education
Department of Education
FREE STATE PROVINCE

GRADE 11

MATHEMATICS

2026 JUNE EXAMINATION

PAPER 2

MARKS: 100

HOURS: 2 HOURS

This question paper consists of 7 pages and a formula sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 5 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless otherwise stated.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are not necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Complete the following identity: $\sin^2 x = \ldots\ldots\ldots\ldots$ (1)

1.2 Hence, simplify the following identity: $\dfrac{(1-\sin x)(1+\sin x)}{\cos x}$ (3)

1.3 Show that: $\dfrac{1-2\sin x\cos x}{\cos x-\sin x}=\cos x-\sin x$ (2)

1.4 Evaluate without using a calculator: $\dfrac{\sin 240^\circ.\cos 330^\circ}{\tan(-225)}$ (5)

1.5 Simplify the following expression: (7)

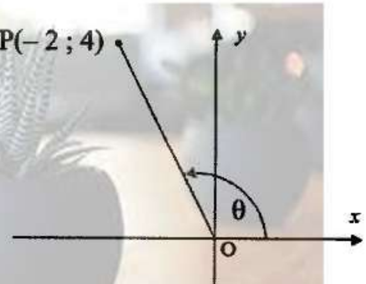
$$\frac{\cos\left(180^0-x\right).\sin\left(x-90^0\right)-1}{\tan^2 x.\cos x.\cos(-x)}$$

[18]

QUESTION 2

2.1 In the diagram below, P has coordinates (– 2 ; 4). θ is an obtuse angle between the x-axis and OP.

Determine, **without the use of a calculator**, the numerical values of the following (leave answers in surd form where necessary),



2.1.1 tan θ (3)

2.1.2 OP (2)

2.1.3 $\sqrt{5}\sin\theta$ (2)

2.2 If $\cos 47^\circ = p$, express the following in terms of p:

2.2.1 $\sin 47^\circ$ (2)

2.2.2 $\tan^2 317^\circ$ (3)

2.2.3 $\cos 133^\circ$ (2)

2.3 Determine the general solutions of the following questions (round off to one decimal place):

2.3.1 $\tan x = 8$ (4)

2.3.2 $2\cos(x+15^\circ) = -0,92$ (4)

2.3.3 $\sin 3x = 0.54$ (4)

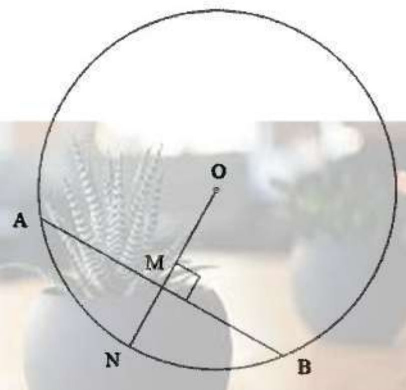
2.4 Solve for x for the specific domain: $\sin x = 3\cos x.\sin x$, for $x \in (-180^\circ;180^\circ)$ (7)

[33]

QUESTION 3

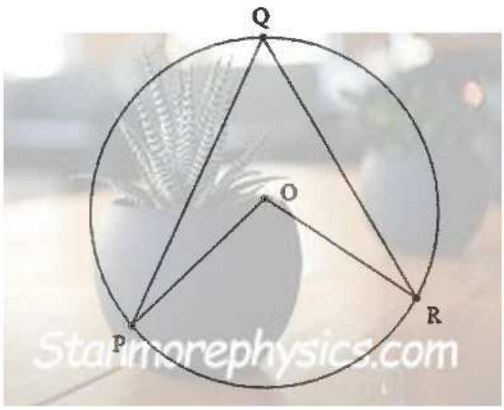
3.1 Complete the theorem:
The line segment drawn from the centre of the circle, perpendicular to the chord, ………….. the chord. (1)

3.2 In the diagram below, ON the radius of circle O, is perpendicular to chord AB at M. ON = 6 cm and AB = 8 cm. Calculate the length of MN (round off to two decimal places where necessary).



