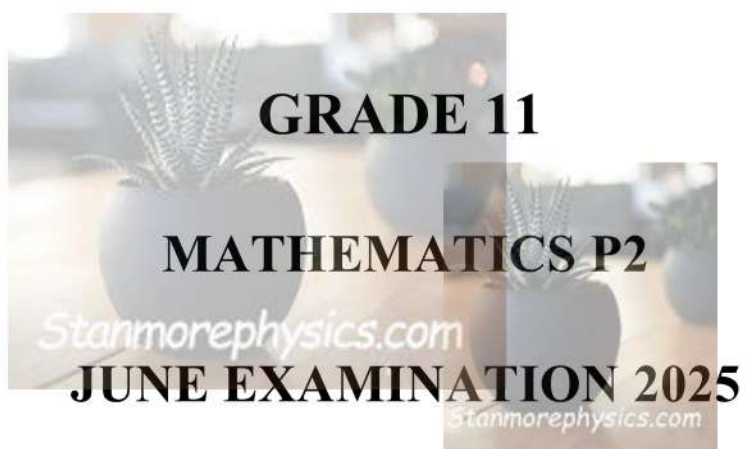




NATIONAL SENIOR CERTIFICATE



MARKS : 100

TIME: : 2 hr

This question paper consists of 8 pages.

