



## GRADE 11 INVESTIGATION 2025

### NATURE OF ROOTS

**THE AIM OF THE INVESTIGATION IS TO DISCOVER THE CHARACTERISTICS OF THE X-INTERCEPTS(ROOTS) OF A QUADRATIC EQUATION.**

<b>DATE</b>	<b>20 FEBRUARY 2025</b>
<b>TOTAL MARKS</b>	<b>50</b>
<b>TIME</b>	<b>1 HOUR</b>

### INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 4 questions. Answer ALL the questions.
2. Show clearly ALL calculations, diagrams, graphs etc. which you have used in determining the answers.
3. Answers only will not necessarily be awarded full marks.
4. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
5. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
6. It is in your own interest to write legibly and to present the work neatly.
7. The answers must be written on this question paper.

FORMULA: 
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Question 1**

Use the quadratic formula to determine the roots of the quadratic equations below.

1.1.  $x^2 - 6x + 9 = 0$



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(3)

1.2.  $x^2 - 4x + 3 = 0$

(3)

1.3.  $x^2 - 4x - 3 = 0$  (leave the answer in surd form).

(3)



(3)

1.5 Using the words given below, classify each of the roots in 1.1 -1.4. (write down the root(s) and then describe them)



Rational	Irrational	Equal	Unequal
Real	Non-real		

	Root (s)	Description	
1.1			(2)
1.2			(2)
1.3			(2)
1.4			(1)

Did you notice that the value under the square root sign determines the description you gave?  
This is called the discriminant ( $\Delta$ )

$$\Delta = b^2 - 4ac$$

Study the two tables above and then complete the following sentences.

If  $\Delta < 0$  then the roots are \_\_\_\_\_

If  $\Delta = 0$  then the roots are \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_

If  $\Delta > 0$  and a perfect square then the roots are \_\_\_\_\_,

\_\_\_\_\_ and \_\_\_\_\_

If  $\Delta > 0$  and not a perfect square then roots are \_\_\_\_\_,

\_\_\_\_\_ and \_\_\_\_\_

(7)

## Question 2

### Application

Without solving for  $x$  discuss the nature of the roots of the following equations.

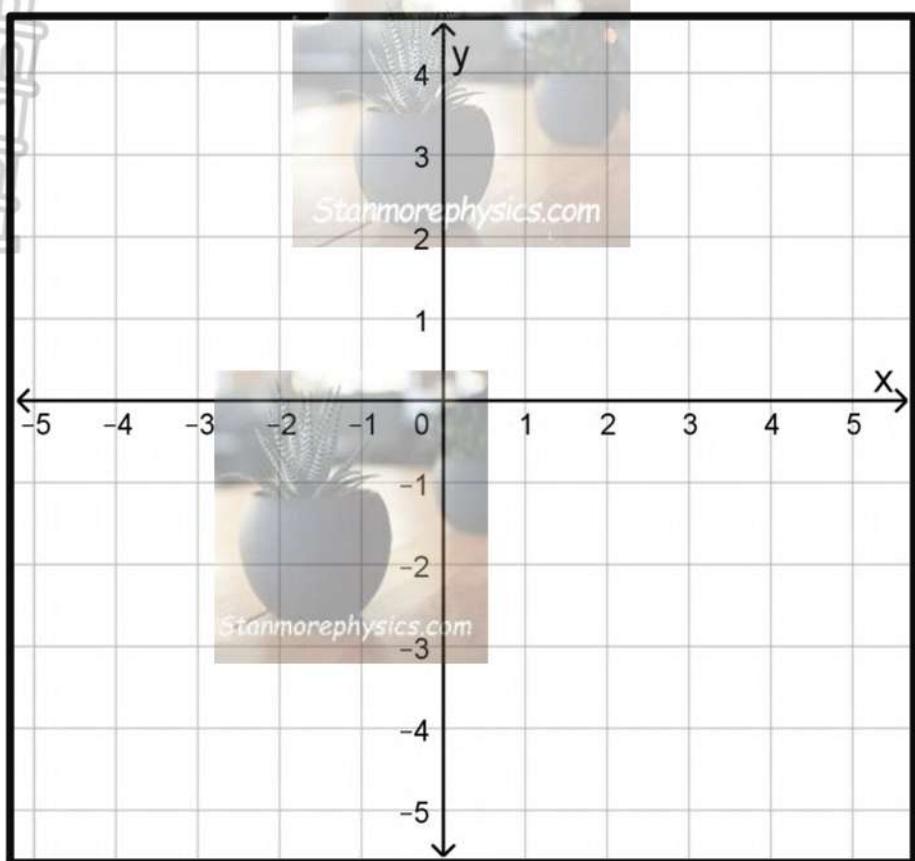
2.1.  $2x^2 + 5x - 3 = 0$

(3)

