KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12 INVESTIGATION 2025

COMPOUND AND DOUBLE ANGLES

"The aim of this task is to discover the compound angle and double angle formulae and to use them to simplify trigonometric expressions and prove identities."

DATE	**25-02-2025**
TOTAL	**50 MARKS**
TIME	**1 HOUR**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. Answer all the questions in all the activities.
2. Answer are to be done on this question paper.
3. Write neatly and legibly.
4. Calculators (where indicated) may be used.
5. No Textbook, notes, exercise books or any other resource may be used when answering this task. The task is written under examination conditions.
6. This investigation is an individual task, and NO GROUP WORK is allowed

Name of Learner		**Class**	

THIS TABLE IS FOR THE EDUCATOR – DO NOT WRITE ANYTHING HERE				
	Part 1	***Part 2***	***Part 3***	***TOTAL***
Total	20	22	8	50
Candidate Mark				
Moderated Mark				

PART 1: COMPOUND ANGLES (20 Marks)

Important formulae you have learnt in previous grades that are needed for this investigation:

1.	Distance formula:	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
2.	Cosine Rule:	$a^2=b^2+c^2-2bc.\cos A$

Definition of a Compound Angle

When two angles are ADDED together, or one angle is SUBTRACTED from another to form a new angle, the new angle is referred to as a COMPOUND ANGLE.

QUESTION 1

1.1: Making a Conjecture on cos(A - B) (5 Marks)

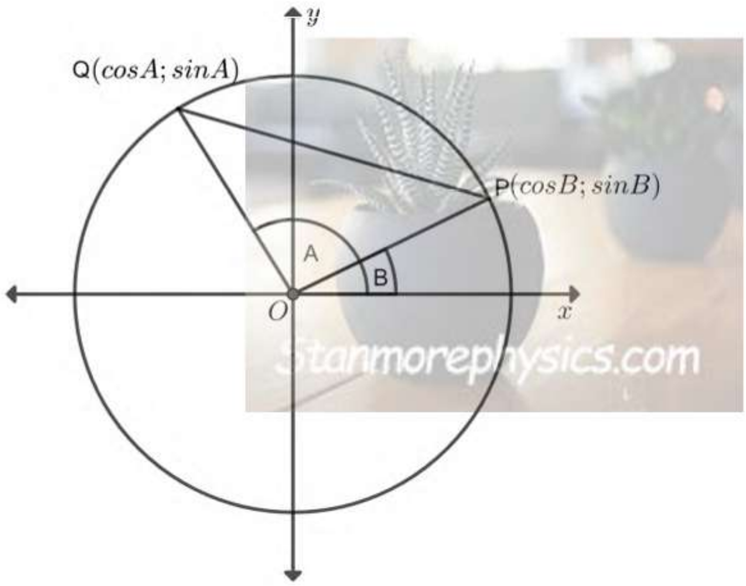
Complete the table below, using the worked example in the first row as a guideline.

You MUST USE YOUR CALCULATOR and round off your answers to three decimal places where applicable.

	Angle A	Angle B	cos(A - B)	$\cos(A)-\cos(B)$	$\cos(A)\cos(B)+\sin(A)\sin(B)$	Marks
E.g.	60°	30°	$0,866$	$-0,366$	$0,866$	
1.1.1	45°	30°				(1)
1.1.2	90°	45°				(1)
1.1.3	120°	60°				(1)
1.1.4	What pattern do you observe in the values of $\cos(A-B)$ and $\cos A\cos B+\sin A\sin B$?					(1)
1.1.5	Based on your pattern, make a conjecture about the identity for $\cos(A-B)$.					(1)

1.2: Proving cos(A - B) Using the Unit Circle (11 Marks)

The diagram below shows a unit circle O on the cartesian plane. P and Q are any points on the circumference with $Q\hat{O}x = A$ and $P\hat{O}x = B$. The coordinates for points P and Q are as indicated. Study the diagram and answer the questions that follow.



#	Question	Marks
1.2.1	Write down the size of $P\hat{O}Q$ in terms of A and B.	(1)
1.2.2	Using the distance formula, show that $PQ^2 = 2 - 2\cos A \cos B - 2\sin A \sin B$. Show all your working.	(3)

1.2.3	Using the Cosine Rule, show that $PQ^2 = 2 - 2\cos(A - B)$. Show all your working.	(3)
1.2.4	Use the two expressions for the value of PQ, provided in 1.2.2 and 1.2.3, to deduce the compound angle formula: $\cos(A - B) = \cos A\cos B + \sin A\sin B$. Show all your working.	(2)
1.2.5	Using $\cos(A - B) = \cos A\cos B + \sin A\sin B$, determine the compound angle for $\cos(A + B)$. Show all working. Hint: $\cos(A + B) = \cos[A - (-B)]$	(2)

1.3: Making a Conjecture on sin(A + B) (4 Marks)

Complete the table below, using the worked example in the first row as a guideline.

You MUST USE YOUR CALCULATOR and round off your answers to three decimal places where applicable.