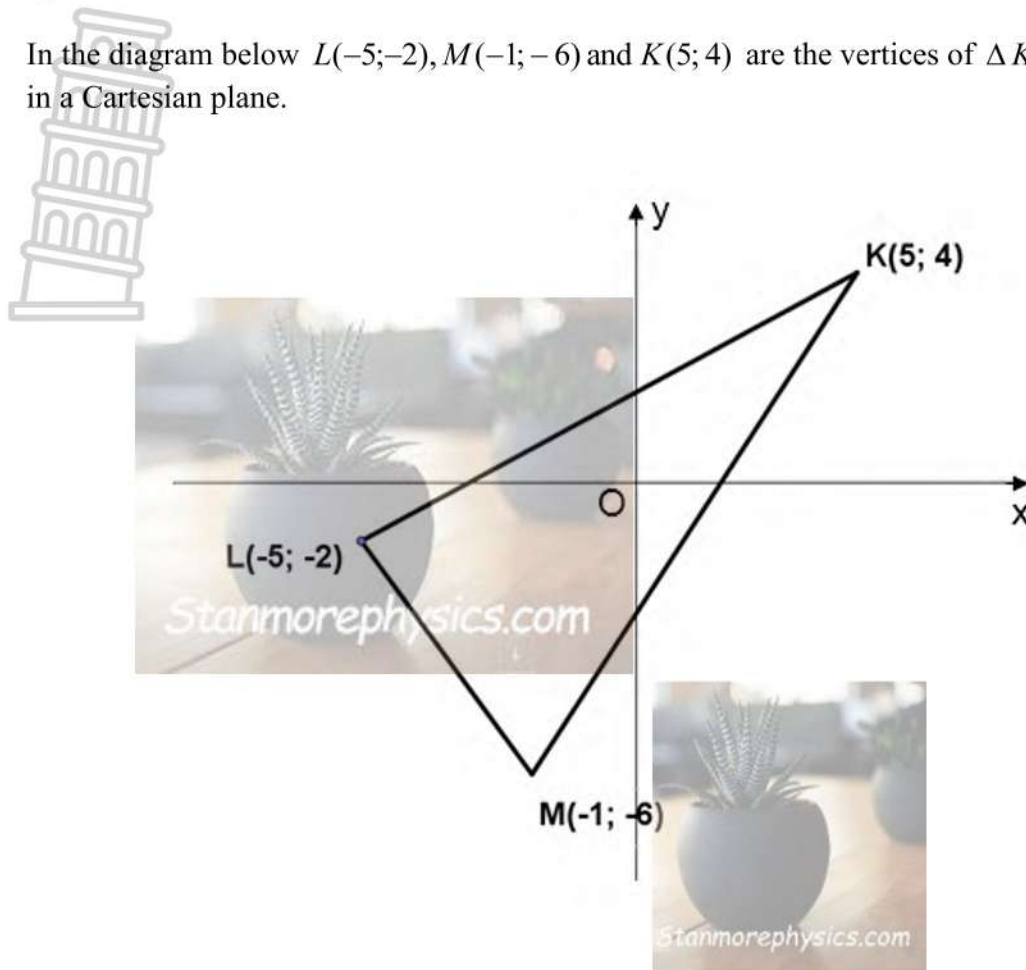
INSTRUCTIONS AND INFORMATION

1. This question paper consists of 5 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary round off your answers to TWO decimal places unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and no graphical) unless stated otherwise.
8. Write neatly and legibly.



QUESTION 1

In the diagram below $L(-5; -2)$, $M(-1; -6)$ and $K(5; 4)$ are the vertices of $\triangle KLM$ in a Cartesian plane.



- 1.1 Calculate the coordinates of N, the midpoint of MK (2)
- 1.2 Determine the gradient of LM (2)
- 1.3 Determine the equation of the line NP parallel to LM passing through N. (3)
- 1.4 Determine the angle of inclination of LM (2)
- 1.5 Show that $\angle KLM = \angle KML$ (5)
- 1.6 Show that $LM = 2NP$, where P is on KL (6)

[20]

QUESTION 2

2.1 If $8 \tan A = -15$ and $\sin A > 0$, determine

Determine without the use of a calculator and with aid of a diagram the value of $\cos A$ (5)

2.2 If $\sin 54^\circ = p$, express the following in terms of p without using a calculator.

2.2.1 $\sin(-126^\circ)$ (4)

2.2.2 $\tan 234^\circ$ (2)

2.3 Simplify without the use of a calculator

2.3.1 $\frac{\cos(180^\circ - x) \cdot \tan(-x) \cdot \sin^2(90^\circ - x)}{\sin(180^\circ - x)} + \sin^2 x$ (7)

2.3.2 $\sin^2 23^\circ + \sin^2 67^\circ$ (2)

[20]**QUESTION 3**

3.1 Determine the general solution of

$$2 \sin(x + 15^\circ) = \cos 203,5^\circ \quad (5)$$

3.2 Solve without using a calculator

$$\cos(2x - 10^\circ) = \sin(x + 40^\circ) \text{ where } x \in [-180^\circ; 180^\circ] \quad (7)$$

3.3 Consider the following identity :

$$\frac{1 + 2 \sin \theta \cdot \cos \theta}{\sin \theta + \cos \theta} = \sin \theta + \cos \theta$$

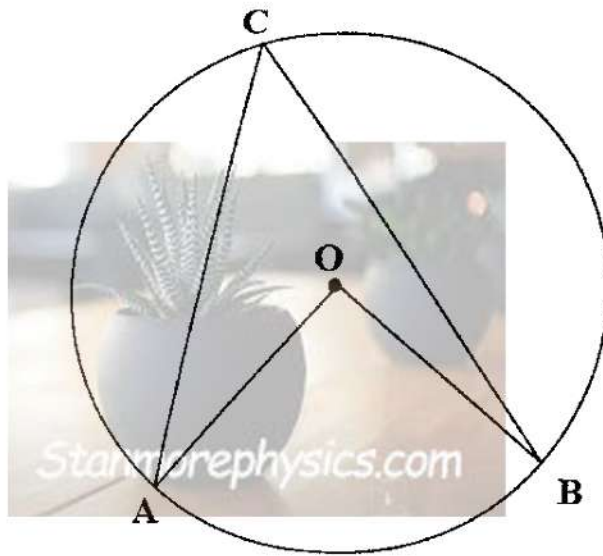
3.3.1 Prove the identity (4)

3.3.2 For which values of x is the identity undefined for $x \in (-180^\circ; 180^\circ)$ (5)

[21]

