KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

JUST IN TIME LEARNER REVISION DOCUMENT

MATHEMATICS

GRADE 11

2026

This document has been compiled by the FET Mathematics Subject Advisors together with Top Teachers.

TABLE OF CONTENTS

TOPIC	1. ALGEBRA

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Rational exponents (fractional exponents): expressions with exponents that are rational numbers

- **Laws of exponents**
  - $a^n \times a^m = a^{m+n}$
  - $\frac{a^m}{a^n} = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(a \times b)^n = a^n \times b^n$
  - $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  - $a^0 = 1$
  - $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$

Quadratic Equations

Highest power of unknown is twice the power of a middle term (usually 2)

Always has 2 **solutions** (known as **roots**)

Standard form is $ax^2 + bx + c = 0$

Step 1: write in **standard form**

Step2: **factorise.**

Step3: equate **each factor to 0**

Step4: **solve**

BEWARE: **NEVER DIVIDE** BY AN **UKNOWN**

Quadratic exponents

It is an equation with three terms, one term has the highest power that is multiple of 2, the middle term has one- half the exponent of the term with a highest power

$x - 2x^{0,5} - 3 = 0$

let $x^{0,5} = k$

$k^2 - 2k - 3 = 0$

$(k-3)(k+1) = 0$

- **NOTE:** when adding or subtracting powers with numerical bases, **factorise.**
- when $a^m = a^n \therefore m = n$ and $a^m = b^m \therefore a = b$
- when $x^{\frac{p}{q}} = t^r \therefore x^{\frac{p}{q} \times \frac{q}{p}} = t^{r \times \frac{q}{p}}$ provided p is not an even number

- **Adding, subtracting, dividing and multiplying surds**
  - $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
  - $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
  - $p\sqrt[n]{a} \pm q\sqrt[n]{a} = p \pm q\sqrt[n]{a}$

- **Factorisation:**
  - common factor
  - trinomial
  - difference of two squares
  - grouping
  - sum and difference of two cubes.

- **Solving surds equation**

Steps:

- isolate the surd
- square both sides
- simplify and solve the equation
- test solutions to an original equation and reject if applicable

<u>using "k" method</u>

- substitute repeated expression with "k".
- solve for k
- substitute back
- solve the original uunknown

- **completing the square**
  steps
  - "take out" the coefficient of x^2 for the first 2 terms $a\left(x^2 + \frac{b}{a}\right)x + c$

- Add and subtract $\left(\frac{1}{2}\times(\textit{the coefficient of } x)\right)^2$ $a\left(x^2+\frac{b}{2a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}\right)+c$
- Factorise the perfect square trinomial and multiply $a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+c$

➢ **Quadratic equation**

from completing the square

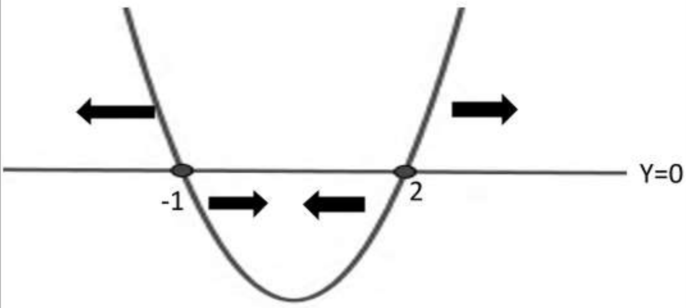
$$a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+c=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}=-\frac{c}{a}$$

If $ax^2+bx+c=0 \therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where $a\neq 0$

➢ **Inequalities(notes)**



- Where $y\geq 0$, above x axis $x\leq -1$ or $x\geq 2$
- Where $y\leq 0$ below the x axis $-1\leq x\leq 2$
- **NOTE:**
  When it is impossible to find a critical value, answer the question using the graphs e.g $3^x(x-3)<0$ .in this expression we want the values of x where the exponential function is **above the x-axis while the linear function is below** or an opposite.

Simultaneous equations	
When solving a pair of simultaneous linear equations, we are, in fact, finding a common point – the point of intersection of the two functions	
Elimination method	**Substitution method**
• Make the coefficients of one of the variables the same in both equations, Eliminate the variable by adding equation (1) and equation (2) together. • Simplify and solve for x • Substitute x back into either original equation and solve for y	• Use the simplest of the two given equations to express one of the variables in terms of the other. • Substitute into the second equation. By doing this we reduce the number of equations and the number of variables by one. • We now have one equation with one unknown variable which can be solved. • Use the solution to substitute back into the first equation to find the value of the other unknown variable.

Nature of roots

$$ax^2+bx+c=0 \therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- $b^2 - 4ac = 0$ equal roots
- $b^2 - 4ac < 0$ non real
- $b^2 - 4ac \geq 0$ real and equal
- $b^2 - 4ac =$ perfect square rational roots
- $b^2 - 4ac \neq$ perfect square irrational roots

ACTIVITIES

1.1 **Simplify**

1.1.1 $\dfrac{5^x.9^{x+1}}{3^{x+2}.15^{x-1}}$ **(4) L2**

1.1.2. $\dfrac{15^{x+2}.45^{1-x}}{3^{3-x}}$ **(4) L2**

1.1.3 $\dfrac{3^{2n+1} - 2.9^n}{2.3^{2n} - 5.3^{2n+1}}$ **(5) L2**

1.1.4 $\dfrac{25^{x+1}.6^x}{10^{x-1}.15^x}$ **(4) L2**

1.1.5 $\dfrac{6^{n+3}.2^{n-1}}{12^{n+2}}$ **(3) L2**

1.1.6 $\dfrac{2^{4t+1}.9^t.6^{2t-1}}{12^{3t}.3^t}$ **(4) L2**

1.2 **Convert the following into exponential form**

1.2.1 $\sqrt{x^3}$ **(1) L1**

1.2.2 $\sqrt[3]{x^2}$ **(1) L1**

1.3 **Solve for x or any given variable**

1.3.1 $6x^{\frac{3}{5}} = 162$ **(3) L1**

1.3.2 $3^{m+2} + 3^{m+1} = 324$ **(4) L2**

1.3.3 $x^{\frac{2}{3}} = 9$ **(4) L2**

1.3.4 $5^{x+1}.5^{x-1} = \dfrac{1}{25}$ **(3) L2**

1.3.5 $2^{2x+1} + 2^{2x-1} = \dfrac{5}{2}$ **(2) L1**

1.3.6 $2^{3x-1} = \dfrac{1}{16}$ **(2) L1**

1.3.7 $x^{\frac{1}{2}} - 2x^{\frac{1}{4}} - 3 = 0$ **(4) L2**

1.3.8 $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} - 4 = 0$ **(4) L2**

1.3.9 $16.2^{x+3} = 8.2^{2x+3}$ **(4) L1**

1.3.10 $x - 4x^{\frac{1}{2}} = 5$ **(5) L2**

1.3.11 $2^{x-2} + 2^x + 2^{x+1} - 52 = 0$ **(4) L2**

1.3.12 $3^x + 3^{x-2} - 10 = 0$ **(4) L2**

1.4 **Simplification**

1.4.1 $\sqrt{18} + \sqrt{50} - \sqrt{8}$ **(3) L1**

1.4.2 $\dfrac{\sqrt{132}}{\sqrt{3}}$ **(3) L1**

1.4.3 $\left(\sqrt{5+3}\right)\left(\sqrt{5-3}\right)$ **(3) L2**

1.4.4 $\left(\sqrt{5} - \sqrt{3}\right)\left(\sqrt{5} + \sqrt{3}\right)$ **(3) L2**

1.4.5 $\dfrac{\sqrt{2}\left(\sqrt{80} - \sqrt{45}\right)}{\sqrt{1000}}$ **(2) L2**

1.4.6 $\sqrt{\dfrac{7^{2022} - 7^{2020}}{12}}$ **(3) L2**

1.5 Solve for x or any given variable

1.5.1 $\sqrt{2t-1}-t=-2$ **(3) L2**

1.5.2 $a=2-\sqrt{2a-5}$ **(4) L2**

1.5.3 $4\sqrt{p-2}=p+1$ **(4) L2**

1.5.4 $\frac{\sqrt{x}}{\sqrt{2}}=3\sqrt{2}$ **(4) L3**

1.5.5 $\sqrt{x}-\sqrt{12+x}=-3$ **(4) L2**

1.5.6 $2\sqrt{2-\frac{p}{2}}+4=p$ **(2) L2**

1.5.7 $2\sqrt{\frac{x}{2}-3}+4=12$ $\sqrt{x}-\sqrt{12+x}=-3$ **(5) L3**

1.5.8 $\sqrt{x-3}=\frac{2x-6}{4}$ **(5) L2**

1.5.9 $\sqrt{2x-1}-\frac{3}{\sqrt{2x-1}}=-2$ **(4) L3**

1.5.10 $\sqrt{x+5}.\sqrt{x-2}=3\sqrt{2}$ **(4) L1**

1.5.11 $3\sqrt{\sqrt{x}+3}=9$ **(4) L3**

1.5.12 $\sqrt{2-7x}+2x=0$ **(4) L2**

1.6 Solve the following equations through factorisation

1.6.1 $x^2=5x$ **(2) L1**

1.6.2 $6x^2-5x=4$ **(3) L1**

1.6.3 $(2x-5)(3x+2)=2(3x-11)$ **(3) L2**

1.6.4 $x+2=\frac{3}{2x-1}$ **(4) L2**

1.6.5 $4x^2=100x$ **(2) L1**

1.6.6 $(4+x)(3-x)=-8$ **(3) L2**

1.6.7 $\frac{x^2-1}{x+1}=2$ **(4) L2**

1.7 Solve for x

1.7.1 $x^2\leq 5x$ **(3) L2**

1.7.2 $6x^2\geq 5x+4$ **(3) L2**

1.7.3 $x(x+1)<2(6+x)$ **(4) L2**

1.7.4 $x(x+1)\geq 6$ **(4) L2**

1.7.5 $(3-x)(x-5)\leq 0$ **(3) L2**

1.7.6 $2^x(x-3)\leq 0$ **(3) L3**

1.7.7 $2^x(x^2-9)<0$ **(3) L3**

1.7.8 $x^2+2x-20<-x-2$ **(4) L2**

1.7.9 $5<x(2x+3)$ **(4) L3**

1.7.10 $4-x>\frac{x^2}{2}$ **(4) L3**

1.8 Solve for x and y simultaneously

1.8.1 $2x^2-3xy=-4$ and $4^{x+y}=2^{y+4}$ **(5) L2**

1.8.2 $x^2-2xy-3y^2=0$ and $y-x=2$ **(6) L2**

1.8.3 $3^{2x}=3^{y-1}$ and $2x+2y=4$ **(5) L2**

1.8.6 $(x-9)^2(y+8)^2=0$ **(4) L2**

1.8.7 $2^{x^2}=\frac{32}{2^y}$ and $x-y-1=0$ **(7) L2**

1.8.8 $x-2y=2$ and $y=(x+1)(x-3)$ **(6) L2**

1.8.4 $y-4=2x$ and $2x^2-3xy+y^2=4$ **(5) L2**

1.8.5 $(3^x-9)\left(\sqrt{y-4}\right)=0$ **(4) L2**

1.8.9 $x^2+5xy+6y^2=0$ calculate the value of $\frac{x}{y}$ **(5) L3**

1.9 **Solve for *x* and correct to 2 decimal places**

1.9.1 $-x+5-2x^2=0$ **(4) L2**

1.9.2 $-x^2=2-7x$ **(4) L2**

1.9.3 $x^2-\frac{2}{3}x-5=0$ **(4) L2**

1.9.4 $x^2=2x+7$ **(4) L2**

1.9.5 $7x^2+18x=9$ **(3) L2**

1.9.6 $x(x+1)=3$ **(5) L2**

1.9.7 $(4-x)(x-7)=59$ **(5) L2**

1.9.8 $\sqrt{2x-1}=\frac{x}{\sqrt{3x+2}}$ **(5) L3**

1.9.9 **Solve for *x* by completing the square**

(a) $x^2=4x$ **(2) L2**

(b) $x^2=2x+3$ **(3) L2**

© $7x^2+18x=9$ **(3) L2**

(d) $x(x+1)=6$ **(3) L2**

1.10 **Nature of roots**

1.10.1 The roots of a quadratic equation are $x=\frac{-10\pm\sqrt{100-4k^2)}}{2k}$ .calculate the values of k for which the roots are non -real. **(2) L2**

1.10.2 For which values of p will $(x+5)^2=1-p^2$ be non-real. **(3) L2**

1.10.3 Given the equation $4x^2+kx+1=0$ has equal roots, determine the value of k (where $k<0$). **(4) L2**

1.10.4 For which values of x will x= $\sqrt{(3-x)(x+5)}$ real? **(4) L2**

1.10.5 Show that $\sqrt{\frac{a^2-b^2}{a+b}}$ is non-real for $b>a$. **(4) L3**

1.10.6 Show that the roots of $2x^2+(1-2n)x-n=0$ are rational and unequal if n is an integer **(3) L3**

1.10.7 $k=\sqrt{(x+1)^2-4}$, where *k* is a real number. Write down the minimum value of *k*. **(1) L1**

1.10.8 Calculate the values of *k*, for which the equation with roots

$$x=\frac{-2\pm\sqrt{4-12(-k+1)}}{6}$$ has real roots. **(3) L2**

1.11 **Miscellaneous**

1.11.1 The roots of $y=\frac{2}{y-1}$ are $y=2$ and $y=-1$. Hence, or otherwise, determine the roots of $x^2+2x-1=\frac{2}{x^2+2x-2}$. **(5) L3**

If $3^{9x}=64$ and $5^{\sqrt{p}}=64$, WITHOUT the use of a calculator, calculate the value of

1.11.2 $\frac{\left[3^{x-1}\right]}{\sqrt{5^{\sqrt{p}}}}$ **(4) L4**

1.11.3 Given the expression: $\frac{8}{1-\frac{1}{\sqrt{2}}}=a+b\sqrt{c}$. Show, WITHOUT the use of a calculator, that $a=bc$ **(5) L3**

1.11.4 If $x=\sqrt{3-2\sqrt{2}}$ and $y=\sqrt{2}-1$, show that $x=y$. **(5) L3**

1.11.5 Calculate a and b if $\sqrt{\frac{5^{2014}-5^{2012}}{6}}=a\left(5^b\right)$ and a is not a multiple of 5. **(4) L4**

1.11.6 Given $a-b=3$ and $3^a+9.3^b=4$, solve simultaneously for *a* and *b*. **(6) L3**

1.11.7 $4^x=8^y$ find the ratio of $x:y$ **(3) L3**

1.12 Word problems: GEMINI AI

1.12.1 The sum of two numbers is 15. The sum of their squares is 117. Determine the two numbers. **(6) L2**

1.12.2 In a right-angled triangle, the hypotenuse is 13 cm. The sum of the other two sides is 17 cm. Calculate the lengths of the two sides. **(6) L2**

1.12.3 The perimeter of a rectangular garden is 34 metres. The total area of the garden is 60 square metres. Calculate the dimensions (length and width) of the garden. **(7) L2**

1.12.4 The product of two consecutive positive integers is 132. Determine the two numbers. **(7) L3**

1.12.5 The length of a rectangle is 3 m more than twice its width. The area of the rectangle is 140 m². Determine the dimensions of the rectangle. **(7) L3**

TOPIC	2. NUMBER PATTERNS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

A linear pattern is a pattern where the difference (d) between consecutive terms is common.

$d = T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = ...$

The general term of a linear pattern can be given by $T_n = dn + c$ where d is the common difference.

A quadratic pattern is a pattern with a common second difference. The **first differences** of a quadratic pattern form a **linear pattern.**

Example: 94 90 82 70 54

-4 -8 -12 -16 ← First differences

-4 -4 -4 ← Second common difference

1. Determining the next term.

To determine the next term, we add the last terms of each of the sequences, i.e. the last term of the quadratic pattern to the last term of the first differences and the second common difference.

The next term is $54 - 16 - 4 = 34$

2. Determining the n^{th} term.

The n^{th} term of the quadratic pattern is given by: $T_n = an^2 + bn + c$

Using this formula to generate the terms we will have:

$a+b+c \quad 4a+2b+c \quad 9a+3b+c \quad 16a+4b+c$

$3a+b \quad 5a+b \quad 7a+b$

$2a \quad 2a$

- $2a =$ the second common difference
- $3a + b =$ the first term of the first differences
- $a + b + c =$ the first term of the quadratic pattern

From the example above:

$2a = -4$	$3a + b = -4$	$a + b + c = 94$
$a = -2$	$3(-2) + b = -4$	$-2 + 2 + c = 94$
	$b = 2$	$c = 94$

$\therefore T_n = -2n^2 + 2n + 94$

4. Determining T_n and n.

(b) Determining which term of the sequence is 81.

$-1206 = -2n^2 + 2n + 94$

It is important to note that $n \in \mathbb{N}$, that means it can **NEVER** be negative, irrational or a fraction.

(a) Determining the 15[th] term of the sequence

$T_{15} = -2(15)^2 + 2(15) + 94$

$T_{15} = -326$

$-2n^2 + 2n + 1300 = 0$

$n^2 - n - 650 = 0$

$(n-26)(n+25) = 0$

$n = 26$ or $n \neq -25$

$\therefore n = 26$

ACTIVITIES

2.1 Given the linear pattern: $7; 2; -3; ...$

- 2.1.1 Write down the next two terms. (2) **L1**
- 2.1.2 Determine the n^{th} term of the linear pattern. (2) **L2**
- 2.1.3 Calculate the value of the 20^{th} term. (2) **L1**
- 2.1.4 Which term in the sequence has a value of -138? (3) **L2**

[9]

2.2 $6; 2x+1$ and $3x-3$ are the first three terms of a linear pattern. Calculate the value of x. (3) **L2**

[3]

2.3 Given the first four terms of a linear patter: $3; x; y; 30$. Calculate the values of x and y. (4) **L3**

[4]

2.4 Consider the linear pattern: 102 ; 97 ; 92 ;........... -133

- 2.4.1 Determine the general term of the pattern. (3) **L2**
- 2.4.2 How many terms are there in the pattern? (3) **L2**

[6]

2.5 In a linear number pattern, the first term is x and a common difference is 5 less than the first term.
Calculate the value of the first term if it is given that the sum of the first three terms of the linear number pattern is 63. (5) **L3**

[5]

2.6 The first term of a linear number pattern is 92 and the constant difference is -4

- 2.6.1 Write down the values of the second and third terms. (2) **L1**
- 2.6.2 Determine an expression for the n^{th} term of the number pattern. (2) **L2**
- 2.6.3 If $T_p + T_q = 0$, determine the value of $(p+q)$ (4) **L4**

[8]

2.7 Consider the linear number pattern: $12; 9; 6; ...$

- 2.7.1 Determine the n^{th} term (general term) of the linear number pattern. (2) **L1**
- 2.7.2 Calculate the tenth term of the linear number pattern. (2) **L1**
- 2.7.3 Is -316 a term in the given linear number pattern? Justify your answer with an appropriate calculation. (2) **L2**
- 2.7.4 The terms of the above linear number pattern are the sequence of the first differences of this quadratic sequence: $4x+y; -2x-1; y+3; ...$
  Calculate the values of x and y. (5) **L3**

[11]

2.8 Given the pattern: $\tan x; \sin x; \sin x.\cos x;...$

2.8.1 Explain how each term is found from the previous term. (1) **L1**

2.8.2 Write down the FOURTH term (T_4) of the pattern. (1) **L1**

[2]

2.9 Consider the linear pattern: x ; 29 ; y ; 41.
Calculate the values of x and y. (4) **L2**

[4]

2.10 The second term of a linear pattern is 9 and the 19^{th} term is 43.
Calculate the value of the first term and the common difference. (3) **L3**

[3]

2.11 Given that the 5^{th} term of a linear pattern is 7 and the sum of the 11^{th} and 17^{th} terms is 68.

2.11.1 Calculate the value of the first term and the common difference. (5) **L3**

2.11.2 Determine T_{25}. (2) **L1**

[7]

2.12 Given a quadratic pattern: $-128;-84;-48;-20;...$

2.12.1 Determine the next TWO terms of the pattern. (2) **L1**

2.12.2 Determine T_n, the general term of the pattern, in the form $T_n = an^2 + bn + c$. (4) **L2**

2.12.3 Given that $T_n = -4n^2 + 56n - 180$, determine the biggest numerical value of T_n. (5) **L3**

2.12.4 Given that $h(n) = T_n + k$. For which values of k will T_n **not** have any positive values. (2) **L3**

[13]

2.13 Given a quadratic pattern: 244;193;148;109;...

2.13.1 Write down the next term of the pattern. (1) **L1**

2.13.2 Determine the formula for the n^{th} term. (4) **L2**

2.13.3 Which term has a value of 508? (4) **L2**

2.13.4 Between which TWO consecutive terms of the quadratic pattern will the first difference be 453? (3) **L3**

2.13.5 Show that all the terms of the quadratic pattern are positive. (4) **L3**

[16]

2.14 Given the number pattern $24;10;0;-6;...$

2.14.1 Show that the above number pattern is quadratic. (2) **L1**

2.14.2 Determine the formula for the n^{th} term. (4) **L2**

2.14.3 Calculate the value of T_{52}. (2) **L1**

2.14.4 Determine the smallest value of the quadratic number pattern. (3) **L3**

2.14.5 Determine the value(s) of n for which the terms of the quadratic number pattern will be positive. (4) **L3**

[15]

2.15 The first four terms of a quadratic pattern are: 10 ; 10 ; 12 ; 16;...

2.15.1 Determine the value of the 29^{th} term. (5) **L2**

2.15.2 Determine the general term for the first differences. (2) **L2**

[7]

2.16 $-10 ; -25 ; -38 ; -49;....$ is a quadratic pattern.

2.16.1 Determine the value of T_{18}. (5) **L2**

2.16.2 Which term(s) has a value of -70? (4) **L2**

2.16.3 Which term is first to be positive? (4) **L3**

[1 3]

2.17 Given: 0; 7; 12 are the third, fourth and fifth terms of a quadratic number pattern.

2.17.1 Calculate the first term of the sequence. (2) **L2**

2.17.2 Determine an expression for the n^{th} term of the pattern. (4) **L2**

2.17.3 Determine which term of the pattern will have the highest value. (3) **L3**

[9]

2.18 The general term of the first differences of a quadratic pattern is given by $T_k = -4k + 29$

2.18.1 Between which two consecutive terms of the quadratic pattern will the first difference be -71? (2) **L2**

2.18.2 If the first term of the quadratic pattern is 15, determine the general term of the quadratic pattern. (4) **L3**

[6]

2.19 The quadratic number pattern: $4; p; 11; q; 22; ...$ has a constant second difference of 1.

2.19.1 Show that $p = 7$ and $q = 16$. (3) **L2**

2.19.2 Determine the general term, T_n, of the quadratic pattern. (4) **L2**

2.19.3 Determine the value of n if $T_n = 232$. (4) **L2**

2.19.4 If the sum of two consecutive terms in the pattern is 1227, calculate the difference between these two terms. (5) **L3**

[16]

2.20 Consider the quadratic number pattern: $-13; -1; 7; ...; -2101$.

2.20.1 Write down the FOURTH and FIFTH term of the quadratic number pattern. (2) **L1**

2.20.2 Show that the general term of the quadratic number pattern is $T_n = -2n^2 + 18n - 29$. (3) **L2**

2.20.3 How many terms are in the above quadratic number pattern? (4) **L2**

2.20.4 Determine the maximum value of Q_n if is given that $Q_n = 2.T_n$. (3) **L3**

2.20.5 Prove that all terms in the sequence of first differences are even numbers. (2) **L3**

[14]

2.21 A quadratic number pattern has a general term $T_n = an^2 + bn - 15$. $T_2 - T_1 = 3$ and $T_3 - T_2 = 7$. Determine the values of a and b. (5) **L3**

[5]

2.22 The first term of a quadratic pattern is 10, the third and fourth terms are -6 and -11 respectively.

2.22.1 Determine the value of the second and fifth term. (4) **L3**

2.22.2 Determine the n^{th} term. (4) **L2**

[8]

2.23 The first four terms of a quadratic pattern are: $23 \; ; x \; ; y \; ; 65$ and the second difference is 2. Show that $x = 35$ and $y = 49$. (5) **L3**

[5]

2.24 The following information is given about a quadratic pattern:

$T_5 = 121$

$T_6 - T_5 = 37$

$T_7 - T_6 = 41$

2.24.1 Write down the first term of the quadratic pattern. (2) **L2**

2.24.2 Determine an expression of the general term. (4) **L2**

[6]

2.25 The first four terms of a quadratic pattern are: $2x - 4 \; ; x + 2 \; ; 3x + 1 \; ; 6x - 1$

2.25.1 Calculate the value of x. (5) **L3**

2.25.2 If $x = 3$, write down the numerical values of the first three terms. (1) **L1**

2.25.3 Hence, determine the n^{th} term of the pattern. (4) **L2**

[10]

2.26 Given the quadratic pattern: $4;9;20;37;...$

2.26.1 Determine the general term of this number pattern. (4) **L2**

2.26.2 State whether the turning point of T_n is a local minimum or local maximum value. Substantiate your answer. (3) **L3**

2.26.3 If $T_n = 3n^2 - 4n + 5$, determine the range of T_n. (3) **L3**

[10]

2.27 A pattern with a constant second difference has $T_n = 4n + 3$ as the general term of its first differences. The first term of the quadratic pattern is 7.
Determine the general term of the quadratic pattern. (5) **L3**

[5]

2.28 A quadratic pattern has a second difference of 2. $T_4 = -25$ and $T_{11} = -158$
Determine the formula for the n^{th} term. (6) **L3**

[6]

2.29 In the diagram below, the first three figures in a pattern that Nonhlanhla is investigating are shown

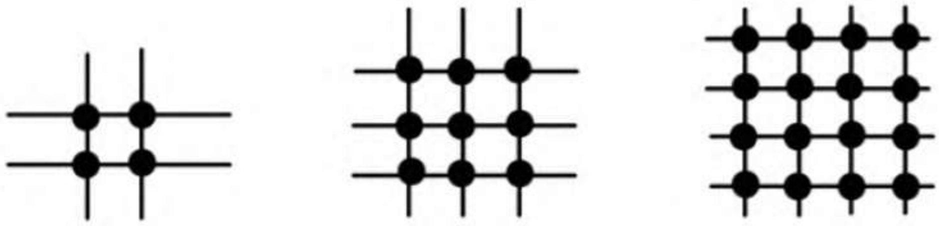


Figure 1 **Figure 2** **Figure 3**

2.29.1 Determine the total number of squares for Figure 5. (1) **L1**

2.29.2 How many dots will there be in Figure 7? (1) **L1**

2.29.3 Considering the n^{th} figure, show that an expression for the number of dots can be written as $(n+1)^2$. (4) **L2**

[6]

2.30 An athlete runs along a straight road. Her distance d from a fixed point P on the road is measured at different times, n, and has the form $d(n) = an^2 + bn + c$. The distances are recorded in the table below.

Time (seconds)	1	2	3	4	5	6
Distance (meters)	17	10	5	2	r	s

2.30.1 Determine the values of r and s. (2) **L1**

2.30.2 Determine the values of a, b and c. (3) **L2**

2.30.3 How far is the athlete from P when $n = 8$. (2) **L2**

2.30.4 Noxolo claims that the pattern will have a minimum value. Moses claims that the pattern will NOT have a minimum value.
Who is correct? Support your answer with an appropriate calculation. (3) **L4**

[10]

TOPIC	3. FUNCTIONS AND GRAPHS		
GUIDELINES, SUMMARY NOTES, & STRATEGIES			
Straight Line	**Parabola**	**Hyperbola**	**Exponential**
$y = mx + c$ m … **gradient** and c … ***y*-intercept**	$y = a(x+p)^2 + q$ **Axis of symmetry with equation** $x = -p$ **Maximum** or **minimum value** $(-p;q)$ Turning point	$y = \dfrac{a}{x+p} + q$ **Vertical asymptote:** **Horizontal asymptote:** $y = q$	$y = a.b^{x+p} + q$ $b > 0$ **and** $b \neq 1$ **Horizontal asymptote with equation** $y = q$
$m < 0$ **… graph is decreasing** $m > 0$ **… graph is increasing** **IF *m* = 0** **THEN** $y = c$ **IF** m is undefined **THEN *x* = *c***	$a < 0$ **… graph faces downwards(concave down) and has a minimum turning point** $a > 0$ **… graph faces upwards(concave up) and has a maximum turning point**	$a < 0$ **… graph is on the second and the fourth quadrant** $a > 0$ **… graph is on the first and the third quadrant**	$a < 0$ **… graph is below the asymptote** $a > 0$ **… graph is above the asymptote**
Domain: $x \in R$ **Range:** $y \in R$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$	**Domain:** $x \in R$, $x \neq -p$ **Range:** $y \in R$, $y \neq q$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$
$y - y_1 = m(x - x_1)$	$y = ax^2 + bx + c$ **Axis of symmetry:** $x_M = \dfrac{-b}{2a}$ **x-intercepts** $y = a(x - x_1)(x - x_2)$ x_1 **and** x_2 **are**	**Axis of symmetry/lines of symmetry:** $\begin{cases} y = x + c \\ y = -x + c \end{cases}$ **OR** $y = \pm(x - p) + q$	

- ✓ Understand reflection about the *y*-axis: $g(x) = f(-x)$ and about the *x*-axis $g(x) = -f(x)$
- ✓ Understand reflection about a line and about a point (use midpoint formula)
- ✓ Understand vertical translation, $h(x) = f(x) + q$, and horizontal translation $h(x) = f(x + p)$
- ✓ If given a sketch of a function $f(x)$, be able to determine the values of x for which: $f(x) < 0$ OR $f(x) > 0$
- ✓ If given the graphs of two functions, $f(x)$ and $g(x)$, on the same set of axes (intersecting graphs), be able to determine the values of x for which:

1. $f(x) = g(x)$
2. $f(x) < g(x)$ or $f(x) - g(x) < 0$
3. $f(x) > g(x)$ OR $f(x) - g(x) > 0$
4. $f(x).g(x) > 0$ or $f(x).g(x) < 0$
5. $\dfrac{f(x)}{g(x)} > 0$

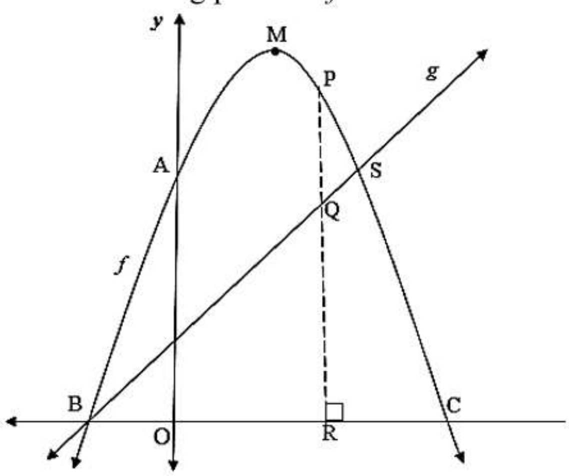
ACTIVITIES

KZN SEPT 2019 (GR 12)

3.1 Given: $f(x) = -2x^2 + x + 6$.

3.1.1	Determine the y-intercept of f.	(1)	**L1**
3.1.2	Determine the x-intercepts of f.	(3)	**L2**
3.1.3	Determine the coordinates of the turning point of f.	(3)	**L2**
3.1.4	Sketch the graph of f showing all the intercepts with the axis and turning points.	(4)	**L2**
3.1.5	Determine the value(s) of k for which $f(x) = k$ will have equal roots.	(2)	**L2**
3.1.6	If the graph of f is shifted 2 units to the right and 1 unit upwards to form the graph of h, determine the equation of h in the form of $y = a(x+p)^2 + q$.	(3)	**L3**

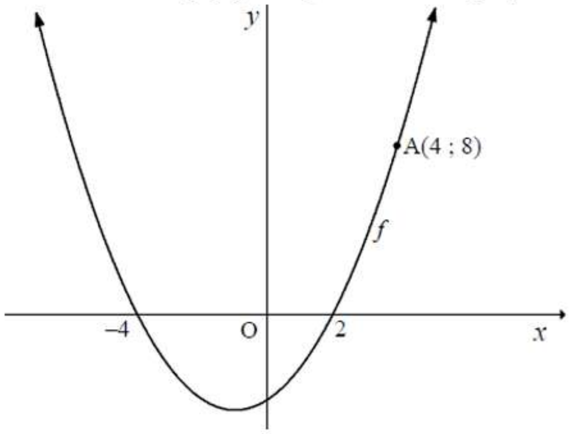
3.2 In the diagram, the graphs of $f(x) = -x^2 + 5x + 6$ and $g(x) = x + 1$ are drawn below.
The graph of f intersects the x-axis at B and C and the y-axis at A.
The graph of g intersects the graph of f at B and S. PQR is perpendicular to the x-axis with points P and Q on f and g respectively.
M is the turning point of f.