	Angle A	Angle B	$\sin(A+B)$	$\sin A + \sin B$	$\sin A\cos B + \cos A\sin B$	Marks
E.g.	60°	30°	1	1.366	1	
1.3.1	45°	30°				(1)
1.3.2	90°	45°				(1)
1.3.3	What pattern do you observe in the values of $\sin(A+B)$ and the values of $\sin A\cos B + \cos A\sin B$?					(1)
1.3.4	Based on your observation, make a conjecture about the identity for $\sin(A+B)$					(1)

PART 2: DOUBLE ANGLES (22 Marks)

QUESTION 2

NB: All graphs should be sketched in the corresponding grid provided.

Using your **CALCULATOR**, go to **TABLE** mode.

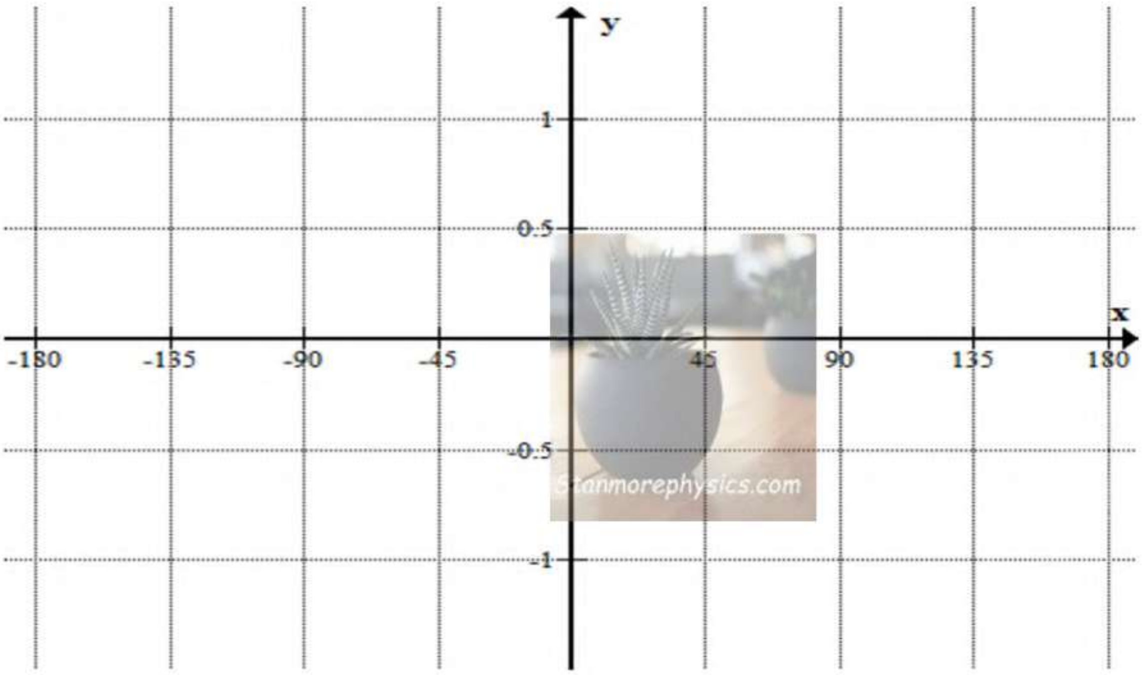
2.1 insert the function $f(x) = 2\cos^2 x - 1$

