGATION
GRADE 10 - 2025
ANALYTICAL GEOMETRY

Duration: $1\frac{1}{2}$ *Hours* **Total: 50 *Marks***

Analytical Geometry is the study of Geometry, using the Cartesian plane.
It is an algebraic approach to the study of Geometry. In this INVESTIGATION, we will address the following concepts:

- The **DISTANCE** between two points.
- The **MIDPOINT** of a line segment.
- The **GRADIENT** of a line.

SURNAME	
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GRADE	
SCHOOL	

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This investigation consists of 4 parts and 14 pages.

2. Answer all the questions.

3. Answers to be done on the question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

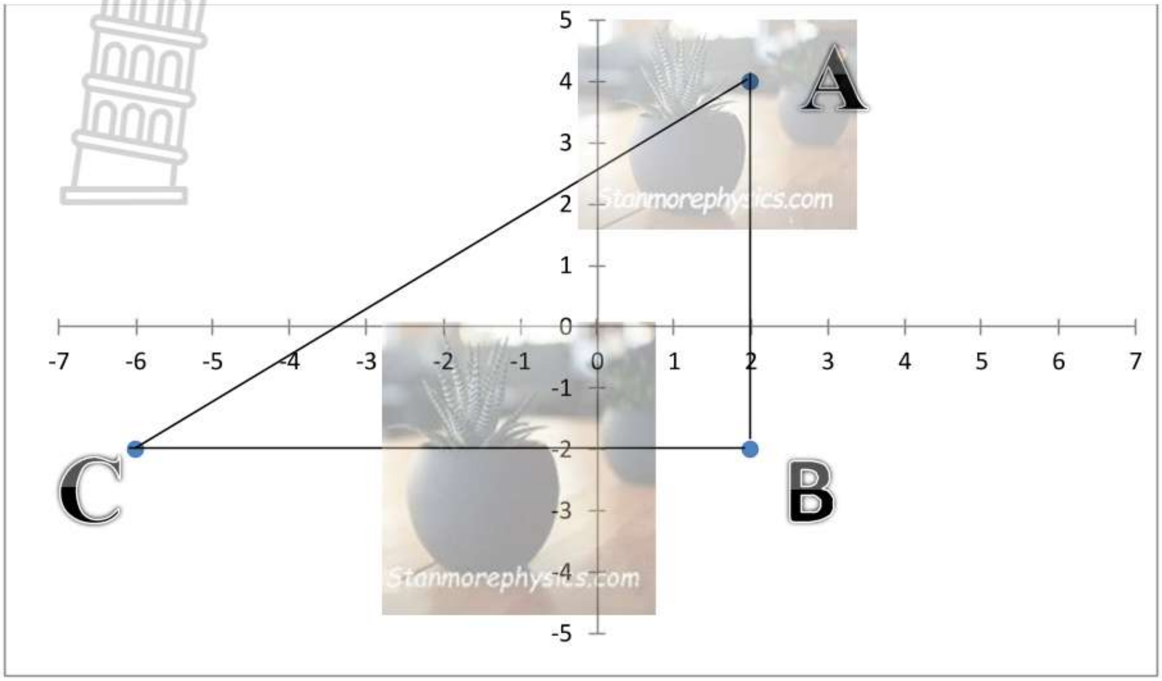
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. Write neatly and legibly.

PART 1: THE DISTANCE BETWEEN TWO POINTS



PART 1.1

1. Consider the right-angle triangle ABC in the Cartesian plane above:

1.1 Write down the co-ordinates of A (_____; _____)

B (_____; _____)

C (_____; _____) [3]

1.2 Now determine how many units from point A to point B is _____________units, and how many units from point B to point C is _______________units.

