education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

JUNE 2025

MARKS: 150

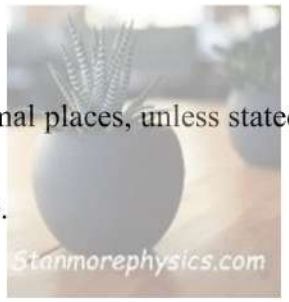
TIME: 3 HOURS

This question paper consists of 12 pages, 1 information sheet and an answer book is provided.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.



QUESTION 1

1.1 The monthly incomes of ten sales assistants are listed below.

R10 000	R14 500	R9 500	R15 000	R12 000
R13 800	R12 250	R14 000	R12 500	R13 000

1.1.1 Calculate the mean of the data. (2)

1.1.2 Calculate the standard deviation of the data. (1)

1.1.3 Calculate the inter-quartile range. (3)

1.1.4 The company undergoes degrading such that it leads to a decrease of R500 in each monthly salary of the ten employees. What effect will the decline have on the following:

(a) Mean (1)
(b) Standard deviation (1)

1.2 In a certain data set, the interval that is one standard deviation from the mean is [350; 380]. Calculate the mean and standard deviation for this data. (3)
[11]

QUESTION 2

The cumulative frequency graph shows the monthly salaries, in thousands of rands, of a sample of 500 shoppers in Johannesburg. The lowest salary is R5 000 and the highest salary is R80 000.

2.1 How many shoppers earned more than R40 000 per month? (1)

2.2 Use the cumulative frequency graph to draw a box-and-whisker plot for the data. (3)

2.3 Describe the skewness of the data. (1)

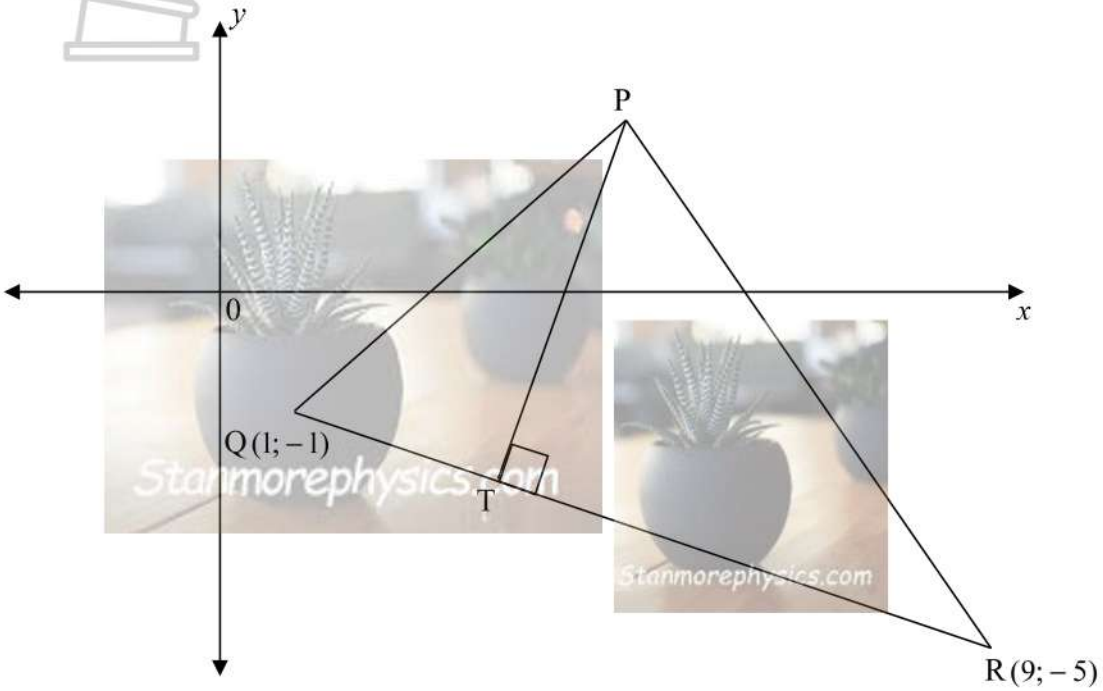
2.4 How high must a salary be to be classified as an outlier? (3)

2.5 Show that none of the salaries is low enough to be classified as an outlier. (1)

[9]

QUESTION 3

3.1 In the diagram, ΔPQR has vertices $P(5\,;2)$, Q$(1\,;-1)$ and R$(9\,;-5)$. PT is the perpendicular height of ΔPQR.



Determine:

3.1.1 the length of QR. (2)

3.1.2 the gradient of PR. (2)

3.1.3 the equation of PT. (3)

3.1.4 the coordinates of T. (5)

3.1.5 the magnitude of $Q\hat{R}P$. (4)

3.2 The equations of two circles with centres A and B are given below:

Circle A: $(x-2)^2+(y-3)^2=9$

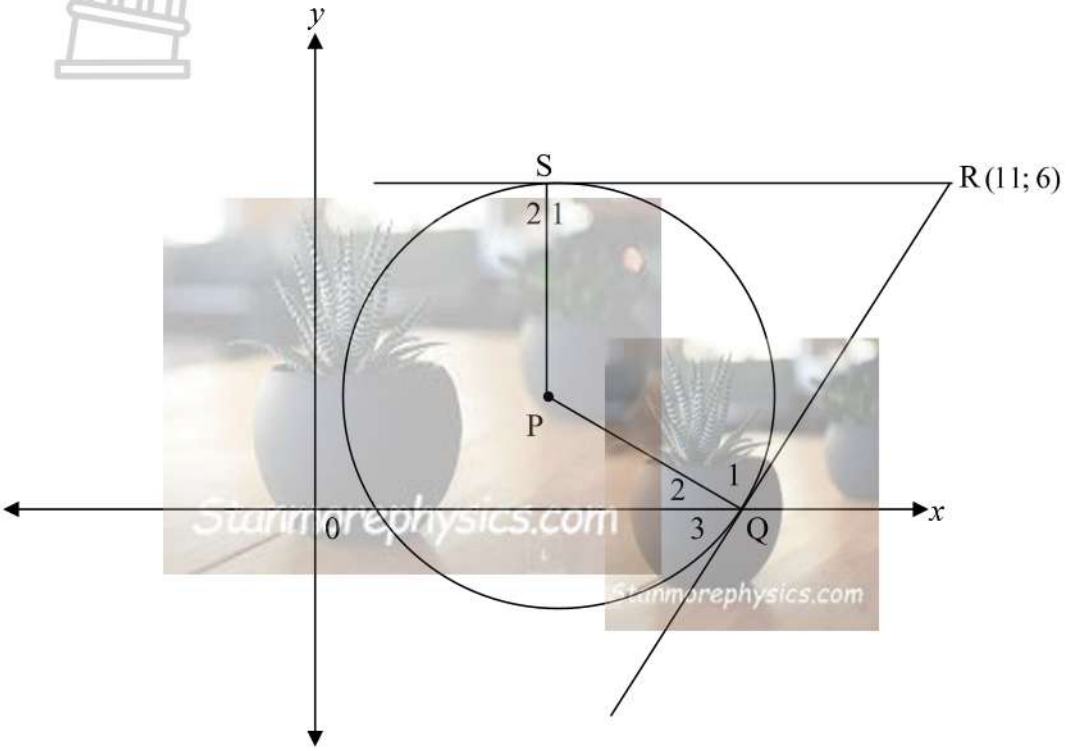
Circle B: $(x-1)^2+(y+1)^2=16$

Without solving for x and y, determine if the circles intersect each other in two points, touch each other, or do not intersect each other. Show all your calculations. (5)

[21]

QUESTION 4

In the diagram below, P is the centre of the circle. PS and PQ are radii. The tangents RS and RQ meet at R(11; 6).



