



## **KWAZULU-NATAL PROVINCE**

**EDUCATION**  
REPUBLIC OF SOUTH AFRICA

### **PROVINCIAL STANDARDISED ASSESSMENT**

**GRADE 12**

**MATHEMATICS P2**

**JUNE 2026**

**MARKS: 150**

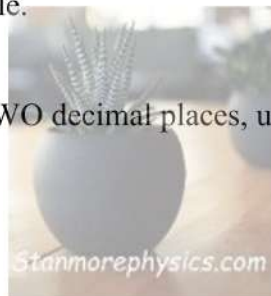
**TIME: 3 hours**

**This question paper consists of 13 pages and an information sheet.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
9. Write neatly and legibly.



**QUESTION 1**

Five data values are represented as follows:

$2d$	$d+1$	$d+2$	$d-3$	$2d-2$
------	-------	-------	-------	--------

- 1.1 If the mean of the data set is 15, show that  $d = 11$ . (3)
- 1.2 Calculate the standard deviation for this data, correct to TWO decimal places. (2)
- 1.3 If a positive number  $p$  is added to each value in the data set, will each of the following increase, decrease or remain the same?
  - 1.3.1 the mean (1)
  - 1.3.2 the standard deviation (1)

[7]

**QUESTION 2**

The ages of the workers at a textile factory in KwaZulu-Natal are represented in the table below.

AGE	FREQUENCY
$25 < x \leq 30$	6
$30 < x \leq 35$	11
$35 < x \leq 40$	17
$40 < x \leq 45$	25
$45 < x \leq 50$	24
$50 < x \leq 55$	10
$55 < x \leq 60$	7
$60 < x \leq 65$	8

- 2.1 Complete the cumulative frequency column of the table in your ANSWER BOOK. (2)
- 2.2 Draw an ogive on the set of axes provided in your ANSWER BOOK to represent the data in the table. (4)
- 2.3 Use your ogive to sketch a box and whisker diagram if the youngest worker is 26 years old and the oldest worker is 65 years years old. (4)
- 2.4 A decision was taken by the management team of the factory to put into place measures that will help to address the problem of youth unemployment in the area.
  - 2.4.1 One of the measures was to allow workers to apply for early retirement from the age of 57. Use the ogive to determine what percentage of the workers would qualify for early retirement. (2)
  - 2.4.2 They also decided to appoint 20 more workers between the ages of 25 and 30. What effect will this action have on the distribution of the ages of the workers? Will it cause the distribution to be more skewed to the left, or to the right? (1)

[13]

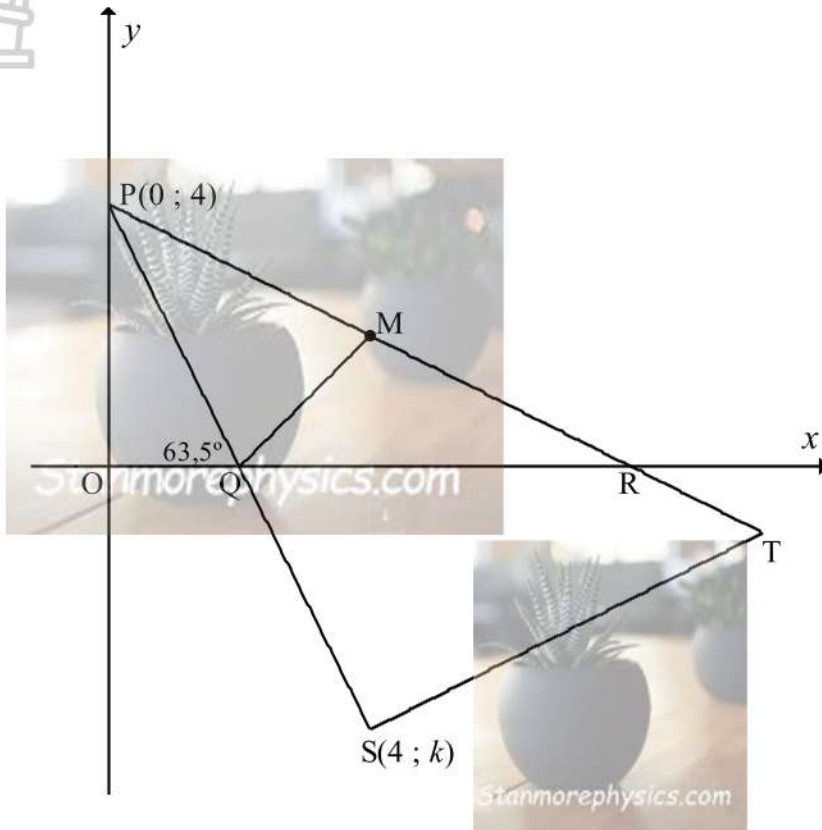
**QUESTION 3**

In the diagram below, triangle PST, with vertices  $P(0;4)$ ,  $S(4;k)$  and T, is shown.

Q and R are the x-intercepts of PS and PT respectively.  $OQ : OR = 1 : 4$ .

M is the midpoint of PR.

$\hat{OQP} = 63,5^\circ$



- 3.1 Determine the gradient of PS, correct to **the nearest integer**. (2)
- 3.2 Write down the equation of PS. (1)
- 3.3 Show that  $k = -4$ . (1)
- 3.4 Determine the length of QR. (4)
- 3.5 Calculate the coordinates of M. (2)
- 3.6 Prove that QM cannot be the perpendicular bisector of PR. (4)
- 3.7 Calculate the area of quadrilateral QSTM, if the coordinates of T are  $(10; -1)$ . (4)

[18]

**QUESTION 4**

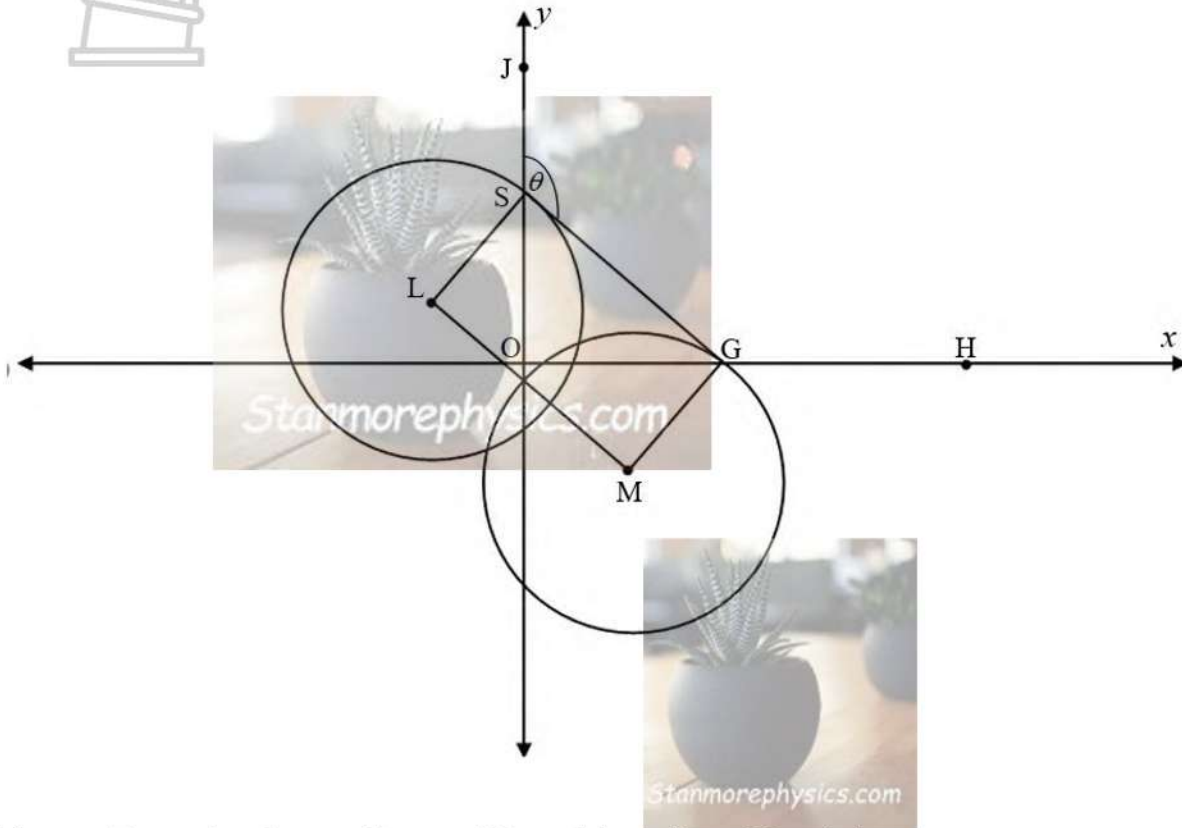
In the diagram below, L and M are the centres of two equal circles.

The equation of the circle centred at L is  $x^2 + 6x + y^2 - 4y = 12$ .

GS is a tangent to both circles, at S and G respectively.

G lies on the  $x$ -axis and S on the  $y$ -axis.

H is a point on the  $x$ -axis to the right of G, and J is a point on the  $y$ -axis above S.



- 4.1 Determine the coordinates of L, and the radius of the circle. (4)
- 4.2 Determine the coordinates of S. (4)
- 4.3 Determine the coordinates of G. (4)
- 4.4 Determine the size of  $\theta$ . (4)
- 4.5 Show that LMGS is a rectangle. (2)
- 4.6 Determine the equation of the circle with centre M in the form  $(x - a)^2 + (y - b)^2 = r^2$ . (3)

[21]

**QUESTION 5**

5.1 If  $\sin 2\alpha = \frac{\sqrt{5}}{3}$ , with  $2\alpha \in [90^\circ ; 270^\circ]$ , determine WITHOUT the use of a calculator the value of:

5.1.1  $\tan 2\alpha$  (3)

5.1.2  $\sin \alpha$  (4)

5.2 Simplify fully:  
 $\tan^2(180^\circ - \theta) \cdot \sin^2(90^\circ + \theta) - \cos(\theta - 180^\circ) \cdot \cos \theta$  (5)

5.3 Given:  $\frac{\cos 5x \cos 3x + \sin 3x \sin 5x}{\cos x - \sin x} = \sin x + \cos x$

5.3.1 Prove the identity. (3)

5.3.2 For which value(s) of  $x$ , where  $x \in [-180^\circ ; 180^\circ]$ , is the above identity undefined? (4)

[19]