3.2.1	Write down the co-ordinates of A.	(1)	**L1**
3.2.2	S is the reflection of A about the axis of symmetry of f. Calculate the coordinates of S.	(2)	**L2**
3.2.3	Calculate the coordinates of B and C.	(3)	**L2**
3.2.4	If PQ = 5 units, calculate the length of OR.	(4)	**L3**
3.2.5	Calculate the coordinates of M	(4)	**L2**
3.2.6	Calculate the maximum length of PQ between B and S.	(4)	**L3**
3.2.7	Determine the values of k for which $g(x) - k$ is a tangent to the graph of f.	(4)	**L4**
3.2.8	Calculate the average gradient between B and S.	(2)	**L2**

KZN MARCH 2026 (GR 12)

3.3 Sketched below is the graph of $f(x) = ax^2 + bx + c$, with x-intercepts of -4 and 2. A(4;8) is a point on the graph.



3.3.1	Write down the coordinates of the image of A after reflection in the axis of symmetry of f.	(2)	**L1**
3.3.2	Show that $a = \frac{1}{2}$, $b = 1$ and $c = -4$	(3)	**L2**
3.3.3	Determine the values of d such that $\frac{1}{2}(x+d)^2 + x = 4 - d$ will have two positive roots.	(3)	**L4**

3.4 The equation of parabola is given by $f\left(x\right)=ax^2+bx+c$.

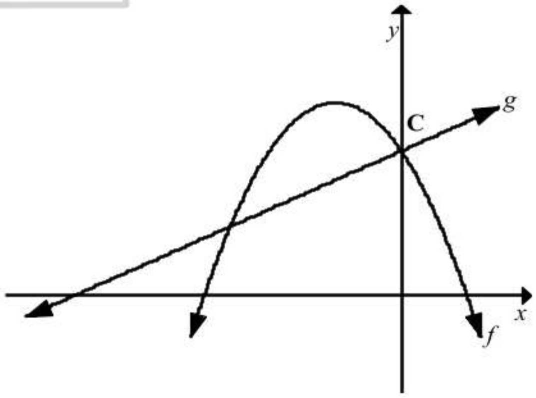
The roots of f are $(m-5)$ and $(m+3)$. The maximum value of f occurs at $x=2$.

3.4.1 Determine the value of m. (2) **L2**

3.4.2 Determine the equation of f in the form $f(x)=ax^2+bx+c$ if it is also given that $f(1)=15$. (4) **L2**

3.4.3 Determine the range of g if $g(x)=f(x)-4$. (3) **L3**

3.5 Given: $f(x)=ax^2+bx+c$ and $g(x)=mx+c$.



If it is given that $f(x).g(x)<0$ for all values of x, where $-6<x<-3$ or $x>2$, determine the value of a in terms of m (show all workings). (5) **L4**

3.6 Given $f(x)=ax^2+bx+c$ with $a>0$; $b<0$; $c>0$ and $b^2-4ac=0$.

3.6.1 Write down the range of f. (1) **L1**

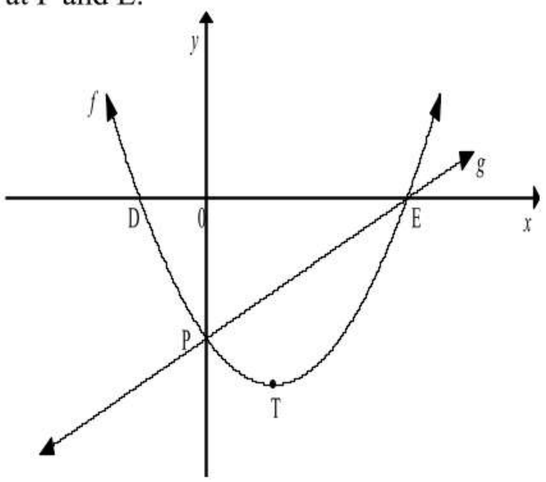
3.6.2 Describe the nature of the roots of f. (2) **L2**

3.6.3 Draw the rough sketch for the graph of f. It is not necessary to label intercepts with the axis and turning point. (3) **L3**

3.6.4 If the values of a and c are 1 and 9 respectively, determine the value of b. (3) **L2**

DBE/JUNE 2024

3.7 The graphs of $f\left(x\right)=x^2-2x-3$ and $g(x)=mx+c$ are drawn below. D and E are the x-intercepts and P is the y-intercept of f. The turning point of f is $T(1;-4)$. The graphs of f and g intersect at P and E.



3.7.1 Write down the range of f. (1) **L1**

3.7.2 Calculate the coordinates of D and E. (3) **L2**

3.7.3 Determine the equation of g. (2) **L2**

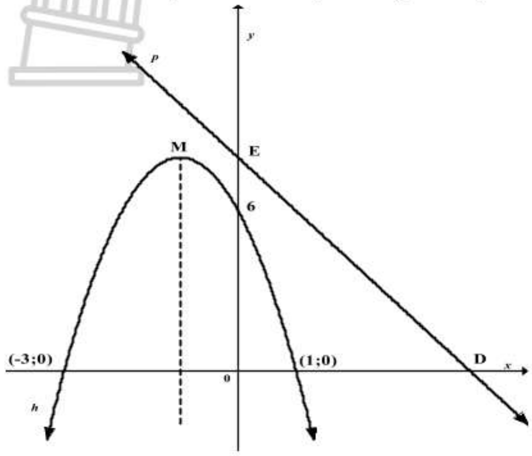
3.7.4 Write down the values of x for which $f(x)-g(x)>0$. (2) **L2**

3.7.5 Determine the maximum vertical distance between h and g if $h(x)=-f(x)$ for $x\in[-2;3]$. (5) **L3**

3.7.6 Given: $k(x)=g(x)-n$. Determine n if k is a tangent to f. (5) **L4**

KZN/JUNE 2024

3.8 The graphs of $h(x)=ax^2+bx+c$ and $p(x)=8-2x$ are sketched below. The x-intercepts of *h* are $(-3;0)$ and (1 ; 0) and the y-intercept of h is (0 ; 6). M is the turning point of h. D and E are the $x-$ and $y-$ intercepts of p respectively.



3.8.1	Write down the coordinates of D.	(1)	L1
3.8.2	Show that $a=-2$, $b=-4$ and $c=6$.	(4)	L3
3.8.3	Calculate the coordinates of M, the turning point of h	(2)	L2
3.8.4	Write down the range of *h*.	(1)	L1
3.8.5	Determine the values of x for which $h(x).p(x)<0$	(3)	L3
3.8.6	If $h(x)=k$, determine the value(s) of k for which:		
	3.8.6.1 roots are non-real	(1)	L3
	3.8.6.2 roots have the same sign.	(2)	L3
3.8.7	Calculate how graph p must be translated so that it becomes a tangent to graph h	(5)	L4

[19]

KZN SEPT 2019 (GR 12)

3.9 Given the equation of the hyperbola: $f(x)=\frac{-4}{2-x}-1$.

3.9.1 Write down the equations of the asymptotes of *f*. (2) L1

3.9.2 Determine the intercepts of the graph of *f* with the axes. (3) L2

3.9.3 Draw the graph of *f*. Show all intercepts with the axes as well as the asymptotes of the graph. (4) L3

3.10 Given: $f(x)=\frac{x+6}{x+2}$.

3.10.1 Express $f(x)$ in the form of $f(x)=\frac{a}{x+p}+q$. (2) L1

3.10.2 Determine the equations of the axes of symmetry. (4) L2

3.10.3 Draw the graph of *f*. Show all intercepts with the axes as well as the asymptotes of the graph. (4) L3

ADAPTED

3.11 Rewrite the equations alongside in the form of: $f(x)=\frac{a}{x+b}+c$

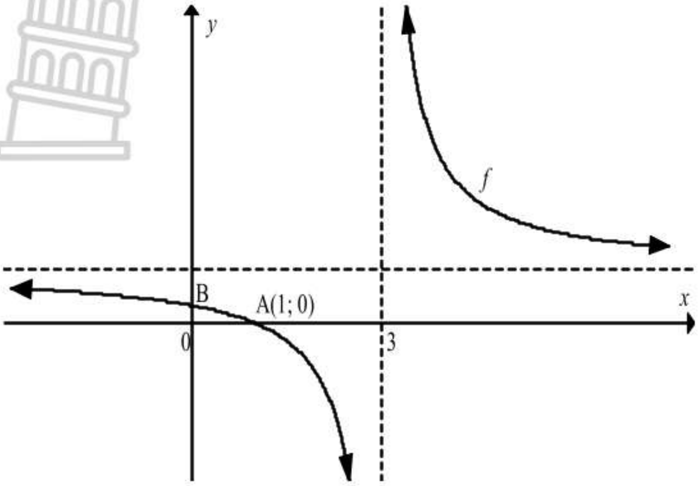
3.11.1 $f(x)=\frac{x-7}{x+5}$ (3) L2

3.11.2 $f(x)=\frac{2x-9}{2-x}$ (3) L2

3.11.3 $f(x)=\frac{5-2x}{4-x}$ (3) L2

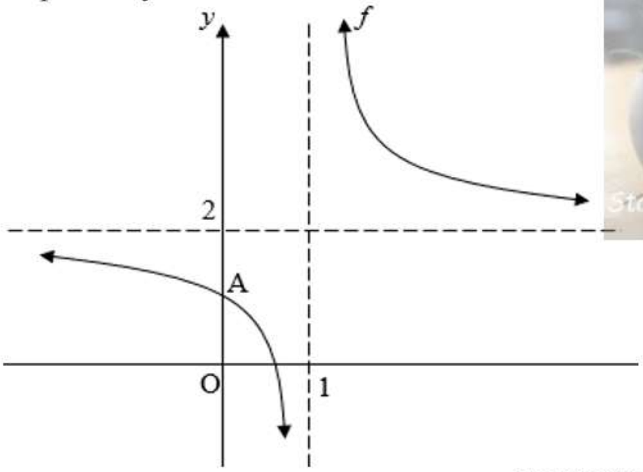
[09]

3.12 In the diagram below, the graph of a hyperbolic function, $f(x)=\dfrac{x+k}{x+p}$, where k is a constant, is drawn. A(1; 0) and B are the x-intercept and the y-intercept of f, respectively. The vertical asymptote goes through the x-axis at 3.



3.12.1	Write down the value of p.	(1)	**L1**
3.12.2	Determine the value of k.	(2)	**L1**
3.12.3	Calculate the coordinates of B.	(2)	**L2**
3.12.4	Determine the values of x for which $x.f(x)\le 0$.	(3)	**L2**
3.12.5	Rewrite the equation of f in the form $f(x)=\dfrac{a}{x+p}+q$.	(2)	**L2**
3.12.6	Determine the coordinates of the image of A, if A is reflected about the axis of symmetry with a negative gradient.	(4)	**L4**

3.13 Given $f(x)=\dfrac{a}{x-b}+c$; $A\left(0\ ;\ \dfrac{1}{2}\right)$ is the y – intercept of the graph. The asymptotes to the graph intersects the x – axis at 1 and the y – axis at 2 respectively.



3.13.1	Write down the equations of the vertical and horizontal asymptotes of f.	(2)	**L1**
3.13.2	Calculate the value of a.	(3)	**L2**
3.13.3	Determine the coordinates of A$^{/}$, the image of A, if it is reflected about (1 ; 2).	(4)	**L3**
3.13.4	Determine the equation of g if $g(x)=f(x-3)$.	(2)	**L2**

EC/NOV 2024

3.14 The lines $y=-x+4$ and $y=x-2$ are the axes of symmetry of the function $f\left(x\right)=\dfrac{-3}{x+p}+q$

3.14.1	Show that $p=-3$ and $q=1$	(4)	**L2**
3.14.2	Calculate the $x-$ intercept of f .	(2)	**L2**
3.14.3	Calculate the $y-$ intercept of f .	(2)	**L1**
3.14.4	Sketch the graph of f . Clearly label all intercepts with the axes and asymptotes.	(3)	**L2**
3.14.5	Write down the domain of g if g is the reflection of f along the line $x=0$.	(3)	**L2**
3.14.6	For which values of x will $x.f\left(x\right)\le 0$?	(3)	**L3**

[17]

3.15 **DBE NOV 2016**

Given: $f(x)=\dfrac{a}{x+p}+q$; $a>0$.

The domain of f is $x\in\mathbb{R}; x\neq -3$, range is $y\in\mathbb{R}; y\neq -2$ and $f(0)<0$.
Use the given information above to sketch the graph of f. (4) **L4**

3.16 Given: $h(x)=4\left(2^{x}\right)+1$

3.16.1 Determine the coordinates of the y – intercept of h. (2) **L2**

3.16.2 Explain why h does not have an x – intercept. (2) **L2**

3.16.3 Draw a sketch graph of h, clearly showing all asymptotes, intercepts with the axes and at least one other point on h. (3) **L2**

3.16.4 Describe the transformation from h to g if $g(x)=4\left(2^{-x}+2\right)$. (2) **L3**

EC SEPT 2022

3.17 Given $g(x)=-2.2^{x-1}+4$.

3.17.1 Determine the domain and range of g (2) **L1**

3.17.2 Determine the x and y intercepts of g (3) **L2**

3.17.3 Draw the graph of g. Show all intercepts with the axes as well as the asymptotes of the graph. (4) **L2**

3.17.4 Write down the equation of the new asymptotes if the graph of g is reflected about the x-axis and then shifted 4 units upward. (2) **L3**

3.17.4 Describe the transformation from g to h if $h(x)=2^{x+1}-4$. (2) **L3**

KZN JUNE 2022

3.18 Given: $h(x)=\left(\dfrac{1}{3}\right)^{x}+4$

3.16.1 Write down the equation of the asymptote of h. (1) **L1**

3.16.2 Draw a sketch of h, clearly indicating the asymptote and intercept with the axis. (3) **L2**

[04]

KZN /JUNE 2024

3.19 The function g is defined as $g(x)=\left(\dfrac{1}{4}\right)^{x}-4$

3.19.1 Write down the equation of asymptote of g. (1) **L1**

3.19.2 Calculate the $y-$ intercept of g . (2) **L1**

3.19.3 Calculate the $x-$ intercept of g . (2) **L2**

3.19.4 Draw a neat sketch of g. Clearly show all the intercepts with the axes and the asymptote. (3) **L2**

3.19.5 Calculate the average gradient of g between $x=0$ and $y=0$ (2) **L3**

3.19.6 Write down the equation of k if it is given that $k(x)=g(x)+4$. (1) **L3**

3.19.7 It is further given that $h(x)=2^{x+3}-4$. Explain in words, how graph g must be transformed to obtain graph h (4) **L3**

HUDSON PARK HIGH SCHOOL JUNE 2018

3.20 Given: $h(x)=2.3^{x}-6$

3.20.1 Sketch the graph of $h(x)$ clearly showing all asymptotes and intercepts. (4) **L2**

3.20.2 Is $h(x)$ an increasing or decreasing function? (1) **L1**

3.20.3 State the range of $h(x)$. (1) **L1**

3.20.4 If $h(x)$ is moved

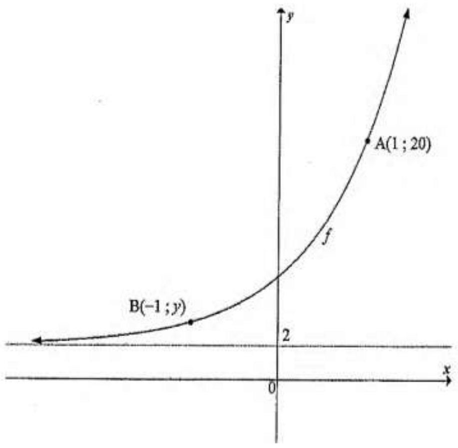
- 5 units vertically downwards; and
- 4 units horizontally to the right to become $g(x)$.

State the equation of $g(x)$ in the form $y=$. (2) **L2**

[08]

DBE NOV 2016

3.21 The sketch below is the graph of $f(x)=2.b^{x+1}+q$. The graph of f passes through the points A(1; 20) and B(-1; y). The line y = 2 is an asymptote of *f*.



3.21.1 Show that the equation of f is $f(x)=2.(3)^{x+1}+2$. (3) **L3**

3.21.2 Calculate the *y*-coordinate of the point B. (1) **L2**

3.21.3 Determine the average gradient of the curve between points A and B. (2) **L2**

3.21.4 A new function *h* is obtained when *f* is reflected about its asymptote. (2) **L**

(a) Determine the coordinates of A', the image of A which lies on *h*. (2) **L2**

(b) Write down the range of *h*. (1) **L1**

(c) Determine the equation of *h*. (4) **L4**

3.21.5 The graph of *f* has undergone a transformation to obtain $g(x)=2.(3)^{x}$.

(a) Describe the transformation that f has undergone in words. (2) **L2**

(b) Hence, determine the range of *g* (1) **L1**

[09]

3.22 Given: $f(x)=\dfrac{2}{x}$ and $g(x)=k^{x}$. The point $(2;9)$ lies on *g*.

3.22.1 Determine the value of *k*. (1) **L1**

3.22.2 Write down equations of the asymptotes of f. (2) **L1**

3.22.3 Draw sketch graphs of *f* and *g* on the same system of axes, clearly indicating all asymptotes and intercepts with the axes. (5) **L3**

3.22.4 For which value(s) of *x* is $f(x).g(x)\le 0$? (2) **L3**

3.22.5 Determine the *x*-coordinates of the points of intersection of *f* and its axis of symmetry that has a positive gradient. (3) **L3**

GP JUNE 2024 (GR 12)

3.23 The sketch below, shows the graphs of $f(x)=-(x-2)^2+9$ and $g(x)=b^{x}$ where b is a constant. D is a turning point of *f* and a point of intersection of *f* and *g*. B is the *y*-intercept. A and C, the *x*-intercepts of f.

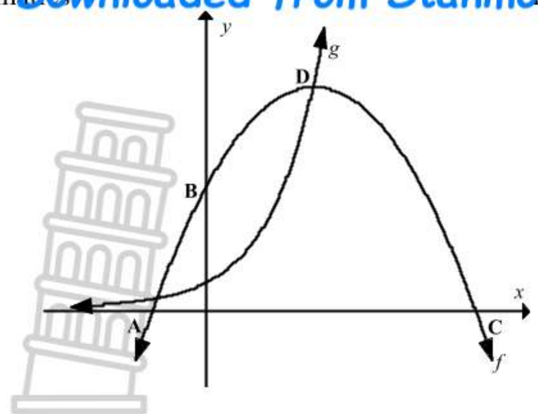
3.23.1 Determine the length of AC. (4) **L2**

3.23.2 Determine the value of b. (2) **L2**

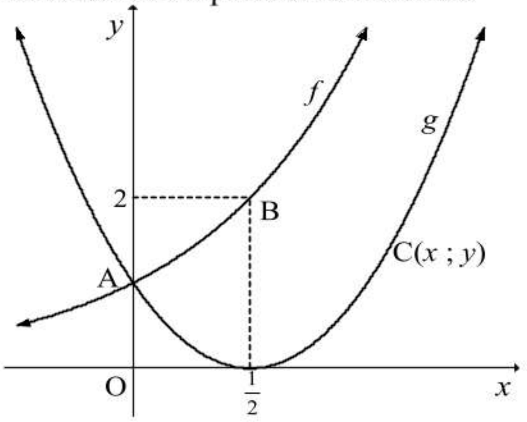
3.23.3 Determine the value of *x* for which $g(x)\ge 9$. (1) **L1**

3.23.4 Write down the equation of *h* if $h(x)=f(x+2)-9$. (2) **L3**

3.23.5 Show, algebraically, that $g\left(x+\dfrac{1}{2}\right)=\sqrt{3}g(x)$. (2) **L3**

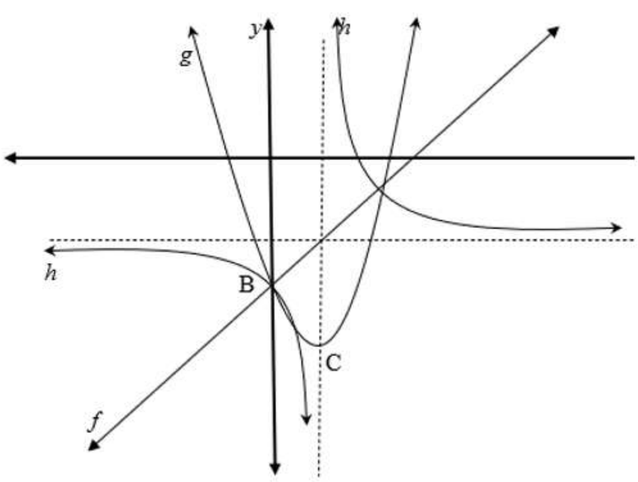


3.24 The graphs of $f(x) = k^x$ and $g(x) = ax^2 + bx + c$ are sketched below. The graphs intersect at A. The graph of *g* touches the *x*-axis at $\left(\frac{1}{2};0\right)$. The coordinates of B, on the graph of *f* are indicated. AC is parallel to the *x*-axis.



3.24.1	Determine the coordinate of A.	(1)	**L1**
3.24.2	Determine the value of *k*.	(2)	**L1**
3.24.3	Show that $a = 4$, $b = -4$ and $c = 1$	(4)	**L2**
3.24.4	Determine the equation of f in the form $y = ...$	(1)	**L1**
3.24.5	Determine the equation of *h* if *h* is reflection of *f* about the *y*-axis	(2)	**L2**
3.24.6	Write down the range of *g*.	(1)	**L1**
3.24.7	For which values of *x* is $g(x) - f(x) \geq 0$?	(2)	**L3**
3.24.8	Use the graphs to determine the value of t for which $f(x) + t = 0$ will have two roots of the same sign.	(3)	**L3**
3.24.9	Write down the new equation of *f* if the *y*-axis is shifted 2 units to the left.	(2)	**L4**

3.25 The graphs of $g(x) = \frac{1}{2}(x-2)^2 - 9$ and $h(x) = \dfrac{a}{x+p} + q$ are sketched below.
The axis of symmetry of graph *g* is the vertical asymptote of graph *h*. The line *f* is an axis of symmetry of graph *h*. B is the *y*-intercept of *h*, *g* and *f*.



3.25.1	Write down the coordinates of C, the turning point of *g*.	(2)	**L1**
3.25.2	Determine the coordinates of B.	(2)	**L1**
3.25.3	Write down the equation of *f*.	(2)	**L2**
3.25.4	Determine the equation of *h*.	(5)	**L3**
3.25.5	Write down the equations of the vertical and horizontal asymptotes of $k(x) = 3h(x) - 2$.	(2)	**L2**
3.25.6	Determine the *x*-intercept of *h*.	(3)	**L2**
3.25.7	For which values of x will: $\dfrac{f(x)}{h(x)} \geq 0$?	(3)	**L3**
3.25.8	Calculate the value(s) of *k* for which $g(x) = f(x) + k$ has two unequal positive roots.	(6)	**L4**

NW NOV 2019

3.26 The function defined as $f(x) = ax^2 + bx + c$ has the following properties:

- f has a minimum/ maximum value at $x = -2,5$
- $f(1) = 0$
- $b^2 - 4ac > 0$
- $f(-2,5) = 6$

Draw a neat sketch graph of *f*. Clearly show all *x*-intercepts and the turning point. (4) **L3**

KZN PRACTICE JUNE 2024 (GR 12)

3.27 The lines $y = x + 1$ and $y = -x - 7$ are the axes of symmetry of the function $f(x) = \dfrac{-2}{x+p} + q$.

2.27.1 Show that $p = 4$ and $q = -3$ (3) **L3**

2.27.2 Calculate the *x*-intercept of *f*. (2) **L2**

2.27.3 Sketch the graph of *f*. Clearly label ALL intercepts with the axes and the asymptotes. (4) **L3**

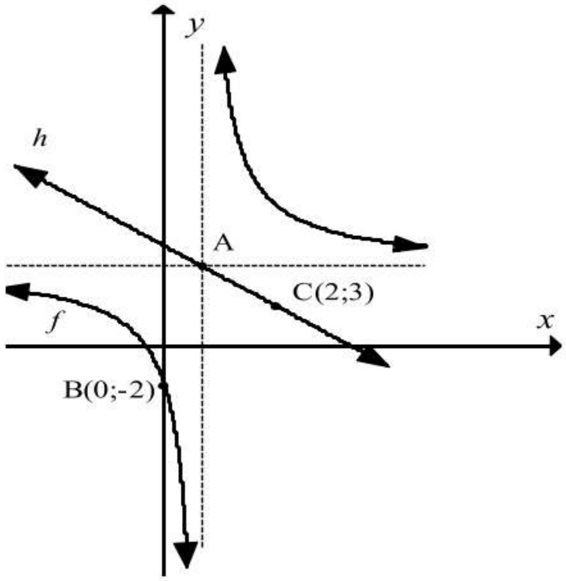
2.27.4 Write down the equation of the vertical asymptote of the graph of h if $h(x) = f(x+5)$. (2) **L2**

2.27.5 Determine the values of x for which $f(x) > 0$. (2) **L2**

2.27.6 Explain how you would use the graph to determine the value(s) of *x* if $\dfrac{-2}{x+4} = -x - 4$. (3) **L3**

FS JUNE 2024 (GR 12)

3.28 In the sketch below the graph of $f(x) = \dfrac{a}{x+p} + 4$ is given.
The asymptotes of *f* intersect at point A.
The graph of f cuts the *y*-axis at $B(0;-2)$.
The axis of symmetry of *f* is the line *h*.
C(2;3) is a point on *h*.



3.28.1 Determine the equation of *h*. (2) **L1**

3.28.2 Determine the coordinates of point A. (2) **L2**

3.28.3 Determine the equation *f*. (3) **L2**

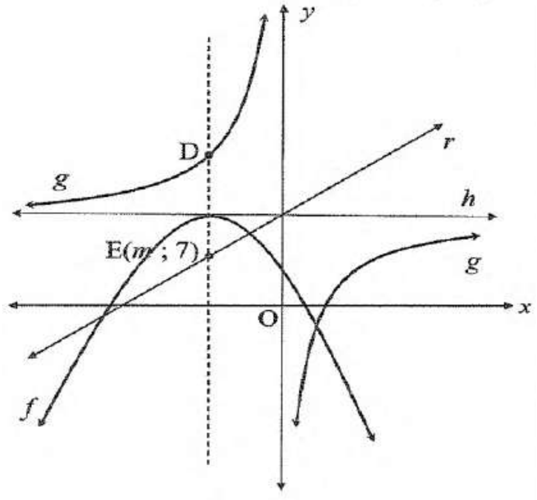
3.28.4 Determine the equations of the asymptotes of $f(x+1)$. (3) **L2**

3.28.5 Write down the coordinates of the image of $D\left(\dfrac{1}{2};0\right)$ if D is reflected about the axis of symmetry $y = x + 3$. (2) **L3**

NW SEPT 2022 (GR 12)

3.29 Below are the graphs of $f(x) = -2(x+p)^2 + q$ and $g(x) = \frac{-3}{x} + n$.

- $h(x) = n$, an asymptote of *g*, is also a tangent to *f*.
- The line $r(x) = x+8$ is an axis of symmetry of *g*.
- $r(x) = x+8$ also intersects the axis of symmetry of f in the point E(m ; 7).