- **Start**: $-180°$; **End**: $=180°$
- **Step**: $45°$

2.1.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$2\cos^2 x - 1$									

2.1.2 Use the table entries to sketch graph of $f(x) = 2\cos^2 x - 1$,where $x \in [-180°;180°]$ (2)

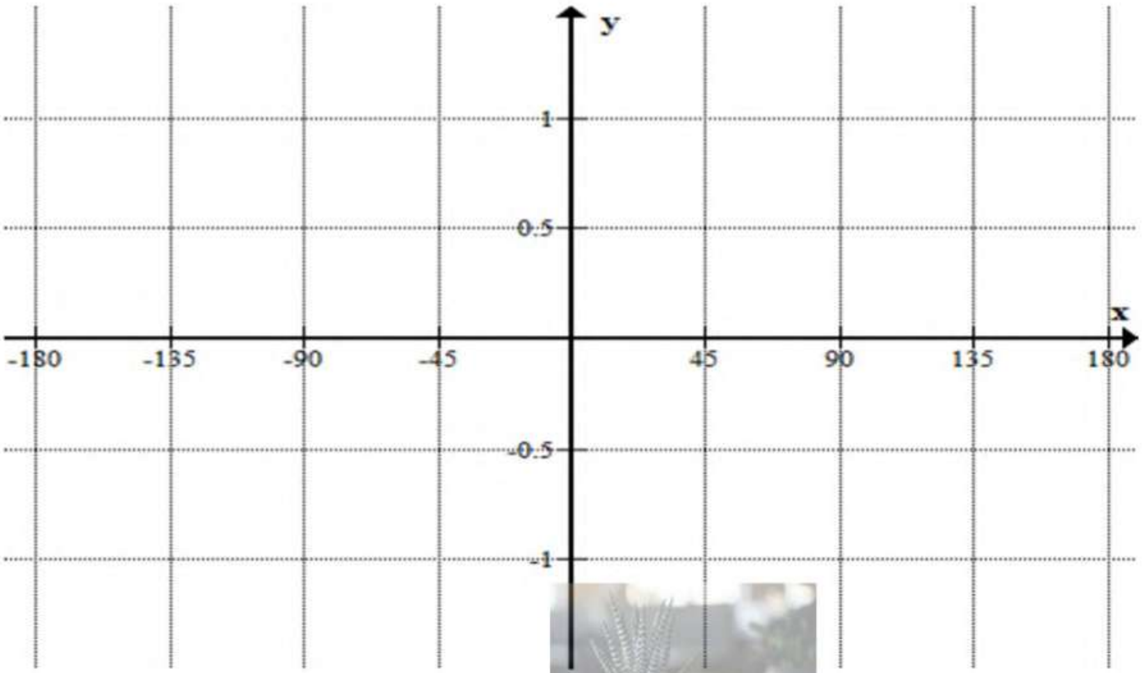


2.2 Using your calculator insert the following function $g(x) = 1 - 2\sin^2 x$

2.2.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$g(x) = 1 - 2\sin^2 x$									

2.2.2 Use the table entries to sketch the graph of, $g(x) = 1 - 2\sin^2 x$ where $x \in [-180°; 180°]$ (2)

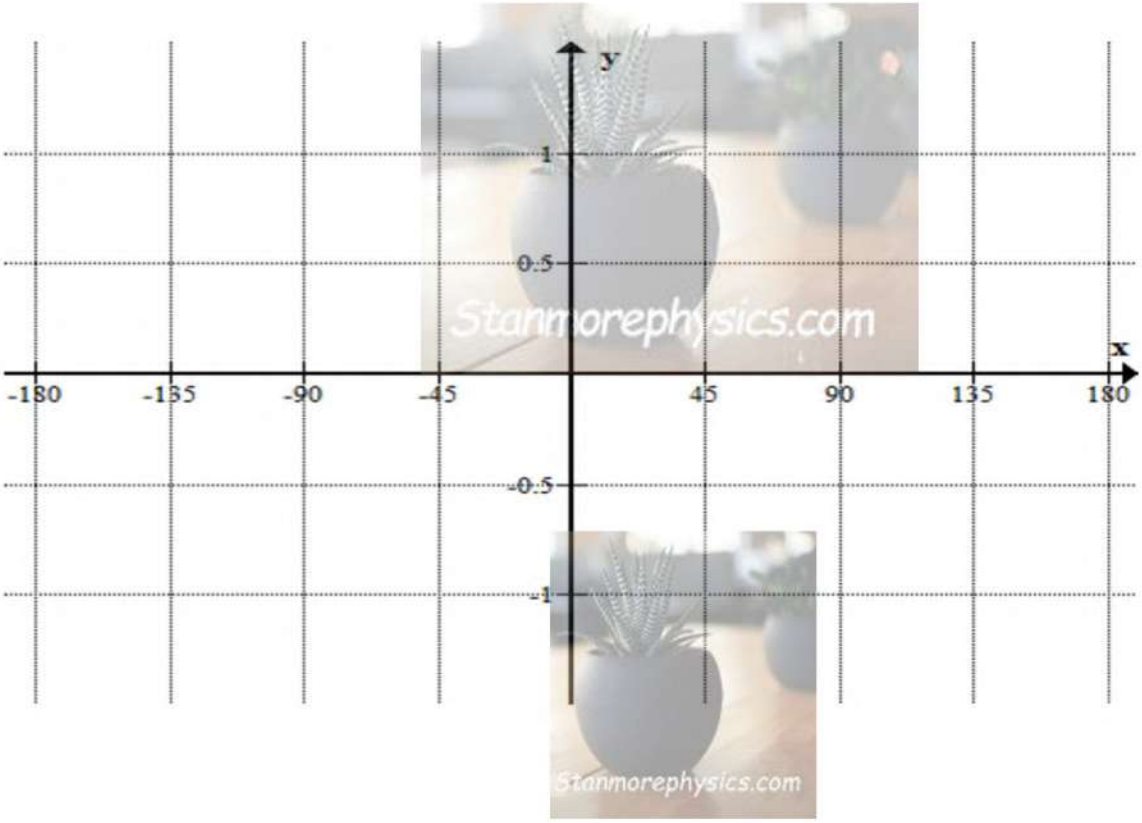


2.3 Using your calculator insert the following function $h(x) = \cos^2 x - \sin^2 x$

2.3.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$h(x) = \cos^2 x - \sin^2 x$									