4.1 Prove that PQRS is a cyclic quadrilateral. (5)

4.2 If the equation of the circle is $x^2 - 8x + y^2 - 4y = -12$, determine the coordinates of P, the centre of the circle, and the length of the radius PQ in surd form. (4)

4.3 If the circle cuts the x-axis at Q, determine the coordinates of Q. (5)

4.4 Determine, with reasons, the length of tangent RS in simplest surd form. (4)

[18]

QUESTION 5

5.1 Expand: $\cos(x+y) = \cdots$ (1)

5.2 Use the identity for $\cos(x+y)$ to derive a formula for $\sin(x-y)$ (3)

5.3 Determine without using a calculator:

$$\frac{\sin(x+10°)\cos(x+340°)+\cos(x+190°)\sin(x-20°)}{\sin 22{,}5°\cos 22{,}5°}$$ (6)

5.4 If $\cos 2\beta = \frac{a}{2}$ and 2β is an acute angle, express the following in terms of a:

5.4.1 $\sin 2\beta$ (2)

5.4.2 $\cos\beta$ (3)

5.5 Given: $f(x) = tan(x-45°)$

5.5.1 Determine the values of x for which $f(x)$ will be undefined if $x \in (-90°, 180°)$. (2)

5.5.2 Prove that: $\tan(x-45°) = \frac{\sin x - \cos x}{\sin x + \cos x}$ (5)

5.5.3 Hence determine without using a calculator, the value of $\tan 15°$ (4)

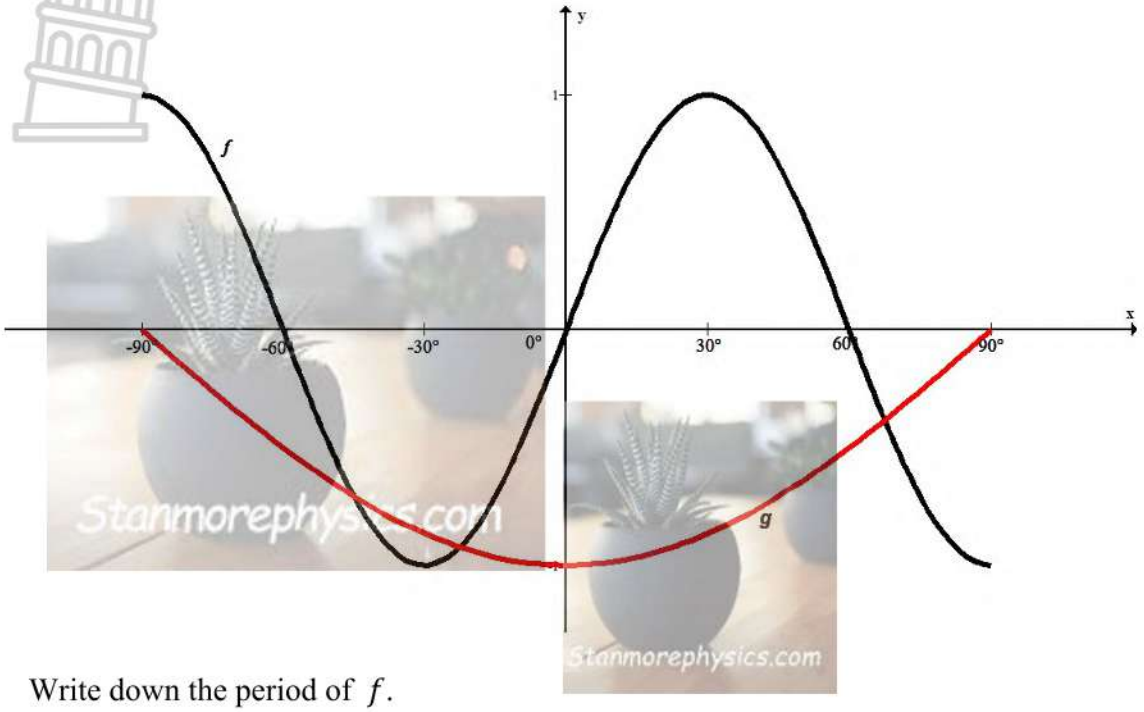
5.6 Determine, without using a calculator, the general solution of the equation:

$$\cos(90°+2x) = \tan^2 495° \cos(-210°)$$ (6)

[32]

QUESTION 6

In the diagram below, the graphs of $f(x) = \sin bx$ and $g(x) = -\cos x$ and are drawn in the interval $-90° \leq x \leq 90°$.



6.1 Write down the period of f. (1)

6.2 Write down the value of b. (1)

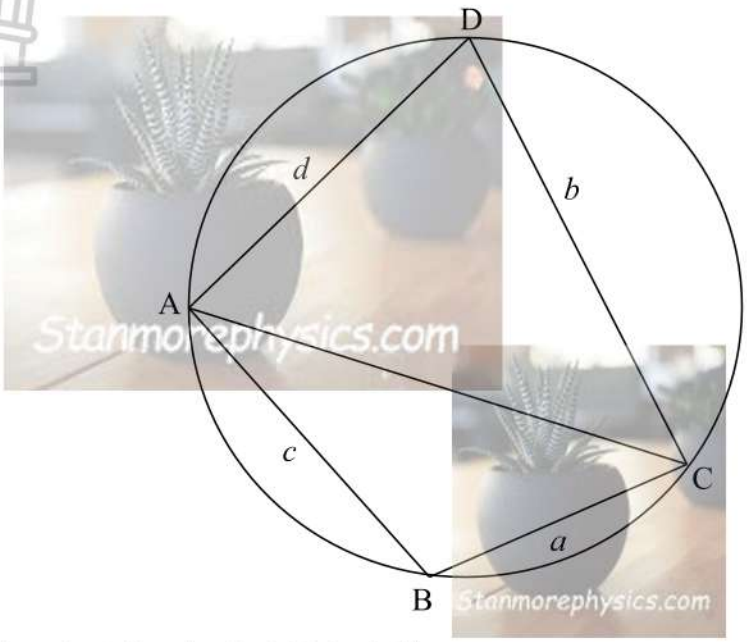
6.3 The general solutions of the equation $\sin bx = -\cos x$ are $x = 67,5° + k.90°$ or $x = 135° + k.180°$ where $k \in Z$. Determine the x-values of the points of intersection of f and g for the given domain. (3)

6.4 Determine the values of x in the given domain for which $\sin b\,x + \cos x < 0$ (4)

6.5 If the graph of $y = \sin b\,x$ is shifted $45°$ right and 2 units up, write down the equation of the new graph. (2)
[11]

QUESTION 7

ABCD is a cyclic quadrilateral in with AB$= c$, BC $= a$, CD $= b$ and AC $= \sqrt{a^2 + ac + c^2}$.