3.29.1 Write down the domain of *g*. (2) **L1**

3.29.2 Calculate the value of *m*. (2) **L1**

3.29.3 Write down the value of *n*. (1) **L1**

3.29.4 Given $f(x) = -2(x+p)^2 + q$, write down the values of *p* and *q*. (2) **L1**

3.29.5 If it is given that $f(x) = -2x^2 - 4x + 6$, calculate the *x*-intercepts of *f*. (3) **L2**

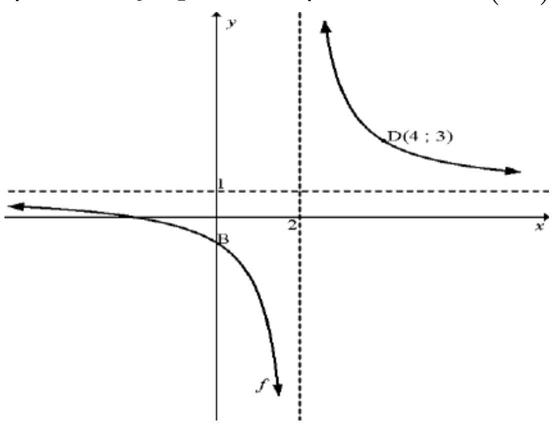
3.29.6 The axis of symmetry of f intersects the graph of *g* at point D. Determine the coordinates of D. (2) **L2**

3.29.7 Determine the equation of *k*(*x*) in the form of $k(x) = \frac{a}{x+t} + s$, if *k* is the reflection of *g* about the line $x = 2$. (3) **L3**

3.29.8 Determine the value of *k* for which the equation $g(x+4) + k = 0$ will have a root less than -5. (3) **L4**

KZN/NOV 2024

3.30 In the diagram, the graph of $f(x) = \frac{a}{x+p} + q$ is drawn. $D(4;3)$ is point on f and B is the $y-$ intercept of f. The asymptotes of f intersect at $(2;1)$



3.30.1 Write down the equations $y = x - 1$ of the asymptotes of f (2) **L1**

3.30.2 Show that the equation of f is $f(x) = \frac{4}{x-2} + 1$ (2) **L3**

3.30.3 Calculate the coordinates of B (2) **L2**

3.30.4 Determine the equation of the axis of symmetry which has a positive gradient. (2) **L2**

3.30.5 Determine the values of x for which $f(x) \leq 0$ (4) **L2**

3.30.6 The graph of f is transformed to obtain the graph of $h(x) = \left(\frac{1}{4}x\right)^{-1}$. Describe the transformation, in words, from f to h. (2) **L3**

[15]

DBE NOV 2017

3.31 Given: $f(x)=\frac{-3}{x+2}+1$ and $g\left(x\right)=2^{-x}-4$

3.31.1 Determine $f(-3)$. (1) **L1**

3.31.2 Determine x if $g(x)=4$. (2) **L2**

3.31.3 Write down the asymptotes of *f*. (2) **L1**

3.31.4 Write down the range of *g*. (1) **L1**

3.31.5 Determine the coordinates of the *x*- and *y*- intercepts of *f*. (5) **L3**

3.31.6 Determine the equation of the axis of symmetry of *f* which has a negative gradient. Leave your answer in the form $y=mx+c$. (2) **L2**

3.31.7 Sketch the graphs of *f* and *g* on the same system of axes. Clearly show ALL intercepts with the axes and asymptotes. (6) **L2**

3.31.8 If it is further given that $f(-1)=g(-1)$, (2) **L4**

Determine the values of *x* for which

$g(x)\geq f(x)$.

[21]

ADAPTED

3.32 Draw the graphs of the following functions, clearly indicating all intercepts (*x* and *y*) on the same system of axes

3.32.1 $f(x)=\frac{-3}{3-x}+1$ and $g(x)=-x^2+3x+4$ (4) **L2**

3.32.2 $f(x)=2^x+1$ and $t(x)=-2x-5$ (4) **L2**

3.32.3 $g(x)=4-x^2$ and $f(x)=-2x+2$ (4) **L2**

3.32.4 $g(x)=\left(\frac{1}{5}\right)^x$ and $g(x)=-x^2-1$ (4) **L2**

3.32.5 $j(x)=3^{-x}$ and $g(x)=2x^2-4x-6$ (4) **L2**

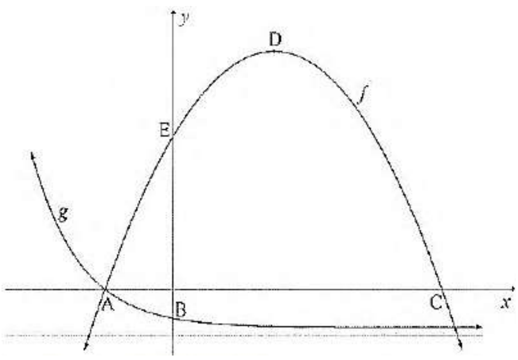
[16]

GRADE 11 DBE NOV 2018

3.33 The diagram below shows the graphs of

$f(x)=-(x-3)^2+25$ and $g(x)=2\left(\frac{1}{2}\right)^{x+1}-4$.

Graph *f* cuts the *x*-axis at A and C, the *y*-axis at E and has a turning point at D.



3.33.1 Write down the equation of the asymptote of *g*. (1) **L1**

3.33.2 Write down the coordinates of D. (2) **L1**

3.33.3 Write down the range of *f*. (1) **L1**

3.33.4 Calculate the length of EB. (4) **L3**

3.33.5 Determine the values of *x* for which *f* is increasing. (2) **L2**

3.33.6 Calculate the average gradient between points A and B. (4) **L3**

3.33.7 Graph *t* is obtained by reflecting *g* about *x*-axis, Write down the range of *t*. (2) **L3**

3.33.8 If $p\left(x\right)=f\left(x\right)+2$, (2) **L2**

Write down the the coordinates of the turning point of *p*.

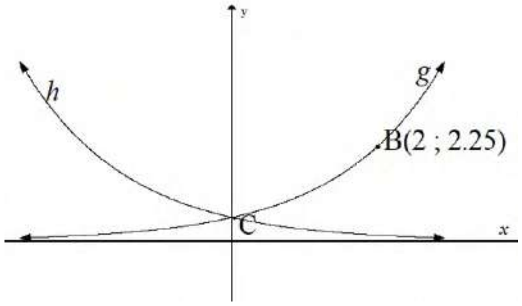
Graph g cuts the x-axis at A and the y-intercept at B.

3.33.9 Determine the value of k for which the straight line $y = 2x+k$ will be a tangent to f. (4) **L4**

[23]

ADAPTED

3.34 In the diagram below, g represents the function $g(x) = a^x, a > 0$.
The graph of h is symmetrical to g about the y-axis. The point $B\left(2;\ 2\frac{1}{4}\right)$ lies on the curve of g and C is the y-intercept of both g and h.



3.34.1 Write down the range of g. (1) **L1**

3.34.2 Calculate the value of a. (2) **L2**

3.34.3 Write down the coordinates B', the image of B which lies on h. (1) **L1**

3.34.4 Determine the equation of h in the form $y = b^x$. (2) **L2**

3.34.5 Write down the coordinates of C. (1) **L1**

3.34.6 Hence, or otherwise, determine the value(s) of x for which $1 < g(x) < 2\frac{1}{4}$. (2) **L2**

3.34.7 B' is the reflection of B in the $y-$axis. Calculate the average gradient between B' and C. (2) **L2**

[11]

DBE/FEB-MARCH 2016

3.35 Given: $f(x) = 2^x + 1$

3.35.1 Determine the coordinates of the $y-$intercept of f. (1) **L1**

3.35.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3) **L2**

3.35.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$. (3) **L2**

3.35.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of h. (1) **L3**

[08]

GRADE 12 DBE NOV 2018

3.36 The function f, defined by $f(x) = \frac{a}{x+p} + q$, has the following properties:
The range of f is $y \in R, y \neq 1$.
The graph f passes through the origin.
$P(\sqrt{2}+2; \sqrt{2}+1)$ lies on the graph of f.

3.36.1 Write down the values of q. (1) **L1**

3.36.2 Calculate the values of a and p. (5) **L3**

3.36.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4) **L2**

3.36.4 Determine the equation of the line of symmetry with a positive y-intercept. (2) **L2**

3.36.5 If point $P(\sqrt{2}+2; \sqrt{2}+1)$ is reflected about the point of intersection of the asymptotes of f, calculate the coordinates of P'. (2) **L2** (3) **L3**

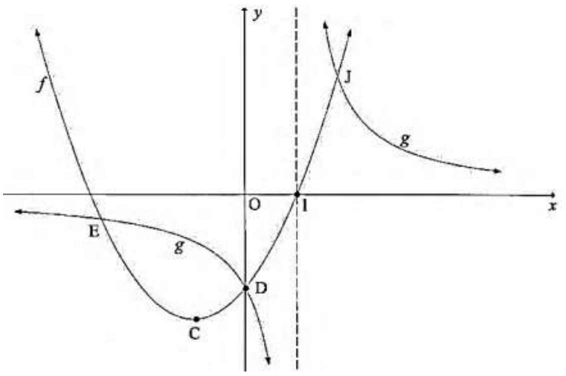
[15]

GRADE 12 DBE NOV 2019

3.37 Below are the graphs of

$f(x) = ax^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

f has a turning point at C and passes through the x -axis at (1; 0). D is the y -intercept of both f and g also intersect each other at E and J. The vertical asymptote of g passes through the x -intercept of f.



3.37.1	Write down the value of p.	(1)	**L1**
3.37.2	Show that a = 3 and b = 2.	(3)	**L3**
3.37.3	Calculate the coordinate of C.	(4)	**L2**
3.37.4	Write down the range of f.	(2)	**L1**
3.37.5	Determine the equation of the line through C that makes an angle of 45° with the positive x -axis. Write your answer in the form y =.....	(3)	**L3**
3.37.6	Determine the value(s) of k for which $f(x)+k$ will be positive.	(2)	**L4**
3.37.7	Determine the value(s) of x for which:		
	(a) $x.f(x) < 0$	(3)	**L2**
	(b) $g(x) \le -3$	(2)	**L2**
3.37.8	The function $h(x) = f(m - x) + q$ has only one x -intercept at $x = 0$. Determine the values of m and q.	(4)	**L4**

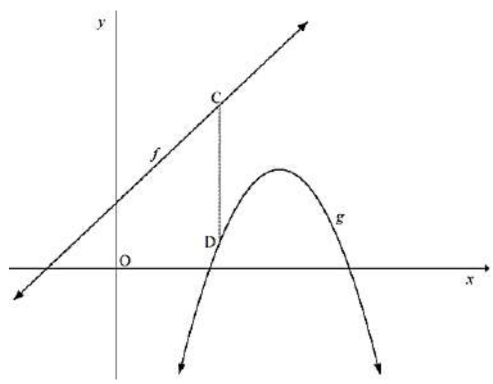
GRADE 12 DBE/ MAY-JUNE 2022

3.38 The graph of passes $g(x) = a\left(\dfrac{1}{3}\right)^x + 7$ through point E(– 2; 10).

3.38.1	Calculate the value of a.	(3)	**L2**
3.38.2	Calculate the coordinates of the y-intercept of g.	(2)	**L1**
3.38.3	Consider: $h(x) = (3)^{-x}$		
	(a) Describe the transformation from g to h.	(2)	**L3**
	(b) Determine the range of h.	(1)	**L1**

DBE NOV 2015

3.39 The sketch below shows the graphs of $f(x) = 2x + 3$ and $g(x) = -2x^2 + 14x + k$.
C is any point on f and D is any point on g, such that CD is parallel to the y-axis.
k is a value such that C lies above D.



3.39.1	Determine the equation of the axis of symmetry of g.	(2)	**L2**
3.39.2	Determine the value(s) of x for which $f(x) \le 0$.	(1)	**L2**
3.39.3	Write down a simplified expression for the length of CD in terms of x and k.	(3)	**L2**
3.39.4	If it is given that the minimum length of CD is 5 units, calculate the value of k.	(4)	**L3**
3.39.5	If it is given that $k = -20$		
	(a) Determine the x-intercepts of g.	(3)	**L2**
	(b) Determine the coordinates of the turning point of g.	(3)	**L2**
	(c) Calculate the value(s) of x for which $f(x) < 2$.	(4)	**L3**
	(d) Write down the domain and range of g.	(2)	**L1**

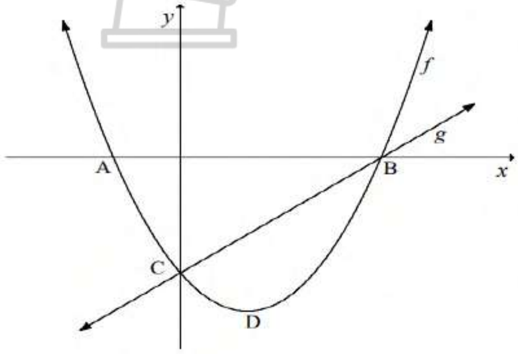
GRADE 11 KZN NOV 2023

3.40

The sketch below shows the graphs of:

$f(x)=\frac{1}{2}(x-2)^2-8$ and $g(x)=x-6$.

- A and B are x-intercepts of f.
- D is a turning point of f.
- C is the y-intercept of both graphs.
- Graphs f and g intersect at B and C.



3.40.1	Calculate the length of AB.	(4)	**L2**
3.40.2	Write down the coordinates of D.	(1)	**L1**
3.40.3	Write down the range of f.	(1)	**L1**
3.40.4	Calculate the average gradient between points A and C.	(3)	**L2**
3.40.5	For which values of k will $\frac{1}{2}x^2-2x-6-k=0$ have two positive roots?	(2)	**L3**
3.40.6	Use the graphs to determine the values of x for which:		
	(a) $f(x)\geq 0$	(2)	**L2**
	(b) $\frac{g(x)}{f(x)}>0$	(2)	**L3**
3.40.7	Determine the maximum value of h if $h(x)=\sqrt{2}^{2-f(x)}$.	(3)	**L3**
3.40.8	Calculate the value(s) of k for which $y=2x+k$ will NOT be a tangent of f.	(5)	**L4**

[23]

TOPIC	4. FINANCE, GROWTH AND DECAY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

TERMINOLOGY:

- **Inflation** – refers to the rate at which prices of goods and services increase over time.
- **Population Growth** – refers to the increase in the number of individuals in population over time.
- **Reducing Balance** – is the method of depreciation that is calculated at a fixed percentage rate of the book value of the assets.
- **Simple growth-** refers to a type of growth that is calculated on the initial/original amount of investment or population.
- **Simple decay –** refers to whereby the value of an asset is reduced by a constant amount each year, which is calculated on the initial amount.
- **Compound growth-** refers to a type of a growth that is calculated on every year amount of investment or population based on recent accumulation.
- **Compound decay -** refers to a type of a growth that is calculated on every year amount of investment or population based on recent depreciations.
- **Nominal interest rate** is the quoted annual interest rate (this interest rate excludes the effect of inflation on price). Nominal can also refer to the advertised or stated interest rate on a loan, without taking into account any fees or compounding of interest.
- **Effective interest rate** – refers to the actual rate of interest that is obtained. The effective annual interest rate can also refer to the interest rate that is actually earned or paid on an investment or loan due to the result of compounding over a given time period once a year
- **Book value –** Refers to the value of an item after depreciation has taken place.
- **Scrap value –** Refers to the value of an item after its useful life.

SIMPLE GROWTH AND COMPOUND GROWTH $(A > P)$

$A \rightarrow$ Final amount. $P \rightarrow$ P amount. $i \rightarrow$ Interest rate. $n \rightarrow$ Period.

Simple growth	Compound growth
$A = P(1 + i \times n)$	$A = P(1+i)^n$

SIMPLE AND COMPOND DECAY $(A < P)$

- **Simple decay** is associated with a decreasing STRAIGHT-LINE graph over years and is also called **<u>linear depreciation method.</u>**
- **Compound decay** is associated with a decreasing EXPONENTIAL graph and is also called **<u>reducing balance method</u>**.

$A \rightarrow$ Final amount $P \rightarrow$ Principal amount $i \rightarrow$ Interest rate $n \rightarrow$ Period

Linear depreciation method	Reducing balance method
$A = P(1 - i \times n)$	$A = P(1-i)^n$

DIFFERENT COMPOUNDING PERIODS

In grade 11, it is possible to earn interest anytime so when interest is not earned once a year, in formula, we need to adjust ***i*** and ***n*** to match with given compounding periods. The table below shows different compounding periods.

Compounding periods	Annually (every year)	Half-yearly (every 6 months)	Quarterly (every 3 month)	Monthly (every 12 months)
Value to be used	$m=1$	$m=2$	$m=4$	$m=12$
Interest rate	i	$\frac{i}{2}$	$\frac{i}{4}$	$\frac{i}{12}$
Period (number of periods)	n	$2n$	$4n$	$12n$

$A \rightarrow$ Final amount $P \rightarrow$ Principal amount $i \rightarrow$ Interest rate $n \rightarrow$ Period

Compound growth	**Compound decay**
$A = P\left(1+\frac{i}{m}\right)^{m \cdot n}$	$A = P\left(1-\frac{i}{m}\right)^{m \cdot n}$

These above formulas can also be used for sake of convenience to learners**.**

NOMINAL AND EFFECTIVE INTEREST

- **Effective interest rate** is noticed where compounding period is taken into consideration. The stated period and compounding period are the same.
- **Nominal interest rate** is an annual rate which financial institutions quote.

To determine the annual effective rate, we use the following formula:

$$1+i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$$

$i_{eff} =$ effective interest rate $m =$ compounding periods

$i_{nom} =$ Nominal interest rate

When working with different compounding periods use the formula:

$$\left(1+\frac{i_{(r)}}{r}\right)^n = \left(1+\frac{i_{(m)}}{m}\right)^m$$

TIMELINES

- Timelines are useful in financial Mathematics when there are several deposits and withdrawals, as well as possible changes in interest rates.
- Timelines provides a visual summary of all the information in an investment fund.

REVISION QUESTIONS

Simple growth and compound growth

4.1 Mr Bhengu wants to invest R15 000 in a savings account. This type of an account will give 10% p.a. simple interest. How much money will Mr Bhengu have after 7 years? (2) **L1**

4.2 A farmer keeps ships. There were 150 ships 5 years ago. Recently the ships are 278. What was the compounded population growth (in % form)? (2) **L2**

4.3 Mthandeni invested R50 000 for 36 months at an interest rate of $x\%$ p.a. compounded annually. His final amount after 4 years was R73 205. Calculate the interest rate, to the **nearest whole number**. (2) **L2**

4.4 Yoli wishes to receive R120 000 in 3,25 years' time. How much money must she invests in an account that offers 8% p.a. compounded monthly. (3) **L2**

4.5 Two friends received an amount of R6 000 each to invest for a period of 5 years. They invest the money as follows:
Mervin 8,5% p.a. simple interest. At the end of the 5 years, Mervin will receive a bonus of exactly 5% of the principal amount.
Haley: 8% p.a. compounded quarterly. Who will have a larger investment after 5 years? Justify your answer with appropriate calculations. (6) **L2**

Simple and compound decay

4.6 Mrs Smith bought a new Smart TV worth R80 000, and the smart TV depreciates at 11% p.a. compounded annually. Determine the scrap value of the car in 3 years' time. (2) **L1**

4.7 A printer's value depreciates according to the reducing balance method over a period of 7 years at a rate of 12% p.a. to R28 607,30. Calculate, to the nearest rand, the original price for the printer. (3) **L2**

4.8 After 2 years the phone is worth $\frac{1}{3}$ of its original value. Use the reducing balance method to calculate the annual rate of depreciation of the phone. (2) **L3**

4.9 An amount of a construction vehicle worth of R150 000 after 6 years' time, depreciated at 12,5% p.a. on the straight-line method. Determine the cost price of vehicle. (3) **L2**

4.10 A car, costing R198 000 has a book value of R102 755, 34 after 3 years. If the value of the car depreciates at $r\%$ p.a. on a reducing balance, calculate r. (4) **L2**

Different compounding periods

4.11 How much will R25 000 worth in 6 years' time in an account that offers 14% p.a. compounded monthly? (3) **L2**

4.12 How much will Qiniso receive in 3 years if he: …
(a) invests R2000 in an account that offers 12% p.a. compounded monthly? (2) **L2**
(b) invests R8 050 in an account that offers 14,25% p.a. compounded quarterly? (3) **L3**
(c) invests R452 000 in an account that offers 9,5% p.a. compounded half-yearly? (3) **L3**

4.13 Exactly 8 years ago Tashil invested R30 000 in an account earning 6,5%p.a., compounded monthly.
(a) How much will he receive if he withdraws his money today? (3) **L2**
(b) Tashil withdrew R10 000 three years after making the initial deposit and re-invested R10 000 five years after making the initial deposit. Calculate the difference between the final amount Tashil will now receive after eight years and the amount he would have received had there not been any transactions on the account after the initial deposit. (7) **L4**

Nominal and effective interests

4.14 Convert 18% p.a. compounded half-yearly to an effective interest rate. (2) **L2**

4.15 Convert the following effective interest rates to nominal interest rates.
(a) 8,5% p.a. to be compounded monthly. (2) **L2**
(b) 33% p.a. to be compounded semi-annually. (2) **L3**

4.16 Calculate the effective interest rate if the interest rate is 9,8%, compounded monthly. (2) **L2**

Timelines

4.18 A savings account was opened with an initial deposit of R24 000. 18 months later R7 000 was withdrawn from the account. Calculate how much money will be in the savings account at the end of 4 years if the interest rate was 10,5% p.a. compounded monthly. (4) **L3**

4.19 Pratham made an initial deposit of R32 000 into an investment account that paid interest at 8,6% p.a. compounded monthly. Another deposit of R23 000 was made 3 years later. The interest rate changed to 10,5% p.a. compounded quarterly 4 years after the initial deposit.

4.19.1 How much was in Pratham's investment account at the end of 4 years? (5) **L2**

4.19.2 At the end of 6 years since he started his investment, Pratham decided to use all his balance as a deposit for a car that cost R220 000 and borrow the rest from the bank. How much did he need to borrow? (4) **L3**

4.20 Siyabonga deposited R25 000 into a savings account with an interest rate of 18% p.a. compounded monthly. Siyabonga withdrew R8 000 from the account 2 years after depositing the initial amount. He deposited another R4 000 into this account $3\frac{1}{2}$ years after the initial deposit. What amount will Siyabonga have 5 years after making the initial deposit in this account? (6) **L4**

MIXED QUESTIONS

4.21 A machine costs R25 000 in 2016. Calculate the book value of the machine after 4 years if it depreciates at 9% p.a. according to the reducing balance method. (3) **L2**

4.22 A company buys a front-end loader at a cost of R1 800 000. The book value of the loader at the end of 4 years is R720 000. Calculate the annual rate of depreciation on the reducing balance method. (5) **L2**

4.23 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (5) **L3**

4.24 Cleaning equipment is bought for R120 000. The value of the equipment depreciates at 15% p.a. on the reducing balance. The inflation rate is 9% p.a.

(a) Calculate the scrap value of the old equipment after 5 years. (3) **L2**

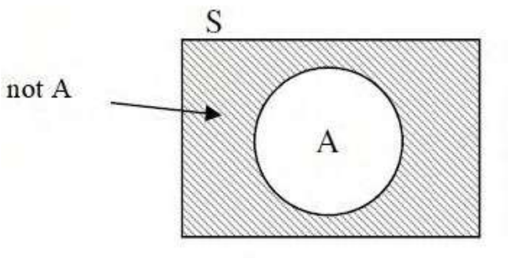
(b) In 5 years' time, the old cleaning equipment will be sold at scrap value. How much will be needed to buy the same new equipment, the proceeds from a sale of an old equipment are used as a deposit. (4) **L4**

4.25 Zipho and Zenkosi are twins, on their 18^{th} birthday they received R15 000 each. They both decided to invest their amounts till they turn 21 years Zipho went to Bank A and receive 13% p.a. compounded monthly. Zenkosi went to Bank B and received $p\%$ p.a. compounded semi-annually. Their investments yield the same amount. Calculate the value of p. (3) **L4**

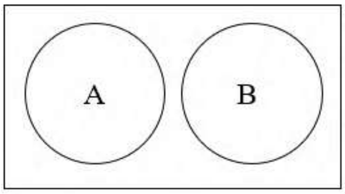
TOPIC	5. GR. 11 PROBABILITY

GUIDELINES, SUMMARY NOTES AND STRATEGIES FOR GR. 11

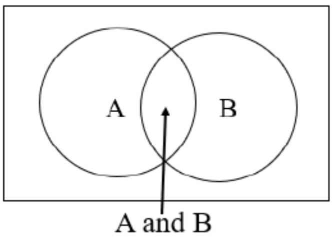
- Mark weighting: Term 3 common test: 20±3 out of 75
  November common test: 25±3 out of 75
- $P(E) = \frac{n(E)}{n(S)}$, where E represents an event and S the sample space.
- Note $0 \leq P \leq 1$. This means that probability **canno**t be negative or above 1.
- General formula: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where 'A and B' are the events that happen at the same time.
- Complementary events: $P(A) + P(\text{not } A) = 1$. It simply means that 'event A' and 'not A' cannot happen at the same time.



- Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$. This means that event A and B cannot happen at the same time. Events A and B are said to be disjoint.

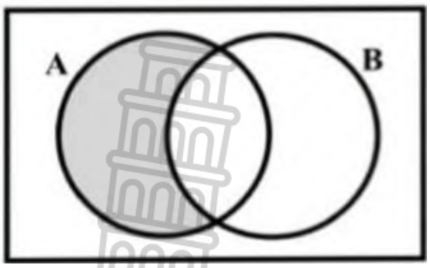


- Not Mutually Exclusive Events: $P(A \text{ and } B) \neq 0$

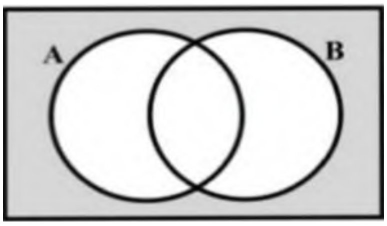


- Independent events: The outcome of the second event is not affected by the outcomes of the first event. For independent events:
  $P(A \text{ and } B) = P(A) \times P(B)$
  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
  $P(A \text{ or } B) = P(A) + P(B) - P(A) \times P(B)$

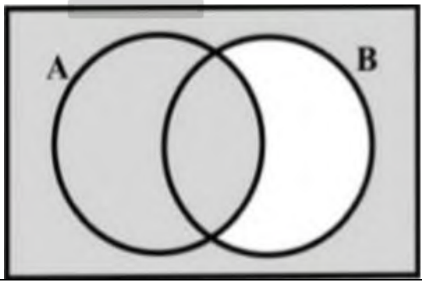
A and (not B) $\rightarrow$ 'A only'



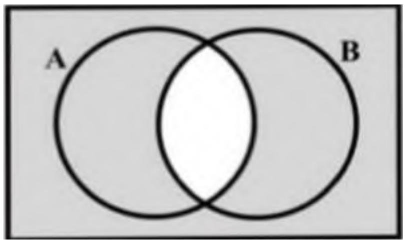
(not A) and (not B) $\rightarrow$ 'not(A or B)'



A or not B



not(A and B)



ACTIVITIES
Mutually Exclusive and Independent Events
KZN/Sept 2024 (GR 11)

5.1 A and B are two events such that:

- P(not A) = 0,55
- P(B) = 0,4
- P(A or B) = 0,67

Are events A and B independent? Justify your answer with relevant calculations (5) **L2**

IEB/Nov 2022 (GR 12)

5.2 It is given that the $P(A) = x$; $P(B) = 0,6 + x$ and $P(A \text{ and } B) = 0,36 - x$.

5.2.1 If events A and B are independent, determine the value of x. (4) **L2**

EC/Nov 2023 (GR 11)

5.3 For any two events A and B, it is given that P(A) = 0,35 and P(A or B) = 0,61. Determine P(B) if:

5.3.1 A and B are mutually exclusive. (3) **L2**

5.3.2 A and B are independent. (4) **L2**

NW/Nov 2024 (GR 11)

5.4 Given: A and B are 2 different events. P(A) = 0,5; P(B) = 0,3; P(A or B) = 0,7

5.4.1 Are the events mutually exclusive? Motivate with appropriate calculations. (3) **L2**

5.4.2 Draw a Venn-diagram representing the situation with different probabilities. (3) **L2**

5.4.3 Calculate P((not A) or B). (2) **L2**

5.4.4 Are events A and B independent? Motivate with appropriate calculations. (4) **L2**

GP/Nov 2023 (GR 11)

5.5 Events A, B and C are such that:

- A and B are independent.
- B and C are independent.
- A and C are mutually exclusive.
- Their probabilities are P(A) = 0,3 , P(B) = 0,4 and P(C) = 0,2

Calculate the probability of the following occurring:

5.5.1 Both A and C (1) **L2**

5.5.2 Both B and C (2) **L2**

5.5.3 At least ONE of A or B. (4) **L2**

EC/Nov 2022 (GR 11)

5.6 Two events A and B are such that:

- P(A) = 0,35
- P(A or B) = 0,75

Determine P(B) if:

5.6.1 A and B are mutually exclusive (3) **L2**

5.6.2 A and B are independent (4) **L2**

Venn Diagrams

NW/Nov 2025 (GR 11)

5.7 Baobab High School has 100 Grade 12 learners. Of these, 35 take Mathematics, 30 take Accounting, and 20 take both subjects.

5.7.1 Are the events "taking Mathematics" (M) and "taking Accounting" (A) mutually exclusive events? Motivate your answer. (2) **L2**

5.7.2 Are the events "taking Mathematics" (M) and "taking Accounting" (A) independent? Justify your answer with the appropriate calculations. (4) **L2**

5.7.3 Draw a Venn diagram representing the above information. (3) **L2**

5.7.4 Use the Venn diagram to determine P((not M) or A). (2) **L2**

KZN/Nov 2024 (GR 11)

5.8 A survey was conducted among 165 Grade 12 learners.
These learners were asked which car brand they preferred among BMW, Audi and VW.
The results of the survey are summarised below:

- 42 learners preferred a BMW.
- 85 learners preferred an Audi.
- 106 learners preferred a VW.
- 18 learners preferred a BMW and an Audi, but not a VW.
- 40 learners preferred a VW and an Audi, but not a BMW.
- x learners preferred a BMW and a VW.
- 13 learners preferred all three car brands.
- 20 learners did not prefer any of these three car brands.

