KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

JUST IN TIME LEARNER REVISION DOCUMENT

MATHEMATICS

GRADE 10

2026

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the contents and to give guidance to teachers.

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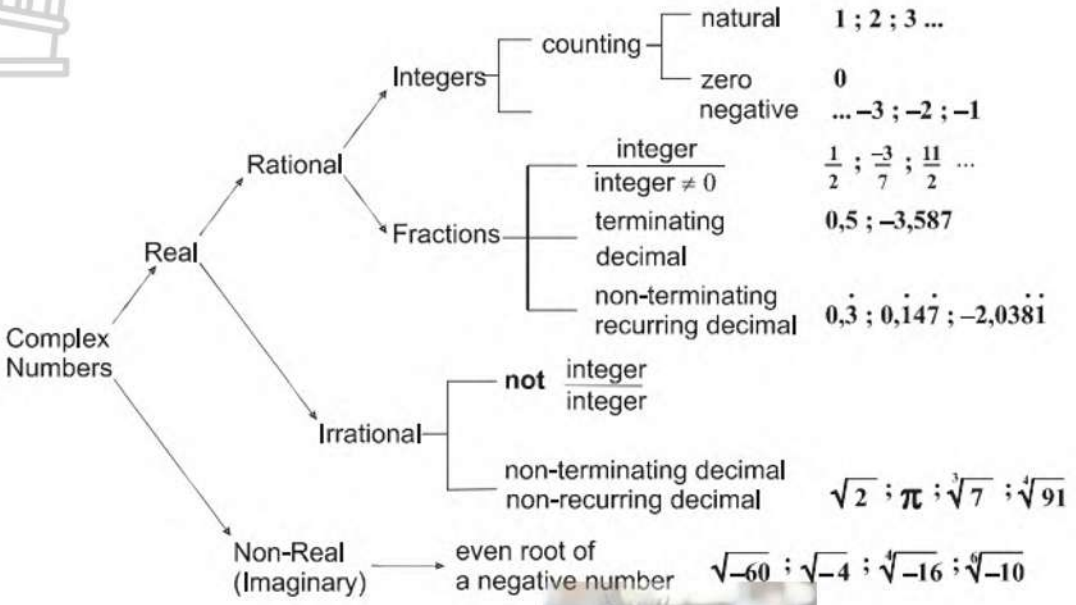
TOPIC 1. ALGEBRA

GUIDELINES, SUMMARY NOTES, & STRATEGIES

➢ **Number system:**

- We can write a rational number as a fraction, $\frac{a}{b}$, where a and $b \in \mathrm{Z}$ and $b \neq 0$.
- We cannot express an irrational number as a fraction.



* Remember: $\frac{\text{any number}}{0}$ is **undefined**

➢ **Algebraic expressions:**

- Multiplying the sum of two or more numbers/variables is the same as multiplying the addends separately.

$$a(b+c) = ab + ac \qquad a(b+c+d) = ab + ac + ad$$

➢ **Factorisation:**

The golden rules of factorisation		
Two terms	**Three terms**	**Four terms**
Step 1: Apply the sign-change rule if necessary **Step 2:** Take out the HCF if it exists. **Step 3:** Apply difference of two squares or sum and difference of two cubes if possible.	**Step 1:** Apply the sign-change rule if necessary. **Step 2:** Take out the HCF if it exists. **Step 3:** Factorise the trinomial.	**Step 1:** Group in pairs and put brackets around each pair separated by the + sign. **Step 2:** Apply the sign-change rule if necessary. **Step 3:** Factorise the pairs. **Step 4:** Take out the common bracket. **Step 5:** Factorise further if needs be.

➢ Simplification of algebraic fractions:

Addition and subtraction	**Multiplication and division**
1. Factorise the denominator 2. Consider the sign-change rule 3. Find the LCD and convert each fraction to an equivalent fraction with the same denominator(LCD)	1. When dividing remember to "tip and times" when required. 2. Factorise the numerator and denominator 3. Consider the sign-change rule 4. Cancel and simplify

4. Write your answer as one fraction	

- **Laws of exponents:**

$a^n \times a^m = a^{m+n}$	$\dfrac{a^m}{a^n} = a^{m-n}$	
$(a^m)^n = a^{mn}$	$(a \times b)^n = a^n \times b^n$	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
$a^0 = 1$	$x^{-n} = \dfrac{1}{x^n}$	

- **Equations:**

Linear equations

Remove the brackets where necessary. The equation will now be in a form of $ax + q = 0$

Add or subtract like terms, and terms with variables one side and constant terms on the other side.

Divide both sides by the coefficient of the variable.

Quadratic equations

Remove any brackets and take all terms on one side of the equals sign so that there is only a 0 on the side. The equation now will be in a form of $ax^2 + bx + c = 0$ or $ax^2 + q = 0$.

Factorise the equation.

We find the solution to the equation by letting each factor equal 0.

Simultaneous equations

When solving a pair of simultaneous linear equations, we are, in fact, finding a common point – the point of intersection of the two lines.

Elimination method

- Make the coefficients of one of the variables the same in both equations, Eliminate the variable by subtracting equation (1) and equation (2) together.
- Simplify and solve for x
- Substitute x back into either original equation and solve for y

Substitution method

- Use the simplest of the two given equations to express one of the variables in terms of the other.
- Substitute into the second equation. By doing this we reduce the number of equations and the number of variables by one.
- We now have one equation with one unknown variable which can be solved.
- Use the solution to substitute back into the first equation to find the value of the other unknown variable.

Literal equations(making a required variable be a subject of the formula)

- Isolate a term with a required variable
- If the unknown variable is in two or more terms, then we take it out as a common factor.
- If required letter is squared, square on both sides and write $\pm$ on RHS
- If the unknown variable is in the denominator, we multiply both sides by the lowest common denominator (LCD) and then continue to solve.

Linear inequalities

- Solve for the unknown
- If multiplied or divided by a negative sign, reverse the inequality sign
- Represent the answer on the number line
- Represent the answer in the interval notation

Word problem solving

- Read the whole question and ask the questions like, which value are we asked to solve?
- Translate the words into algebraic expressions by rewriting the given information in terms of the variables.
- Set up an equation or system of equations to solve for the variable.
- Solve the equation algebraically using substitution and check the solution.

ACTIVITIES

1.1 State whether the following expressions are rational, irrational or non-real.

1.1.1 $\sqrt{30}$ (1) **L1**

1.1.2 6,5 (1) **L1**

1.1.3 $\sqrt{-87}$ (1) **L1**

1.1.4 2,3333333…. (1) **L1**

1.2 If $x \in \{0;1;2;3;4\}$, write down a value of x such that $K = \sqrt{\frac{9}{4-x}}$ is:

1.2.1 Undefined (1) **L2**

1.2.2 Rational (1) **L2**

1.2.3 Irrational (1) **L2**

1.3 Write down two rational numbers between $\sqrt{3}$ and $\sqrt{11}$ (show your workings) (2) **L2**

1.4 Use your knowledge of number relationships to answer the following:

1.4.1 Write 0.1111111111…. as a fraction (2) **L2**

1.4.2 The value of $\sqrt{42}$ lies between two integers. Find these two integers without finding the exact value of $\sqrt{42}$, show all your workings. (2) **L2**

1.4.3 Determine which integer from 1.4.2 is closest to $\sqrt{42}$, without finding the actual value of $\sqrt{42}$. Give a reason for your answer. (1) **L2**

1.4.5 If $x = \frac{1}{9}$, express the recurring decimal fraction $0,\dot{3}$ in terms of x. (2) **L2**

1.5 Consider the surd $\sqrt{73}$.

1.5.1 Prove that $\sqrt{73}$ is not an integer, without finding its exact value. (3) **L2**

1.5.2 If $x = 2\sqrt{73}$, which two integers will x lie between? (2) **L2**

1.6 Express the recurring decimal $0,2\dot{8}\dot{8}$ as a fraction by answering the next two questions:

1.6.1 Let $x = 0,2\dot{8}\dot{8}$. Write down the value of $100x$ to **four** decimal places. (1) **L2**

1.6.2 Determine the value of $99x$ using your answer to 1.6.1 and solve for x. (4) **L2**

1.7 The following inequality is given: $\sqrt{20} < x \leq \sqrt{46}$.

1.7.1 Illustrate this inequality on a number line showing only integral values. Mark off the approximate positions of $\sqrt{20}$ and $\sqrt{46}$. (3) **L2**

1.7.2 If x is an integer, what values of x would satisfy the inequality? (2) **L2**

1.8 Write $0.2424242424.$ in the form $\frac{a}{b}$ where a and b are integers. (3) **L2**

1.9 Expand the following expressions:

1.9.1 $(2x-1)(x^2-5x+6)$ (2) **L2**

1.9.2 $(x-y)(x^2+xy+y^2)$ (2) **L2**

1.9.3 $\left(\frac{x}{2}-\frac{y}{3}\right)\left(\frac{x}{2}+\frac{y}{3}\right)$ (2) **L2**

1.9.4 $(1-t^4)(1-t^2)(1+t^2)$ (4) **L2**

1.9.5 $(x-1)^2-(2x+1)^2$ (3) **L2**

1.10 Simplify the following expressions:

1.10.1 $\frac{5^{2x-1}\times 25^{x+1}.5}{25^{2x}}$ (4) **L2**

1.10.2 $p-[2p-(3p-(4p-5p))]$ (1) **L2**

1.10.3 $(2k+1)(k+1)^2$ (3) **L2**

1.10.4 $\frac{4^x.8^{x+1}}{32^{x+1}}$ (4) **L2**

1.10.5 $(c+3)^3$ (3) **L2**

1.10.6 $4^{x+3}.8^{1-x}.2^{x-9}$ (3) **L2**

1.10.7 $\frac{27^x-1}{9^x-1}\div\left(3^{2x}+3^x+1\right)$ (4) **L3**

1.11 Simplify the following expressions fully:

1.11.1 $\frac{25^n\times 3^{2n-1}}{75^n\times 15^{n-1}}$ (3) **L2**

1.11.2 $\left(\frac{x^{10}}{y^{-5}}\right)^{-\frac{2}{5}}$ (2) **L2**

1.11.3 $\frac{12^x.15^x.5}{2^{-1}.18^x.10^{x+1}}$ (5) **L2**

1.12 Simplify as far as possible:

1.12.1 $\frac{2^x-2^{x-2}}{2^{x+1}-2^x}$ (4) **L2**

1.12.2 $\frac{4^{m-1}.81^{m+1}}{18^m.18^{m-1}}$ (4) **L2**

1.12.3 $\frac{96^{n+1}.4^{-n}}{8^n.32.3^{n-1}}$ (7) **L2**

1.12.3 $\frac{2^{2n+2}-2^{2n+1}}{4^{n+1}}$ (3) **L2**

1.12.5 $\frac{3^t-3^{t-2}}{2\times 3^t-3^t}$ (3) **L2**

1.12.6 $\dfrac{2a+4}{a^2-4} \div \dfrac{3a-6}{a^2-4a+4}$ (3) **L3**

1.12.7 $\dfrac{x^2-5x+6}{x^2-3x} \div \dfrac{3x^2-7x+2}{x^2-9}$ (4) **L2**

1.12.8 $\dfrac{x^2-5x+6}{8-x^3} \div \dfrac{x^2-9}{x^3+2x^2+4x}$ (6) **L2**

1.12.9 $\sqrt{\dfrac{2^x+2^{x+2}}{2^x.5^{-1}}}$ (3) **L3**

1.12.10 $(k^{-2}-s^{-2}) \div (k^{-1}+s^{-1})$ (4) **L3**

1.12.11 $\dfrac{x^3+1}{x^2-x+1}$ (3) **L2**

1.12.12 $\dfrac{2x^3+16y^3}{8y-4x} \times \dfrac{x^2-4y^2}{x^2+4xy+4y^2}$ (5) **L3**

1.12.13 $\dfrac{2}{x-3}+\dfrac{5}{x+3}+\dfrac{9}{x^2-9}$ (5) **L2**

1.12.14 $\dfrac{3}{a-4}+\dfrac{2}{a+3}-\dfrac{21}{a^2-a-12}$ (5) **L3**

1.12.15 $\dfrac{2}{a-1}-\dfrac{3}{1-a}-\dfrac{2}{a^2-1}$ (5) **L3**

1.12.16 $\dfrac{5}{x+4}+\dfrac{2x^2+18}{x-3} \times \dfrac{x+3}{x^2(x+3)+9x+27}-\dfrac{6x-4}{x^2+x-12}$ (8) **L3**

1.12.17 $\dfrac{8a^3-27}{6a^2-27a+27}$ (4) **L2**

1.12.18 $\dfrac{3x^2+x-10}{x^3+8} \times \dfrac{x^2-2x+4}{(3x-5)^2}$ (4) **L2**

1.13 Factorise fully:

1.13.1 x^2y-x (1) **L1**

1.13.2 $1+y^2-10$ (2) **L1**

1.13.3 $3a^2b^3+9ab^2$ (1) **L1**

1.13.4 x^3-4x (2) **L1**

1.13.5 $6x^2+3y-9x-2xy$ (3) **L2**

1.13.6 $t^2(k-s)-k+s$ (3) **L2**

1.13.7 $p^2-24p+144$ (3) **L2**

1.13.8 $x^2+2xy+y^2$ (3) **L2**

1.13.9 $3x^2+9x-2xy-6y$ (2) **L2**

1.13.10 $x+4y-3ax-12ay$ (2) **L2**

1.13.11 $(a-b)^2-b^2$ (2) **L2**

1.13.12 $4p^2+7pq-2q^2$ (2) **L2**

1.13.13 $5x^3+40$ (3) **L2**

1.13.14 $x^3-\dfrac{1}{x^3}$ (3) **L2**

1.13.15 $\frac{1}{27}x^3+216$ (2) **L2**

1.14 Factorise fully:

1.14.1 a^2-b^2-a+b (3) **L2**

1.14.2 $2x^4-128$ (3) **L2**

1.14.3 $2p+p^2-3$ (2) **L2**

1.14.4 $\frac{x^2}{81}-p^2$ (2) **L2**

1.14.5 $a^2-5ab+6b^2$ (3) **L2**

1.14.6 $10x^2-23xy+9y^2$ (3) **L2**

1.14.7 $\frac{1}{2}x^2-\frac{1}{2}x-3$ (3) **L2**

1.14.8 $8x^2+14xy-15y^2$ (3) **L2**

1.14.9 $2a^2-14a+20$ (3) **L2**

1.15 Solve for x in the following equations:

1.15.1 $x^2-2x=3$ (3) **L2**

1.15.2 $-500x(x-5)=0$ (2) **L2**

1.15.3 $2-x=3(5x-10)$ (3) **L2**

1.15.4 $(x-3)(2-x)=-2x$ (3) **L2**

1.15.5 $2.4^x=16$ (2) **L2**

1.15.6 $5^{2x}=26-5^0$ (2) **L2**

1.15.7 $\frac{x+2}{3}-\frac{1}{4}=\frac{1}{6}x$ (3) **L2**

1.15.8 $(x+4)(x-2)=7$ (3) **L2**

1.15.9 $2x+3y=7$ and $y=4x$ (NB: you are not required to solve for y.) (2) **L2**

1.16 Solve for x:

1.16.1 $x^2-81=0$ (2) **L2**

1.16.2 $x^2+6x=27$ (3) **L2**

1.16.3 $2^{x+1}=\frac{1}{4}$ (3) **L2**

1.16.4 $k=t\left(1+\frac{x}{200}\right)^r$ (in terms of t, k and r) (4) **L3**

1.16.5 $\frac{x-1}{x-2}-\frac{x}{x-1}=\frac{2x}{x^2-3x+2}$ (5) **L3**

1.16.6 $6x^2-6=5x$ (4) **L2**

1.16.7 $\frac{5}{3x-12}=\frac{x+4}{x^2+8x+16}$ (5) **L3**

1.16.8 $\frac{5}{4}\left(x-\frac{1}{5}\right)=\frac{3}{2}\left(2x+\frac{4}{3}\right)-4$ (4) **L2**

1.16.9 $\frac{3}{x-2}+\frac{x+3}{4-x^2}=\frac{6}{x+2}$ (6) **L2**

1.16.10 $5^{x+2}=\frac{1}{125}$ (3) **L2**

1.16.11 $4^{x+2}-2^{2x}=15$ (4) **L2**

1.16.12 $-3<\frac{-x+4}{6}\leq 3$ (3) **L2**

1.16.13 $3^{-2x-1}.7^{3x-2}.9^{x+1}=147$ (**Hint**: simplify left hand side first.) (5) **L3**

1.16.14 $-2\leq 3-\frac{x}{3}\leq 5$, where $x\in Z$. Represent your solution on a number line. (3) **L3**

1.16.15 $2.5^{x-1}-250=0$ (3) **L2**

1.17 Solve x and y simultaneously:

1.17.1 $7x+3y=13$ and $2x-3y=-4$ (5) **L2**

1.17.2 $x+2y=1$ and $2x+3y=6$ (5) **L2**

1.17.3 $3x+5y=-7$ and $4x+7y=-10$ (5) **L2**

1.17.4 $\frac{x}{2}+\frac{y}{3}=9$ and $\frac{1}{3}x-\frac{1}{2}y=-\frac{1}{2}$ (5) **L2**

1.17.5 $7x-3y=41$ and $3x-y=17$ (5) **L2**

1.17.6 $2y-3x=7$ and $4y-5x=21$ (5) **L2**

1.17.7 $2x-y-8=0$ and $x-3y+1=0$ (5) **L2**

1.17.8 $2x+\frac{1}{3}y=1$ and $3x+5y=6$ (5) **L3**

1.17.9 $4x+3y=5$ and $2x-\frac{3}{4}y=1$ (5) **L3**

1.17.10 $\frac{y-2}{3}-\frac{x-1}{2}=1$ and $\frac{y-4}{5}-\frac{x-3}{4}=1$ (6) **L3**

1.18 Make the variable required **(indicated in brackets)** be the subject of the formula:

1.18.1 $a^2=u^2+2sv$ (v) (4) **L2**

1.18.2 $k=\frac{mp^2}{gr}$ (m) (3) **L2**

1.18.3 $A=P(1+i.n)$ (n) (3) **L2**

1.18.4 $s=r+tv^2$ (v) (3) **L2**

1.18.5 $s=ut+\frac{1}{2}av^2$ (v) (2) **L2**

1.18.6 $Tn=a+(n-1)d$ (d) (2) **L2**

1.18.7 $c=\sqrt{a+2x}$ (x) (2) **L2**

1.19 Solve the following inequalities and represent the solution on a number line:

1.19.1 $x+18 \leq 9-2x$ (3) **L2**

1.19.2 $5(x-1) > 7(x-1)$ (3) **L2**

1.19.3 $-11 \leq 2x+1 < 3$ (3) **L2**

1.19.4 $-4 \leq x-3 \leq \frac{1}{2}$ (3) **L2**

1.19.5 $\frac{x}{3}-\frac{x}{2} > 1$ (3) **L2**

1.19.6 $\frac{3y+2}{4}-\frac{y-6}{3} > 0$ (4) **L3**

1.19.7 $(x+3)(x-4) \leq (x-3)(x+4)$ (4) **L2**

1.9.8 $\frac{x}{3}-14 > 14-\frac{x}{7}$ (3) **L2**

1.20 Word problems

1.20.1 The difference between two numbers is 5. Six times the smaller number is equal to four times the greater number. Find the numbers (4) **L2**

1.20.2 Tabelo is currently four times as old as his daughter, Linda. 6 years from now, Tabelo will be three times as old as Linda. Calculate Linda's age currently (4) **L3**

1.20.3 A shop sells bicycles and tricycles. In total there are 7 cycles (cycles include both bicycles and tricycles) and 19 wheels. Determine how many of each there are, if a bicycle has two wheels and a tricycle has three wheels. (3) **L2**

1.20.4 The area of a rectangular field is equal to 300 square meters. Its perimeter is equal to 70 meters. Find the length and width of this rectangle. (4) **L3**

1.20.5 Four children have small toys. The first child has 1/10 of the toys, the second child has 12 more toys than the first, the third child has one more toy of what the first child has and the fourth child has double the third child. How many toys are there? (5) **L4**

1.20.6 In a shop, the cost of 4 shirts, 4 pairs of trousers and 2 hats is R560. The cost of 9 shirts, 9 pairs of trousers and 6 hats is R1290. What is the total cost of 1 shirt, 1 pair of trousers and 1 hat? (6) **L4**

1.20.7 A man is six times older than his son is now. In twenty-four years' time he will be double his son's age. How old was he when his son was born? (4) **L3**

1.20.8 If the area of a particular square is given as $4x^2-40x+100$ square units, determine the value of x. (4) **L2**

TOPIC 2. NUMBER PATTERNS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

A linear pattern is a pattern where the difference (d) between consecutive terms is common.

Example: 7 11 15 19 23

4 4 4 4

From the above example, the first term $a = 7$ and the common difference $d = 4$.

1. Determining the next term

To determine the next term in the linear pattern, we add the common difference to the previous term.

$T_3 = 15 = T_2 + d = 11 + 4$

2. Determining the general term or the nth term.

The general term can be used to generate all the terms of a sequence.

The general formula of a linear pattern can be written in the form:

$T_n = dn + c$ where d is the common difference and $c = a - d$

From the above example $d = 4$ and $c = a - d = 7 - 4 = 3$ $\therefore T_n = 4n + 3$

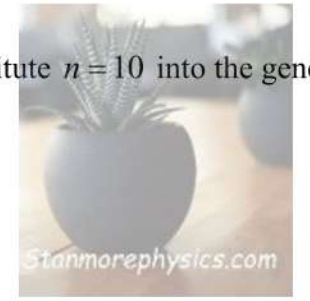
3. Using the general term to determine T_n and n.

(a) If we are to calculate the 10th term or T_{10}, we substitute $n = 10$ into the general term.

$T_n = 4n + 3$

$T_{10} = 4(10) + 3$

$T_{10} = 43$



(b) If we are to calculate which term of the sequence is 99, we equate $n = 14$.

$99 = 4n + 3$ $\therefore n = 24$

$T_{24} = 99$

It is important to note the square and cube numbers.

In a pattern 1 ; 4; 9...

$T_1 = 1 \times 1$, $T_2 = 2 \times 2$, $T_3 = 3 \times 3$ which draws a conclusion of the general term $T_n = n \times n = n^2$

In a pattern: 1 ; 8; 27..

$T_1 = 1 \times 1 \times 1$, $T_2 = 2 \times 2 \times 2$, $T_3 = 3 \times 3 \times 3$ which therefore means $T_n = n \times n \times n = n^3$

ACTIVITIES

2.1 Consider the linear pattern: $8;14;20...$

2.1.1	Write down the next term.	(1)	**L1**
2.1.2	Determine the n^{th} term.	(3)	**L2**
2.1.3	Calculate the 13^{th} term.	(2)	**L1**
2.1.4	Which term of the pattern has a value of 176?	(3)	**L2**
		[09]	

2.2 Consider the linear pattern: $3;8;13...$

2.2.1	Write down the next term.	(1)	**L1**
2.2.2	Determine the n^{th} term.	(3)	**L2**
2.2.3	Calculate the 13^{th} term.	(2)	**L1**
2.2.4	Which term of the pattern has a value of 268?	(3)	**L2**

2.3 Given a linear pattern: $268;263;258;....$

2.3.1	Write down the next two terms.	(2)	**L1**
2.3.2	Determine the n^{th} term.	(3)	**L2**
2.3.3	Determine T_{29}.	(2)	**L1**
2.3.4	Which term of the pattern is first to be negative?	(3)	**L3**
		[10]	

2.4 Consider the linear pattern: $-122;-111;-100$

2.4.1	Determine the n^{th} term.	(3)	**L2**
2.4.2	Which term is first to be positive?	(3)	**L3**
		[06]	

2.5 $1;5;9;13.....$ are the first four terms of a linear pattern.

2.5.1	Write down the next two terms.	(2)	**L1**
2.5.2	Determine T_{19}.	(4)	**L3**
2.5.3	Is 402 a term in this pattern? Justify your answer.	(3)	**L3**
		[09]	

2.6 $37;34;31$ are the first three terms of a linear pattern.

2.6.1	Write down the next two terms.	(2)	**L1**
2.6.2	Determine the smallest value of n for which $T_n > -14$	(4)	**L3**
2.6.3	Which term in the pattern is first to be negative?	(3)	**L3**

2.7 Consider the linear pattern: $23;a;31.....$

2.7.1	Write down the value of a.	(2)	**L2**
2.7.2	Determine the n^{th} term.	(3)	**L2**
2.7.3	Calculate the value of the 31^{st} term.	(2)	**L1**
2.7.4	Which term of the pattern has a value of 231?	(3)	**L3**
		[10]	

2.8 Consider the linear pattern: $-2;x;y;-14...$ and

	Calculate the values of x and y.	(4)	**L3**
		[04]	

2.9 Consider the linear pattern: $x;2x+1;4x-1....$

2.9.1	Calculate the value of x.	(3)	**L3**
2.9.2	Write down the numerical values of the first three terms.	(1)	**L1**

2.9.3 Determine the n^{th} term. (3) **L2**

2.9.4 Determine the numerical value of the 60^{th} term. (2) **L1**

[09]

2.10 Given the linear pattern: $-x-2;-2x+1;-50+3x....$

2.10.1 Calculate the value of x. (3) **L3**

2.10.2 Write down the numerical values of the first three terms (1) **L1**

2.10.3 Determine the n^{th} term. (3) **L2**

2.10.4 Determine the numerical value of the first term that is less than -100. (2) **L1**

[09]

2.11 $t+3;3t-6;27-t;-10+5t$ are the first four terms of a linear pattern.

2.11.1 If $t=7$ determine the numerical values of the 5^{th} and 6^{th} terms. (3) **L2**

2.11.2 Determine the n^{th} term. (3) **L2**

2.11.3 Calculate the value of T_{81} (2) **L1**

2.11.4 Which term has a value of 500? (3) **L3**

2.11.5 Determine the largest value of n for which $T_n < 1007$. (4) **L3**

[15]

2.12 The figure below shows a ball that is rolling from the top of the stairs all the way down. Not all the steps are shown. The steps are made of squares.

2.12.1 Determine the number of squares that make the 5^{th} step. (1) **L1**

2.12.2 Determine the formula for the number of squares in the n^{th} term. (3) **L2**

2.12.3 If the last step has 35 squares. How many steps will it take the ball to reach the floor? (3) **L3**

[07]

2.13 The table below represents PATTERN A and PATTERN B

Position of term (n)	**1**	**2**	**3**	**4**
PATTERN A	0	1	2	3
PATTERN B	0	1	4	9

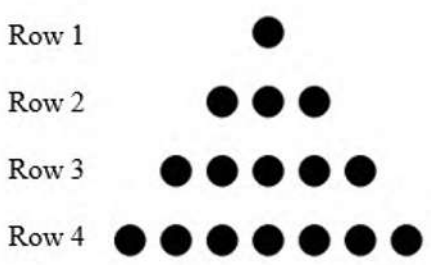
2.13.1 Write down the next term of PATTERN A. (1) **L1**

2.13.2 Determine the formula for the general term of PATTERN A. (3) **L2**

2.13.3 Hence or otherwise, write down the formula for the general term of PATTERN B. (1) **L2**

[05]

2.14 Consider a geometric pattern below.



2.14.1 How many dots will make the 5th row? (1) **L1**

2.14.2 Determine the general term for the number of dots in the n^{th} term. (2) **L3**

2.14.3 Which row will be made of 99 dots? (3) **L3**

[06]

2.15 If a pattern **LAUGHEDLAUGHEDLAUGHED**….is continued in this way, what will be the 203rd letter? (2) **L4**

[02]

2.16 Imagine your grandpa and his four friends training for a boxing day held at your school in order to raise funds for a grade 10 trip. The trainer group them in pairs. Your grandpa and one of his friends are in group A and the others are in group B.

- Group A trains for 15 minutes on day one, they add 20 minutes on the following day and day after.
- Group B trains for 140 minutes on day 1. They are too tired on day two that they train 10 minutes less than that on the previous day. They continue to train 10 minutes less than the previous day each day.