7.1 Complete the following in ΔABC: $AC^2 = \cdots$ (1)

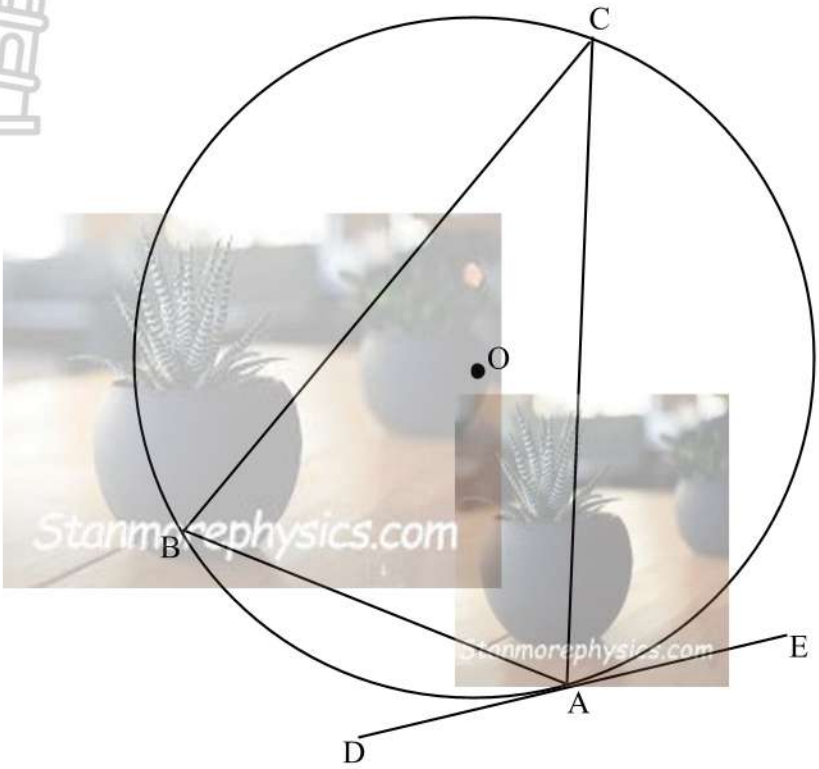
7.2 Calculate the numerical value of $\hat{B}$. (4)

7.3 Show that Area ABCD $= \frac{1}{2}(ac + bd)\sin B$. (5)

[10]

QUESTION 8

In the diagram below. O is the centre of the circle. A, B and C are points on the circle. DE is a tangent to the circle at A.

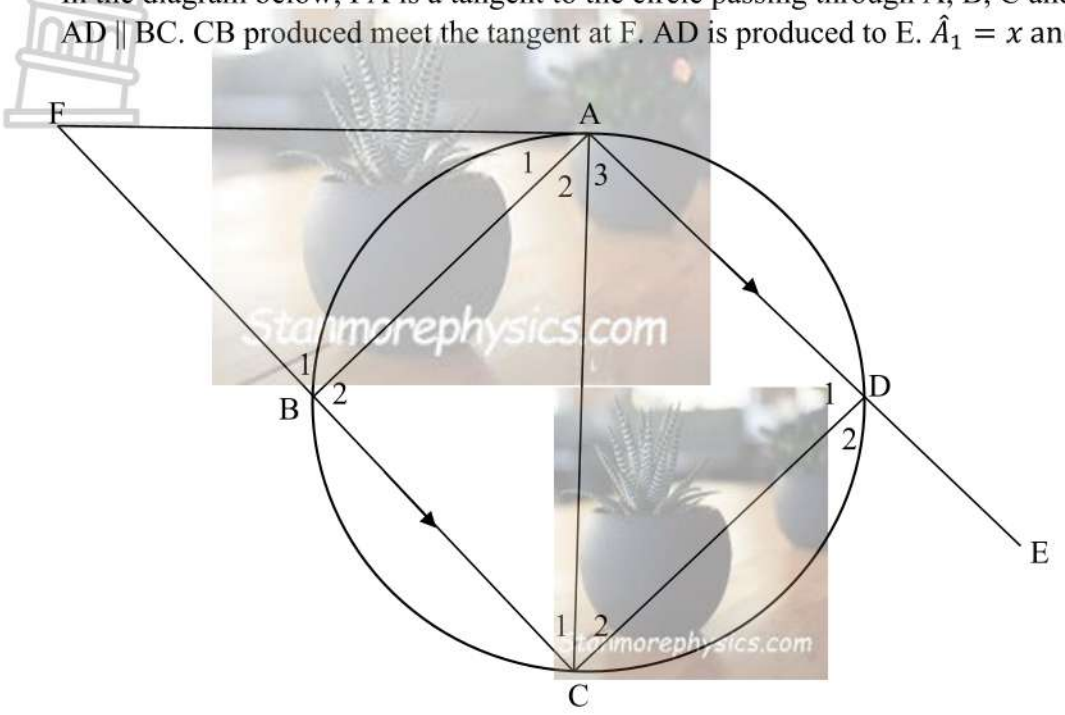


Prove the theorem which states that $C\hat{A}E = A\hat{B}C$ **[5]**

QUESTION 9

9.1 Complete the statement below:
Exterior angle of a cyclic quadrilateral is equal to the ……. (1)

9.2 In the diagram below, FA is a tangent to the circle passing through A, B, C and D.
AD || BC. CB produced meet the tangent at F. AD is produced to E. $\hat{A}_1 = x$ and $\hat{D}_2 = y$.



9.2.1 Determine, with reasons, two angles equal to x. (4)

9.2.2 Determine, with reasons, two angles equal to y. (4)

9.2.3 Show that $\hat{F} = \hat{C}_2$ (3)

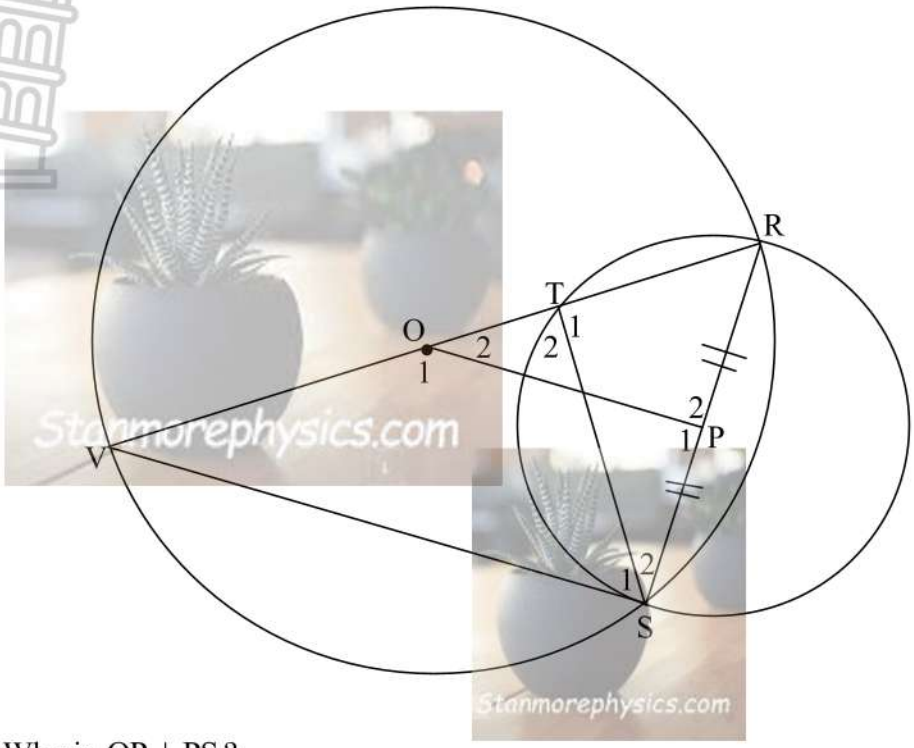
9.2.4 Express $\hat{A}_2$ in terms of x and y. (2)