[2]

If we look at the two points C and B we can formulate the distance HORIZONTALLY to be:

CB = (x value of B) – (x value of C)

CB = (2) – (- 6)

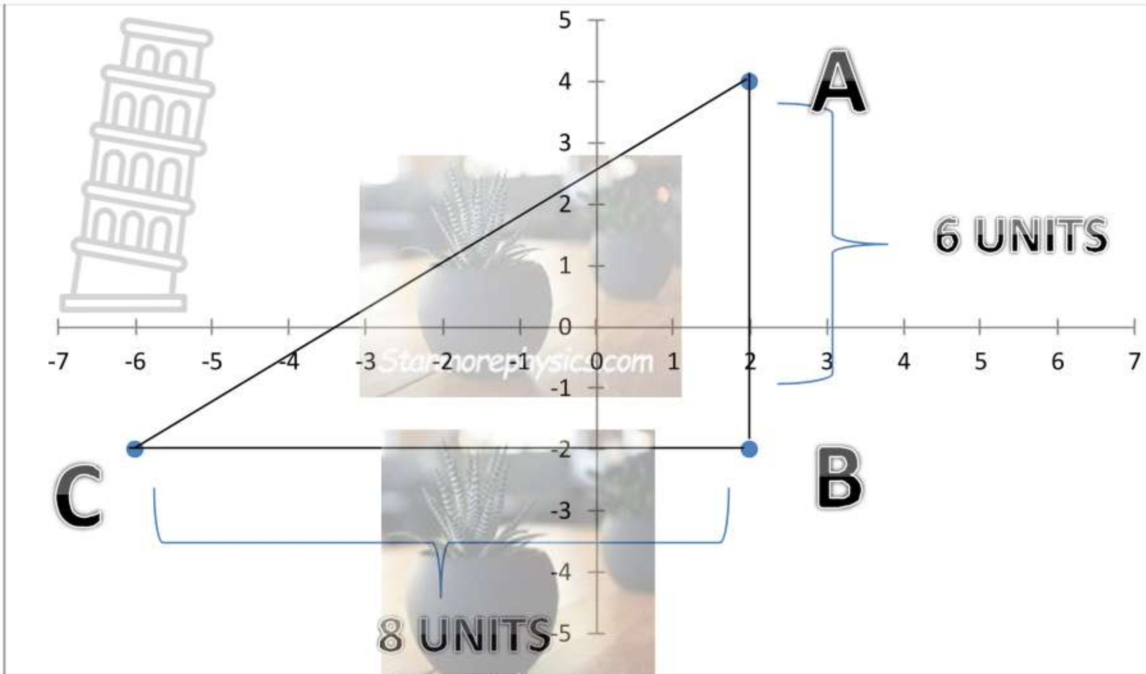
CB = 8 units

If we look at the two points A and B we can formulate the distance VERTICALLY to be:

AB = (y value of A) – (y value of B)

AB = (4) – (-2)

AB = 6 units



Suppose that we wish to calculate the length of line segment AC, which is the hypotenuse,
We would then use the Pythagoras formula which is:

$$AC^2 = BC^2 + AB^2$$

Keep in mind

CB = (x value of B) – (x value of C) AND AB = (y value of A) – (y value of B)

CB = (2) – (-6) AB = (4) – (-2)