2.16.1 Which group will train more on day 6? Justify your answer by relevant calculations. (3) **L3**

2.16.2 Can this continue indefinitely? Motivate your answer. (2) **L2**

[05]

2.17 Consider the pattern: $\frac{1}{5}$; $\frac{3}{7}$; $\frac{5}{9}$; $\frac{7}{11}$.....

2.17.1 Write down the next two terms. (2) **L2**

2.17.2 Determine the general term of the pattern. (4) **L3**

[06]

2.18 Consider the linear pattern: 4 ; 6 ; 8...

2.18.1 Show that all terms in the pattern will be even. (3) **L2**

2.18.2 Calculate the 100^{th} term. (2) **L1**

2.18.3 Is 301 part of the pattern? Justify. (2) **L1**

[07]

2.19 The table below represents PATTERN A and PATTERN B

Position of term (n)	**1**	**2**	**3**	**4**
PATTERN A	3	5	7	9
PATTERN B	9	25	49	81

2.19.1 Write down the next term of PATTERN A. (1) **L1**

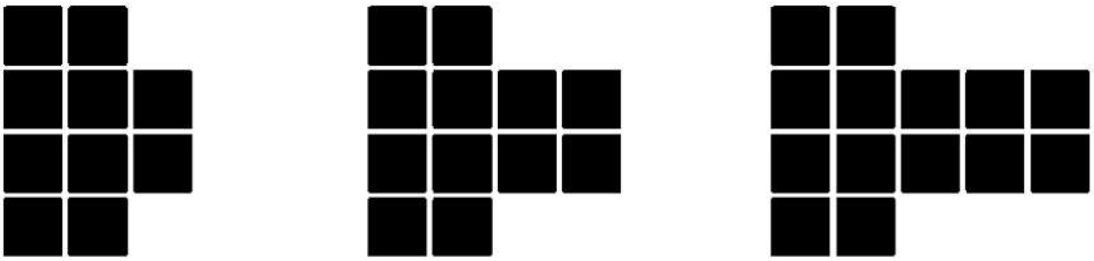
2.19.2 Determine the formula for the general term of PATTERN A. (3) **L2**

2.19.3 Hence or otherwise, write down the formula for the general term of PATTERN B. (1) **L2**

2.19.4 Determine the 25th term of PATTERN B. (2) **L1**

[7]

2.20 Consider the pattern below.



Term 1 Term 2 Term 3

2.20.1 Draw the pattern for the 4th term. (1) **L1**

2.20.2 Determine the formula for the general term of pattern. (2) **L2**

2.20.3 Which term will have 88 squares? (3) **L3**

[06]

FUNCTIONS AND GRAPHS

GUIDELINES, SUMMARY NOTES AND STRATEGIES FOR GR. 10:

FROM CAPS DOCUMENT: GR. 10 FUNCTIONS AND GRAPHS:

- Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable).
- Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$ where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$ where $b > 0, b \neq 1$.

Characteristics	Straight line	Parabola	Hyperbola	Exponential
Standard form	$y = ax + q$ / $y = mx + c$	$y = ax^2 + q$	$y = \frac{a}{x} + q$	$y = a.b^x + q$
a	$a > 0$: The graph increases from left to right. $a < 0$: The graph decreases from left to right.	$a > 0$: The graph has a minimum turning point at $(0 : q)$. $a < 0$: The graph has a maximum turning point at $(0 : q)$.	$a > 0$: The graph lies in the first and third quadrants (decreasing function) $a < 0$: The graph lies in the second and fourth quadrants (increasing function)	For, $b > 0$, $a > 0$: The graph curves upwards. $a < 0$: The graph curves downwards. For, $0 < b < 1$, $a > 0$: The graph curves downwards. $a < 0$: The graph curves upwards.
q	Vertical shift: $q > 0$ upward $q < 0$ downward	Vertical shift $q > 0$ upward $q < 0$ downward	Vertical shift $q > 0$ upward $q < 0$ downward	Vertical shift $q > 0$ upward $q < 0$ downward
asymptotes	None	None	Vertical asymptote: $x = 0$ Horizontal asymptote: $y = q$	Only the horizontal asymptote: $y = q$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}, x \neq 0$	$x \in \mathbb{R}$
Range	$y \in \mathbb{R}$	$a > 0, [q; \infty)$ $a < 0, (-\infty : q]$	$y \in \mathbb{R}, y \neq q$	$y > q$ if $a > 0$ $y < q$ if $a < 0$
Axis of symmetry	None	$x = 0$	$y = x + q$ or $y = -x + q$	None

Interpretation	
	$f(x) > 0$: graph lies above the line $y = 0$.
	$f(x) < 0$: graph lies below the line $y = 0$.
	$f(x) = g(x)$: point of intersection.
	$f(x) \geq g(x)$: one graph lies above the other.
	$f(x) < g(x)$: one graph lies below the other.
	$f(x).g(x) \leq 0$: one graph lies above $y = 0$ and the other below $y = 0$
	$f(x).g(x) \geq 0$: both graphs lies above $y = 0$ or below $y = 0$

ACTIVITIES: FUNCTIONS AND GRAPHS

Grade 10 – GP/June 2024

3.1 Consider the functions $f(x) = ax^2 + c$ and $g(x) = bx + q$. f and g intersect at points $A(-1;-3)$ and $B(2;0)$

3.1.1 Calculate the values of a and c. (4) **L2**

3.1.2 Calculate the values of b and q. (3) **L2**

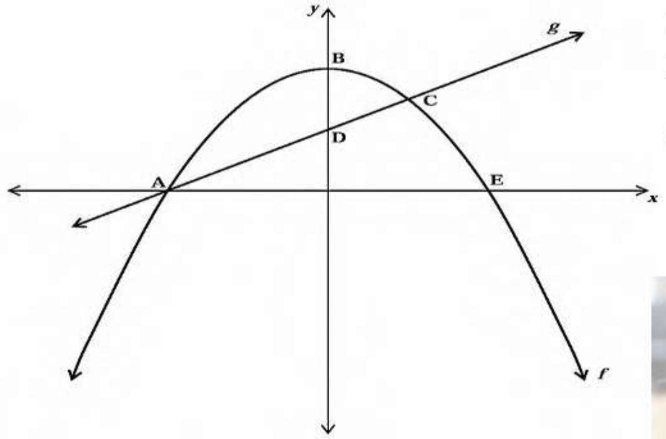
3.1.3 Sketch graphs f and g on the same set of axes. (6) **L2**

3.1.4 Determine the value of $g(x) - f(x)$. (2) **L2**

.1.5 For which values of x is $g(x) \geq f(x)$? (2) **L3**

Grade 10 – GP/June 2022

3.2 The graphs of $f(x) = -x^2 + 4$ and $g(x) = mx + 2$ are sketched. A and E are the x-intercepts of f. B and D are the y-intercepts of f and g respectively. f and g intersect at A and C.



3.2.1 Determine the coordinates of points B and D. (2) **L1**

3.2.2 Write down the range of f. (1) **L1**

3.2.3 Determine the length of AE. (4) **L2**

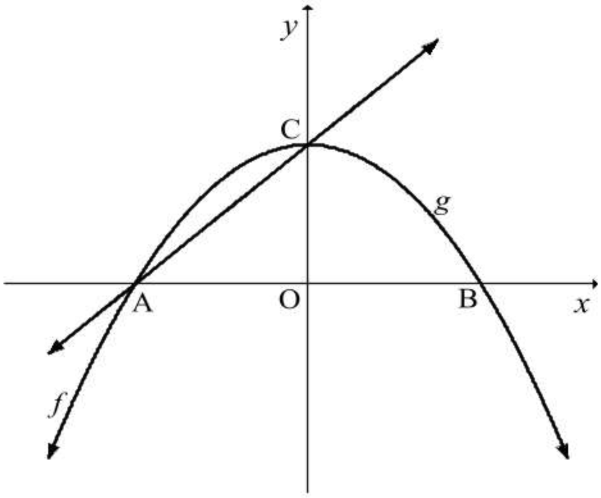
3.2.4 Calculate the value of m. (2) **L1**

3.2.5 Determine the coordinates of A and C, the points of intersection of f and g. (3) **L3**

3.2.6 If $k(x) = g(-x)$, determine the values of x such that $f(x) \geq k(x)$. (2) **L3**

Grade 10 – KZN/Nov 2023

3.3 The diagram shows the graphs of $g(x) = -x^2 + 4$ and $f(x) = mx + c$. A and B are the x-intercepts of g and C is the y-intercept of g. Graph f passes through A and C.



3.3.1 Write down the coordinates of C. (1) **L1**

3.3.2 Determine the coordinates of A and B. (3) **L2**

3.3.3 Determine the values of m and c. (2) **L2**

3.3.4 How should g be transformed so that the new graph will have a y-intercept of -2 and still have a maximum turning point? (2) **L4**

3.3.5 For which values of x is $f(x) < g(x)$? (2) **L2**

3.3.6 For which values of x is $f(x).g(x) \leq 0$? (2) **L2**

Grade 10 – DBE/Nov 2018

3.4 The equation of the function $g(x)=\frac{a}{x}+q$ passes through the point $(3;2)$ and has a range of $y\in(-\infty;1)\cup(1;\infty)$.

3.4.1 Determine the equation of g. (3) **L2**

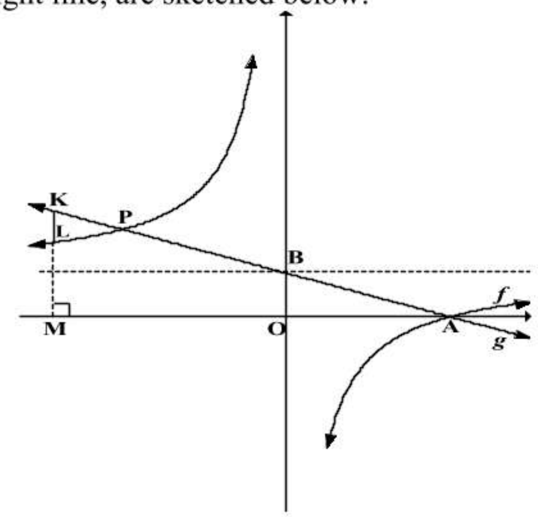
3.4.2 Determine the equation of h, the axis of symmetry of g which has a positive gradient. (2) **L2**

3.4.3 Sketch the graphs of g and h on the same system of axes. Clearly show ALL the asymptotes and intercepts with the axes. (4) **L2**

3.4.4 Write the equation of the asymptotes of f if $f(x)=-g(x)+5$. (3) **L3**

Mind Action Series

3.5 The graph of f, a hyperbola, and $g(x)=-\frac{x}{4}+1$, a straight line, are sketched below:



3.5.1 Determine the lengths of OA and OB. (2) **L1**

3.5.2 Write down the equations of the asymptotes of f. (2) **L1**

3.5.3 Determine the equation of f. (3) **L2**

3.5.4 If $OM=8$ units, determine the length of KL. (4) **L3**

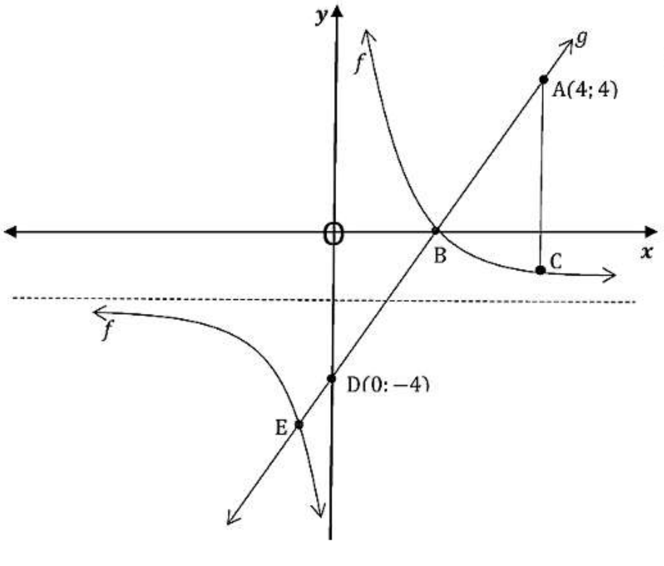
3.5.5 Determine the coordinates of P. (4) **L3**

3.5.6 For which values of x is $f(x).g(x)>0$? (3) **L3**

3.5.7 For which values of x is $\frac{f(x)}{g(x)}\le 0$? (3) **L3**

Grade 10 – NW/Nov 2024

3.6 The graphs of $f(x)=\frac{4}{x}-2$ and $g(x)=mx+c$ are drawn below. $A(4;4)$, B, $D(0;-4)$ and E are points on g. B is the x-intercept of both f and g. B and E are the points of intersection of f and g. AC is parallel to the y-axis, with C a point on f.



3.6.1 Determine the values of m and c. (2) **L1**

3.6.2 Write down the range of f. (2) **L1**

3.6.3 Determine the coordinates of the point(s) of intersection of the two graphs algebraically. (4) **L3**

3.6.4 Determine the value(s) of x for which $g(x)\ge f(x)$. (2) **L2**

3.6.5 Calculate the length of AC. (3) **L3**

Grade 10 – KZN/Nov 2024

3.7 Given that $f(x) = -\frac{2}{x} + 4$

3.7.1 Determine the equation of the horizontal asymptote of f. (1) **L1**

3.7.2 Determine the coordinates of the x-intercept of f. (2) **L1**

3.7.3 Sketch f on a Cartesian plane. Show all the intercepts with the axes and the asymptotes. (3) **L2**

3.7.4 Determine the equation of g, if g is formed by shifting the graph of f 7 units down. (1) **L2**

Grade 10 – GP/Nov 2023

3.8 Consider the exponential function: $t(x) = 2.2^{-x} - 4$

3.8.1 Write down the range of t. (1) **L1**

3.8.2 Calculate the x-intercept of t (2) **L2**

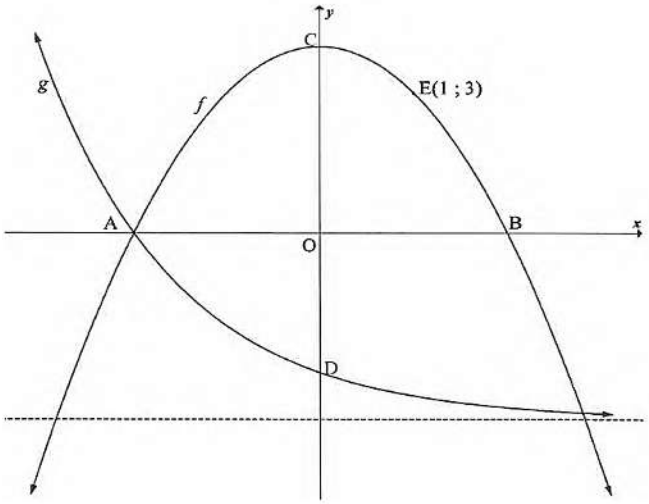
3.8.3 Calculate the y-intercept of t (1) **L1**

3.8.4 Sketch the graph of t on a system of axes. Clearly show all intercepts with the axes and the asymptote. (3) **L2**

3.8.5 Describe the transformation of t to h, if $h(x) = 2.2^{x}$. (2) **L2**

Grade 10 – DBE/Nov 2018

3.9 Sketched below are the graphs of $f(x) = ax^2 + q$ and $g(x) = \left(\frac{1}{2}\right)^x - 4$. A and B are the $x-$intercepts of f. The graphs intersect at A and point E(1;3) lies on f. C is the turning point of f and D is the y-intercept of g.



3.9.1 Write down the coordinates of D. (2) **L1**

3.9.2 Write down the range of g. (1) **L1**

3.9.3 Calculate the coordinates of A. (2) **L2**

3.9.4 Calculate the value of a and q. (4) **L2**

3.9.5 Determine the length of CD. (2) **L1**

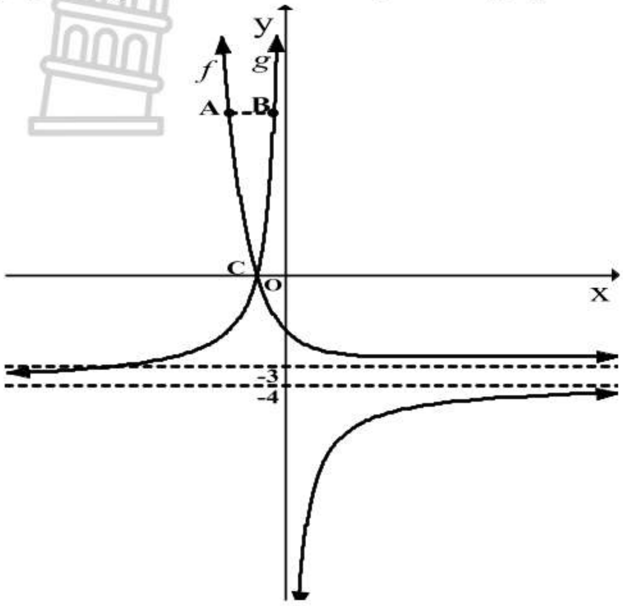
3.9.6 Determine the equation of a straight line through A and D. (3) **L2**

3.9.7 For which values of x is $f(x) > 0$? (2) **L2**

3.9.8 For which values of x is f decreasing? (1) **L2**

Grade 10 – GP/Nov 2022

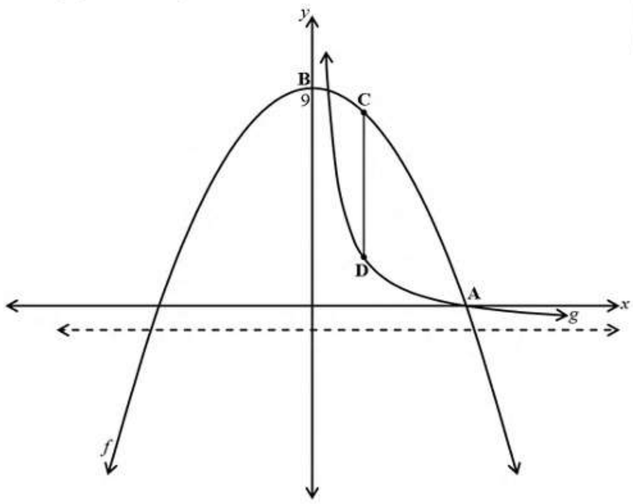
3.10 The sketch below shows the graphs of $f(x)=a^x-3$ and $g(x)=-\frac{4}{x}-4$. A$(-2;6)$ is a point on the graph of f and B is a point on the graph of g. C is the x-intercept of the graphs.



3.10.1	Write down the domain of f.	(1)	**L1**
3.10.2	Write down the range of g.	(1)	**L2**
3.10.3	Write down the equation of the asymptote of f.	(1)	**L1**
3.10.4	Show that the value of a is 3^{-1}.	(2)	**L2**
3.10.5	Determine the coordinates of C.	(3)	**L2**
3.10.6	Write down the values of x for which $g(x)>f(x)$.	(2)	**L2**
3.10.7	Determine the equation of the straight line through A and C.	(4)	**L2**
3.10.8	Determine the equation of the axis of symmetry of g with a negative gradient.	(2)	**L2**
3.10.9	If AB is a horizontal line, calculate the length of AB.	(3)	**L3**

Grade 10 – GP/November 2023

3.11 The graphs of $f(x)=ax^2+q$ and $g(x)=\frac{3}{x}-1$; for $x>0$, are drawn below. A is the x-intercept and point of intersection of both graphs. B$(0;9)$ is the turning point of f and CD is a vertical line.



3.11.1	Write down the range of f.	(1)	**L1**
3.11.2	Write down the domain of g.	(1)	**L1**
3.11.3	Write down the equation of the horizontal asymptote of g.	(1)	**L1**
3.11.4	Write down the value(s) of x such that f is decreasing.	(1)	**L1**
3.11.5	Determine the values of a and q.	(4)	**L2**
3.11.6	If $h(x)=-f(x)-3$, write down the Equation of h.	(2)	**L2**
3.11.7	If $h(x)=-f(x)-3$, write down the Coordinates of the turning point of h.	(1)	**L1**
3.11.8	Write down the equation of the line of symmetry of g, which has a positive gradient.	(2)	**L2**
3.11.9	Calculate the length of CD if the equation of line CD is $x=1$.	(4)	**L3**
3.11.10	Determine the value(s) of x such that $f(x)>x+3$.	(4)	**L4**

3.12 Draw a sketch graph of $t(x)=a.b^x+q$, where $a>0;\ b>1$ and $q<0$. (3) **L3**

Grade 10 – DBE/November 2015

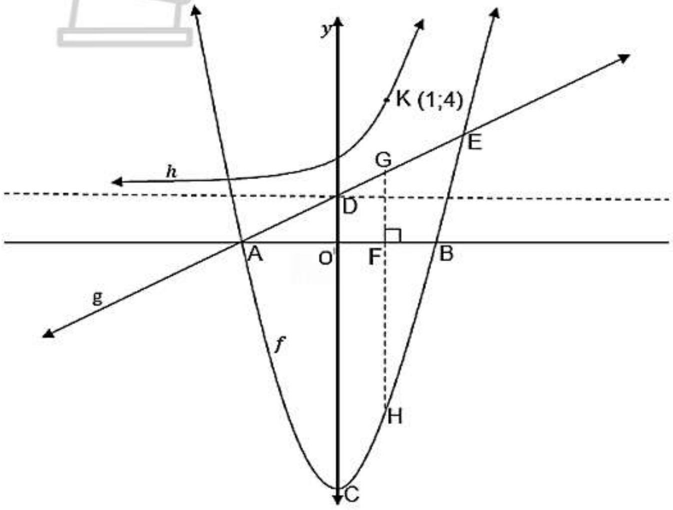
3.13 Given: $f(x)=ax^2+c$. f passes through the $x-$axis at $(d-5)$ and $(d-1)$, where $d\in\mathbb{R}$.

3.13.1 Determine the value of d. (2) **L3**

3.13.2 Determine the values of a and c if it is also given that $f(1)=-9$. (4) **L3**

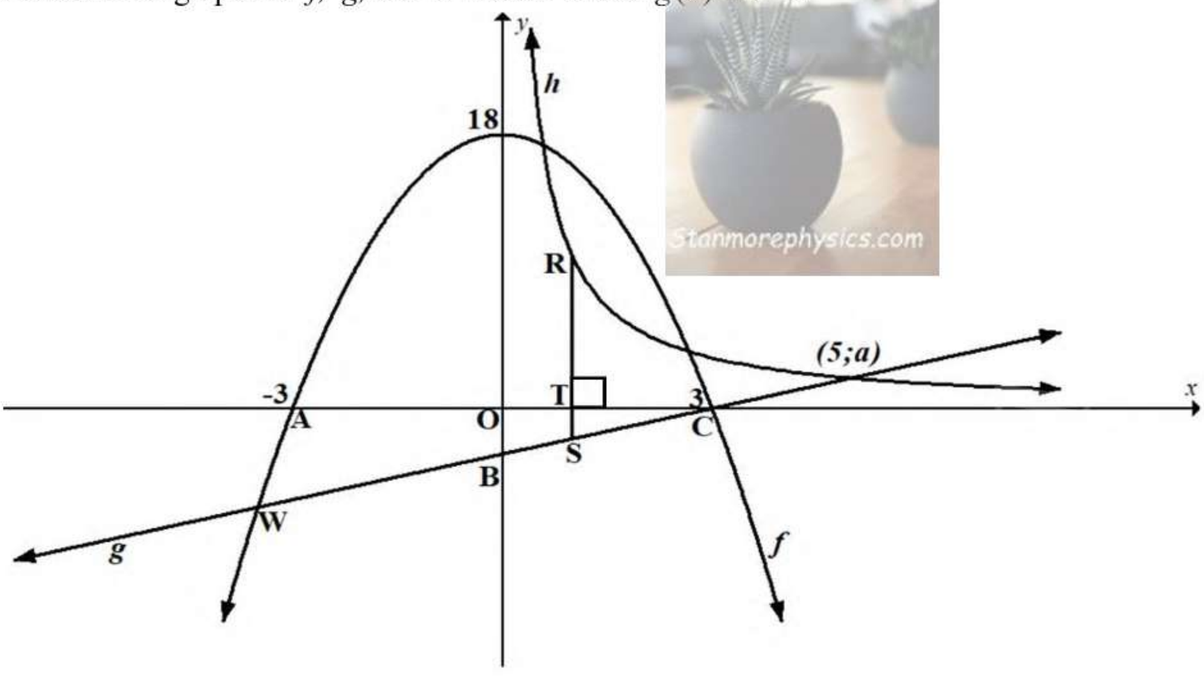
Grade 10 – GP/November 2020

3.14 In the given sketch, the following graphs are represented:
$f(x)=3x^2-12$, $g(x)=x+2$ and $h(x)=a^x+q$.
A and B are the x-intercepts and C is the turning point of f. D is the y-intercept of g.
$K(1;4)$ is a point on the graph of h.



3.14.1	Calculate the length of GH if $OF=1$ unit.	(3)	**L3**
3.14.2	Determine the coordinates of E, the point of intersection of f and g	(5)	**L3**
3.14.3	Determine the equation of h.	(3)	**L2**
3.14.4	Determine k if k is the reflection of f about the x-axis.	(2)	**L2**
3.14.5	For which value(s) of x will $f(x)\geq 0$?	(2)	L3

3.15 Consider the graphs of f, g, and h below. Given $g(x)=x-3$



3.15.1	Determine the equation of f.	(3)	**L2**
3.15.2	Determine the coordinates of B	(1)	**L1**
3.15.3	Determine the equation of the hyperbola, h.	(3)	**L3**
3.15.4	Determine the length of RS if OT = 1	(4)	**L3**
3.15.5	If the equation of f is $f(x)=-2x^2+18$ determine the coordinates of W.	(6)	**L2**
3.15.6	Give the range of $f(x)$	(2)	**L1**

[19]

TOPIC 4. FINANCE

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Interest is calculated in two different ways: **Simple interest and Compound Interest.**

Simple Interest: Interest is calculated only on the money invested or borrowed, which is called the principal amount, the capital, or the present value. It is used for short-term loans (hire purchase accounts) and investments.

Compound Interest: interest is calculated on the principal amount and the interest that is added to the principal amount at regular time intervals. It is used with long- term loans and investments.

Simple interest & Compound interest formulae:

Simple interest	**Compound interest**
Simple Interest = $A = P(1+in)$ ***P*** = the original amount/Capital/Principal amount/Present value $i = \frac{r}{100}$ or $i = r\%$ (r = interest rate per annum as decimal fraction) ***n*** = number of years **A** = final amount/ accumulated amount	**Compound Interest =** $A = P(1+i)^n$ ***P*** = the original amount/ Capital/Principal amount/Present value $i = \frac{r}{100}$ or $i = r\%$ (r = interest rate per annum as decimal fraction) ***n*** = number of years **A** = final amount/ accumulated amount

Deposit: A sum of money paid as a first instalment on an item with the understanding that the balance will be paid at a later stage.

Discount: Is a reduction in price from the original price of a product or service, it is often expressed in percentage form.

***Instalment*:** A sum of money due as one of several equal payments for something, spread over an agreed period of time.

Hire purchase loan agreement is a short-term loan. Interest is calculated as ***simple interest*** on the full amount of the loan over the entire repayment period.

Inflation is the average increase in the cost of goods and services over a period of time. We can use ***compound interest formula*** to calculate the expected increase in the cost of goods and services over a period of time.

Exchange rates: There are different money systems in different countries. ***Currency*** is the term used to describe the particular money system of a country. Here are the currencies of a few countries:

Country	**Currency used**	**Symbol for the currency**
South Africa	Rand	R
United State of America	US dollar	$
United Kingdom	British Pound	£
Several European countries	Euro	€

Population growth is the increase in the number of individuals in a population. The formula for calculating exponential growth of a population is similar to the compound interest formula:

$$P_{future} = P_{present}\left(1+\frac{r}{100}\right)^n$$

$P_{present}$ = present size of the population

P_{future} = future size of the population

r = average population growth rate expressed as a percentage

ACTIVITIES

4.1 Samuel invested *R*50 000 in the African Bank for 4 years at 10,05% simple interest. How much money did he receive at the end of the investment period? (3) **L1**

4.2 A student buys a Laptop for R11 000 on a hire purchase agreement. He agrees to pay a deposit of 20% and the balance over a period of two years in monthly installments at 14% simple interest p.a. He also takes out an insurance premium of R68,50 per month. Determine the amount of each installment. (5) **L2**

4.3 Read the advertisement below.

Buy a Hisense 575L French Door Fridge for only R928 per month.

You have 36 months to pay.
No deposit is required.

4.3.1 Calculate the total amount to be paid over a period of 36 months. (1) **L1**

4.3.2 The monthly instalment, quoted in the advertisement, is calculated on a hire purchase agreement which charges interest of 9,5% p.a. on the cash price of the fridge. Show that the price of the fridge is R25 999. (3) **L3**

4.3.3 Calculate the total interest paid over a period of 36 months if the fridge is bought with this hire purchase agreement. (1) **L1**

4.3.4 The fridge is insured at 11,5% p.a of the cash price. The total insurance is calculated and then split up over 36 months. It is then added to the monthly instalment. Calculate the new monthly instalment if the customer wants to insure the fridge. (3) **L3**

4.3.5 The cost of the fridge is subject to inflation and increases to a cash price of R27 499 after 2 years. Calculate the annual inflation rate. (4) **L2**

4.4 Seven years ago, Mr Gina decided to invest R18 000 in a bank account that paid simple interest at 4,5% p.a.

4.4.1 Calculate how much interest Mr Gina has earned over the 7 years. (2) **L1**

4.4.2 Mr Gina wants to buy a television set that costs R27 660,00 now. If the average rate of inflation over the last 5 years was 6,7% p.a., calculate the cost of the television set 5 years ago. (3) **L2**

4.4.3 At what rate of simple interest should Mr Gina have invested his money 7 years ago if he intends buying the television set now using only his original investment of R18 000 and the interest earned over the last 7 years? (3) **L2**

4.5 Jabu wants to buy a computer costing R7950, on a hire-purchase agreement. The conditions of the agreement are:

- Jabu must pay a deposit of 25% of the purchase price.
- Interest is charged at 15% per annum simple interest on the balance.
- He must also pay a compulsory monthly insurance premium of R70,75.
- The balance is to be settled in monthly instalments.