**QUESTION 6**

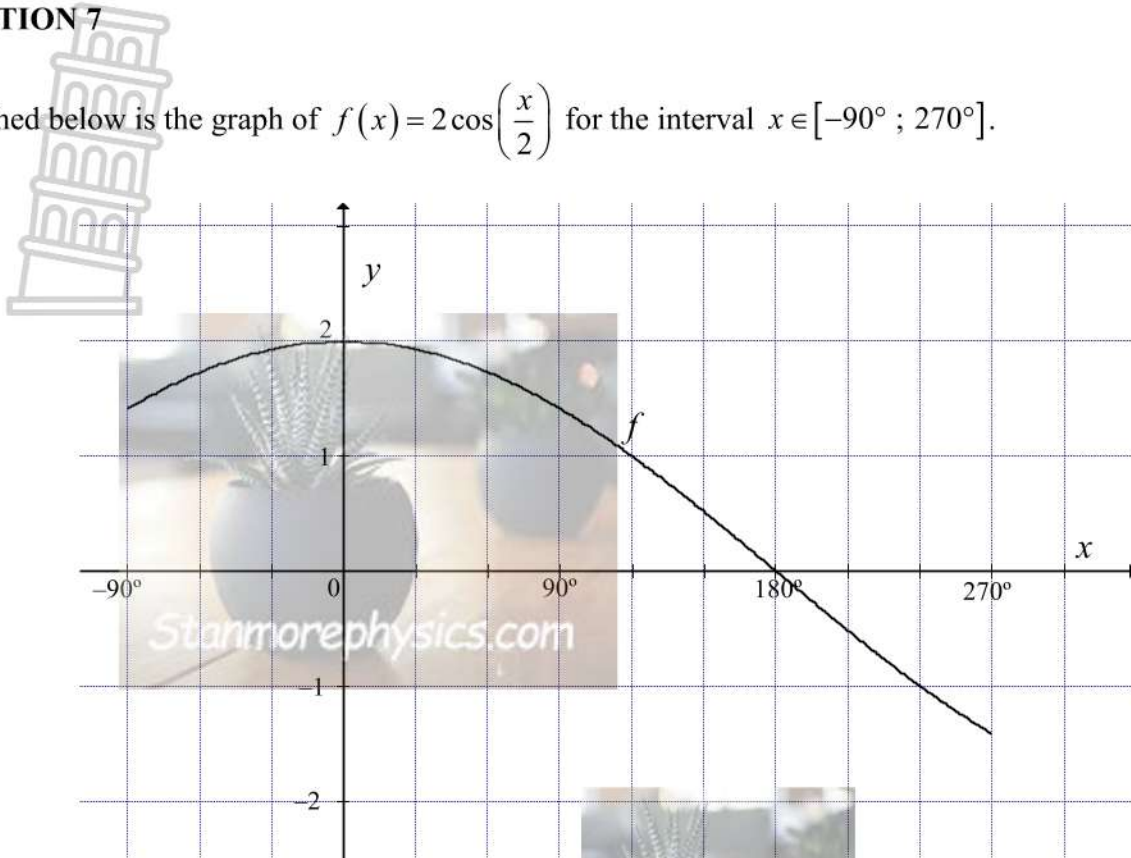
6.1 Determine the general solution of the following equation:  
 $2 \cos 2x - \sin x + 1 = 0$  (7)

6.2 The expression  $\sqrt{3} \cos \beta + \sin \beta$  can be rewritten in the form  $k \sin(m^\circ + \beta)$ .  
 Determine the values of  $k$  and  $m$ .  
 Show all your working details. (4)

[11]

**QUESTION 7**

Sketched below is the graph of  $f(x) = 2 \cos\left(\frac{x}{2}\right)$  for the interval  $x \in [-90^\circ ; 270^\circ]$ .



7.1 Write down the period of  $f$ . (1)

7.2 Sketch the graph of  $g$  if  $g(x) = \sin(x - 60^\circ)$  on the grid in your ANSWER BOOK for the interval  $x \in [-90^\circ ; 270^\circ]$ .  
Clearly indicate all intercepts with the axes, turning points and end points. (4)

7.3 Describe the transformation from  $g$  to  $h$ , if  $h(x) = -\cos x$ . (2)

7.4 Use the graphs to determine the values of  $x$  in the interval  $x \in [-90^\circ ; 270^\circ]$  for which:

7.4.1  $2 \cos\left(\frac{x}{2}\right) \geq 0$ ? (2)

7.4.2  $f(x) \cdot g(x) > 0$  (3)

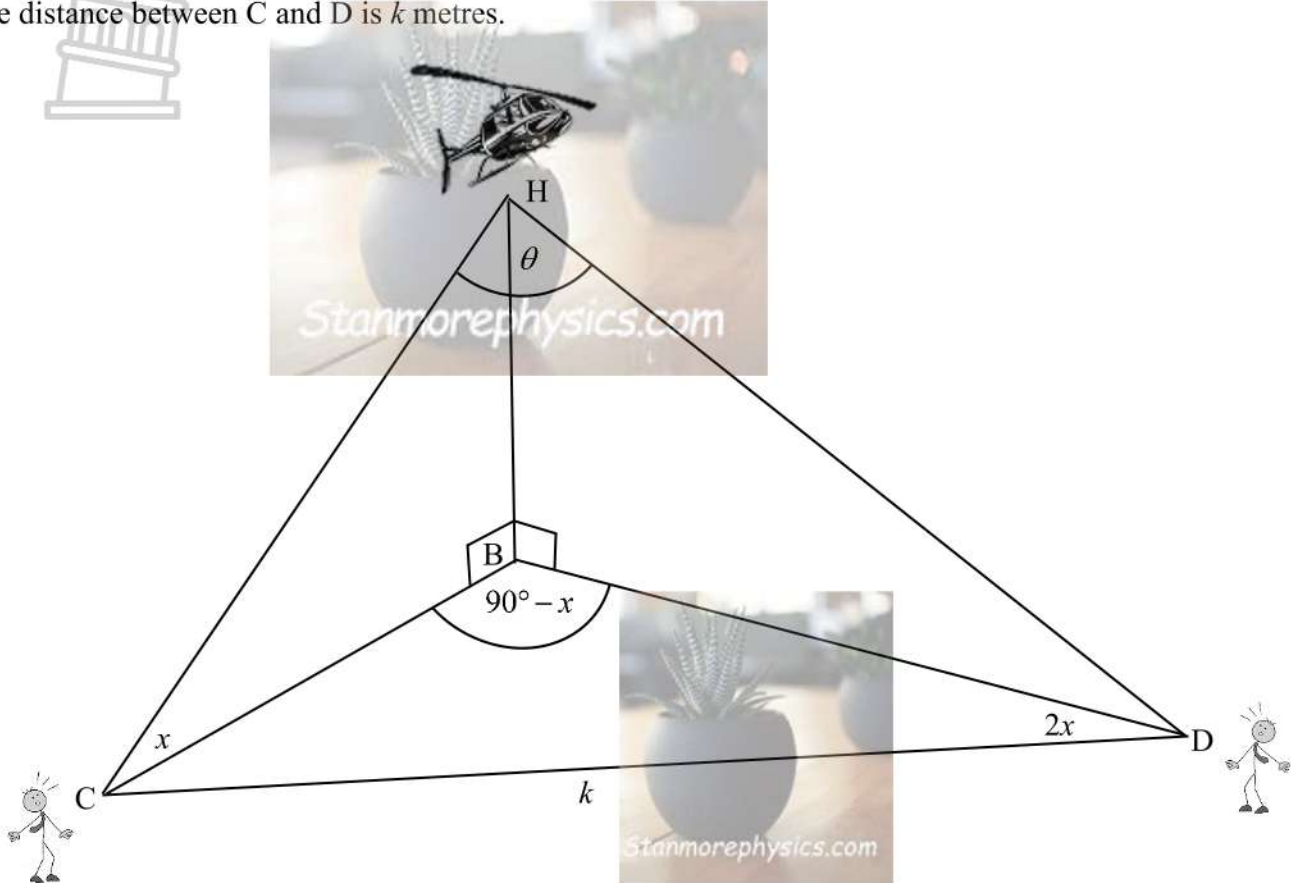
**[12]**

**QUESTION 8**

A helicopter H is hovering directly above point B on the ground. Two people, standing at point C and point D look up to view the helicopter. B, C and D are in the same horizontal plane.

The angle of elevation from C to H is  $x$ .  $\hat{C}BD = 90^\circ - x$ ,  $\hat{C}DB = 2x$  and  $\hat{C}HD = \theta$ .

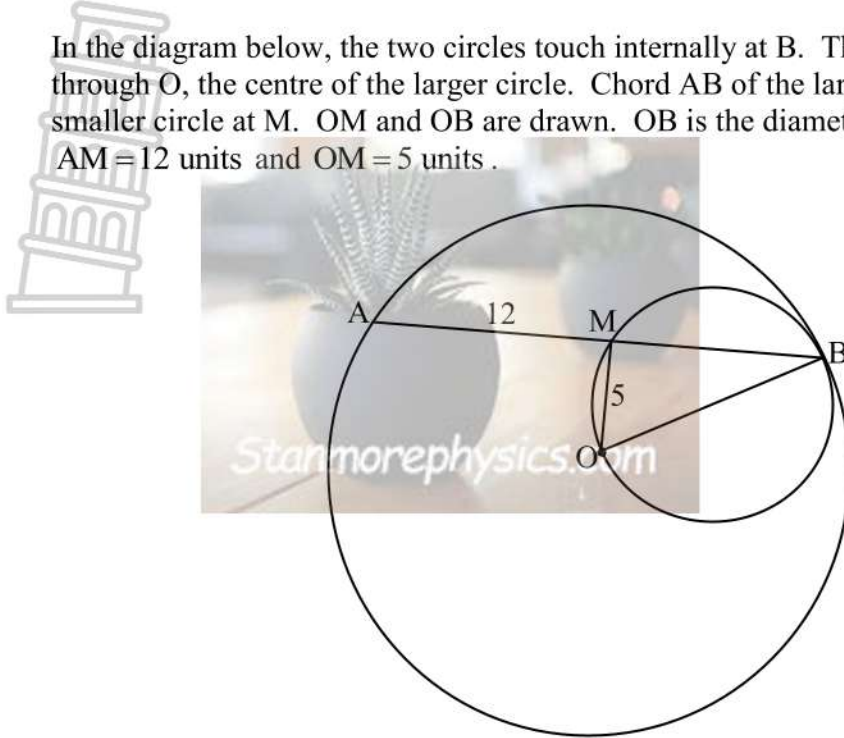
The distance between C and D is  $k$  metres.