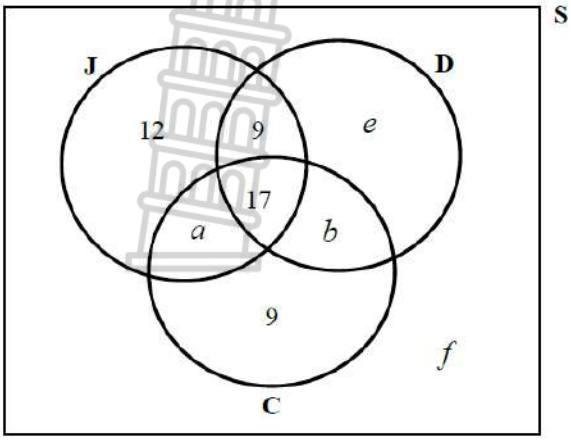
Draw a Venn diagram to represent the above information. (4) **L2**

KZN/Sept 2024 (GR 11)

5.9 A survey was done among a group of 100 tourists to find out which city in South Africa they loved during their visit. They chose from Johannesburg (J), Durban (D) and Cape Town (C). The results of the survey are listed below:

- 17 said they loved all three cities.
- 30 said they loved Durban and Cape Town
- 27 said they loved Johannesburg and Cape Town
- 52 said they loved Durban.
- 9 said they loved only Johannesburg and Durban
- 12 said they loved Johannesburg only.
- 9 said they loved Cape Town only.

The above information is represented in the partially completed Venn Diagram alongside.



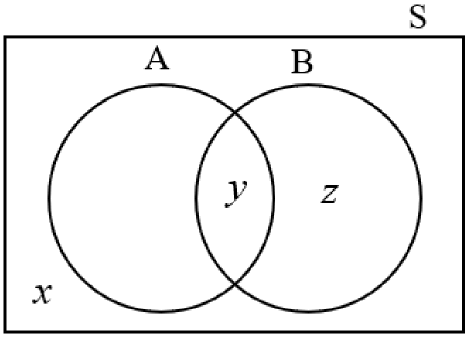
5.9.1 Write down the values of a, b, e and f. (4) **L2**

5.9.2 Calculate the probability that a tourist selected at random loved Cape Town or both Johannesburg and Durban (3) **L2**

IEB/Nov 2024 (GR 12)

5.10 In the Venn diagram alongside, A and B are two sets within a sample space S.

- n(S) = 80
- n(B) = 20 and P(A) = 0,5
- 4P(B) = 5P(A and B)
- x, y and z are the number of elements in each set.



Determine the values of x, y and z. (7) **L3**

Gemini AI

5.11 In a group of 60 learners, it is found that they participate in two main extra-curricular activities: Drama (D) and Debating (B).

- The probability that a learner chosen at random participates in neither activity is $\frac{1}{6}$.
- The probability that a learner participates in Drama is 0,5.
- The probability that a learner participates in Debating is 0,6.

5.11.1 Calculate the number of learners who participate in **neither** activity. (1) **L1**

5.11.2 Let the number of learners who participate in **both** activities be x. Use a Venn diagram to represent this information in terms of x. (3) **L2**

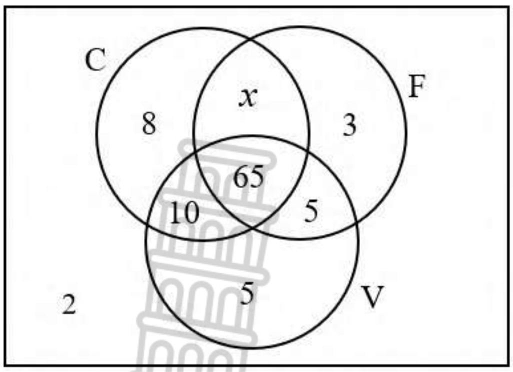
5.11.3 Calculate the value of x. (2) **L2**

5.11.4 Are the events "participating in Drama" and "participating in Debating" **independent**? Show all calculations to justify your answer. (3) **L2**

GP/Nov 2023 (GR 11)

5.12 A leadership camp was organised for 103 Grade 11 learners. The learners were asked to indicate their meal preferences.
They could choose from Chicken (C), Fish (F) or Vegetables (V).
The following information was collected:

- 2 learners do not eat chicken, fish or vegetables.
- 5 learners eat only vegetables.
- 8 learners eat only chicken.
- 23 learners do not eat fish.
- 3 learners eat only fish.



Let the number of learners who eat chicken or fish be x

5.12.1 Calculate the value of x. (1) **L1**

5.12.2 Calculate the probability that a learner chosen at random eats any TWO of the given food choices. (2) **L2**

Contingency Tables

KZN/Nov 2025 (GR 11)

5.13 A survey was conducted amongst 45 males and 85 females who were randomly selected. This survey was about the brand of clothes they would love to buy, between Gucci (G) and Louis Vuitton (LV). The results are summarised below:

- $3y$ females said they would love to buy Louis Vuitton.
- y males said they would love to buy Gucci.
- The total number of males who said they would love to buy Louis Vuitton is double the total number of females who said they would love to buy Gucci.

5.13.1 Calculate the total number of females who said they would love to buy Gucci (6) **L3**

5.13.2 Are the events of being a male and having said that they would love to buy one of the brands independent? Justify your answer with calculations. (4) **L2**

Crawford College/Sept 2023 (GR 12)

5.14 The sports coordinator at Crawford has reported the following data about sport and gender in her campus.

	Play sport	Do not play sport	TOTAL
Female	65	42	107
Male	36	68	104
TOTAL	101	110	211

Are the events 'male' and 'play sport' independent? Show all calculations to support your answer. (5) **L2**

Gemini AI (GR 11)

5.15 A survey was conducted among 200 Grade 11 learners to determine their preference between two different cafeteria lunch options: **Option A (Healthy Bowls)** and **Option B (Burgers)**. The results were partially recorded in the contingency table below.

	Option A	Option B	TOTAL
Boys	45	x	
Girls	y	30	
TOTAL	105		200

5.15.1 Determine the values of x and y and hence complete the contingency table. (3) **L1**

5.15.2 A learner is chosen at random. Calculate the probability that the learner is a girl and prefers Option A. (2) **L1**

5.15.3 Determine whether the event of being a Boy and the event of preferring Option A are independent or dependent. Support your answer with a full mathematical calculation. (5) **L2**

5.15.4 If two learners are chosen at random, one after the other without replacement, calculate the probability that at least one of them prefers Option B. (4) **L3**

Durban Girls College/Sept 2022 (GR 12)

5.16 A survey was conducted among 100 teenagers and 60 parents to determine how many of them watch TV over weekends. Their responses are shown in the partially completed table below.

	Watched TV during the weekends	Did not watch TV during the weekends	**TOTALS**
Teenagers	80	a	
Parents	48	12	
Totals	b	32	160

5.16.1 Calculate the values of a and b. (2) **L1**

5.16.2 Are the events "being a teenager" and "did not watch TV during weekends" independent of each other?
Show calculations to prove your answer. (4) **L2**

Tree Diagrams

NW/Nov 2025 (GR 11)

5.17 A bag contains a total of 25 balls, which are red or blue. The exact number of each colour is unknown. However, it is known that the probability of drawing a red ball, setting it aside, and then drawing another red ball is $\frac{11}{60}$.

Determine how many red and how many blue balls are in the bag.
(Hint: Let the number of red balls be x.) (6) **L3**

KZN/Nov 2024 (GR 11)

5.18 A Grade 11 teacher collected the mathematics workbook from each of the 25 learners in her class. Some of the learners in her class are boys and the remainder are girls. The teacher randomly selected the first book from the pile and marked it. The teacher again randomly selected the second book from the pile and marked it. The probability that the first two books selected belonged to boys is $\frac{7}{20}$

Calculate the number of girls in the class. (5) **L3**

KZN/Sept 2024 (GR 11)

5.19 Nelisiwe had a small box of 80 Smarties sweets.

40% of the Smarties are green, $\frac{3}{20}$ of them are blue and the remainder are red.

Nelisiwe picks a Smartie out of the box, takes note of the colour and then eats it.
She then picks out a second Smartie and notes the colour before eating it.

5.19.1 How many red Smarties were in the box at the beginning? (2) **L2**

5.19.2 Represent the above situation by means of a tree diagram. Indicate the probabilities associated with each branch and the possible outcomes. (3) **L2**

5.19.3 Calculate the probability that Nelisiwe will pick two Smarties of the same colour. (3) **L2**

NSC/Nov 2024 (GR 12)

5.20 At a kiosk, 120 people buy either a cup of coffee or a bottle of water. The chance of rain on any given day is 75%. The chance of a person buying a cup of coffee on a rainy day is three times the chance of the person buying coffee on a non-rainy day.

The probability of a person buying coffee on any given day is $\frac{7}{12}$.

Calculate the number of cups of coffee that will be sold on a non-rainy day. (4) **L2**

NW/Nov 2024 (GR 11)

5.21 A parking area has 14 Volkswagen and 18 BMW cars parked. There are no other cars. During the afternoon, two cars are stolen – one during the early afternoon and the other one later on.

5.21.1 Represent the situation using a tree diagram, indicating probabilities on all branches. Also indicate the possible outcomes. (4) **L2**

5.21.2 Determine, using the tree diagram, the probability that:

(a) both stolen cars are BMWs. (2) **L2**

(b) at least ONE Volkswagen will be stolen. (2) **L2**

TOPIC:	6. STATISTICS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Definitions & Terminology

Discrete data: These are distinct values (individual numerical data) which are countable.

Continuous data: These are numerical data which are uncountable and which are arranged in groups or class intervals.

Grouped data: this is a set of numerical data arranged in class intervals (groups) where the observations in each group are counted to give the frequencies.

Central tendency: The measures of central tendency are the 3 different averages, i.e. the mean, median and mode.

- **Mean** ($\bar{x}$) – the sum of a set off data divided by the number of data items, the balance point of a distribution of data. $\bar{x} = \frac{\sum x}{n}$ for ungrouped data, $\bar{x} = \frac{\sum f.x}{\sum f}$ in a frequency table and $\tilde{x} = \frac{\sum f_i.x_i}{\sum f_i}$ for grouped data
- **Median** – the middle data item when the data is listed in order. 50% of the data lies below and 50% lies above the median
- **Mode and modal class** – the data item which occurs most often. For grouped data, the modal class is the interval with the highest frequency
- **Dispersion-** The measures of dispersion, i.e. the range, interquartile range, variance and standard deviation are used to measure the spread and variability of the data.
- **Range** – the difference between the highest and lowest value (Range = highest value – lowest value)
- **Interquartile range (IQR)** – the difference between the upper and lower quartile (IQR = $Q_3 - Q_1$). It contains 50% of the data set and therefore it is affected by outliers.
- **Variance** – The average of the square of the deviations of each data item from the mean. It measures the variability of the data.
- **Standard Deviation** – the measure of deviation from the mean, it is the square root of the variance. It is the most common measure of dispersion.

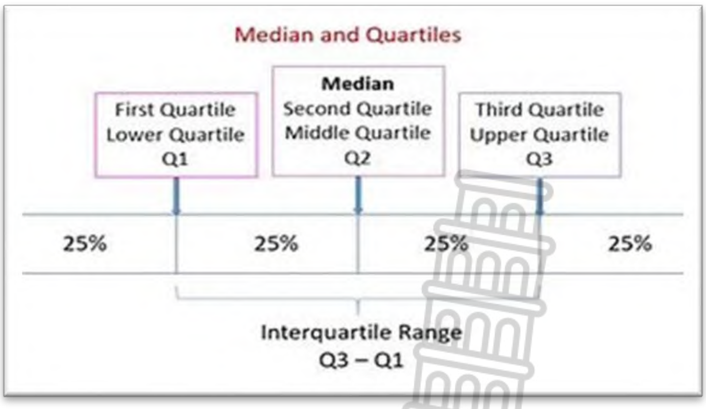
Quartiles – These divide the set of data into 4 equal parts and helping to understand the distribution and spread of data. The quartiles are:

- Lower quartile (Q_1): The 25th percentile.
- Median (Q_2): The 50th percentile.
- Upper quartile (Q_3): The 75th percentile.

$Q_1 = \frac{1}{4}(n+1)^{th}$ position of the data set.

$Q_2 = \frac{1}{2}(n+1)^{th}$ position of the data set.

$Q_3 = \frac{3}{4}(n+1)^{th}$ position of the data set.



A **percentile** is a measure that tells us what percentage (%) of the total frequency scored at or below this measure.

The position of the m^{th} percentile is $\frac{m(n+1)}{100}$

An **Ogive** (**Cumulative Frequency curve**) is a graphical representation of the cumulative frequency of the data set. An *S* shaped curve will be formed.

A Box and whisker diagram, displays the five number summary **(min, Q_1, Q_2, Q_3 and max)**

FS/Sept 2023 (Gr-12)

6.1 To celebrate Pi Day at school, learners participate in a competition to write down the value of Pi (π), up to the most correct decimal places. Eleven learners make it to the final round of the competition, where their number of correct decimal places is counted. The judges stop counting after the first mistake. The results of the eleven learners are shown in the table below.

63	**79**	**50**	**74**	**75**	**66**	**150**	**86**	**72**	**74**	**60**

6.1.1 Calculate the mean of the data (2) **L1**

6.1.2 Calculate the standard deviation for the given data (1) **L1**

6.1.3 Calculate the number of results that lie outside ONE standard deviation of the mean (3) **L2**

6.1.4 Identify the outlier(s) in the given data (1) **L1**

6.1.5 The result with the number of the most correct decimal places is increased by k %, while the result with the number of the lowest correct decimal places is decreased by t %. The other nine results remain unchanged.
Only one of the options below correctly reflects the new range of the data in terms of k and t. Choose the answer and write only the letter next to the question number

A. $100 + k - t$ B. $150k - 50t$

C. $150k + 50t$ D. $100 + \frac{3}{2}k + \frac{1}{2}t$ (2) **L3**

6.1.6 It was established that a judge made a mistake with one of the six lowest results. The result was corrected and changed to double its original value. How will this change impact the median of the data? Motivate your answer. (3) **L4**

[12]

6.2 The following table represents the marks achieved by 65 grade 11 learners in Mathematics test out of 40 marks. (3) **L2**

Interval	**Frequency**	**Cumulative frequency**
$5 \leq x < 10$	5	
$10 \leq x < 15$	9	
$15 \leq x < 20$	14	
$20 \leq x < 25$	17	
$25 \leq x < 30$	11	
$30 \leq x < 35$	7	
$35 \leq x < 40$	2	

6.2.1 Complete the cumulative frequency table above. (2) **L2**

6.2.2 Draw an ogive (cumulative frequency graph) for the above data. (3) **L2**

6.2.3 The school decided to reward learners who obtained 80% and above. How many learners were awarded? (3) **L2**

[8]

6.3 The mean age of the first 13 spectators who went to St George's Park to watch an ODI (South Africa versus Australia) cricket match is 27. The 13 ages are given below:

20 32 25 14 x **38 22 30 19 28 34 40 25**

6.3.1 Calculate the value of x. (2) **L2**

6.3.2 Hence, determine the standard deviation for the ages. (1) **L1**

6.3.3 Determine how many of the spectators had an age which is within one standard deviation of the mean. (2) **L2**

[5]

6.4 In the grid below a, b, c, d, e, f and g represent values in the data set written in ascending order. No value in the data set is repeated.

a	b	c	d	e	f	g

Determine the value of a, b, c, d, e, f and g if:

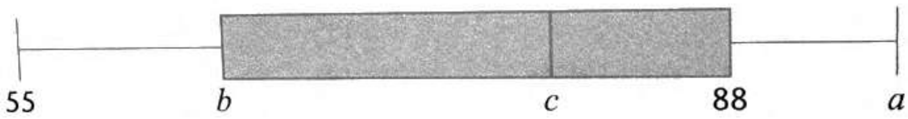
- The maximum is 42
- The range is 35
- Median is 23
- The difference between the median and the upper quartile is 14
- The interquartile range is 22
- $e = 2c$
- The mean is 25 [7] **L3**

Platinum Mathematics Gr 11

6.5 The percentage marks for a class of 19 learners for a mathematics test were:

81; 80; 74; 75; 57; 55; 91; 88; a; 76; 61; 60; 83; 89; b; 69; 80; 90; 80

The box-and-whisker diagram shows the five-point summary



6.5.1 If the range of the data is 40 and the interquartile range is 23, determine the values of a, b, and c. (4) **L3**

6.5.2 Calculate the standard deviation and variance of the above data using a calculator (2) **L1**

6.5.3 Describe the skewedness of the distribution. (2) **L1**

[8]

6.6 Ten students wrote a theoretical Life Science test (T) out of 45 and completed a Practical Life Science test (P) out of 50. Their marks are shown on the summary below.

Test	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
T	38	39	43	26	32	39	9	31	41	17
P	39	37	43	24	33	34	14	29	35	21

6.6.1 List the five number summary for both the Theoretical Life Science test (T) and Practical Life Science test (4) **L2**

6.6.2 List the outlier(s) in Theoretical Life Sciences test (2) **L2**

6.6.3 Calculate the mean and the standard deviation for the theoretical test using your calculator (4) **L1**

6.6.4 How many scores fall within on standard of the mean for the theoretical tests? (3) **L2**

[13]

EC Nov 2011 P2

6.7 The following are the ages of the first 12 people who went to vote in Lota voting station on 18 May 2011:

31 60 25 19 44 53 25 36 42 18 49 55

6.7.1 Determine the mode (1) **L1**

6.7.2 Determine the five number summary (5) **L2**

6.7.3 Determine the inter-quartile range (1) **L2**

6.7.4 Use the information in question above to draw a box and whisker diagram. (3) **L2**

6.7.5 Determine the mean and standard deviation (1) **L2**

6.7.6 At another voting station the mean age was 37 and the standard deviation was 20. How do the ages of voters at this station compare to those of Lota? Motivate your answer by referring to the mean and standard deviation of the ages. (4) **L4**

[17]

6.8 **EC Nov 2011 P2**

The following table represents the percentages of 75 grade 11 learners of Future Private school:

Interval	**Frequency**	**Cumulative Frequency**
$10 \leq x < 20$	3	
$20 \leq x < 30$	6	
$30 \leq x < 40$	10	
$40 \leq x < 50$	12	
$50 \leq x < 60$	15	
$60 \leq x < 70$	13	
$70 \leq x < 80$	9	
$80 \leq x < 90$	5	
$90 \leq x < 100$	2	

6.8.1 Complete the cumulative frequency above. (3) **L2**

6.8.2 Draw the ogive (cumulative frequency curve) for the above data. (3) **L2**

6.8.3 The ogive curve is used to determine the median of the percentages. The school decides that 5% should be added on. What is the new median? (2) **L3**

[8]

6.9 The table below shows the marks of 200 matrics in Mathematics.

Marks	Frequency	Cumulative Frequency
$0 \leq x < 20$		22
$20 \leq x < 30$		68
$30 \leq x < 40$		142
$40 \leq x < 50$		179
$50 \leq x < 60$		200

6.9.1 Complete the frequency column of the table. (2) **L2**

6.9.2 Write down the modal class of the data (2) **L1**

6.9.3 Draw an ogive curve for the data. (4) **L2**

6.9.4 The top 20% of the learners were allowed entry to university. Determine the required mark for entry to university. Indicate your answer with the letter A on the graph. (2) **L3**

[10]

DBE/September 2021(2)

6.10 The table below shows the marks that 15 learners from one particular school obtained in the aptitude test.

Marks (as a %)	62	58	78	85	74	48	74	84	100	46	80	92	60	90	92

6.10.1 Calculate the mean mark obtained by these learners (2) **L1**

6.10.2 Calculate the standard deviation of these learners' marks (1) **L1**

6.10.3 Calculate the number of these learners whose marks lie more than one standard deviation above the mean (2) **L2**

6.10.4 The final Grade 11 marks (as a percentage) obtained by a group of learners was analysed. The one standard deviation interval about the mean was calculated as (82.7; 94.1). **L3**

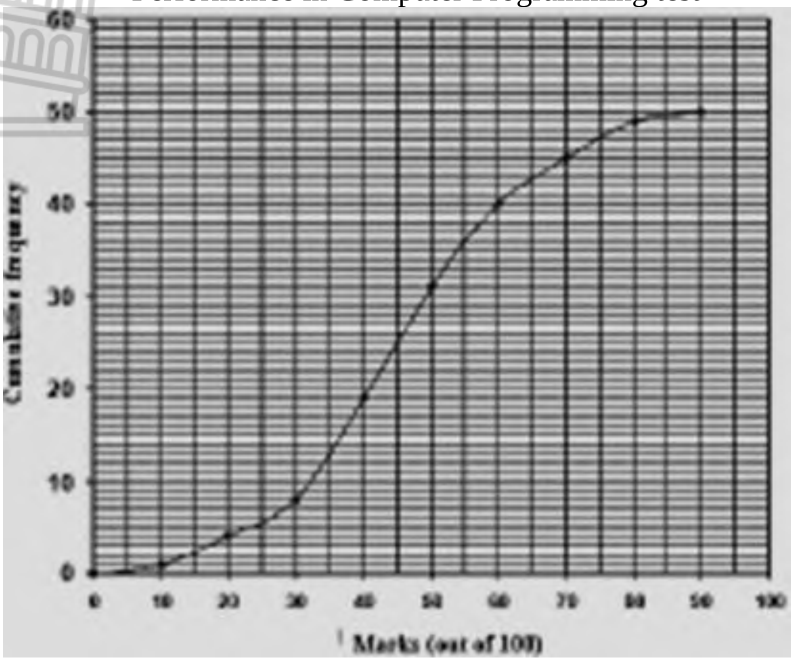
Calculate the standard deviation for the final Grade 11 marks. (3)

[8]

DoE/November 2009(1)/Gr 12

6.11 The ogive (cumulative frequency graph) shows the performance of students who took a test in basic programming skills. The test had a maximum of 100 marks.

Performance in Computer Programming test



6.11.1 How many students took the test? (1) **L1**

6.11.2 Only the top 25% of the students are allowed to do an advanced course in programming. Determine the cut-off mark to determine the top 25%. (1) **L2**

6.11.3 Construct a frequency table for the information given in the ogive above. (3) **L2**

6.11.4 Use the graph to estimate the following, indicating where you read off your answers:

i) The median

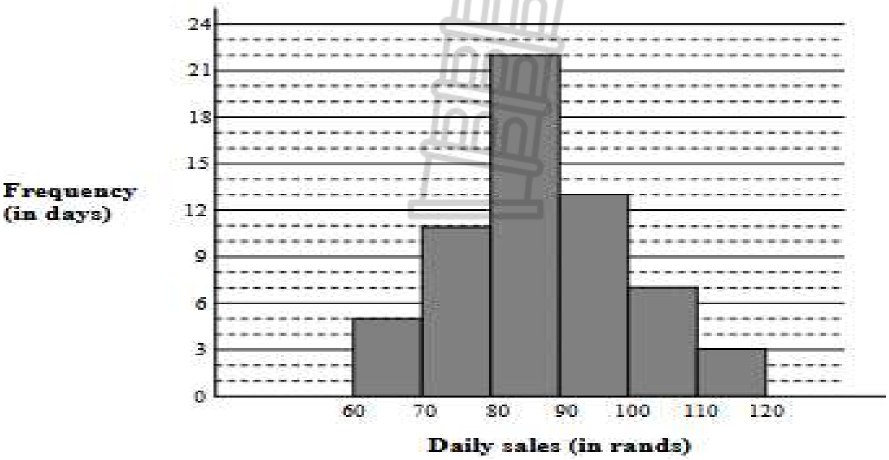
ii) The upper Quartile

iii) The 70th percentile

(3) **L3**

DoE/November 2008 /Gr12

6.12 A group of learners wrote a standardised English test that was scored out of 60. The results were represented in a cumulative frequency graph below. A street vendor has kept a record of sales for November and December 2007. The daily sales in rands are shown in the histogram below.



6.12.1 Complete the cumulative frequency table for the sales over November and December. (3) **L2**

6.12.2 Draw an ogive for the sales over November and December (1) **L2**

6.12.3 Use your ogive to determine the median value for the daily sales. Explain how you obtain your answer. (2) **L3**

[9]

DBE/NOVEMBER 2013 Gr12

6.13 The Grade 10 classes of the three schools wrote a term test. All three schools have the same number of learners in Grade 10. The results of the test have been summarised in the table below.

	SCHOOL A	SCHOOL B	SCHOOL C
Mean	9.8	9.8	14.8
Standard deviation	2.3	3.1	2.3

6.13.1 In which school (A, B, or C) is the majority of the results more widely spread around the mean? Give a reason for your answer. (2) **L3**

6.13.2 What is the difference in the spread around the respective means of the marks in School A and School C? (1) **L3**

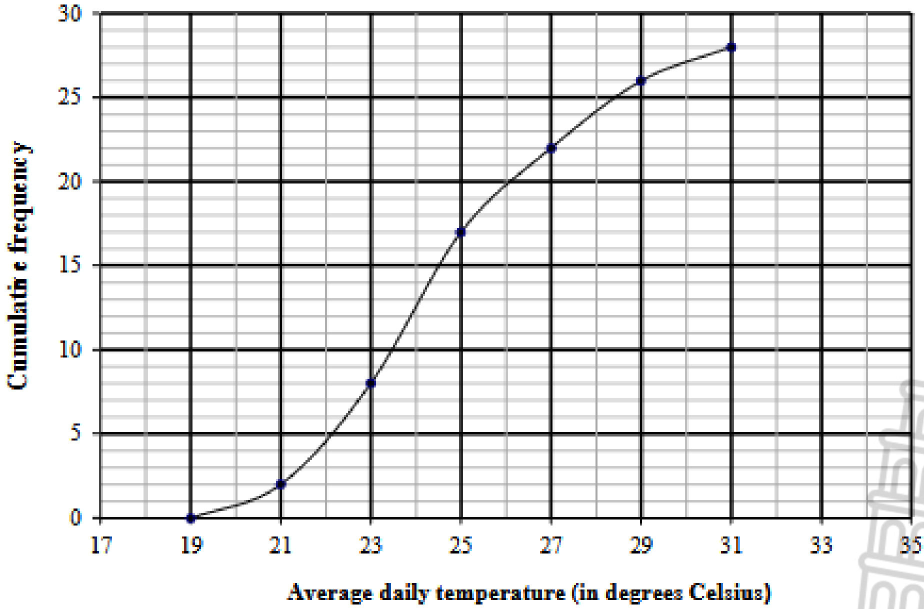
6.13.3 Explain how the marks of School A must be adjusted to match marks of School C (3) **L3**

6.13.4 If each mark in School C is lowered by 10%, explain the effect it will have on the mean and standard deviation of this school. (2) **L2**

[7]

6.14 The 2012 Summer Olympic Games was held in London. The average daily temperature, in degrees Celsius, was recorded for the duration of the Games. A cumulative frequency graph (ogive) of this data is shown below.

Cumulative frequency graph of average daily temperature recorded



6.14.1 Over how many days was the 2012 Summer Olympic Games held? (1) **L1**

6.14.2 Estimate the percentage of days that the average daily temperature was less than 24°C. (2) **L2**

6.14.3 Complete the frequency table for the data above. (3) **L3**

6.14.4 Hence, draw a frequency polygon of the data. (4) **L3**

FS Sept Mock 2023 Gr12

6.15 The speeds, in kilometres per hour, of cyclists that passed a point on the route of the Ironman Race were recorded and summarised in the table below:

Speed (km/h)	Frequency (f)	Cumulative Frequency
$0 < x \leq 10$	10	10
$10 < x \leq 20$	A	30
$20 < x \leq 30$	45	B
$30 < x \leq 40$	72	C
$30 < x \leq 40$	D	170

6.15.1 Complete the above table by filling in the missing LETTERS (3) **L2**

6.15.2 Draw a cumulative frequency curve for the above data. (2) **L2**

6.15.3 Indicate clearly on the graph where the estimates of the lower quartile (Q1) and median (M) speed can read off. Write down these estimates. (2) **L3**

6.15.4 Draw a box and whisker diagram for the data. (2) **L2**

6.15.5 Use your data to estimate the number of cyclists that passed the point with speeds greater than 35 km/h. (1) **L2**

[10]

TOPIC	7. ANALYTICAL GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Analytical Geometry is the Geometry on the Cartesian plane.

- It is an algebraic approach to the study of Geometry "using coordinates".

In this topic we will address the following concepts:

- The **distance** between two points /length of a line segment.
- The **midpoint** of a line segment.
- The **gradient** of a line segment.
- Angle of **inclination**.
- **Equation** of a line segment.
- **Areas** of polygons.

Tip: Think of a **DISTANCE** formula!!

- When required to calculate the **length/distance** of a line segment.
- When **given** the distance …
- When proving equal lengths.

❖ Suppose we have the line segment AB:

1. The distance/length between two points (AB):

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

Tip: Use the **midpoint formula** if:
given the end points of the line segment and if it is **indicated**

- or **stated** on the statement that the coordinate between those points is the midpoint!

NB: We **DON'T** assume if a point is in the middle or not, we authorised by statements or **geometric facts to** use midpoint formula!