2.3.2 Use the table entries to sketch the graph of $h(x) = \cos^2 x - \sin^2 x$ where $x \in [-180°; 180°]$ (2)

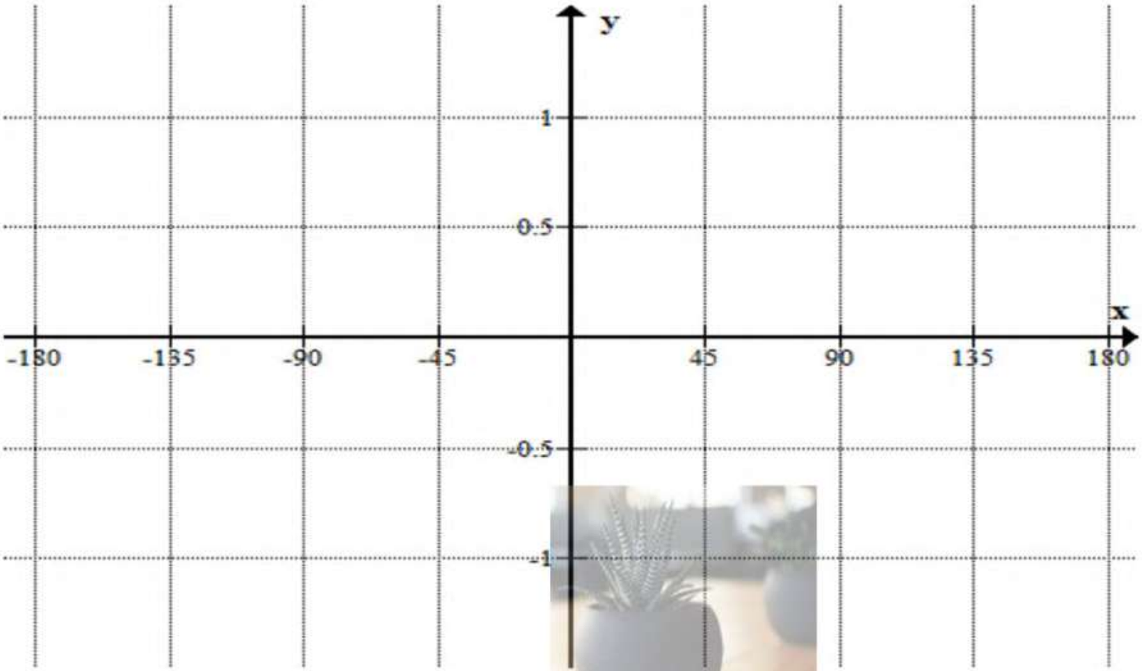


2.4 Using your calculator insert the following function $j(x) = \cos 2x$

2.4.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$j(x) = \cos 2x$									

2.4.2 Use the table entries to sketch the graph of $j(x) = \cos 2x$, where $x \in [-180^\circ; 180^\circ]$ (2)



2.5 What conclusion can be made from f, g, h and j ? (2)

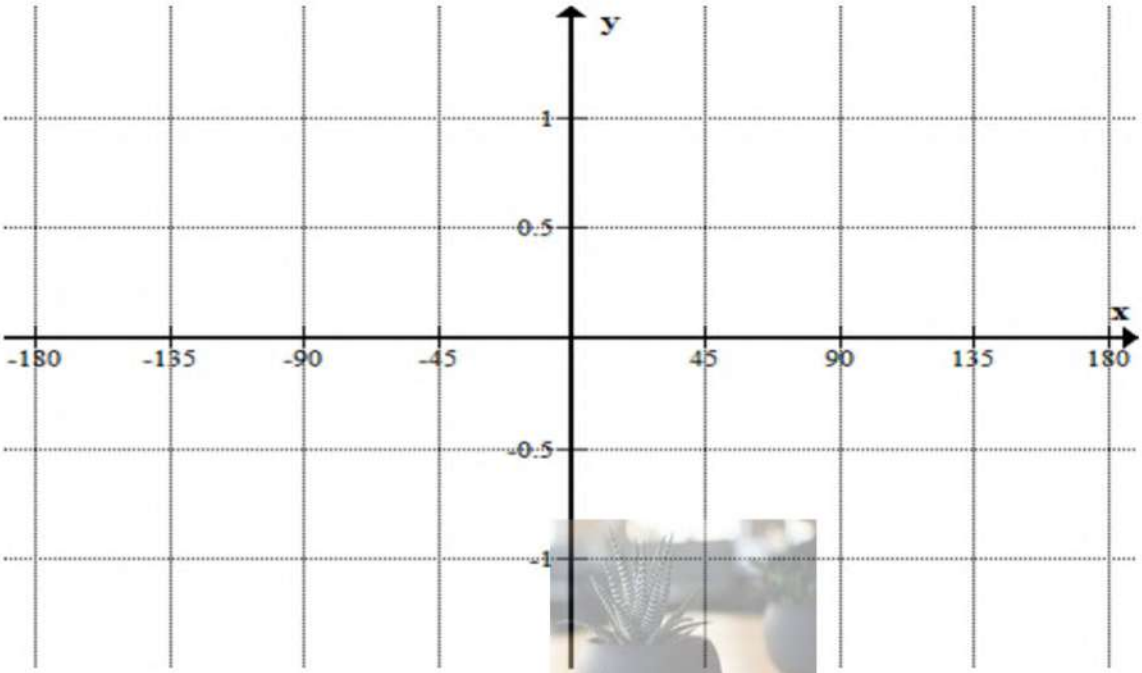
QUESTION 3

3.1 Using your calculator insert the following function $u(x) = 2\sin x \cos x$

3.1.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$u(x) = 2\sin x \cos x$									

3.1.2 Use the table entries to sketch the graph of , $u(x) = 2\sin x \cos x$ where $x \in [-180°;180°]$ (2)



3.2 Using your calculator insert the following function $v(x) = \sin 2x$

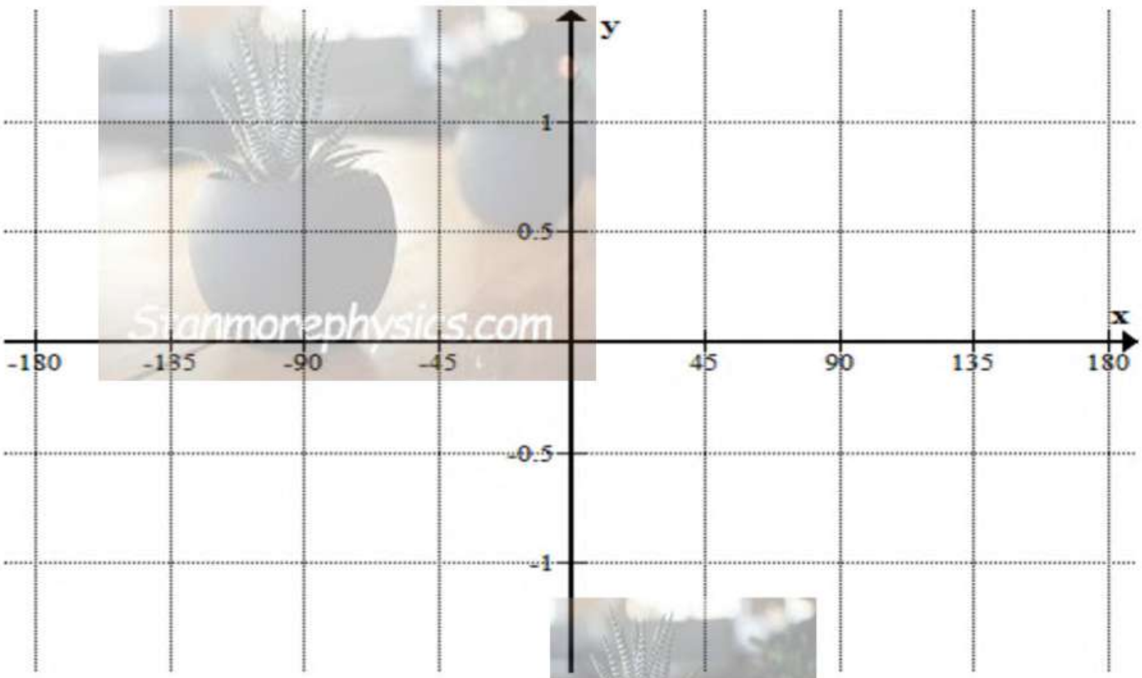
(1)

3.2.1 Complete the following table:

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$v(x) = \sin 2x$									

(2)

3.2.2 Use the table entries to sketch the graph of $v(x) = \sin 2x$ where $x \in [-180°; 180°]$



3.3 What general conclusion can you make from u and v ? (2)

PART 3: APPLICATION OF COMPOUND AND DOUBLE ANGLE IDENTITIES (8 Marks)

QUESTION 4