9.2.5 Hence calculate the value of y for which AC is a tangent to the circle passing through the points A, B and F. (3)

[17]

QUESTION 10

In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.



10.1 Why is $OP \perp PS$? (1)

10.2 Prove that:

10.2.1 $VS = 2OP$ (3)

10.2.2 $\Delta ROP \,|||\, \Delta RVS$ (5)

10.2.3 $\Delta RVS \,|||\, \Delta RST$ (3)

10.2.4 $4PS^2 = RV.RT$ (4)

[16]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \; ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} \; ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In \; \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \; \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha \,.\cos\beta + \cos\alpha \,.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha \,.\cos\beta - \cos\alpha \,.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \,.\cos\beta - \sin\alpha \sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha \,.\cos\beta + \sin\alpha \sin\beta$$

$$\cos 2\alpha = cos^2\alpha - sin^2\alpha \qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= 1 - 2sin^2\alpha$$

$$= 2cos^2\alpha - 1$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_1 - x)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \; of \; B) = P(A) + P(B) - P(A \; en \; B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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NSC Answer Book
NSS-Antwoordeboek

SCHOOL NAME *SKOOL NAAM*	

NAME AND *SURNAME* *NAAM EN VAN*		**CLASS** *KLAAS*	
DATE *DATUM*			
SUBJECT NAME *VAKNAAM*	**MATHEMATICS P2/*WISKUNDE V2***		

MARKER/*NASIENER*					**MODERATOR'S INITIALS IN RELEVANT BLOCK/** *MODERATORPARAAF IN BETROKKE BLOKKIE*																			
Question *Vraag*	**Marks** *Punte*			**Marker's Code & Initials**	**Marks** *Punte*			**SM**	**Marks** *Punte*			**DCM**	**Marks** *Punte*			**CM**	**Marks** *Punte*			**IM**	**Marks** *Punte*			**EM**
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CONTROLLED AND CERTIFIED CORRECT **(SURNAME AND INITIALS OF EA)** ***GEKONTROLEER EN AS KORREK GESERTIFISEER (VAN EN VOORLETTERS VAN EA)***		READ INSTRUCTIONS ON THE NEXT PAGE. *LEES INSTRUKSIES OP DIE VOLGENDE BLADSY.*

This answer book consists of 18 pages.
Hierdie antwoordboek bestaan uit 18 bladsye.

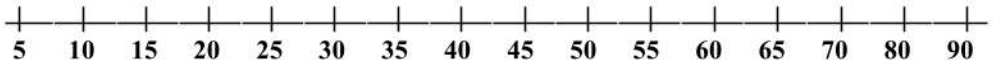
QUESTION 1 / VRAAG 1

R9 500	R10 000	R12 000	R12 250	R12 500
R13 000	R13 800	R14 000	R14 500	R15 000

	Solution / Oplossing	Marks / Punte
1.1.1		(2)
1.1.2		(1)
1.1.3		(3)
1.1.4(a)		(1)
(b)		(1)
1.2		(3)
		[11]

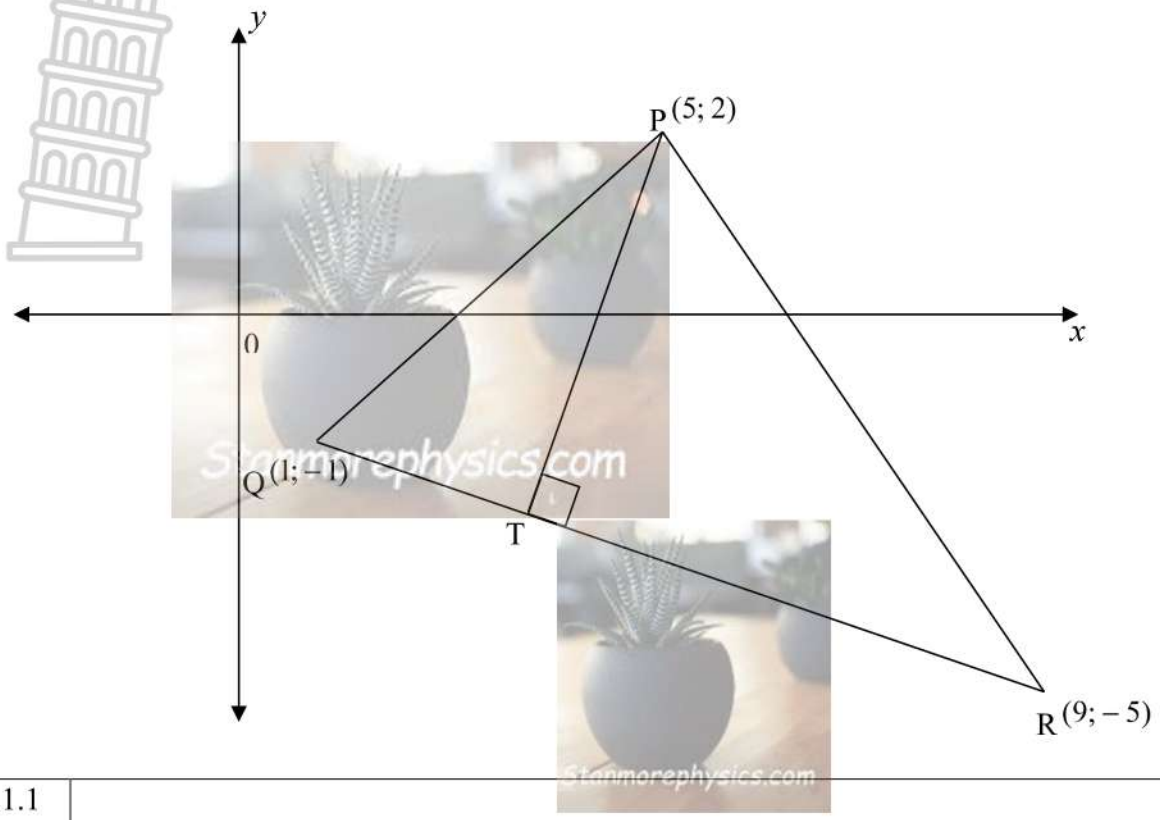
Extra space / Addisionele spasie

QUESTION 2 / VRAAG 2

2.1		(1)
2.2		(3)
2.3		(1)
2.4		(3)
2.5		(1)
		[9]