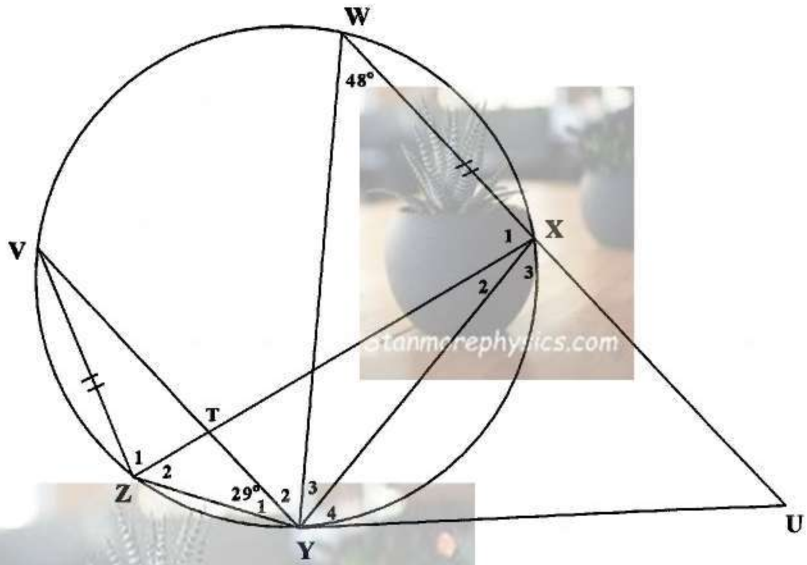
(5)

3.3 Prove the theorem: The angle subtended by an arc at the centre of a circle is twice the size of the angle subtended by the same arc at the circle.



(5)

3.4 In the diagram below, V, W, X, Y and Z lie on the circle. WX meets tangent YU at U. VY intersect XZ at T. VZ = WX, $\hat{W} = 48^\circ$ and $\hat{Y}_1 = 29^\circ$.



3.4.1 Determine the following angles giving reasons:

(a) $\hat{Y}_4$ (2)

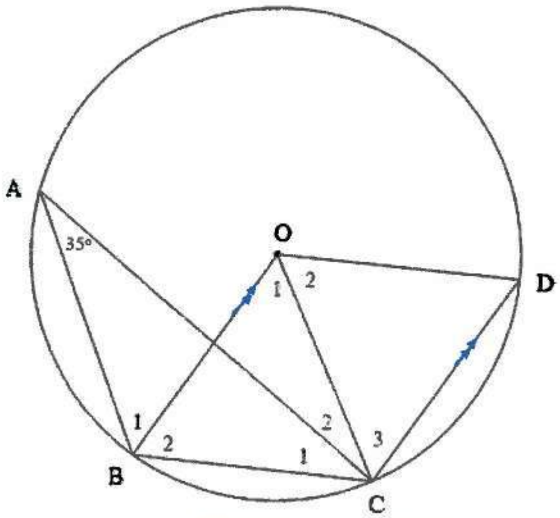
(b) $\hat{Z}_2$ (2)

(a) $\hat{Y}_3$ (2)

[17]

QUESTION 4

4.1 In the diagram below, O is the centre of the circle and points A, B, C and D lie on the circumference of the circle. OB//DC and $\hat{A} = 35°$. Calculate the following angles giving reasons.

