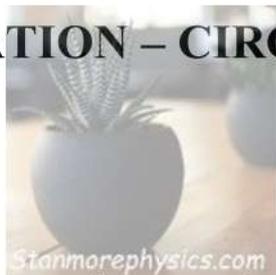




## GRADE 11 MATHEMATICS

### INVESTIGATION – CIRCLE THEOREMS



*The aim of this task is to explore some theorems in circle geometry, to attempt to prove some of the theorems and to apply these theorems to calculate the sizes of unknown angles.*



DATE            MARCH 2021  
TOTAL           50 MARKS  
TIME            2 HOURS

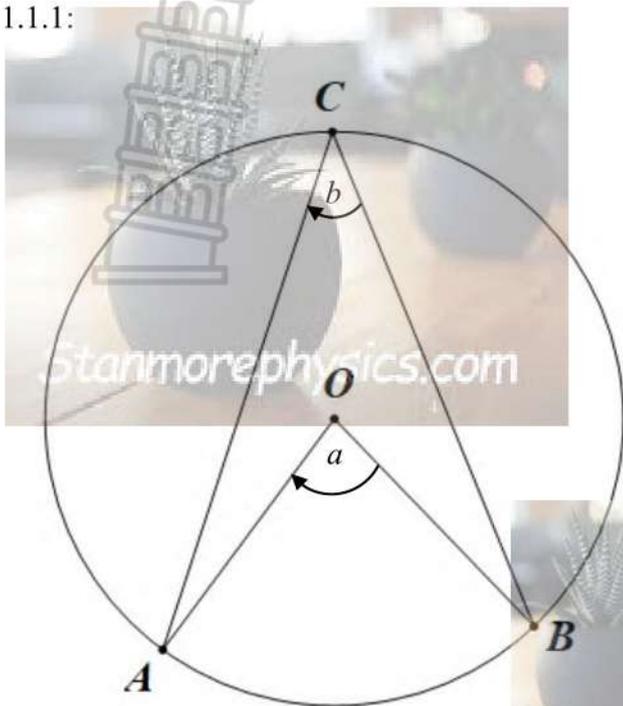
#### INSTRUCTIONS

1. *This question paper consists of 5 pages. Answer all questions.*
2. *A protractor is needed in answering some of the questions.*
3. *Write your answers on the question paper, in the spaces provided for this purpose.*
4. *Provide reasons for all statements.*
5. *Write neatly and legibly.*

**PART A: INVESTIGATIONS AND PROOFS:** [Downloaded From Stanmorephysics.com](http://www.stanmorephysics.com)

1.1 In each of the diagrams below, O is the centre of the circle. In each case measure the angles marked  $a$  (angle at the centre) and  $b$  (angle at the circumference), record your measurements, and answer the questions that follow.

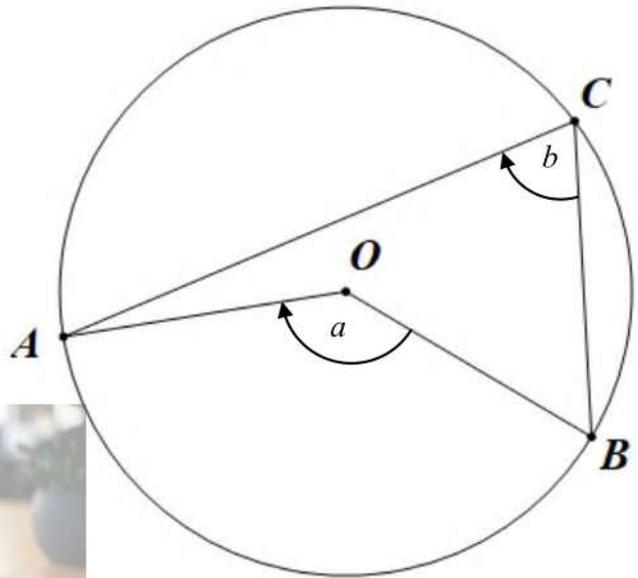
1.1.1:



Angle	Size	What do you notice about the sizes of $a$ and $b$ ?
$a$		
$b$		

(2)