Extra space/ Addisionele spasie

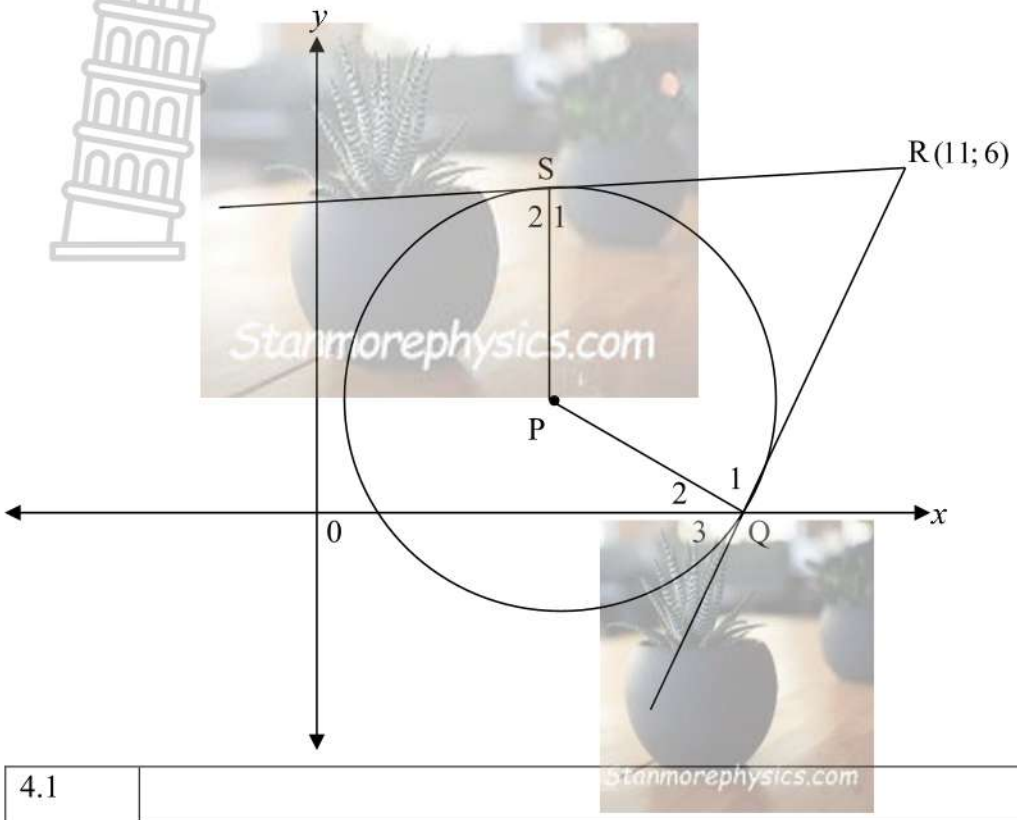
QUESTION 3 / VRAAG 3



3.1.1		(2)
3.1.2		(2)
3.1.3		(3)
3.1.4		

		(5)
3.1.5		(4)
3.2		(5)
		[21]

QUESTION 4 / VRAAG 4



4.1		(5)
4.2		(4)

4.3		(5)
4.4		(4)
		[18]

Extra space/ Addisionele spasie

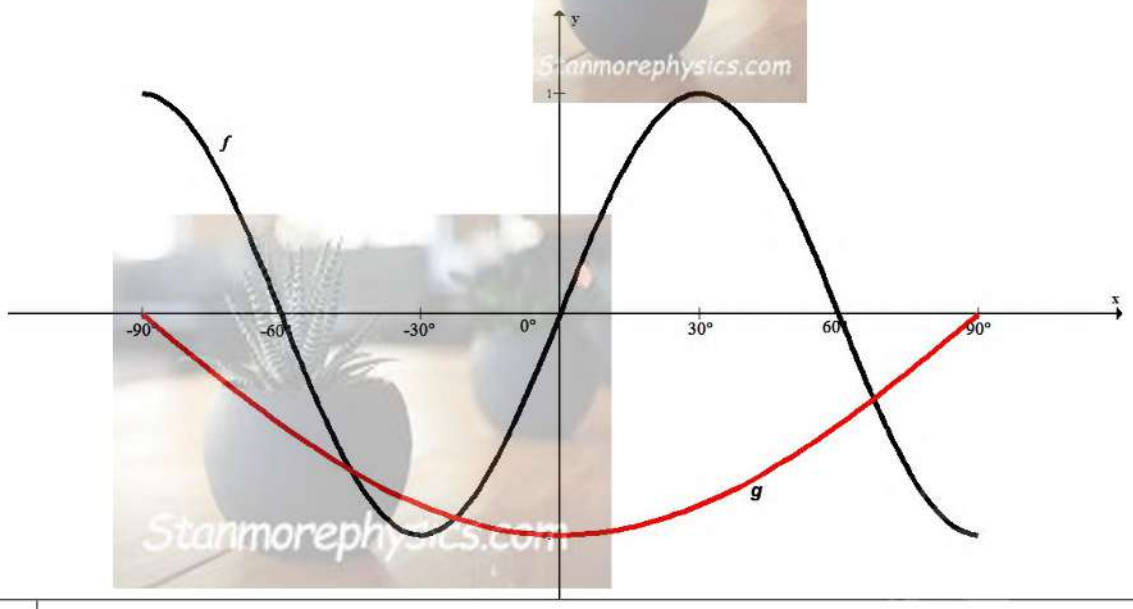
QUESTION 5 / VRAAG 5

5.1		(1)
5.2		(3)
5.3		(6)
5.4.1		(2)

5.4.2		(3)
5.5.1		(2)
5.5.2		(5)
5.5.3		(4)