CB = 8 units AB = 6 units

Therefore $AC^2 = CB^2 + AB^2$

$$AC^2 = ((x_1) - (x_2))^2 + ((y_1) - (y_2))^2$$

$$AC = \sqrt{((x_1) - (x_2))^2 + ((y_1) - (y_2))^2}$$

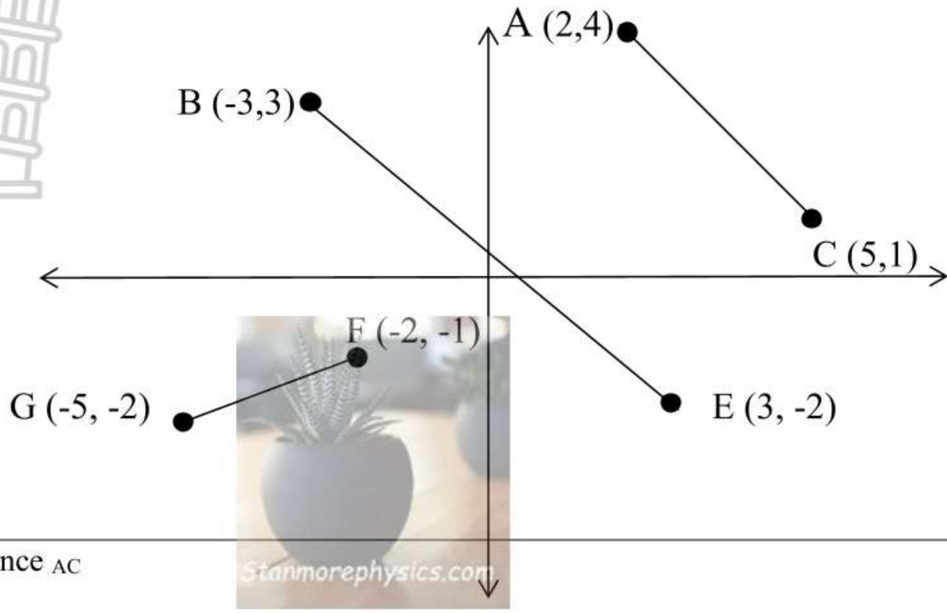
We can generalise this concept to create what is known as the "*Distance Formula*".

The formula to calculate the length of a line between points

$$\sqrt{(Ax - Bx)^2 + (\mathrm{Ay} - \mathrm{By})^2}$$

PART: 1.2

Now use the formula to calculate the lengths of the lines….



1.2.1 Distance $_{AC}$

[2]

1.2.2 Distance $_{BE}$

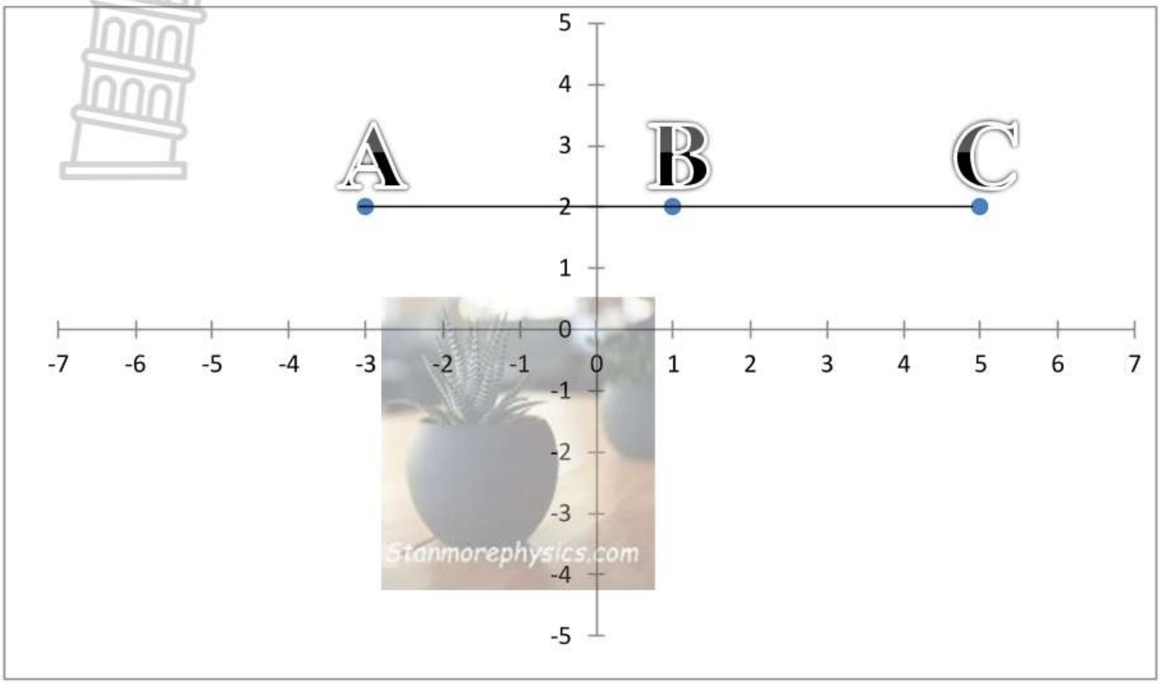
[2]