QUESTION 4

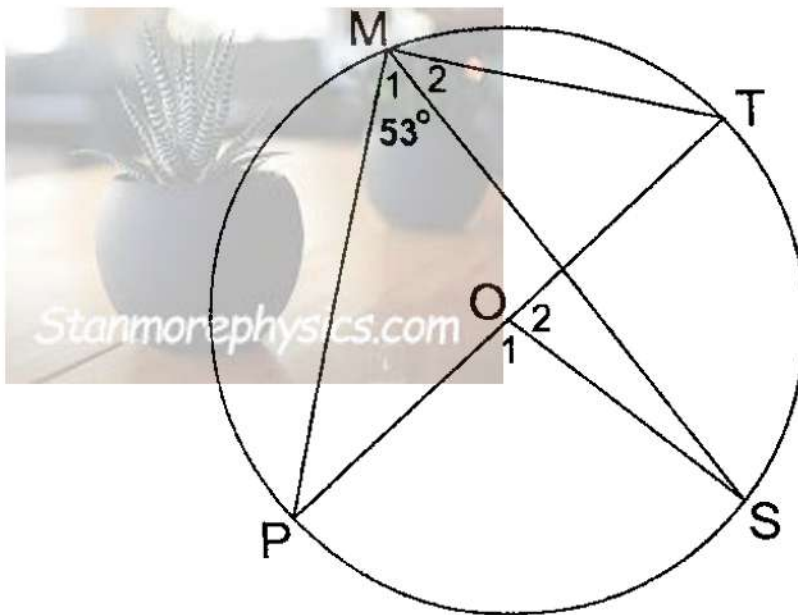
4.1 In the diagram below A, B and C are points on the circle



Use the diagram to prove the Theorem which states that : if O is the centre of the circle, then $\hat{AOB} = 2\hat{C}$

(6)

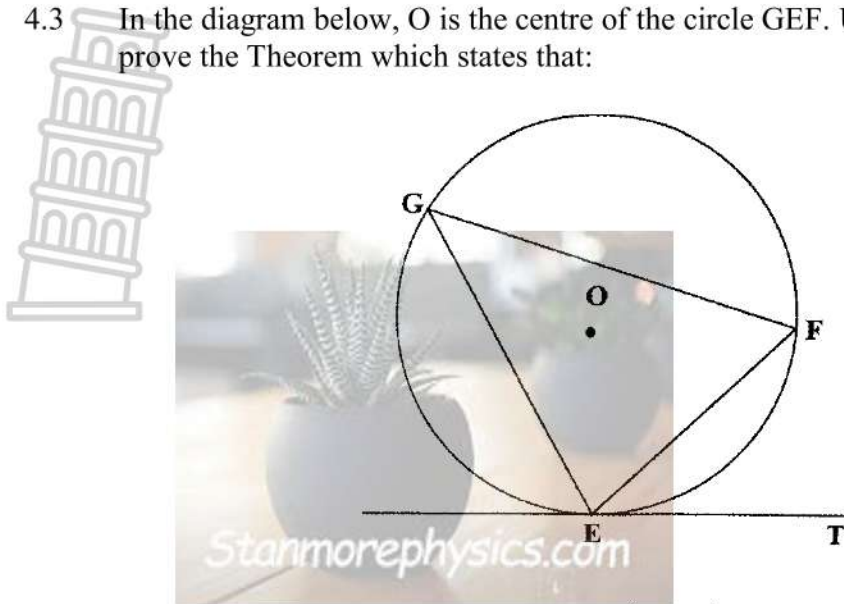
4.2 M, P, S and T are points on a circle with centre O, PT is a diameter. MP, MT, MS are drawn. $\hat{M} = 53^\circ$



