**KWAZULU-NATAL PROVINCE**

EDUCATION
REPUBLIC OF SOUTH AFRICA

# GRADE 11 MATHEMATICS INVESTIGATION 2023

# NATURE OF ROOTS

*"The aim of this task is to investigate the Nature of the Roots of Quadratic equations."*

| DATE | 22 FEBRUARY 2023 |
|------|------------------|
| TOTAL | 50 |
| TIME | 2 hours |

## INSTRUCTIONS

1. Answer all the questions for each Activity

2. Write your answers in the spaces provided **on this question paper**.

3. You may use an approved scientific calculator (non-programmable and nongraphical), unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Write neatly and legibly.

6. This is an individual task, and NO GROUP WORK is allowed.

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**Pre-requisite Knowledge for this task:**

- Grade 9 & 10 knowledge on Number systems
- Understanding of the concept of 'roots'
- The graph of the parabola $y = ax^2 + q$ from Gr. 10

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Name of Learner: _____

Name of School: _____

| _____ |
|---|
| **50** |

1

## PART A: THE ROOTS OF A QUADRATIC EQUATION

### RECAP OF WORK DONE IN EARLIER GRADES: NUMBER SYSTEM

**In earlier grades you have learnt about the Number System, as follows:**

```
                          ALL NUMBERS

          REAL NUMBERS              NON-REAL NUMBERS

   RATIONAL              IRRATIONAL
   NUMBERS               NUMBERS
```
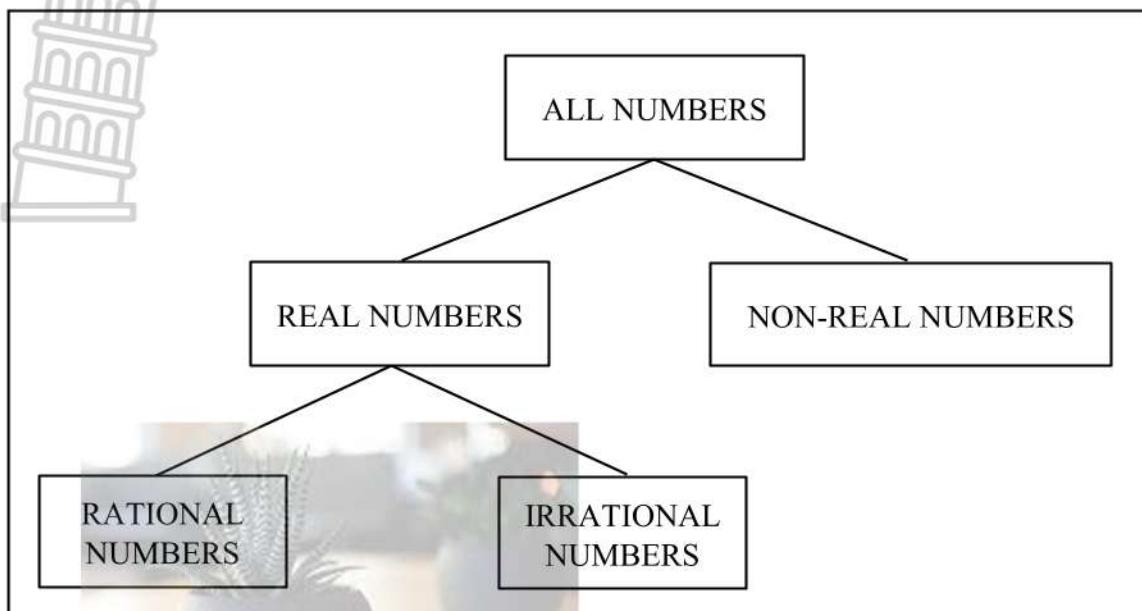
### ACTIVITY 1: [ 4 Marks ]

Classify each of the numbers below, by placing a ✓ in each appropriate block:

|     | Number | Non-real | Real | Rational | Irrational |
|-----|--------|----------|------|----------|------------|
| 1.1 | $\dfrac{-3+\sqrt{13}}{2}$ | | | | |
| 1.2 | $\dfrac{6+\sqrt{16}}{8}$ | | | | |
| 1.3 | $\dfrac{7+\sqrt{-9}}{2}$ | | | | |
| 1.4 | $\dfrac{-1+\sqrt{0}}{10}$ | | | | |

(4)

### SOLVING QUADRATIC EQUATIONS:

To solve any quadratic equation $ax^2 + bx + c = 0$, the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used.