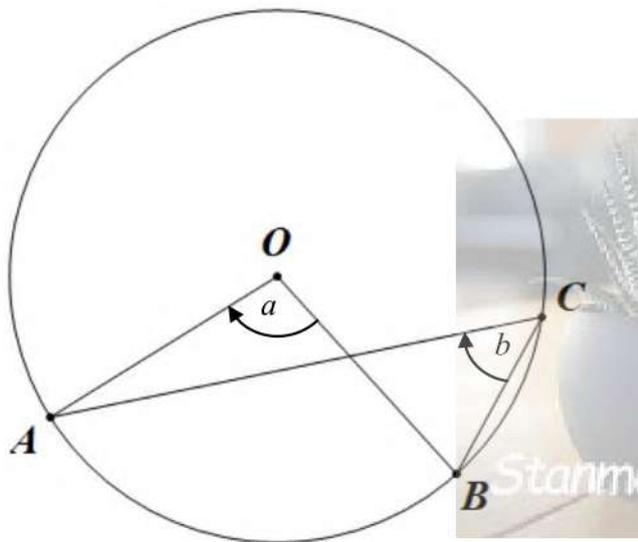
1.1.2



Angle	Size	What do you notice about the sizes of $a$ and $b$ ?
$a$		
$b$		

(2)

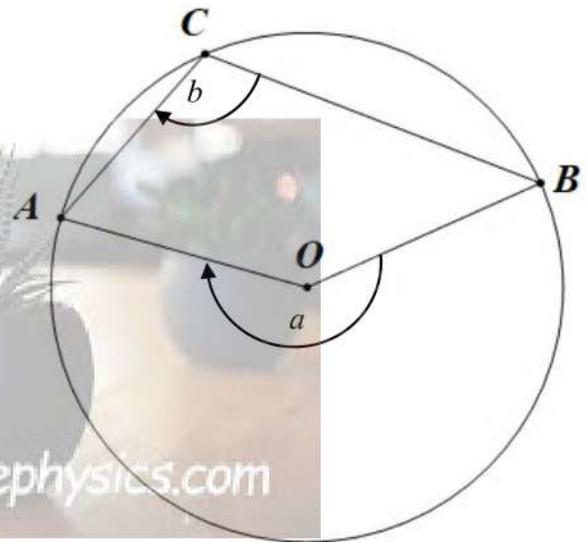
1.1.3



Angle	Size	What do you notice about the sizes of $a$ and $b$ ?
$a$		
$b$		

(2)

1.1.4



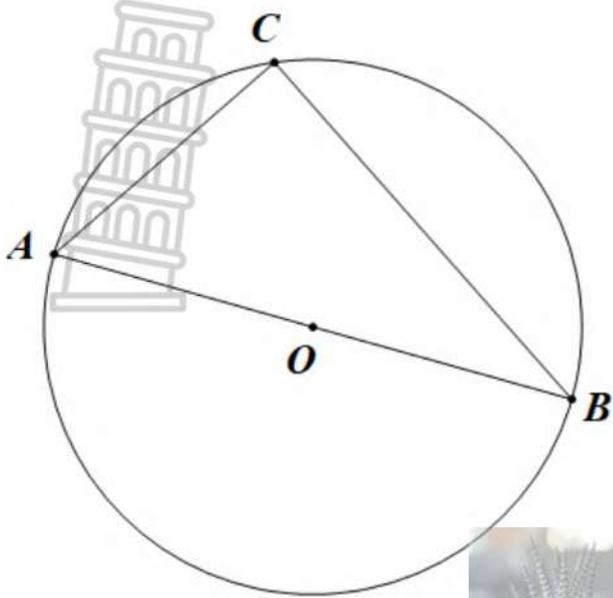
Angle	Size	What do you notice about the sizes of $a$ and $b$ ?
$a$		
$b$		

(2)

1.2 What do you observe concerning the angle at the centre and the angle at the circumference?

.....(2)

2. In the sketch  $O$  is the centre of the circle and  $AOB$  is a diameter. Therefore arc  $ACB$  is a semi-circle, and  $\hat{C}$  is an **angle in a semi-circle**. Measure  $\hat{C}$  and record your value in the table below.:



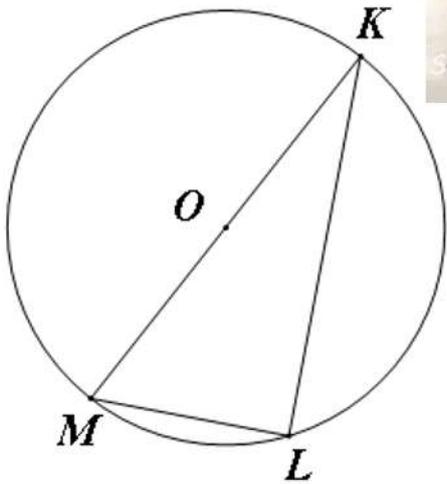
Now follow the following steps to draw and measure two more angles in semicircle  $ACB$ :