2.2.  $2x^2 - 7x = -8$

(3)

3.1.1. Sketch the graph of  $f(x) = x^2 - 4$  using point by point plotting. Show all intercepts and the turning point clearly.



(3)

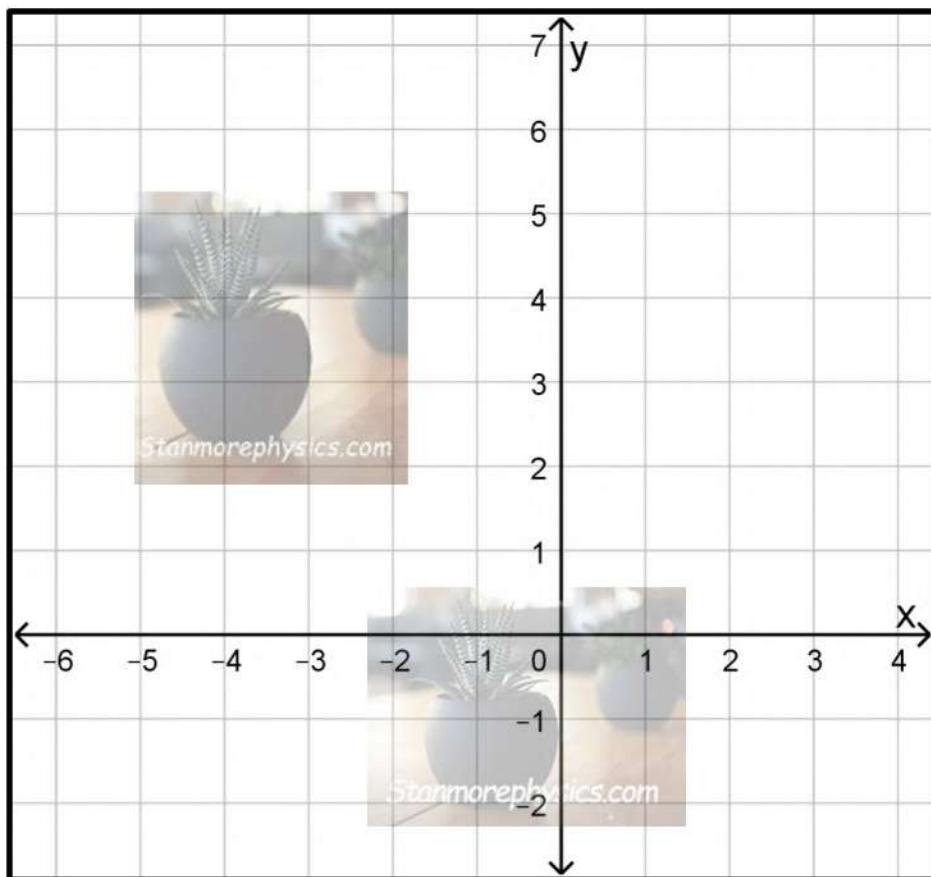
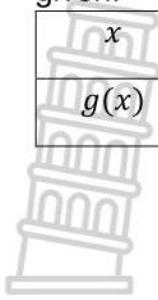
3.1.2. Study the graph and then write down the value(s) of the roots of  $f(x)$ .  
Discuss the nature of these roots

(2)

### 3.2.1. *Downloaded from Stanmorephysics.com*

Hint: Complete the table below and then plot the values in the table on the set of axis given.