The solutions to the equation are also called the ROOTS of the equation.
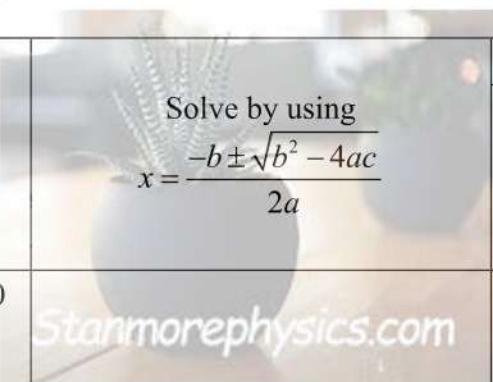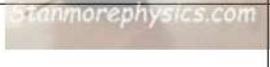
For example the roots of the equation $3x^2 + 5x - 4 = 0$ are $x = \dfrac{-5 \pm \sqrt{5^2 - 4(3)(-4)}}{2(3)} = \dfrac{-5 \pm \sqrt{73}}{6}$

The two roots can also be written separately as: $x = \dfrac{-5 + \sqrt{73}}{6}$ or $x = \dfrac{-5 - \sqrt{73}}{6}$.

2

**ACTIVITY 2:  [ 12 Marks ]**

Complete the table below:

| | Equation | Solve by using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | Are the roots of the equation….. | | |
|---|---|---|---|---|---|
| | | | Equal or unequal? | Real or Non-real? | Rational or Irrational? (provided roots are real) |
| 2.1 | $4x^2 + 11x + 6 = 0$ | | | | |
| 2.2 | $3x^2 + 6x + 1 = 0$ | | | | |
| 2.3 | $x^2 - 6x + 9 = 0$ | | | | |
| 2.4 | $2x^2 + 4x + 10 = 0$ | | | | |

(12)

The **NATURE OF THE ROOTS** of a quadratic equation refers to whether the roots are
- equal or unequal;
- real or non-real;  and
- rational or irrational (in the case of real roots).

**DISCRIMINANT:**

From the above examples it is clear that the nature of the roots is determined by the "the number inside the square root sign", i.e. $b^2 - 4ac$.

$b^2 - 4ac$ is called the discriminant and given the symbol $\Delta$.

## ACTIVITY 3: [ 4 Marks ]

Calculate the discriminant for each of the equations in Activity 2:

| Equations | $4x^2 + 11x + 6 = 0$ | $3x^2 + 6x + 1 = 0$ | $x^2 - 6x + 9 = 0$ | $2x^2 + 4x + 10 = 0$ |
|---|---|---|---|---|
| $\Delta = b^2 - 4ac$ | | | | |

(4)

## ACTIVITY 4: [ 4 Marks ]

Study the examples in Activity 2, and then complete the table below:

| If….. | Roots are real or non-real? | Roots are equal or unequal? | Roots are rational or irrational? (provided roots are real) |
|---|---|---|---|
| $\Delta = 0$ | | | |
| $\Delta > 0$ and a perfect square | | | |
| $\Delta > 0$ and not a perfect square | | | |
| $\Delta < 0$ | | | |

(4)

## ACTIVITY 5: [ 8 Marks ]

| 5.1    Determine the nature of the roots of each of the following equations, without solving the equations: | |
|---|---|
| 5.1.1    $3x^2 - 4x + 4 = 0$ | 5.1.2    $2x^2 + 5x = 4$ |
| 5.1.3    $2x^2 - x - 15 = 0$ | 5.1.4    $4x^2 + 49 = 28x$ |
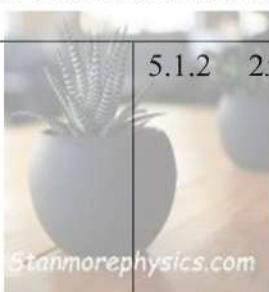
4

(8)

**PART B: LINK BETWEEN A PARABOLA AND THE ROOTS OF A QUADRATIC EQUATION**

**THE PARABOLA GRAPH**

In Gr. 10 you have studied the parabola with equation $y = ax^2 + c$.