1.2.3 Distance $_{GF}$

[2]

[11]

PART 2: THE MIDPOINT OF A LINE SEGMENT



PART 2.1

2. Consider the line segment AC in the Cartesian plane above:

2.1 Write down the co-ordinates of A (_____; _____)

B (_____; _____)

C (_____; _____) [3]

2.2 Now determine how many units from point A to point B is _____________units, and how many units from point B to point C is _______________units.____________[2]

<u>If we look at the numbers -3 and 5 for example</u>.

Halfway between -3 and 5 is 1.

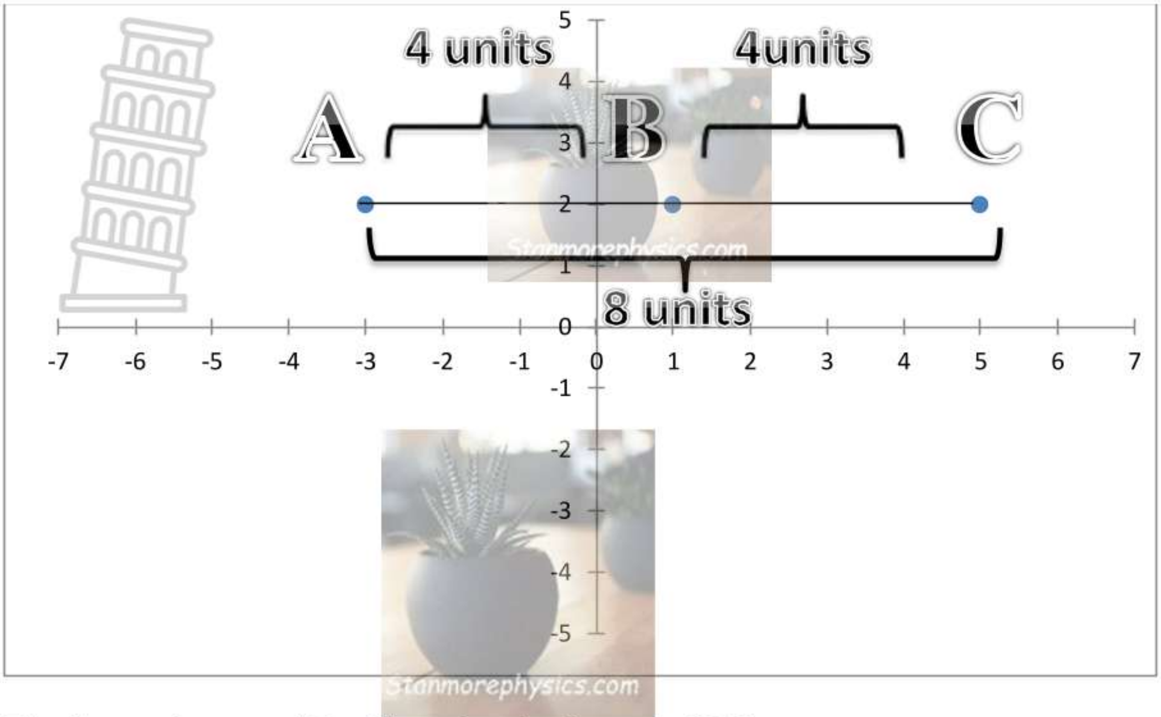
How do we get to 1?

One way is to take the distance between -3 and 5 (which is 8),

half that (which is 4),

then add this to -3 and you will get 1

(You could also subtract 4 from 5 to get to 1).



The distance between AB will equal to the distance of BC
Which means B is the mid-point of line A
We can conclude that the coordinates of B will be determined by adding the two
x-values and then dividing them by 2, AND adding the two y-values and then dividing them by 2

Which means:

the average of the two x values
and
the average of the two y values
will be the midpoint of that line.