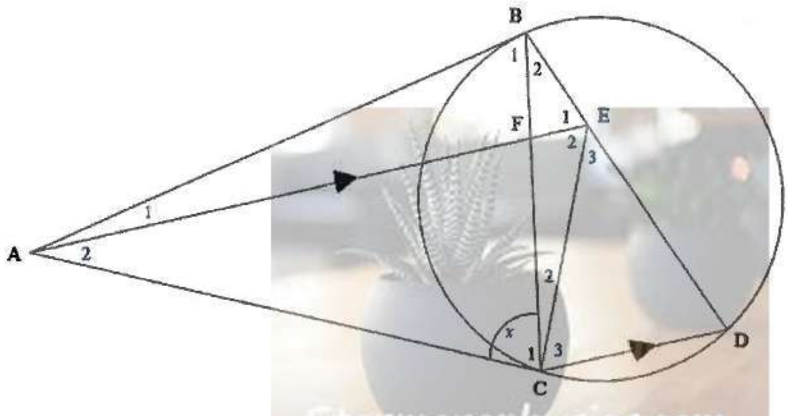


4.1.1 $\hat{O}_1$ (2)

4.1.2 $\hat{B}_2$ (4)

4.1.3 $\hat{O}_2$ (3)

4.2 In the diagram below, AB and AC are the tangents to the circle at the points A and B. AE//CD with E on BD. $A\hat{C}B = x$.



4.2.1 Name, with reasons, three other angles each equal to x. (6)

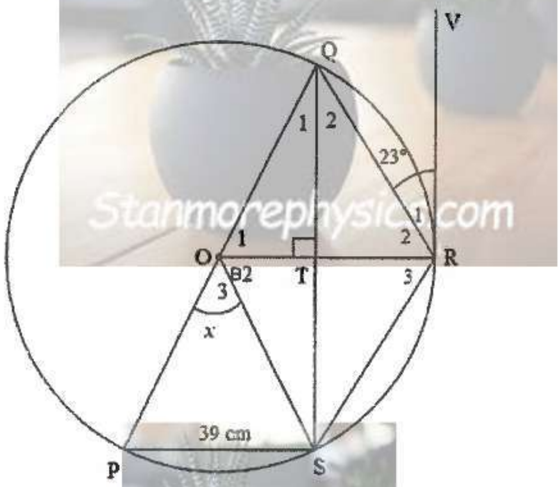
4.2.2 Prove that ABEC is a cyclic quadrilateral. (2)

4.2.3 Prove that ΔCDE is an isosceles triangle. (4)

[21]

QUESTION 5

5.1 In the diagram below, PQ is the diameter of circle O. OR ⊥ QS at T and RV is a tangent to the circle at R. S is the point on the circumference.
$Q\hat{R}V = 23°$
$P\hat{O}S = x$
PS = 39 cm



5.1.1 Prove that: $\Delta QTR \equiv \Delta STR$. (3)

5.1.2 Determine the sizes of the following angles, giving reasons:

(a) $\hat{R}_2$ (2)

(b) $\hat{P}$ (2)

5.1.3 Determine the size of $\hat{O}_1$ in terms x (give reasons). (3)

5.1.4 Calculate the length of OT. (1)
[11]

TOTAL: 100

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}\ ; r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ; -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

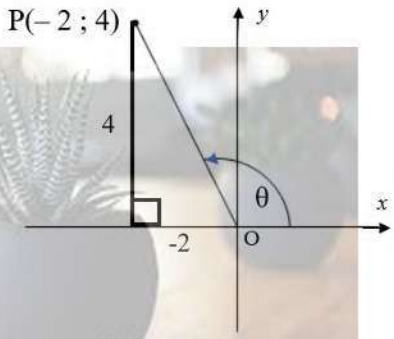
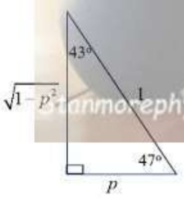
$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