$x$	-1	0	1	2	4
$g(x)$					



3.2.2. Use the graph to determine the roots of  $g(x)$ . Discuss the nature of these roots

(3)

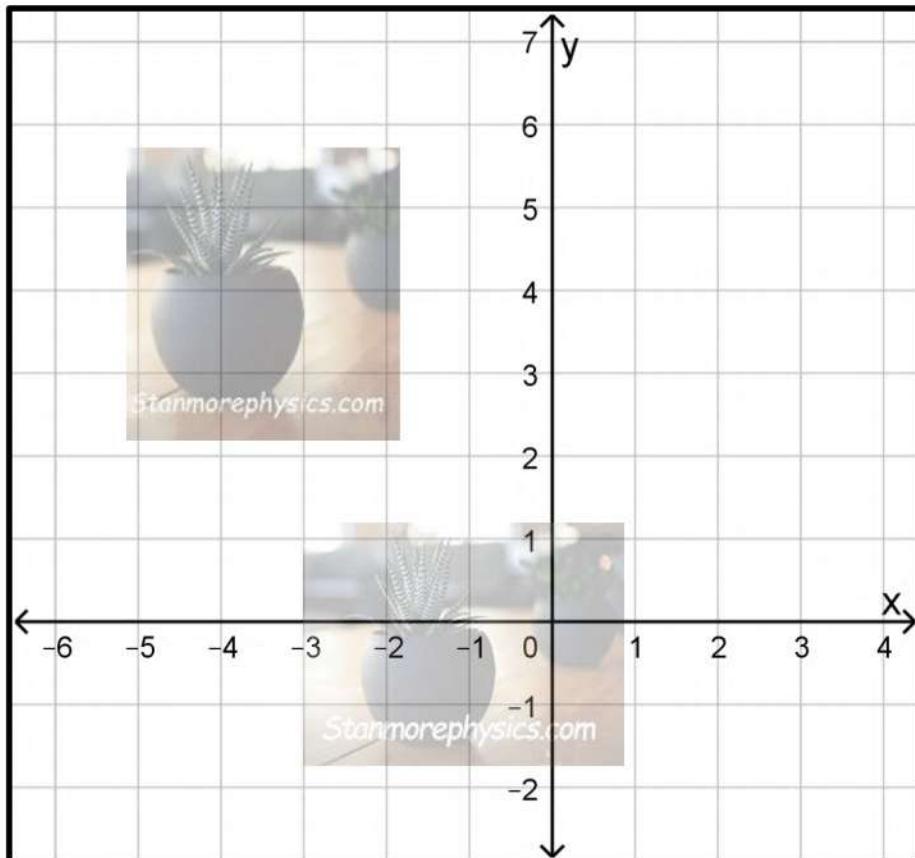
(2)

### 3.2.3. *Downloaded from Stanmorephysics.com*

Sketch the graph of  $h(x) = x^2 + 4$

Hint: Complete the table below and then plot the values in the table on the set of axis given.

$x$	-1	0	1	2	4	5
$h(x)$						

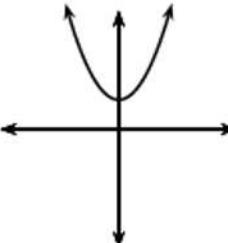
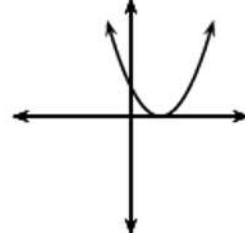
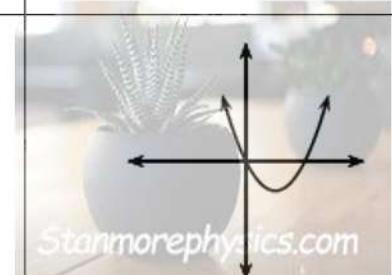


(3)

3.2.4. Does  $h(x)$  have real roots? Explain your answer.

(2)

Complete the following table summarising the results from question 3

Nature of roots	Discriminant	Function ( $a > 0$ )
Roots are _____	$\Delta < 0$	
Roots are real and equal	$\Delta = 0$	
Roots are _____ and _____	$\Delta > 0$ $\Delta$ is a rational number	

(3)

**TOTAL MARKS: 50**



**MATHEMATICS – GRADE 11  
INVESTIGATION 2025- MEMO**

MARKS: 50

DURATION: 1 hour

**AIMS OF THE INVESTIGATION:** To investigate nature of roots ( $x$ -intercepts) of a quadratic equation

FORMULA: 
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



**Question 1**

Use the quadratic formula to determine the roots of the quadratic equations below.