4.5.1 Calculate the balance after Jabu pays the deposit. (2) **L1**

4.5.2 If the balance is to be paid off in 24 months, calculate Jabu's total monthly instalment. (4) **L2**

4.6 If the exchange rate between Rand and American Dollar is *R*11, 83: $1, and the exchange rate between Rand and British pound is *R*14, 95: £1, find the exchange rate between $ and £. (4) **L3**

4.7 A new small car cost R39 000. If the rate of inflation is 10,25% p.a. how much will the same type of car cost in 5 years' time (4) **L2**

4.8 A diamond ring is said to be an investment. A diamond ring bought in 1976 cost R300. In 1997 the ring was valued at R 2000

4.8.1 Determine the average rate of increase. (3) **L2**

4.8.2 If the average rate of inflation over the same period is 12% can this be considered an investment? Give reason for your answer (4) **L3**

4.9 Mr Ngubane went to House and Home furniture store to buy a dining room suit that worth R34 999. As a regular customer, the manager gave him a discount of 12%. He paid a deposit of R10 000 and the balance on Hire-purchase agreement at an interest rate of 13% p.a. for 36 months.

4.9.1 Calculate the value of the discount received? (1) **L1**

4.9.2 What is the monthly instalment Mr Ngubane must pay? (2) **L3**

4.10 The following advertisement appeared with regards to buying a bicycle on a hire-purchase agreement loan:

- *Purchase price* *R5 999*
- *Required deposit* *R600*
- *Loan term* *Only 18 months, at 8% p.a. simple interest*

4.10.1 Calculate the monthly amount that a person must budget for in order to pay for the bicycle. (4) **L4**

4.10.2 How much interest does one have to pay over the full term of the loan (1) **L1**

4.11 The Department of Education provides teachers (t) for schools in the ratio of 1:35 to learners (l).

4.11.1 Draw up a table representing this information. (Use units of 10 for the teachers, $t \in [10;40]$) (3) **L3**

4.11.2 Use the table to determine a mathematical rule to represent this information in the form $t = \ldots$ (2) **L2**

4.11.3 How many teachers will be provided in a school of 1 200 learners? (2) **L2**

4.11.4 If the average gross salary of a teacher is R232 500 p.a. How much will the government spend on their salaries per year in a school of 800 learners? (3) **L3**

4.11.5 If there was no money for teachers, how much would each learner have to pay per year to cover this expense. (2) **L3**

4.12 Given below is the table with the fuel prices for 2013

PETROL	REEF	
	UNLEADED	
	93	**95**
03 July 2013	1300	1323
05 June 2013	1216	1239
01 May 2013	1224	1247
03 April 2013	1297	1320
06 March 2013	1287	1308
06 February 2013	1206	1227
02 January 2013	1165	1186

4.12.1 Determine the rate of inflation per month from January to March for unleaded 93 petrol. (4) **L2**

4.12.2 In July the government claims that the rate of inflation for petrol for the last two months is less than 5%. Prove that this is correct. (4) **L2**

4.12.3 Mark says that his pocket money should increase according to the rate of petrol inflation. If he got R200 in January, what amount should he receive in July? (2) **L2**

4.13 Brent Crude Oil cost $47, 71 a barrel.

4.13.1 Calculate the cost in rand, of importing a barrel when the exchange rate is R13, 39 to the dollar. (2) **L1**

4.13.2 If the rand weakens, will the cost of importing Brent oil increase or decrease? (1) **L2**

4.14 Anne deposits R5 000 in an account and four years later she deposits R4 500 in the same account. If this account earns interest that is compounded annually at 9% p.a. for the first three years and 8% p.a. for the rest of the period, how much will Anne have in the account at the end of 10 years? (4) **L3**

4.15 Aneliswa decides to invest some money at a compound interest rate of 14% p.a. How much money (to the nearest rand) should be invested if she wants to receive a final amount of R85 000 in eight years' time? (3) **L2**

4.16 The mid-year population for South Africa in 2024 was 58 347 857. If the average population growth rate is 1,56%, calculate the size of the population 5 years back. (3) **L3**

TOPIC 5. PROBABILITY

FROM CAPS DOCUMENT: GR. 10 PROBABILITY:

- Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome.
- Venn diagrams as an aid to solving probability problems.
- Mutually exclusive events and complementary events.
- The identity for any two events A and B: P(A or B) = P(A) + P(B) – P(A and B)
- **Probability**: The measure of chance or likelihood of an event occurring.

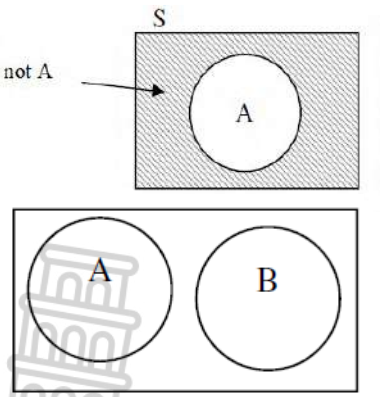
GUIDELINES, SUMMARY NOTES, & STRATEGIES

- Mark Weighting: **Term 3** common test, $\frac{15\pm3}{75}$ **November** final paper, $\frac{15\pm3}{100}$

TERMINOLOGY:

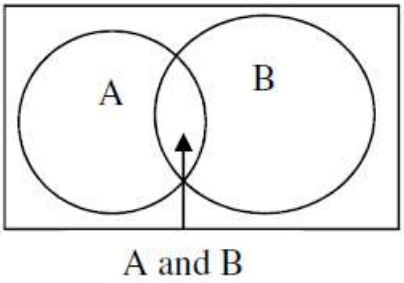
- EVENT: An event is an activity that has outcomes.
  - ✓ For example, rolling an even number is an event with outcomes.
- OUTCOME: Is the possible result of an event.
  - ✓ For example, the event rolling an even number has 2, 4 and 6 as possible results.
- SAMPLE SPACE: The set of all possible outcomes.

- Probability = $\frac{\text{number of favourable outcomes}}{\text{number of posible outcomes}}$ OR $P(A)=\frac{n(E)}{n(S)}$, where $0\le P\le 1$. This simple means that probability **canno**t be negative or above 1.
- We can express probabilities as **fractions, decimals or percentages.**
- General formula: $P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$, where 'A and B' are the events that happen at the same time.
- Complimentary events: $P(A)+P(\text{not } A)=1$. It simple means that event A and not A cannot happen at the same time.

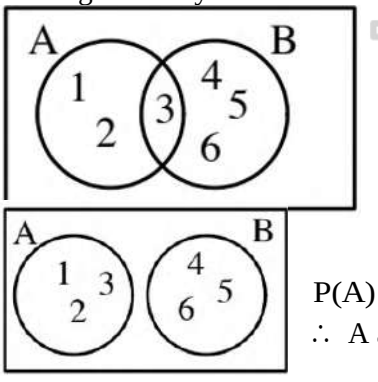


- Mutually Exclusive Events: $P(A \text{ or } B)=P(A)+P(B)$. Event A and B cannot happen at the same time. Events A and B are said to be disjoint.
  $P(A \text{ and } B)=0$

- Not Mutually Exclusive Events:
  $P(A \text{ and } B)\neq 0$



- A and B are **exhaustive** if $P(A \text{ or } B)=1$. Mutually exclusive events and not mutually exclusive events can be exhaustive if together they contain all elements of sample space.

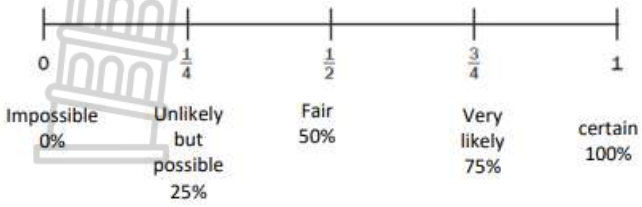


Not mutually exclusive

Mutually exclusive.

P(A) + P(B) = 1
∴ A and B are complimentary

- Probability Scale: A scale which can be used to plot the probability of an event happening. It starts at 0% (impossible event), 50% (equally likely) in the middle, and all the way through to 100% (certain events).



$0 \le P \le 1$

Complementary rule

P(not A) = 1 – P(A)

Addition rule

- *Mutually exclusive*
  P(A or B) = P(A) + P(B)
- *General rule*
  P(A or B) = P(A) + P(B) – P(A and B)

ACTIVITIES

Grade 10 – KZN/Nov 2025

5.1 If P(D) = 0,25 ; P(K) = 0,5 ; and P(D or K) = 0,625

5.1.1 Calculate P(D and K) . (3) **L2**

5.1.2 Are events D and K complementary? Give a reason. (2) **L2**

Grade 10 – NW/Nov 2025

5.2 Consider events A and B. $P(A) = 0,43$, $P(B) = 0,37$ and P(A and B) = 0,1591.

5.2.1 Determine whether events A and B are complementary. (2) **L2**

5.2.2 Are events A and B mutually exclusive? Explain your answer. (2) **L2**

5.2.3 Draw a Venn diagram to represent the information above. (4) **L3**

Grade 10 – NW/Nov 2025

5.3 Two different events, A and B, form part of an investigation.

- $P(\text{A}) = \frac{2}{3}$; $P(\text{not } B) = \frac{3}{5}$;
- $P(\text{A or B}) = \frac{4}{7}$

Determine the value of:

5.3.1 P(not A) (2) **L2**

5.3.2 P(B) (2) **L1**

5.3.3 P(A and B) (3) **L2**

Grade 10 – KNZ/Sept 2024

5.4 Given: $P(\text{A}) = 0,4$; $P(\text{B}) = 0,5$ and $P(\text{A and B}) = 0,25$. Determine:

5.4.1 P(not B) (1) **L2**

5.4.2 P(A or B) (2) **L2**

5.4.3 P(not A and B) (3) **L3**

Grade 10 – KNZ/Sept 2025

5.5 Two events A and B are complementary. $P(\text{not B}) = 0,45$.

5.5.1 Copy and complete the statement: $P(\text{A}) + P(\text{B}) = ...$ (1) **L1**

5.5.2 Write down the value of:

5.5.2.1 P(A and B) (1) **L2**

5.5.2.2 P(A) (1) **L2**

The Answer Series

5.6 If P(not A) = 0,3 , P(A or B) = 0,8 and P(A and B) = 0,4 , determine:

5.6.1 P(A) (2) **L2**

5.6.2 P(B) (2) **L2**

5.6.3 P((not A) or B)) (3) **L3**

Grade 10 – DBE/Nov 2025

5.7 A survey was conducted among 100 learners to determine their participation in two subjects: **Mathematics (M)** and **History (H)**. The following results were obtained:

- 42 learners participate in Mathematics.
- 35 learners participate in History.
- x learners participate in both subjects.
- 38 learners did not participate at all.

5.7.1 Draw a Venn diagram to represent the above information (3) **L2**

5.7.2 Determine the number of learners who participated in both subjects (2) **L2**

5.7.3 Determine the probability that a learner selected at random participates in….

5.7.3.1 Neither Mathematics nor History. (1) **L1**

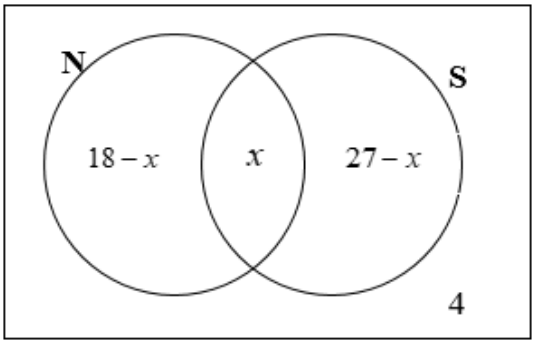
5.7.3.2 Mathematics or History. (1) **L1**

Grade 10 – KNZ/Sept 2025

5.8 In a certain class of 33 girls:

- 18 play netball (N)
- 27 play soccer (S)
- 4 do not play netball or soccer
- An unknown number(x) play both netball and soccer.

The information is represented in the Venn diagram alongside.



5.8.1 Calculate the value of x. (3) **L2**

5.8.2 If a girl from this class is chosen at random, calculate the probability that she:

5.8.2.1 does not play netball or soccer. (1) **L1**

5.8.2.2 plays only netball. (2) **L2**

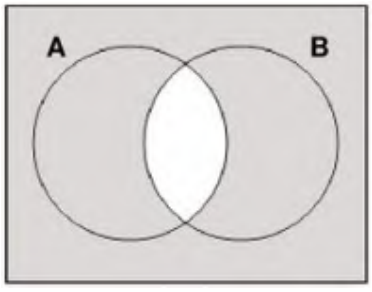
Grade 10 –KNZ/Sept 2025

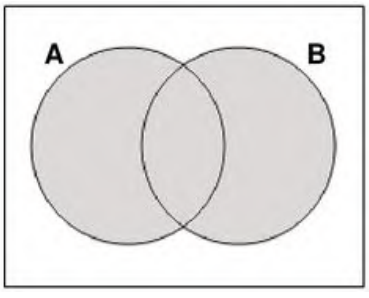
5.9 There are 120 customers at a shop in KwaZulu Natal. 111 customers bought rice (R), 74 customers bought flour (F) and 67 bought both rice and flour.

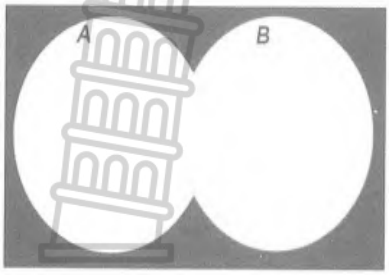
5.9.1 Represent the above information in a Venn diagram. (4) **L2**

5.9.2 Hence, calculate P((not R) and F) (2) **L2**

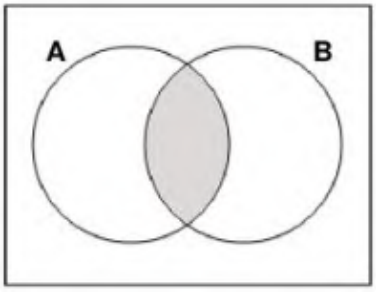
5.10 What expression BEST represent that shaded area of the following Venn diagrams?

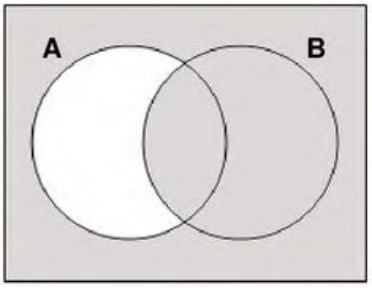
a. 

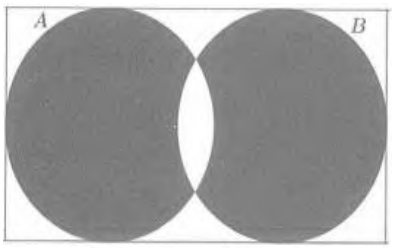
b. 

c. 

(3) **L1**

d. 

e. 

f. 

(3) **L1**

Mind Action Series

5.11 A twelve-side die is rolled. Suppose that the following events are given;

$A = \{\text{multiples of } 3\}$ and $B = \{\text{multiples of } 2\}$

5.11.1 Draw a Venn diagram to illustrate the information above (4) **L2**

5.11.2 Use your Venn diagram to determine;

(a) P(not A and B) (2) **L3** (b) P(A or not B) (2) **L3**

(c) P(not(A and B)) (2) **L3** (d) P((not A) and (not B)) (2) **L3**

(e) P((not A) or (not B)) (2) **L3** (f) P(not(A or B)) (2) **L3**

Maths Sharp

5.12 A die is thrown 30 times, study the results below before answering the questions that follow:

4	3	6	2	1	1	5	6	2	5
3	3	5	2	3	1	1	6	3	2
1	6	2	1	5	2	1	4	1	2

5.12.1 Draw up a frequency table for the results. (3) **L2**

5.12.2 What is the theoretical probability of rolling a 4? (1) **L1**

5.12.3 What was the relative frequency of rolling a 4 in this experiment? (1) **L1**

5.12.4 Do you think the dice is loaded (i.e., it is made to always land on a specific number)? Give a reason for your answer. (2) **L2**

5.13 State which of the following sets of events are mutually exclusive:

a) Event 1: The learners in Grade 10 in the swimming team
Event 2: The learners in Grade 10 in the debating team

b) Event 1: The learners in Grade 8
Event 2: The learners in Grade 12

c) Event 1: The learners who take Mathematics in Grade 10
Event 2: The learners who take Physical Science in Grade 10 (1) **L1**

Grade 10 – GP/Nov 2023

5.14 In a random experiment, A and B are different events.
Suppose P(A) = 0.4, P(B) = *k*

5.14.1 If P(A or B) = 0,6, for which value of *k* are A and B mutually exclusive? (2) **L2**

5.14.2 For which value of *k* are A and B complementary events? (2) **L2**

5.15 In a class of 60 learners, 8 learners take History (**H**), 41 learners take Mathematics (**M**) and 16 learners take neither of these two subjects.

5.15.1 Calculate the number of learners that do both History and Mathematics. (2) **L2**

5.15.2 Draw a Venn diagram to illustrate the above information. (4) **L2**

5.15.3 Determine:

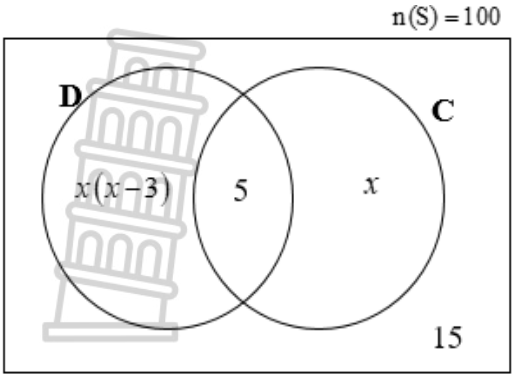
5.15.3.1 The probability of a learner taking History ONLY. (1) **L2**

5.15.3.2 The probability that a learner does not take Mathematics. (2) **L2**

5.16 The probability that a learner is also a science learner is 0,927.
Determine the number of learners that take Science. (2) **L3**

The Answer Series

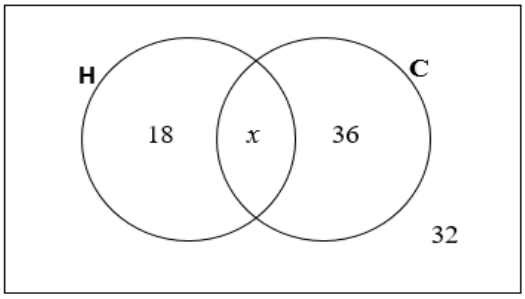
5.17



The Venn diagram alongside shows information about the number of learners who owns dogs and/or cats or neither.
There are 100 learners in total.

Determine the probability that a learner owns a dog ONLY. (4) **L4**

5.18



There is a vendor at a sports match selling hot dogs and cans of cooldrink. The hotdogs cost R40 each, and the cooldrinks cost R15 each.

The vendor received a total of R2 855. Determine the number of people who bought both a hotdog and a cooldrink.

(4) L4

Grade 10 – KZN/ Nov 2024

5.19 The letters the word **COORDINATES** were put into a hat. A letter is chosen at random.
Determine the probability that:

3.19.1 the letter N is chosen. (1) **L2**
3.19.2 the letter O is chosen. (1) **L2**

5.20 Balls numbered from 1 to 10 were placed in a container.

5.20.1 From the ball numbered 1 to 10, list the numbers that will be event A, the factors of 18. (1) **L1**
5.20.2 From the ball numbered 1 to 10, list the numbers that will be event B, all the odd numbers. (1) **L1**
5.20.3 Draw a Venn diagram to illustrate the above information. (3) **L2**
5.20.4 Determine P((not A) or B) (2) **L3**

TOPIC **6. STATISTICS**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

- The word data is the plural of the word *datum* which means "a piece of information''.
- In earlier grades you dealt with the following graphs: bar graphs, histograms, pie charts, broken-line graphs.
- We focus on the ungrouped data (but arranged) and grouped data (in the interval form).
- In order to make sense of the ungrouped data, we need to organise the data in an ascending numerical order.
- We can use a stem-and-leaf diagram and tally table to organise data.

MEASURES OF CENTRAL TENDENCY OF LISTS OF DATA

- Mean is the sum of all the values divided by the total number of values.
- The mean is usually written as $\bar{x}$ (often read as "x bar").

$$\bar{x} = \frac{\text{sum of all values}}{\text{total number of values}}$$

- Median is the middle value when all values are placed in ascending or descending order.
- Mode is the data item that occurs most frequently.

QUARTILES AND THE FIVE NUMBER SUMMARY

- Quartiles are the three values Q_1, Q_2 and Q_3 that divide a data set into four approximately equal parts. Each part consists of approximately 25% of the elements of the data set**.**

  HOW TO FIND THE QUARTILES:

  i) Put the data items in order and find the median (Q_2).

  ii) Find the midpoint of the data items to the left of the median. This is the lower quartile (Q_1).

  iii) Find the midpoint of the data items to the right of the median. This is the upper quartile (Q_3).

- The five number summary consists of 5 items

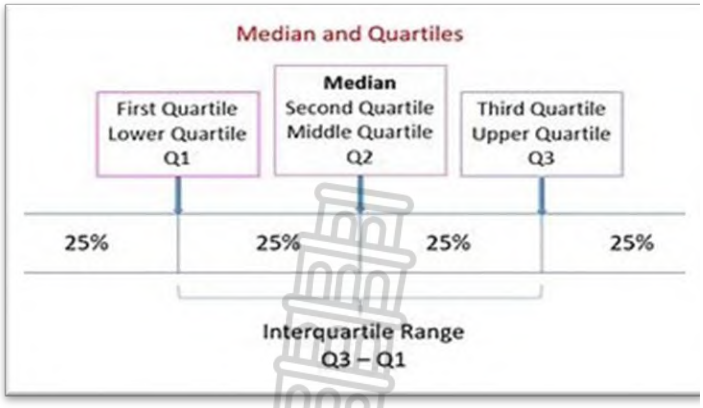
  1) The minimum value in the data set;

  2) Q_1, the lower quartile; $\text{Position} = \frac{1}{4}(n+1)$

  3) Q_2, the median; $\text{Position} = \frac{1}{2}(n+1)$

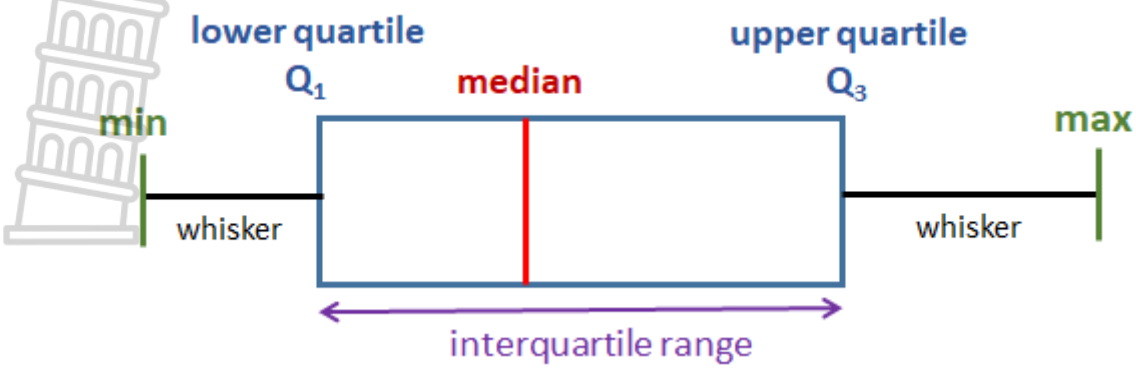
  4) Q_3, the upper quartile; $\text{Position} = \frac{3}{4}(n+1)$

  5) The maximum value in the data set.



BOX AND WHISKER DIAGRAMS

- A graphical representation of the five number summary is known as a box and whisker diagram, also sometimes called a box plot.



PERCENTILES

- Percentiles are values from 0 to 99 that tell you the percentage of the marks that are less than a particular mark.
- Some special percentiles: median is at the 50^{th} percentile, the lower quartile is at the 25^{th} percentile and the upper quartile is at the 75^{th} percentile.

MEASURES OF DISPERSION

- A measure of dispersion or measure of spread tells you how spread out the data is.
- The range is the difference between the largest and smallest items of data.
  Range=Max Value – Min Value
- The interquartile range (or IQR) is the difference between the upper quartile and the lower quartile.
  $IQR=Q_3 - Q_1$
- The semi-interquartile range is half of the interquartile range.

  Semi-IQR= $\frac{1}{2} \times IQR$

ACTIVITIES

6.1 Nineteen girls were required to complete a puzzle as quickly as possible. Their times (in seconds) were recorded and are shown below.

14 15 16 16 17 17 18 18 19 19 19 20 21 21 22 23 24 24 29

6.1.1 Identify the median time taken by the girls to complete the puzzle. (1) **L1**

6.1.2 Determine the lower and upper quartiles for the data. (2) **L1**

6.1.3 Draw a box and whisker diagram to represent the data. (2) **L2**

6.2 The five-number summary of the time (in seconds) taken by 19 boys to complete the same puzzle is (15;19 ; 23 ; 26 ; 30).

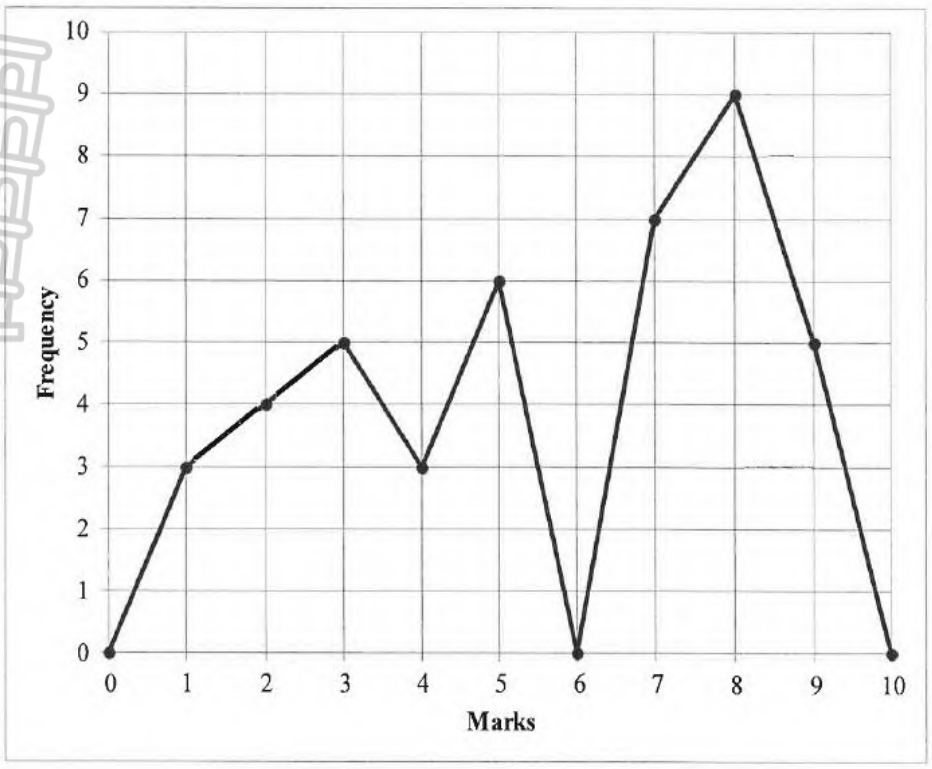
6.2.1 Calculate the interquartile range for the time taken by the boys. (2) **L2**

6.2.2 If only one boy took 19 seconds to complete the puzzle, what percentage of the boys took at least 19 seconds to complete the puzzle? (1) **L3**

6.2.3 In which group, the girls or the boys, did a larger number of learners complete the puzzle in less than 23 seconds? Justify your answer. (2) **L3**

[10]

6.3 The line graph below shows test marks out of 10 obtained by a Grade 10 class.



6.3.1 Write down the above information in a frequency column table. (2) **L2**

6.3.2 How many learners wrote the test? (1) **L2**

6.3.3 Calculate the range for the data. (2) **L1**

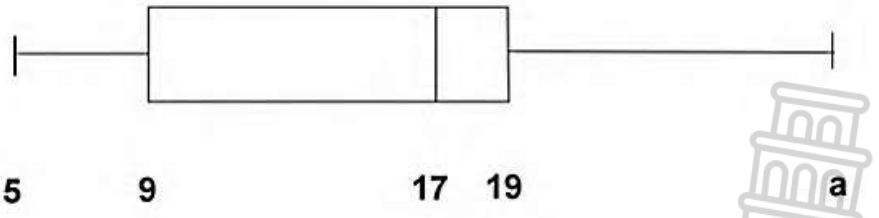
6.3.4 Calculate the mean for the test. (3) **L2**

6.3.5 Determine the median for the data. (3) **L2**

6.3.6 Draw a box and whiskers diagram for the data. (3) **L2**

[14]

6.4 A second-hand car dealer's sales per month for March 2013 to February 2014 is given by the box and whiskers diagram below



6.4.1 If the range of the sales is 25, calculate the value of a. (1) **L2**

6.4.2 Calculate the semi-interquartile range of the data (2) **L2**

6.4.3 If he sells less than 9 cars per month, his profit is not enough to cover all his expenses. How many months was his profit not enough (2) **L3**

6.4.4 Approximately what percentage of the year did he sell more than 19 cars a month? (1) **L2**

[06]

6.5 The figures in the table below are the ages, to the nearest year, of a random sample of 30 people negotiating a mortgage in a bank. Use the data below to answer the questions that follow.