More generally, a parabola is the graph of any quadratic expression, i.e. $y = ax^2 + bx + c$.

The shape of a parabola:

| For $a > 0$: | | For $a < 0$: | |
|---|---|---|---|
| | "Happy Graph" | | "Sad Graph" |
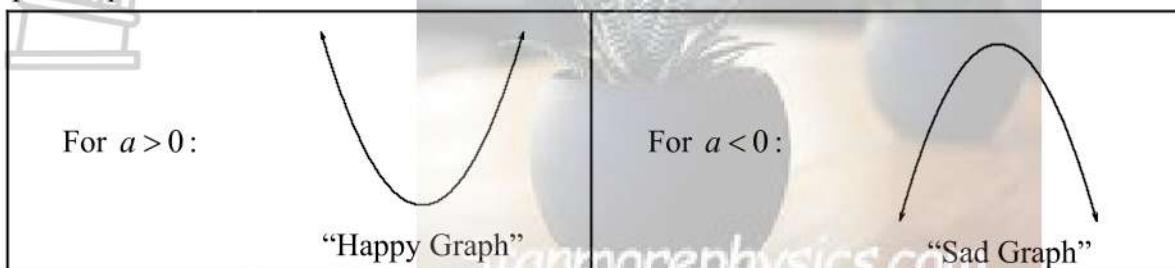
To determine the $x$-intercepts of a parabola, solve for $x$ in $ax^2 + bx + c = 0$.

**ACTIVITY 6: [ 2 Marks ]**

Explain why the roots of $ax^2 + bx + c = 0$ will be the $x$-intercepts of the parabola.

…………………………………………………………………………………………………………………… (2)

**ACTIVITY 7: [ 6 Marks ]**

Complete the table below:
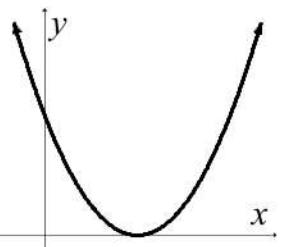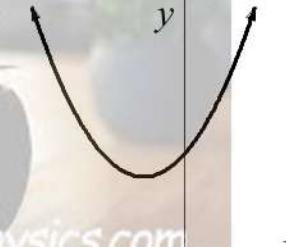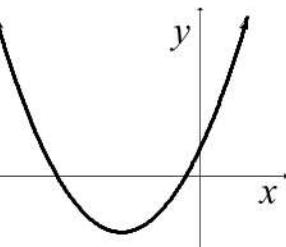
Three sketches of parabolas are given.

Each of them represents one of the following three equations:

- $y = 3x^2 + 6x + 1$
- $y = x^2 - 6x + 9$
- $y = 2x^2 + 4x + 10$

Complete the table by filling in the equation of each parabola (choose from the list above), and then motivating your choice by referring to nature of the roots.

Use your answers from Activity 2 and Activity 3.  You do not need to do any further calculations.

| | Parabola | Equation of Parabola $y = ax^2 + bx + c$ (choose from the list above) | No. of $x$-intercepts | Are the roots of $ax^2 + bx + c = 0$ real or non-real? |
|---|---|---|---|---|
| 7.1 | | | | |
| 7.2 | | | | |
| 7.3 | | | | |

(6)

**PART C: APPLICATION**

**ACTIVITY 8: [ 10 Marks ]**

8.1 Sketch graphs of three parabolas with general equation $y = ax^2 + bx + c$ are shown below.
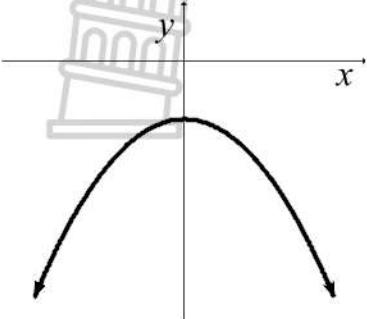
| Graph A | Graph B | Graph C |
|---|---|---|
|  |  |  |
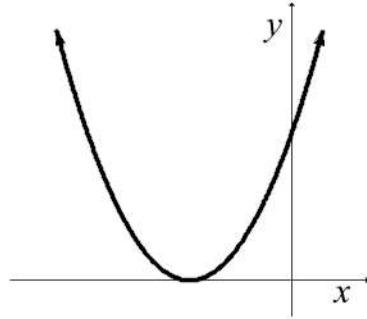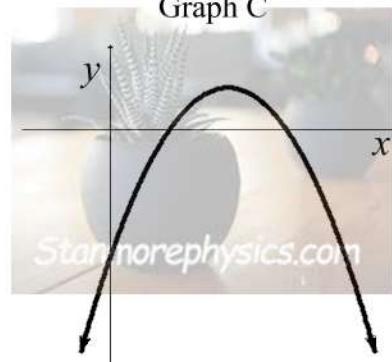
Match each of the statements below to one of these graphs. Write only the letter A, B or C in each case.