1.1.  $x^2 - 6x + 9 = 0$

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$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)} \checkmark \text{a substitution}$$

$$x = \frac{6 \pm \sqrt{0}}{2} \checkmark$$

$$x = 3 \checkmark \text{ca}$$

1.2.  $x^2 - 4x + 3 = 0$

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$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)} \checkmark \text{a substitution}$$

$$x = \frac{4 \pm \sqrt{4}}{2} \checkmark$$

$$x = 3 \text{ or } x = 1 \checkmark \text{ca}$$

(3)

(3)

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)} \checkmark \text{a substitution}$$

$$x = 2 \pm \sqrt{7} \checkmark \text{ca} \checkmark \text{ca}$$

(3)

1.4.  $x^2 - 4x + 7 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)} \checkmark \text{a substitution}$$

$$x = \frac{4 \pm \sqrt{-12}}{2} \checkmark$$

No real solution  $\checkmark$ 

(3)

1.5 Using the words given below, classify each of the roots in 1.1 -1.4.

(write down the root(s) and then classify)

Rational	Irrational	Equal	Unequal
Real	Non-real		

	Root (s)	Description	
1.1	$x = 3$	Real, Rational, Equal $\checkmark \checkmark$	(2)
1.2	$x = 3$ or $x = 1$	Real, rational, unequal $\checkmark \checkmark$	(2)
1.3	$x = 2 \pm \sqrt{7}$	Real, Irrational, unequal $\checkmark \checkmark$	(2)
1.4	$x = \frac{4 \pm \sqrt{-12}}{2}$	Non-real $\checkmark$	(1)

Did you notice that the value under the square root sign determines the description you gave?

This is called the discriminant ( $\Delta$ )

$$\Delta = b^2 - 4ac$$

Study the two tables above and the complete the following sentences.

If  $\Delta < 0$  then the roots are Non-real

If  $\Delta = 0$  then the roots are Real, Rational and equal.

If  $\Delta > 0$  and a perfect square then the roots are Real, rational and unequal.

If  $\Delta > 0$  and not a perfect square then roots are Real, irrational and unequal.

(7)

## Question 2

Application

Calculate the value for  $\Delta$  and then discuss the nature of the roots of the following equations.

Use the descriptive words in the table in 1.5.

2.1.  $2x^2 + 5x - 3 = 0$

$$\Delta = b^2 - 4ac$$

$$\Delta = (5)^2 - 4(2)(-3) \checkmark$$

$$\Delta = 49 \checkmark$$

*The roots are real, rational and unequal.✓*

(3)