29	26	31	42	38	45	35	37	38	38
36	39	49	40	32	32	34	27	56	29
33	31	33	52	44	32	30	38	42	33

6.5.1 Copy and complete the frequency table below.

FREQUENCY TABLE

Age Intervals	Tally	Frequency
$25 \le x < 33$		
$33 \le x < 41$		
$41 \le x < 49$		
$49 \le x < 57$		

(4) **L2**

6.5.2 Use the information from the frequency table to draw a histogram. (2) **L2**

6.5.3 Calculate the estimated mean of the age interval. (3) **L2**

[09]

6.6 Thirty one students wrote a Grade 10 Mathematics test that was out of 60 marks. Their marks are recorded in the stem and leaf diagram below.

1	2 3 4 5 6 6 7
2	0 0 1 2
3	0 1 2 5 7 9
4	4 6 8 9 9
5	0 2 2 6 8 8 8
6	0

6.6.1 Calculate the five numbers summary of the above data. (5) **L2**

6.6.2 Draw a box whisker plot for the above data. (3) **L2**

6.6.3 Calculate the mean for the above data. (2) **L2**

6.6.4 What percentage of learners achieved a mark between 49 and 59? (2) **L2**

[12]

6.7 A mathematics teacher records the time taken by the learners of his Grade 10 class to complete a class test.

Time in minutes	Number of learners	Midpoints	Midpoint×Frequency
$10 < t \le 20$	2		
$20 < t \le 30$	5		
$30 < t \le 40$	7		
$40 < x \le 50$	8		
$50 < x \le 60$	18		
$60 < x \le 70$	20		

6.7.1 Determine the modal class of the given data. (2) **L1**

6.7.2 Complete the midpoint column of the data above. (2) **L1**

6.7.3 In which interval does the median lie? (2) **L2**

6.7.4 Calculate the estimated mean of the given data. Show all your calculations. (4) **L2**

[10]

6.8 The following mathematics test marks were recorded for a Grade 10A class of 28 students

MARKS	FREQUENCY	MIDPOINTS	MIDPOINT × FREQUENCY
$20 < x \le 30$	2	25	50
$30 < x \le 40$	3	35	105
$40 < x \le 50$	11	45	495
$50 < x \le 60$	7	55	A
$60 < x \le 70$	3	B	195
$70 < x \le 80$	2	75	150
$80 < x \le 100$	0	90	0

6.8.1 Complete the table above by filling in the two missing values. (2) **L1**

6.8.2 Calculate an estimate of the mean mark. (2) **L2**

6.8.3 Write down the modal class. (1) **L1**

6.8.4 In which interval does the median lie? (2) **L2**

6.8.5 In which interval does the 80th percentile lie? (2) **L2**

[09]

6.9 The following Mathematics test marks of a Grade 10B class are recorded below:

45 49 50 51 51 53 54 57 57 59 60 64
65 66 70 71 73 74 75 76 83 89 89

6.9.1 Write down the median mark for this class. (1) **L1**

6.9.2 Calculate the interquartile range mark for this class. (3) **L2**

6.9.3 Represent the data on a box and whisker diagram. (3) **L2**

6.9.4 Comment on the distribution of the data with reference to the box and whisker diagram. (1) **L1**

[08]

6.10 A tuck shop at a particular school sells soft drink cans. The economic friendly club of this school collected soft drink cans for recycling for a period of 20 days. The number of cans collected was recorded and the data is given below:

48 50 52 59 60 68 73 76 76 76
78 79 80 81 82 82 84 91 92 98

6.10.1 Determine the median of the cans collected. (1) **L1**

6.10.2 Determine the value of the upper and lower quartiles. (2) **L2**

6.10.3 Calculate the interquartile (IQR) range of the data. (2) **L2**

6.10.4 Write down the minimum and maximum value of the data. (1) **L1**

6.10.5 Represent the 5 number summary on a box and whisker diagram. (3) **L2**

6.10.6 Comment on the box and whisker diagram. (1) **L1**

[10]

6.11 Telkom conducted a survey regarding the duration of telephone calls made by people in a certain community. The information was then tabulated as indicated below:

Duration (min)	**No of calls (*f1*)**	**Midpoint (*x1*)**	***(f1)* × *(x1)***
$2 \leq t < 5$	47	3,5	164,5
$5 \leq t < 8$	139	6,5	903,5
$8 \leq t < 11$	211	9,5	2004,5
$11 \leq t < 14$	102	12,5	1275
$14 \leq t < 17$	58	15,5	899
$17 \leq t < 20$	19	**A**	**B**

6.11.1 Calculate the values of A and B. (2) **L1**

6.11.2 Determine the approximate mean for the duration of the telephone calls. (3) **L2**

6.11.3 In which interval does the 75th percentile lie? (2) **L2**

6.12 A baker keeps a record of the number of scones that he sells each day. The data for 19 days is shown below:

31 36 62 74 65 63 60 34 46 56
37 46 40 52 48 39 43 31 66

6.12.1 Determine the mean of the given data. (2) **L2**

6.12.2 Rearrange the data in ascending order and then determine the median. (2) **L1**

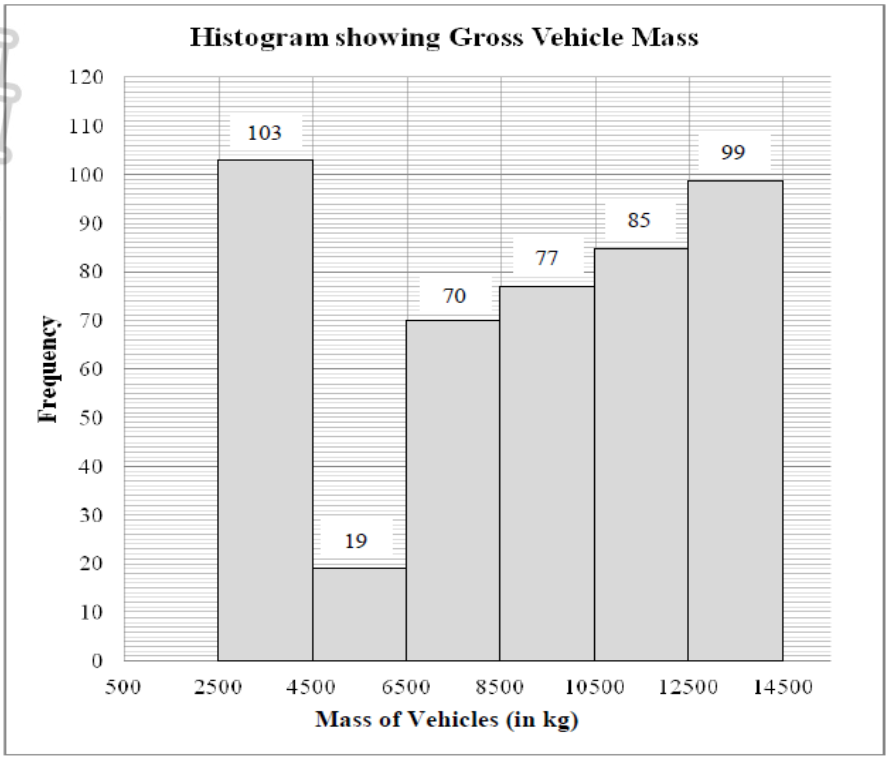
6.12.3 Determine the lower and upper quartiles for the data. (2) **L2**

6.12.4 Draw a box and whisker diagram to represent the data. (2) **L2**

[08]

6.13 Traffic authorities are concerned that heavy vehicles (trucks) are often overloaded. In order to deal with this problem, a number of weighbridges have been set up along the major routes in

South Africa. The gross (total) vehicle mass is measured at these weigh bridges. The histogram below shows the data collected at a weighbridge over a month.



6.13.1 Write down the modal class of the data. (1) **L1**

6.13.2 Estimate the mean gross vehicle mass for the month. (5) **L3**

6.13.3 Which of the measures of central tendency, the modal class or the estimated mean, will be most appropriate to describe the data set? Explain your choice. (2) **L3**

[08]

6.14 Amahle recorded the amount of data (in MB) that she had used daily in the past 15 days.

3 10 12 13 15 15 16 18 24 26 34 40 42 43 45

6.14.1 Calculate the range of the data. (2) **L2**

6.14.2 Calculate the mean data usage. (2) **L2**

6.14.3 Write down the five number summary of the data. (3) **L2**

6.14.4 Draw a box and whisker diagram to represent the data. (3) **L2**

6.14.5 Determine the interquartile range. (3) **L2**

[13]

6.15 At a School Athletics Day, a stopwatch was used to find the time it took a group of athletes to A run the 200m event. The results were recorded in a frequency table below.

Class intervals (time in seconds)	Frequency (number of athletes)
$20 \leq x < 30$	6
$30 \leq x < 40$	16
$40 \leq x < 50$	21
$50 \leq x < 60$	8

6.15.1 How many athletes ran the 200m event? (1) **L1**

6.15.2 Calculate the estimated mean. (4) **L2**

6.15.3 Identify in which class interval the thirtieth (30^{th}) percentile lies. (2) **L2**

6.15.4 Represent the data on a histogram. (3) **L2**

[10]

6.16 The following Life Sciences marks, for a test out of 50, were recorded from 15 grade 10 learners at a certain school: 10 13 15 17 18 20 23 24 26 28 28 29 39 48 49

6.16.1 Write down the median. (1) **L1**

6.16.2 Determine the mean of the data. (2) **L2**

6.16.3 Determine the range of the data. (1) **L2**

6.16.4 Determine the interquartile range of the data. (3) **L2**

6.16.5 Draw a box and whisker diagram to represent the data above. (3) **L2**

6.16.6 The pass mark for the Life Sciences test was 30%. How many learners passed? (2) **L2**

[12]

6.17 The following table represents the percentage of monthly income spent on petrol and car expenses by fifty people.

Percentage	Frequency	Midpoint	Midpoint × frequency
$12 \leq p < 18$	8	15	a
$18 \leq p < 24$	20	b	420
$24 \leq p < 30$	12	27	c
$30 \leq p < 36$	d	33	264
$36 \leq p < 42$	2	39	e

6.17.1 Determine the values of a, b, c, d and e. (5) **L2**

6.17.2 Calculate the estimated mean. (2) **L2**

6.17.3 Determine the modal class. (1) **L1**

6.17.4 Determine the interval containing the median. (2) **L2**

[10]

6.18 As part of the 2009 Census@School, the 26 Grade 10 A learners measured their heights.
The girls' heights (in centimetres) were:

150 150 153 155 156 158 160 161 164 164 166 170 170

The boys' heights in centimetres were:

140 142 151 157 158 159 160 162 165 180 180 180 180

6.18.1 Find the five number summary and the interquartile range for the girls and for the boys. (4) **L2**

6.18.2 On the same number line draw two box and whisker diagrams to illustrate the girls' heights and the boys' heights. (4) **L2**

6.18.3 Use the five number summaries, the interquartile ranges and the box and whisker diagrams to write down two conclusions you can make about the heights' of the girls and the boys. (2) **L3**

[10]

6.19 The company HEALTHCRAZY conducted a survey at a gym in KwaZulu-Natal, to find out which age group most frequently uses their health supplements.

AGE	**FREQUENCY**
$20 \leq x < 30$	29
$30 \leq x < 40$	28
$40 \leq x < 50$	21
$50 \leq x < 60$	15
$60 \leq x < 70$	10
$70 \leq x < 80$	2

6.19.1 Identify the modal class for the data. (1) **L1**

6.19.2 Determine the interval in which the median lies. (1) **L2**

6.19.3 Estimate the mean age of the people that use the health supplements. (4) **L2**

6.19.4 Identify in which class interval the thirtieth (30^{th}) percentile lies. (2) **L2**

[08]

TOPIC 7. ANALYTICAL GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Analytical Geometry is the Geometry on the Cartesian plane,

- It is an algebraic approach to the study of Geometry "using coordinates".

In this topic, we will address the following concepts:

- The **distance** between two points (length of a line segment).
- The **midpoint** of a line segment.
- The **gradient** of a line.

1. The distance between two points:

- The formula to calculate the **distance /length** of a line segment between points A and B, is given by:

$$d_{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

$$(d_{AB})^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

2. The midpoint of a line segment

- The midpoint of a line segment when given the coordinates of two points is given by:

$$M_{AB}\left(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2}\right)$$

- When given the midpoint (M) and required to calculate the coordinate the formula is given by:

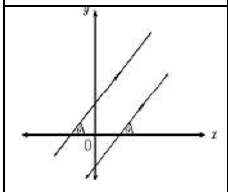
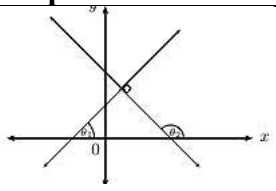
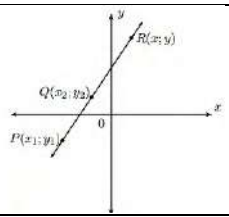
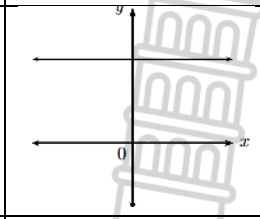
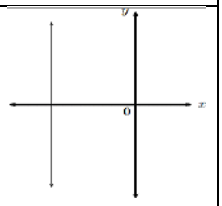
$$x_M = \frac{x_A + x_B}{2}; y_M = \frac{y_A + y_B}{2}$$

3. The gradient of a line:

- The gradient of a line is given by

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

More about gradient:

Parallel Lines	Perpendicular Lines	Collinear Points	Horizontal Line	Vertical Line
				
$m_1 = m_2$	$m_1 \times m_2 = -1$	$m_{PQ} = m_{QR} = m_{PR}$	$m = 0$	m is undefined

2. EQUATION OF A LINE: $y - y_1 = m(x - x_1)$ OR $y = mx + c$ / $y = ax + q$

Determine equation of a line when given:

- The gradient and the y-intercept
- The gradient and one point on the line
- Line passing through two points

- Line through one point and parallel or perpendicular to a given line
- Coordinates of a point on a Cartesian Plane

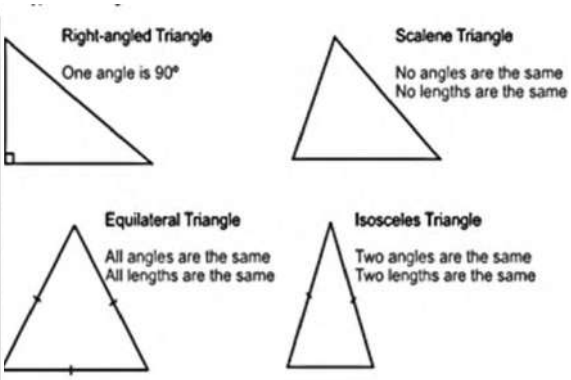
REMEMBER!!!

- Every point on the y - axis has $x = 0$
- Every point on the x - axis has $y = 0$
- In analytical geometry we represent geometric figures on the Cartesian plane and use coordinates of points to derive and prove important facts about these figures.

REMEMBER!!!

- Properties of triangles
- Pythagoras theorem
- Properties of Quadrilaterals
- Areas and perimeters of basic polygons

Property		Rectangle	Square	Parallelogram	Rhombus	Trapezium
Sides	All Sides are equal	×	✓	×	✓	×
	Opposite Sides are equal	✓	✓	✓	✓	×
	Opposite Sides are parallel	✓	✓	✓	✓	✓
Angles	All angles are equal	✓	✓	×	×	×
	Opposite angles are equal	✓	✓	✓	✓	×
	Sum of two adjacent angles is 180	✓	✓	✓	✓	×
Diagonals	Bisect each other	✓	✓	✓	✓	×
	Bisect perpendicularly	×	✓	×	✓	×



ACTIVITIES

7.1 Determine the **distance** between the two given points that follow:

7.1.1 $C\left(5;\frac{1}{2}\right)$ and $D\left(\frac{1}{3};\frac{1}{4}\right)$ (3) **L1**

7.1.2 $E\left(8;\frac{1}{6}\right)$ and $F\ (-13;-2)$ (3) **L1**

7.1.3 $G\left(-12;-\frac{4}{3}\right)$ and $H(-5;-1)$ (3) **L1**

[09]

7.2 Determine the missing variable's value(s) if the length and points are given

7.2.1 $D_{WX} = 7.3276\ldots$units $W(1:-2)$ and $X\left(a;\frac{1}{6}\right)$ (3) **L2**

7.2.2 $D_{YZ}=\frac{\sqrt{205}}{3}$ units $Y(4;-1)$ and $Z\left(-\frac{1}{3};b\right)$ (3) **L2**

7.2.3 $D_{AB}=\sqrt{178}$ units $A(1;c)$ and $B(4;16)$ (3) **L2**

[09]

ACTIVITY 2

7.3. Determine the midpoint for each of the following pairs of coordinates

7.3.1 $A(-1;-9)$ and $B(1;3)$ (2) **L1**

7.3.2 $C\left(-\frac{1}{2};-2\right)$ and $D\left(-1\frac{1}{2};7\right)$ (2) **L1**

7.3.3 $E(3;6)$ and $F(5;-5)$ (2) **L1**

[06]

7.4 Determine the other coordinate if the first coordinates and the midpoint is given

7.4.1 $K(-2;1)$, $L\ (x;y)$, $M_{KL}\ (-6;-6)$ (3) **L2**

7.4.2 $N(8;3)$, $N\ (x;y)$, $M_{NO}(6;5)$ (3) **L2**

7.4.3 $P(1;-8)$, $Q(x;y)$, $M_{PQ}\left(\frac{1}{5};\frac{1}{4}\right)$ (3) **L2**

[09]

ACTIVITY 3

7.5. Given two points on the line, determine the equation of the line

7.5.1 $A\left(\frac{4}{5};-5\right)$ and $B\left(-\frac{1}{3};-\frac{1}{3}\right)$ (2) **L2**

7.5.2 $C\left(1\frac{1}{2};3\right)$ and $D\left(-1;-\frac{1}{2}\right)$ (2) **L2**

7.5.3 $E(-3;4)$ and $F\left(-3;-\frac{1}{4}\right)$ (2) **L2**

[06]

7.6. 7.6.1 Determine the gradient of the new line, given that $m_{AB} = 2$:

(a) Is parallel to AB (2) **L2**

(b) Is perpendicular to AB (3) **L2**

7.6.2 Determine the gradient of the new line, given that $m_{AB} = -\frac{1}{3}$:

(a) Is parallel to AB (2) **L2**

(b) Is perpendicular to AB (3) **L2**

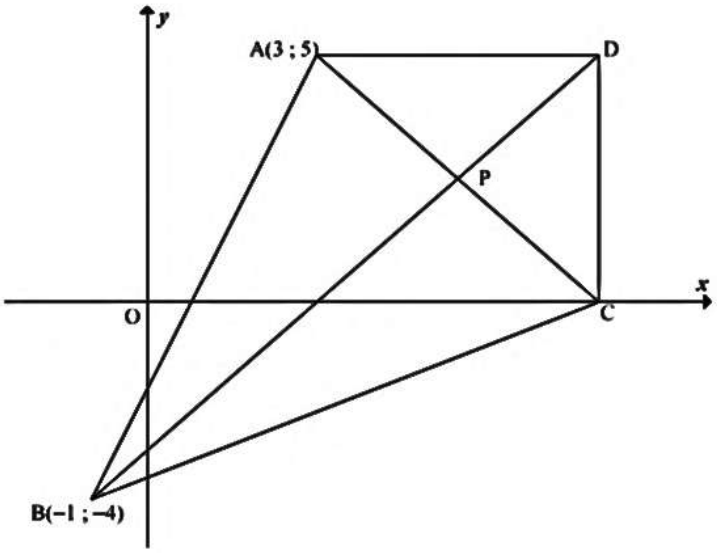
[10]

DBE/ NOVEMBER 2016

7.7 Show that a triangle ABC, with vertices A $(1;1)$; B $(3;6)$ and C $(6;3)$, is an isosceles triangle (4) **L2**

[04]

7.8 In the diagram below, ABCD is a kite with A $(3;5)$ and B $(-1;-4)$ and AB $=$BC. D is a point such that AD is parallel to the $x-axis$ and AD $=5$units. CD is perpendicular to the $x-axis$. The diagonals intersect at P



7.8.1 Show that the coordinates of C are $(8;0)$ (2) **L2**

7.8.2 Write down the coordinates of point P (2) **L2**

7.8.3 Calculate the gradient of line BD (2) **L2**

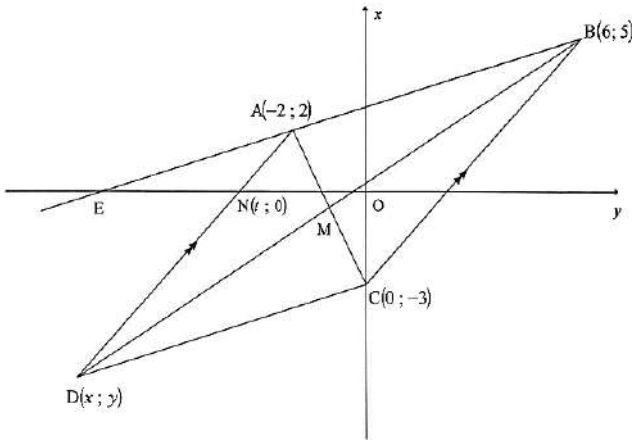
7.8.4 Calculate the length of line AC (2) **L2**

7.8.5 Calculate the area of the kite ABCD (3) **L2**

[11]

DBE/NOVEMBER 2018

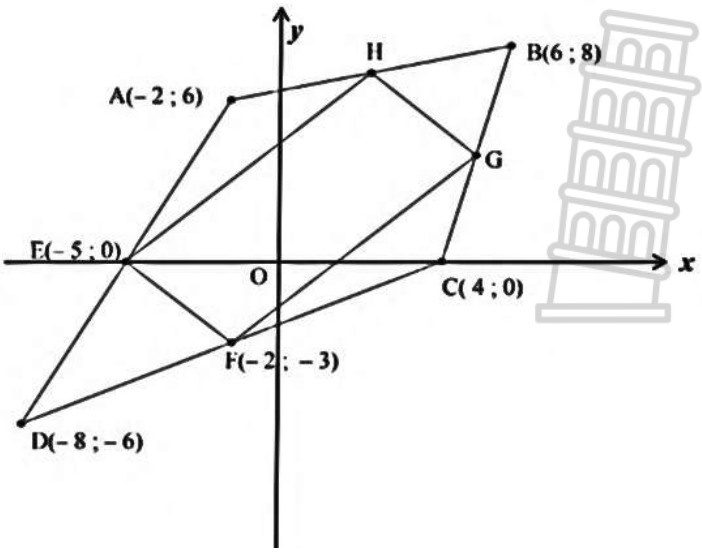
7.9 In the diagram, A $(-2;2)$, B $(6;5)$, C $(0;-3)$ and D $(x;y)$ are the vertices of a quadrilateral having AD || BC. BA produced has an x-intercept at E. BD and AC intersect at M. N $(t;0)$ is a point on A



7.9.1	Calculate the gradient of BC	(2)	**L1**
7.9.2	Determine the equation of AD	(3)	**L3**
7.9.3	Determine value of t	(2)	**L1**
7.9.4	Calculate the length of AN	(2)	**L1**
7.9.5	If DC is defined by $y=\frac{3}{8}x-3$, determine the coordinates of DC	(2)	**L2**
7.9.6	Prove that ABCD is a parallelogram	(3)	**L2**
7.9.7	Calculate the coordinates of M	(3)	**L1**
		[17]	

EC/NOVEMBER 2019

7.10 In the diagram below, H and G- are the midpoints of AB and BC respectively. The coordinates of A $(-2;6)$, B $(6;8)$, C $(4;0)$, D $(-8;-6)$, E $(-5;0)$ and F $(-2;-3)$ are given. The diagram is not necessarily drawn to scale



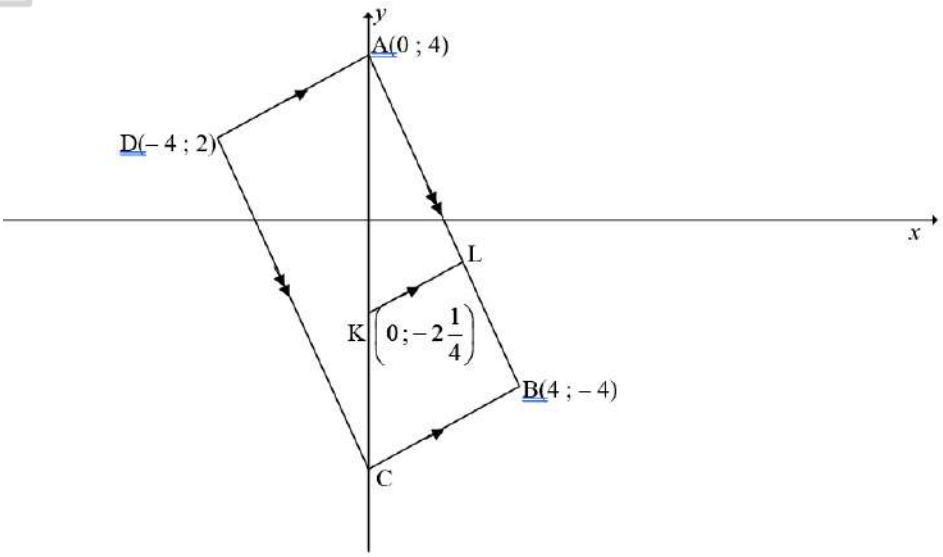
7.10.1	Show by calculation that AB = BC	(5)	**L1**
7.10.2	If it is further given that AD = DC, what type of quadrilateral is ABCD? Motivate your answer.	(2)	**L1**
7.10.3	Determine the coordinates of G and H	(5)	**L3**

7.10.4 If line BD is drawn and it is also given that EH || BD, prove that $\Delta AEH \;|||\; \Delta CDB$ (4) **L2**

[16]

DBE/NOVEMBER 2015

7.11 In the diagram, C is a point on the y axis such that A $(0;4)$, B $(4;-4)$, C and D $(-4;2)$ are vertices of parallelogram ABCD. K is the point $\left(0;-2\frac{1}{4}\right)$ and L is a point on AB such that KL || CB



7.11.1 Calculate the length of diagonal DB (3) **L2**

7.11.2 Calculate the coordinates of M, the midpoint of DB (3) **L1**

7.11.3 Calculate the gradient of AD (3) **L2**

7.11.4 Prove that AD $\perp$ AB. (3) **L3**

7.11.5 Give reason why parallelogram ABCD is a rectangle (1) **L2**

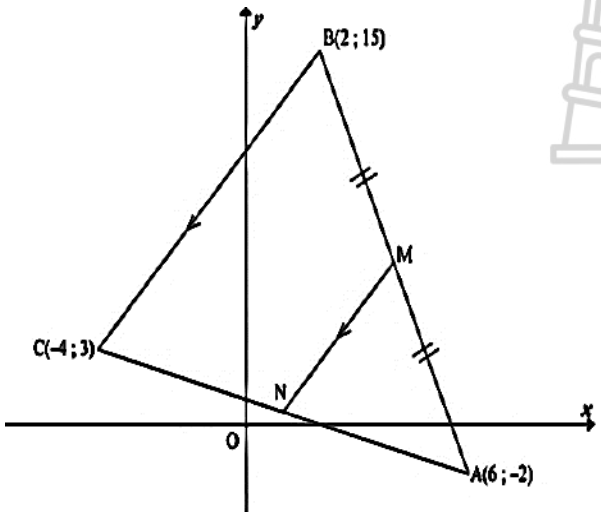
7.11.6 Determine the equation of KL in the form $y = mx + c$ (2) **L2**

7.11.7 Write down, with reasons, the coordinates of C (3) **L3**

[18]

DBE/NOVEMBER 2016

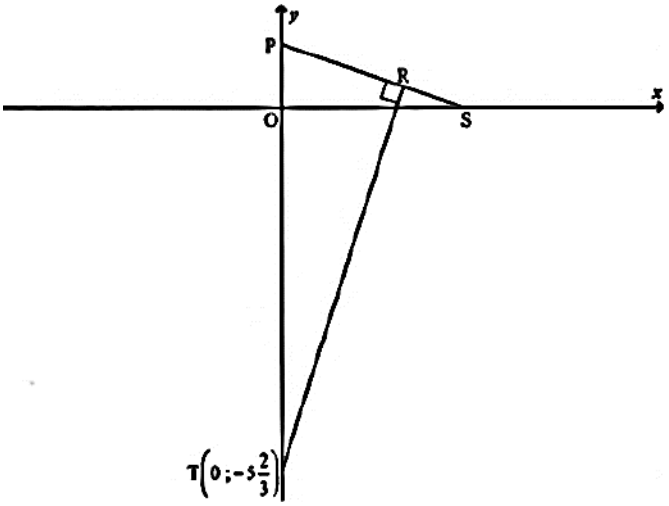
7.12 In the diagram, A $(6;-2)$, B $(12;15)$ and C $(-4;3)$ are the vertices of ΔABC. M is the midpoint of AB. N is a point on AC such that MN || BC.