- 8.1 Show that  $CB = 2k \sin x$  (3)
  - 8.2 Hence, show that HC is  $2k \tan x$  (2)
  - 8.3 If  $k = 40$  m,  $x = 23^\circ$  and  $HD = 30$  m, calculate the size of angle  $\theta$ , correct to two decimal places. (4)
- [9]**

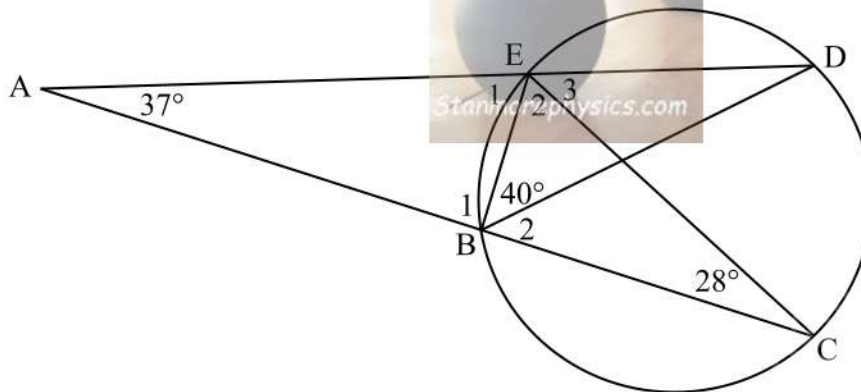
**QUESTION 9**

- 9.1 In the diagram below, the two circles touch internally at B. The smaller circle passes through O, the centre of the larger circle. Chord AB of the larger circle cuts the smaller circle at M. OM and OB are drawn. OB is the diameter of the smaller circle. AM = 12 units and OM = 5 units.



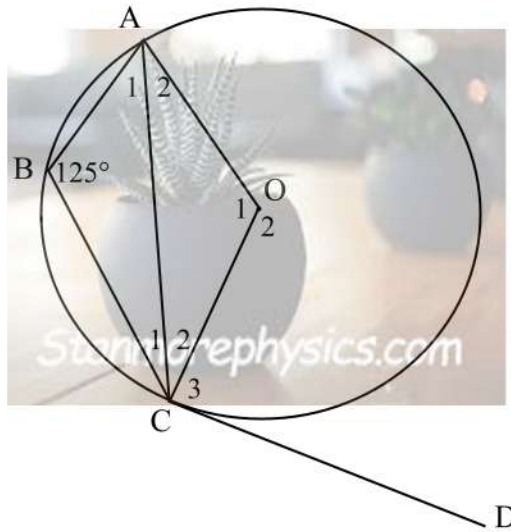
Calculate, with reasons, the length of the radius of the larger circle. (4)

- 9.2 In the diagram below, AED and ABC are two straight lines. B, C, D and E lie on the circumference of the circle. BE, EC and BD are drawn.



Calculate, with reasons, the size of  $\hat{B}_1$ . (4)

- 9.3 In the diagram below, O is the centre of the circle and A, B and C are points on the circle. CD is a tangent to the circle at C.  $\hat{B} = 125^\circ$ .



Calculate, with reasons, the size of  $\hat{C}_2$ .

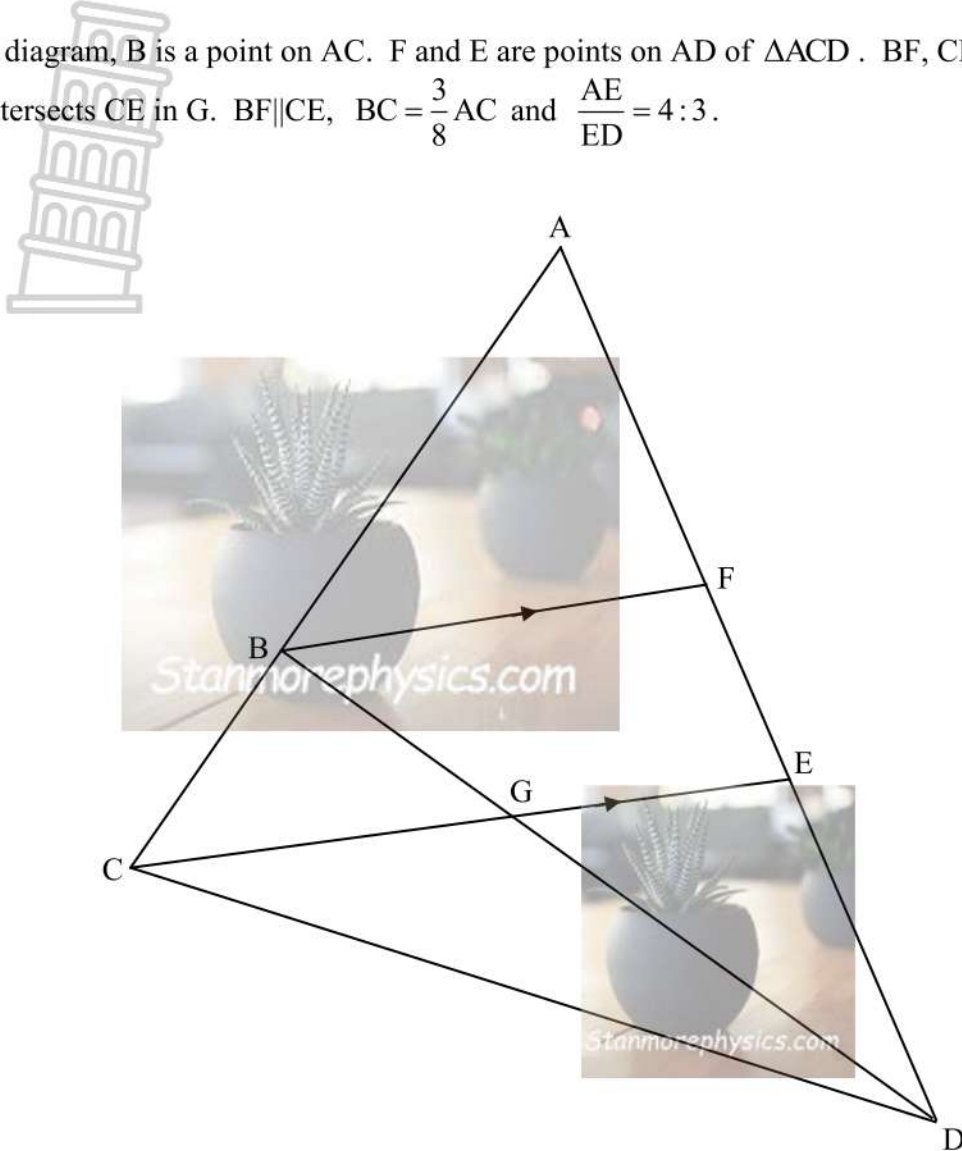
(4)

[12]



**QUESTION 10**

In the diagram, B is a point on AC. F and E are points on AD of  $\triangle ACD$ . BF, CE and BD are drawn. BD intersects CE in G.  $BF \parallel CE$ ,  $BC = \frac{3}{8} AC$  and  $\frac{AE}{ED} = 4:3$ .



Calculate, with reasons, the value of:

10.1  $\frac{AF}{FE}$  (2)

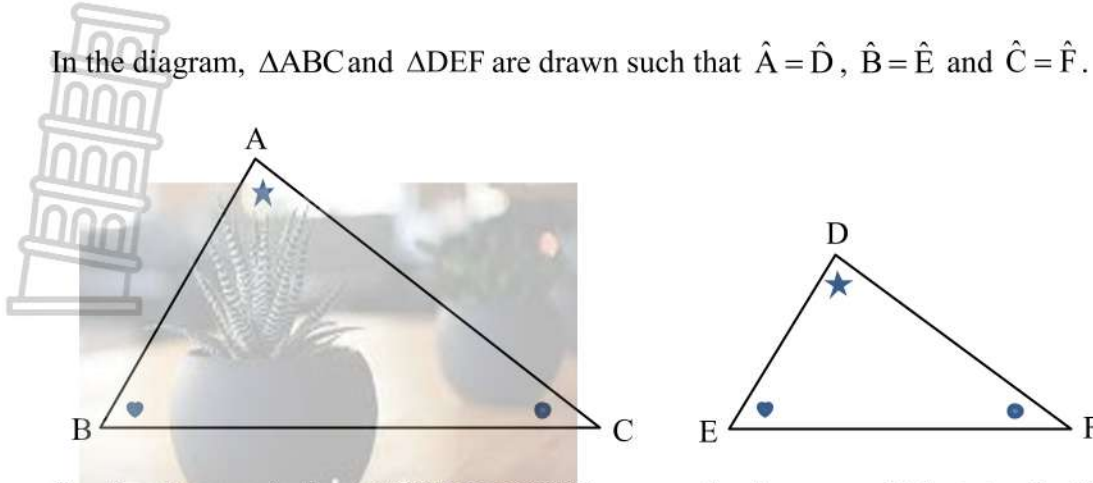
10.2  $DG:GB$  (3)

10.3  $\frac{\text{Area of } \triangle BCG}{\text{Area of } \triangle DGC}$  (2)

[7]

**QUESTION 11**

11.1 In the diagram,  $\triangle ABC$  and  $\triangle DEF$  are drawn such that  $\hat{A} = \hat{D}$ ,  $\hat{B} = \hat{E}$  and  $\hat{C} = \hat{F}$ .



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is:

$$\frac{DE}{AB} = \frac{DF}{AC}$$

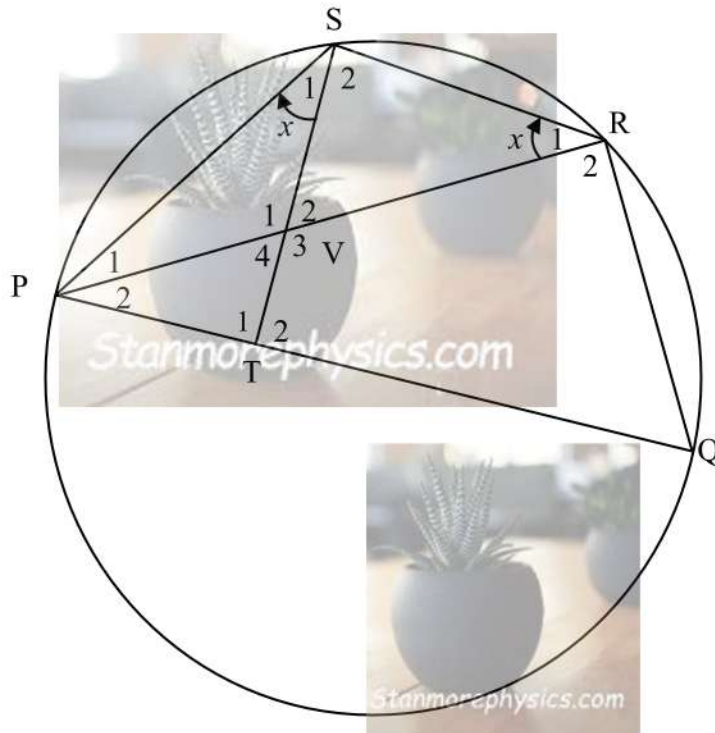
(6)



11.2

In the diagram below:

- P, Q, R and S are points on the circumference of the circle.
- PQ is the diameter.
- PR is drawn.
- ST is drawn, where T is a point on PQ.
- V is the point of intersection between ST and PR.
- $\hat{S}_1 = \hat{R}_1 = x$ .