4.1	Prove that $\dfrac{\cos 2\beta + 1}{\cos \beta} = \dfrac{2\sin\beta}{\tan\beta}$	(4)
4.2	WITHOUT using a calculator, calculate the value of: $\dfrac{\sin 105^\circ}{\cos 15^\circ}$	(4)
	GRAND TOTAL:	**50**

Extra space:

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MARKING GUIDELINES

PART 1: COMPOUND ANGLES (20 Marks)

QUESTION 1

1.1: Making a Conjecture on cos(A - B) (5 Marks)

	Angle A	Angle B	cos(A - B)	$\cos(A)-\cos(B)$	$\cos(A)\cos(B)+\sin(A)\sin(B)$	Marks
E.g.	60°	30°	$0,866$	$-0,366$	$0,866$	
1.1.1	45°	30°	0.966	-0.159	0.966	✓ A(1)
1.1.2	90°	45°	0.707	-0.707	0.707	✓ A(1)
1.1.3	120°	60°	0.5	-1	0.5	✓ A(1)
1.1.4	What pattern do you observe in the values of $\cos(A-B)$ and $\cos A\cos B+\sin A\sin B$?					✓CA(1)
	They are the same.					
1.1.5	Based on your pattern, make a conjecture about the identity for $\cos(A-B)$.					✓A (1)
	$\cos(A-B)=\cos A\cos B+\sin A\sin B$					

1.2: Proving cos(A - B) Using the Unit Circle (11 Marks)

#	Question	MG
1.2.1	$P\hat{O}Q = x\hat{O}Q - x\hat{O}P = (A-B)$	✓ A $P\widehat{O}Q=(A-B)$ (1)
1.2.2	$PQ^2=(\cos A-\cos B)^2+(\sin A-\sin B)^2$ $=\cos^2 A-2\cos A\cos B+\cos^2 B+\sin^2 A-2\sin A\sin B+\sin^2 B$ $=\cos^2 A+\sin^2 A+\cos^2 B+\sin^2 B-2\cos A\cos B-2\sin A\sin B$ $=1+1-2\cos A\cos B-2\sin A\sin B$ $=2-2\cos A\cos B-2\sin A\sin B$	✓A Correct use of distance formula. ✓A Both square identities ✓A Simplification (3)