2. The midpoint of a line segment

$$M_{AB}\left(\frac{x_A + x_B}{2};\frac{y_A + y_B}{2}\right)$$

$$x_M = \frac{x_A + x_B}{2};\ y_M = \frac{y_A + y_B}{2}$$

Tip: We use this formula:

- If required to calculate the coordinate of one end point of the line segment and **GIVEN** the coordinate of the midpoint(M) …

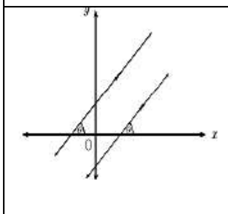
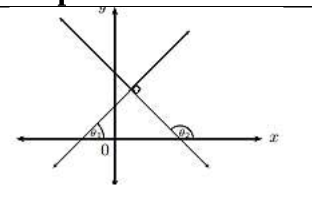
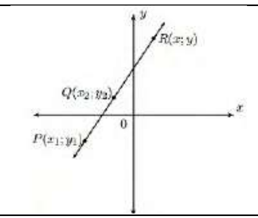
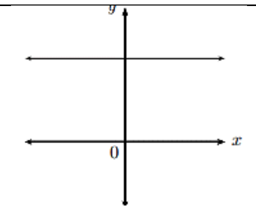
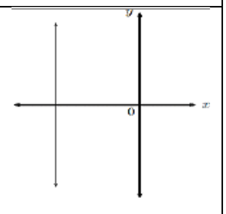
3. The gradient of a line:

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

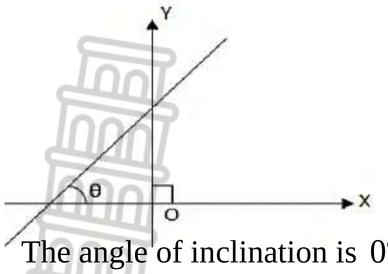
Tip: THINK of a gradient formula:

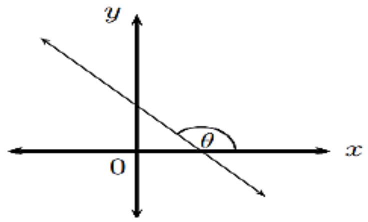
- When required to calculate the gradient of a line.
- "When **given** the gradient …"
- Collinear points…

More about gradient:

Parallel Lines	Perpendicular Lines	Collinear Points	Horizontal Line	Vertical Line
				
$m_1 = m_2$	$m_1 \times m_2 = -1$	$m_{PQ} = m_{QR} = m_{PR}$	$m = 0$	m is undefined

4. THE INCLINATION OF A LINE: $\text{m} = \tan\theta$





- The angle of inclination is $0° \le \theta \le 180°$
- Positive gradients (m) have an angle of inclination $0° < \theta < 90°$ (Acute angle).
- Negative gradients have an angle of inclination $90° < \theta < 180°$ (Obtuse angle).

5. EQUATION OF A STRAIGHT LINE: $y - y_1 = m(x - x_1)$ OR $y = mx + c \,/\, y = ax + q$

Determine equation of a line when given:

- The gradient and the y-intercept
- The gradient and one point on the line
- Line passing through two points

Coordinates of a point on a Cartesian Plane

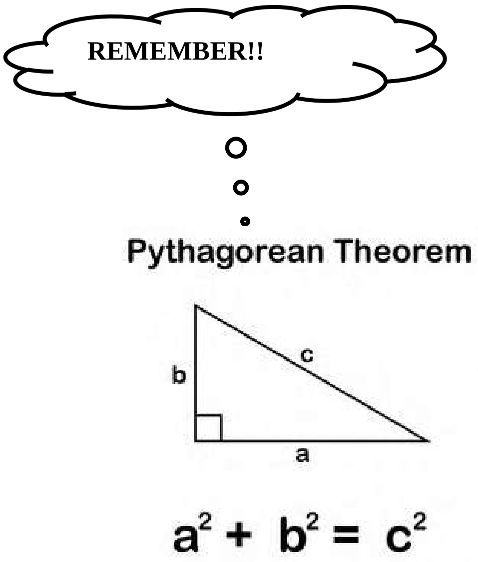
- Every point on the y - axis has $x = 0$
- Every point on the x - axis has $y = 0$
- In analytical geometry we represent geometric figures on the Cartesian plane and use coordinates of points to derive and prove important facts about these figures.

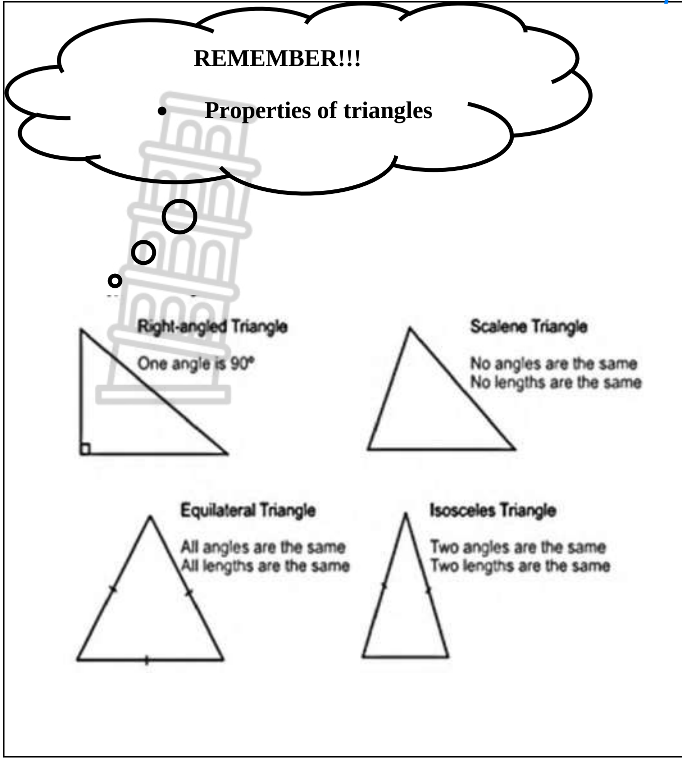
6. Areas of polygons.

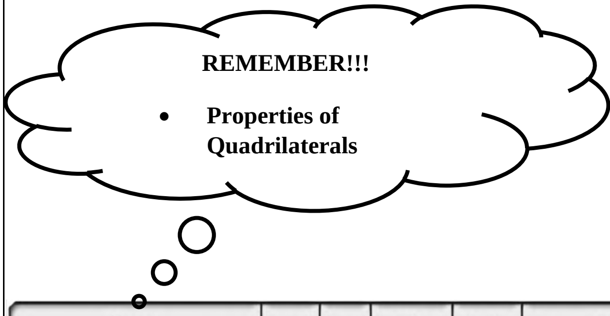
Area of a right angled triangle: $\frac{1}{2} \times base \times \perp height$

For non-right angled triangle use area rule

Area of trapezium : $\frac{1}{2}(\text{A+B})h$; where A and B are || sides and h is a $\perp$ height .



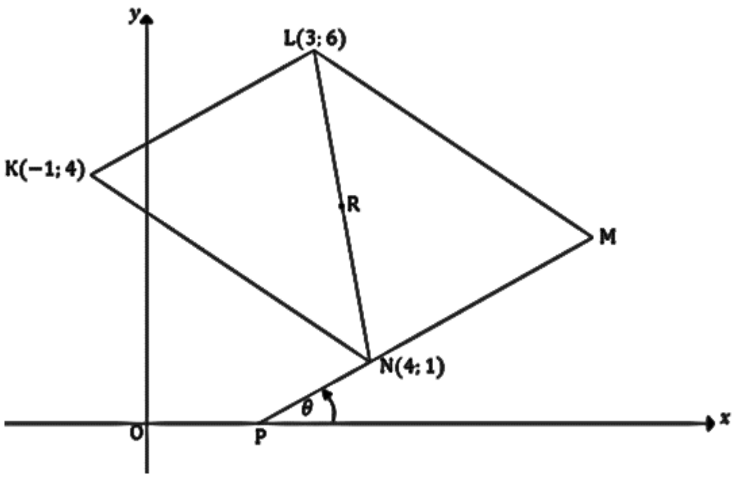




	Property	Rectangle	Square	Parallelogram	Rhombus	Trapezium
Sides	All Sides are equal	✗	✓	✗	✓	✗
	Opposite Sides are equal	✓	✓	✓	✓	✗
	Opposite Sides are parallel	✓	✓	✓	✓	✓
Angles	All angles are equal	✓	✓	✗	✗	✗
	Opposite angles are equal	✓	✓	✓	✓	✗
	Sum of two adjacent angles is 180	✓	✓	✓	✓	✗
Diagonals	Bisect each other	✓	✓	✓	✓	✗
	Bisect perpendicularly	✗	✓	✗	✓	✗

DBE/SEPTEMBER 2019

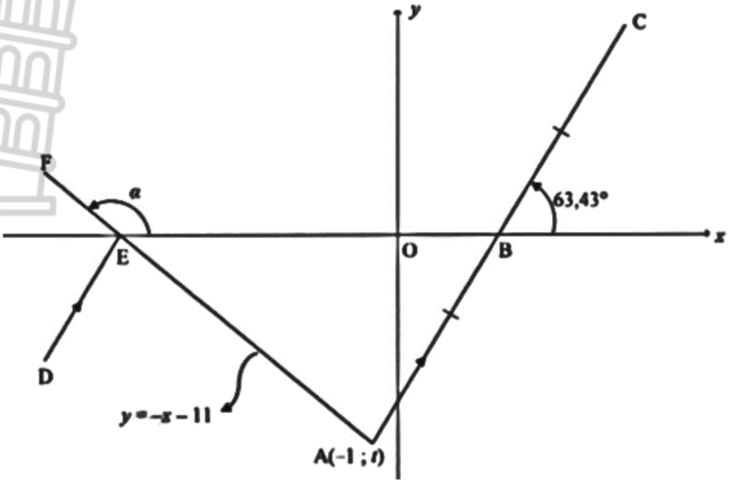
7.1 In the diagram below, $K(-1;4)$; $L(3;6)$ M and $N(4;1)$ are vertices of a parallelogram. R is the midpoint of LN and P is the x - intercept of the line MN produced to P.



7.1.1	Determine the gradient of KL	(2)	**L1**
7.1.2	Calculate the coordinates of R	(3)	**L1**
7.1.3	Calculate the coordinates of M	(4)	**L2**
7.1.4	Determine the equation of NM in the form $y = mx + c$	(3)	**L2**
7.1.5	Calculate the coordinates of P	(2)	**L2**
7.1.6	Find the size of θ, the inclination of PM.	(2)	**L2**
7.1.7	Calculate the size of $K\hat{P}N$.	(4)	**L3**
		[20]	

DBE/NOVEMBER 2018

7.2 In the diagram below, the equation of line AF is $y = -x - 11$. B, a point on the x-axis is the midpoint of the straight line joining $A(-1;t)$ and C. The angles of inclination of AF and AC are α and $63{,}43°$ respectively. AF cuts the x- axes at E. D is a point such that $DE \parallel AC$.

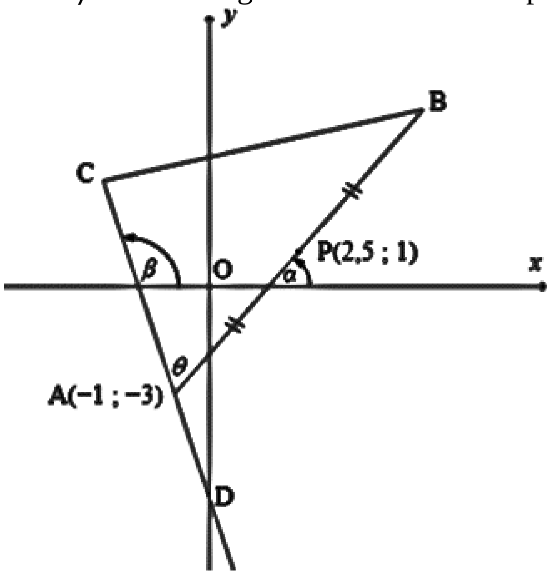


7.2.1	Calculate the:		
	a) value of t.	(2)	**L1**
	b) size of α.	(2)	**L2**
	c) gradient of AC, to the nearest whole number.	(2)	**L2**
7.2.2	Determine the equation of AC in the form $y = mx + k$.	(2)	**L2**
7.2.3	Calculate the:	(3)	**L2**
	a) coordinates of C.	(3)	**L3**
	b) Size of $F\hat{E}D$.	(3)	**L3**
		[17]	

DBE/FEB./MAR./2014

7.3 In the diagram below, A $A(-1;-3)$ B and C are the vertices of a triangle. P (2,5 ; 1) is the midpoint of AB. CA extended cuts the y- axis at D. The equation of CD is $y = -3x + k$.

$C\hat{A}B = \theta$. α and β are the angles that B and AC respectively make with the x- axis.

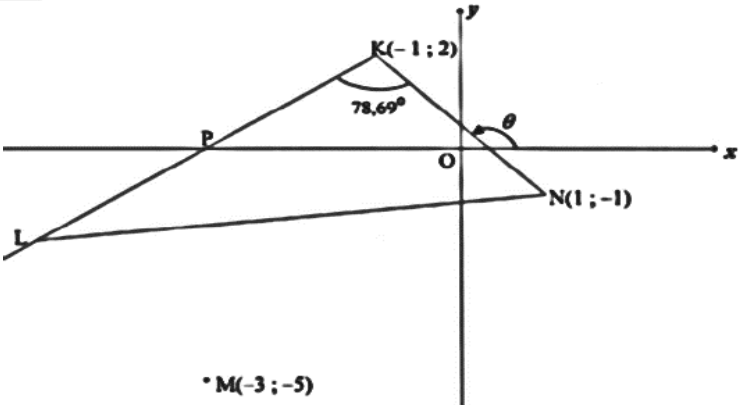


7.3.1	Determine the value of k.	(2)	**L1**
7.3.2	Determine the coordinates of B.	(2)	**L2**
7.3.3	Determine the gradient of AB.	(2)	**L1**

7.3.4	Calculate the size of θ.	(5)	L2
7.3.5	Calculate the length of AD. Leave your answer in surd form.	(2)	L3
7.3.6	If AC = 2AD and AB = $\sqrt{113}$, calculate the length of CB.	(5)	L3
		[18]	

DBE/NOVEMBER 2018

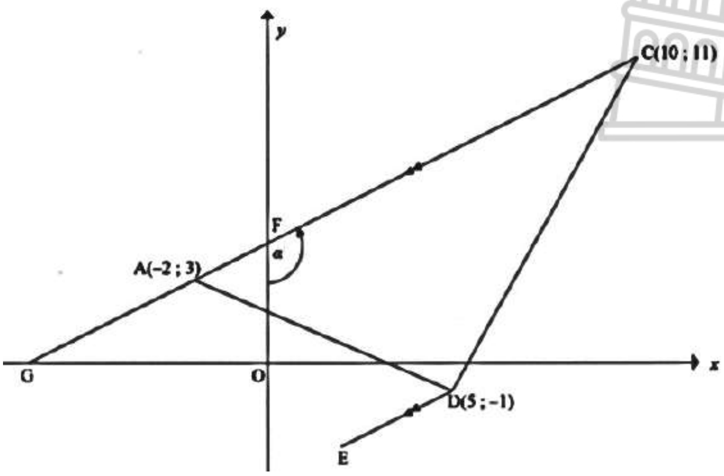
7.4 In the diagram, K$(-1;\ 2)$; L and N (1; 1) are vertices of ΔKLN and $L\hat{K}N = 78{,}69°$. KL intersects the x-axis at P and KL is produced. The inclination of KN is θ. The coordinates of M are $(-3; -5)$.



7.4.1	Calculate:		
	a) the gradient of KN.	(2)	L1
	b) the size of θ.	(2)	L2
7.4.2	Show that the gradient of KL is equal to 1.	(2)	L3
7.4.3	Determine the equation of the straight line KL in the form $y = mx + c$.	(2)	L2
7.4.4	Calculate the length of KN.	(2)	L1
7.4.5	It is further given that KN = LM.		
	a) Calculate the possible coordinates of L.	(5)	L4
	b) Determine the coordinates of L if it is given that KLMN is a parallelogram.	(4)	L3
7.4.6	T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN.	(4)	L4
		[23]	

DBE/NOVEMBER 2015

7.5 In the diagram, A$(-2\ ;\ 3)$, C(10; 11) and D$(5; -1)$ are the vertices of ΔACD. CA intersects the $y-axis$ at F and CA produced cuts the $x-axis$ at G. The straight line DE is drawn parallel to CA and $C\hat{F}O = \alpha$.



7.5.1	Calculate the gradient of the line AC.	(2)	L1

7.5.2 Determine the equation of line DE in the form $y = mx + c$. (3) **L2**

7.5.3 Determine the size of α. (3) **L3**

7.5.4 B is a point on the first quadrant such that ABDE, in that order forms a rectangle. Calculate, giving reasons the: (5) **L3**

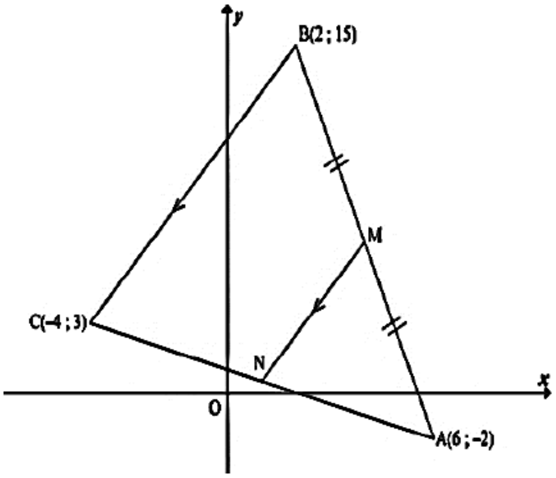
a) coordinates of M, the midpoint of BE. (3) **L2**

b) length of diagonal BE. (3) **L2**

[19]

DBE/NOVEMBER 2016

7.6 In the diagram, $A(6;-2)$, $B(12;15)$ and $C(-4;3)$ are the vertices of Δ ABC. M is the midpoint of AB. N is a point on AC such that $MN \parallel BC$.



7.6.1 Determine the coordinates of M, the midpoint of AB. (2) **L1**

7.6.2 Determine the gradient of line MN. (3) **L2**

7.6.3 Hence, or otherwise, determine the equation of line MN, in the form $y = mx + c$. (2) **L2**

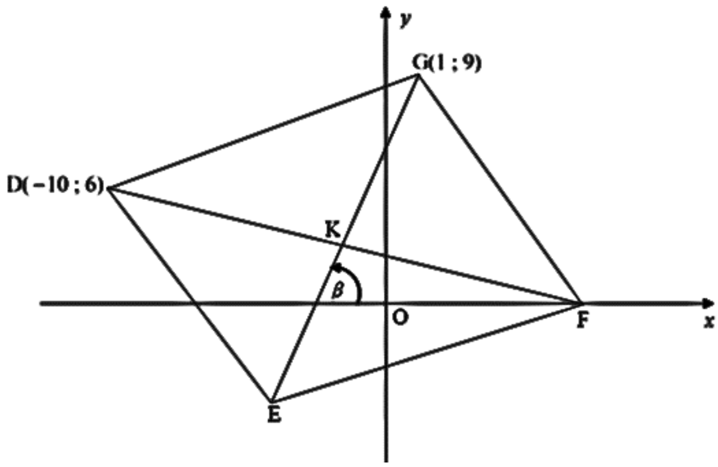
7.6.4 Calculate with reasons, the coordinates of point N. (4) **L3**

7.6.5 If ABCD (in that order) is a parallelogram, determine the coordinates of point D. (4) **L3**

[15]

KZN/2024

7.7 In the diagram below, D $(-10;\ 6)$;E, F and G $(1;9)$ are the vertices of a quadrilateral. F is a point on the $x-axis$. The diagonals of the quadrilaterals bisect each other at K. The equation of diagonal EG is $3x - y + 6 = 0$ and β is the angle if inclination of EG.



7.7.1 Calculate the size of β. (3) **L2**

7.7.2 Calculate the coordinates of F if the equation of DF is $x + 3y = 8$. (2) **L2**

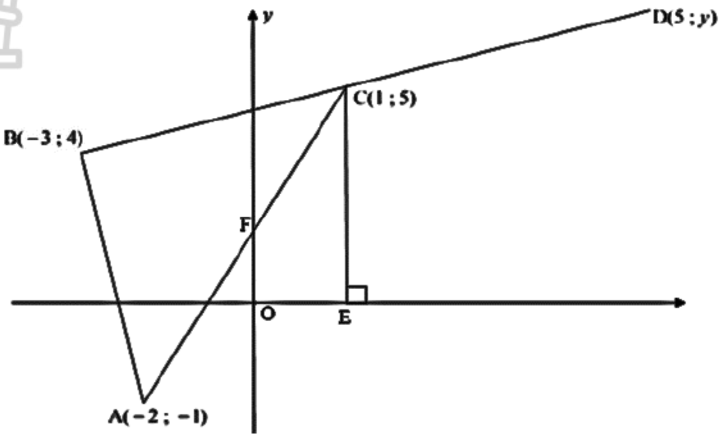
7.7.3 Calculate the coordinates of E. (4) **L3**

7.7.4 Prove that DEFG is a rhombus. (3) **L3**

[12]

KZN/2024

7.8 In the diagram, A(–2 ; –1), B(–3 ; 4) and C(1; 5) are the vertices of a triangle. BC is produced to D(5 ; y), AC cuts the y- axis at F. E is a point on the x- axis such that CE is perpendicular to the $x-$axis.



7.8.1 Calculate the length of AC. Leave answer in simplified surd form. (2) **L1**

7.8.2 Calculate the gradient of BC. (2) **L1**

7.8.3 Calculate the value of y if B, C and D are collinear. (3) **L2**

7.8.4 If H is a point such that AH $\perp$ BC, determine the equation of AH. (3) **L3**

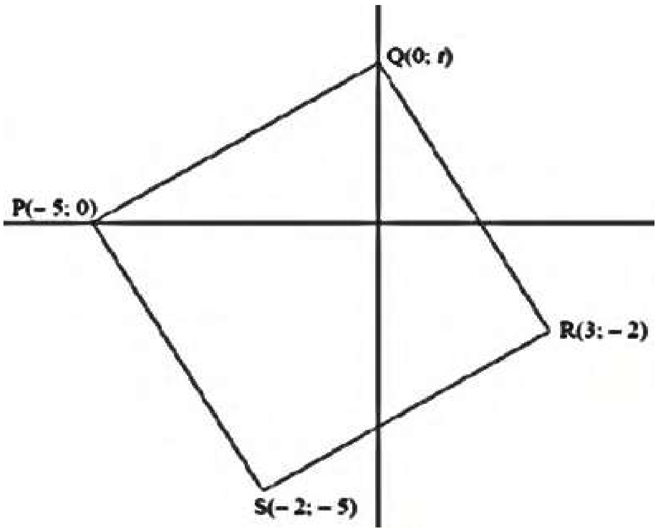
7.8.5 Calculate the coordinate of G if CBAG, in that order, is a parallelogram. (3) **L3**

7.8.6 Calculate the area of CEOF. (4) **L2**

[17]

FS/JUNE/2024

7.9 PQRS is a quadrilateral with vertices P (–5 ; 0); Q (0 ; t); R (3 ; –2) and S (–2 ; –5). P is on the $x-$axis and Q is on the $y-$axis.



7.9.1 Determine the length of PS and RS. (4) **L2**

7.9.2 Calculate the coordinates of M, the midpoint of PR. (2) **L1**

7.9.3 Show that the value of t = 3, if it is given that PR and PS bisect each other. (1) **L3**

7.9.4 Determine the gradient of PQ. (2) **L2**

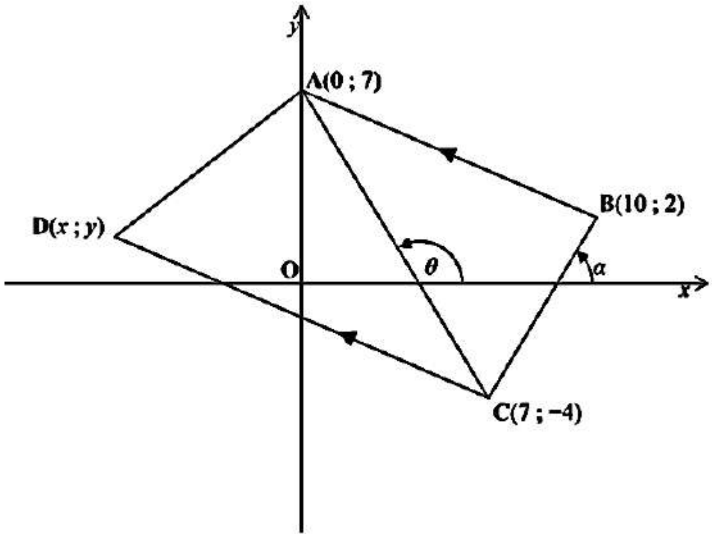
7.9.5 Calculate the size of $P\hat{Q}R$. (4) **L3**

7.9.6 What type of a quadrilateral is PQRS? Justify your answer. (3) **L4**

[16]

GAUTENG NOVEMBER 2022

7.10 In the diagram, the points A(0 ; 7), B(10 ; 2), $C(7;-2)$ and $D(x;y)$ form quadrilateral ABCD. $AB \parallel CD$, with AC joined. The angles of inclination for AC and BC are is θ and α respective.



Determine:

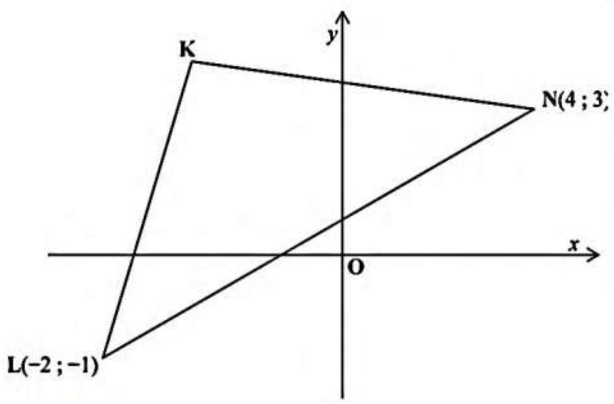
7.10.1 the length of AB (Leave your answer in simplified surd form). (3) **L1**

7.10.2 the size of α and θ. (4) **L2**

7.10.3 the magnitude (size) of $B\hat{C}D$. (4) **L2**

[11]

7.11 Consider ΔKLN drawn below. KL has the equation $y = 5x + 9$ while KN has the equation $5y + x - 19 = 0$.



7.11.1 Show that the coordinates of K are $(-1;4)$. (3) **L3**

7.11.2 Show that KL $\perp$ KN. (3) **L2**

7.11.3 Hence, or otherwise, determine the area of ΔKNL. (4) **L2**

7.11.4 Determine the equation of the perpendicular bisector of LN. (4) **L3**

7.11.5 If L, N and P(7 ; y) are collinear, find y. (2) **L2**

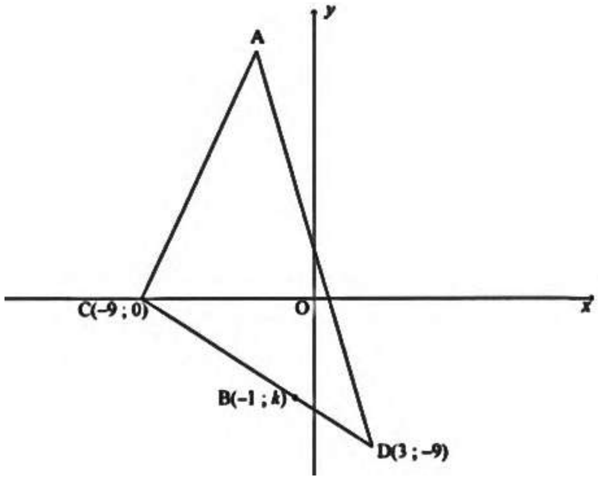
7.11.6 Determine the coordinates of Q, if KLQN is a parallelogram. (2) **L3**

7.11.7 Explain, with geometric reasons, why KLQN is a square. (2) **L3**

[20]

DBE/NOVEMBER 2024

7.12 In the diagram below, ΔACD has the vertices A, $D(3;-9)$ and $C(-9;0)$, where A is a point in the second quadrant. $B(-1;k)$ lies on side DC.

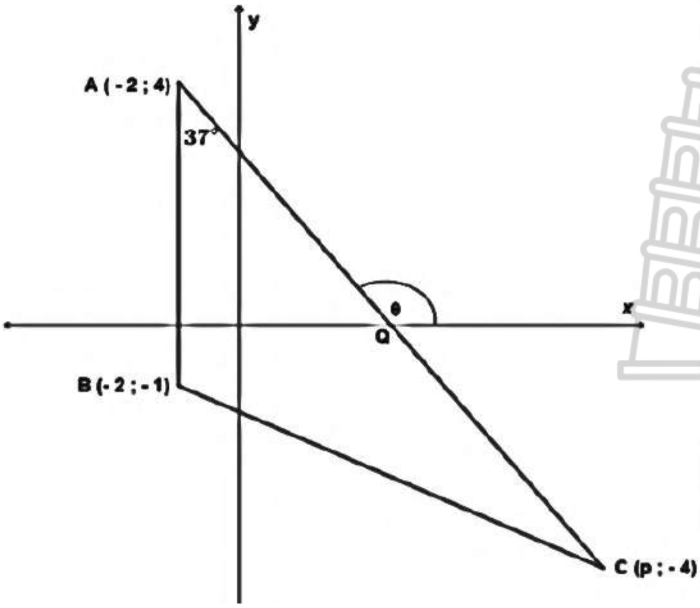


7.12.1 Calculate the gradient of DC.	(2)	**L1**
7.12.2 Determine the equation of DC in the form $y = mx + c$.	(2)	**L2**
7.12.3 Show that k = -6.	(1)	**L2**
7.12.4 Calculate the length of DC.	(2)	**L2**
7.12.5 Calculate the ratio of $\frac{DB}{DC}$.	(2)	**L2**
7.12.6 If it is further given that the gradient of AD is -4 and the length of AD is $\sqrt{612}$ units, calculate the coordinates of A.	(6)	**L3**

[15]

FS/JUNE2024

7.13 In the diagram below, A $(-2;4)$, B$(-2;-1)$ and C$(p;-4)$ are the vertices of ΔABC .Q is the $x-$intercept of the line AC. θ is the angle of inclination of AC and $B\hat{A}C = 37°$.