11.2.1 Prove that  $\triangle PSV \parallel \triangle PRS$ . (3)

11.2.2 Prove that VTQR is a cyclic quadrilateral. (4)

11.2.3 Prove that  $\triangle PRQ \parallel \triangle PTV$ . (3)

11.2.4 Hence, prove that  $ST^2 = PT(PQ - PT)$ . (5)

[21]

**TOTAL: 150**

**INFORMATION SHEET: MATHEMATICS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \quad A = P(1-ni) \quad A = P(1-i)^n \quad A = P(1+i)^n$$

$$T_n = a + (n-1)d \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r-1} ; r \neq 1 \quad S_\infty = \frac{a}{1-r} ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \quad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In  $\Delta ABC$ :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$



$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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### PROVINCIAL STANDARDISED ASSESSMENT

GRADE 12

MATHEMATICS P2

Stanmorephysics.com

JUNE 2026

**SPECIAL ANSWER BOOK**

NAME OF CANDIDATE: \_\_\_\_\_

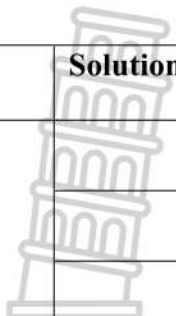
150

TIME: 3 hours

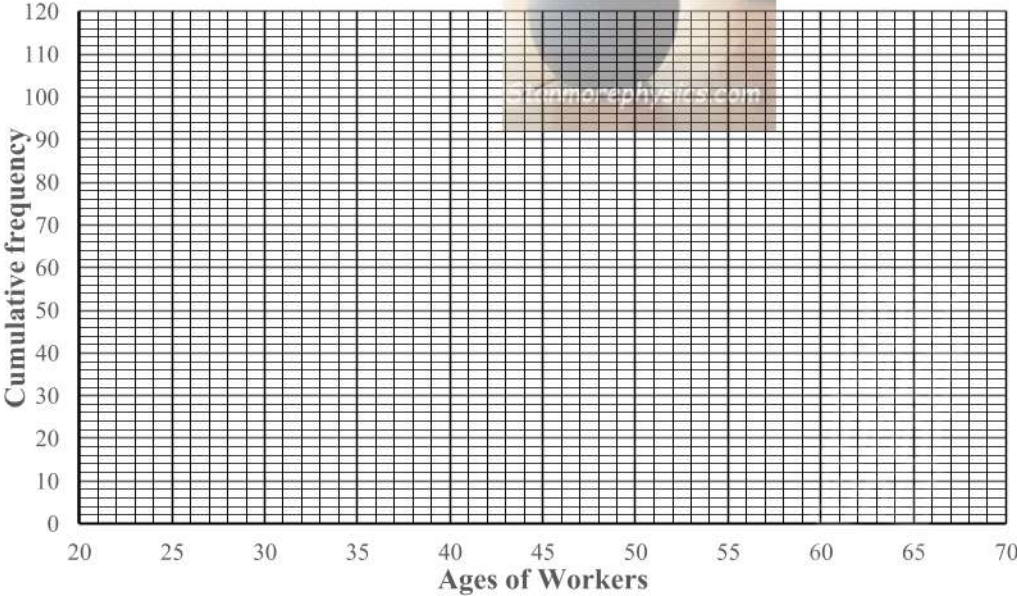
This answer book consists of 23 pages.

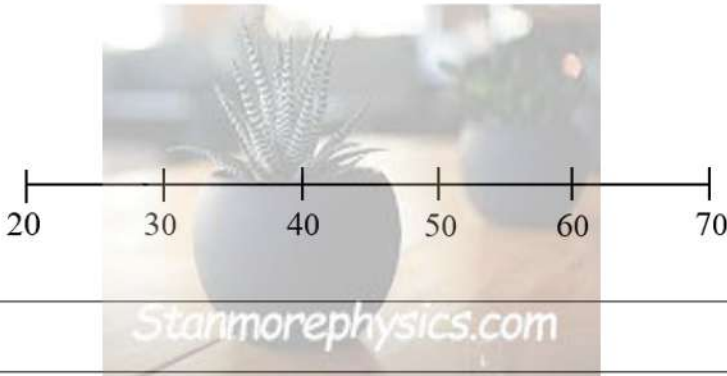


**QUESTION 1**

	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>
1.1		(3)
1.2		(2)
1.3.1		(1)
1.3.2		(1)
	<b>[7]</b>	

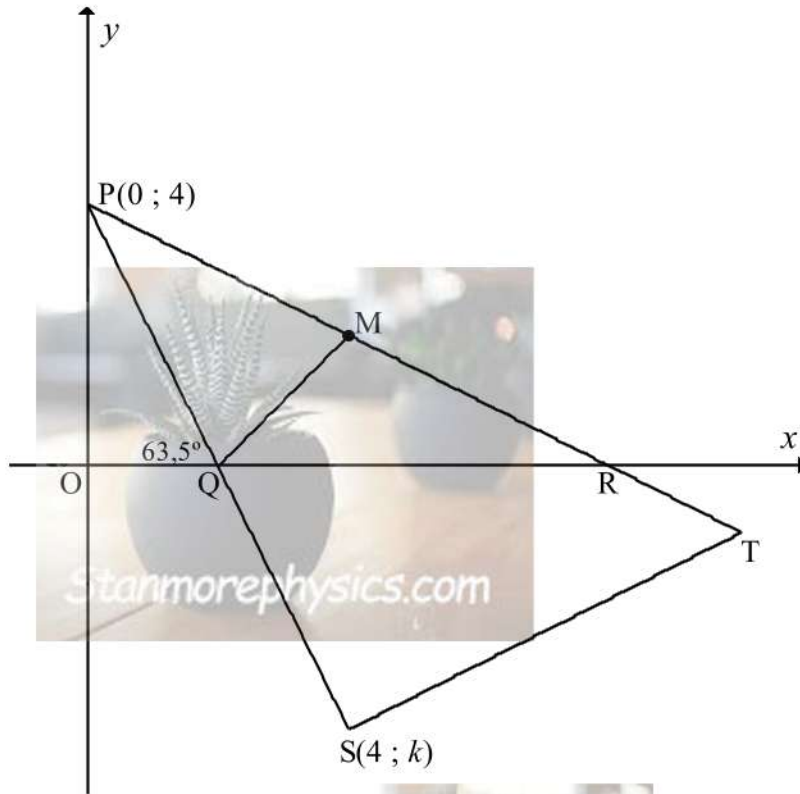


**QUESTION 2**

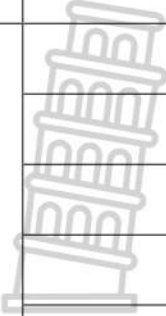


	<b>Solution/Oplossing</b>	<b>Marks/ Punte</b>																											
2.1	<table border="1"> <thead> <tr> <th>AGE</th> <th>FREQUENCY</th> <th>CUM. FREQ</th> </tr> </thead> <tbody> <tr> <td><math>25 &lt; x \leq 30</math></td> <td>6</td> <td></td> </tr> <tr> <td><math>30 &lt; x \leq 35</math></td> <td>11</td> <td></td> </tr> <tr> <td><math>35 &lt; x \leq 40</math></td> <td>17</td> <td></td> </tr> <tr> <td><math>40 &lt; x \leq 45</math></td> <td>25</td> <td></td> </tr> <tr> <td><math>45 &lt; x \leq 50</math></td> <td>24</td> <td></td> </tr> <tr> <td><math>50 &lt; x \leq 55</math></td> <td>10</td> <td></td> </tr> <tr> <td><math>55 &lt; x \leq 60</math></td> <td>7</td> <td></td> </tr> <tr> <td><math>60 &lt; x \leq 65</math></td> <td>8</td> <td></td> </tr> </tbody> </table>	AGE	FREQUENCY	CUM. FREQ	$25 < x \leq 30$	6		$30 < x \leq 35$	11		$35 < x \leq 40$	17		$40 < x \leq 45$	25		$45 < x \leq 50$	24		$50 < x \leq 55$	10		$55 < x \leq 60$	7		$60 < x \leq 65$	8		(2)
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2.2	<p style="text-align: center;"><b>Ages of Workers at a Textile factory in KZN</b></p> 	(4)																											

	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>
2.3		(4)
2.4.1		(2)
2.4.2		(1)
		<b>[13]</b>

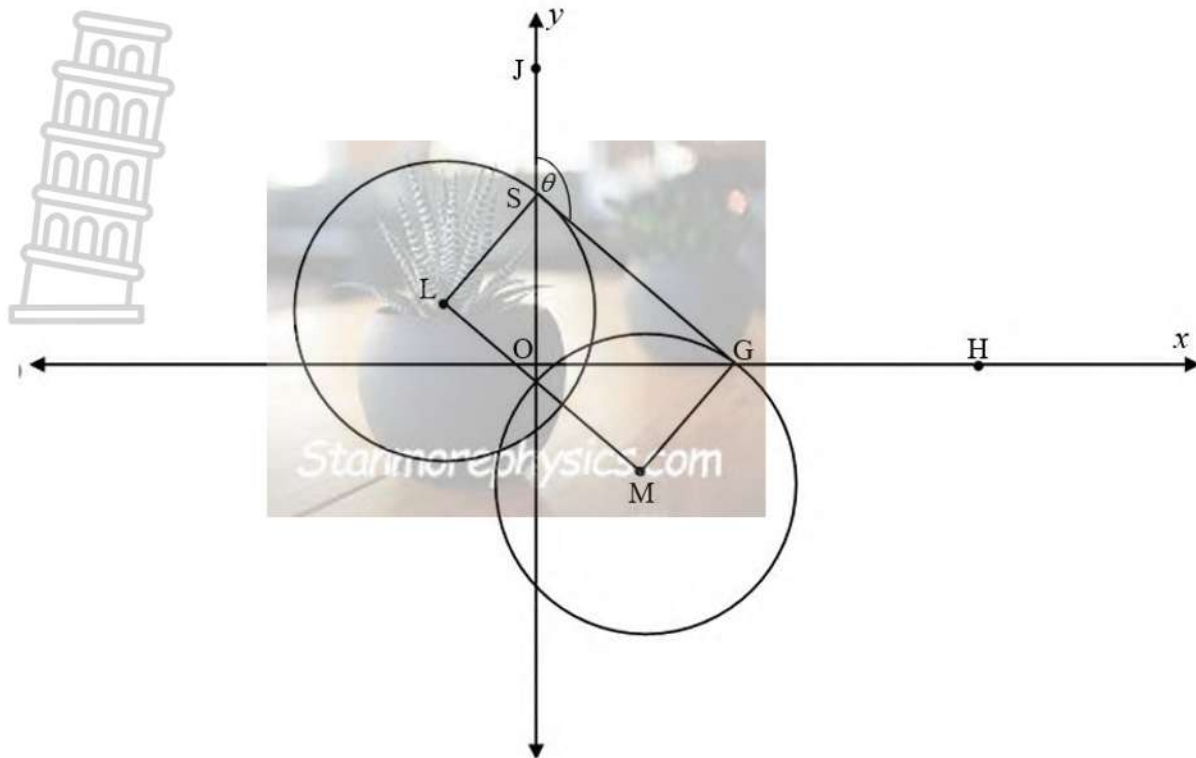
**QUESTION 3**



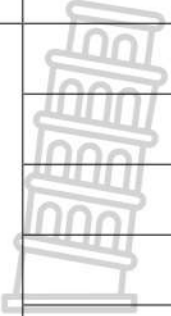

	<b>Solution/Oplissing</b>	<b>Marks/Punte</b>
3.1		(2)
3.2		(1)
3.3		(1)