Determine with reasons, the size of \hat{O}_2

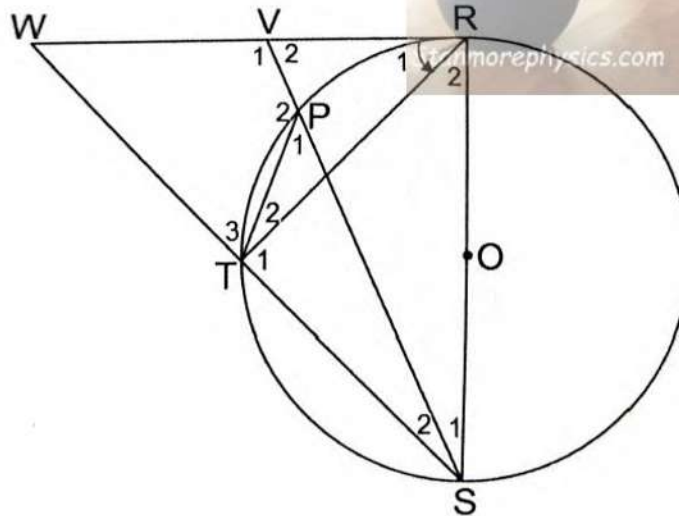
(4)

4.3 In the diagram below, O is the centre of the circle GEF. Use the diagram to prove the Theorem which states that:



If ET is a tangent to the circle, then $\hat{FET} = \hat{G}$ (7)

4.4 RS is a diameter of the circle with centre O. Chord ST is produced to W. Chord SP produced Meets the tangent RW at V. $\hat{R}_1 = 55^\circ$.



Calculate the size of :

4.4.1 \hat{WRS} (1)

4.4.2 \hat{W} (2)

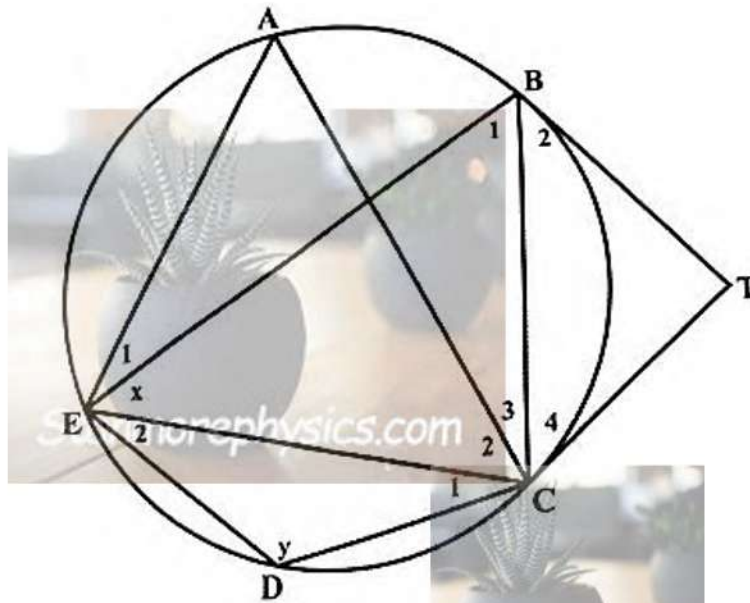
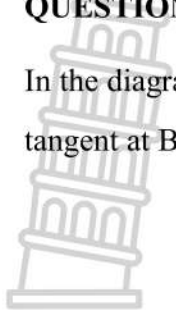
4.4.3 \hat{P}_1 (3)

4.4.4 Prove that: $\hat{V}_1 = \hat{PTS}$ (4)

[27]

QUESTION 5

In the diagram below, a circle is drawn passing through A, B, C, D and E. The tangent at B and C meet at T. $\hat{BEC} = x$ and $\hat{D} = y$.



Express each of the following in terms of x and/or y .