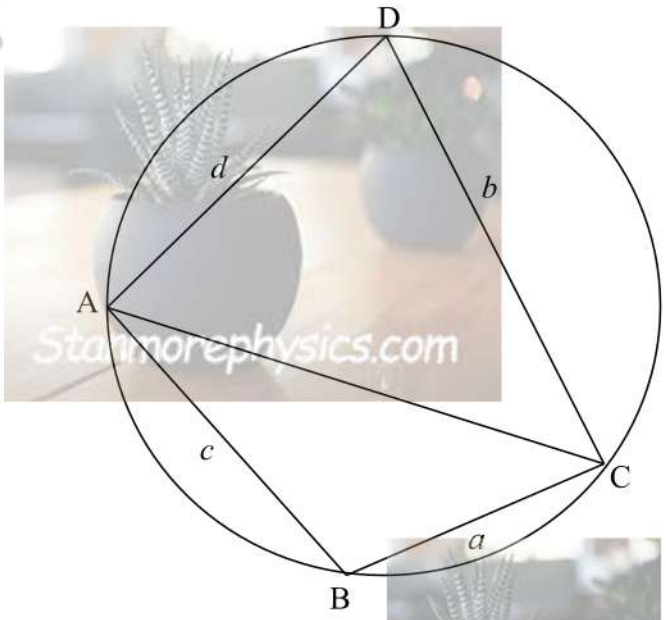
5.6		(6) **[32]**

QUESTION 6 / VRAAG 6



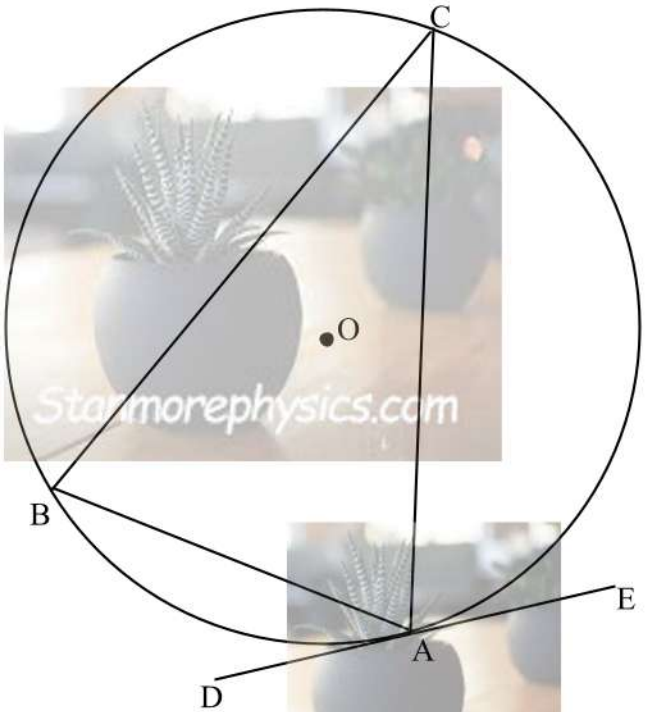
6.1		(1)
6.2		(1)
6.3		(3)
6.4		(4)
6.5		(2)
		[11]

QUESTION 7 / VRAAG 7



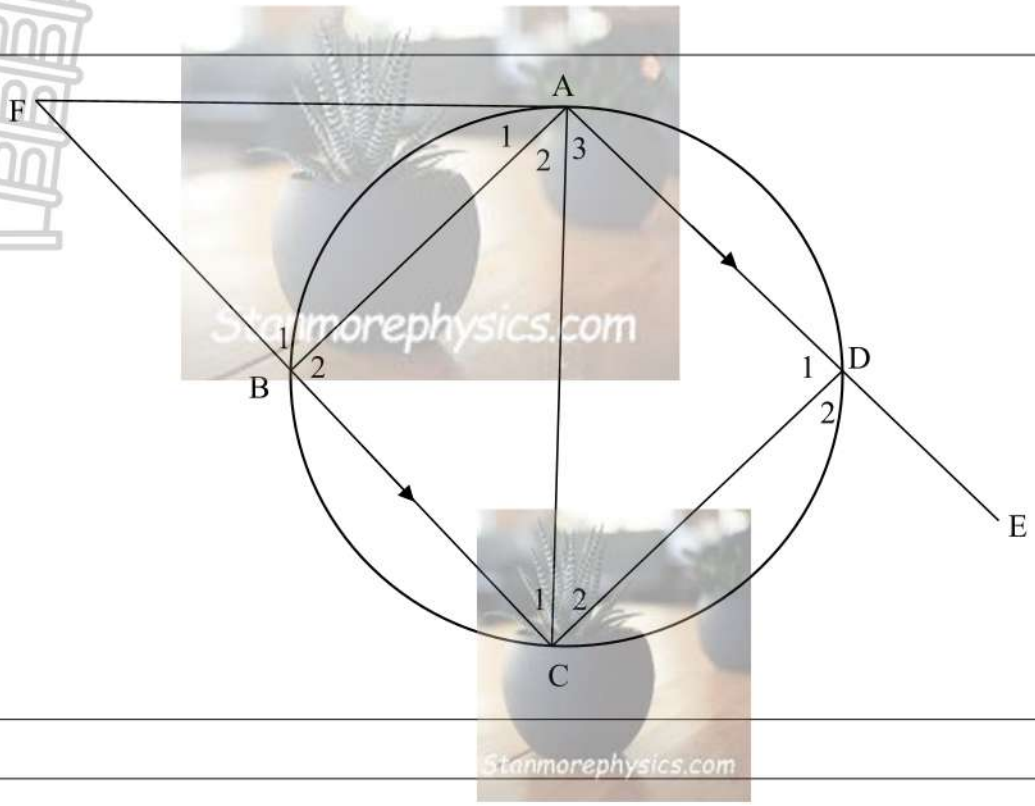
7.1		(1)
7.2		(4)
7.3		(5)
		[10]

QUESTION 8 / VRAAG 8



8.		(5)
		[5]

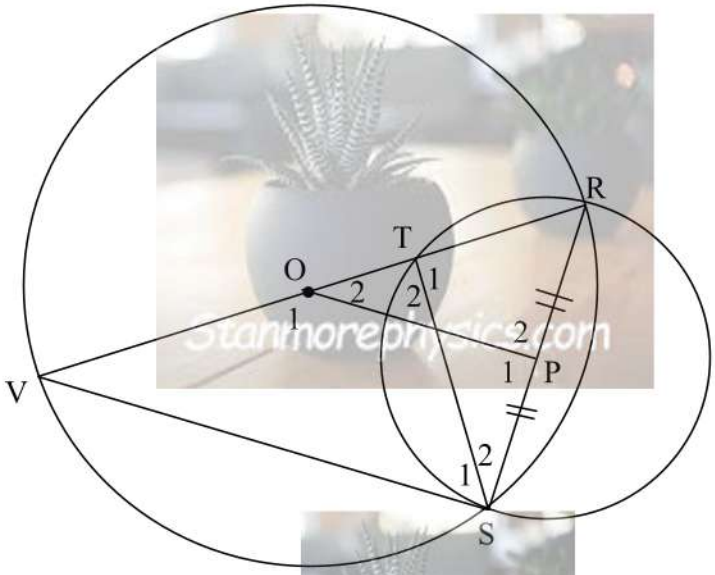
QUESTION 9 / VRAAG 9

9.1		(1)
9.2		
9.2.1		(4)
9.2.2		(4)
9.2.3		(3)
9.2.4		(2)

9.2.5		(3)
		[17]

Additional space / Addisionele spasie

QUESTION 10 / VRAAG 10



10.1		(1)
10.2.1		(3)
10.2.2		(5)

10.2.3		(3)
10.2.4		(4)
		[16]

Extra space / Addisionele spasie

ADDITIONAL SPACE / ADDISIONELE SPASIE

TOTAL / TOTAAL:	**150**

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REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE 12 / GRAAD 12

MATHEMATICS PAPER 2 / WISKUNDE V2

JUNE/JUNIE 2025

MARKING GUIDELINE / NASIENRIGLYN

MARKS/PUNTE: 150

This marking guideline consist of 13 pages.

Hierdie nasienriglyn bestaan uit 13 bladsye.

Pease turn over/ Blaai asb om

NOTE:

- If a candidate answers a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guideline.
- Assuming values/answer in order to solve a problem is unacceptable.