7.12.1 Determine the coordinates of M, the midpoint of AB (2) **L1**
7.12.2 Determine the gradient of line MN (3) **L2**
7.12.3 Hence, or otherwise, determine the equation of line MN, in the form $y = mx + c$ (2) **L2**
7.12.4 Calculate, with reasons, the coordinates of point N (4) **L2**
7.12.5 If ABCD (in that order) is a parallelogram, determine the coordinates of point D (4) **L3**
[15]

DBE/NOVEMBER 2015

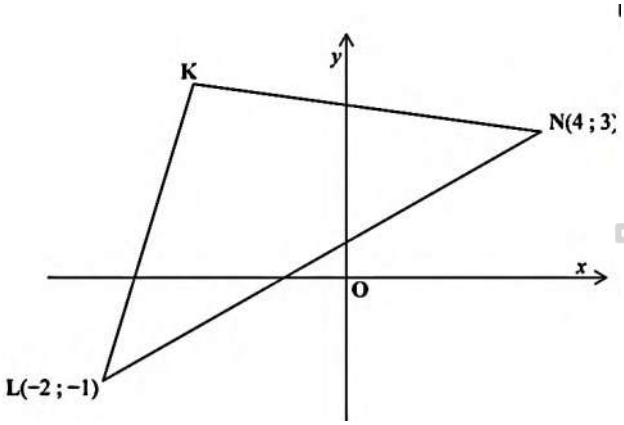
7.13 In the diagram, the straight-line SP is drawn having S and P as its x- and y-intercepts respectively. The equation of SP is $x + ay - a = 0, a > 0$. It also given that OS = 3OP. The straight-line RT is drawn with R on SP and RT $\perp$ PS. RT cuts the y-axis in T $\left(0; -5\frac{2}{3}\right)$.



7.13.1 Calculate the coordinates of P (2) **L2**
7.13.2 Calculate the value of a (2) **L3**
7.13.3 Determine the equation of RT in the form $y = mx + c$ if it is given that $a = 3$ (3) **L3**
7.13.4 Calculate the coordinates of R, the point where PS and TR meet (4) **L2**
7.13.5 Calculate the area of ΔPRT if it is given that R $\left(2; \frac{1}{3}\right)$ (3) **L3**
7.13.6 Calculate, giving reasons, the radius of a circle passing through the points P, R and T (2) **L4**
[16]

GAUTENG NOVEMBER 2022

7.14 Consider ΔKLN drawn below. KL has the equation $y = 5x + 9$ while KN has the equation $5y + x - 19 = 0$.



7.14.1 Show that the coordinates of K are $(-1; 4)$ (3) **L3**
7.14.2 Show that KL $\perp$ KN (3) **L2**
7.14.3 Hence, or otherwise, determine the area of ΔKNL (4) **L2**
7.14.4 Determine the equation of the perpendicular bisector of LN (4) **L3**
7.14.5 If L, N and P(7 ; y) are collinear, find y (2) **L2**

7.14.6 Determine the coordinates of Q, if KLQN is a parallelogram (2) **L3**

7.14.7 Explain, with geometric reasons, why KLQN is a square (2) **L3**

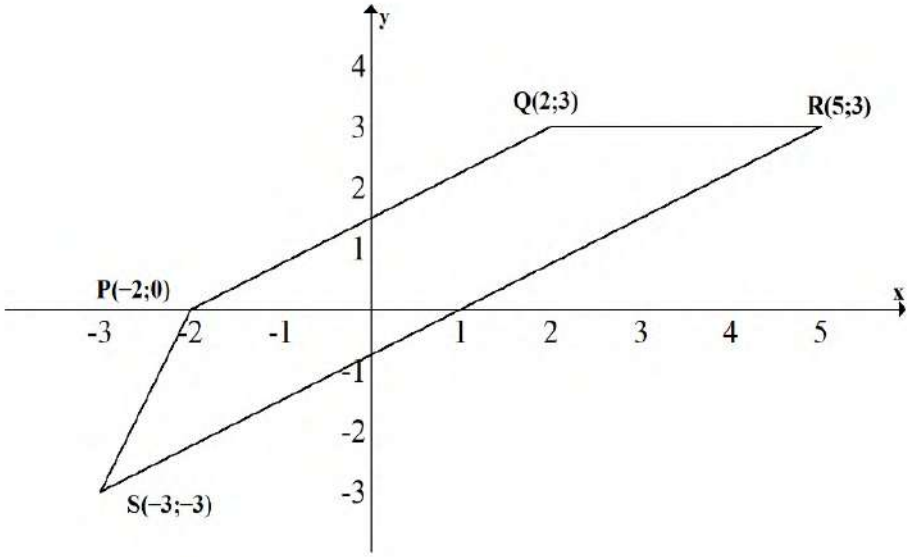
[26]

SIYAVULA

7.15 ΔPQR has vertices $P(1;8)$, $Q(8;7)$ and $R(7;0)$. Show through calculations that ΔPQR is a right-angled isosceles triangle. (6) **L3**

[06]

7.16 PQRS are the points $(-2;0),(2;3)(5;3)$ and $(-3;-3)$ respectively.



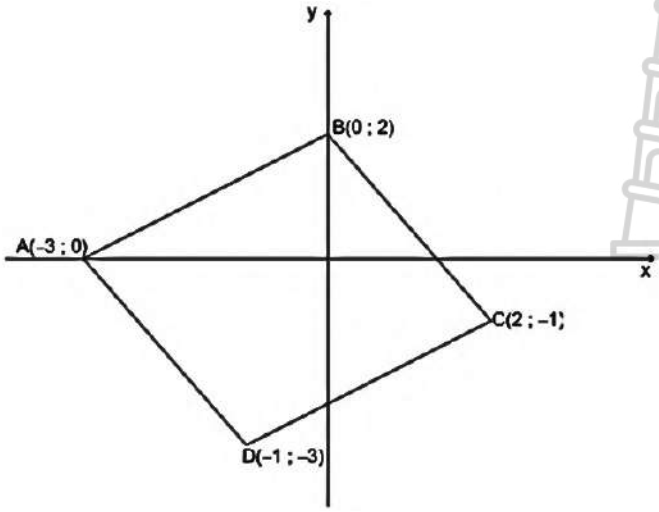
7.16.1 Show that SR=2PQ (4) **L2**

7.16.2 What kind of quadrilateral is PQRS (4) **L2**

[08]

KZN/November 2024

7.17 In the cartesian plane below $A(-3;0)$, $B(0;2)$, $C(2;-1)$ and $(-1;-3)$ are points on a quadrilateral.



7.17.1 Determine the coordinates of R, the midpoint of AC (3) **L2**

7.17.2 Calculate the gradient of the line AB (2) **L2**

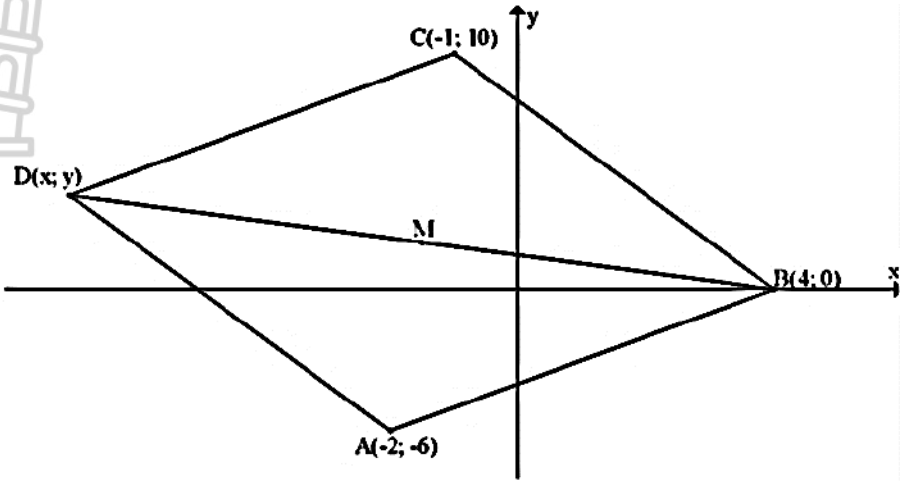
7.17.3 Calculate the length of AC (3) **L2**

7.17.4 If ABCD is a rhombus, calculate the area of ABCD (5) **L2**

[13]

EC/NOVEMBER 2020

7.18 ABCD is a parallelogram with A $(-2;-6)$, B(4 ; 0), C $(-1;0)$ and D $(x;y)$ as shown below



7.18.1 Calculate the length of BC (2) **L1**

7.18.2 Determine the gradient of AB (2) **L1**

7.18.3 Determine the equation of CD (3) **L2**

7.18.4 Determine the coordinates of M, the midpoint of BD (3) **L1**

7.18.5 HENCE, OR OTHERWISE, determine the values of x and y (3) **L1**

[13]

NW/JUNE 2018

7.19 A $(-1;-1)$, B(2 ; 0) and C(5 ; p) are three points on the Cartesian plane.

Determine the value(s) of p if:

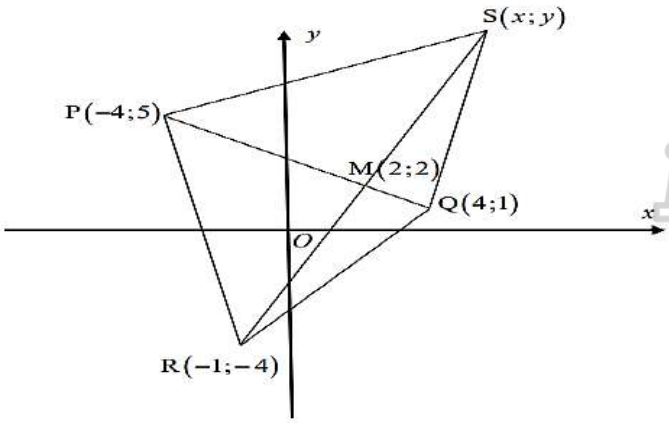
7.19.1 A, B, and C are collinear (2) **L1**

7.19.2 AB is perpendicular to BC (3) **L2**

7.19.3 The length of BC is 5 units (3) **L2**

[08]

7.20 In the diagram P(-4 ; 5), Q(4 ; 1) and R(-1 ; -4) are the vertices of a triangle in the Cartesian plane with M on PQ. M(2 ; 2) is a midpoint of straight line RS.



7.20.1 Determine the gradient of PQ (2) **L1**

7.20.2 Show that $P\widehat{M}S = 90°$ (3) **L2**

7.20.3 Determine the coordinates of S (3) **L2**

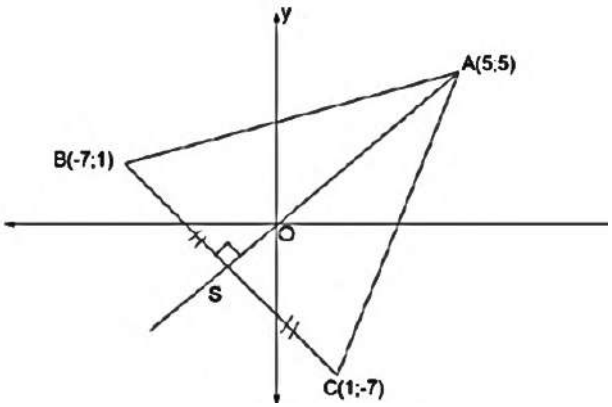
7.20.4 Prove that ΔQRS is isosceles (3) **L2**

7.20.5 Determine the area of ΔPRS (5) **L2**

[16]

GP/November 2021

7.21 In the cartesian Plane below, points $A(5;5)$, $B(-7;1)$, $C(1;-7)$ are the vertices of a triangle. Point S is the midpoint of BC. The line AS is perpendicular to BC



7.21.1 Determine the coordinates of S (3) **L2**

7.21.2 Determine the gradient of the line AC (2) **L2**

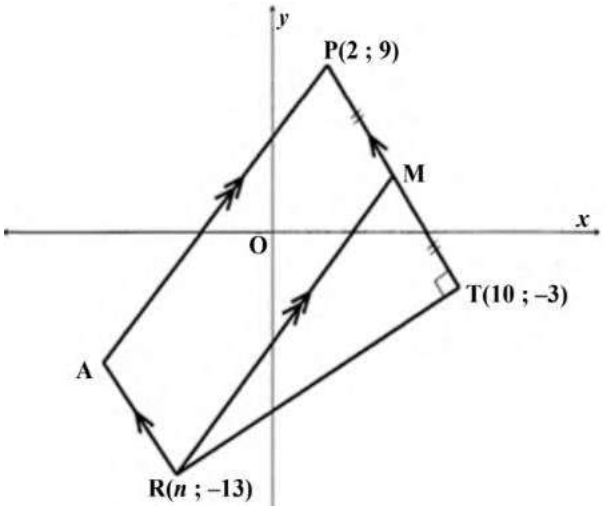
7.21.3 Determine the distance of BC (3) **L1**

7.21.4 Determine the area of ΔABC (5) **L2**

[12]

GP/November 2023

7.22 In the diagram below, P $(2;9)$, A, R $(n;-13)$ and M are vertices of a parallelogram PARM. PTM is a straight line such that M is a midpoint of PT. T $(10;-3)$ is a point such that PT $\perp$ RT.



7.22.1.(a) Determine:

7.22.1. The length of PT (Leave your answer in surd form) (2) **L2**

7.22.2 The gradient of PT (2) **L2**

7.22.3 The gradient of AR (1) **L2**

7.22.4 The coordinates of M (3) **L2**

7.22.1.(b) Determine the equation of PM in the form $y = mx + c$ (3) **L2**

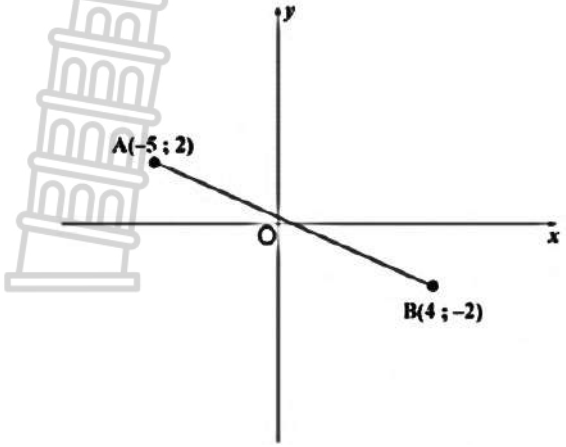
7.22.1.(c) Show that $n = 5$ (3) **L3**

7.22.1.(d) Calculate the area of ΔRMT (4) **L2**

[18]

GP/June 2024

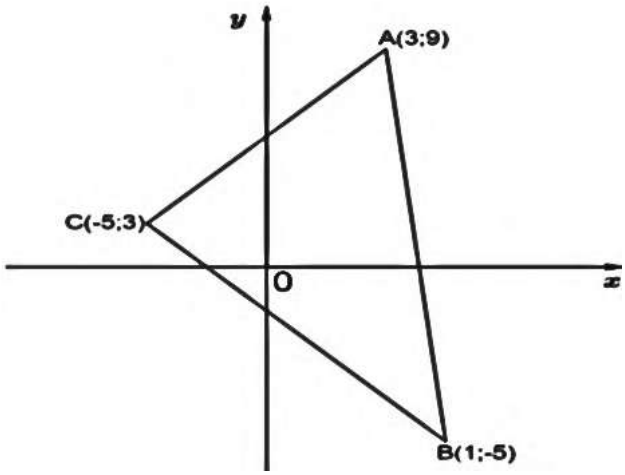
7.23 In the figure below, A $(-5;2)$ and B $(4;-2)$ are points on a line segment AB



7.23.1	Determine the length of AB	(3)	**L2**
7.23.2	Determine the coordinates of the midpoint of AB	(3)	**L2**
7.23.3	Determine the gradient of line AB	(3)	**L2**
		[09]	

GP/June 2023

7.24 Study the diagram below and answer the questions that follow.



7.24.1	Calculate the midpoint of a line segment AB	(3)	**L2**
7.24.2	Calculate the gradient of the line AC	(2)	**L2**
7.24.3	Calculate the gradient of the line CB	(2)	**L2**
7.24.4	Show that AC is perpendicular to CB	(1)	**L3**
7.24.5	Determine the length of the line segment AC	(2)	**L2**
7.24.6	Determine the length of the line segment CB	(2)	**L2**
7.24.7	Determine the area of ΔABC	(2)	**L2**
		[14]	

TOPIC 8. TRIGONOMETRY

FROM CAPS DOCUMENT: GR. 10 TRIGONOMETRY:

- Define the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$ using a right-angled triangle ($S_{OH}C_{AH}T_{OA}$).
- Define the reciprocals of the trigonometric ratios $\text{cosec}\,\theta$, $\sec\theta$ and $\cot\theta$ using a right-angled triangle.
- Derive values of the trigonometric ratios for the special angles (without using a calculator). $\theta \in \{0°;30°;45°;60°;90°\}$.
- Solve simple trigonometric equations for angles between $0°$ and $90°$.
- Extend the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for $0° \le \theta \le 360°$.
- Use diagrams to determine the numerical values of ratios for angles from $0°$ to $360°$.
- Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $0° \le \theta \le 360°$
- Determination of the period, amplitude and range of the graphs.
- Study the effect of on the graphs defined by $y = a\sin\theta + q$, $y = a\cos\theta + q$ and $y = a\tan\theta + q$ for $0° \le \theta \le 360°$.
- Sketch graphs, find equations of given graphs and interpret graphs.

Solve two-dimensional problems involving right-angled triangles.

1. **Right angled triangles:**

✓ Triangle with one angle equals to $90°$. The side opposite to the right angle (the longest side) is called the hypotenuse.

Learners:

✓ Need to be able to use the Pythagoras Theorem to calculate the length of the missing side given two sides.

✓ Be able to label the sides of a right-angled triangle as **opposite, adjacent and hypotenuse.** The hypotenuse is the **only** side that is fixed. The other two sides are labelled as **opposite** or **adjacent** depending on which angle you are using as the point of reference.

2. **Defining trigonometric ratios:**

✓ Define the trigonometric ratios $\sin\alpha, \cos\alpha,$ and $\tan\alpha$ using a right-angled triangle.

✓ In a right-angled triangle: $\sin\alpha = \dfrac{\text{opposite}}{\text{hypotenuse}}, \cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan\alpha = \dfrac{\text{opposite}}{\text{adjacent}}$.

✓ SOH CAH TOA helps you to remember these definitions.

✓ Define the reciprocals of the trigonometric ratios; $\text{cosec}\theta = \dfrac{1}{\sin\theta}, \sec\theta = \dfrac{1}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$ using right-angled triangles. $\text{cosec}\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}, \sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$ and $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

3. **Calculating the trigonometric ratios of a given angle:**

Learners:

✓ Should be able to calculate the trigonometric ratios of a given angle.

✓ Must be able to round off answers correctly to a specific degree of accuracy.

✓ Need to able to make correct substitutions, demonstrate sound calculator skills and make a general statement based on the answers obtained.

4. **Solving simple trigonometric equations:**

Learners must be able to:

✓ Determine the size of an angle when given its *sine, cosine, and tangent ratio.*

✓ Derive values of the trigonometric ratios for the special cases (without using a calculator)

- ✓ Need to be able to determine the length of a side by creating an equation using a ratio and the given angle.
- ✓ Be able to determine the size of an angle by creating an equation involving a ratio of the known sides and an unknown angle.
- ✓ Solve problems involving angles of elevation and depression.

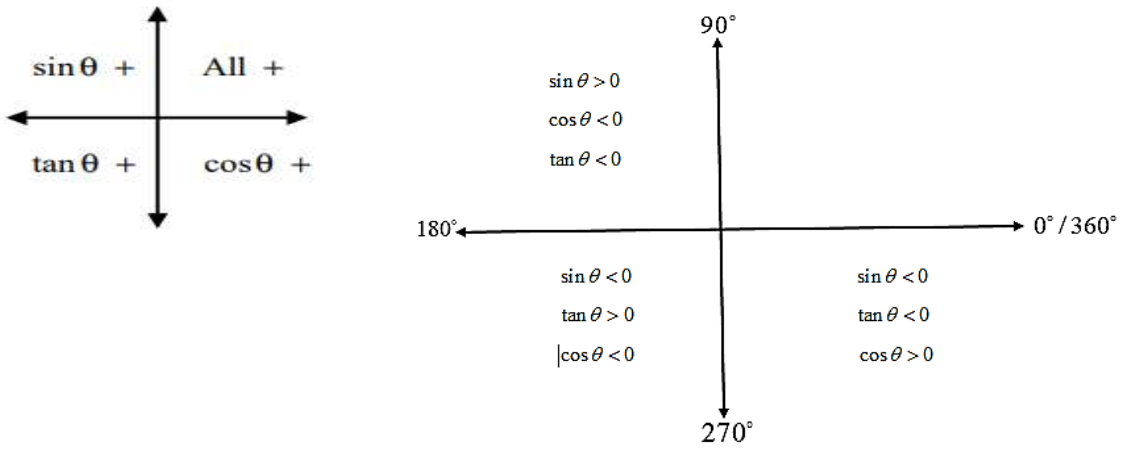
5. Angles in the Cartesian plane:

- ✓ Define trig functions of α using a Cartesian plane:

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r},\ \cos\alpha = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r},\ \tan\alpha = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

Important notes:

- ➢ All trigonometric functions are positive in the first quadrant.
- ➢ $\sin\theta$ is positive in the second quadrant and $\tan\theta$ and $\cos\theta$ are negative.
- ➢ $\tan\theta$ is positive in the third quadrant and $\sin\theta$ and $\cos\theta$ are negative.
- ➢ $\cos\theta$ is positive in the fourth quadrant and $\sin\theta$ and $\tan\theta$ are negative.



- ✓ Use diagrams to determine the numerical values of ratios for angles between $0°$ to $360°$.

6. Sketching graphs of the trigonometric functions:

- ✓ Point by point plotting of graphs defined by $y = \sin\theta, y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [0°; 360°]$

Learners need to know and follow these steps when sketching a trig function:

- ➢ Use a table method of specific values of θ and the corresponding y-values.
- ➢ At least know the basic shape of the graph.
- ➢ Draw the asymptotes (where applicable) using the broken line (dotted line).

- ✓ Study the effect of ***a*** and ***q*** on the graphs defined by $y = a\sin\theta + q$, $y = a\cos\theta + q$ and $y = a\tan\theta + q$ for $\theta \in [0°; 360°]$

Summary of the trigonometric functions

For graphs of the form $y = a\sin x + q$ and $y = a\cos x + q$:

The value of a (ignoring negative signs) represents the vertical stretch of the graph as we as the Amplitude.

If a is negative, then there is a reflection in the x-axis.

The value of q represents a vertical shift of the graph of $y = \sin x$ or $y = \cos x$ up or down.

The period of these graphs is $360°$.

For graphs of the form $y = \tan x + q$:

The value of a (ignoring negative signs) represents a vertical stretch of the graph $y = \tan x$ from the

x-axis. The critical points for $y = a\tan x$ are $(45°; a), (135°; -a), (225°; a)$ *and* $(315°; -a)$.

If a is negative, then there is a reflection in the x-axis.

The value of q represents a vertical shift of the graph $y = \tan x$ up or down.

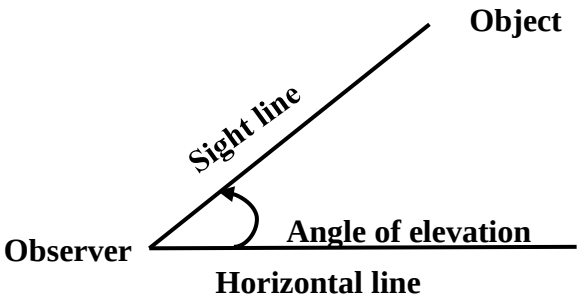
The equations of the asymptotes are $x = 90° + k.180°$ where k represents integer values.
The x-intercepts are at the points $(0° + k.180°; 0)$ where k represents integer values.
The period of these graphs is $180°$.

7. Interpretation of trigonometric functions:
Determine:

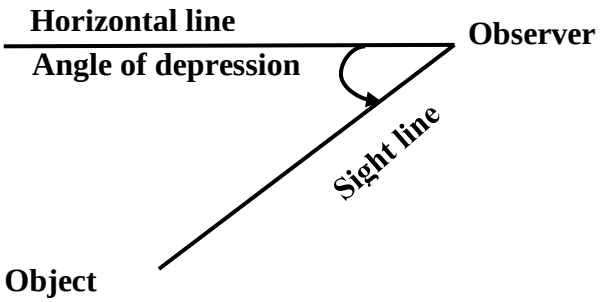
- The minimum value and maximum value.
- The range and domain of the graph.
- The amplitude of the graph.
- Determining the equations of sketched graphs.
- Interval in which the graph increases or decreases.
- x-value(s) at the point of intersection of the graphs.
- The length/distance of a vertical and horizontal line (between two points).

Solving 2D problems:

Angle of elevation represents the angle from the horizontal line upward to an object. An observer's line of sight would be above the horizontal line.

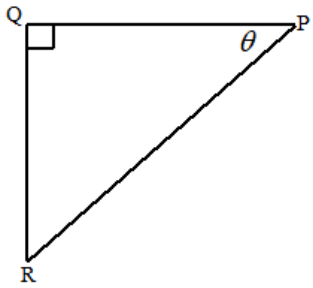


✓ **Angle of depression** represents the angle from the horizontal line downward to an object. An observer's line of sight would be below the horizontal line.



ACTIVITIES

8.1 In the diagram, ΔPQR is a right-angled triangle. $\hat{Q} = 90°$, $\hat{P} = \theta$, $PQ = 5$ units and $PR = 13$ units.



Determine the numerical value of:

8.1.1 QR (2) **L1**

8.1.2 $\cos\theta$ (2) **L1**

8.1.3 $\cot\theta$ (2) **L1**

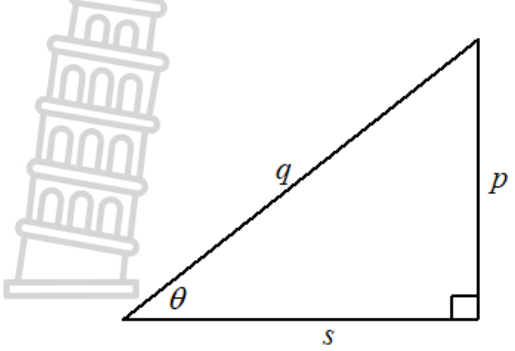
8.1.4 $\sin(90° - \theta)$ (2) **L2**

8.1.5 $\cos(90° - \theta)$ (1) **L2**

8.1.6 $\text{cosec}(90° - \theta)$ (2) **L2**

8.1.7 Write down the trigonometric ratio that is equal to $\frac{QR}{PR}$. (1) **L1**

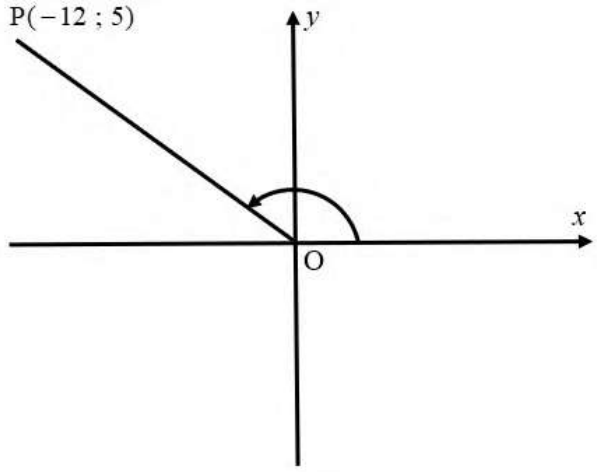
8.2 A right-angled triangle with sides p, q and s is shown below:



Determine the following in terms of p, q and s.