- Choose any other point on semicircle  $ACB$  and name it point  $D$ .
- Draw  $AD$  and  $BD$ .
- Measure  $\hat{D}$ .
- Choose a third point on semicircle  $ACB$  and name it point  $E$ .
- Draw  $AE$  and  $BE$ .
- Measure  $\hat{E}$ .

Angle	$\hat{C}$	$\hat{D}$	$\hat{E}$
Size			

[2]

3. In the sketch  $O$  is the centre of the circle and  $KOM$  is a diameter.

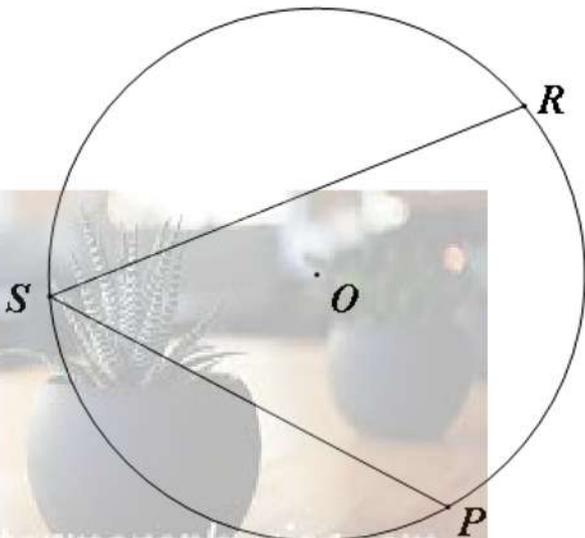


Prove that  $\hat{L}$  is equal to  $90^\circ$  :  
(Do not measure  $\hat{L}$ , but use knowledge of Geometry facts and theorems to show why  $\hat{L} = 90^\circ$ .)

.....  
 .....  
 .....  
 .....  
 .....

[4]

4. In the sketch  $O$  is the centre of the circle and  $\hat{S}$  is an angle in (major) segment  $PR$ .



Measure  $\hat{S}$  and record the value in the table below.

Now, follow the steps below to draw and measure two more angles in segment  $PR$ .

- Choose any other point on arc  $PSR$  (same side of  $PR$  as  $S$ ) and name it point  $T$ .
- Draw  $PT$  and  $TR$ .
- Measure  $\hat{T}$  and record in the table.
- Repeat for a third point  $Q$ .

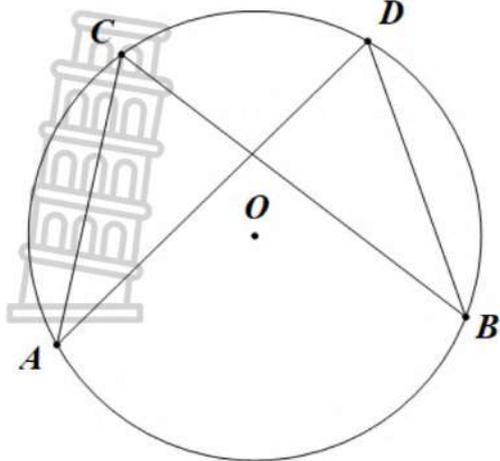
Angle	$\hat{S}$	$\hat{T}$	$\hat{Q}$
Size			

Complete:  
Angles in the same .....

are .....

[3]

5. In the sketch  $O$  is the centre of the circle and  $\hat{C}$  and  $\hat{D}$  are angles in segment  $AB$  of the circle (or: angles subtended by chord  $AB$ ).



(Do not measure  $\hat{C}$  and  $\hat{D}$ , but use knowledge of Geometry facts and theorems to show why  $\hat{C} = \hat{D}$ .)

Hint: Draw in  $AO$  and  $BO$ .

.....

.....

.....

.....

.....

[3]

6. In the sketch below  $ABCD$  is a cyclic quadrilateral, i.e. a quadrilateral with all four of its sides chords of the circle.