5.1 \hat{B}_2 (2)

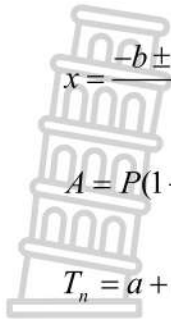
5.2 \hat{C}_4 (3)

5.3 \hat{T} (3)

5.4 \hat{A} (2)

5.5 \hat{B}_1 (2)
[12]

INFORMATION SHEET: MATHEMATICS



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c \quad y - y_1 = m(x - x_1)$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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GRADE 11

MATHEMATICS P2

JUNE EXAMINATION 2025

Stanmorephysics.com

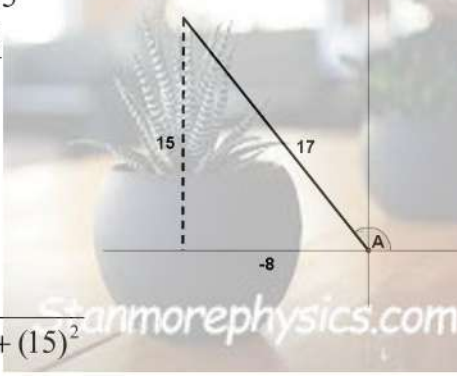
MARKING GUIDELINE

Stanmorephysics.com

MARKS : 100

This Marking Guideline consists of 7 pages

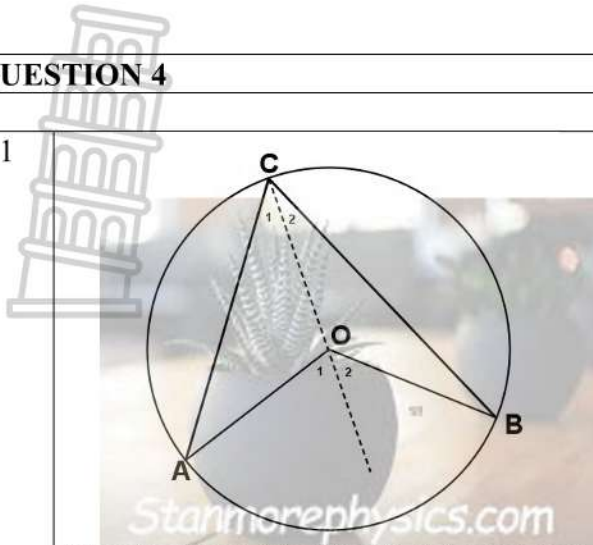
QUESTION 1		
1.1	$N\left(\frac{5-1}{2}; \frac{4-6}{2}\right)$ $= (2; -1)$	✓ x – coordinate ✓ y – coordinate (2)
1.2	$m_{ML} = \frac{-6+2}{-1+5}$ $= -1$	✓ subst. in corr. orm ✓ answer (2)
1.3	$m_{NP} = m_{ML} \quad (\text{NP// ML})$ $= -1$ $y = -x + c$ $-1 = -(2) + c$ $c = 1$ $y = -x + 1$	✓ $m_{NP} = -1$ ✓ subst. ✓ answer (3)
1.4	$\tan \theta = -1$ $\theta = 135^\circ$	✓ $\tan \theta = -1$ ✓ answer (2)
1.5	$LK = \sqrt{(5+5)^2 + (4+2)^2}$ $= 2\sqrt{34}$ $MK = \sqrt{(5+1)^2 + (4+6)^2}$ $= 2\sqrt{34}$ $\therefore LK = MK$ $\angle KLM = \angle KML \quad (\text{L's opp.} = \text{sides})$	✓ subst. in corr. Form ✓ LK value ✓ subst. in corr. Form ✓ MK value ✓ $LK = MK$ (5)
1.6	$LM = \sqrt{(-5+1)^2 + (-2+6)^2}$ $= 4\sqrt{2}$ <p>But $P\left(\frac{5-5}{2}; \frac{4-2}{2}\right)$ (mid-point theorem, PN//ML)</p> $P(0;1)$ $\therefore PN = \sqrt{(2-0)^2 + (-1-1)^2}$ $= 2\sqrt{2}$ $\therefore LM = 2PN$	✓ subst in corr. Form. ✓ LM value ✓ Reason ✓ P coordinates ✓ subst. in corr. form. ✓ PN value (6)
		[20]