2.2.  $2x^2 - 7x = -8$

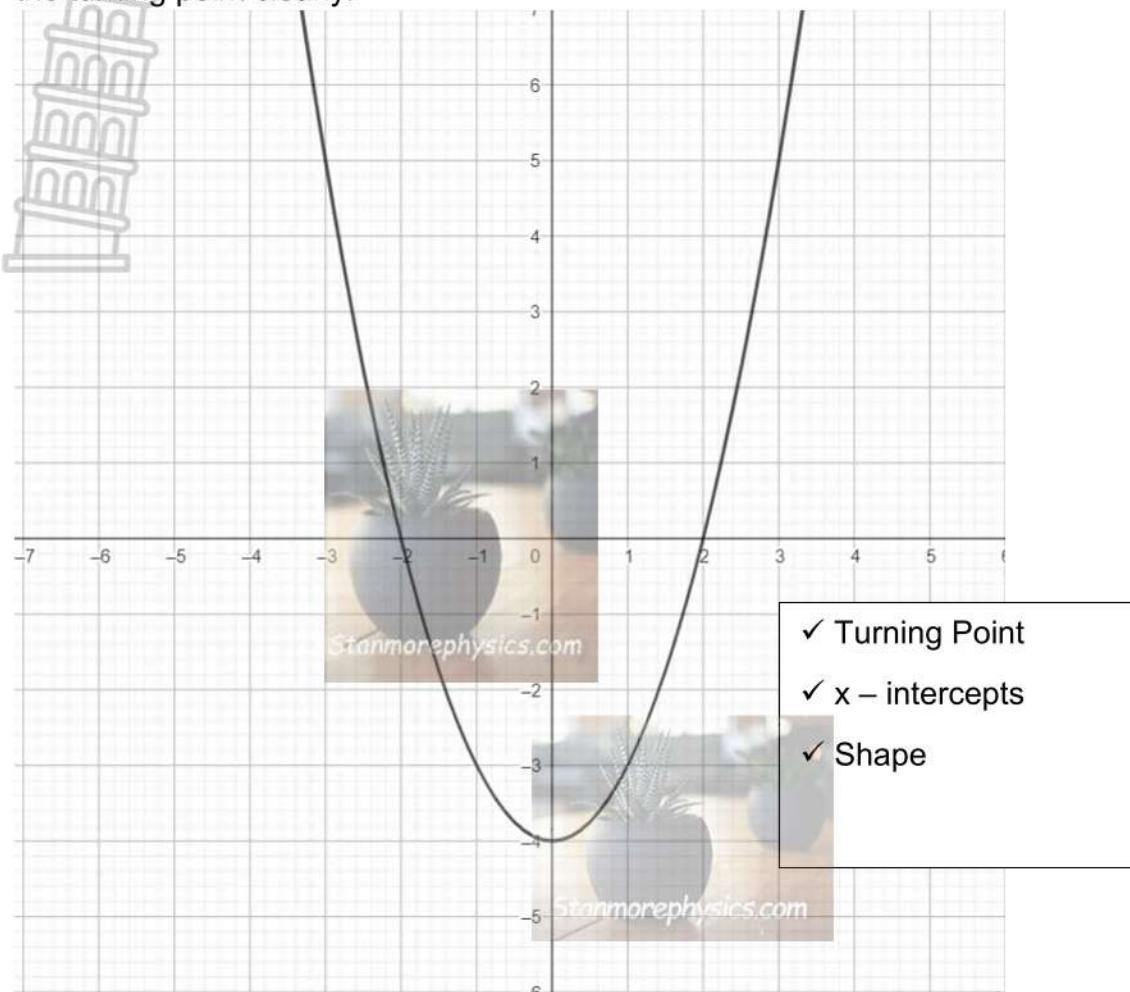
$$\Delta = (-7)^2 - 4(2)(8)$$

$$\Delta = -15$$

The roots are non-real.

(3)

3.1.1. Sketch the graph of  $f(x) = x^2 - 4$  using point by point plotting. Show all intercepts and the turning point clearly.



(3)