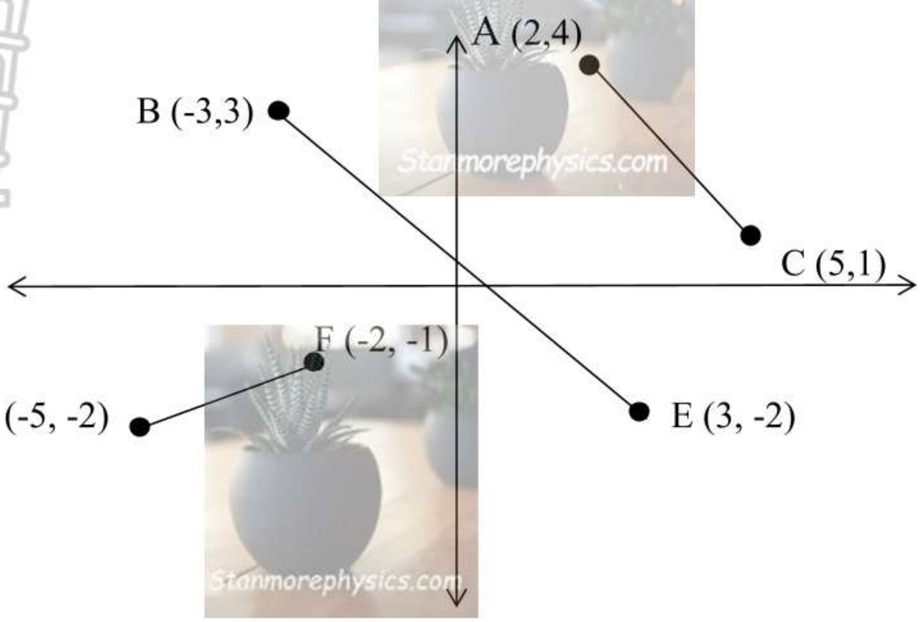
We can generalise this concept to create what is known as the "*MIDPOINT Formula*"

The formula to calculate the MIDPOINT between points

$$\left(\frac{X_1 + X_2}{2} + \frac{Y_1 + Y_2}{2} \right)$$

PART: 2.2

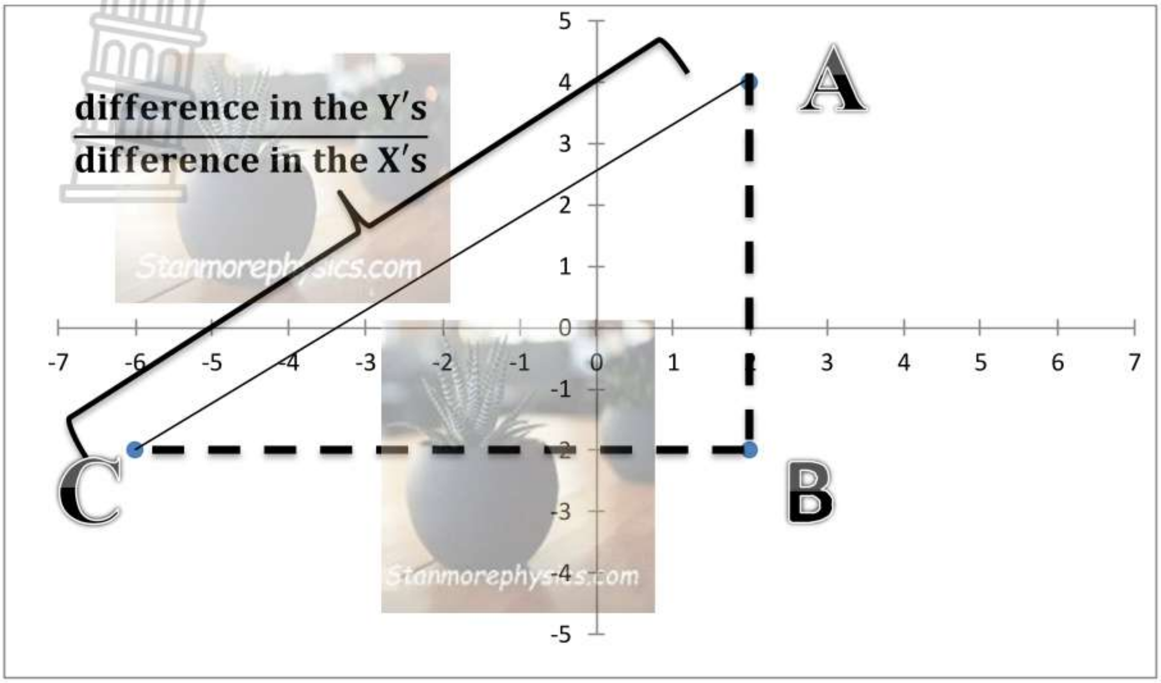
Now use the formula to calculate the MIDPOINTS of the lines….