QUESTION 2		
2.1	$8 \tan A = -15$ $\tan A = -\frac{15}{8}$  $r = \sqrt{(-8)^2 + (15)^2}$ $= 17$ $\cos A = -\frac{8}{17}$	<ul style="list-style-type: none"> ✓ dividing by 8 ✓ r - value ✓✓ diagram ✓ cos A - value
2.2.1	$\sin 54^\circ = p$ $x = \sqrt{1 - p^2}$ $\sin(-126^\circ)$ $= -\sin 126^\circ$ $= -\sin 54^\circ$ $= -p$	<ul style="list-style-type: none"> ✓ x - value ✓ - sin 126 ✓ - sin 54° ✓ answer
2.2.2	$\tan 234^\circ$ $= \tan 54^\circ$ $= \frac{p}{\sqrt{1 - p^2}}$	<ul style="list-style-type: none"> ✓ tan 54° ✓ answer
2.3.1	$\frac{\cos(180^\circ - x) \cdot \tan(-x) \cdot \sin^2(90^\circ - x)}{\sin(180^\circ - x)} + \sin^2 x$ $= \frac{(-\cos x) \cdot (-\tan x) \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \frac{-\cos x \cdot -\frac{\sin x}{\cos x} \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \cos^2 x + \sin^2 x$ $= 1$	<ul style="list-style-type: none"> ✓ - cos x ✓ - tan x ✓ cos² x ✓ sin x ✓ - $\frac{\sin x}{\cos x}$ ✓ simplification ✓ 1
2.3.2	$\sin^2 23^\circ + \sin^2 67^\circ$ $= \sin^2 23^\circ + \cos^2 23^\circ$ $= 1$	<ul style="list-style-type: none"> ✓ cos² 23° ✓ 1
[20]		

QUESTION 3		
<p>3.1</p>	$2 \sin(x + 15^\circ) = \cos 203,5^\circ$ $2 \sin(x + 15^\circ) = -0.92$ $\sin(x + 15^\circ) = -0.46 \quad RA = 27.39^\circ$ $x + 15^\circ = 180^\circ + 27.39^\circ + k.360^\circ$ <p>or</p> $x + 15^\circ = 360^\circ - 27.39^\circ + k.360^\circ \quad k \in Z$ $x = 192.39^\circ + k.360^\circ \text{ or } x = 317.61^\circ + k.360^\circ$ $k \in Z$ <p style="text-align: center;">OR</p> $2 \sin(x + 15^\circ) = \cos 203,5^\circ$ $2 \sin(x + 15^\circ) = -0.92$ $\sin(x + 15^\circ) = -0.46$ $x + 15^\circ = \sin^{-1}(-0.46) + k.360^\circ \text{ or}$ $x + 15^\circ = 180^\circ - \sin^{-1}(-0.46) + k.360^\circ$ $x = -42.39^\circ + k.360^\circ \text{ or } x = 192.39^\circ + k.360^\circ$ $k \in Z$	$\checkmark -0.92$ $\checkmark -0.46$ $\checkmark x = 192.39^\circ + k.360^\circ$ $\checkmark x = 317.61^\circ + k.360^\circ$ $\checkmark k \in Z$ <p style="text-align: center;">OR</p> $\checkmark -0.92$ $\checkmark -0.46$ $\checkmark x = -42.39^\circ + k.360^\circ$ $\checkmark x = 192.39^\circ + k.360^\circ$ $\checkmark k \in Z$ <p style="text-align: right;">(5)</p>
<p>3.2</p>	$\cos(2x - 10^\circ) = \sin(x + 40^\circ)$ $\cos(2x - 10^\circ) = \cos[90^\circ - (x + 40^\circ)]$ $\cos(2x - 10^\circ) = \cos(50^\circ - x)$ $2x - 10^\circ = 50^\circ - x + k,360^\circ \text{ or}$ $2x - 10^\circ = -50^\circ + x + k,360^\circ \quad k \in Z$ $3x = 60^\circ + k,360^\circ \text{ or } x = -40^\circ + k.360^\circ \quad k \in Z$ $x = 20^\circ + k.120^\circ$ $x = -100^\circ; -40^\circ, 20^\circ; 140^\circ$	$\checkmark \cos[90^\circ - (x + 40^\circ)]$ $\checkmark 2x - 10^\circ = 50^\circ - x + k,360^\circ$ $\checkmark 2x - 10^\circ = -50^\circ + x + k,360^\circ$ $\checkmark \text{simplification}$ $\checkmark x = -100^\circ$ $\checkmark x = 20^\circ$ $\checkmark x = -40^\circ$ <p style="text-align: right;">(7)</p>
<p>3.3.1</p>	$L.H.S = \frac{1 + 2 \sin \theta \cdot \cos \theta}{\sin \theta + \cos \theta}$ $= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cdot \cos \theta}{\sin \theta + \cos \theta}$ $= \frac{(\sin \theta + \cos \theta)(\sin \theta + \cos \theta)}{\sin \theta + \cos \theta}$ $= \sin \theta + \cos \theta = RHS$	$\checkmark \text{identity}$ $\checkmark \checkmark \text{factorizing}$ $\checkmark \text{answer}$ <p style="text-align: right;">(4)</p>
<p>3.3.2</p>	$\sin \theta + \cos \theta = 0$ $\sin \theta = -\cos \theta$ $\frac{\sin \theta}{\cos \theta} = -1$ $\tan \theta = -1$ $\theta = \tan^{-1}(-1) + 180^\circ \cdot k$ $\theta = -45^\circ, 135^\circ,$	$\checkmark \text{transposing}$ $\checkmark \text{dividing by cos}$ $\checkmark \text{identity}$ $\checkmark -45^\circ$ $\checkmark 135^\circ$ <p style="text-align: right;">(5)</p>
[21]		