| | |
|---|---|
| 8.1.1 $b^2 - 4ac = 16$ : Graph ……….. | 8.1.2 $b^2 = 4ac$ : Graph ……….. |
| 8.1.3 $b^2 - 4ac = -7$ : Graph ……….. | |

(3)

8.2 Determine the value(s) of $m$ for which the equation $mx^2 - 2x + 4 = 0$ will have real roots.

(3)

8.3 Determine two values of $p$ for which the equation $x^2 + 6x - 8p = 0$ will have rational roots.

(4)

**TOTAL: 50 MARKS**

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# GRADE 11 MATHEMATICS INVESTIGATION 2023

# NATURE OF ROOTS

# MARKING GUIDELINE

# TOTAL:  50 MARKS

.

This marking guideline consists of 6 pages.

**ACTIVITY 1 [ 4 Marks ]**

|  | Number | Non-real | Real | Rational | Irrational | |
|---|---|---|---|---|---|---|
| 1.1 | $\dfrac{-3+\sqrt{13}}{2}$ | | ✓ | | ✓ | One mark for each sub-question:  If all the ticks are in correct positions |
| 1.2 | $\dfrac{6+\sqrt{16}}{8}$ | | ✓ | ✓ | | |
| 1.3 | $\dfrac{7+\sqrt{-9}}{2}$ | ✓ | | | | |
| 1.4 | $\dfrac{-1+\sqrt{0}}{10}$ | | ✓ | ✓ | | (4) |

**ACTIVITY 2 [ 12 Marks ]**

|  | Equation | Solve by using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ | Equal or unequal? | Real or Non-real? | Rational or Irrational? |
|---|---|---|---|---|---|
| 2.1 | $4x^2+11x+6=0$ | $x=\dfrac{-11\pm\sqrt{11^2-4(4)(6)}}{2(4)}$ $=-\dfrac{3}{4}\ \text{or}\ -2$ | unequal | real | rational |
| 2.2 | $3x^2+6x+1=0$ | $x=\dfrac{-6\pm\sqrt{6^2-4(3)(1)}}{2(3)}$ $=\dfrac{-3\pm\sqrt{6}}{3}$ $=-0,18\ \text{or}\ -1,82$ | unequal | real | irrational |
| 2.3 | $x^2-6x+9=0$ | $x=\dfrac{6\pm\sqrt{(-6)^2-4(1)(9)}}{2(1)}$ $=3\ \text{or}\ 3$ | equal | real | rational |
| 2.4 | $2x^2+4x+10=0$ | $x=\dfrac{-4\pm\sqrt{4^2-4(2)(10)}}{2(2)}$ $=\dfrac{-4\pm\sqrt{-64}}{4}\ \ \text{OR}$ $=-1\pm\sqrt{-4}$ | unequal | non-real | |
| **FOR EACH SUBQUESTION:** | | ✓ for substitution ✓ for answer $4\times(2)=(8)$ | ✓ for all answers correct $4\times(1)=(4)$ | | |

**ACTIVITY 3 [ 4 Marks ]**

| Equation: | $4x^2 + 11x + 6 = 0$ | $3x^2 + 6x + 1 = 0$ | $x^2 - 6x + 9 = 0$ | $2x^2 + 4x + 10 = 0$ | 1 mark per correct answer: $4 \times (1) = (4)$ |
|---|---|---|---|---|---|
| $\Delta = b^2 - 4ac$ | 25 | 24 | 0 | $-64$ | |