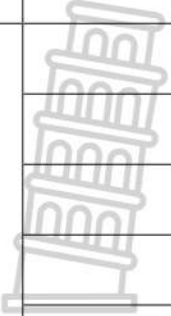
	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>						
3.4								
(4)								
3.5								
(2)								
3.6	 Stanmorephysics.com							
(4)								
3.7								
(4)								
		<b>[18]</b>						

**QUESTION 4**



	<b>Solution/Oplissing</b>		<b>Marks/ Punte</b>
4.1			(4)
4.2			(4)

	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>
4.3		(4)
4.4	 Stanmorephysics.com	(4)
4.5		(2)

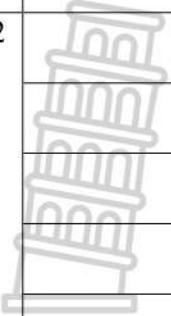
	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>
4.6		(3)
		<b>[21]</b>



**QUESTION 5**

	<b>Solution/Oplossing</b>	<b>Marks/ Punte</b>
5.1.1		(3)
5.1.2		(4)
5.2		(5)
5.3.1		(3)



	<b>Solution/Oplossing</b>	<b>Marks/ Punte</b>
5.3.2		
		<b>[19]</b>



**QUESTION 6**

	<i>Solution/Oplossing</i>	<b>Marks/ Punte</b>
6.1		
	(7)	
6.2		
	(4)	
		<b>[11]</b>



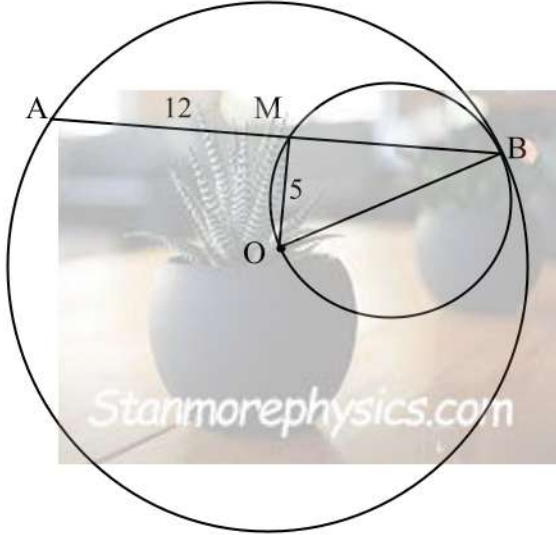
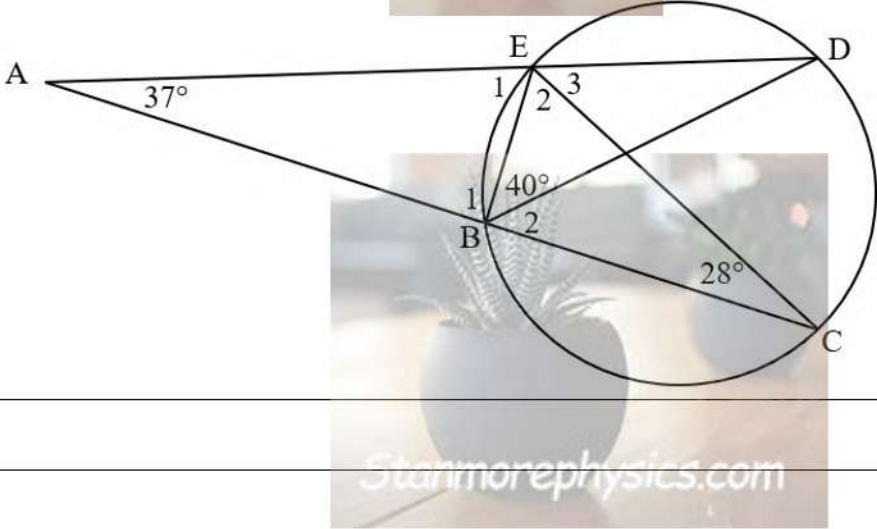
**QUESTION 7**

	Solution/Oplissing	Marks/ Punte
7.1		
		(1)
7.2		(4)
7.3		(2)
7.4.1		(2)
7.4.2		(3)
		<b>[12]</b>



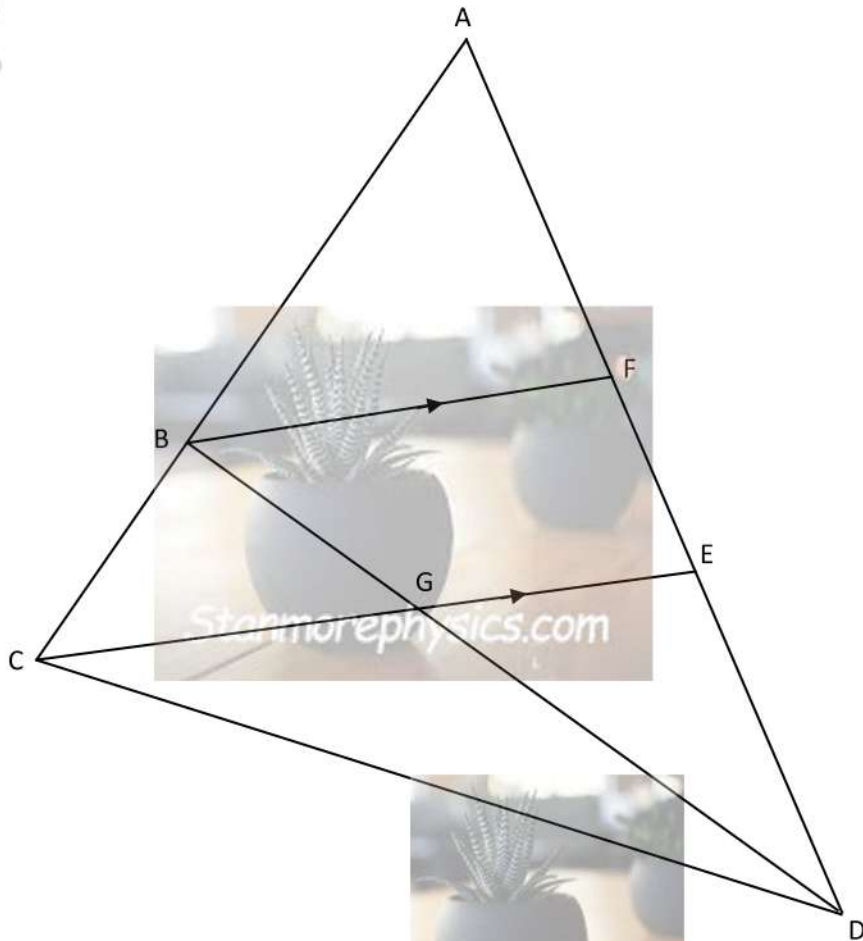
	<b>Solution/Oplossing</b>	<b>Marks/ Punte</b>
8.2	 <i>Stanmorephysics.com</i>	(2)
8.3	 <i>Stanmorephysics.com</i>	(4)
		<b>[9]</b>

**QUESTION 9**



	Solution/Oplissing	Marks/ Punte
9.1		
9.2		(4)



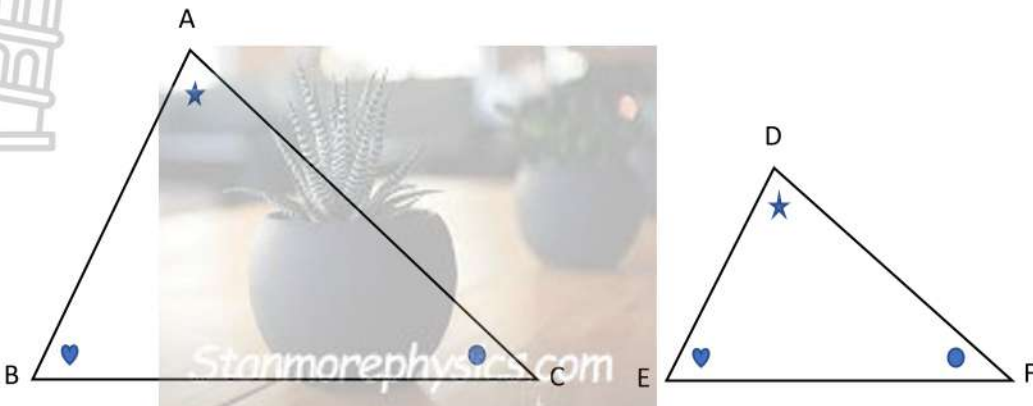
**QUESTION 10**



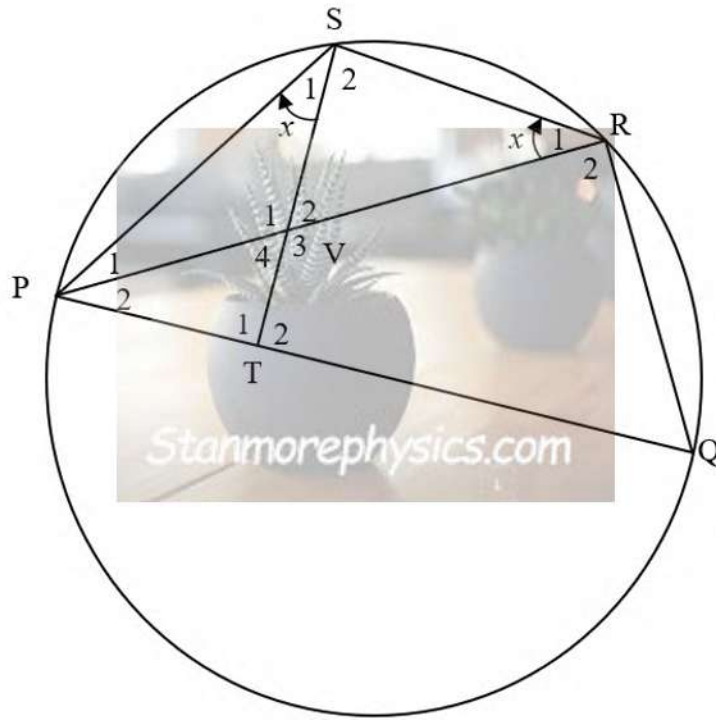
	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>
10.1		(2)