QUESTION 4

4.1



Construction : join C to O and produced

In $\triangle AOC$

$$\hat{C}_1 = \hat{A} \quad (OC = OA, \text{ radii})$$

$$\hat{O}_1 = \hat{C}_1 + \hat{A} \quad (\text{ext. } \angle \text{ of a } \triangle)$$

$$\therefore \hat{O}_1 = 2\hat{C}_1$$

In $\triangle BOC$

$$\hat{C}_2 = \hat{B} \quad (OC = OB, \text{ radii})$$

$$\hat{O}_2 = \hat{C}_2 + \hat{B} \quad (\text{ext. } \angle \text{ of a } \triangle)$$

$$\therefore \hat{O}_2 = 2\hat{C}_2$$

$$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$$

$$\therefore \hat{O} = 2\hat{C}$$

$$\hat{AOB} = 2\hat{C}$$

✓ construction



✓ S/R

✓ S

(6)

4.2

$$\hat{O}_1 = 106^\circ \quad (\angle \text{ at centre} = 2 \times L \text{ at circum.})$$

$$\therefore \hat{O}_2 = 74^\circ \quad (\angle \text{'s on a str, line})$$

OR

$$\hat{M}_2 = 37^\circ \quad (\angle \text{ subt. by diameter})$$

$$\therefore \hat{O}_2 = 74^\circ \quad (\angle \text{ at centre} = 2 \times \angle \text{ at circum.})$$

✓ S ✓ R

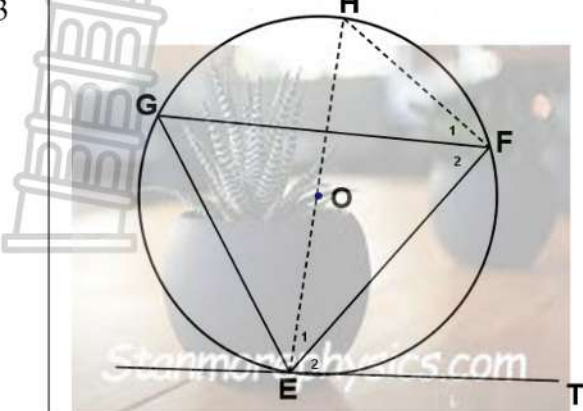
✓ S ✓ R

✓ S ✓ R

✓ S ✓ R

(4)