2.2.1 Midpoint $_{AC}$ [2]
2.2.2 Midpoint $_{BE}$ [2]
2.2.3 Midpoint $_{GF}$ [2]

[11]

PART 3: THE GRADIENT OF A LINE



PART 3.1

3. Consider the line segment AC in the Cartesian plane above:

3.1 Write down the co-ordinates of A (_____; _____)
C (_____; _____) [2]

3.2 Now determine the difference between the y values of AC __________ and the difference between the x values of AC ________ [2]

The Gradient measures the steepness and direction of a line. A line can either slant up, slant down, be horizontal (gradient is zero) or be vertical (gradient is undefined). The symbol used for gradient is m. The above diagram line AC is slanting up which means it has a positive gradient. If a line is slanting down means it has a negative gradient.

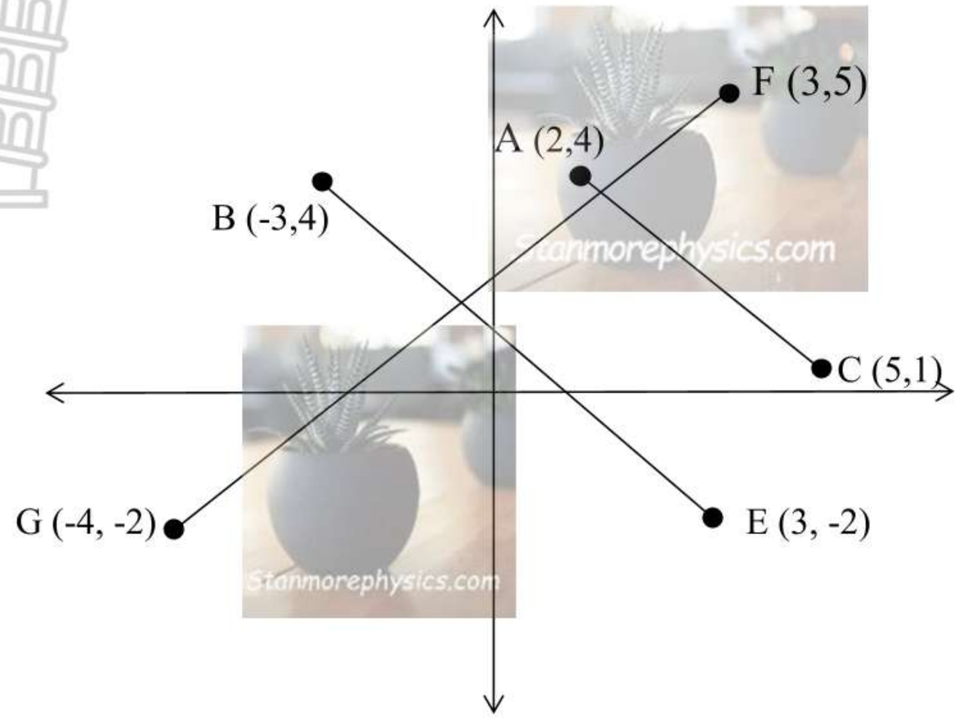
We can generalise this concept to create what is known as the "*gradient Formula"*

The formula to calculate the gradient between points

$$\mathbf{m} = \frac{\textbf{difference in the Y's}}{\textbf{difference in the X's}}$$

PART: 3.2

Now use the formula to calculate the gradients of the lines….