	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>	
10.2			
		(3)	
10.3	 Stanmorephysics.com		
		(2)	
		[7]	

**QUESTION 11**

	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>
11.1	 <p>The diagram shows two triangles, <math>ABC</math> and <math>DEF</math>. Triangle <math>ABC</math> has vertices <math>A</math>, <math>B</math>, and <math>C</math>. Vertex <math>A</math> is marked with a blue star, vertex <math>B</math> with a blue heart, and vertex <math>C</math> with a blue circle. Triangle <math>DEF</math> has vertices <math>D</math>, <math>E</math>, and <math>F</math>. Vertex <math>D</math> is marked with a blue star, vertex <math>E</math> with a blue heart, and vertex <math>F</math> with a blue circle. The triangles are similar. A watermark 'Stanmorephysics.com' is visible over the diagram.</p>	(6)

11.2



	<b>Solution/Oplissing</b>	<b>Marks/ Punte</b>
11.2.1		(3)
11.2.2		(4)







# KWAZULU-NATAL PROVINCE

EDUCATION  
REPUBLIC OF SOUTH AFRICA

## PROVINCIAL STANDARDISED ASSESSMENT

GRADE 12

MATHEMATICS P2

JUNE 2026

MARKING GUIDELINES

MARKS: 150

These marking guidelines consist of 15 pages.

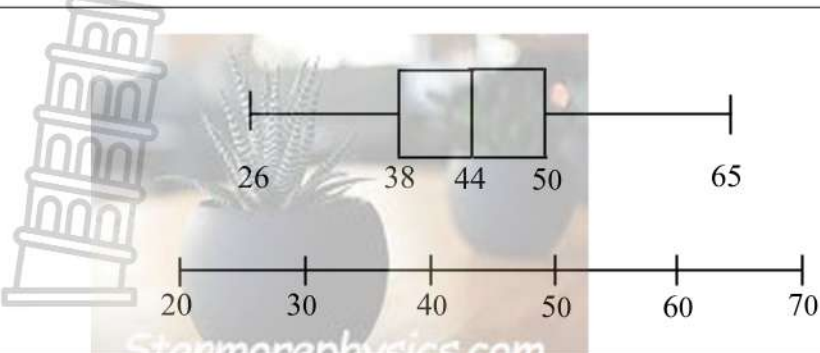
**QUESTION 1**

1.1	$\frac{7d-2}{5} = 15$ $7d-2 = 75$ $7d = 77$ $d = 11$	✓A $7d-2$ ✓A equation ✓A simplification	(3)	
1.2	22 ; 12 ; 13 ; 8 ; 20 $\sigma = 5,22$	Answer only: Full marks	✓A data values ✓CA answer	(2)
1.3.1	The mean will increase	✓A increase	(1)	
1.3.2	The standard deviation will remain the same	✓A remain the same	(1)	
			<b>[7]</b>	

**QUESTION 2**

2.1	<table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th>AGE</th> <th>FREQUENCY</th> <th>CUM. FREQ</th> </tr> </thead> <tbody> <tr> <td><math>25 &lt; x \leq 30</math></td> <td>6</td> <td>6</td> </tr> <tr> <td><math>30 &lt; x \leq 35</math></td> <td>11</td> <td>17</td> </tr> <tr> <td><math>35 &lt; x \leq 40</math></td> <td>17</td> <td>34</td> </tr> <tr> <td><math>40 &lt; x \leq 45</math></td> <td>25</td> <td>59</td> </tr> <tr> <td><math>45 &lt; x \leq 50</math></td> <td>24</td> <td>83</td> </tr> <tr> <td><math>50 &lt; x \leq 55</math></td> <td>10</td> <td>93</td> </tr> <tr> <td><math>55 &lt; x \leq 60</math></td> <td>7</td> <td>100</td> </tr> <tr> <td><math>60 &lt; x \leq 65</math></td> <td>8</td> <td>108</td> </tr> </tbody> </table>	AGE	FREQUENCY	CUM. FREQ	$25 < x \leq 30$	6	6	$30 < x \leq 35$	11	17	$35 < x \leq 40$	17	34	$40 < x \leq 45$	25	59	$45 < x \leq 50$	24	83	$50 < x \leq 55$	10	93	$55 < x \leq 60$	7	100	$60 < x \leq 65$	8	108	✓A 34          ✓CA 108	(2)
AGE	FREQUENCY	CUM. FREQ																												
$25 < x \leq 30$	6	6																												
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2.2	<p style="text-align: center;"><b>Ages of Workers at a Textile factory in KZN</b></p>	✓A grounding ogive  ✓CA plotting at upper limits  ✓CA accurate points  ✓CA smooth curve	(4)																											

GRADE 12  
Marking Guidelines

2.3		<p>Plotted on box and whisker diagram:</p> <ul style="list-style-type: none"> <li>✓A 26 and 65</li> <li>✓CA 38 (accept 37 – 39)</li> <li>✓CA 44 (accept 43 – 45)</li> <li>✓CA 50 (accept 49 – 51)</li> </ul> <p style="text-align: right;">(4)</p>
2.4.1	<p><math>108 - 96 = 12</math> workers</p> <p><math>\frac{12}{108} \times 100 = 11\%</math></p>	<ul style="list-style-type: none"> <li>✓A 12 (accept 11 – 13)</li> <li>✓CA 11%</li> </ul> <p style="text-align: right;">(2)</p>
2.4.2	<p>more skewed to the right</p>	<ul style="list-style-type: none"> <li>✓A answer</li> </ul> <p style="text-align: right;">(1)</p>
<b>[13]</b>		



## QUESTION 3

3.1	Angle of inclination of PS = $116,5^\circ$ $m_{PS} = \tan 116,5^\circ = -2$	✓A angle of inclination ✓CA answer (2)
3.2	$y = -2x + 4$	✓CA answer (1)
3.3	Substitute S(4;k) in: $y = -2x + 4$ $k = -2(4) + 4$ $= -4$	✓CA substitution (1)
3.4	At Q: $0 = -2x + 4$ $x = 2$ $\therefore Q(2; 0)$ and OQ = 2 units since OQ : QR = 1 : 4 R(8 ; 0) and OR = 8 units $\therefore QR = 6$ units	✓CA $0 = -2x + 4$ ✓CA $x = 2$  ✓CA OR = 8 ✓CA QR = 6 (4)
3.5	$M\left(\frac{0+8}{2}; \frac{4+0}{2}\right)$ M(4 ; 2)	✓CA x-coordinate ✓CA y-coordinate (2)
3.6	$m_{QM} = \frac{2-0}{4-2} = 1$ $m_{PR} = \frac{0-4}{8-0} = -\frac{1}{2}$ M is the midpoint of PR, but, $m_{QM} \times m_{PR} = 1 \times -\frac{1}{2} = -\frac{1}{2}$ $-\frac{1}{2} \neq -1$ $\therefore$ BM cannot be the perpendicular bisector	✓CA $m_{QM}$ ✓CA $m_{PR}$  ✓CA calculating $m_{QM} \times m_{PR}$ ✓CA $-\frac{1}{2} \neq -1$ (4)
3.7	MS = $2 - (-4) = 6$ units Area of $\Delta MQS = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 6 \times 2 = 6 \text{ units}^2$ Area of $\Delta MTS = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 6 \times 6 = 18 \text{ units}^2$ Area of quadrilateral QSTM = Area of $\Delta MQS$ + Area of $\Delta MTS$ = $6 + 18 = 24 \text{ units}^2$	✓CA length of MS ✓CA Area of $\Delta MQS$ ✓CA Area of $\Delta MTS$  ✓CA answer (4)

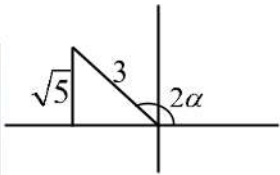
**[18]**

**QUESTION 4**

<p>4.1</p>	$(x+3)^2 + (y-2)^2 = 12+9+4$ $(x+3)^2 + (y-2)^2 = 25$ $L(-3; 2)$ $r = 5$	<p>✓A LHS of equation ✓A RHS of equation</p> <p>✓CA <math>L(-3; 2)</math> ✓CA <math>r = 5</math></p> <p style="text-align: right;">(4)</p>
<p>4.2</p>	$(-3-0)^2 + (2-y)^2 = 25$ $29+4-4y+y^2 = 25$ $y^2 - 4y - 12 = 0$ $(y-6)(y+2) = 0$ $y = 6 \text{ or } y = -2$ $\therefore S(0; 6)$	<p>✓CA substitution</p> <p>✓CA standard form ✓CA factors</p> <p>✓CA coordinates of S</p> <p style="text-align: right;">(4)</p>
<p>4.3</p>	$m_{LS} = \frac{6-2}{0-(-3)}$ $m_{LS} = \frac{4}{3}$ $m_{SG} = -\frac{3}{4}$ $y = -\frac{3}{4}x + 6$ <p>At G: <math>0 = -\frac{3}{4}x + 6</math>      <b>OR</b>      <math>m_{SG} = \frac{6-0}{0-x} = -\frac{3}{4}</math></p> $\frac{3}{4}x = 6$ $x = 8$ $\therefore G(8; 0)$	<p>✓CA <math>m_{LS}</math></p> <p>✓CA <math>m_{SG} = -\frac{3}{4}</math></p> <p>✓CA substitute in equation of SG <b>OR</b> substitute in gradient of SG</p> <p>✓CA coordinates of G</p> <p style="text-align: right;">(4)</p>
<p>4.4</p>	$\tan \hat{S}GH = -\frac{3}{4}$ $\hat{S}GH = 143,13^\circ$ $\hat{O}SG + 90^\circ = 143,13^\circ \text{ [ext } \angle \text{ of } \triangle OSG]$ $\hat{O}SG = 53,13^\circ$ $\therefore \theta = 126,87^\circ$	<p>✓CA <math>\tan \hat{S}GH = -\frac{3}{4}</math></p> <p>✓CA <math>\hat{S}GH = 143,13^\circ</math></p> <p>✓CA <math>\hat{O}SG = 53,13^\circ</math></p> <p>✓CA answer</p> <p style="text-align: right;">(4)</p>