QUESTION 1

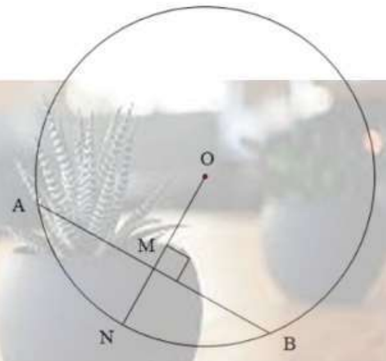
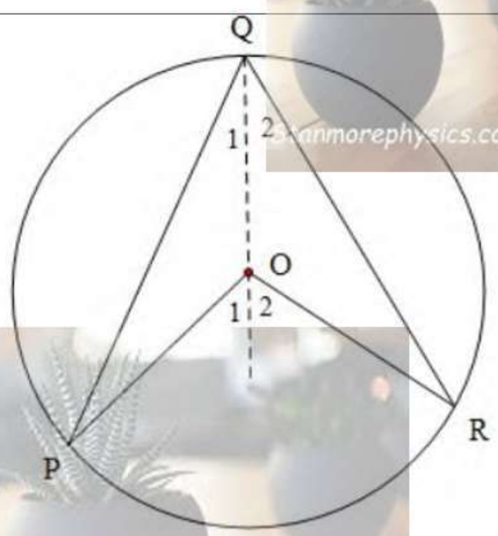
1.1	$\sin^2 x = 1 - \cos^2 x$	✓answer (1)
1.2	$\frac{(1-\sin x)(1+\sin x)}{\cos x}$ $= \frac{1-\sin^2 x}{\cos x}$ $= \frac{\cos^2 x}{\cos x}$ $= \cos x$	✓product ✓identity ✓answer (3)
1.3	$\frac{1-2\sin x\cos x}{\cos x - \sin x} = \cos x - \sin x$ $\text{LHS} = \frac{\cos x^2 - 2\sin x.\cos x + \sin^2 x}{\cos x - \sin x}$ $= \frac{(\cos x - \sin x)(\cos x - \sin x)}{\cos x - \sin x}$ $= \cos x - \sin x$	✓identity ✓factorization (2)
1.4	$\frac{\sin 240^o.\cos 330^o}{\tan(-225)}$ $= \frac{-\sin 60^o.\cos 30^o}{-\tan 45}$ $= \frac{-\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}}{-1}$ $= \frac{3}{4}$	✓ $-\sin 60^o$ ✓ $\cos 30^o$ ✓ $-\tan 45$ ✓substitution ✓answer (5)
1.5	$\frac{\cos(180^o - x).\sin(x-90^o)-1}{\tan^2 x.\cos x.\cos(-x)}$ $= \frac{-\cos x.(-\cos x)-1}{\tan^2 x.\cos x.\cos x}$ $= \frac{\cos^2 x - 1}{\frac{\sin^2 x}{\cos^2 x}.\frac{\cos^2 x}{1}}$ $= \frac{-\sin^2 x}{\sin^2 x}$ $= -1$	✓ $-\cos x.$ ✓ $-\cos x.$ ✓ $\cos x$ ✓ $\frac{\sin^2 x}{\cos^2}$ ✓identity ✓simplification ✓answer (7)
		[18]