8.2.1	$\tan\theta$	(2)	**L1**
8.2.2	$\cot(90°-\theta)$	(2)	**L2**
8.2.3	$-\sin\theta$	(2)	**L1**
8.2.4	$\sin(90°-\theta)$	(1)	**L1**
8.2.5	$\sec^2\theta$	(2)	**L1**
8.2.6	If it is given that $p=12$ and $\theta=35°$, determine the numerical value of q.	(2)	**L2**

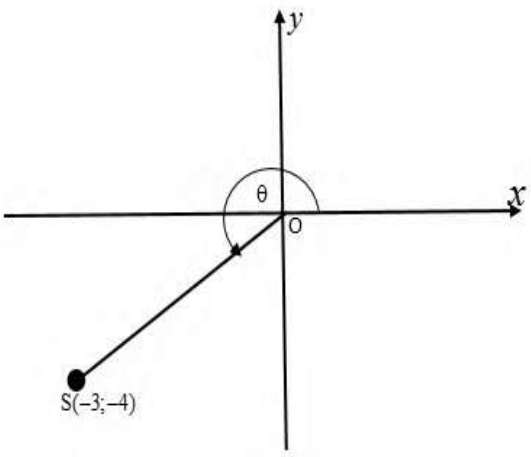
8.3 In the diagram below, $P(-12;5)$ is a point in the Cartesian plane such that $X\hat{O}P=\theta$



Determine the following:

8.3.1	Length of OP.	(2)	**L2**
8.3.2	$\tan\theta$	(3)	**L2**
8.3.3	$(\sin\theta+\cos\theta)^2$	(3)	**L2**
8.3.4	$\cos 60°.\cos\theta$	(2)	**L2**
8.3.5	$4\sin\theta.\cos\theta$	(3)	**L1**
8.3.6	$1-\tan\theta$	(2)	**L1**

8.4 $S(-3;-4)$ is a point on the Cartesian plan such that OS makes an angle of θ with the positive x-axis.



Calculate the following WITHOUT using a calculator:

8.4.1	The length of OS	(2)	**L1**
8.4.2	The value of $\sec\theta+\sin^2\theta$	(3)	**L2**
8.4.3	The value of $\cos\theta+\sin^2\theta$	(3)	**L2**

8.5 If $A=20°$ and $B=55°$, use a calculator to evaluate the following (rounding off to TWO decimal places):

8.5.1	$\cos(B-A)$	(2)	**L1**
8.5.2	$\cos B-\cos A$	(2)	**L1**

8.5.3 $\sin^2 B$ (2) **L1**

8.5.4 $\sin(A+B)$ (2) **L1**

8.5.5 $\sin A + \sin B$ (2) **L1**

8.5.6 $\tan^2 B$ (2) **L1**

8.5.7 $2\text{cosec}A + \sin 5B$ (3) **L1**

8.6 If $A = 65°$ and $B = 38°$, determine, rounding off to TWO decimal places:

8.6.1 $\sin(A-B)$ (2) **L1**

8.6.2 $\cos 2B$ (2) **L1**

8.6.3 $\sec\frac{1}{3}A$ (3) **L1**

8.7 If $\alpha = 50°$ and $\beta = 25°$, determine, rounding off to TWO decimal places:

8.7.1 $\tan\alpha$ (2) **L1**

8.7.2 $\text{cosec}(\alpha-\beta)$ (2) **L1**

8.7.3 $\text{cosec}\alpha - \text{cosec}\beta$ (3) **L1**

8.7.4 $\cos(\alpha+\beta)$ (2) **L1**

8.8 In which quadrant does the terminal arm of the angle θ lie if:

8.8.1 $\sin\theta < 0$ and $\cos\theta < 0$ (1) **L1**

8.8.2 $\tan\theta < 0$ and $\cos\theta > 0$ (1) **L1**

8.8.3 $\cos\theta < 0$ where $\sin\theta > 0$ (1) **L1**

8.9 Given $13\sin x = 5$ where $90° < x < 180°$, with the aid of a diagram, determine:

8.9.1 $\cos x$ (3) **L2**

8.9.2 $\text{cosec}^2 x$ (3) **L2**

8.9.3 $1-\sin^2 x$ (3) **L2**

8.9.4 $\cot\theta + \tan\theta$ (3) **L2**

8.10 Given $7\tan\theta - 5 = 0$ where $\theta \in (90°;270°)$, with the aid of a diagram, determine:

8.10.1 $\sin\theta$ (3) **L2**

8.10.2 $\cot\theta$ (2) **L2**

8.10.3 $5\text{cosec}\theta$ (2) **L2**

8.10.4 $\cos\theta.\tan\theta$ (3) **L2**

8.11 If $\tan\beta = 2,4$ and $\sin\beta < 0$, determine the following with the aid of a diagram:

8.11.1 $\sin\beta$ (3) **L2**

8.11.2 $\cos^2\beta + \sin^2\beta$ (3) **L2**

8.11.3 $\dfrac{\sin\beta}{\cos\beta}$ (3) **L2**

8.11.4 $\dfrac{\sec\beta}{\text{cosec}\beta}$ (3) **L2**

8.11.5 $\dfrac{\text{cosec}45°\cot\beta}{\sec 45°.\sin 30°}$ (5) **L2**

8.12 Solve for x in the following where $x \in [0°;90°]$

8.12.1 $\cos x = \frac{1}{2}$ (2) **L2**

8.12.2 $\frac{7\text{cosec}x}{17} = 1$ (4) **L2**

8.12.3 $-\tan\left(\frac{1}{2}x + 23°\right) = -1$ (4) **L2**

8.12.4 $\sin x = \sin 21{,}5°$ (3) **L2**

8.12.5 $\cos x = \sin 36°$ (3) **L2**

8.12.6 $2\sin x = \tan 38°$ (3) **L2**

8.12.7 $\cot x = \frac{\sin 10°}{12}$ (3) **L2**

8.12.8 If $\cos x = \frac{3}{4}$ and $0° < x < 90°$, determine the value of $\tan x$ (4) **L3**

8.12.9 If $\tan\theta = \frac{6}{8}$ and $\sin\theta < 0$, determine the value of $\sec\theta - \text{cosec}\theta$ (3) **L4**

8.13 Determine the numerical values of the following WITHOUT using a calculator.

8.13.1 $\frac{\cos 60° \times \sin 30° \times 2\sec 60°}{\cot^2 30°}$ (6) **L2**

8.13.2 $\frac{\tan 45° \times \sin^2 45°}{\text{cosec}45°.\cot 60°}$ (6) **L2**

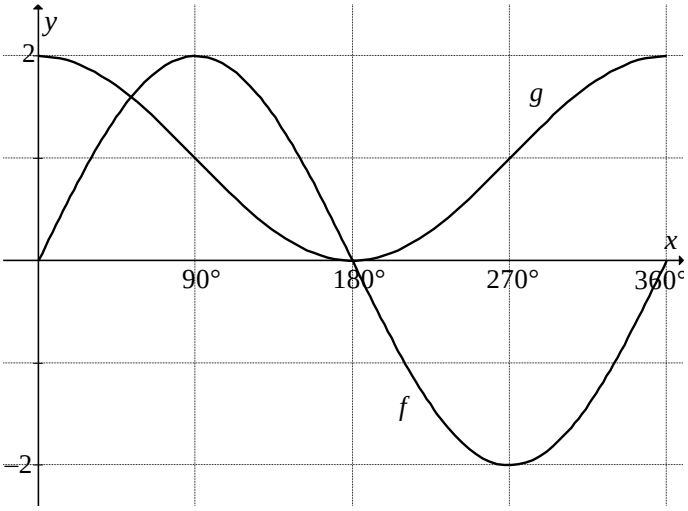
8.13.3 $\frac{\sin 60° \times \cos 30° \times 2\text{cosec}30°}{\tan^2 30°}$ (6) **L2**

8.13.4 $\frac{\text{cosec}45°.\sin 60°}{\sin 45°.\cos 30°}$ (5) **L2**

8.13.5 $\frac{\sin 45°.\tan^2 60°}{\cos 45°} + \sin 30°$ (5) **L2**

KZN NOV 2024

8.14 The graphs of $f(x) = p\sin x$ and $g(x) = \cos x + q$ for $x \in [0°;360°]$ are sketched below:



8.14.1 Determine the values of p and q. (1) **L1**

8.14.2 Write down the amplitude of g. (2) **L1**

8.14.3 Write down the range of f. (3) **L1**

8.14.4 For which values of x will $f(x) > 0$? (3) **L2**

8.14.5 The graph of g is reflected about the x-axis and then shifted 2 units upwards to obtain the graph of h. Write down the equation of h. (4) **L2**

8.15 Consider the function $f(x) = -3\tan x$.

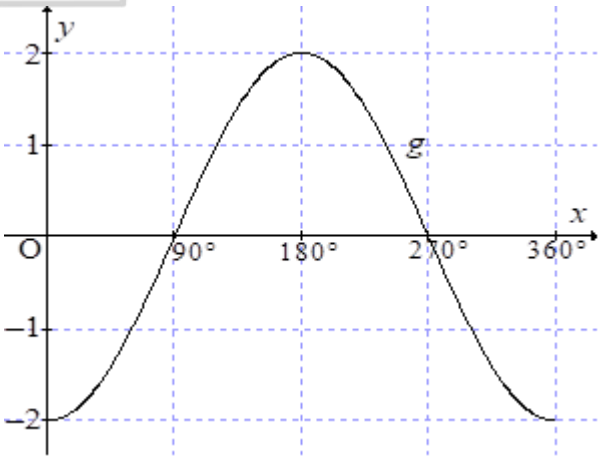
8.15.1 Sketch the graph of f for $0° \le x \le 360°$. Clearly show ALL the intercepts with the axes and the asymptotes. (4) **L2**

8.15.2 Write down the period of f. (3) **L1**

8.15.3 Write down the equation of h if h is the reflection of f about the x-axis. (2) **L2**

DBE NOV 2018

8.16 Sketched below is the graph of $g(x) = a.\cos b\theta$.



8.16.1 Write down the values of a and b. (2) **L1**

8.16.2 Use the graph to determine the value(s) of x for which $g(x) > 0$. (2) **L2**

8.16.3 Determine the range of h if h is the image of g after g is shifted down 2 units. (2) **L2**

8.16.4 Determine using the graph, the value of:

$$-2\left(\cos 0° + \cos 1° + \cos 2° + ... + \cos 358° + \cos 359° + \cos 360°\right)$$ (2) **L4**

KZN NOV 2025

8.17 Given: $g(x) = \cos x + 1$ and $f(x) = 2\sin x$ for $x \in [0°; 360°]$.

8.17.1 Sketch the graphs of f and g on the same set of axes. (6) **L2**

8.17.2 Write down the period of f. (2) **L1**

8.17.3 Determine the range of g. (2) **L1**

8.17.4 For which values of x will $\frac{g(x) - f(x)}{3} = 1$? (3) **L2**

8.18 Consider the function $f(x) = -\sin x$:

8.18.1 Write down the period of *f*. (1) **L1**

8.18.2 Sketch the graph of f for $x \in [0°; 360°]$, clearly showing ALL the intercepts with the axes. (3) **L2**

8.18.3 Determine the values of x for which $f(x) = 0$. (3) **L2**

8.19 Given that $f(x) = -2\cos x$ and $g(x) = -\tan x + 1$

8.19.1 Sketch the two functions on the same system of axes for $x \in [0°; 360°]$, clearly showing ALL the intercepts with the axes. (6) **L2**

8.19.2 Write down the range of *f*. (1) **L1**

8.19.3 Write down the period of *g*. (1) **L1**

8.19.4 Write down the value/s of *x* for which *g* is undefined. (2) **L2**

8.20 Given that $f(x) = -\cos x$ and $g(x) = 2\sin x + 1$

8.20.1 Sketch the graphs of f and g on the same system of axes for $x \in [0°;360°]$, clearly showing ALL the intercepts with the axes. (6) **L2**

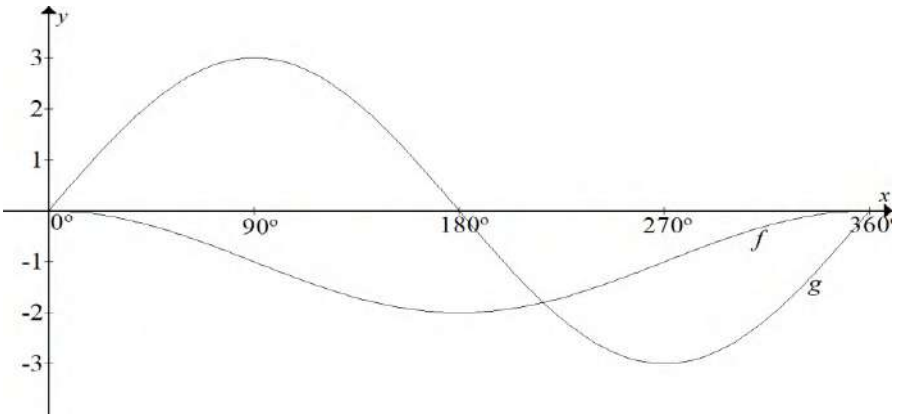
8.20.2 Hence, or otherwise determine the following:

8.20.2.1 The amplitude of g. (1) **L1**

8.20.2.2 The range of g. (1) **L1**

8.20.2.3 The value(s) of x for which $f(x).g(x) \geq 0$ (2) **L2**

8.21 The graphs of $f(x) = \cos x + q$ and $g(x) = a\sin x$ for $x \in [0°;360°]$ are drawn below.

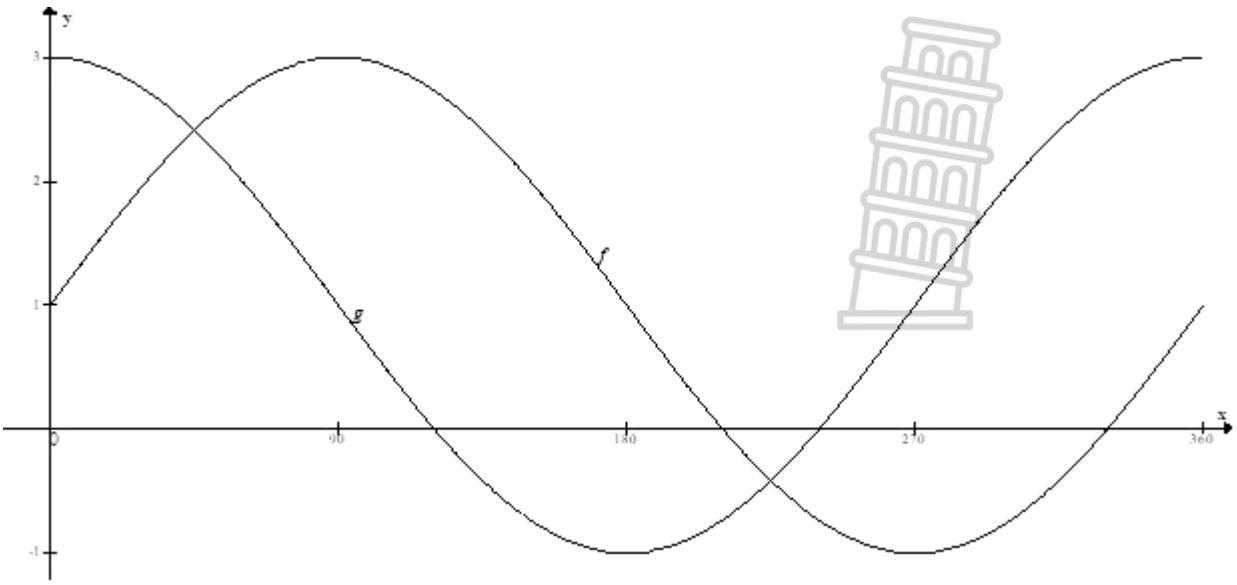


8.21.1 Determine the values of a and q. (2) **L2**

8.21.2 What is the range of f. (1) **L1**

8.21.3 For what value(s) of x is $f(x) = 0$ (2) **L2**

8.22 The graphs of $f(x) = a\sin x + p$ and $g(x) = b\cos x + r$ are shown below.



8.22.1 Determine the values of a, b, p and r. (4) **L2**

8.22.2 Write down the domain and range of f. (2) **L1**

8.22.3 What is the period of g. (1) **L1**

8.22.4 Write down the amplitude of f. (1) **L1**

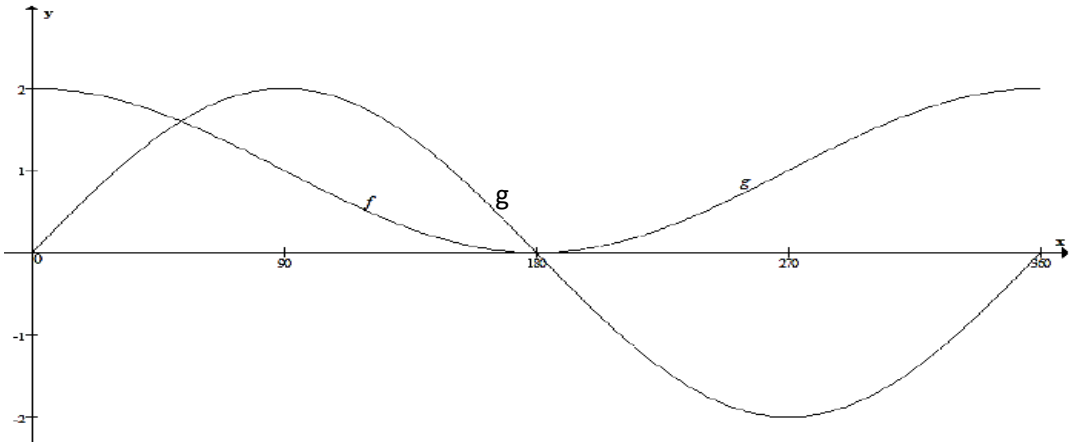
8.23 Given that $f(x) = \tan x$ and $g(x) = -\sin x$ for $x \in [0°;360°]$, answer the following questions.

8.23.1 Sketch the graphs of f and g on the same system of axes for $x \in [0°;360°]$, clearly showing ALL the intercepts with the axes. (6) **L2**

8.23.2 Determine the value(s) of x for which $f(x) = g(x)$ (3) **L2**

8.23.3 If the graph of g is shifted 2 units down to form $h(x)$, write down the range of h. (3) **L3**

8.24 The graphs of $f(x)$ and $g(x)$ are sketched below. Determine:



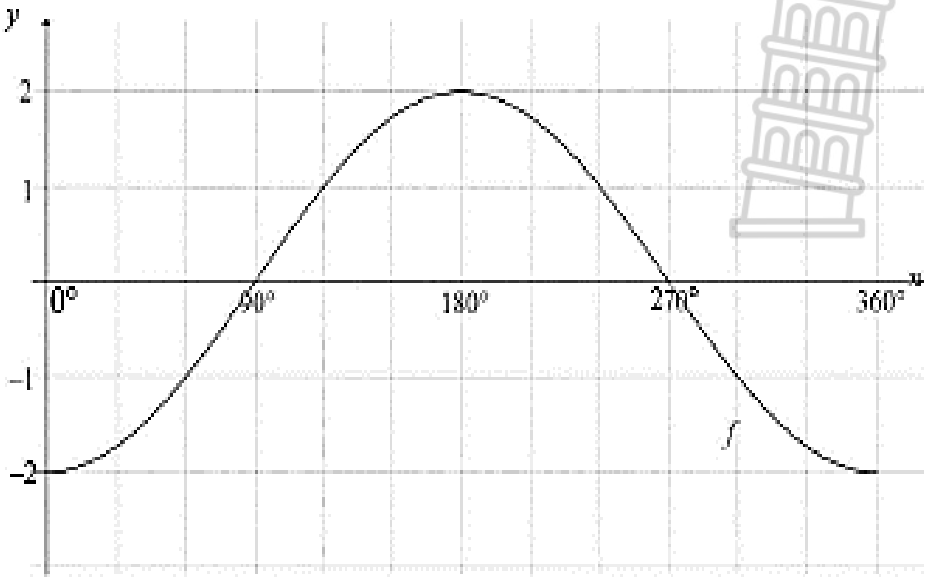
8.24.1 The equations of f and g. (2) **L2**

8.24.2 The range of f and g. (2) **L1**

8.24.3 The amplitude of f and g. (2) **L1**

8.24.4 The period of f and g. (2) **L1**

8.25 In the diagram below, the graph of $f(x) = -2\cos x$ for the interval $0° \le x \le 360°$



8.25.1 Write down the amplitude of f. (1) **L1**

8.25.2 Write down the maximum value of $f(x)+2$. (1) **L3**

8.25.3 On the same system of axes, sketch the graph of for the interval $g(x)=\sin x+1$ for the interval $0^\circ \le x \le 90^\circ$ (3) **L2**

8.25.4 Referring to your graphs, answer the following questions:

8.25.4.1 The value of $f(180^\circ)-g(180^\circ)$ (3) **L3**

8.25.4.2 Write down the values of x for which $f(x)<g(x)$. (4) **L3**

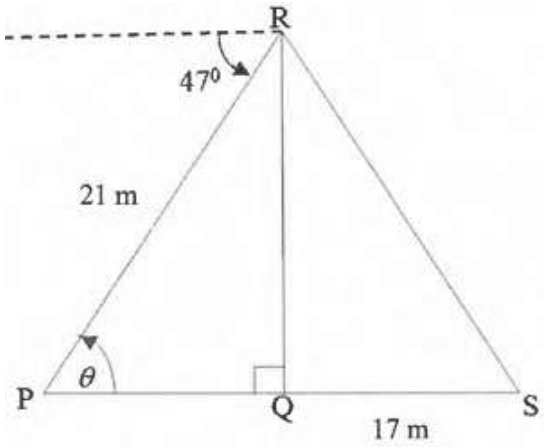
8.25.5 The graph of f is reflected about the *x*-axis, and shifted 2 units down to form the graph of *h*. Determine:

8.25.5.1 The equation of *h*. (2) **L2**

8.25.5.2 The range of *h*. (1) **L1**

DBE NOV 2017

8.26 RQ is a vertical pole. The foot of the pole, Q, is on the same horizontal plane as P and S. The pole in anchored with wire cables RS and RP. The angle of depression from the top of the pole to point P is $47°$. PR is 21 m and QS is 17 m. $RPQ=\theta$.



8.26.1 Write down the size of θ. (1) **L1**

8.26.2 Calculate the length of RQ. (2) **L2**

8.26.3 Hence, calculate the size of $\hat{S}$. (3) **L2**

8.26.4 If P, Q and S lie in a straight line, how far apart are the anchors of the wire cables? (3) **L2**

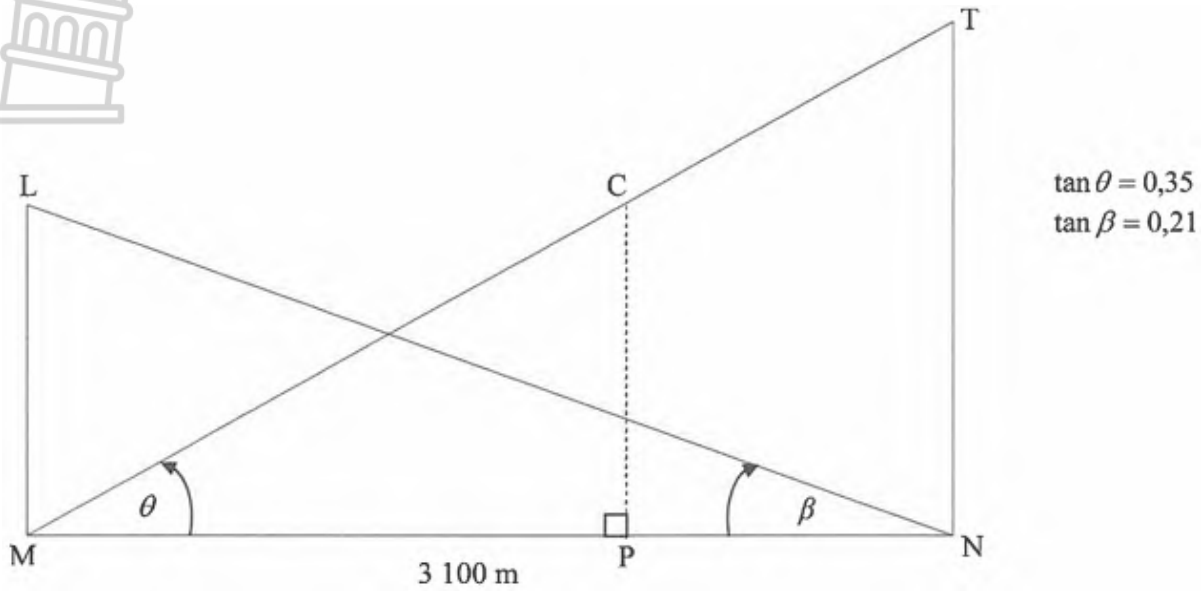
DBE NOV 2016

8.27 The diagram below represents a cross-section of the peaks of Table Mountain, T and Lions Head, L, above sea level. M and N are directly below peaks L and T respectively such that MPN lies on the same horizontal plane at sea level and P is directly below C.

$MN = 3100\,m$

The angle of elevation to L from N is β and the angle of elevation to T from M is θ.

It is given that $\tan\theta = 0,35$ and $\tan\beta = 0,21$.



8.27.1 Calculate the ratio of LM:TN (2) **L1**

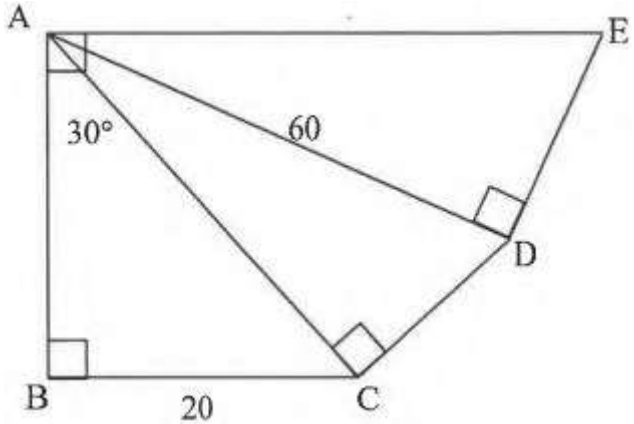
8.27.2 A cable car, C, travelling from the top of the Table Mountain, T, follows a path along TCM.

8.27.2.1 Calculate the angle formed ($M\hat{T}N$) between the cable and the vertical height TN. (2) **L2**

8.27.2.2 If the cable car, C, travels along the cable such that $TC = 400\,m$, calculate the height of the cable car above the sea level at that instant. (5) **L3**

DBE NOV 2018

8.28 In the diagram below ΔABC, ΔACD and ΔADE are right-angled triangles. $B\hat{A}E = 90°$, $B\hat{A}C = 30°$. $BC = 20$ units and $AD = 60$ units.



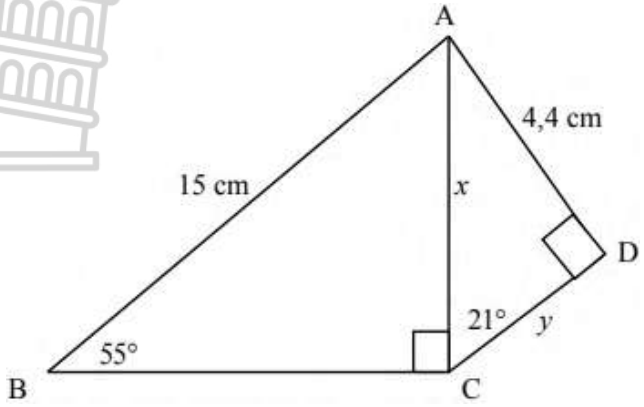
8.28.1 Length of AC. (2) **L2**

8.28.2 The size of $C\hat{A}D$. (2) **L2**

8.28.3 Length of DE. (3) **L2**

EC/NOV2019

8.29 In the diagram below, $AB = 15$ units, $AD = 4,4$ units, $\hat{B} = 55°$, $A\hat{C}D = 21°$ and $A\hat{D}C = 90°$.



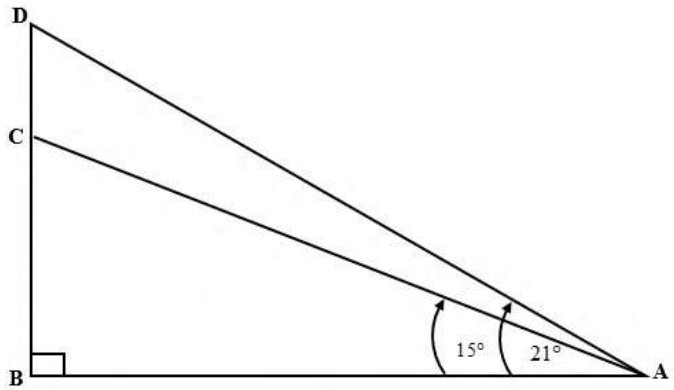
Determine the value of:

8.29.1 x (2) **L2**

8.29.2 y (2) **L2**

GP/NOV2022

8.30

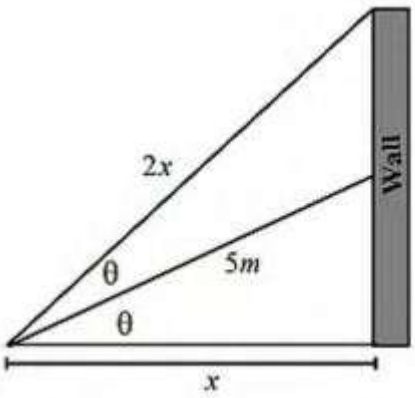


In the diagram alongside, a person is standing at a point A which is 30 m away from point B. point A and point B are on the same horizontal plane. The angle of elevation of the first storey window at point C from point A is $15°$. The angle of elevation of the second window at point D from point A is $21°$.