**ACTIVITY 4 [ 4 Marks ]**

| If….. | Roots are real or non-real? | Roots are equal or unequal? | Roots are rational or irrational? (provided roots are real) | 1 mark per row completed correctly: $4 \times (1) = (4)$ |
|---|---|---|---|---|
| $\Delta = 0$ | real | equal | rational | |
| $\Delta > 0$ and a perfect square | real | unequal | rational | |
| $\Delta > 0$ and not a perfect square | real | unequal | irrational | |
| $\Delta < 0$ | non-real | | | |

**ACTIVITY 5 [ 8 Marks ]**

| 5.1 | Determine the nature of the roots of each of the following equations: | | |
|---|---|---|---|
| 5.1.1 $3x^2 - 4x + 4 = 0$ | | 5.1.2 $\quad 2x^2 + 5x = 4$ $2x^2 + 5x - 4 = 0$ | |
| $\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-4)^2 - 4(3)(4) \\ &= -32\end{aligned}$ Roots are non-real (and unequal) | $\checkmark \ \Delta = -32$ $\checkmark$ Roots are non-real | $\begin{aligned}\Delta &= b^2 - 4ac \\ &= (5)^2 - 4(2)(-4) \\ &= 57\end{aligned}$ Roots are real, irrational and unequal | $\checkmark \ \Delta = 57$ $\checkmark$ Roots are irrational and unequal |
| 5.1.3 $2x^2 - x - 15 = 0$ | | 5.1.4 $\quad\quad 4x^2 + 49 = 28x$ $4x^2 - 28x + 49 = 0$ | |
| $\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-1)^2 - 4(2)(-15) \\ &= 121\end{aligned}$ Roots are real, rational and unequal | $\checkmark \ \Delta = 121$ $\checkmark$ Roots are rational and unequal. | $\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-28)^2 - 4(4)(49) \\ &= 0\end{aligned}$ Roots are real, rational and equal | $\checkmark \ \Delta = 0$ $\checkmark$ Roots are rational and equal |

(8)
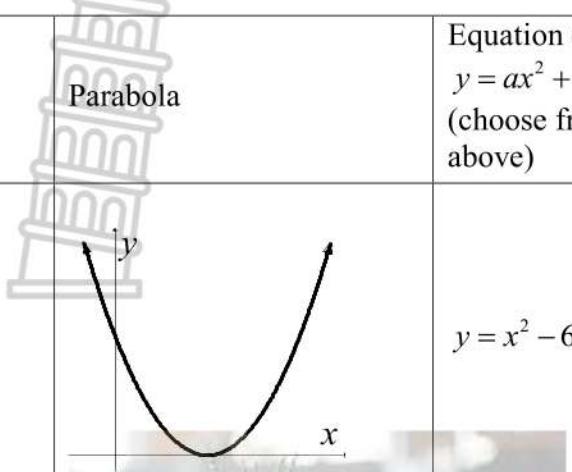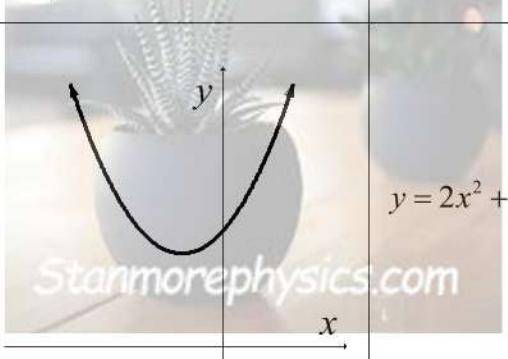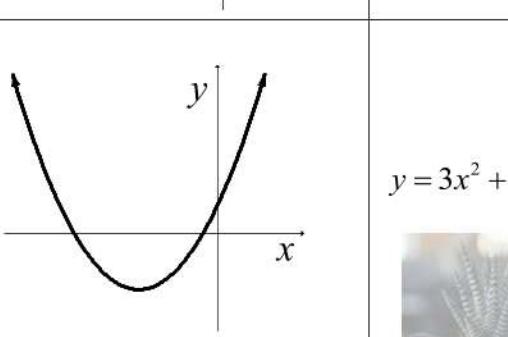
**ACTIVITY 6 [ 2 Marks ]**

| *Explain why the roots of $ax^2 + bx + c = 0$ will be the x-intercepts of the parabola.* | |
|---|---|
| Because the *y*-value is equal to 0 on the *x*-axis | $\checkmark \ \checkmark$ answer (2) |