QUESTION 2

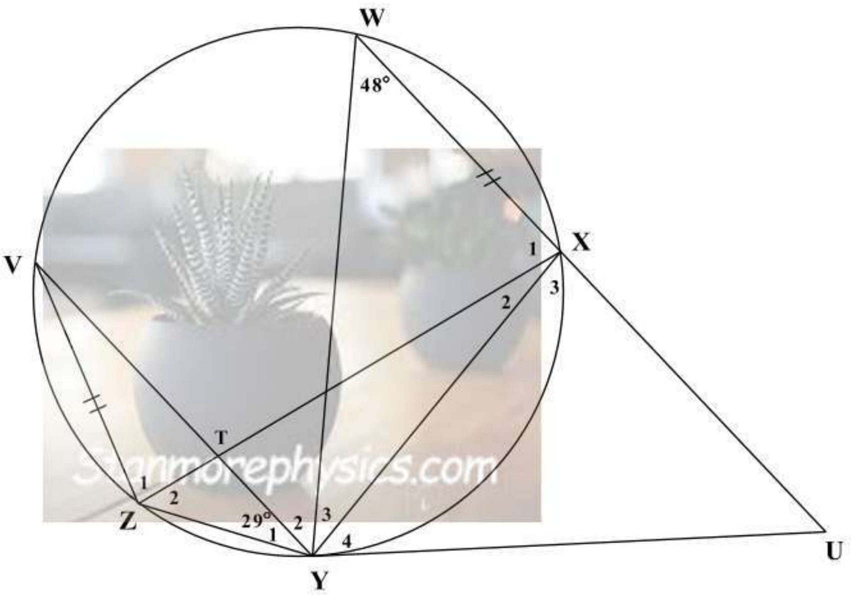
		
2.1.1	$\tan\theta = -\frac{4}{2} = -2$	✓substitute ✓answer (2)
2.1.2	$OP^2 = (4)^2 + (-2)^2$ $= 20$ $= \sqrt{20}$ $= 2\sqrt{5}$	✓pyth theorem ✓answer (2)
2.1.3	$\sqrt{5}\sin\theta = \sqrt{5}.\frac{4}{2\sqrt{5}}$ $= 2$	✓substitution ✓answer (2)
2.2.1	$1 = p^2 + y^2$ $y^2 = 1 - p^2$ $y = \sqrt{1-p^2}$ $\sin 47^o = \sqrt{1-p^2}$ 	✓substitution ✓value of y ✓answer (3)
2.2.2	$\tan^2 317^o = (-\tan 43^o)^2$ $= \left(\frac{p}{\sqrt{1-p^2}}\right)^2$ $= \frac{p^2}{1-p^2}$	✓reduction ✓substitution ✓answer (3)
2.2.3	$\cos 133^o = -\cos 47$ $= -p$	✓reduction ✓answer (2)
2.3.1	$\tan x = 8$ $ref\angle = \tan^{-1} 8$ $= 82{,}87^o$ I $x = 82{,}87^o + k.360^o$ III $x = 180^o + 82{,}9^o + k.360^o$ $= 262{,}87^o$ $k \in \mathbb{Z}$	✓ref $\angle$ ✓first quad ✓third quad ✓ $k \in \mathbb{Z}$ (4)

2.3.2	$2\cos(x+15^o)=-0,92$ $\cos(x+15^o)=-\dfrac{0,92}{2}$ $ref\angle=\cos\left(\dfrac{0.92}{2}\right)$ $=62,61^o$ II / III $x+15^o=180^o-62,61^o+k.360^o$ / $x+15=180^0+62,61^o+k.360^o$ $x=102.39^o+k.360^o$ / $x=227,61^o+k.360^o$ $k\in\mathbb{Z}$	✓std form ✓ref $\angle$ ✓second quad ✓third quad (4)
2.3.3	$\sin 3x=0.54$ $ref\angle=\sin^{-1}0,54$ $=32,68^o$ I / II $3x=32,68^o+k.360^o$ / $3x=180^o-32.68+k.360^o$ $x=10,89^o+k.120^o$ / $x=49,11^o+k.120^o$ $k\in\mathbb{Z}$	✓ref $\angle$ ✓first quad ✓second quad ✓ $k\in\mathbb{Z}$ (4)
2.4	$\sin x=3\cos x.\sin x$ $\sin x-3\cos x.\sin x=0$ $\sin x(1-3\cos x)=0$ $\sin x=0$ / $\cos x=\dfrac{1}{3}$ $ref\angle=0^o$ $ref\angle=70,53^o$ $x=k.360^o$ / $k.180^o$ or I / IV $x=70,53^o+k.360^o$ / $x=360^o-70,53+k.360^o$ $x=289,47^o+k.360^o$ $k\in\mathbb{Z}$ $x\in\{-70,53^o;0^o;70,53^o\}$	✓transposing ✓factorising ✓both std forms ✓both ref $\angle$'s ✓solutions for sin x ✓solutions for cos x ✓all x-value (7)
		[33]