(4)

<p>4.3</p> 		
	<p>Construction : draw diam. EH then join H to F</p> $\hat{F}_1 + \hat{F}_2 = 90^\circ \quad (\angle \text{ subt. by diam.})$ $\hat{E}_1 + \hat{H}_2 = 90^\circ \quad (\text{sum of L's in a } \Delta)$ <p>Also $\hat{E}_1 + \hat{E}_2 = 90^\circ \quad (\text{tan } \perp \text{ radius})$</p> $\hat{E}_1 + \hat{H}_2 = \hat{E}_1 + \hat{E}_2$ $\hat{E}_2 = \hat{H}_2$ <p>but $\hat{G} = \hat{H}_2 \quad (\angle \text{s on the same segm.})$</p> $F \hat{E} T = \hat{G}$	<p>✓ constr.</p> <p>✓ S/R</p> <p>✓ S/R</p> <p>✓ S/R</p> <p>✓ S</p> <p>✓ S ✓R</p> <p>(7)</p>
<p>4.4.1</p>	$W \hat{R} S = 90^\circ \quad (\text{tan } \perp \text{ radius})$	<p>✓ S/R</p> <p>(1)</p>
<p>4.4.2</p>	$\hat{T}_1 = 90^\circ \quad (\angle \text{ subt. by diam.})$ $W + \hat{R}_1 = \hat{T}_1 \quad (\text{ext. } \angle \text{ of } \Delta)$ $\hat{W} = 90^\circ - 55^\circ$ $= 35^\circ$	<p>✓ S/R</p> <p>✓ answer</p> <p>(2)</p>
<p>4.4.3</p>	$\hat{P}_1 = \hat{R}_2 \quad (\angle \text{'s a the same segm.})$ $\hat{R}_2 = 90^\circ - 55^\circ$ $= 35^\circ$ $\therefore \hat{P}_1 = 35^\circ$	<p>✓ S/R</p> <p>✓ S</p> <p>✓ S</p> <p>(3)</p>
<p>4.4.4</p>	$\hat{P}_1 = W = 35^\circ \quad (\text{proved})$ <p>$\therefore PVWT$ is a cyclic quad. (conv., ext $\angle =$ int. opp. \angle)</p> $\hat{V}_1 = \hat{P}_1 \quad (\text{ext. } \angle \text{ of a cyclic quad.})$	<p>✓ S</p> <p>✓ S ✓R</p> <p>✓ R</p> <p>(4)</p>
		<p>[27]</p>

QUESTION 5		
5.1	$\hat{B}_2 = x$ (tan – chord Theorem)	✓ S ✓ R (2)
5.2	$BT = CT$ (tang. from same point)	✓ S ✓ R
	$\hat{C}_4 = x$ (∠'s opp. = sides)	✓ S/R (3)
5.3	$\hat{T} = 180^\circ - \hat{B}_2 - \hat{C}_4$ (sum of ∠'s of a Δ)	✓ S ✓ R
	$\therefore \hat{T} = 180^\circ - 2x$	✓ S (3)
5.4	$\hat{A} = 180^\circ - y$ (opp. ∠'s of a cyclic quad.)	✓ S ✓ R (2)
5.5	$\hat{B}_1 = 180^\circ - y$ (opp. ∠'s of a cyclic quad.)	✓ S ✓ R (2)
	OR	OR
	$\hat{B}_1 = 180^\circ - y$ (∠ on the same segment)	✓ S ✓ R (2)
		[12]

TOTAL: 100 MARKS