Measure each of the angles:

$\hat{A} = \dots\dots\dots^\circ$        $\hat{B} = \dots\dots\dots^\circ$

$\hat{C} = \dots\dots\dots^\circ$        $\hat{D} = \dots\dots\dots^\circ$

Complete:

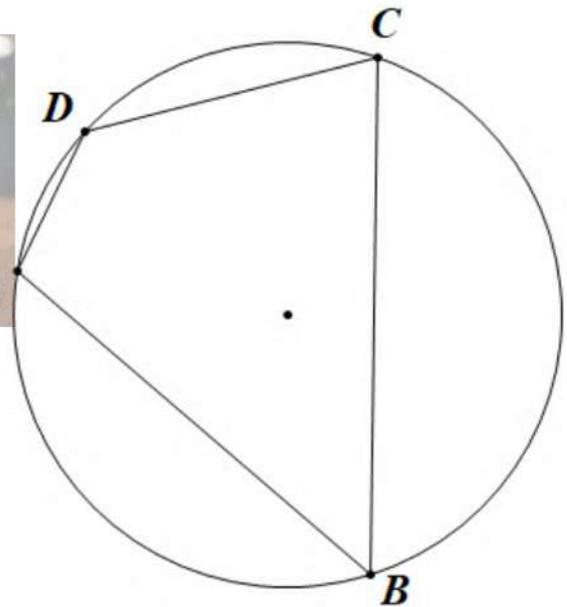
..... + ..... =  $180^\circ$

and also:

..... + ..... =  $180^\circ$

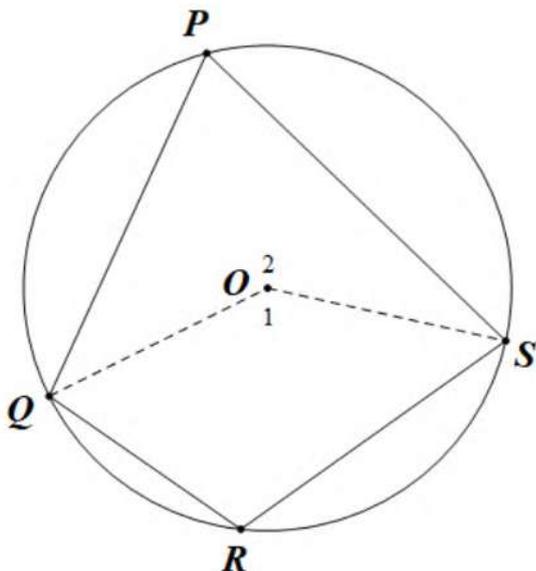
Therefore:

The ..... angles of a cyclic quadrilateral are .....



[6]

7. In the sketch  $PQRS$  is a cyclic quadrilateral.  $O$  is the centre of the circle and  $QO$  and  $OS$  are drawn.



Complete the following proof, also giving reasons for statements where required.

$\hat{O}_1 = 2 \times \text{angle } \dots\dots\dots$

[Reason: .....] ]

$\hat{O}_2 = 2 \times \text{angle } \dots\dots\dots$

[Reason: .....] ]

$\therefore \hat{O}_1 + \hat{O}_2 = 2 \times \text{angle } \dots\dots\dots + 2 \times \text{angle } \dots\dots\dots$

Also:  $\hat{O}_1 + \hat{O}_2 = \dots\dots\dots^\circ$  [Reason: .....] ]

$\therefore 2 \times \text{angle } \dots\dots\dots + 2 \times \text{angle } \dots\dots\dots = \dots\dots\dots^\circ$

$\therefore \text{angle } \dots\dots\dots + \text{angle } \dots\dots\dots = \dots\dots\dots^\circ$

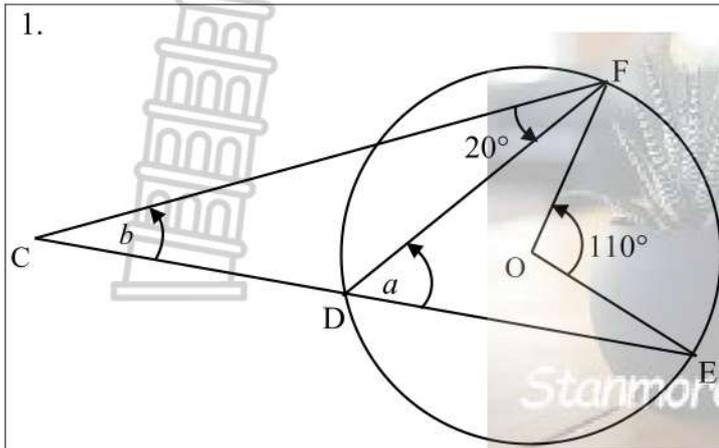
[7]