1.2.3	$PQ^2 = OP^2 + OQ^2 - 2OP.OQ\cos(A-B)$ $=(1)^2+(1)^2-2(1)(1)\cos(A-B)$ $=2-2\cos(A-B)$	✓A Correct use of cosine rule. ✓A $(1)^2 + (1)^2$ ✓A $2(1)(1)\cos(A-B)$ (3)
1.2.4	$2-2\cos(A-B)=2-2\cos A\cos B-2\sin A\sin B$ $-2\cos(A-B)=-2(\cos A\cos B+\sin A\sin B)$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$	✓A Equation ✓A $-2\cos(A-B)$ $=-2(\cos A\cos B+\sin A\sin B)$ (2)
1.2.5	$\cos(A+B)=\cos[A-(-B)]$ $=\cos A\cos(-B)+\sin A\sin(-B)$ $=\cos A\cos B-\sin A\sin B$	✓A $\cos A\cos(-B)+\sin A\sin(-B)$ ✓A $\cos A\cos B-\sin A\sin B$ (2)

1.3: Making a Conjecture on sin(A + B) (4 Marks)

	Angle A	Angle B	$\sin(A+B)$	$\sin A+\sin B$	$\sin A\cos B+\cos A\sin B$	Marks
E.g.	60°	30°	1	1.366	1	
1.3.1	45°	30°	0.966	1.207	0.966	✓ A (1)
1.3.2	90°	45°	0.707	1.707	0.707	✓ A (1)
1.3.3	What pattern do you observe in the values of $\sin(A+B)$ and the values of $\sin A\cos B+\cos A\sin B$?					
	They are the same or They are equal.					✓ A Answer(1)
1.3.4	Based on your observation, make a conjecture about the identity for $\sin(A+B)$					
	$\sin(A+B)=\sin A\cos B+\cos A\sin B$					✓ A(1)

PART 2: DOUBLE ANGLES (22 Marks)

QUESTION 2

All graphs should be sketched in the corresponding grid provided.

Using your **CALCULATOR**, go to **TABLE** mode.

2.1 insert the function $f(x) = 2\cos^2 x - 1$

- **Start**: $-180°$; **End**: $=180°$
- **Step**: $45°$

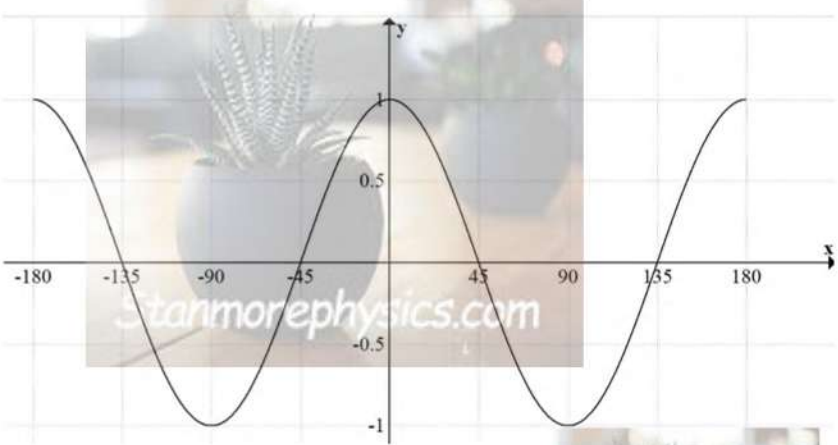
2.1.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$2\cos^2 x - 1$	1	0	-1	0	1	0	-1	0	1

A✓

(2)

2.1.2 Use the table entries to sketch graph of $f(x) = 2\cos^2 x - 1$,where $x \in [-180°;180°]$



A✓Plot
A✓Shape

2.2 Using your calculator insert the following function $g(x) = 1 - 2\sin^2 x$

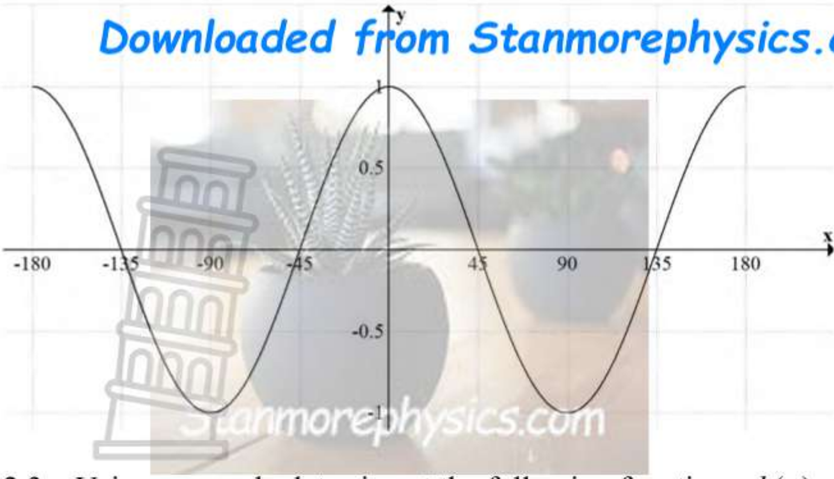
2.2.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$g(x) = 1 - 2\sin^2 x$	1	0	-1	0	1	0	-1	0	1