QUESTION 3

3.1	bisect	✓answer (1)
3.2	 	
3.2.1	AM = MB = 4 cm (line from centre ⊥ to chord) $0B^2 = MB^2 + 0M^2$ $(6)^2 = (4)^2 + OM^2$ (Pythagoras theorem) $OM^2 = 36 - 16$ $OM^2 = 20$ $OM = 4{,}47$ MN = 6 – 4,47 = 1,53 cm	✓S ✓R ✓substitution ✓value of OM ✓value of MN (5)
3.3	 	
	Construction QO and produce. $\hat{O}_1 = \hat{Q}_1 + \hat{P}$ (ext ∠ of Δ) $\hat{O}_1 = \hat{Q}_1 + \hat{Q}_1$ (∠'s opp equal sides, OQ and OP are radii) $\hat{O}_1 = 2\hat{Q}_1$ Similarly $\hat{O}_2 = 2\hat{Q}_2$ $\hat{O}_1 + \hat{O}_2 = 2(\hat{Q}_1 + \hat{Q}_2)$ $\therefore P\hat{O}R = 2 \times P\hat{Q}R$	✓construction ✓S/R ✓S/R ✓simplification ✓conclusion (5)

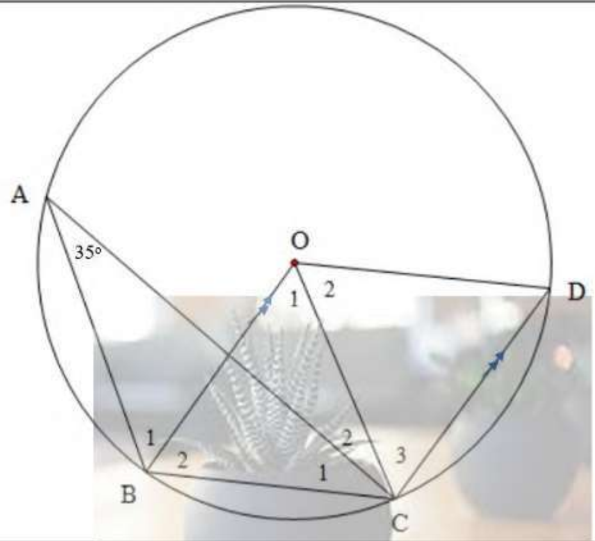
3.4



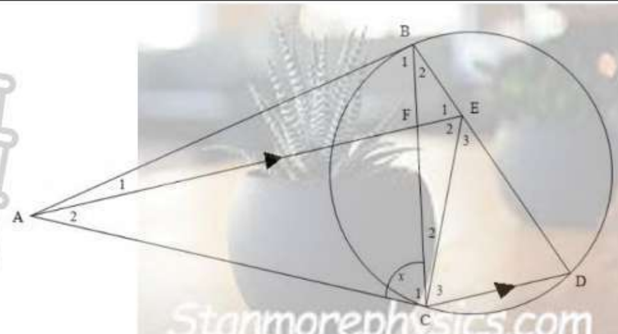
3.4.1(a)	$\hat{Y}_4 = 48^\circ$ (tan chord theorem)	✓S ✓R (2)
3.4.1(b)	$\hat{Z}_2 = 48^\circ$ (tan chord theorem/ $\angle$'s in the same segment)	✓S ✓R (2)
3.4.1(c)	$\hat{Y}_3 = 29^\circ$ ($\angle$'s subtended by = chords)	✓S ✓R (2)
		[17]