Determine the distance between the two windows. (6) **L3**

KZN/NOV2023

8.31 A handyman attempts to reach the roof of a hall with a ladder 5 metres in length. Unfortunately, the ladder is too short, and a new ladder will be required. Suppose that the ladder needed to reach the top is double the distance from the foot of the ladder to the wall. Also, the angle between his current ladder and the ground will need to be equal the angle between the two ladders.

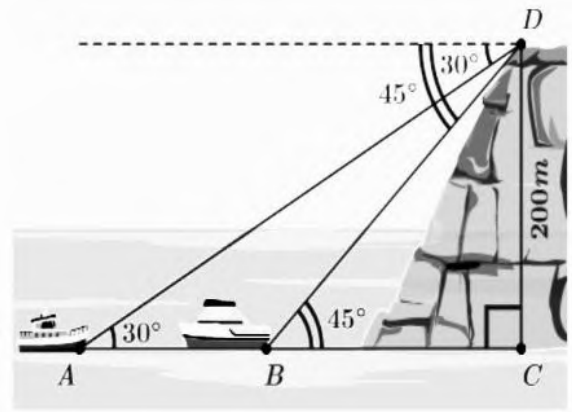


8.31.1 Calculate the value of θ. (4) **L3**

8.31.2 Hence, or otherwise, determine what length the ladder should be to get the handyman to the roof? (4) **L2**

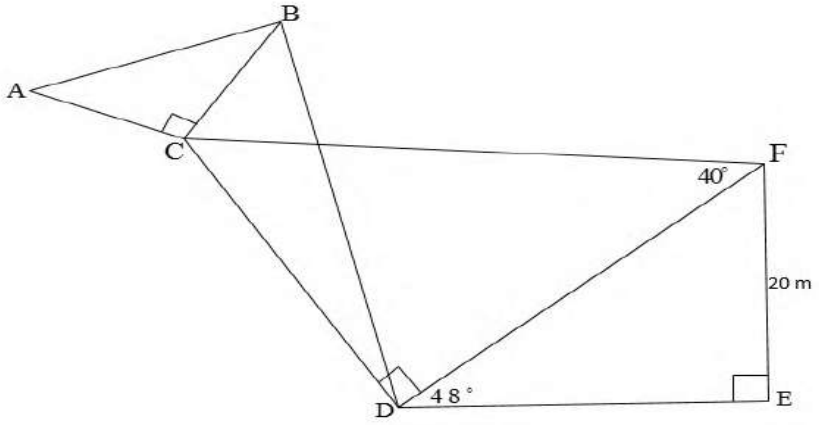
NW/NOV2023

8.32 An observer at the top of the cliff, 200 m above the see level, observes the angle of elevation of the two ships to be $45°$ for ship A, and $30°$ for ship B, as shown in the sketch below:



Determine the size of $B\hat{D}C$. (2) **L2**

8.33 The diagram below represents a crane. Use the information in the diagram to calculate the length of cable **CF.** (5) **L3**



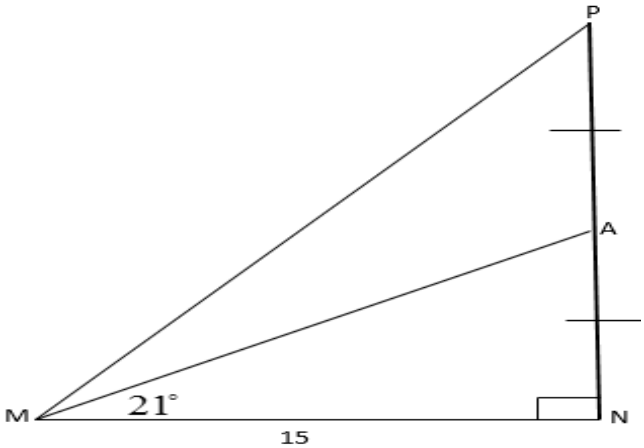
8.34 In the sketch below, ΔMNP is drawn having a right angle at N and $MN = 15$ units. A is the midpoint of PN and $A\hat{M}N = 21°$.

Calculate :

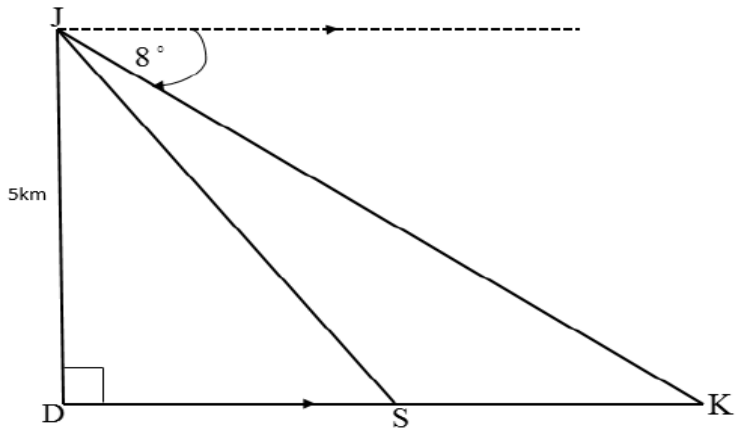
8.34.1 AN (3) **L3**

8.34.2 $P\hat{M}N$ (3) **L3**

8.34.3 MP (3) **L3**



8.35 An aeroplane at J is flying directly over point D on the ground at a height of 5 kilometres. It is heading to land at point along the route from D to K.



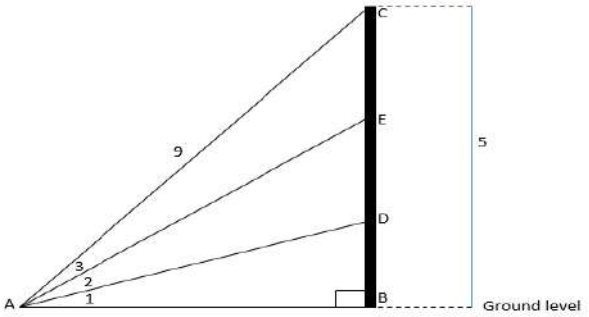
8.35.1 Write down the size of $J\hat{K}D$. (1) **L3**

8.35.2 Calculate the distance DK , correct to the nearest metre. (3) **L2**

8.35.3 If the distance SK is 8 kilometres, calculate the distance DS. (1) **L1**

8.35.4 Calculate the angle of elevation from point S to J, correct to ONE decimal place. (2) **L2**

8.36 In the diagram below, B is the foot of multi-story building. Three people, D, E and C, are standing at the windows on the three different floors. They are all looking at object A on the ground, which is in the same horizontal plane as B. $AC = 9$ units, $BC = 5$ and $\hat{A}_1 = \hat{A}_2 = \hat{A}_3$



Calculate the:

8.36.1 Size of $C\hat{A}B$ (2) **L2**

8.36.2 Length of AE (5) **L4**

8.36.3 Length of DE (4) **L4**

TOPIC 9. EUCLIDEAN GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

LEARNERS ARE EXPECTED TO KNOW:

1) PRIOR KNOWLEDGE

a) Angles – Types of angles according to their values (Acute angles, Right angles, Obtuse angles, Reflex angles and angles around a point)

b) Geometry of parallel lines (Corresponding angles, Alternate angles, co-interior angles, vertically opposite angles)

c) Types of triangles (Acute angled, Obtuse angled, right angled, Isosceles triangle, Equilateral, scalene)

d) Triangle angle geometry (Sum of interior angles, exterior angle of triangle)

e) Pythagoras Theorem

f) Congruency of triangles (SAS, RHS, SAA, SSS)

g) Similarity (Equiangular triangles and sides in equal proportions)

2) GRADE 10 CONTENT

a) Properties of quadrilaterals (Parallelogram, Rectangle, Rhombus, Square, Kite and Trapezium)

b) Investigate and make conjunctures about the properties of sides, angles, diagonals, and areas of quadrilaterals.

c) Proofs of theorems

i) The opposite sides and angles of a parallelogram are equal.

ii) The diagonals of a parallelogram bisect each other.

iii) If one pair of a quadrilateral are both equal and parallel, then the quadrilateral is a parallelogram.

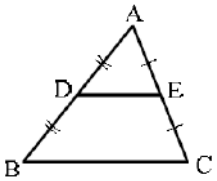
iv) The diagonals of a rectangle are equal.

v) The diagonals of a rhombus bisect each other at right angles and bisect the interior angles of the rhombus.

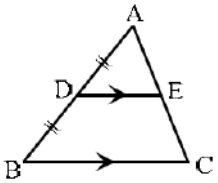
d) Mid-point theorem

i) Acceptable reason for "a line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side" can be abbreviated "line through midpt. || to 2nd side".

The Midpoint Theorem can be stated in the following two ways:



If AD=DB and AE=EC, then DE || BC.



If AD=DB and DE || BC, then AE=EC.

Note: DO NOT state **"converse midpoint theorem"**

e) How to prove:

i) Lines are parallel/perpendicular.

ii) A triangle is right angled.

iii) A triangle is isosceles

iv) A quadrilateral is a parallelogram.

v) A parallelogram is a rectangle/ rhombus/ square.

3) ACCEPTABLE REASONS ACCORDING TO EXAM GUIDELINES

See Examination Guidelines Grade 12, 2021

4) STRATEGIES

a) Learners need to master numerical questions before moving on to proof type questions.

b) PROVING A TRIANGLE IS A RIGHT TRIANGLED TRIANGLE.

Method 1: Show two sides of the triangle are perpendicular

Method 2: Calculate the distances of all three sides and then test the Pythagorean's theorem to show the three lengths make the Pythagorean's theorem true.

c) PROVING A QUADRILATERAL IS A PARALLELOGRAM

Method 1: Show that the diagonals bisect each other by showing the same midpoint of the diagonals

Method 2: Show both pairs of opposite sides are parallel.

Method 3: Show both pairs of opposite sides are equal by using distance.

Method 4: Show one pair of sides is both parallel and equal.

d) PROVING A QUADRILATERAL IS A RECTANGLE

Prove that it is a parallelogram first, then:

Method 1: Show that the diagonals are congruent/equal.

Method 2: Show that it has a right angle for corner angles.

e) PROVING A QUADRILATERAL IS A RHOMBUS

Prove that it is a parallelogram first, then:

Method 1: Prove that the diagonals are perpendicular.

Method 2: Prove that a pair of adjacent sides are equal.

Method 3: Prove that all four sides are equal.

f) **PROVING THAT A QUADRILATERAL IS A SQUARE**

Method 1: Prove the diagonals bisect each other, are equal and perpendicular.

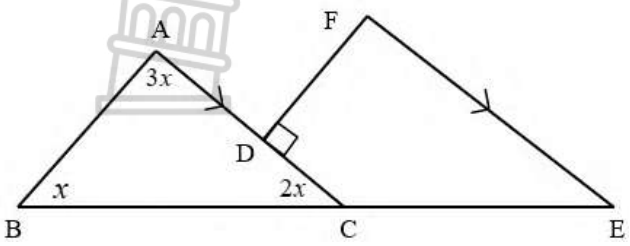
Method 2: Prove the adjacent sides are equal and perpendicular.

ACTIVITIES

NUMERICAL QUESTIONS

9.1 GP NOVEMBER 2023 PAPER 2 QUESTION 8

In the diagram below, a polygon $ABEFD$ is drawn. AD is produced to a point, C, that lies on BC. $AC \parallel FE$. It is further given that $A = 3x$, $B = x$ and $ACB = 2x$.



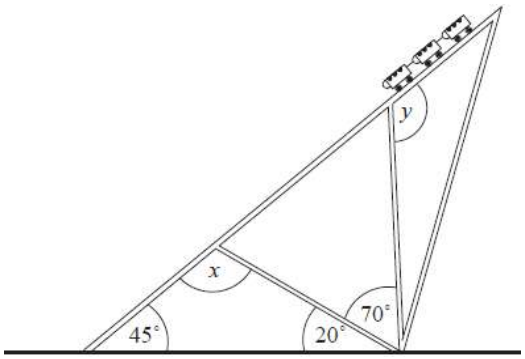
9.1.1 Determine, with reasons, the numerical value of x. (2) **L2**

9.1.2 What type of triangle is ΔABC? Give a reason for your answer. (2) **L1**

[04]

9.2 HSB NOVEMBER 2014 PAPER 2

The diagram shows the Space Shuttle ride at a theme park.



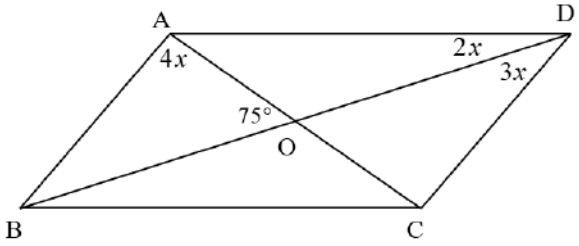
9.2.1 Calculate the value of x (2) **L1**

9.2.2 Calculate the value of y (2) **L1**

[04]

9.3 HSB NOVEMBER 2014 PAPER 2 - AMENDED

In the figure below, $ABCD$ is a parallelogram with $BAO = 4x$, $ADB = 2x$, $BDC = 3x$ and $AOB = 75°$.



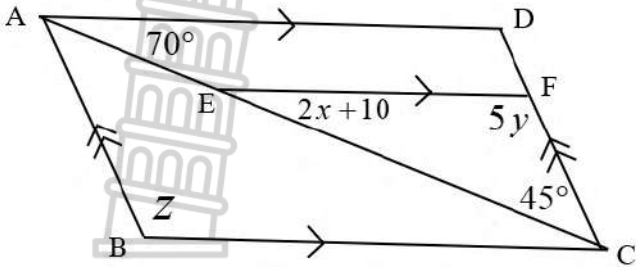
9.3.1 Calculate the size of ABD in terms of x. (1) **L1**

9.3.2 Show that the value of x is $15°$ (3) **L2**

9.3.3 Hence or otherwise, calculate the magnitude of BCD. (3) **L2**

[07]

9.4 **IEB JUNE 2012 PAPER 2 QUESTION 4**

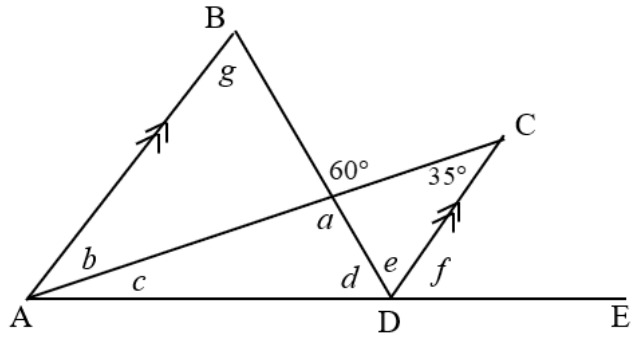


9.4.1 Carefully study the diagram alongside and calculate, with reasons, the values of x, y and z. (3) **L2**

[03]

9.5 **IEB NOVEMBER 2013 PAPER 2 QUESTION 4.1**

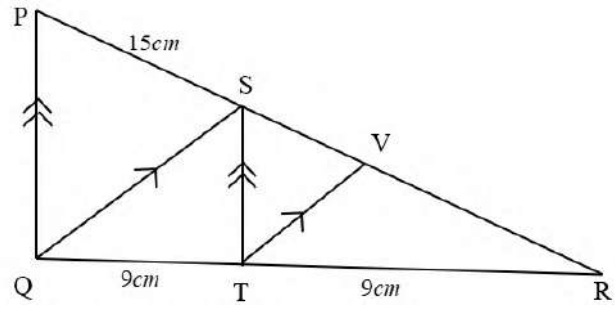
In the diagram alongside, lines AB and CD. Lines AD and CD are equal in length.



9.5.1 Find the size of angles a to g, giving reasons (7) **L2**

[07]

9.6 **IEB NOVEMBER 2013 PAPER 2 QUESTION 4.2**



In the diagram below, $QS \parallel TV$, $PQ \parallel ST$, $QT = TR = 9cm$ and $PS = 9cm$

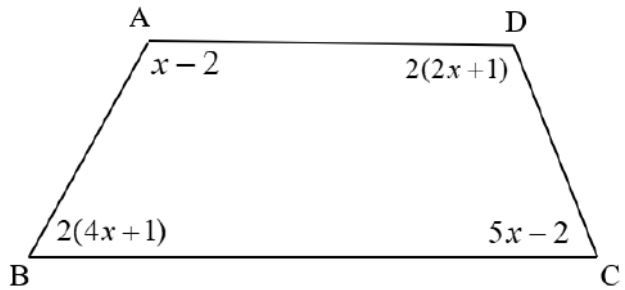
9.6.1 Prove $VR = 7,5cm$ (4) **L2**

9.6.2 Calculate PQ if $PQ = \frac{16}{5}VR$ (2) **L2**

9.6.3 Hence prove that $P\hat{Q}R = 90°$ (2) **L3**

[08]

9.7 **IEB NOVEMBER 2013 PAPER 2 QUESTION 8**

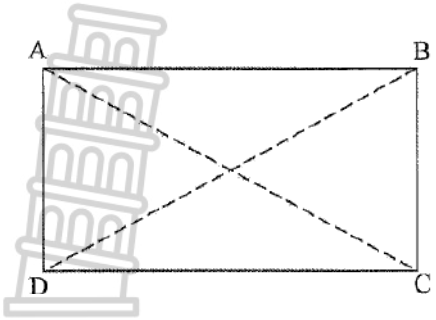


The angles of quadrilateral ABCD are $x-2$, $2(4x+1)$, $5x-2$ and $2(2x+1)$ respectively.

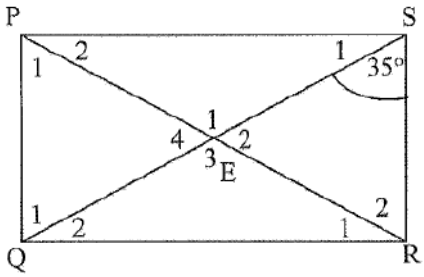
9.7.1 Show that the quadrilateral is a trapezium (9) **L3**

[09]

9.8 NSC JUNE 2015 PAPER 2 QUESTION 6



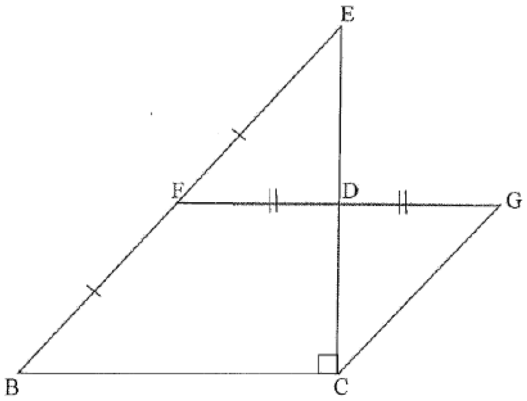
9.8.1 In the diagram alongside, ABCD is a rectangle. Use the diagram to prove the theorem which states that the diagonals of a rectangle are equal. (6) **L2**



9.8.2 In the diagram alongside, PQRS is a rectangle with E being the point of intersection of the diagonals. $\hat{QSR} = 35°$. Calculate the size of $\hat{RPS}$ i.e. $\hat{P}_2$ (4) **L2**

[10]

9.9 NSC JUNE 2015 PAPER 2 QUESTION 7



In the figure below, BCE is a right-angled triangle. F is the midpoint of BE and D is a point on CE. FD is produced by its own length to G and forms parallelogram BCGF. $FD = 30mm$ and $DE = 25mm$.

9.9.1 Why is $ED = DC$ (2) **L1**

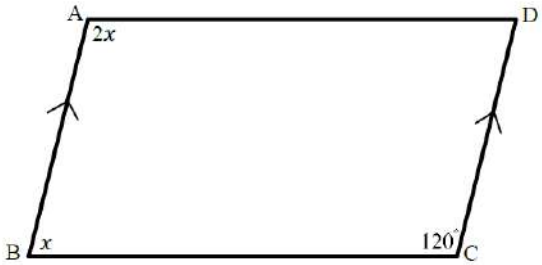
9.9.2 Prove that EFCG is a rhombus (2) **L2**

9.9.3 Calculate the area parallelogram BCGF (2) **L2**

9.9.4 Prove, by calculation, that: (2) **L2**
The area of rhombus = the area of parallelogram BCGF

[08]

9.10 GP JUNE 2018 PAPER 2 QUESTION 4



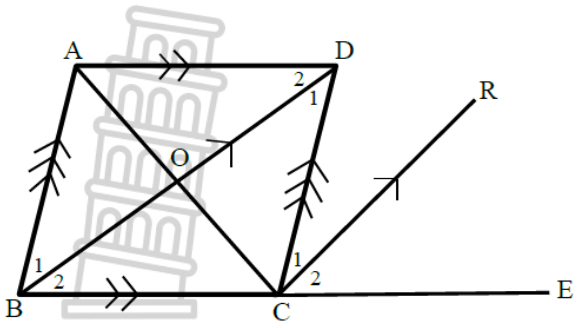
In the diagram alongside, quadrilateral ABCD, $AB \parallel CD$. $\hat{A} = 2x$, $\hat{B} = x$ and $\hat{C} = 120°$

9.10.1 Prove that $AD \parallel BC$. (4) **L2**

9.10.2 What type of quadrilateral is ABCD? (2) **L2**
Give a reason for your answer.

[06]

9.11 **GP JUNE 2018 PAPER 2 QUESTION 5**



In the diagram alongside, $ABCD$ is a parallelogram.

CR bisects $D\hat{C}E$ and $CR \parallel BD$.

Prove that:

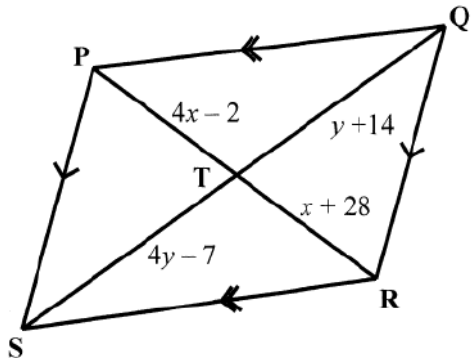
9.11.1 $BC = CD$ (5) **L3**

9.11.2 $ABCD$ is a rhombus (3) **L2**

9.11.3 If it is given that $BD = 24cm$ and $AB = 13cm$, then $AC = 10cm$ (5) **L2**

[13]

9.12 **KZN JUNE 2020 PAPER 2 QUESTION 5.2**

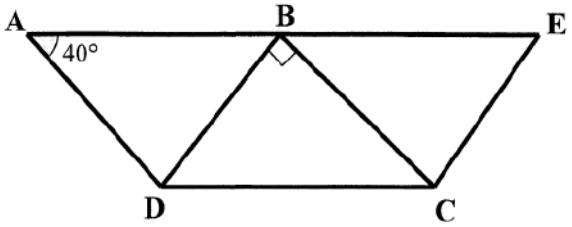


In the diagram alongside, $PQRS$ is a parallelogram. $PT = 4x - 2$, $TR = x + 28$, $ST = 4y - 7$ and $TQ = y + 14$

9.12.1 Determine, with reasons, the values of x and y (4) **L2**

[04]

9.13 **KZN MARCH 2020 PAPER 2 QUESTION 5.3**



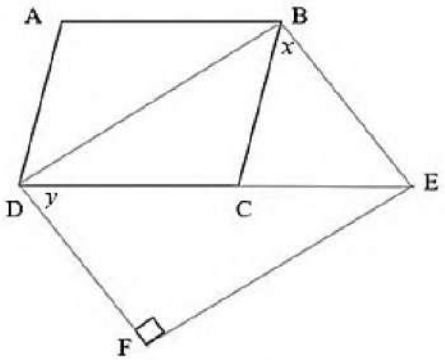
In the diagram alongside, $ABCD$ and $BECD$ are parallelograms with common base DC.

$BC \perp BD$ and $D\hat{A}B = 40°$

9.13.1 Determine the size of $B\hat{E}C$ (4) **L2**

[04]

9.14 **EC NOV 2020 PAPER 2 QUESTION 5**

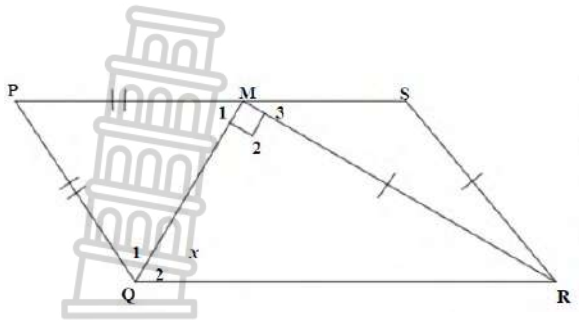


In the diagram alongside, $ABCD$ is a parallelogram such that $AD = BE$, $A = 124°$, ED bisects BEF and $BEFD$ is a quadrilateral.

9.14.1 Calculate, with reasons, the values of x and y. (6) **L3**

[06]

9.15 **GP JUNE 2022 PAPER 2 QUESTION 6**



PQRS is a parallelogram with M on PS such that PM=PQ and $SM = SR$. $QMR = 90°$ and $Q_2 = x$

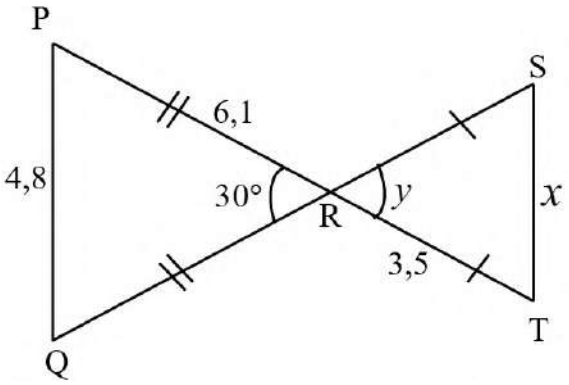
9.15.1 Determine, with reasons, two other angles which are equal to x (4) **L2**

9.15.2 Determine the size of M_3 in terms of x (2) **L2**

9.15.3 Calculate the numerical value of x (2) **L2**

[08]

9.16 **SIYAVULA**



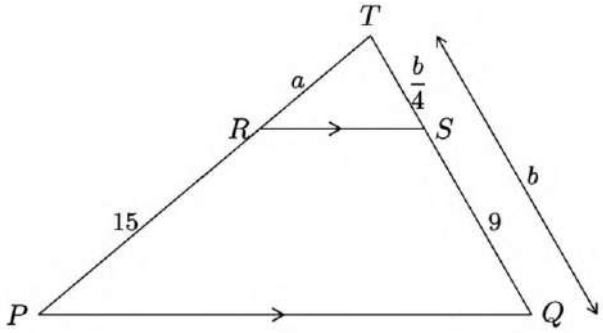
In the diagram, $PR = RQ$, $RS = RT$.

9.16.1 With a reason, write down the size of y (2) **L1**

9.16.2 Calculate the length of x. (7) **L4**

[09]

9.17 **SIYAVULA**



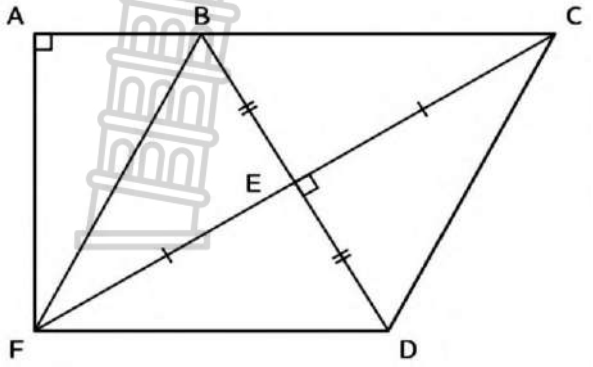
In the sketch alongside, $RS \parallel PQ$, $RT = a$, $RP = 15$, $TS = \frac{b}{4}$, $SQ = 9$ and $TQ = b$.