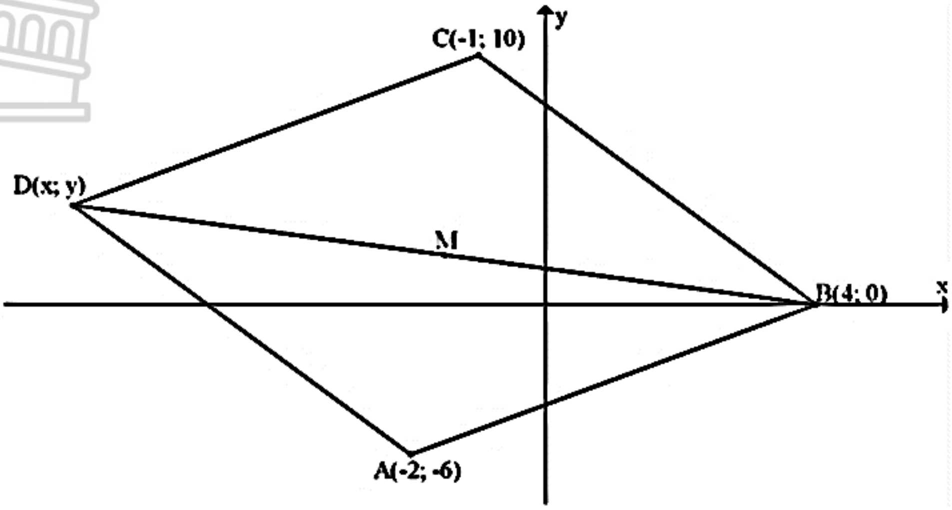
7.13.1 Determine the size of θ.	(2)	**L2**
7.13.2 Calculate the equation of line AC.	(3)	**L2**
7.13.3 Calculate the coordinate of Q, the $x-$intercept of AC.	(2)	**L2**

7.13.4 Determine the integral value of p if the point A, Q and C are collinear. (3) **L3**

7.13.5 Determine the equation of the line parallel to BC and passing through point A. (5) **L3**

[15]

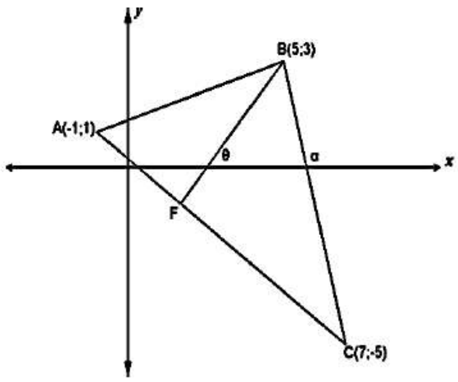
7.14 ABCD is a parallelogram with A $(-2;-6)$, B(4 ; 0), C $(-1;0)$ and D $(x;y)$ as shown below.



7.14.1	Calculate the length of BC.	(2)	**L1**
7.14.2	Determine the gradient of AB.	(2)	**L1**
7.14.3	Determine the equation of CD.	(3)	**L2**
7.14.4	Determine the coordinates of M, the midpoint of BD.	(3)	**L1**
7.14.5	HENCE, OR OTHERWISE, determine the values of x and y.	(3)	**L1**
		[13]	

GAUTENG/JUNE 2018

7.15 In the sketch below the coordinates of the vertices of Δ ABC are A $(-1;1)$, B(5 ; 3) and C(7 ; -5). Point F is a point on AC such that AF = CF. Line BF and line BC make angles θ and α respectively with the x-axis as indicated in the sketch.



7.15.1	Calculate the gradient of the line BC.	(2)	**L1**
7.15.2	Calculate the coordinates of the point F.	(2)	**L1**
7.15.3	Determine the equation of the median BF.	(3)	**L2**
7.15.4	Calculate the size of $F\hat{B}C$. (Rounded off to ONE decimal figure).	(5)	**L3**
7.15.5	If the coordinates of point K is (6 ; p), calculate the value of p if $A\hat{F}K = 90°$.	(4)	**L4**
7.15.6	Calculate the coordinates of point T, given that ABCT is a parallelogram.	(2)	**L3**
7.15.7	Prove that the diagonals of quadrilateral ABCT bisect each other.	(2)	**L3**

7.15.8 Determine the perimeter of parallelogram LMNO which is an enlargement by a scale factor of TWO of parallelogram ABCT. (5) **L3**

[25]

NW/JUNE 2018

7.16 A$(-1;-1)$, B$(2\ ;0)$ and C$(5\ ;\ p)$ are three points on the Cartesian plane.

Determine the value(s) of p if:

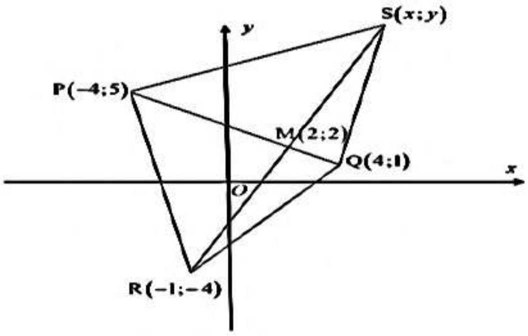
7.16.1 A, B, and C are collinear. (2) **L1**

7.16.2 AB is perpendicular to BC. (3) **L2**

7.16.3 The length of BC is 5 units. (3) **L2**

[08]

7.17 In the diagram P$(-4\ ;5)$, Q$(4\ ;1)$ and R$(-1\ ;\ -4)$ are the vertices of a triangle in the Cartesian plane with M on PQ. M$(2\ ;2)$ is a midpoint of straight line RS.



7.17.1 Determine the gradient of PQ. (2) **L1**

7.17.2 Show that $P\hat{M}S = 90°$. (3) **L2**

7.17.3 Determine the coordinates of S. (3) **L2**

7.17.4 Prove that ΔQRS is isosceles. (3) **L2**

7.17.5 Determine the area of ΔPRS. (5) **L3**

[16]

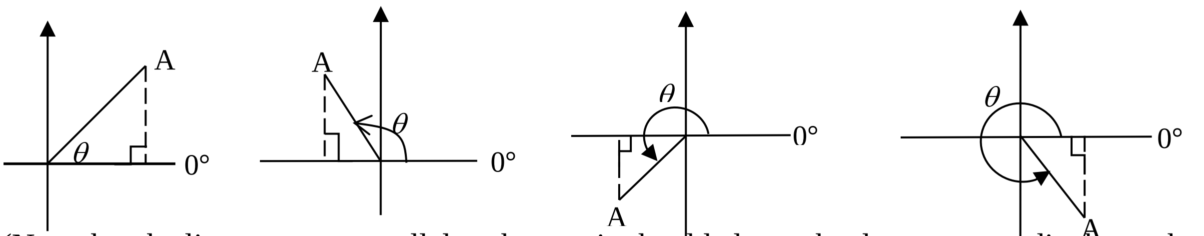
TOPIC	8. TRIGONOMETRY
	50 MARKS FROM PAPER 2
GUIDELINES, SUMMARY NOTES & STRATEGIES	
SUB-TOPIC	**Use of diagrams to determine the numerical values of ratios for angles from 0° to 360°.**
Concepts involved	• Definition of trig ratios • Interpretation of interval notation • Knowing in which quadrant a trig ratio is positive or negative (CAST RULE) • Application of Pythagoras theorem • Reduction formulae • Special angles • Identities

Approach to determine the numerical values of ratios for angles from 0° to 360°.

1. Write the given equation in a form of a simple trig ratio, for example,

   If $b\cos\theta - a = 0$ then $\cos\theta = \frac{a}{b}$; If $b\sin\theta - a = 0$ then $\sin\theta = \frac{a}{b}$; If $b\tan\theta - a$ then $\tan\theta = \frac{a}{b}$

2. Draw the sketch in the correct quadrant, using the given interval/restriction and where the trig ratio holds, for example



(Note that the line segment parallel to the y-axis should always be drawn perpendicular to the *x*-axis)

3. Fill in the known details in the diagram drawn in the correct quadrant.
4. Use Pythagoras theorem to calculate the value of the unknown side. Decide whether the value is positive or negative. Remember the radius is always positive
5. Use the diagram to answer the questions asked based on the diagram
6. Do not use a calculator.

SUB-TOPIC	**Trigonometric Identities**

It must be noted that learners need to be able to derive and apply the fundamental identities.

Two fundamental identities to be used:	**Also note:**	**From the square identity:**
(i) The quotient identity; $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (ii) Square identity: $\sin^2\theta + \cos^2\theta = 1$	$\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$	$\sin^2\theta = 1 - \cos^2\theta = (1-\cos\theta)(1+\cos\theta)$ $\cos^2\theta = 1 - \sin^2\theta = (1-\sin\theta)(1+\sin\theta)$

Tips on how to approach identities:

1. Choose the more complicated side (either LHS or RHS).
2. Express ***tanθ*** in terms of ***sinθ*** and ***cosθ***.
3. Simplify your chosen side **ALGEBRAICALLY** using the identities (in any of the forms), as far as possible or until it is the same/equal as/to the other side. (Sometimes it is necessary and sufficient to simplify both sides separately).
4. It is usually helpful to keep an eye on your required result (RHS/LHS) as it will give you the clue as to which identities would be the best to use?

SUB-TOPIC	Reduction formulae
Concepts involved	• Reduction of angles greater than $90°$ to acute angles using the following reduction formulae ($180° \pm ; 360° \pm$ and $90° \pm$) • For function values ($90° \pm$), each function changes to their co- function. • The **quadrant** should be identified and then the **sign** of the trig ratio in that quadrant should be noted. • Trigonometric identities. • Basic algebraic manipulations.

Reduction formulae for ($180° \pm$ and $360° \pm$)

S (second quadrant, $(180° - \theta)$):

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

$\tan(180° - \theta) = -\tan\theta$

A (first quadrant, θ; $(360° + \theta)$):

$\sin\theta = \sin\theta$

$\cos\theta = \cos\theta$

$\tan\theta = \tan\theta$

T (third quadrant, $(180° + \theta)$):

$\sin(180° + \theta) = -\sin\theta$

$\cos(180° + \theta) = -\cos\theta$

$\tan(180° + \theta) = \tan\theta$

C (fourth quadrant, $(360° - \theta)$):

$\sin(360° - \theta) = -\sin\theta$

$\cos(360° - \theta) = \cos\theta$

$\tan(360° - \theta) = -\tan\theta$

Reduction formulae for (and $90° +$)

$(90° + \theta)$:

$\sin(90° + \theta) = \cos\theta$

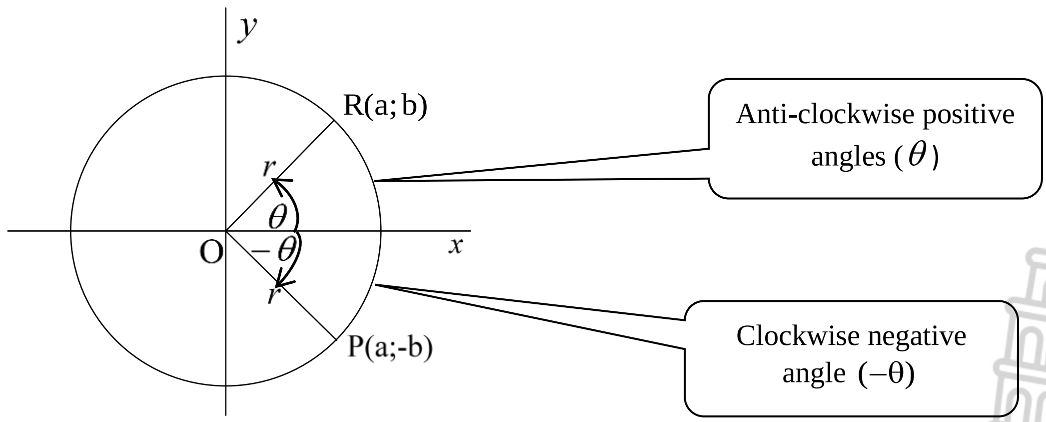
$\cos(90° + \theta) = -\sin\theta$

$(90° - \theta)$:

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

SUB-TOPIC	Negative Angles



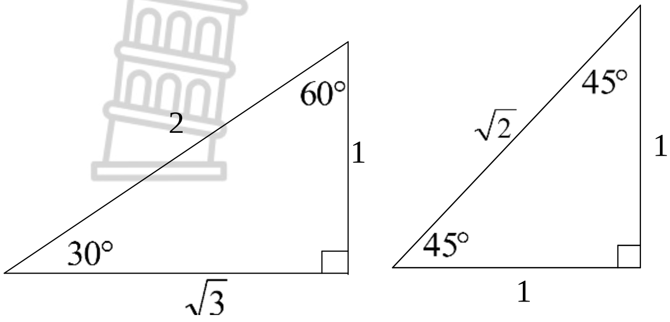
- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$

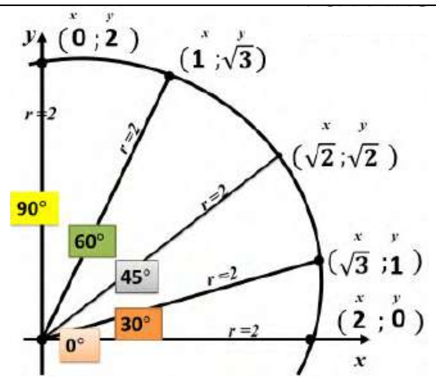
NB: Remember to change negative angles $(-\theta)$ to positive angles (θ)

SUB-TOPIC	Special Angles

- Be careful – Using a calculator is not allowed when dealing with special angles.

For example: Using a diagram: $\tan 30° = \frac{1}{\sqrt{3}}$. Using a calculator: $\tan 30° = \frac{\sqrt{3}}{3}$





F. SUBTOPIC	Solving Trig Equations And General Solution,

Hints for the solution of trigonometric equations:

- Simplify the equation by using identities where possible.
- Simplify the equation as much as possible until you have a trigonometric ratio of one angle equal to a constant/value.
- Factorization, as in the solution of algebraic equations, is often used. Look out for:
  - Common factor
  - Difference of two squares
  - Quadratic trinomials
  - Grouping of terms
- Take care not to divide by an unknown variable.
- Use the CAST rules to determine in which quadrants the angle will be.

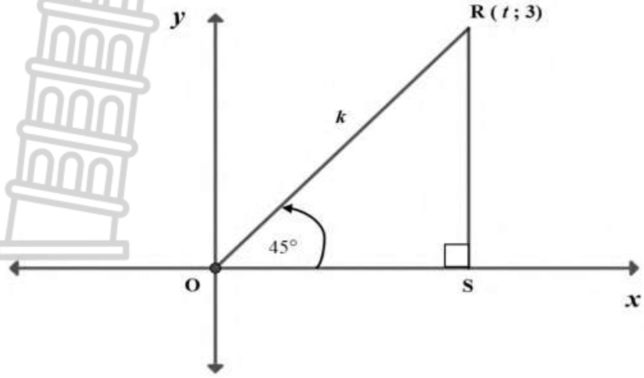
SUBTOPIC	Solutions to triangles/2D problems

Strategies for solving 2 dimensional problems:

- Fill in all the given information on the diagram.
- Problems on 2D require you to solve a combination of triangles in one plane
- Identify the right angled-triangle first (try to use Pythagoras or trig ratios in this triangle).
- Start to work in triangles with more information to find the common side.
- Label the side/angle as soon as you know its value.
- Use basic geometric results e.g. Exterior angle of a triangle, corresponding angle, co-interior angles, alternate angles, etc.

ACTIVITIES
USE OF DIAGRAMS
Grade 11 EC/Nov 2025

8.1



In the diagram alongside, O is the origin. $OR = k$ and $R\hat{O}S = 45°$. $R(t:3)$ is a point in the first quadrant and S lies on the x-axis such that $OS \perp RS$.

Without using a calculator, determine the value(s) of the following:

8.1.1 t (2) **L2**

8.1.2 k (2) **L2**

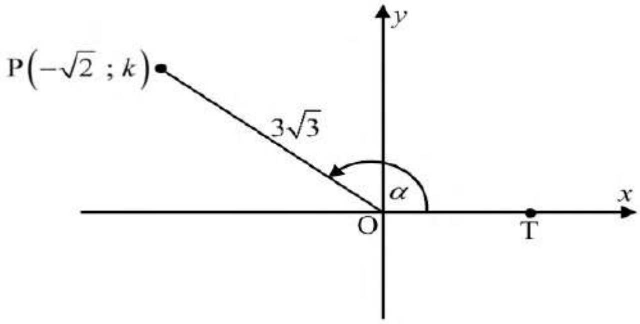
8.2 Given: $8\cos^2\alpha - 2 = 0$

8.2.1 Determine the value of $\cos\alpha$ where $\cos\alpha > 0$ and $0° < \alpha < 180°$. (2) **L1**

8.2.2 Hence, or otherwise, use a diagram to determine the value of $4(\tan^2\alpha - \cos^2\alpha)$. (3) **L3**

Grade 11 KZN/Nov 2025

8.3 In the diagram below, P is a point $(-\sqrt{2};k)$ such that $OP = 3\sqrt{3}$ units. $T\hat{O}P = \alpha$ is an obtuse angle.



Without using a calculator, determine the values of the following:

8.3.1 k (2) **L2**

8.3.2 $\tan(\alpha - 180°)$ (2) **L2**

8.3.3 $\cos\beta$, if it is further given that $\alpha + \beta = 180°$ (2) **L3**

8.4 If $\sin 34° = p$, determine, in terms of p, the value of $\tan 484°$ **without using a calculator**. (2) **L2**

Grade 11 FS/June 2025

8.5 If $7\sin A - 3 = 0$ and $90° < A < 270°$, determine the following **without using a calculator** and with the aid of a sketch:

8.5.1 $\cos A$ (4) **L2**

8.5.2 $\dfrac{\sin A}{\tan A}$ (2) **L2**

8.6 If $\cos 26° = k$, express the following in terms of k:

8.6.1 $\sin 26°$ (2) **L1**

8.6.2 $\tan 296°$ (2) **L2**

Grade 11 GP JN PLC/Nov 2025

8.7 If $\sin 19° = p$, determine the following in terms of p:

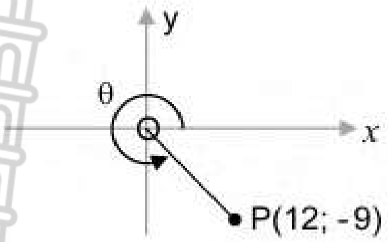
8.7.1 $\cos 19°$ (3) **L2**

8.7.2 $\tan(-559°)$ (2) **L2**

8.7.3 $\cos^2 71° - 1$ (2) **L2**

Grade 11 The Answer Series

8.8 In the diagram below, $P(12;-9)$ is a point on OP.



8.8.1 Determine, without the use of a calculator, the value of $\sin\theta$. (2) **L2**

8.8.2 If $R(4;a)$ is a point on OP, determine the value of a. (4) **L3**

Grade 11 KZN/March 2026

8.9 If $3\tan\theta+2=0$ and $\sin\theta>0$, calculate the following with the use of a diagram and without using a calculator:

8.9.1 $\cos\theta$ (4) **L2**

8.9.2 $13\cos^2\theta-\sqrt{13}\sin\theta$ (3) **L2**

8.10 Given $\cos\beta=k$ and $\sin\beta=2k$ for $\beta\in[180°;270°]$, calculate, without the use of a calculator, the possible value(s) of k, leaving your answer in surd form. (4) **L3**

Grade 11 LP/March 2025

8.11 If $\sin 17°=a$, **without using a calculator**, express the following in terms of a:

8.11.1 $\tan 17°$ (3) **L2**

8.11.2 $\sin 107°$ (2) **L2**

8.11.3 $\cos^2 253°+\sin^2 557°$ (4) **L3**

Grade 11 NW/June 2025

8.12 Given $\sin\theta=-\frac{3}{5}$ and $\tan\theta<0$, **without using a calculator**, determine:

8.12.1 $\cos\theta$ (3) **L2**

8.12.2 $\tan(180°-\theta)$ (2) **L2**

8.12.2 $\cos(90°+36{,}87°)$ if $\theta=-36{,}87°$ (3) **L3**

8.13 If $\cos 25°=t$, express $2\sin 785°-\cos 240°-\sin^2 115°$ in terms of t, and leave your answer in the form at^2+bt+c. (6) **L3**

Grade 11 NW/Nov 2025

8.14 If $\sin A=-\frac{3}{5}$ and $270°<A<360°$, determine, **without using a calculator**, the value of:

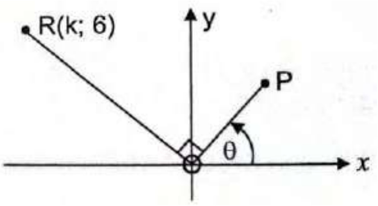
8.14.1 $\cos A$ (3) **L2**

8.14.2 $\tan(A-180°)$ (2) **L2**

8.15 If $\tan\alpha=p$, $\sin\alpha<0$ and $p>0$, determine $\sin\alpha$ in terms of p. (4) **L3**

Grade 11 The Answer Series

8.16 In the diagram alongside, P is a point in the first quadrant such that $5\sin(90°-\theta)-3=0$



Determine the value of:

8.16.1 $\sin\theta$ (5) **L3**

8.16.2 k (5) **L4**

REDUCTIONS, SPECIAL ANGLES & NEGATIVE ANGLES

Grade 11 EC/Nov 2025

8.17 Simplify $\dfrac{\cos x.\tan x}{2\left[\sin x.\cos\left(x-90°\right)+\cos x.\cos\left(-x\right)\right]}$ to a single trigonometric ratio of x. (6) L2

8.18 Simplify $\dfrac{\cos 42°\sin 48°-\tan^2\left(-45°\right)}{\cos^2 132°}$ without using a calculator. (5) L3

Grade 11 KZN/Nov 2025

8.19 Simplify the following without the use of a calculator:

$$\frac{\tan 205°.\cos 315°.\sin\left(-45°\right)}{\sin 210°.\cos 150°.\tan 25°}$$ (5) L3

8.20 Simplify the following to a single trigonometric term:

$$\sin\left(180°+x\right).\cos\left(x-90°\right)-\frac{\sin x}{\sin\left(90°-x\right).\tan\left(180°-x\right)}$$ (6) L2

Grade 11 FS/June 2025

8.21 Simplify the following without the use of a calculator:

$\tan 330°-\sin 120°\cos 240°$ (5) L2

8.22 Simplify the following to a single trigonometric ratio:

$$\frac{\sin\left(180°-\theta\right)\cos\left(-\theta\right)\cos 25°}{\sin\left(90°+\theta\right)\sin 425°}$$ (5) L2

Grade 11 GP JN PLC/Nov 2025

8.23 Simplify without using a calculator:

$$\frac{\sin\left(-x\right).\tan\left(x-360°\right).\sin\left(450°-x\right)}{\cos 180°}+\cos^2\left(x-180°\right)$$ (8) L3

Grade 11 The Answer Series

8.24 Simplify $\dfrac{\sin A.\cos A.\tan A}{1-\cos^2 A}$ (3) L2

8.25 Evaluate without using a calculator:

$$\frac{\tan 300°+\cos\left(90°+x\right)}{\sin x+2\cos\left(-30°\right)}$$ (6) L2

Grade 11 LP/March 2025

8.26 Simplify the following fully without the use of a calculator:

$$\frac{\cos\left(-225°\right).\sin 135°+\sin 330°}{\tan 225°}$$ (6) L2

8.27 Simplify the following fully:

$$\frac{\sin\left(180°-A\right).\tan A.\sin\left(90°+A\right)}{\tan\left(180°+A\right).\sin\left(-A\right).\cos\left(-A\right)}$$ (6) L2

Grade 11 NW/June 2025

8.28 Simplify: $\dfrac{\sin\left(-\theta\right)\sin\left(180°-\theta\right)+\cos\left(90°+\theta\right)}{1-\sin\left(360°-\theta\right)}$ (6) L2

Grade 11 NW/Nov 2025

8.29 Determine the value of:

$$\frac{\sin\left(-120°\right).\tan 330°}{\cos\left(360°-x\right).\sin\left(90°+x\right)+\sin^2\left(180°+x\right)}$$ (8) L2

IDENTITIES

Grade 11 EC/Nov 2025

8.30 Prove that $\tan\beta - \sin\beta.\cos\beta = \tan\beta.\sin^2\beta$ (3) **L2**

Grade 11 KZN/Nov 2025

68.31 Prove that $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{2}{\cos x}$ (4) **L2**

Grade 11 FS/June 2025

8.32 Prove the following identity: $\frac{\sin\theta - \tan\theta.\cos^2\theta}{\cos\theta - 1 + \sin^2\theta} = \tan\theta$ (4) **L3**

Grade 11 GP JN PLC/Nov 2025

8.33 Prove the following identity $\left(\frac{1}{\sin\beta} + \frac{1}{\tan\beta}\right)^2 = \frac{1+\cos\beta}{1-\cos\beta}$ (4) **L3**

Grade 11 The Answer Series

8.34 Show that $\frac{\sin^m\theta - \cos^m\theta}{\tan^m\theta - 1} = \cos^m\theta$ (3) **L4**

8.35 Hence, show that:

$\frac{\sin\theta - \cos\theta}{\tan\theta - 1} \times \frac{\tan^2\theta - 1}{\sin^2\theta - \cos^2\theta} \times \frac{\sin^3\theta - \cos^3\theta}{\tan^3\theta - 1} \times \frac{\tan^4\theta - 1}{\sin^4\theta - \cos^4\theta} \times .. \times \frac{\sin^{2007}\theta - \cos^{2007}\theta}{\tan^{2007}\theta - 1} = \cos^{1004}\theta$ (3) **L4**

Grade 11 LP/March 2025

8.36 Prove that $\frac{\sin x}{1+\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x}$ (3) **L2**

Grade 11 KZN/March 2026

8.37 Prove that $\frac{1}{\tan x}(\cos x + \tan x.\sin x) = \frac{1}{\sin x}$ (5) **L2**

Grade 11 NW/June 2025

8.38 Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta.\sin^2\theta$ (5) **L2**

Grade 11 NW/Nov 2025

8.39 Prove the following identity: $\frac{(2-2\sin^2 x)(1+2\tan^2 x)}{1+\sin^2 x} = 2$ (6) **L3**

Grade 11 The Answer Series

8.40 Prove that $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2\tan\theta$ (5) **L3**

8.41 Prove that $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = \frac{2}{\cos\theta}$ (4) **L3**

8.42 Show that $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$ (4) **L3**

Adapted

8.43 Show that $\frac{\sin\theta - \sin\theta\cos\theta}{\sin^2\theta} = \frac{\sin\theta}{1+\cos\theta}$ (5) **L4**

8.44 Prove that $\frac{1+2\sin x\cos x}{\cos^2 x - \sin^2 x} = \frac{1+\tan x}{1-\tan x}$ (6) **L4**

8.45 $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \sin x\cos x = 1$ (3) **L2**

8.46 $\frac{\cos^2 x - \sin^2 x}{1-\tan^2 x} = \cos^2 x$ (5) **L3**

TRIGONOMETRIC FUNCTIONS

Grade 11 EC/Nov 2012

8.47 Given $f(x) = -\sin x$ and $g(x) = \cos(x - 30°)$

8.47.1 Write down the maximum value of $3.g(x)$. (1) **L1**

8.47.2 Sketch the graphs of f and g on the same system of axes for $x \in [-180°;180°]$ (6) **L2**

8.47.3 Use your graph to determine the values of x, for $x \in [-180°;180°]$, for which $f(x) - g(x) \leq 0$. (4) **L2**

8.47.4 Answer the following questions:

(i) Write down the equation of h if h is the translation of g by $60°$ to the right and 1 unit up. (2) **L2**

(ii) Determine the maximum value of $h(x) - f(x)$ (2) **L2**

8.47.5 Explain why the reflection of f in the x-axis and the reflection of f in the y-axis will result in the same graph. (2) **L2**

Grade 11 EC/ Nov 2016

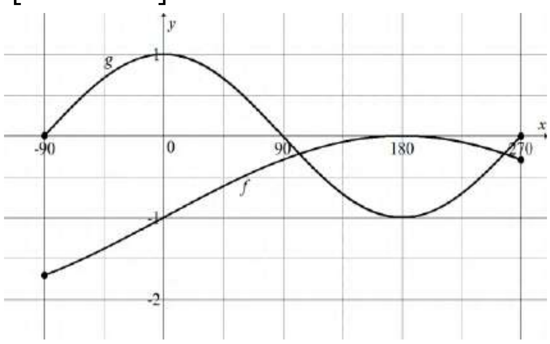
8.48 On the same of axes draw $f(x) = -\cos(45° - x)$ and $g(x) = \tan(-x)$ for the interval of $-90° \leq x \leq 180°$. (6) **L2**

8.48.1 For which values of x is $f(x) - g(x) \leq 0$ for $x \in [-90°;90°]$? (2) **L2**

8.48.2 Write down the equation of $h(x) = -f(x - 45°)$. (2) **L2**

Grade 11 EC/Nov 2025

8.49 Given $f(x) = a\sin\left(\frac{x}{2}\right) - 1$ and $g(x) = \cos x + q$, $x \in [-90°; 270°]$



Use the graphs to answer the following questions where $x \in [-90°; 270°]$:

8.49.1 Determine the values of a and q. (2) **L2**

8.49.2 Write down the range of f. (2) **L2**

8.49.3 Write down the amplitude of g. (1) **L1**

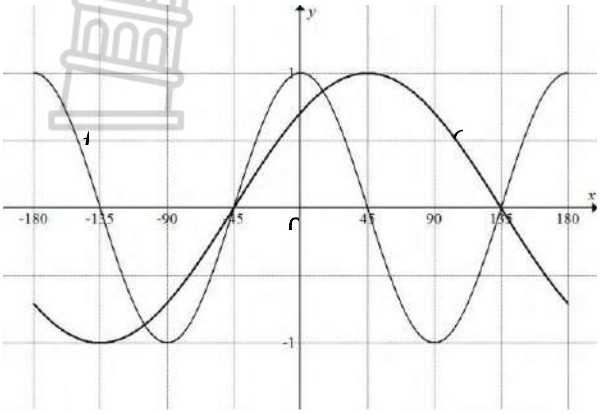
8.49.4 Write down the value(s) of x where $f(x) - g(x) = -2$. (1) **L2**