Total for Part A: 35 marks

**PART B: APPLICATION** *Downloaded from Stanmorephysics.com*

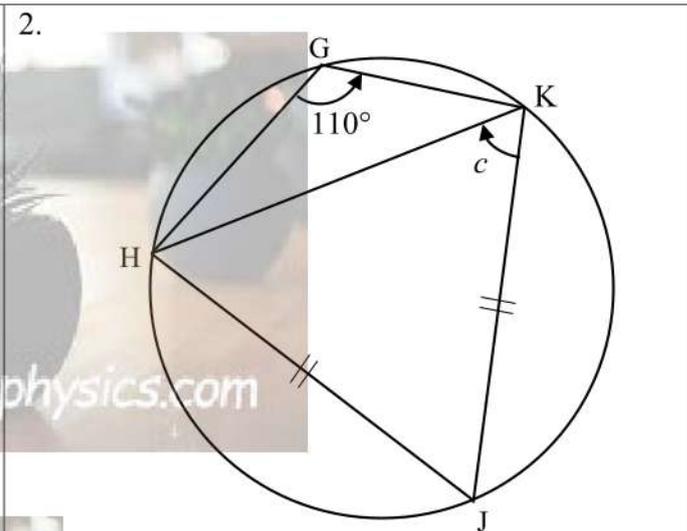
In each sketch O is the centre of the circle.

Calculate the sizes of the angles marked *a* up to *f*. Give reasons for all your statements.



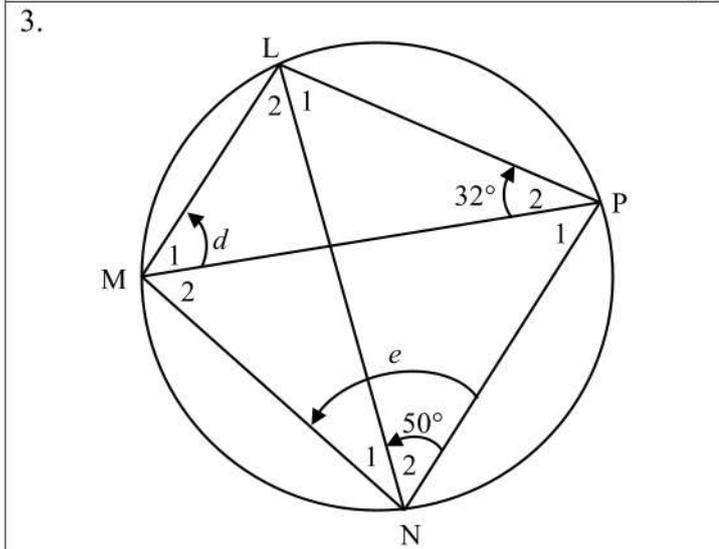
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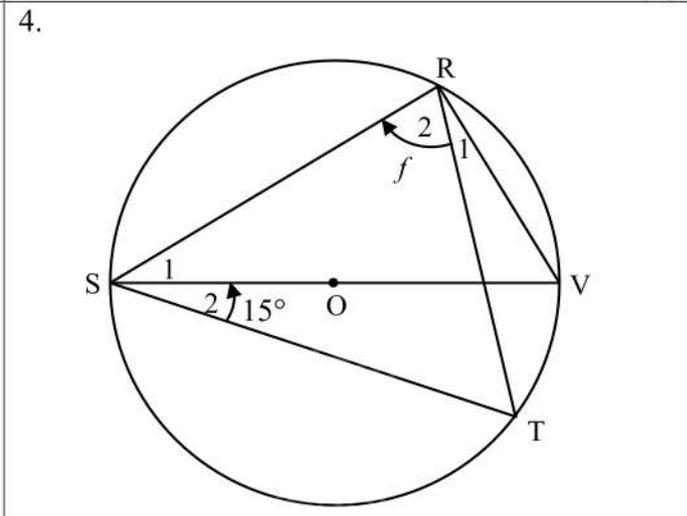
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Total for Part B: 15 marks

**TOTAL: 50 marks**