4.5	$\hat{L}\hat{S}\hat{G} = \hat{M}\hat{G}\hat{S} = 90^\circ$ $\therefore LS \parallel MG$ $LS = MG$ $\therefore LMGS$ is a parallelogram Also: $LMGS$ is a rectangle	$[\tan \perp \text{rad}]$ $[\text{co-int. } \angle\text{s suppl.}]$ $[\text{equal radii}]$ $[\text{one pair opp. sides = and } \parallel]$ $[\text{a parm. with an angle of } 90^\circ]$	$\checkmark$ A showing $LMGS$ is a parm. $\checkmark$ A showing $LMGS$ is a rectangle (2)
4.6	Using translation: Coordinates of M: $(5; -4)$ Equation of circle with centre M: $(x-5)^2 + (y+4)^2 = 25$ <b>OR</b> Midpoint of LG = $\left(\frac{-3+8}{2}; \frac{2+0}{2}\right) = \left(\frac{5}{2}; 1\right)$ The diagonals of $LMGS$ bisect since $LMGS$ is a rectangle. Let M be the point $(a; b)$ . $\frac{a+0}{2} = \frac{5}{2}$ and $\frac{b+6}{2} = 1$ $\therefore a = 5$ and $b = -4$ $M(5; -4)$ $(x-5)^2 + (y+4)^2 = 25$	$\checkmark$ CA $x$ -coordinate of M $\checkmark$ CA $y$ -coordinate of M $\checkmark$ CA answer <b>OR</b> $\checkmark$ CA $x$ -coordinate of M $\checkmark$ CA $y$ -coordinate of M $\checkmark$ CA answer (3)	(3) <b>[21]</b>

## QUESTION 5

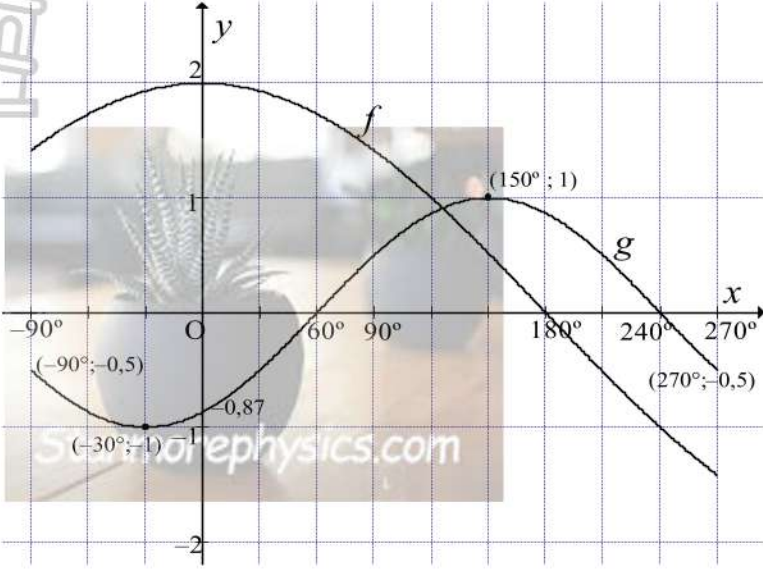
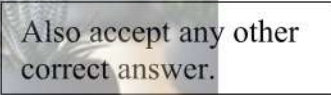
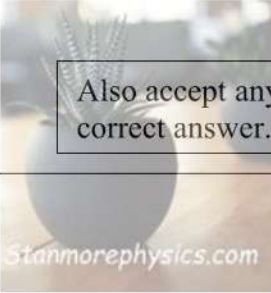
5.1.1	$\sin 2\alpha = \frac{\sqrt{5}}{3}$ $x^2 = 3^2 - (\sqrt{5})^2 \quad [\text{Pythagoras}]$ $x = -2$ $\tan 2\alpha = -\frac{\sqrt{5}}{2}$ 	✓A substitution ✓A value of $x$ ✓CA answer (3)
5.1.2	$\cos 2\alpha = -\frac{2}{3}$ $1 - 2\sin^2 \alpha = -\frac{2}{3}$ $2\sin^2 \alpha = \frac{5}{3}$ $\sin^2 \alpha = \frac{5}{6}$ $\sin \alpha = \sqrt{\frac{5}{6}}$	✓CA value of $\cos 2\alpha$ ✓CA double $\angle$ expansion (CA is on $-\frac{2}{3}$ ) ✓CA simplification ✓CA answer (4)
5.2	$\tan^2(180^\circ - \theta) \cdot \sin^2(90^\circ + \theta) - \cos(\theta - 180^\circ) \cdot \cos \theta$ $= (-\tan \theta)^2 \cdot (\cos \theta)^2 - (-\cos \theta) \cdot \cos \theta$ $= \tan^2 \theta \cdot \cos^2 \theta + \cos^2 \theta$ $= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta + \cos^2 \theta$ $= \sin^2 \theta + \cos^2 \theta$ $= 1$	✓A $-\tan \theta$ ✓A $\cos \theta$ ✓A $-\cos \theta$ ✓CA quotient identity ✓CA answer (5)

5.3.1	<p>LHS</p> $= \frac{\cos 5x \cos 3x + \sin 3x \sin 5x}{\cos x - \sin x}$ $= \frac{\cos 5x \cos 3x + \sin 5x \sin 3x}{\cos x - \sin x}$ $= \frac{\cos(5x - 3x)}{\cos x - \sin x}$ $= \frac{\cos 2x}{\cos x - \sin x}$ $= \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)(\cos x - \sin x)}$ $= \frac{\cos x - \sin x}{\cos x - \sin x}$ $= \cos x + \sin x$ <p>= RHS</p>	<p>✓A using compound <math>\angle</math> expansion</p> <p>✓A double <math>\angle</math> expansion</p> <p>✓A factorisation</p> <p>(3)</p>
5.3.2	<p>Undefined when: <math>\cos x - \sin x = 0</math></p> $\cos x = \sin x$ $\tan x = 1$ $x = 45^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ <p>For <math>x \in [-180^\circ; 180^\circ]</math>: <math>x = -135^\circ</math> or <math>x = 45^\circ</math></p>	<p>✓A <math>\cos x - \sin x = 0</math></p> <p>✓A <math>\tan x = 1</math></p> <p>✓CA answer ✓CA answer</p> <p>(4)</p>
<b>[19]</b>		

## QUESTION 6

6.1	$2 \cos 2x - \sin x + 1 = 0$ $2(1 - 2 \sin^2 x) - \sin x + 1 = 0$ $2 - 4 \sin^2 x - \sin x + 1 = 0$ $-4 \sin^2 x - \sin x + 3 = 0$ $4 \sin^2 x + \sin x - 3 = 0$ $(4 \sin x - 3)(\sin x + 1) = 0$ $\sin x = \frac{3}{4} \quad \text{or} \quad \sin x = -1$ $x = 48,59^\circ + k \cdot 360^\circ, \quad k \in \mathbb{Z}$ $\text{or } x = 131,41^\circ + k \cdot 360^\circ, \quad k \in \mathbb{Z}$ $\text{or } x = 270^\circ + k \cdot 360^\circ, \quad k \in \mathbb{Z}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <math>k \in \mathbb{Z} :</math> at least once.         </div>	<p>✓A double <math>\angle</math> identity</p> <p>✓CA standard form</p> <p>✓CA factorisation</p> <p>✓CA both equations</p> <p>✓CA <math>x = 48,59^\circ + k \cdot 360^\circ</math></p> <p>✓CA <math>x = 131,41^\circ + k \cdot 360^\circ</math></p> <p>✓CA or <math>x = 270^\circ + k \cdot 360^\circ</math></p> <p style="text-align: right;">(7)</p>
6.2	$\sqrt{3} \cos \beta + \sin \beta$ $= 2 \left( \frac{\sqrt{3}}{2} \cos \beta + \frac{1}{2} \sin \beta \right)$ $= 2(\sin 60^\circ \cos \beta + \cos 60^\circ \sin \beta)$ $= 2 \sin(60^\circ + \beta)$ $\therefore k = 2 \quad \text{and} \quad m = 60$	<p>✓A <math>2 \left( \frac{\sqrt{3}}{2} \cos \beta + \frac{1}{2} \sin \beta \right)</math></p> <p>✓A <math>\sin 60^\circ</math> and <math>\cos 60^\circ</math></p> <p>✓A <math>k = 2</math></p> <p>✓A <math>m = 60</math></p> <p style="text-align: right;">(4)</p>
<b>[11]</b>		

QUESTION 7

7.1	period = $720^\circ$	✓A answer (1)
7.2		✓A shape ✓A turning points ✓A intercepts with axes ✓A end points (4)
7.3	translation of $30^\circ$ to the right <b>OR</b> translation of $390^\circ$ to the right <b>OR</b> translation of $330^\circ$ to the left	 ✓A number of degree translated ✓A direction of translation (2)
7.4.1	$-90^\circ \leq x \leq 180^\circ$ <b>OR</b> $x \in [-90^\circ ; 180^\circ]$	 ✓A ✓A answer <b>OR</b> ✓A ✓A answer (2)
7.4.2	$60^\circ < x < 180^\circ$ or $240^\circ < x \leq 270^\circ$ <b>OR</b> $x \in (60^\circ ; 180^\circ)$ or $(240^\circ ; 270^\circ]$	✓CA answer ✓CA ✓CA answer <b>OR</b> ✓CA answer ✓CA ✓CA answer (3)
<b>[12]</b>		