QUESTION 4

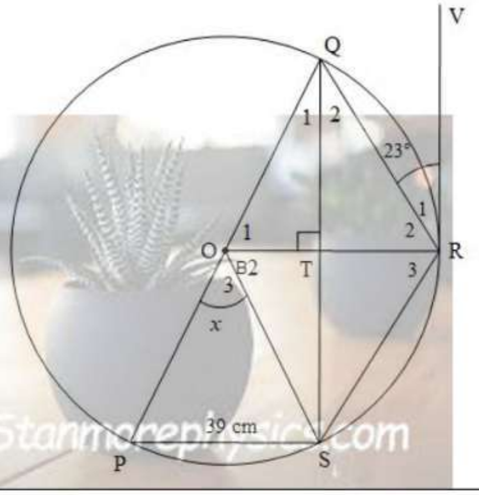
4.1



4.1.1	$\hat{O}_1 = 70^\circ$ ($\angle$at centre = 2 $\times$ $\angle$at the cirm)	✓S ✓R	(2)
4.1.2	$O_1 + \hat{B}_2 + \left(\hat{C}_1 + \hat{C}_2\right) = 180^\circ$ (sum of $\angle$'s of triangle) $\hat{B}_2 + \hat{B}_2 + 70^\circ = 180^\circ$ ($\angle$' opp = sides, radii) $2\hat{B}_2 = 110^\circ$ $\hat{B}_2 = 55^\circ$	✓S/R ✓S ✓R ✓answer	(4)
4.1.3	$\hat{O}_1 = \hat{C}_3 = 70^\circ$ (alt $\angle$'s, BO ∥ CD) $\hat{O}_2 + \hat{D} + \hat{C}_3 = 180^\circ$ ($\angle$'s of a triangle) $\hat{O}_2 + 70^\circ + 70^\circ = 180^\circ$ ($\angle$'s opp = sides) $\hat{O}_2 = 40^\circ$	✓S/R ✓S/R ✓answer	(3)

4.2	 	
4.2.1	$\hat{D} = x$ (tan chord theorem) $\hat{E}_1 = x$ (corresponding $\angle$'s, $AE \parallel CD$) $\hat{B}_1 = x$ ($\angle$'s opp = sides, AB = AC, tangents from same point)	✓S ✓R ✓S ✓R ✓S ✓R (6)
4.2.2	$\hat{E}_1 = \hat{C}_1 = x$ (proved) $\therefore$ ABEC is a cyclic quad ($\angle$'s subtended by the same line are =)	✓S ✓R (2)
4.2.3	$\hat{B}_1 = \hat{E}_2$ ($\angle$'s in the same segment) $= \hat{C}_3$ (alt $\angle$'s, $AE \parallel CD$) But $\hat{D} = \hat{B}_1$ (proved) $\therefore \hat{C}_3 = \hat{D}$ $\therefore \Delta CDE$ is an isosceles Δ (sides opp = $\angle$'s)	✓S ✓R ✓S/R ✓S ✓R (4)
		[21]

QUESTION 5

5.1		
5.1.1	QT = TS (line from centre ⊥ to chord. $Q\hat{T}R = S\hat{T}R = 90^\circ$ (given) TR is common side. $\Delta QTR \equiv \Delta STR$ (SAS)	✓S/R ✓S ✓condition (3)
5.1.2(a)	$\hat{R}_2 = 67^\circ$ (tan ⊥ radius)	✓S ✓R (2)
5.1.2(b)	$\hat{P} + \hat{R}_2 + \hat{R}_3 = 180^\circ$ (opp ∠'s of a cyclic quad) $\hat{P} + 67^\circ + 67^\circ = 180^\circ$ $\hat{P} = 46^\circ$	✓S/R ✓answer (2)
5.1.3	$\hat{O}_3 = 2\hat{Q}_1$ (∠ at the centre = 2 × ∠ at the cirm) $\hat{O}_1 = 180^\circ - 90^\circ - 2x$ (sum of ∠'s of Δ) $\hat{O}_1 = 90^\circ - 2x$	✓S/R ✓S/R ✓answer (3)
5.1.4	OT = 19,5 cm (mid-point theorem)	✓ answer (1)
		[11]
		TOTAL: 100