A✓

(2)

2.2.2 Use the table entries to sketch the graph of , $g(x) = 1 - 2\sin^2 x$ $x \in [-180°;180°]$
where



A✓Plot
A✓Shape

2.3 Using your calculator insert the following function $h(x)=\cos^2 x-\sin^2 x$

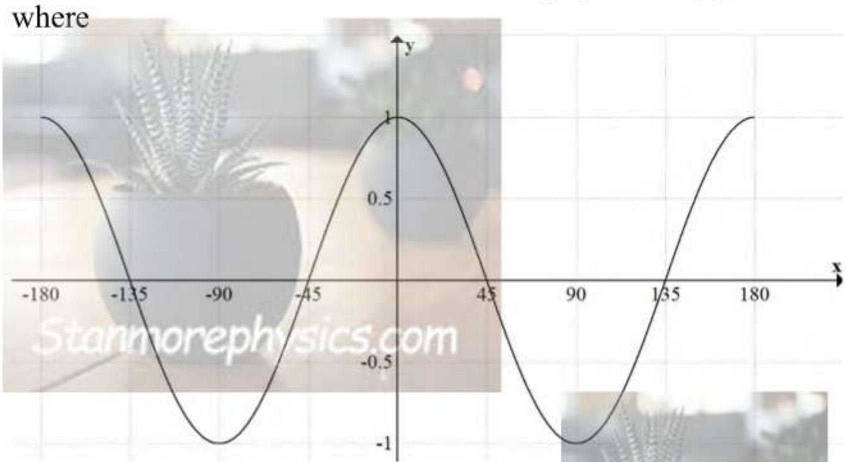
2.3.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$h(x)=\cos^2 x-\sin^2 x$	1	0	-1	0	1	0	-1	0	1

A✓

(2)

2.3.2 Use the table entries to sketch the graph of $h(x)=\cos^2 x-\sin^2 x$ where $x\in\left[-180^\circ;180^\circ\right]$



A✓Plot
A✓Shape

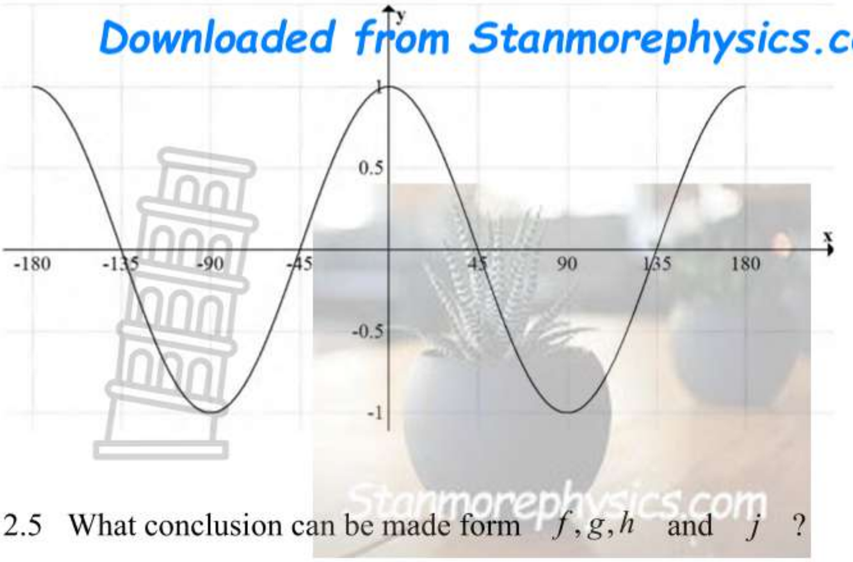
2.4 Using your calculator insert the following function $j(x)=\cos 2x$

2.4.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$j(x)=\cos 2x$	1	0	-1	0	1	0	-1	0	1

A✓

2.4.2 Use the table entries to sketch the graph of $j(x)=\cos 2x$, where $x\in\left[-180^\circ;180^\circ\right]$ (2)



A✓Plot
A✓Shape

2.5 What conclusion can be made form f, g, h and j ? (2)

A✓
A✓

QUESTION 3
They are the same graph

3.1 Using your calculator insert the following function $u(x) = 2\sin x\cos x$

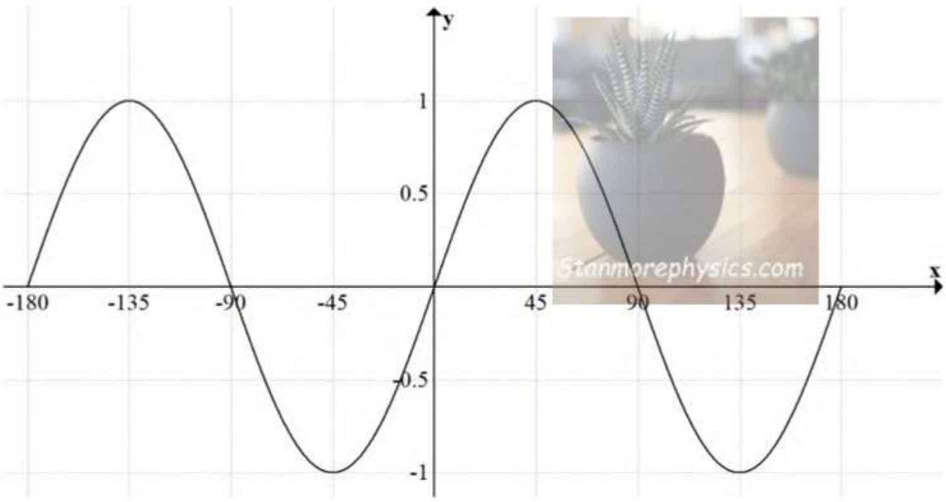
3.1.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$u(x) = 2\sin x\cos x$	0	1	0	-1	0	1	0	-1	0