3.2.1 Gradient $_{AC}$	[2]
3.2.2 Gradient $_{BE}$	[2]
3.2.3 Gradient $_{GF}$	[2]

3.3 What do you notice about the gradient of AC and gradient BE   [1]
3.4 Now multiply the gradients of GF and gradients of AC and write down what you notice   [1]

[12]

<u>RULE:</u>

when the **gradients of the two lines are multiplied gives you -1** those lines are **perpendicular**

when the **gradients of the two lines are equal** those lines are **parallel**

<u>therefore, from part 3.2:</u>

AC is parallel to BE

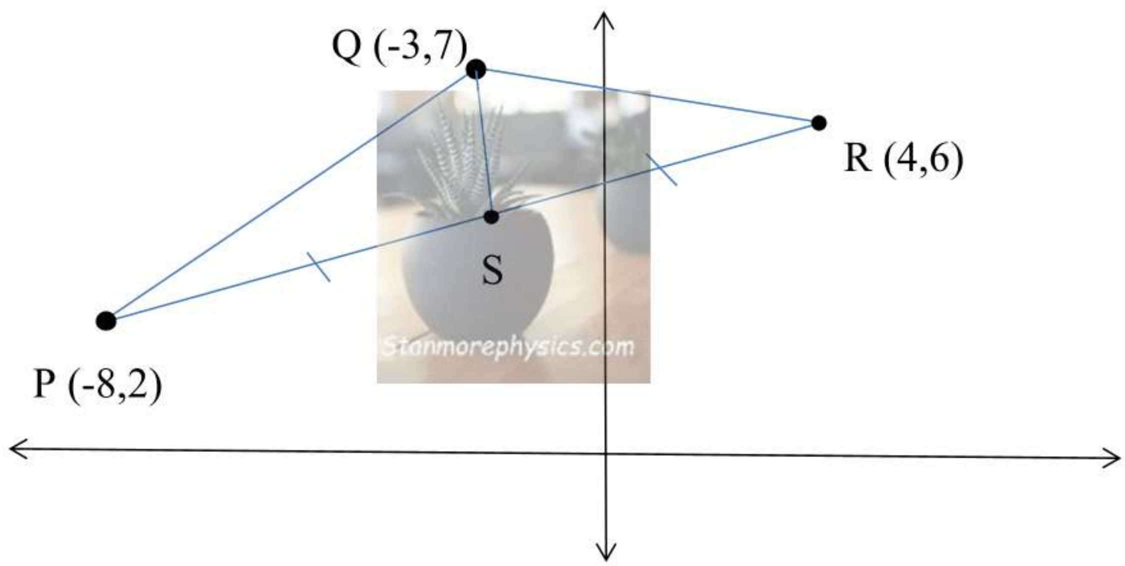
AND

AC is perpendicular to GF

PART 4: MIXED APPLICATION

Use the formulas from part 1, part 2 and part 3 to solve the following:

In the cartesian plane below, P (-8,2), Q (-3,7) and R (4,6) are points on a triangle with point S the midpoint of line segment PR.