9.17.1 Calculate the length of TQ. (2) **L2**

9.17.2 Calculate the length of RT. (4) **L3**

[06]

9.18 **SIYAVULA**

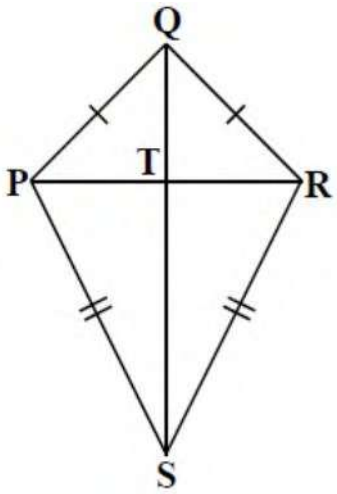


In the diagram alongside, $FAC = CED = 90°$, $AB = 8cm$, $BF = 17cm$, $BE = ED$ and $FE = EC$.

9.18.1 Determine the area of $ACDF$. (7) **L4**

[07]

9.19 **NSC NOVEMBER 2015 PAPER 2 QUESTION 9**

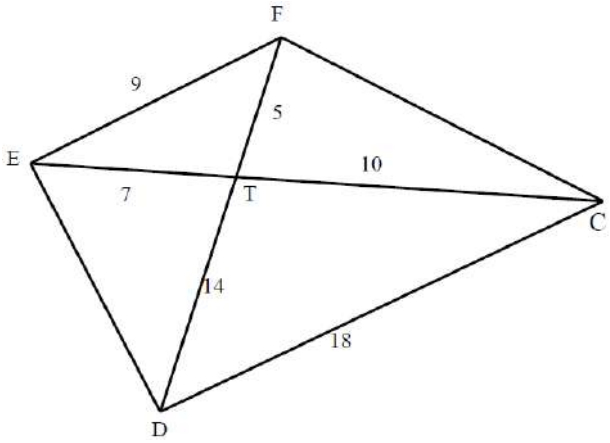


Consider the diagram alongside. It shows kite $PQRS$, in which $PQ = QR$ and $PS = SR$. The diagonals of the kite intersect at T. $PQ = 10cm$, $PR = 16cm$ and $QS = 21cm$.

9.19.1 Determine the perimeter of the kite. Show all your workings. (4) **L3**

[04]

9.20 **EC NOV 2020 PAPER 2 QUESTION 5**



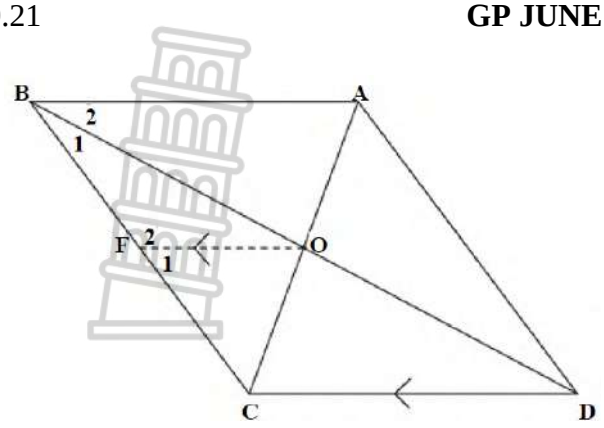
In the diagram alongside, $FT = 5cm$, $ET = 7cm$, $EF = 9cm$, $CT = 10cm$, $DT = 14cm$ and $CD = 18cm$.

9.20.1 Prove that $\Delta EFT \,|||\, \Delta DCT$ (3) **L2**

9.20.2 If it is further given that $DFC = TDC$, prove that $FEC = TFC$. (3) **L2**

[06]

9.21 **GP JUNE 2024 PAPER 2 QUESTION 5**



$ABCD$ is a rhombus with the diagonals $AC = 16cm$ and $DB = 30cm$

9.21.1 Calculate the length of BC (4) **L3**

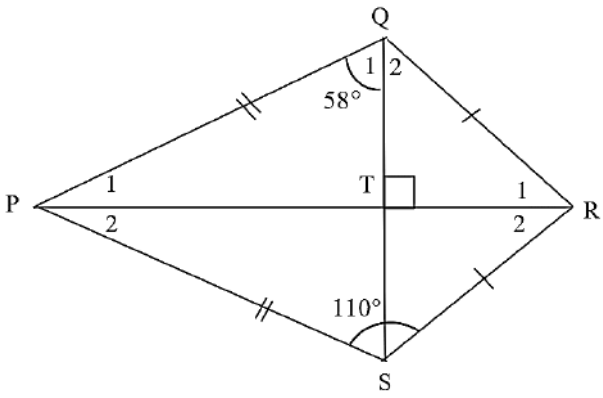
9.21.2 Calculate OF, if OF is parallel to DC with F on BC. (3) **L2**

9.21.3 Determine the size of OBF if $OFC = 64°$ (3) **L2**

9.21.4 Determine the area of the rhombus $ABCD$ (2) **L2**

[12]

9.22 **KZN JUNE 2022 PAPER 2 QUESTION 8**



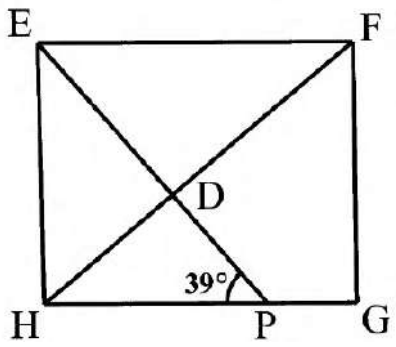
In the quadrilateral $PQRS$, $Q_1 = 58°$ and $P\hat{S}R = 110°$. $QS \perp PR$, $QR = RS$ and $PQ = PS$.
Use the diagram to calculate, with reasons, the size of:

9.22.1 P_1 (2) **L1**

9.22.2 R_2 (3) **L2**

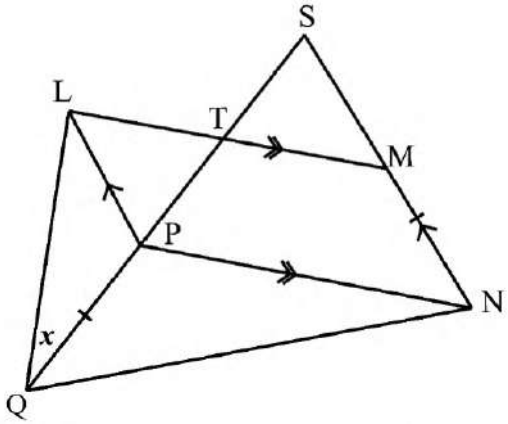
[05]

9.23 **KZN JUNE 2022 PAPER 2 QUESTION 9**



In the diagram below, $EFGH$ is a square and P is a point on HG such that $EPH = 39°$. EP and FH intersect at D.

9.23.1 Calculate the size of EDF (4) **L2**



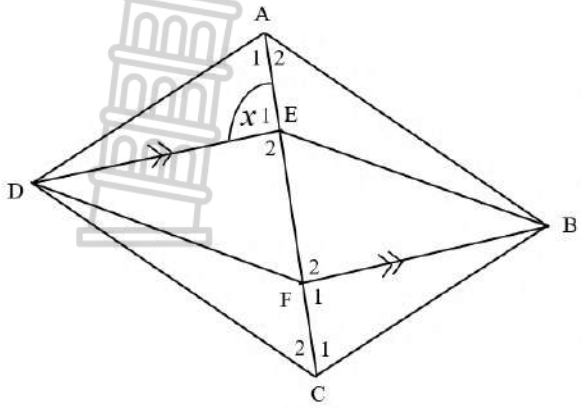
In the diagram alongside, $LP \parallel SN$, $LM \parallel PN$ and $PQ = MN$. Let $LQS = x$

9.23.2 Use the diagram to prove, with reasons, that $Q\hat{S}N = 2LQS$ (5) **L3**

[09]

NON-NUMERICAL QUESTIONS

9.24 **KZN SEPT 2018 QUESTION 3**



$ABCD$ is a parallelogram. Let $E_1 = x$

Prove:

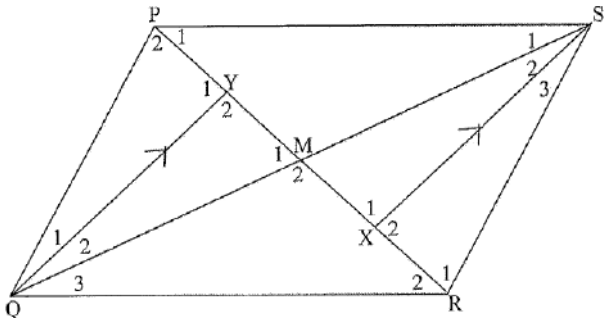
9.24.1 $E_1 = F_1$ (3) **L2**

9.24.2 $\Delta AED \equiv \Delta CFB$ (4) **L2**

9.24.3 $DEBF$ is a parallelogram (3) **L2**

[10]

9.25 **NSC JUNE 2015 PAPER 2 QUESTION 8**



In the diagram alongside, $PQRS$ is a parallelogram with diagonals PR and QS. QY and XS are drawn such that $QY \parallel XS$

Use the diagram to prove each of the following:

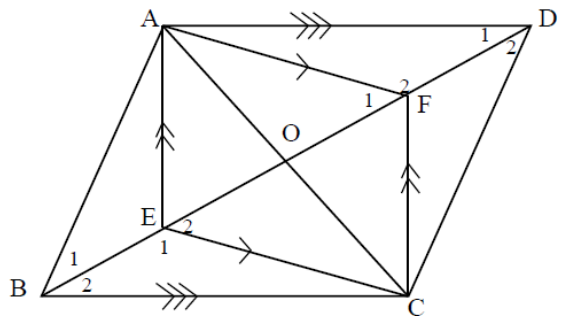
9.25.1 $\Delta QPY \equiv \Delta SRX$ (6) **L3**

9.25.2 $QYSX$ is a parallelogram (3) **L2**

9.25.3 $YM = MX$ (2) **L2**

[11]

9.26 **GP JUNE 2017 PAPER 2 QUESTION 5**



In the figure alongside, $AECF$ is a parallelogram and $AD \parallel BC$.

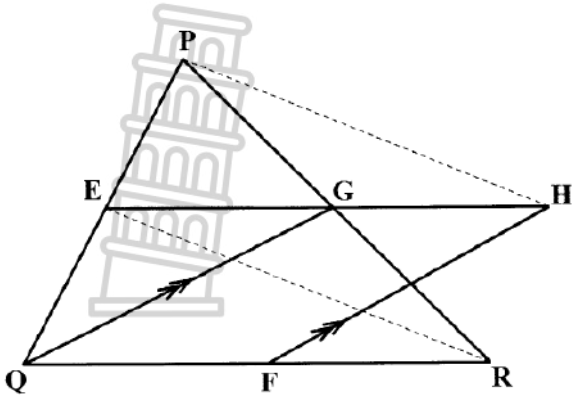
Prove that:

9.26.1 $E_1 = F_2$ (3) **L2**

9.26.2 $ABCD$ is a parallelogram (5) **L3**

[08]

9.27 **KZN JUNE 2019 PAPER 2 QUESTION 3**



In the diagram alongside, ΔPQR has E, F and G the midpoints of PQ, QR and PR respectively. $OG \parallel FH$

Prove:

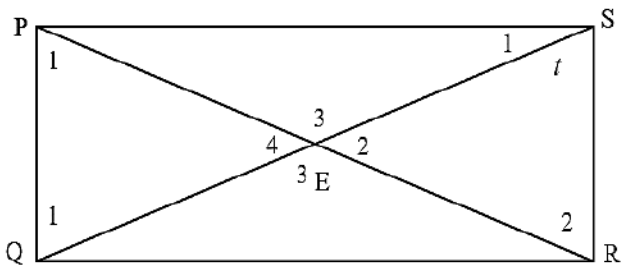
9.27.1 $QGHF$ is a parallelogram (3) **L2**

9.27.2 $EG = GH$ (3) **L2**

9.27.3 $ER \parallel PH$ (3) **L2**

[09]

9.28 **KZN JUNE 2024 PAPER 2 QUESTION 5**



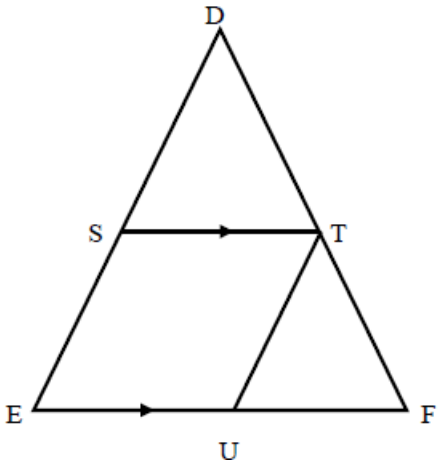
In the diagram alongside, $PQRS$ is a rectangle with E being the point of intersection of diagonals. $Q\hat{S}R = t$

9.28.1 Calculate the size of E_4 in terms of t (4) **L2**

[04]

9.29 **EC NOV 2020 PAPER 2 QUESTION 5**

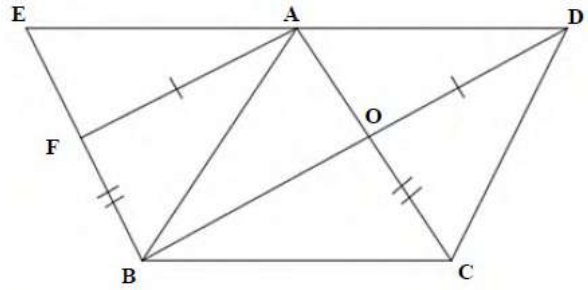
In ΔDEF, $DS = SE$, $EU = UF$ and $ST \parallel EF$



9.29.1 Prove that $SEUT$ is a parallelogram (4) **L3**

[04]

9.30 **GP JUNE 2022 PAPER 2 QUESTION 8**



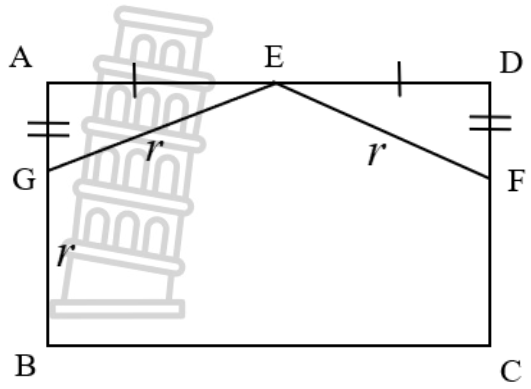
$ABCD$ is a parallelogram. BD and AC intersect at O. $AF = OD$, $CO = FB$. DA and BF produced meet at E.

9.30.1 Prove that $BOAF$ is a parallelogram. (4) **L2**

9.30.2 Prove that $AD = EA$. (4) **L3**

[08]

9.31 **MIND ACTION SERIES**



$ABCD$ is a parallelogram with $AE = ED$ and $AD = a$. $GE = EF = BG = r$ and $AB = b$.

$DF = \frac{1}{2}\sqrt{4r^2 - a^2}$

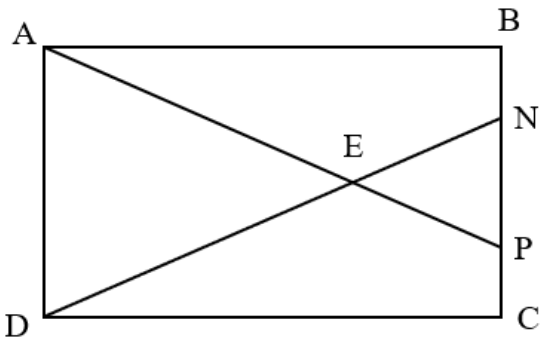
9.31.1 Prove that $ABCD$ is a rectangle. (4) **L3**

9.31.2 Express AG in terms b and r. (1) **L1**

9.31.3 Prove that $r = \dfrac{a^2 + 4b^2}{8b}$ (2) **L2**

[07]

9.32 **KZN MARCH 2020 PAPER 2 QUESTION 6**



In the diagram alongside, rectangle $ABCD$ is given with $AP = DN$

9.32.1 Prove that $\Delta ABP \equiv \Delta DCN$ (4) **L2**

9.32.2 Prove that $AE = DE$ (4) **L2**

[08]

TOPIC 10. MEASUREMENTS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

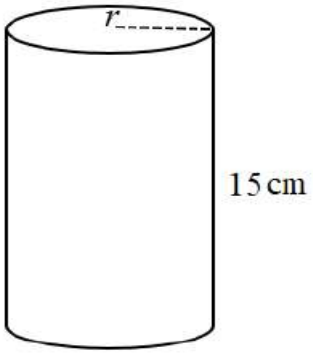
According to the Curriculum Statement, the learners should:

- Revise the volume and surface area of right prisms and cylinders.
- Study the effect on volume and surface area when multiplying any dimension by a constant factor k.
- Calculate the volume and surface areas of spheres, right pyramids, and right cones.
  - For pyramids, the bases must either be an equilateral triangle or a square.
  - Problem types must include composite figures (combination of the different figures).

ACTIVITIES

[LIMPOPO NOV 2023 PAPER 2 QUESTION 10]

10.1 The right cylinder below has the height of $15\ cm$ and the radius r



If the volume of the cylinder is $750{,}0\ cm^3$, calculate:

10.1.1	Its radius.	(2)	**L2**
10.1.2	The base area of the cylinder in terms of π	(2)	**L2**
10.1.3	The surface area of the cylinder.	(2)	**L2**
		[06]	

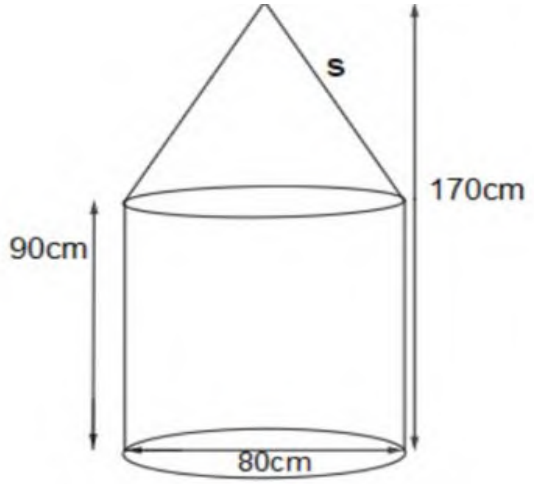
[KZN UGU TEST 2023 QUESTION 4]

10.2 A right pyramid has a square base of side $80\ cm$ and a height of $90\ cm$,

10.2.1	Determine the length of the slant height.	(3)	**L1**
10.2.2	Calculate the volume of the pyramid.	(3)	**L2**
10.2.3	Calculate the total surface area of the pyramid.	(3)	**L2**

KZN UGU TEST 2023 QUESTION 5

10.3 A composite structure for storing grains consists of a cylindrical base (close the bottom) and a conical top. As shown below. The height of the structure is $170\ cm$ and the height and diameter of the cylindrical base is $90\ cm$ and $80\ cm$ respectively. The slant height of the conical top is s.



You may make use of the following formulae

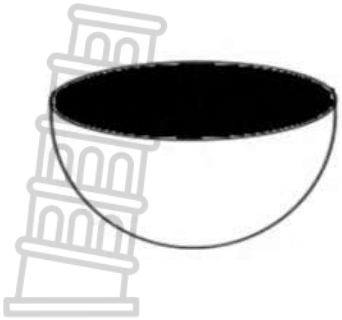
$$V = \frac{1}{3} area\ of\ base \times h$$

$$TSA = \pi r^2 + \pi rs$$

10.3.1	Write down the: a) Radius of the conical top. b) Height of the conical top.	(1) (1)	**L1**
10.3.2	Determine the lateral surface area of the conical top.	(4)	**L2**
10.3.3	Calculate the total surface area of the entire structure.	(4)	**L2**
10.3.4	Calculate the volume of the entire structure.	(4)	**L2**
		[14]	

KZN UGU TEST 2023 QUESTION 6

10.4 The surface area of the hemisphere below is $233\ cm^2$.



You may make use of the following formulae

$V = \frac{4}{3}\pi r^3$

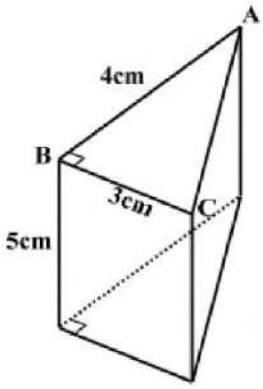
$TSA = 4\pi r^2$

10.4.1 Calculate the radius of the hemisphere, to the nearest integer. (3) **L2**

[03]

LP SEKHUKHUNE EAST TEST 2023 QUESTION 3

10.5 The diagram shows a piece of cheese cut in a form of a triangular prism. The dimensions are indicated on the diagram.



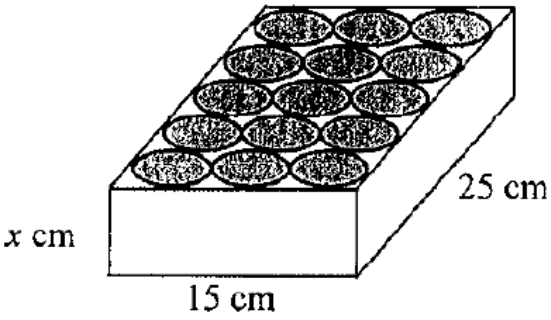
10.5.1 Determine the volume of the cheese. (3) **L2**

10.5.2 Calculate the total surface area of the wrapping paper around the cheese. (2) **L2**

[05]

LP SEKHUKHUNE EAST TEST 2023 QUESTION 4

10.6 An open rectangular cardboard box has the following dimensions: length $25\ cm$, breadth $15\ cm$ and height $x\ cm$. The volume of the box is $3000\ cm^3$. Fifteen (15) identical cans of cold drink fit snugly into the box, as shown in the diagram below. The box and the cans are of equal height. (ignore the thickness of the cardboard in your calculations.)



10.6.1 Calculate the height $x\ cm$ of the box. (3) **L2**

10.6.2 Calculate the radius of the can. (2) **L1**

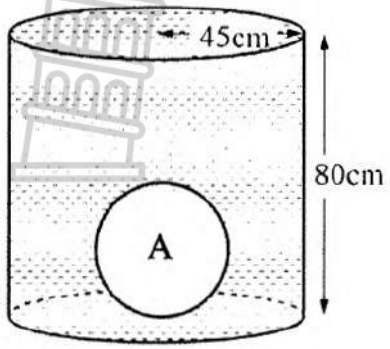
10.6.3 If a can is filled to the top, calculate the volume of cold drink contained in the can. (2) **L2**

10.6.4 Calculate the volume of the space in between all the cans in the box. (2) **L2**

[09]

KZN SEPTEMBER 2019 QUESTION 6

10.7 The diagram below shows a sphere, with a diameter of $30\ cm$, placed inside an empty cylindrical tank of radius $45\ cm$ and height $80\ cm$. The cylinder is then filled with water.

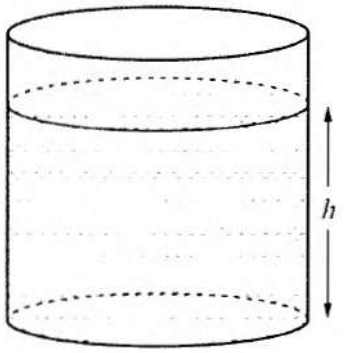


Volume of sphere $= \frac{4}{3}\pi r^3$
Volume of cone $= \frac{1}{3}\pi r^2 h$
Volume of a cylinder $= \pi r^2 h$

10.7.1 Calculate the volume of the metal sphere, indicated in the diagram as A. (2) **L1**

10.7.2 Calculate the volume of water required to fill the tank. (3) **L2**

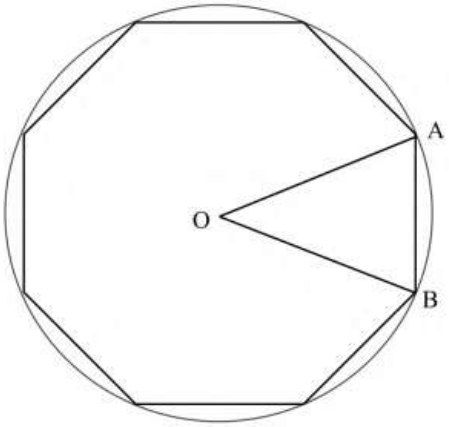
10.7.3 The sphere is removed from the tank as seen in the diagram below. Calculate h, the height of the water remaining in the tank. (3) **L2**



10.7.4 The sphere is then melted down and the metal is made into a solid cone of height $25\ cm$. Calculate the radius of the cone. (2) **L2**

[10]

10.8



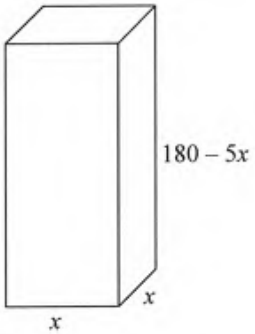
The diagram shows the regular octagon inscribed in a circle of radius r cm and with centre O. A and B are two vertices of the octagon. AO, BO and AB are drawn.

10.8.1 Show that the area of the octagon is $2\sqrt{2}r^2\text{cm}^2$ (4) **L3**

10.8.2 If $r = 5\text{cm}$, calculate the perimeter of the octagon (3) **L2**

[7]

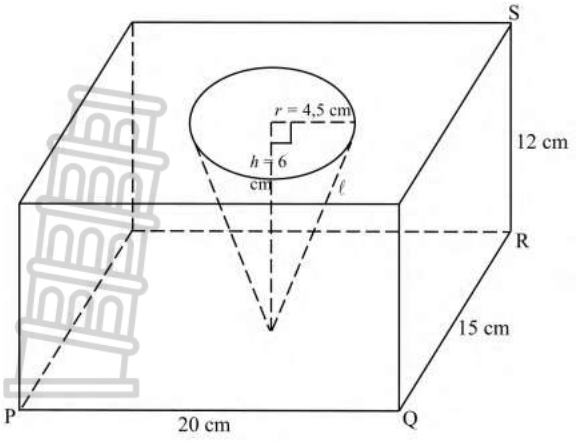
10.9



In the diagram, the right rectangular prism has a square base of x cm and the height of $(180-5x)$ cm.

Calculate the maximum surface area of the prism (5) **L3**

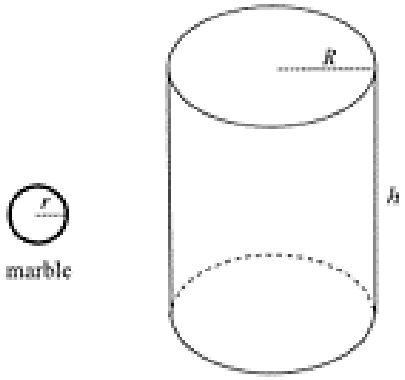
10.10



The solid was made by drilling the right circular cone out of a right rectangular prism (i.e. the cone is removed from the prism)
P, Q, R, and S are the vertices of Prism such that $PQ = 20$ cm, $QR = 15$ cm, $RS = 12$ cm.
The radius of the cone, r, is 4.5 cm and the height of the cone, h, is 6 cm. The slant height of the cone is l.

Calculate the total surface area of the solid. (7) **L4**

10.11



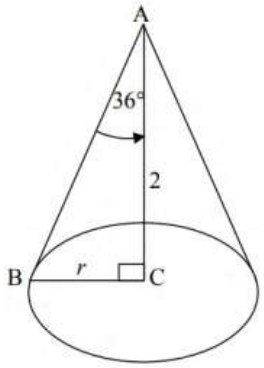
The cylindrical container shown will be filled with spherical marbles. the container has a radius of R cm and a perpendicular height of h cm. The volume of the container is 300 cm^3. The radius of each marble is 0.75 cm.

10.11.1 Show that the height of the container, h, is given by $h = \frac{300}{\pi R^2}$ (2) **L2**

10.11.2 The container is filled with 100 marbles. Then water is poured into the container until the water reaches the top. Calculate the volume of water that was poured into the container. (3) **L4**

[5]

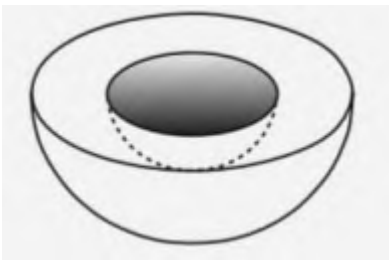
10.12



The perpendicular height, AC, of the cone is 2 metres and the radius is r. AB is the slant height.
$B\hat{A}C = 36°$

Calculate the total surface area of the cone. (6) **L3**

10.13



The sketch shows a hemisphere with smaller hemisphere cut out:
the radius of the larger hemisphere is 5 cm and the radius of the smaller hemisphere is 3 cm. Calculate the:

10.13.1 volume (3) **L2**
10.13.2 surface area of the shape. (4) **L3**

[7]