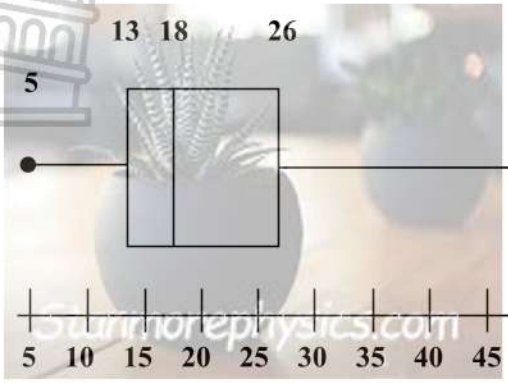
LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyn van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

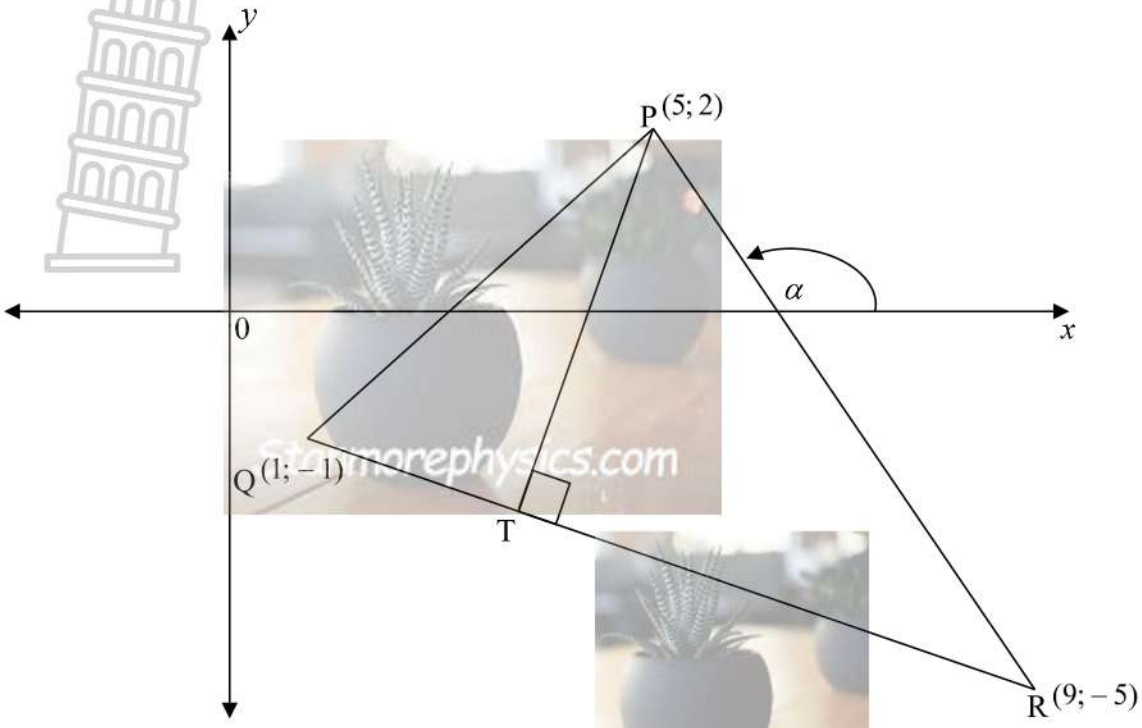
QUESTION/VRAAG 1

1.1.1	$\bar{x} = \frac{126\,550}{10}$ $= \text{R}12\,655$	✓ Method/Metode ✓Answer/Antwoord (2)
1.1.2	Standard deviation is 1 725,17 Standaardafwyking is 1 725,17	✓Answer/Antwoord (1)
1.1.3	R9 500 R10 000 R12 000 R12 250 R12 500 R13 000 R13 800 R14 000 R14 500 R15 000 $Q_1 = \text{R}12\,000$ $Q_3 = \text{R}14\,000$ $\text{IQR} = \text{R}2\,000$	✓ Q_1 / K_1 ✓ Q_3 / K_3 ✓ IQR / IKV (3)
1.1.4(a)	Decrease by R500 / Smaller by R500 / Lower by R500 Verminder met R500 / Word kleiner met R500 / Verlaag met R500	✓ Answer/Antwoord (1)
(b)	Remain the same / Bly dieselfde	✓ Answer/Antwoord (1)
1.2	$\bar{x} - \sigma = 350$ $\bar{x} + \sigma = 380$ $\therefore 2\bar{x} = 730$ $\therefore \bar{x} = 365$ $\therefore \sigma = 15$	✓ Substitution / Vervang ✓$\bar{x}$ ✓ σ (3)
		[11]

QUESTION/VRAAG 2

2.1	60	✓ Answer/Antwoord (1)
2.2		✓ Min: 5 and/en Max: 80 ✓ Q_1 / K_1: 13 ✓ Q_3 / K_3: 26 (3)
2.3	Positively skewed / skewed to the right. Positief skeef / Skeef na regs.	✓Answer/Antwoord (1)
2.4	$(Q_1 - 1,5 \times IQR\ ;\ Q_2 + 1,5 \times IQR)$ $= (13 - 1,5 \times 13\ ;\ 26 + 1,5 \times 13)$ $= (-6,5\ ;\ 45,5)$ A salary must be greater than R45 500.	✓IQR/IKV = 13 ✓$(-6,5\ ;\ 45,5)$ ✓ > R45 500 (3)
2.5	The lowest salary is R4 000, which lies in the interval $(-6,5\ ;\ 45,5)$. Therefore, none of the salaries is low enough to qualify as an outlier. Die kleinste salaris is R4 000, wat in die interval $(-6,5\ ;\ 45,5)$ lê. Daarom is die kleinste salaris nie laag genoeg om 'n uitskieter te wees nie.	✓ Answer/Antwoord. (1)
		[9]