A✓

(2)

3.1.2 Use the table entries to sketch the graph of , $u(x) = 2\sin x\cos x$ where $x \in [-180°; 180°]$



A✓Plot
A✓Shape

3.2 Using your calculator insert the following function $v(x) = \sin 2x$

3.2.1 Complete the following table: (1)

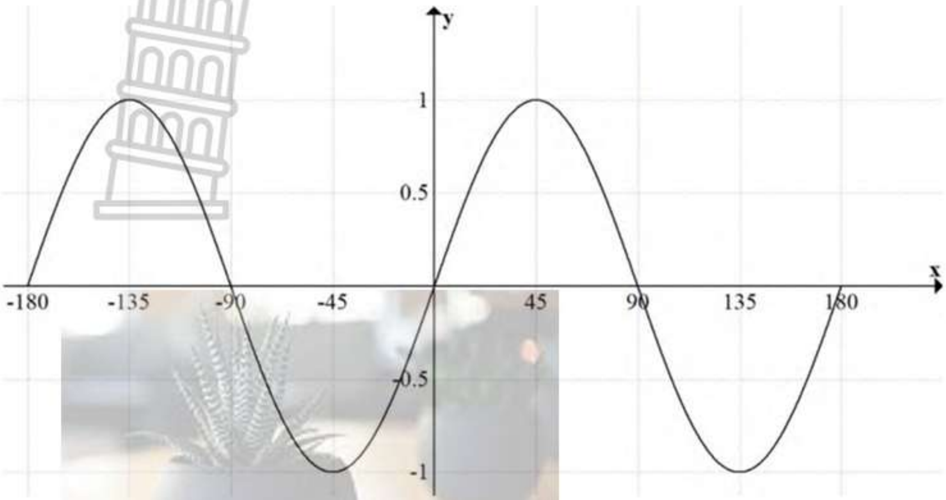
x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$

A✓

$v(x) = \sin 2x$	0	1	0	-1	0	1	0	-1	0

(2)

3.2.2 Use the table entries to sketch the graph of $v(x) = \sin 2x$ where $x \in \left[-180^{\circ}; 180^{\circ}\right]$



A✓Plot
A✓Shape

(2)

3.3 What general conclusion can make from u and v

They are the same graph

A✓
A✓

PART 3: APPLICATION OF COMPOUND AND DOUBLE ANGLE IDENTITIES (8 Marks)

QUESTION 4

4.1	L.H.S. $= \dfrac{2\cos^2\beta - 1 + 1}{\cos\beta}$	✓ A $2\cos^2\beta - 1$
	$= \dfrac{2\cos^2\beta}{\cos\beta}$	
	$= 2\cos\beta$	✓ CA Simplification $(2\cos\beta)$
	R.H.S. $= 2\sin\beta.\dfrac{\cos\beta}{\sin\beta}$	✓ A $\frac{\cos\beta}{\sin\beta}$
	$= 2\cos\beta$	✓ CA Simplification $(2\cos\beta)$
	$\therefore$ L.H.S.=R.H.S.	**(4)**

4.2	$\dfrac{\sin(60^\circ + 45^\circ)}{\cos(45^\circ - 30^\circ)} = \dfrac{\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ}{\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ}$	✓A $\begin{Bmatrix} \sin 105^\circ = \sin(60^\circ + 45^\circ) \\ \cos 15^\circ = \cos(45^\circ - 30^\circ) \end{Bmatrix}$
	$= \dfrac{(\frac{\sqrt{3}}{2})(\frac{1}{\sqrt{2}}) + (\frac{1}{2})(\frac{1}{\sqrt{2}})}{(\frac{1}{\sqrt{2}})(\frac{\sqrt{3}}{2}) + (\frac{1}{\sqrt{2}})(\frac{1}{2})}$	✓A $\left\{\dfrac{\sin 6\ ^\circ \cos 4\ ^\circ + \cos 60^\circ \sin 45^\circ}{\cos 4\ ^\circ \cos\ \ ^\circ + \sin 4\ ^\circ \sin 30^\circ}\right.$ ✓A $\left\{\dfrac{(\frac{\sqrt{3}}{2})(\frac{1}{\sqrt{2}}) + (\frac{1}{2})(\frac{1}{\sqrt{2}})}{(\frac{1}{\sqrt{2}})(\frac{\sqrt{3}}{2}) + (\frac{1}{\sqrt{2}})(\frac{1}{2})}\right.$
	$= \dfrac{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}}{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}}$	✓CA Answer (1) **(4)**
	$= 1$	
		GRAND TOTAL: 50