8.49.5 For which values of x is $f(x).g(x) \leq 0$? (2) **L2**

8.49.6 The graph of h is obtained by shifting the graph of g by $90°$ to the left. Determine the equation of h in its simplest form. (2) **L3**

Grade 11 DBE/Nov 2014

8.50 In the diagram below the graph of $f(x)=a\cos bx$ and $g(x)=\sin(x+p)$ are drawn for $x\in[-180°;180°]$



8.50.1 Write down the value of a, b and p. (3) **L2**

8.50.2 For which values of x in the given interval does the graph of f increase as the graph of g increases? (2) **L2**

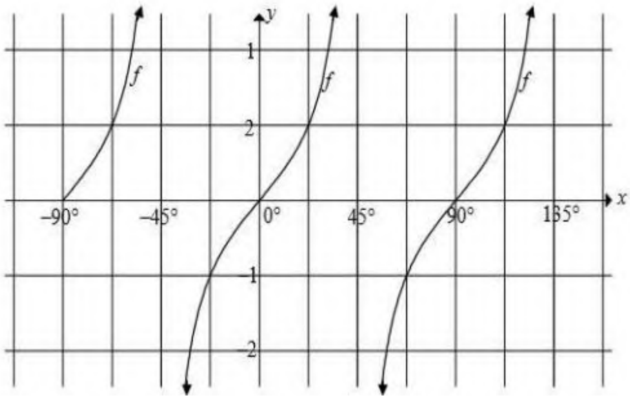
8.50.3 Write down the period of $f(2x)$ (2) **L2**

8.50.4 Determine the minimum value of h if $h(x)=3f(x)-1$ (2) **L2**

8.50.5 Determine how the graph of g must be transformed to form the graph of k, where $k(x)=-\cos x$ (2) **L2**

Grade 11 DBE/Nov 2015

8.51 In the diagram, the graph of $f(x)=\tan bx$ is drawn for the interval $-90°\leq x\leq 135°$.



8.51.1 Determine the value of b. (1) **L1**

8.51.2 Determine the value of x in the interval $0°\leq x\leq 135°$ for which $f(x)\leq -1$ (2) **L2**

8.51.3 Graph h is defined as $h(x)=\tan b(x+55°)$. Write down the equation of the asymptote of h in the interval $-90°\leq x\leq 135°$. (2) **L3**

TRIGONOMETRIC EQUATIONS AND GENERAL SOLUTIONS

Grade 11 DBE/Nov 2007

8.52 Solve for x:
$5^{\tan x}=125$ if $x\in[0°;360°]$ (5) **L2**

8.53 Determine the general solution for $\sin x.(2\cos x-1)=0$ (7) **L2**

Grade 11 EC/Nov 2013

8.54 Determine the general solution of $\sqrt{\tan\theta}=x+\frac{1}{x}$ if $x^2+\frac{1}{x^2}=1$ (6) **L3**

Grade 11 DBE/Nov 2015

8.55 Determine the general solution of $\frac{1-\cos^2 2x}{4\cos(90°+2x)}=0,21$ (8) **L3**

Grade 11 EC/Nov 2015

8.56 Determine the general solution of $2\cos^2 x+5\sin x=4$ (6) **L3**

Grade 11 KZN/Nov 2025

8.57 Determine the general solution of $\tan x.\sin x=\frac{3}{\sin x}-\cos x$ (6) **L3**

DBE NOV 14

8.58 Determine the general solution of $\sin 2x = 4\cos 2x$ (5) **L2**

Grade 11 DBE/Exemplar 2013

8.59 Given $\frac{8}{\sin^2 A} - \frac{4}{1+\cos A} = \frac{4}{1-\cos A}$

For which values of A in the interval $0° \le A \le 360°$ is the given identity undefined. (3) **L2**

8.60 Determine the general solution of $8\cos^2 x - 2\cos x - 1 = 1$ (6) **L3**

Grade 11 DBE/Nov 2017

8.61 Determine the general solution of $\sin(x - 30°) = \cos 2x$ (5) **L2**

DBE/Exemplar 2013

8.62 Solve for θ if $0° \le \theta \le 360°$: $\frac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)} = 0,5$ (8) **L3**

Grade 11 DBE/Nov 2014

8.63 For which values of x in the interval $0° \le x \le 360°$ will the following identity be undefined?

$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{2}{\cos x}$ (2) **L2**

Grade 11 DBE/Nov 2013

8.64 If $x \in [-180°; 180°]$, give two values of x for which the following identity is undefined:

$\frac{\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x} = 2\tan x$ (2) **L2**

Mock P2 FS/May 2024

8.65 If x and y are acute angles such that $\tan\left(\frac{x+y}{2}\right) = 1$ and $\cos(x - y) = \frac{\sqrt{3}}{2}$, determine the values of x and y. (5) **L4**

Grade 11 Study & Master HG

8.66 Solve for θ in the interval $\theta \in [0°; 180°]$, without using a calculator:

$\tan^2\theta = \frac{\sin(-120°).\tan(-330°)}{\cos 240°}$ (7) **L4**

8.67 Solve for x in the interval $x \in [0°; 360°]$, without using a calculator: $4\sin^2 x - 3 = 0$ (4) **L3**

Grade 11 EC/ Nov 2025

8.68 Determine the general solution of: $3\sin\theta = -2\cos^2\theta$ (6) **L3**

Grade 11 NW/June 2024

8.69 Calculate the value of θ, if $0° < \theta < 180°$, **without using a calculator**:

$\sin\theta = \sqrt{\frac{(9)^{\cos 300°}}{\left(\frac{1}{4}\right)^{\sin 150°}.(8)^{\tan 225°}}}$ (8) **L4**

Grade 12 LP/March 2026 (Adapted)

8.70 Given that: $\sin(A - B) - \sin(A + B) = -2\sin A \sin B$.

Determine the solution for $\sin 4x - \sin 10x = \sin 3x$, for $x \in [0°; 30°]$. (7) **L4**

Grade 11/Mind Action Series

8.71 Determine the general solution for:

8.71.1 $\cos\theta = -\cos 2\theta$ (5) **L3**

8.71.2 $2 - 11\sin\theta\cos\theta + 3\sin^2\theta = 0$ (7) **L4**

8.71.3 $5\sin^2\theta + 3\sin\theta\cos\theta - 4 = 0$ (7) **L4**

8.71.4 $1 + \sin\theta\cos\theta - 7\cos^2\theta = 0$ (7) **L4**

8.71.5 $\tan\theta = \cos\theta$ (9) **L4**

8.71.6 $\tan\alpha = \sin\alpha$ (7) **L3**

8.71.7 $2\sin^2 x + \cos x - 1 = 0$ (6) **L3**

8.71.8 $\sin(x + 20°) = \cos 2x$ (6) **L3**

8.71.9 $\sin 2\theta = \cos(\theta - 50°)$ (6) **L3**

8.71.10 $\sin(x - 20°) = -\cos(2x + 30°)$ (7) **L3**

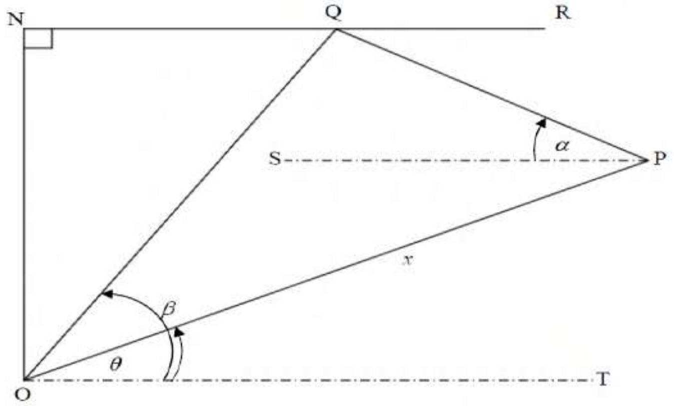
8.71.11 $\sin\theta\cos^2\theta = \sin^3\theta$ (5) **L2**

8.71.12 $\sin^2 x + 2\cos^2 x - 1 = 2\sin x\cos x$ (8) **L3**

PROBLEMS IN 2-DIMENSIONS

Grade 11 EC/Nov 2025

8.72 A man standing on a horizontal plane at point O observes two helicopters positioned at points P and Q. The line of sight from point O to each helicopter form the angles of elevation θ and β with the horizontal plane. Additionally, a second man located in the helicopter at point P observes the helicopter at point Q and his line of sight forms an angle of elevation α with a horizontal plane at P, measured along line PQ. $ON \perp NR$ and $OP = x$.



8.72.1 Show that $O\hat{Q}P = 180° - (\beta + \alpha)$. (2) **L2**

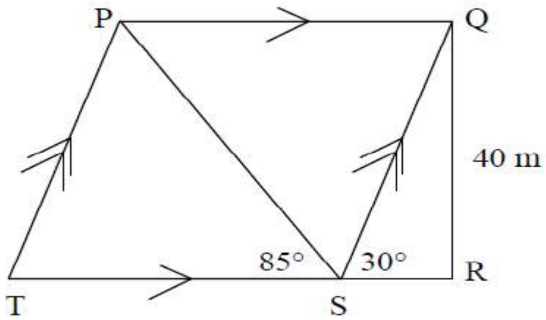
8.72.2 Show that $OQ = \dfrac{x\sin(\alpha + \theta)}{\sin(\beta + \alpha)}$ (3) **L2**

8.72.3 Determine the value of x if $\theta = 15°$, $\beta = 60°$, $\alpha = 30°$ and $OQ = 5\sqrt{2}$. (2) **L2**

Grade 11 EC/Nov 2012

8.73 Trapezium PQRT is a plot of land bought by a farmer. RST is a straight line. ΔQRS is a right-angled triangle at R and PQST is a parallelogram.

$QR = 40$ m, $P\hat{S}T = 85°$ and $Q\hat{S}R = 30°$.



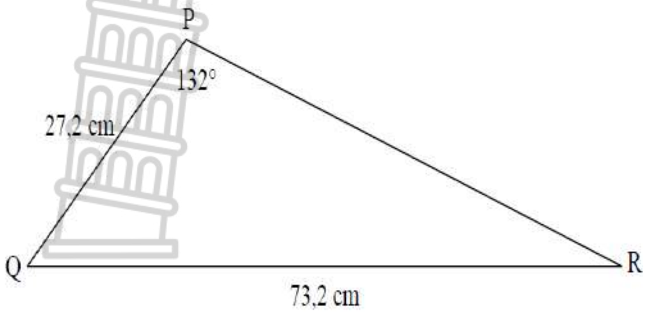
8.73.1 Calculate the length of QS. (2) **L2**

8.73.2 Calculate the length of PQ. (3) **L2**

8.73.3 Determine the area of the trapezium PQRT. (5) **L3**

Grade 11 Exemplar 2013

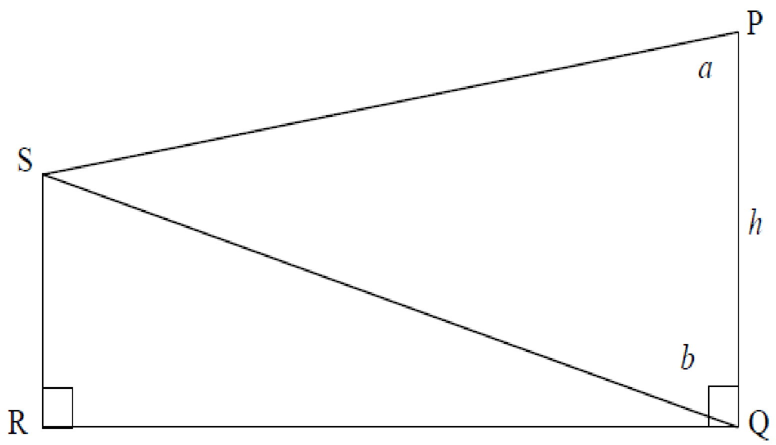
8.74 In ΔPQR, $\hat{P} = 132°$, $PQ = 27,2$ cm and $QR = 73,2$ cm.



8.74.1 Calculate the size of $\hat{R}$ (3) **L2**

8.74.2 Calculate the area of ΔPQR (3) **L2**

8.75 In the figure below, $S\hat{P}Q = a$, $P\hat{Q}S = b$ and $PQ = h$. PQ and SR are perpendicular to RQ.



Determine the distance SQ in terms of a, b and h. (3) **L2**

TOPIC	9. EUCLIDEAN GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

1. WAYS IN WHICH EUCLIDEAN GEOMETRY IS TESTED

1. Completing a statement of a theorem in words.
2. Determining the value of an angle in **two ways:** numerical and/or in terms of the variable(s)
3. **Proofs in riders:** Direct and indirect proofs
4. **Examinable proofs** to be known
   4.1 Line from the centre ⊥ chord
   4.2 Line from centre to midpoint of chord
   4.3 Angle at the centre is twice the angle at the circumference
   4.4 Opposite angles of a cyclic quad are supplementary
   4.5 Tan chord theorem

2. COMPLETING A STATEMENT OF A THEOREM IN WORDS.

- Know by heart all the theorems and be able to complete the statement.

3. DETERMINING THE VALUE OF AN ANGLE

- Know all the theorems about **lines**, **triangles** and **circles (Centre** group, **non-centre** group, **tangent** group and **cyclic quad** group**)**.
- Every statement must come with a reason and reasons must be stated according to the list of acceptable reasons from the exam guidelines
  e.g. for isosceles triangles the acceptable reason is: ∠s opp = sides

4. PROOFS IN RIDERS

Know how theorems and their converses are being formed in diagrams.

- When given 3 points on the circumference look out for a possibility of a triangle. If one side is produced then you may expect exterior angle of a triangle. If there is a tangent on the circle, then there is a possibility of having a Tan Chord Theorem
- When given 4 or 5 points on the circumference then there is a possibility that 4 points may be joined and then there is a cyclic quad. In a case that one side is produced then you may expect exterior angles of a cyclic quad.
- Start with a given angle linking with what is required to prove
- Visualization: Mind picture of diagrams of theorems

5. DIRECT AND INDIRECT PROOFS IN RIDERS.

- In Geometry we mostly use angles to prove questions.

1. Direct proof question: Prove $\hat{A} = \hat{B}$

2. Indirect proof question: Prove that a line is parallel to another line.

Remember in Euclidean geometry- we mostly use angles to prove. This question is not asking about the angles directly. Here we need to prove sides but using angles i**ndirectly. Why indirectly?** Because we mostly use angles to prove.

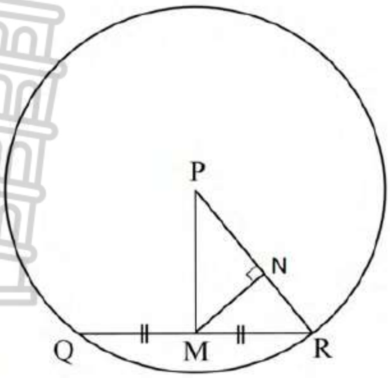
∴ First, we need to change this question to be direct, and then prove. If we say it must be direct we mean that it must ask to prove angles 1st, then conclude by stating the sides that are parallel

NB!!!!!

- Do not make any assumption e.g. do not assume that a line is a tangent or a diameter, unless you are told that it is.
- Look for key words in the statement such as **centre, parallel lines, tangents, cyclic quadrilaterals, bisects, etc**.
- Continuously update the diagram as you read the statement and as you find the angles.
- When proving theorems, **no construction, no marks.**
- You will not always be told that you have a cyclic quadrilateral. Therefore, check lines joining four points on the circumference.
- For every statement there **must** be a reason.

ACTIVITIES

9.1



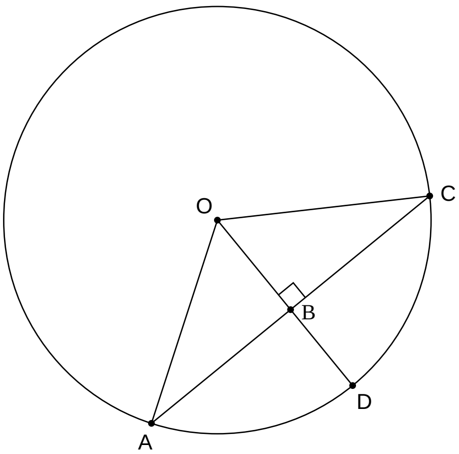
P is the centre of the circle with radius 73 units. M is the midpoint of chord QR, N is a point on PR so that PN= 40 units. $MN \perp PR$.

9.1.1 Give a reason why $PM \perp QR$. (1) **L1**

9.1.2 Determine the length of MR (to the nearest whole number), giving reasons. (4) **L3**

[5]

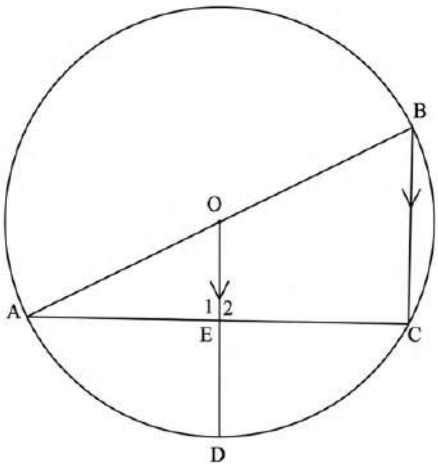
9.2



In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B. $OB = 2x$ units and $AC = 8x$ units.

Show that the length of BD is $2x\left(\sqrt{5}-1\right)$ (5) **L3**

9.3

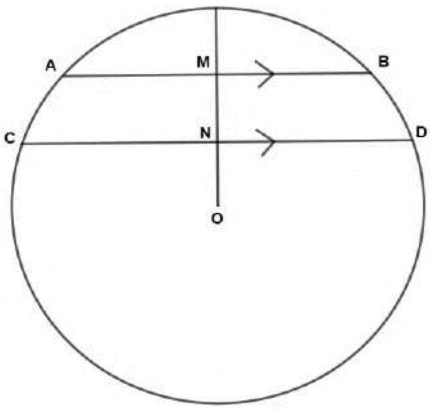


AB is a diameter of circle O. OD is drawn parallel to chord BC and intersects AC at E.

ED = 4 cm and AC = 16 cm.

Calculate the length of AB. (5) **L3**

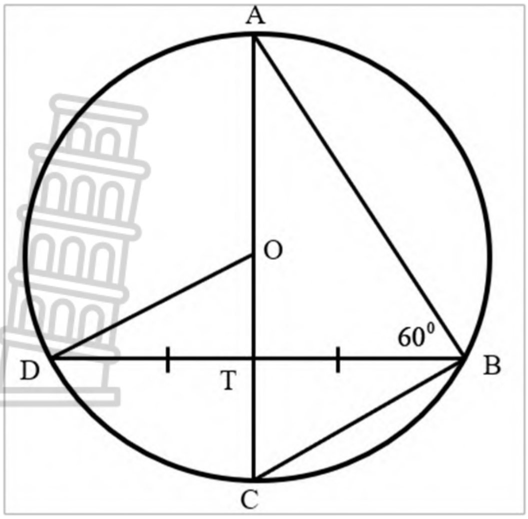
9.4



AB and CD are two parallel chords on the same side of the centre O of a circle. The shortest distance between AB and CD is 1 cm . The length of AB is 6 cm and of CD is 8 cm. M is the midpoint of AB and the line OM intersects CD at N which is the midpoint of CD.

Calculate the length of the radius of the circle (4) **L4**

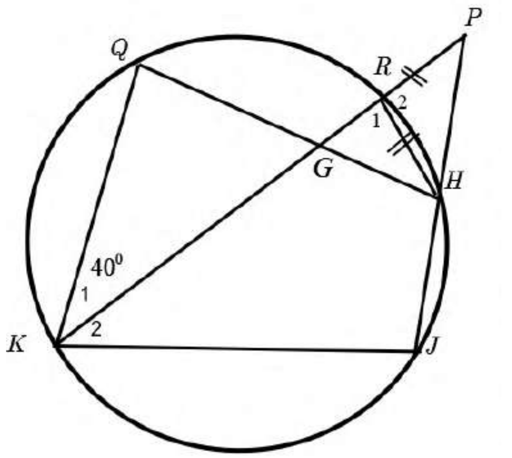
9.5



Given circle with centre O, $DT = TB$ and $A\hat{B}D = 60^0$.

9.5.1	Determine $T\hat{B}C$.	(1)	**L2**
9.5.2	Show that $OD \parallel BC$.	(4)	**L3**
		[5]	

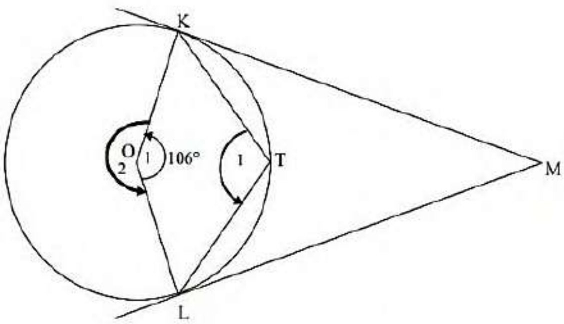
9.6



In the diagram below, points Q, H, J and K lie on a circle. RK bisects $\hat{K}$ and RH=RP KR and JH produced meet at P.
$\hat{K}_1 = 40^0$.
Prove that:

9.6.1	RH bisects $G\hat{H}P$.	(4)	**L2**
9.6.2	JK= JP	(3)	**L2**
9.6.3	$\hat{Q} = J\hat{K}Q$.	(3)	**L3**
		[10]	

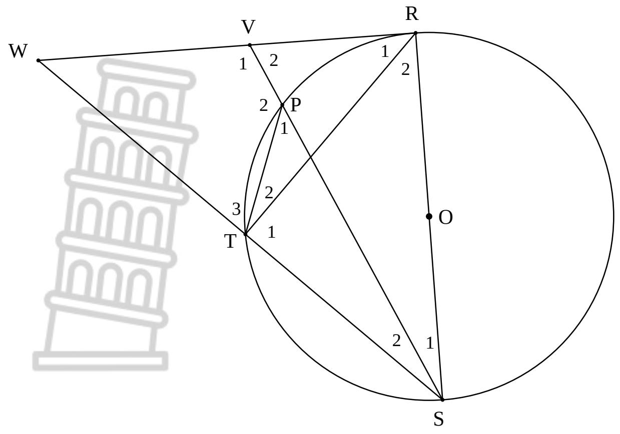
9.7



In the diagram, O is the centre of the circle. KM and LM are tangents to the circle at K and L respectively. T is a point on the circumference of the circle. KT and TL are joined. $\hat{O}_1 = 106°$

9.7.1	Calculate, with reasons, the size of $\hat{T}_1$.	(3)	**L2**
9.7.2	Prove that the quadrilateral OKML is a kite.	(3)	**L2**
9.7.3	Prove that quadrilateral OKML is a cyclic quadrilateral.	(3)	**L2**
9.7.4	Calculate, with reasons, the size of $\hat{M}$.	(2)	**L2**
		[11]	

9.8



In the diagram below, RS is a diameter of the circle centred at O. Chord ST is produced to W. Chord SP produced meets the tangent RW at V. $\hat{R}_1 = 50°$

Calculate the sizes of the following angles.

9.8.1 $\hat{R}_2$ (3) **L2**

9.8.2 $\hat{W}$ (3) **L3**

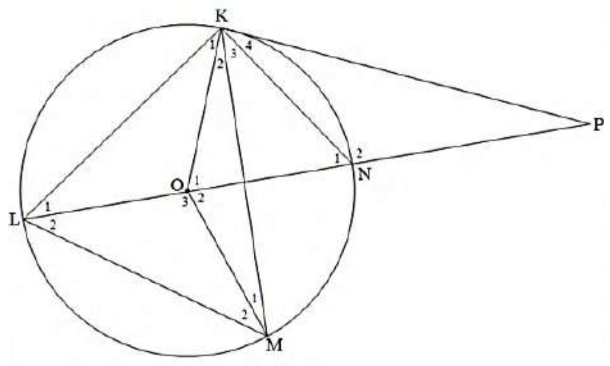
9.8.3 $\hat{P}_1$ (2) **L2**

9.8.4 Prove that $\hat{V}_1 = P\hat{T}S$ (4) **L4**

9.8.5 Hence, prove that WVPT is a cyclic quadrilateral. (1) **L1**

[13]

9.9



In the diagram, O is the centre of the circle and KP is the tangent to the circle. LN, the diameter of the circle, is extended to meet KP at P. Straight lines OK, OM, KM and KN are drawn.

9.9.1 Write down two angles each equal to 90°. (2) **L1**

9.9.2 If $\hat{K}_4 = x$, write down the following angles in terms of x, giving reasons.

(a) $\hat{L}_1$ (2) **L1**

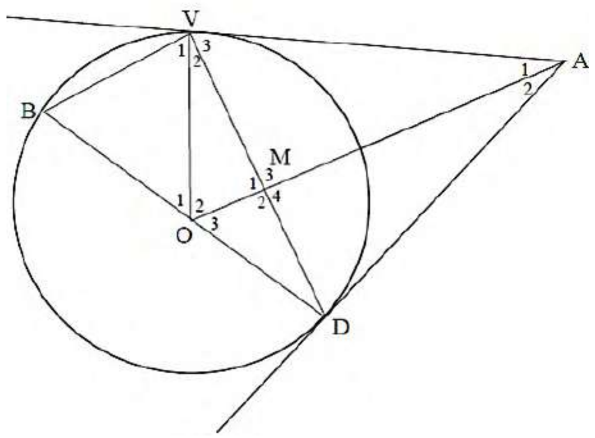
(b) $\hat{K}_1$ (2) **L1**

(c) $\hat{O}_1$ (2) **L2**

9.9.3 Join MP, which is a tangent to the circle, and hence prove that KOMP is a cyclic quadrilateral. (3) **L3**

[11]

9.10



From a point A outside the circle, centre O, two tangents AD and AV are drawn. AO and VD intersect at M. BOD is a diameter of the circle. BV and VO are drawn, $V\hat{A}D = 40°$.

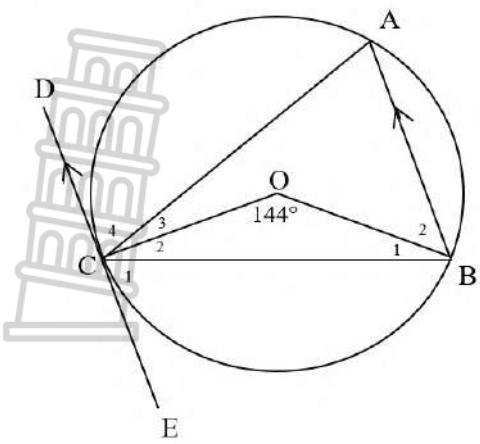
9.10.1 Prove that quadrilateral VODA is cyclic. (2) **L2**

9.10.2 Calculate the magnitude of $\hat{O}_1$ (2) **L3**

9.10.3 Prove that $BV \parallel OA$. (5) **L3**

[9]

9.11



In the figure O is the centre of the circle.
DE is a tangent to the circle at C.
$DE \parallel AB$ and $C\hat{O}B = 144°$

Giving reasons, determine the value of :

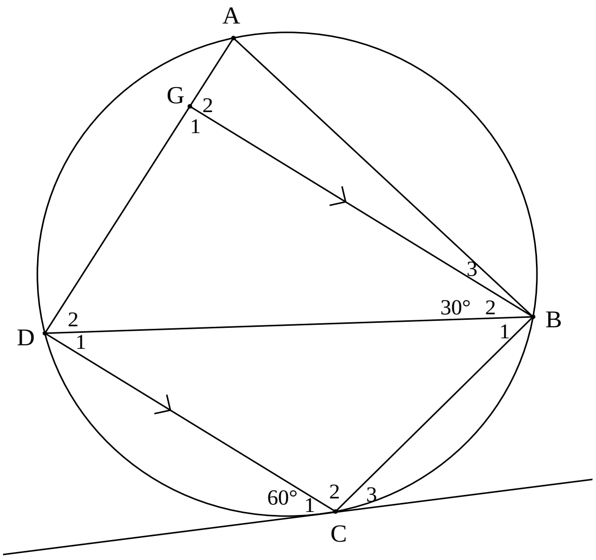
9.11.1 C_1 (5) **L2**

9.11.2 B_1 (4) **L2**

9.11.3 (3) **L2**

[12]

9.12



In the diagram, ABCD is a cyclic quadrilateral. G is a point on AD such that $BG \parallel CD$. ECF is a tangent to the circle at C. BD is a chord to the circle. $G\hat{B}D = 30°$ and $\hat{C}_1 = 60°$.