**ACTIVITY 7:  [ 6 Marks ]**

| | Parabola | Equation of Parabola $y = ax^2 + bx + c$ (choose from the list above) | No. of $x$-intercepts | Are the roots of $ax^2 + bx + c = 0$ real or non-real? |
|---|---|---|---|---|
| 7.1 |  | $y = x^2 - 6x + 9$ | One (Also accept: Two equal $x$-intercepts) | Real |
| 7.2 |  | $y = 2x^2 + 4x + 10$ | Zero $x$-intercepts | Non-real |
| 7.3 |  | $y = 3x^2 + 6x + 1$ | Two $x$-intercepts | Real |
| **FOR EACH SUBQUESTION:** | | ✓ for the correct equation chosen: $3 \times (1) = (3)$ | ✓ for both answers correct: $3 \times (1) = (3)$ | |

(6)

**ACTIVITY 8:** [ 10 Marks ]

8.1 Sketch graphs of three parabolas with general equation $y = ax^2 + bx + c$ are shown below.

| Graph A | Graph B | Graph C |
|---|---|---|
|  |  |  |
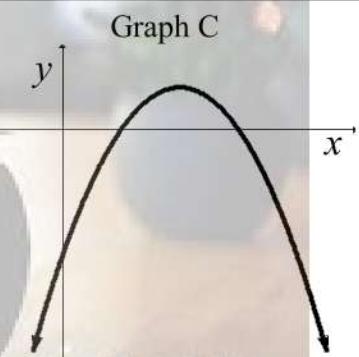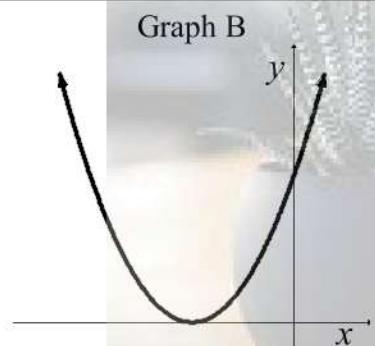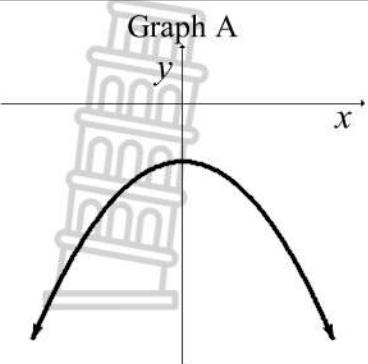
Match each of the statements below to one of these graphs. Write only the letter A, B or C in each case.

| 8.1.1 $b^2 - 4ac = 16$ : **Graph C** | 8.1.2 $b^2 = 4ac$ : **Graph B** |
|---|---|
| 8.1.3 $b^2 - 4ac = -7$ : **Graph A** | One mark for each correct graph chosen: <br> $3 \times (1) = (3)$ |

| 8.2 *Determine the value(s) of m for which the equation* $mx^2 - 2x + 4 = 0$ *will have real roots*: <br><br> $\Delta = b^2 - 4ac$ <br> $\quad = (-2)^2 - 4(m)(4)$ <br> $\quad = -16m + 4$ <br> For real roots: $\Delta \geq 0$ <br> $\quad -16m + 4 \geq 0$ <br> $\qquad\qquad m \leq \dfrac{1}{4}$ | ✓ $\Delta = -16m + 4$ <br> ✓ $\Delta \geq 0$ <br><br> ✓ answer <br><br> (3) |
|---|---|

| 8.3 *Determine two values of p for which the equation* $x^2 + 6x - 8p = 0$ *will have rational roots.* <br><br> $\Delta = b^2 - 4ac$ <br> $\quad = (6)^2 - 4(1)(-8p)$ <br> $\quad = 36 + 32p$ <br> For roots to be rational: $\Delta$ has to be a perfect square. <br> Two possible values of $p$: <br> $p = 0$ <br> $p = 2$ <br> Also accept other values of $p$ that will make $\Delta$ a perfect square. | ✓ $\Delta = 36 + 32p$ <br> ✓ $\Delta$ has to be a perfect square. <br><br> ✓ answer <br> ✓ answer <br><br> (4) |
|---|---|

**TOTAL: 50 MARKS**