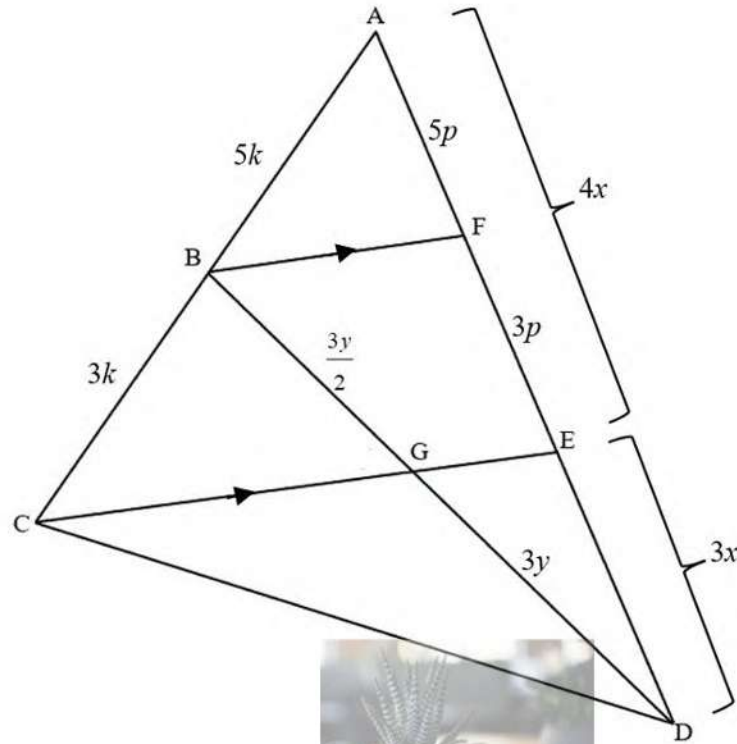
## QUESTION 8

8.1	$\frac{CB}{\sin 2x} = \frac{k}{\sin(90^\circ - x)}$ $CB = \frac{k \cdot \sin 2x}{\sin(90^\circ - x)}$ $CB = \frac{k \cdot 2 \sin x \cos x}{\cos x}$ $CB = 2k \sin x$	<p>✓A application of sine rule</p> <p>✓A double <math>\angle</math> expansion</p> <p>✓A co-ratio</p> <p>(3)</p>
8.2	$\cos x = \frac{CB}{HC}$ $\cos x = \frac{2k \sin x}{HC}$ $HC = \frac{2k \sin x}{\cos x}$ $HC = 2k \tan x$	<p>✓A trig ratio</p> <p>✓A <math>HC = \frac{2k \sin x}{\cos x}</math></p> <p>(2)</p>
8.3	$HC = 2(40) \tan 23^\circ$ $CD^2 = HC^2 + HD^2 - 2(HC)(HD) \cos \theta$ $40^2 = (2(40) \tan 23^\circ)^2 + 30^2 - 2(2(40) \tan 23^\circ)(30) \cos \theta$ $\cos \theta = \frac{[2(40) \tan 23^\circ]^2 + 30^2 - 40^2}{2[2(40) \tan 23^\circ](30)}$ $\cos \theta = 0,2224046183\dots$ $\theta = 77,15^\circ$	<p>✓A applying cosine rule</p> <p>✓A substitution</p> <p>✓CA value of <math>\cos \theta</math></p> <p>✓CA answer</p> <p>(4)</p>
		<b>[9]</b>

**QUESTION 9**

<p>9.1</p>	<p> <math>\hat{O}MB = 90^\circ</math>  <math>MB = AM</math>  <math>= 12 \text{ units}</math>  <math>OB^2 = OM^2 + MB^2</math>  <math>= 5^2 + 12^2</math>  <math>\therefore \text{radius} = OB = 13 \text{ units}</math> </p> <p><b>OR</b></p> <p> <math>\hat{O}MB = 90^\circ</math>  <math>\therefore \hat{O}MA = 90^\circ</math>  <math>OA^2 = OM^2 + AM^2</math>  <math>= 5^2 + 12^2</math>  <math>\therefore \text{radius} = OA = 13 \text{ units}</math> </p>	<p> <math>\checkmark S \checkmark R</math>  <math>\checkmark S/R</math>    <math>\checkmark CA \text{ answer}</math> </p> <p><b>OR</b></p> <p> <math>\checkmark S \checkmark R</math>  <math>\checkmark S/R</math>    <math>\checkmark CA \text{ answer}</math> </p> <p>(4)</p>
<p>9.2</p>	<p> <math>\hat{D} = \hat{C}</math>  <math>= 28^\circ</math>  <math>\hat{B}_1 = 180^\circ - (28^\circ + 37^\circ + 40^\circ)</math>  <math>= 75^\circ</math> </p>	<p> <math>\checkmark S \checkmark R</math>  <math>\checkmark R</math>  <math>\checkmark CA \text{ answer}</math> </p> <p>(4)</p>
<p>9.3</p>	<p> <math>\hat{A}CD = \hat{B}</math>  <math>= 125^\circ</math>  <math>\hat{C}_3 = 90^\circ</math>  <math>\hat{C}_2 = 125^\circ - 90^\circ = 35^\circ</math> </p> <p><b>OR</b></p> <p> <math>\hat{O}_2 = 2 \times \hat{B}</math>  <math>= 250^\circ</math>  <math>\hat{O}_1 = 360^\circ - \hat{O}_2</math>  <math>= 360^\circ - 250^\circ = 110^\circ</math>  <math>\hat{C}_1 = \hat{A}_1</math>  <math>\hat{C}_2 = \frac{125^\circ - 110^\circ}{2} = 35^\circ</math> </p>	<p> <math>\checkmark R</math>  <math>\checkmark S</math> size of <math>\hat{A}CD</math>  <math>\checkmark S/R</math>  <math>\checkmark CA \text{ answer}</math> </p> <p><b>OR</b></p> <p> <math>\checkmark R</math>  <math>\checkmark S</math> size of <math>\hat{O}_2</math>  <math>\checkmark S</math> size of <math>\hat{O}_1</math>    <math>\checkmark CA \text{ answer}</math> </p> <p>(4)</p>
<b>[12]</b>		

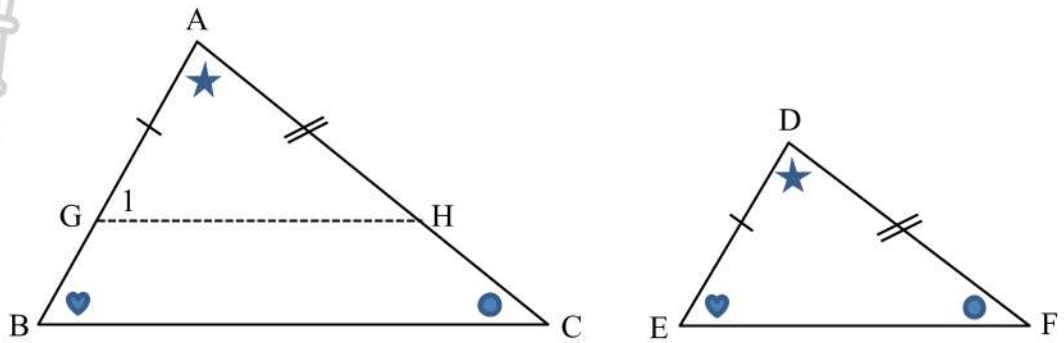
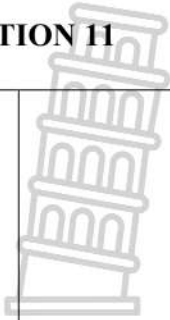
QUESTION 10



10.1	$\frac{AF}{FE} = \frac{5}{3}$ <p>[ prop. theorem, <math>BF \parallel CE</math> ]</p>	<p>✓S ✓R</p> <p>(2)</p>
10.2	<p>From sketch: <math>8p = 4x</math></p> $\therefore p = \frac{x}{2}$ <p>And: <math>FE = \frac{3x}{2} = 1,5x</math></p> $ED:FE = 3x:1,5x$ $= 2:1$ $\therefore DG:GB = 2:1$ <p>[ prop theorem, <math>BF \parallel CE</math> ]</p>	<p>✓S</p> <p>✓S</p> <p>✓S/R</p> <p>(3)</p>
10.3	$\frac{\text{Area of } \triangle BCG}{\text{Area of } \triangle DGC} = \frac{\frac{1}{2}(GB) \cdot \text{height}}{\frac{1}{2}(GD) \cdot \text{height}}$ $= \frac{GB}{GD}$ <p>[same height]</p> $= \frac{1}{2}$	<p>✓ substitution</p> <p>✓CA answer</p> <p>(2)</p>
<p>[7]</p>		

QUESTION 11

11.1



Mark off G and H on AB and AC respectively, such that  $AG = DE$  and  $AH = DF$ .

In  $\triangle AGH$  and  $\triangle DEF$ :

1.  $\hat{A} = \hat{D}$  [given]

2.  $AG = DE$  [construction]

3.  $AH = DF$  [construction]

$\therefore \triangle AGH \equiv \triangle DEF$  [s ;  $\angle$  ; s]

$\therefore \hat{G}_1 = \hat{E}$  [ $\equiv \Delta$ s]

$GH \parallel EF$  [corresponding  $\angle$ s are =]

In  $\triangle ABC$ :  $\frac{AG}{AB} = \frac{AH}{AC}$  [prop. theorem;  $GH \parallel EF$ ]

Also:  $\frac{DE}{AB} = \frac{DF}{AC}$  [ $AG = DE$ ;  $AH = DF$ ]

✓ construction

✓S ✓R

✓ S/R

✓S ✓R

(6)

11.2.1	In $\triangle PSV$ and $\triangle PRS$ : 1. $\hat{P}_1 = \hat{P}_1$ [common] 2. $\hat{S}_1 = \hat{R}_1$ [given; both = $x$ ] 3. $\hat{V}_1 = \hat{S}_R$ [sum of $\angle$ s of $\triangle$ ] $\therefore \triangle PSV \parallel \triangle PRS$ [ $\angle$ ; $\angle$ ; $\angle$ ]	✓S ✓S ✓R (3)
11.2.2	$\hat{R}_2 = 90^\circ$ [ $\angle$ in semi-circle] $\therefore \hat{S}RQ = 90^\circ + x$ $\hat{S}PT = 90^\circ - x$ [opp. $\angle$ s of a cyclic quad.] $\hat{T}_1 = 90^\circ$ [sum of $\angle$ s of $\triangle$ ] $= \hat{R}_2$ $\therefore VTQR$ is a cyclic quadrilateral [converse: ext. $\angle$ of cyclic quad]	✓S/R ✓S/R ✓S ✓R (4)
11.2.3	In $\triangle PRQ$ and $\triangle PTV$ : 1. $\hat{P}_2 = \hat{P}_2$ [common] 2. $\hat{R}_2 = \hat{T}_1$ [proved; both = $90^\circ$ ] 3. $\hat{Q} = \hat{V}_4$ [sum of $\angle$ s of $\triangle$ ] $\therefore \triangle PRQ \parallel \triangle PTV$ [ $\angle$ ; $\angle$ ; $\angle$ ]	✓S ✓S ✓R (3)
11.2.4	From 11.2.1: $\frac{PS}{PR} = \frac{PV}{PS}$ [ $\parallel \triangle$ s] $\therefore PS^2 = PR \cdot PV$ From 11.2.3: $\frac{PR}{PT} = \frac{PQ}{PV}$ [ $\parallel \triangle$ s] $\therefore PT \cdot PQ = PR \cdot PV$ $\therefore PT \cdot PQ = PS^2$ [both = $PR \cdot PV$ ] But: $ST^2 = PS^2 - PT^2$ [Pythagoras] $ST^2 = PT \cdot PQ - PT^2$ $ST^2 = PT(PQ - PT)$	✓S ✓S ✓S ✓S (5)
[21]		

TOTAL: 150 MARKS