3.1.2. Use the graph to determine the roots of  $f(x)$ . Discuss the nature of these roots

$x = -2$  or  $x = 2$   ca

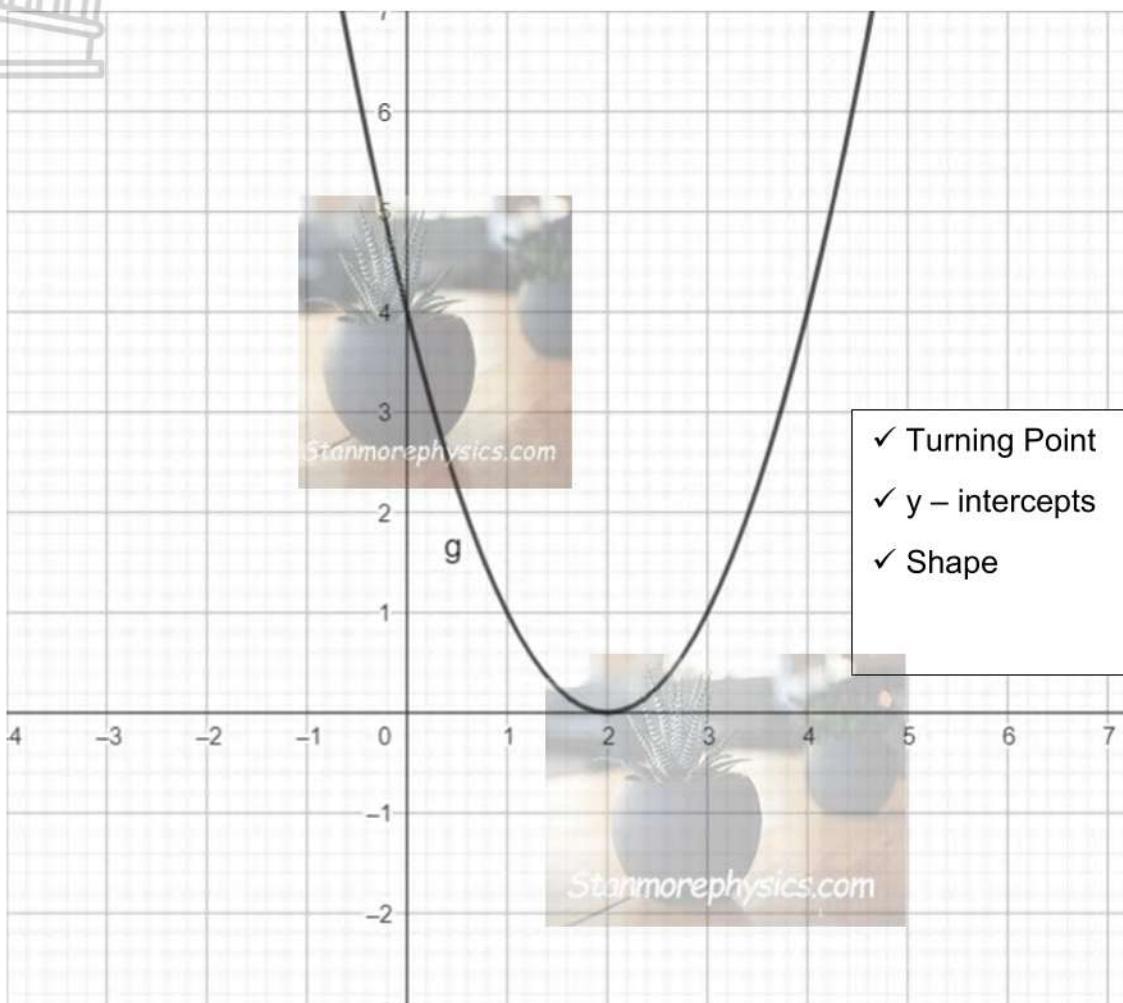
Real, rational and unequal.

(2)

3.2.1. Sketch the graph of  $g(x) = x^2 - 4x + 4$

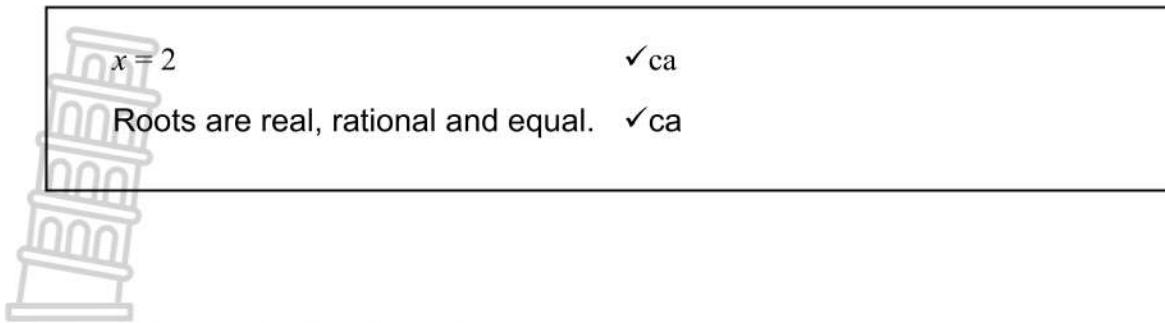
Hint: Complete the table below and then plot the values in the table on the set of axis given.

$x$	-1	0	1	2	4
$g(x)$	9	4	1	0	4



(3)

3.2.2. **Downloaded from Stanmorephysics.com** Sketch the graph of the function  $y = x^2 - 4x + 2$ . Discuss the nature of these roots