9.12.1 Calculate with reasons, the size of:

(a) $\hat{D}_1$ (1) **L1**

(b) $\hat{B}_1$ (2) **L1**

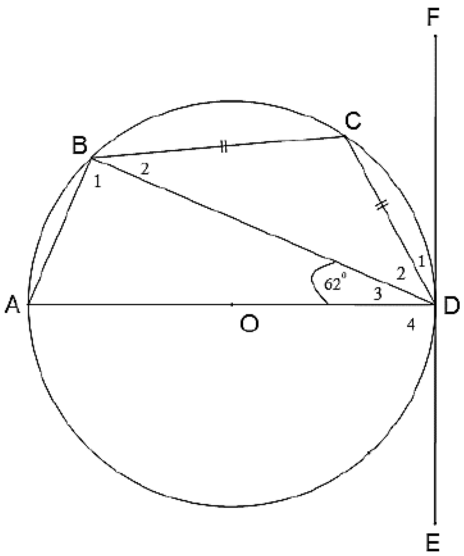
(c) $\hat{C}_2$ (1) **L2**

(d) $D\hat{A}B$ (2) **L3**

9.12.2 Is BD a diameter to the circle? Motivate your answer. (2) **L2**

[8]

9.13



9.13.1 Complete this statement:
A line drawn from the centre of a circle is ………………… to the tangent. (1) **L1**

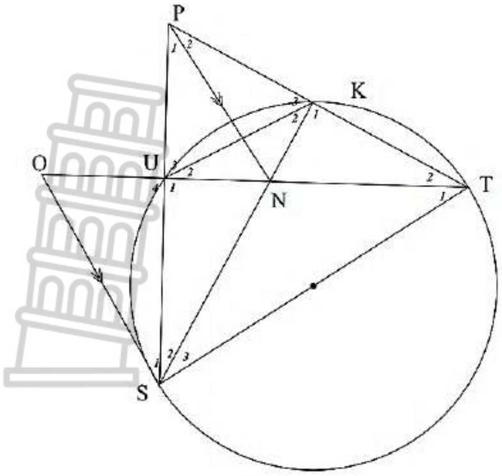
In the diagram below, AOD is a diameter of the circle and EDF is a tangent to the circle at D. $A\hat{D}B = 62°$ and $BC = CD$.
Calculate, with reasons, the numerical value of:

9.13.2 $B\hat{C}D$ (4) **L3**

9.13.3 $C\hat{D}F$ (4) **L2**

[9]

9.14

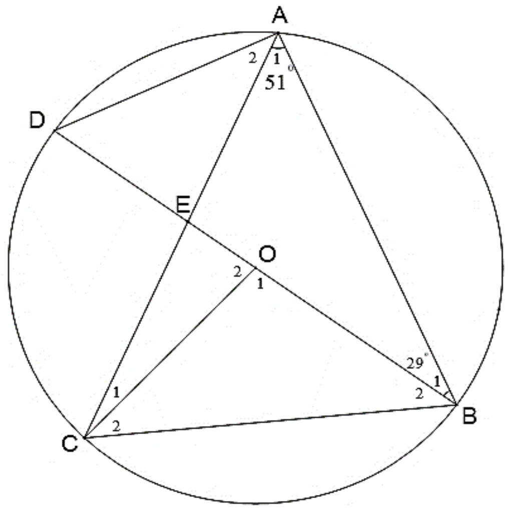


ST is a diameter of the circle. OS || PN, TO bisects $S\hat{T}P$.

Prove that:

9.14.1 PUNK is a cyclic quadrilateral	(5)	**L2**
9.14.2 SO is a tangent to circle KUST	(6)	**L3**
9.14.3 POST is a cyclic quadrilateral	(4)	**L3**
	[15]	

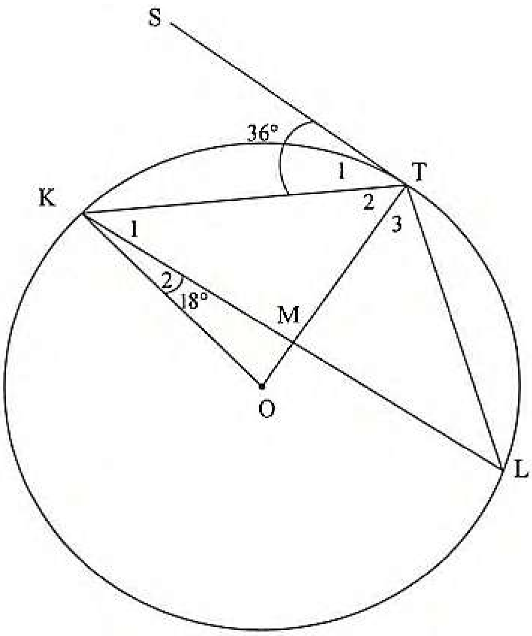
9.15



In the diagram, O is the centre of the circle. Points A, B, C and D lie on the circumference of the circle. BOD is a diameter.
AC and BD intersect at E.
$\hat{A}_1 = 51°$ and $\hat{B}_1 = 29°$.
Determine (giving reasons) the size of:

9.15.1 $\hat{O}_1$.	(2)	**L1**
9.15.2 $\hat{A}_2$.	(2)	**L2**
9.15.3 $\hat{D}$.	(1)	**L1**
9.15.4 $\hat{C}_1$	(3)	**L3**
	[8]	

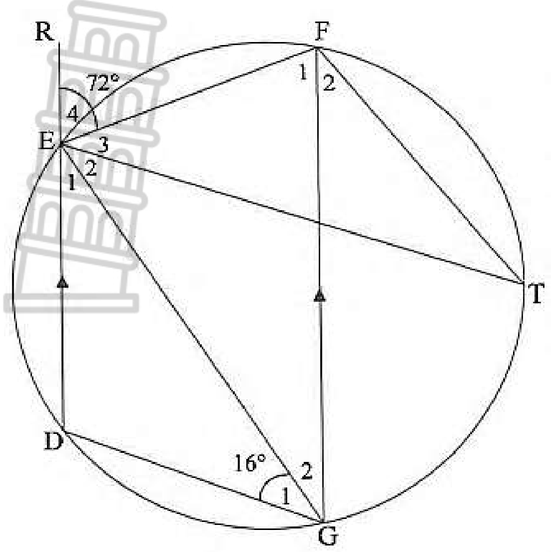
9.16



In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersect KL at M.
ST is a tangent to the circle at T.
$S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.

9.16.1 Determine, with reasons, the size of:		
(a) $\hat{T}_2$	(2)	**L2**
(b) $\hat{L}$	(2)	**L1**
(c) $K\hat{O}T$	(2)	**L3**
9.16.2 Prove, giving reasons, that $KM = ML$.	(3)	**L3**
	[9]	

9.17

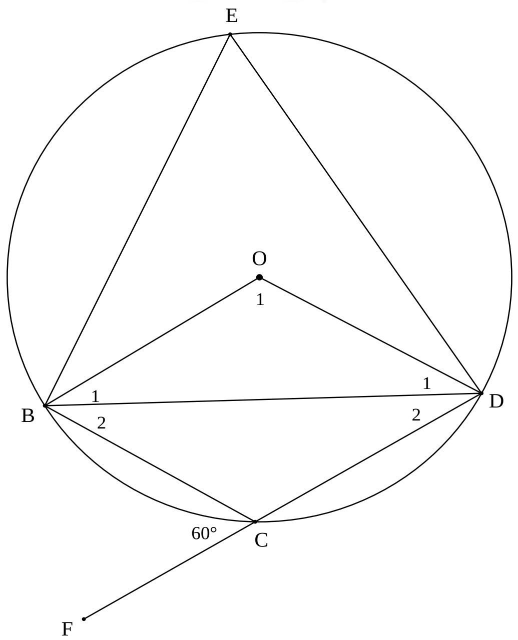


In the diagram, DEFG is a cyclic quadrilateral with $DE \parallel GF$. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72°$ and $\hat{G}_1 = 16°$.

Determine, with reasons, the size of the following angles:

9.17.1	$D\hat{G}F$	(2)	**L1**
9.17.2	$\hat{T}$	(2)	**L2**
9.17.3	$G\hat{E}F$	(2)	**L2**
		[6]	

9.18

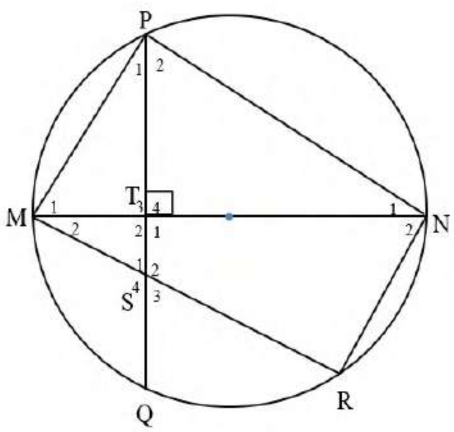


The diagram shows a circle with centre O. BEDC is a cyclic quadrilateral. OB, OD and BD are drawn. DCF is a straight line. $B\hat{C}F = 60°$.

Determine, giving reasons, the size of the following angles:

9.18.1	$\hat{E}$	(2)	**L1**
9.18.2	$\hat{O}_1$	(2)	**L1**
9.18.3	$\hat{D}_1$	(2)	**L2**
		[6]	

9.19

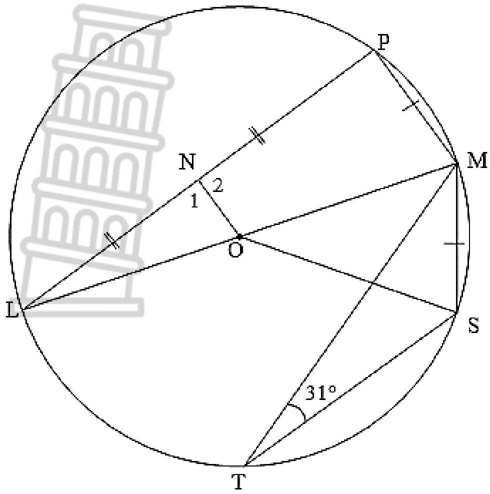


In the given sketch, MN is a diameter of the circle. MPNR is a cyclic quadrilateral and $PQ \perp MN$.

Prove:

9.19.1	TSRN is a cyclic quadrilateral	(4)	**L2**
9.19.2	MP is a tangent to the circle through PTN	(4)	**L3**
		[8]	

9.20



In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn.

$PM = MS$ and $M\hat{T}S = 31°$

9.20.1 Determine, with reasons, the size of each of the following angles:

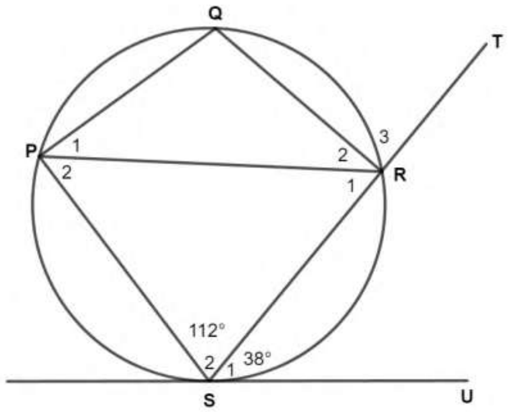
(a) $M\hat{O}S$ (2) **L1**

(b) $\hat{L}$ (2) **L1**

9.20.2 Prove that $ON = \frac{1}{2}MS$. (4) **L3**

[8]

9.21



In the diagram, PQRS is a cyclic quadrilateral. SU is a tangent to the circle at S and chord SR is produced to T. $PQ = QR$, $\hat{S}_1 = 38°$ and $\hat{S}_2 = 112°$. Determine, with reasons, the size of the following angles:

9.21.1 $\hat{Q}$ (2) **L1**

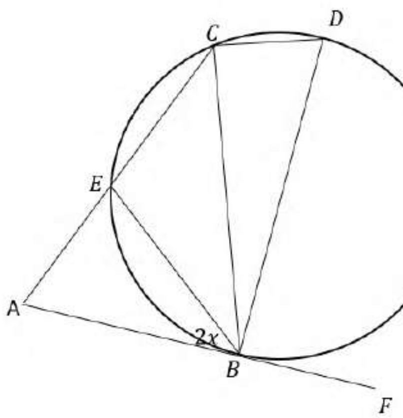
9.21.2 $\hat{R}_2$ (2) **L2**

9.21.3 $\hat{P}_2$ (2) **L1**

9.21.4 $\hat{R}_3$ (2) **L2**

[8]

9.22



In the sketch, BD is a diameter and ABF is a tangent.

ABE=$2x$ and AB=EB

Determine, with reasons, the size of the following angles in terms of x:

9.22.1 BEA (2) **L1**

9.22.2 ECB (2) **L2**

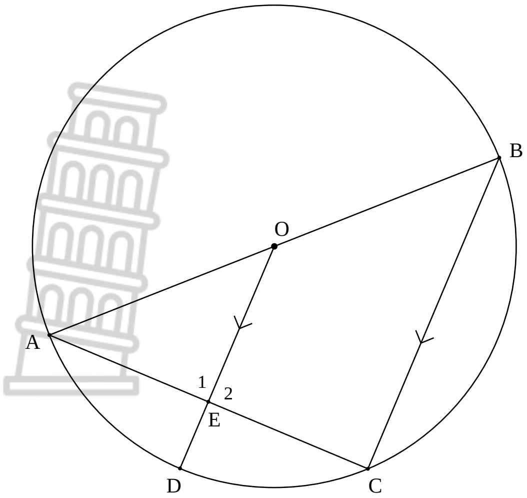
9.22.3 D (2) **L2**

9.22.4 DCB (2) **L2**

9.22.5 CEB (2) **L2**

[10]

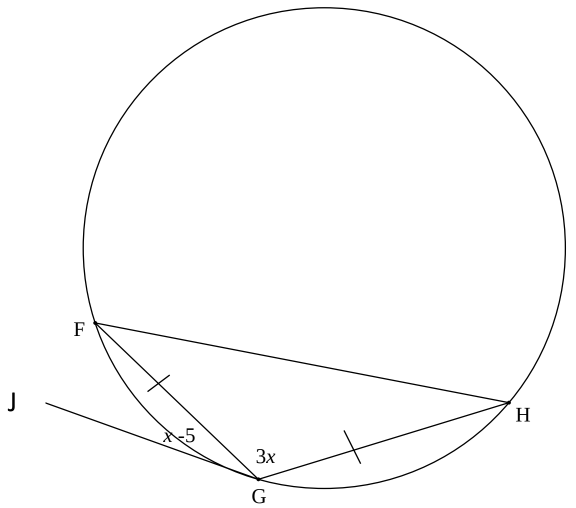
9.23



O is the centre of the circle. A, B, C and D are points on the circumference. AOB is a straight line. AC and BC are drawn. OD is drawn parallel to BC, and intersects AC at E. The radius of the circle is 10 cm, and $AC = 12 \text{ cm}$

Calculate the length of ED. (6) **L3**

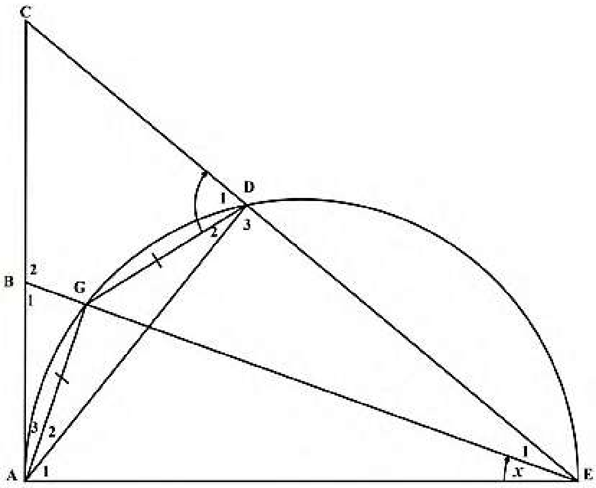
9.24



In the diagram below, JG is a tangent to the circle FGH at G. FG, GH and FH are drawn. GH = FG, $J\hat{G}F = x - 5$ and $F\hat{G}H = 3x$.

Calculate the value of x. (5) **L3**

9.25



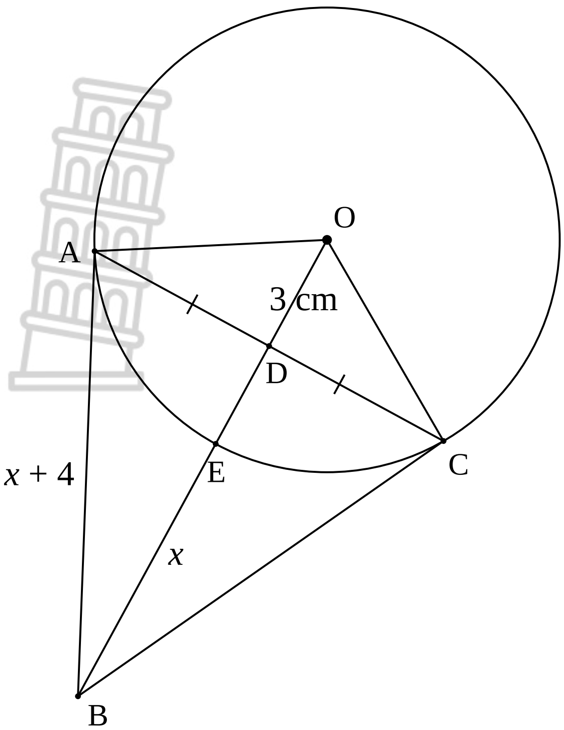
A, G, D and E are points on a semi-circle having AE as the diameter. CA is a tangent to the semi-circle at A. ED produced meets the tangent at C. AG, GD and EG are drawn. EG produced meets the tangent at B. $AG = GD$. $A\hat{E}B = x$.

9.25.1 Name, with reasons, THREE other angles each equal to x. (5) **L3**

9.25.2 Prove that BCDG is a cyclic quadrilateral. (5) **L3**

[10]

9.26



In the diagram below O is the centre of the circle. Quadrilateral ABCO is drawn with A and C on the circumference. AB is a tangent to the circle at point A and BC is a tangent to the circle at C. D is the midpoint of chord AC and . $AD = DC$. E is the point on the circumference of the circle with centre O.

The length of:

$AC = 8\text{cm}$,

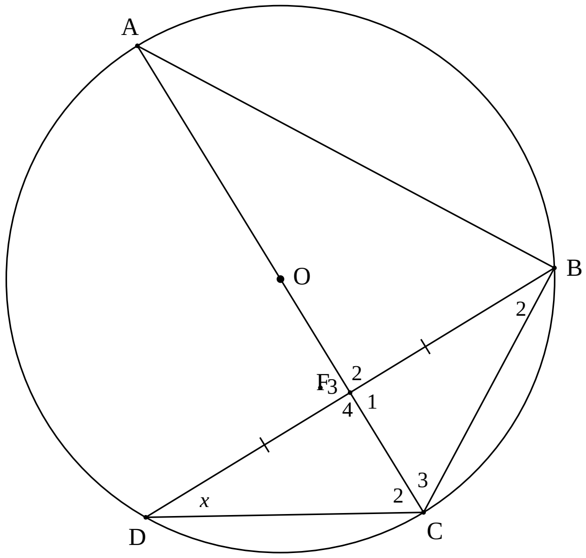
$OD = 3\text{cm}$,

$BE = x$ and

$AB = x + 4$

9.26.1	Write down the size of $O\hat{D}C$ with a reason.	(2)	**L1**
9.26.2	Calculate the length of OA.	(2)	**L1**
9.26.3	Write down the size of $O\hat{A}B$ and give a reason for your answer.	(2)	**L1**
9.26.4	Calculate the value of x if $x > 1$.	(4)	**L3**
		[10]	

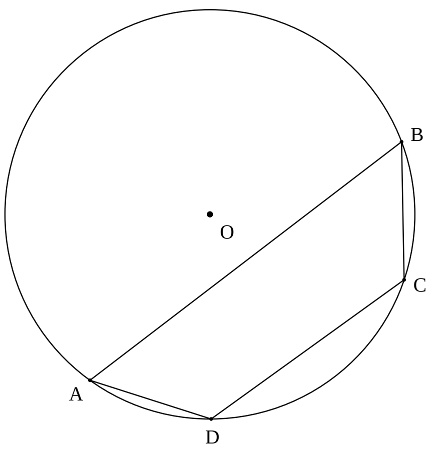
9.27



In the diagram, A, B and D lie on the circle with centre O. AOFC and DFB are straight lines. $DF = FB$, $\hat{D} = x$.

9.27.1	Determine, with reasons. the size of EACH of the following in terms of x.		
	(a) $\hat{A}$	(2)	**L1**
	(b) $\hat{C}_3$	(3)	**L2**
9.27.2	Prove, giving reasons, that $\hat{F}_2 = \hat{F}_3$	(2)	**L2**
		[7]	

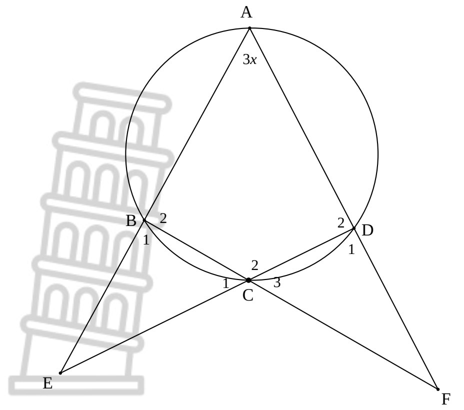
9.28



In the diagram, O is the centre of the circle. ABCD is a cyclic quadrilateral.

Use the diagram to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is to prove that $\hat{B} + \hat{D} = 180°$. (5) **L4**

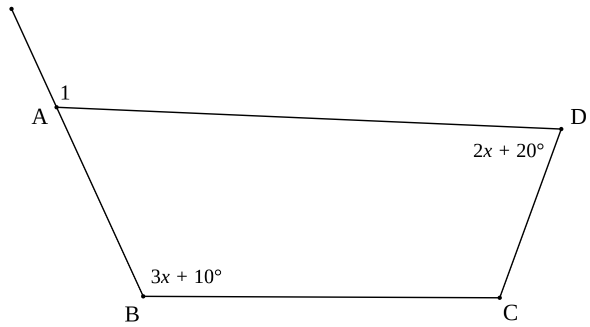
9.29



In the diagram, ABCD is a cyclic quadrilateral. AB and DC are produced to meet at E. AD and BC are produced to meet at F. $A\hat{F}B = 2x$, $D\hat{A}B = 3x$ and $A\hat{E}D = x$.

Determine, giving reasons, the value of x. (6) **L4**

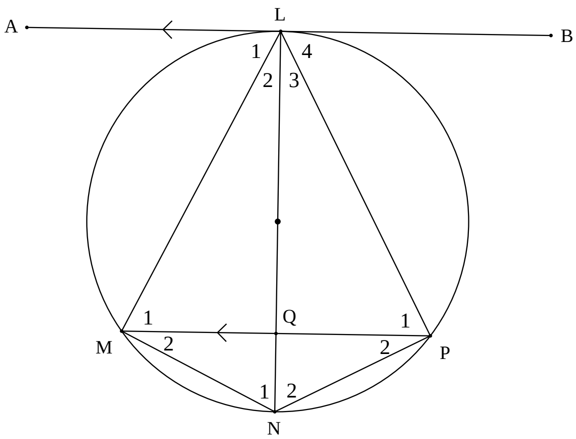
9.30



In the diagram, $\hat{A}_1 = \hat{C}$, $\hat{B} = 3x + 10°$ and $\hat{D} = 2x + 20°$. Calculate, with reasons, the value of x.

(5) **L4**

9.31



ALB is a tangent to circle LMNP. ALB || MP. Prove that:

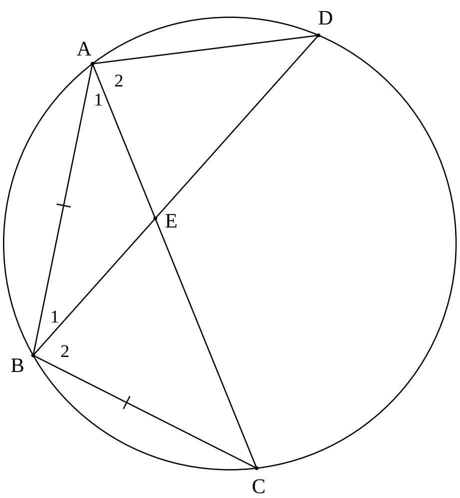
9.31.1 LM = LP (5) **L3**

9.31.2 LN bisects $M\hat{N}P$ (3) **L2**

9.31.3 LM is a tangent to circle MNQ. (3) **L3**

[11]

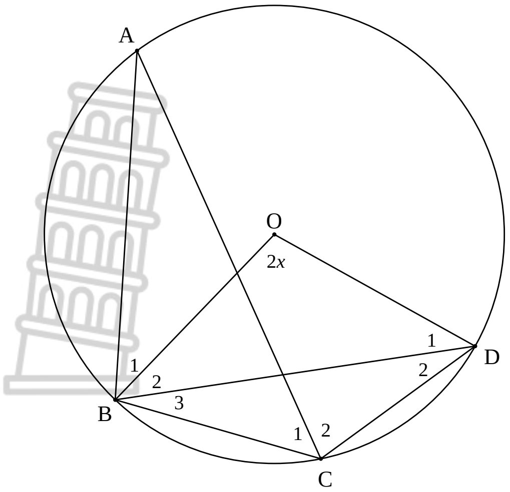
9.32



In the circle ABCD, AB = BC.

Prove that AB is a tangent to the circle AED. (4) **L3**

9.33

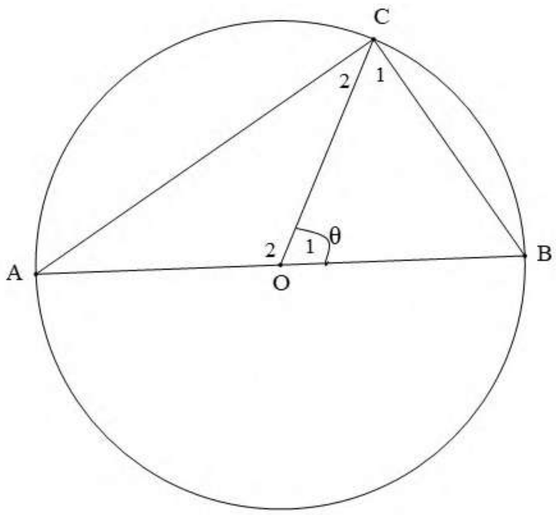


In the diagram, O is the centre of the circle through A, B, C and D.
$BC = CD$ and $B\hat{O}D = 2x$.

Determine the following angles in terms of x.

9.33.1	$\hat{B}_2$	(3)	**L2**
9.33.2	$B\hat{C}D$	(3)	**L3**
9.33.3	$\hat{A}$	(4)	**L4**
		[10]	

9.34

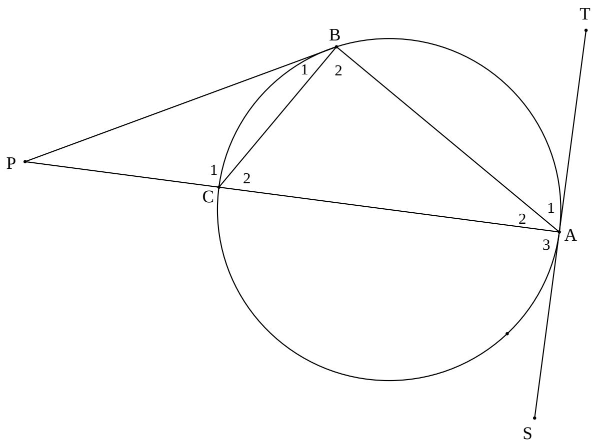


In the figure, AB is a diameter of the circle with centre O and radius r. C is a point on the circle such that $C\hat{O}B = \theta$ (acute angle).

Prove that:

9.34.1 $BC = \dfrac{r\sin\theta}{\cos\frac{\theta}{2}}$ (3) **L3**

9.35

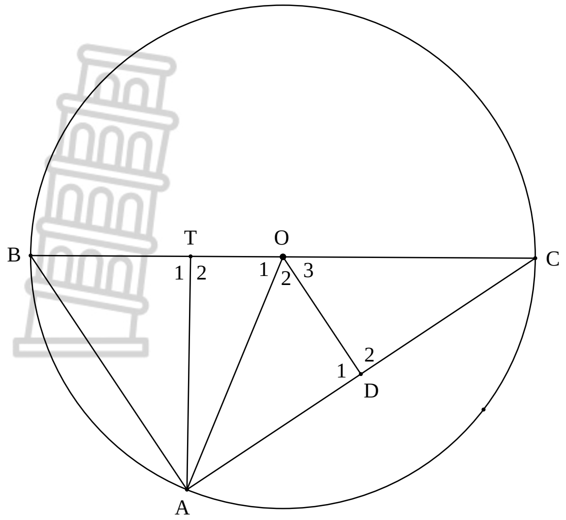


In the diagram, AC is a diameter of the circle and PB is a tangent. AC produced meets the tangent at P. TAS is also a tangent. $\hat{P} = x$ and $\hat{C}_1 = y$.

Prove that:

9.35.1	$\hat{A}_2 + \hat{A}_3 = y$	(4)	**L3**
9.35.2	$x + 2y = 270°$	(4)	**L4**
		[8]	

9.36



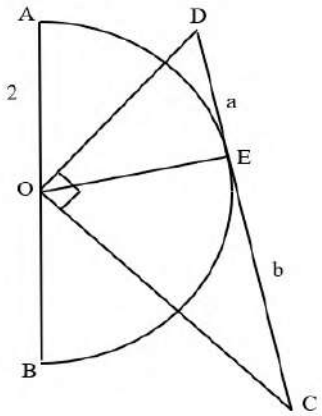
BC is a diameter of the circle ABC with centre O. $OD \perp AC$ and $AT \perp BOC$. $BT = x$, $TO = 5$ and $AD = \sqrt{42}$.

9.36.1 Prove that $OA = 7$ (5) **L4**

9.36.2 Calculate the length of AB. (4) **L4**

[9]

9.37

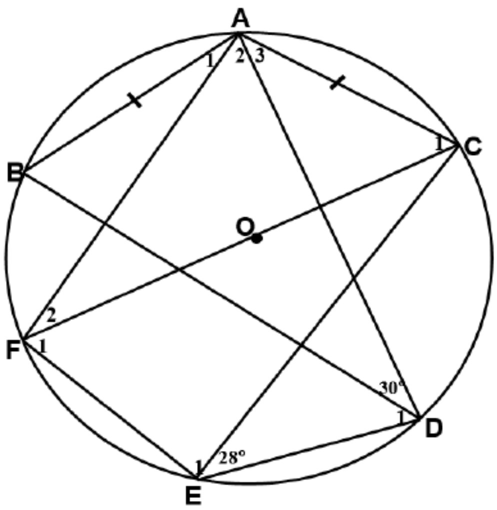


AB is the diameter of semicircle AOBE, centre O. DEC is a tangent to the circle at E and $DO \perp OC$, AO = 2, DE = a and EC = b.

9.37.1 Prove that $a.b = 4$ (6) **L4**

[6]

9.38



In the diagram, O is the centre of the circle.

Chords AB = AC. $C\hat{E}D = 28°$ and $A\hat{D}B = 30°$

Calculate with reasons the following angles:

9.38.1 $\hat{E}_1$ (2) **L1**

9.38.2 $\hat{A}_2$ (3) **L2**

9.38.3 $\hat{F}_2$ (2) **L2**

[7]