4.1Calculate the gradient of PR   [2]
4.2 Calculate the distance of PQ and QR   [3]

4.3 Hence what type of triangle is Δ PQR

[1]

4.4 Determine the co-ordinates of point S

[2]

4.5 Prove that PR is perpendicular to QS

[4]

4.6 Hence calculate the area of triangle PQR

[4]

[16]

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Marking Guideline

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PART 1: THE DISTANCE BETWEEN TWO POINTS

PART 1.1

Question	Solution	Marks
1.1	A (2,4) ✓ B (2, -2) ✓ C (-6, -2) ✓	[3]
1.2	A to B = 6 units✓ B to C = 8 units✓	[2]

PART 1.2

Question	Solution	Marks
1.2.1	$d_{AC} = \sqrt{(2-5)^2 + (4-1)^2}$✓ $= 3\sqrt{2}$✓	[2]
1.2.2	$d_{BE} = \sqrt{(-3-3)^2 + (3--2)^2}$✓ $= \sqrt{61}$✓	[2]
1.2.3	$d_{GF} = \sqrt{(-5--2)^2 + (-2--1)^2}$✓ $= \sqrt{10}$✓	[2]

PART 2: THE MIDPOINT OF A LINE SEGMENT

PART 2.1

Question	Solution	Marks
2.1	A (-3,2) ✓ B (1,2) ✓ C (5,2) ✓	[3]
2.2	A to B = 4 units✓ B to C = 4 units✓	[2]

PART 2.2

Question	Solution	Marks
2.2.1	$M_{AC} = (\frac{2+5}{2} ; \frac{4+1}{2})$✓ $M_{AC} = (\frac{7}{2} ; \frac{5}{2})$✓	[2]
2.2.2	$M_{BE} = (\frac{-3+3}{2} ; \frac{3+-2}{2})$ ✓ $M_{BE} = (0 ; \frac{1}{2})$ ✓	[2]
2.2.3	$M_{GF} = (\frac{-5+-2}{2} ; \frac{-2+-1}{2})$ ✓ $M_{GF} = (\frac{-7}{2} ; \frac{-3}{2})$ ✓	[2]

PART 3: THE GRADIENT OF A LINE

PART 3.1

Question	Solution	Marks
3.1	A (2,4) ✓ C (-6, -2) ✓	[2]
3.2	Difference between the y values of ac = 6 or -6✓ Difference between the x values of ac = 8 or -8✓	[2]

PART: 3.2

Question	Solution	Marks
3.2.1	$m_{AC} = \frac{4-1}{2-5}$ ✓ $m_{AC} = -1$ ✓	[2]
3.2.2	$m_{BE} = \frac{4--2}{-3-3}$ ✓ $m_{BE} = -1$ ✓	[2]
3.2.3	$m_{GF} = \frac{5--2}{3--4}$ ✓ $m_{GF} = 1$ ✓	[2]
3.3	They are equal or they are the same✓	[1]

3.4	$-1 \times 1 = -1$ When you multiply the two gradients you get -1.✓	[1]

PART 4: MIXED APPLICATION

Question	Solution	Marks
4.1	$m_{PR} = \frac{2-6}{-8-4}$ $or\ m_{PR} = \frac{6-2}{4--8}$ ✓ $m_{PR} = \frac{1}{3}$ $or\ m_{PR} = \frac{1}{3}$✓	[2]
4.2	$d_{PQ} = \sqrt{(-8--3)^2 + (2-7)^2}$✓ $= 5\sqrt{2}$ ✓ $d_{QR} = \sqrt{(4--3)^2 + (6-7)^2}$ $= 5\sqrt{2}$ ✓	[3]
4.3	Isosceles triangle✓	[1]
4.4	$M_{PR} = (\frac{-8+4}{2} ; \frac{2+6}{2})$ ✓ $S = (-2 ; 4)$ ✓	[2]
4.5	$m_{QS} = \frac{7-4}{-3--2}$✓ $m_{QS} = -3$ ✓ $\frac{1}{3} \times -3 = -1$✓ $\therefore PR\ is\ perpindicular\ to\ QS$✓	[4]

4.6	$d_{PR} = \sqrt{(-8-4)^2 + (2-6)^2}$ $= 4\sqrt{10}$ ✓ $d_{QS} = \sqrt{(-3--2)^2 + (7-4)^2}$ $= \sqrt{10}$ ✓ $Area = \frac{1}{2} \times 4\sqrt{10} \times \sqrt{10}$ ✓ $= 20$ ✓	[4]