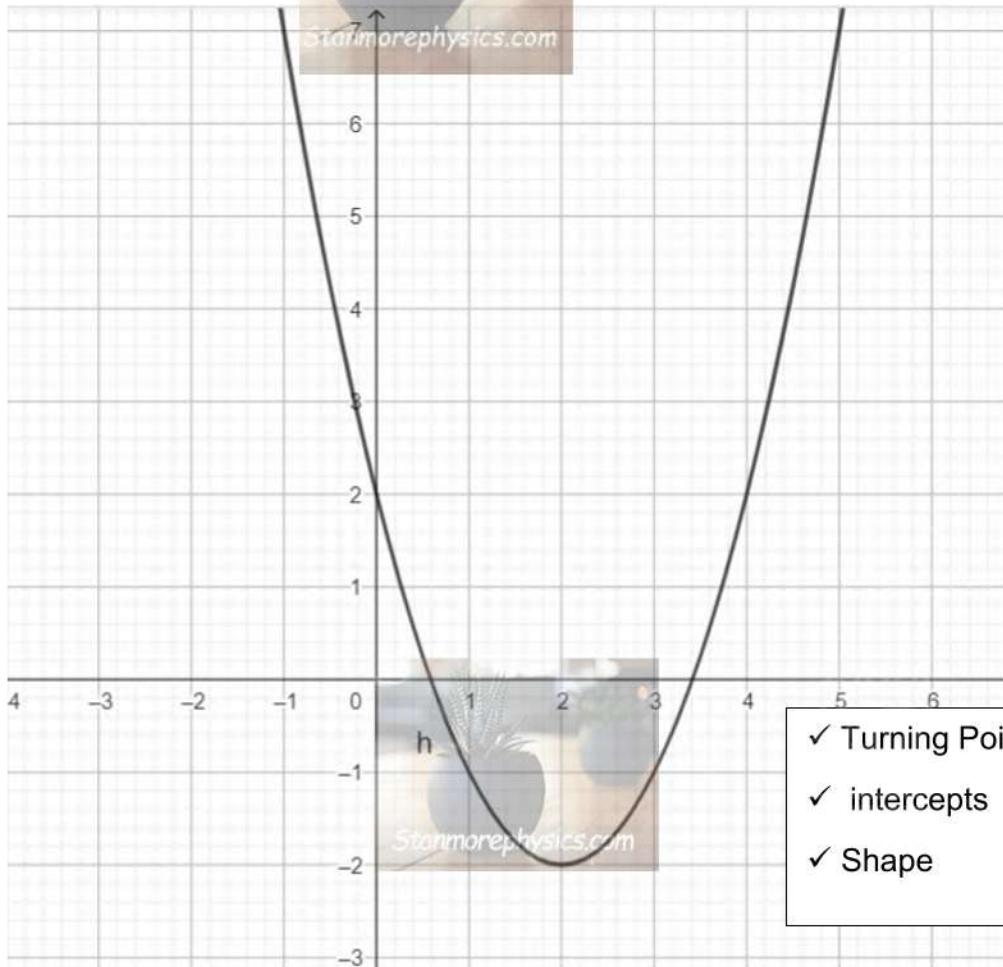
(2)

3.2.3.

Sketch the graph of  $h(x) = x^2 - 4x + 2$ .

Hint: Complete the table below and then plot the values in the table on the set of axis given.

$x$	-1	0	1	2	4	5
$h(x)$	7	2	-1	-2	2	7



(3)

Yes.

✓a

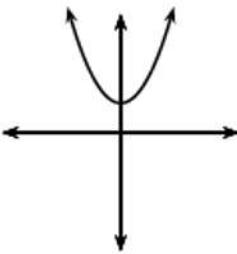
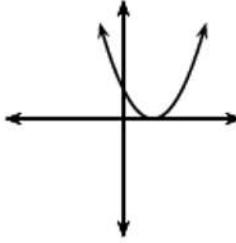
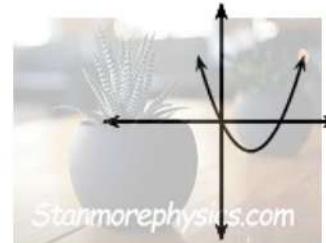
The graph cuts the x – axis.

✓

(2)

**Question 4**

Complete the following table summarising the results from question 3

Nature of roots	Discriminant	Function ( $a > 0$ )
<u>Roots are non – real</u> ✓a		
<u>Roots are real and equal</u>	$\Delta = 0$ ✓a	
<u>Roots are Real and unequal</u> ✓a	$\Delta > 0$ $\Delta$ is a rational number	

(3)

**TOTAL MARKS: 50**