QUESTION/VRAAG 3

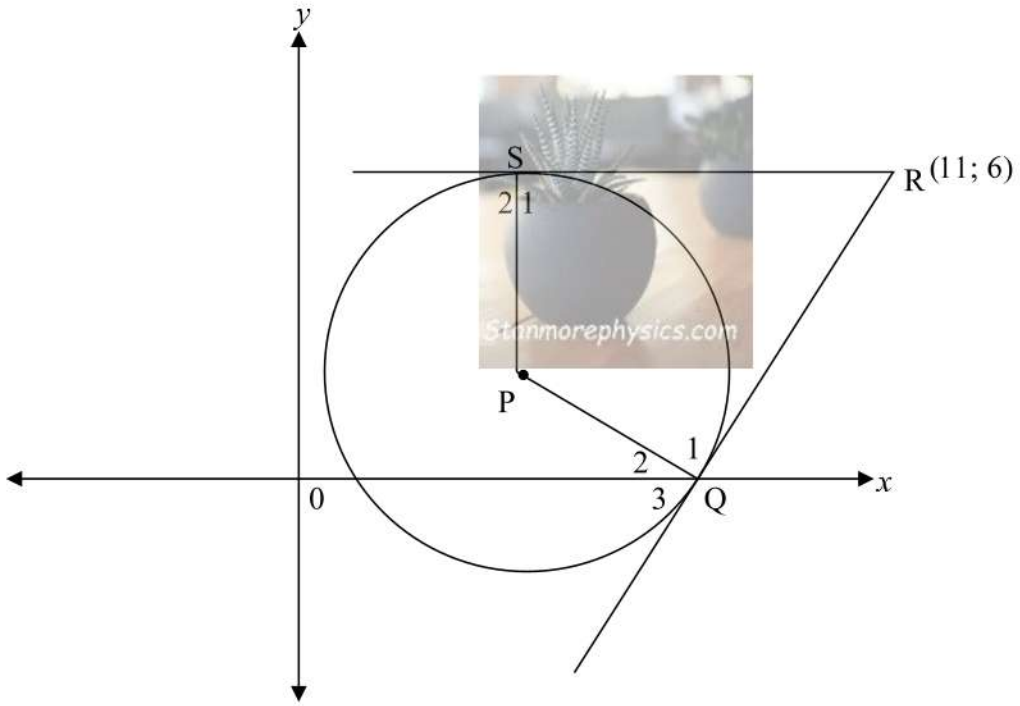


3.1.1	$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(1-9)^2 + (-1+5)^2}$ $= 4\sqrt{5} = 8{,}94$	✓ Substitution into the correct formula / Vervang in korrekte formule. ✓ Answer/Antwoord (2)
3.1.2	$m_{PR} = \frac{-5-2}{9-4}$ $= -\frac{7}{4}$	✓ Substitution into the correct formula / Vervang in korrekte formule. ✓ Answer/Antwoord (2)
3.1.3	$m_{QR} = \frac{-5-(-1)}{9-1} = \frac{-4}{8} = -\frac{1}{2}$ $m_{PT} = 2 \quad (PT \perp QR)$ $y - 2 = 2(x-5)$ $\therefore y - 2 = 2x - 10$ $\therefore y = 2x - 8$	✓ m_{QR} and gradient of PT / m_{QR} en helling van PT ✓corrrect substitution into formula/ Vervanging in korrekte formule ✓ $y = 2x - 8$ (3)

3.1.4	$m_{QR} = -\frac{1}{2}$ $y-(-1) = -\frac{1}{2}(x-1)$ $\therefore y+1 = -\frac{1}{2}x+\frac{1}{2}$ $\therefore y = -\frac{1}{2}x-\frac{1}{2}$ $\therefore -\frac{1}{2}x-\frac{1}{2} = 2x-8$ $\therefore -x-1 = 4x-16$ $\therefore -5x = -15$ $\therefore x = 3$ $\therefore y = 2(3)-8 = -2$ $\therefore$ T$(3;-2)$	✓ correct substitution into fomula for QR / korrekte vervanging in vergelyking vir QR ✓ $y = -\frac{1}{2}x-\frac{1}{2}$ ✓ $-\frac{1}{2}x-\frac{1}{2} = 2x-8$ ✓ $x = 3$ ✓ $y = -2$ (5)
3.1.5	PT $= 2\sqrt{5}$; $TR = 3\sqrt{5}$ $\tan T\hat{R}P = \frac{2\sqrt{5}}{3\sqrt{5}}$ $T\hat{R}P = 33{,}69°$ $\therefore Q\hat{R}P = 33{,}69°$ **OR / OF** RP $= \sqrt{65}$; PT $= 2\sqrt{5}$ $\sin T\hat{R}P = \frac{2\sqrt{5}}{\sqrt{65}}$ $T\hat{R}P = 33{,}69°$ $\therefore Q\hat{R}P = 33{,}69°$ **OR / OF** $\cos T\hat{R}P = \frac{3\sqrt{5}}{\sqrt{65}}$ $T\hat{R}P = 33{,}69°$ $\therefore Q\hat{R}P = 33{,}69°$	✓ PT ✓ TR ✓ Substitution / Vervang ✓ Answer/Antwoord (4) **OR / OF** ✓ PT ✓ RP ✓ Substitution /Vervang ✓ Answer/Antwoord (4) **OR / OF** ✓ TR ✓ RP ✓ Substitution / Vervang ✓ Answer/Antwoord (4)
3.2	$r_A = 3$ and A$(2;3)$ $r_B = 4$ and B$(1;-1)$ Sum of radii / Som van radiuses = 4 + 3 = 7	✓ radius A and/en B ✓ sum of radii / som van radiusse

	Distance between A and B: $AB^2 = (1-2)^2 + (-1-3)^2 = 17$ $AB = \sqrt{17} = 4{,}1231...$ $r_A + r_B = 7 > \sqrt{17}$ The circles intersect each other.	✓ subst. into distance formula / vervang in afstandformule ✓ $AB = \sqrt{17}$ ✓ correct conclusion / korrekte gevolgtrekking (5)
		[21]

QUESTION/VRAAG 4



4.1	$\hat{S}_1 = 90°$ [tan ⊥ radius/ raaklyn loodreg op radius] $\hat{Q}_1 = 90°$ [tan ⊥ radius/ raaklyn loodreg op radius] $\therefore \hat{S}_1 + \hat{Q}_1 = 180°$ ∴ PQSR is a cyclic quad. [opp ∠s of quad suppl./ teenoorgestelde hoeke van vierhoek aanvulled]	✓ $\hat{S}_1 = 90°$ ✓ R ✓ $\hat{Q}_1 = 90°$ ✓ $\hat{S}_1 + \hat{Q}_1 = 180°$ ✓ R (5)
4.2	$x^2 - 8x + y^2 - 4y = -12$ $x^2 - 8x + 16 + y^2 - 4y + 4 = -12 + 20$ $(x-4)^2 + (y-2)^2 = 8$ $P(4;2)$; $PQ = \sqrt{8} = 2\sqrt{2}$ *units*	✓ Completing a square / Voltooi die vierkant ✓ Simplification/Vereenvoudig ✓ $P(4;2)$ ✓ $PQ = \sqrt{8} = 2\sqrt{2}$ (4)
4.3	$(x-4)^2 + (y-2)^2 = 8$	

	$(x-4)^2+(0-2)^2=8$ $(x-4)^2=4$ $x-4=\pm 2$ $x=2+4 \quad or \quad x=-2+4$ $x=6 \quad or \quad x=2$ $Q(6;0)$	✓ Substitution / Vervanging ✓ Simplification/Vereenvoudig ✓ Simplification/Vereenvoudig ✓ $x-values$ ✓ $P(6;0)$ (5)
4.4	$RS=RQ$ $[tangents\ from\ the\ same\ point]$ [raaklyne uit dieselfde punt] $RQ=\sqrt{(11-6)^2+(6-0)^2}$ $RQ=\sqrt{25+36}$ $RQ=\sqrt{61}$ = RS	✓ S ✓ R ✓ Substitution / Vervanging ✓ RS (4)
		[18]

QUESTION/VRAAG 5

5.1.	$cos\,(x+y)=cos\,x\,cos\,y-sin\,x\,sin\,y$	✓ Answer / Antwoord (1)
5.2	$\sin(x+y)=\cos[90^0-(x-y)]$ $=\cos[(90^0-x)+y]$ $=\cos(90^0-x)\cos y-\sin(90^0-x)\sin y$ $=\sin x\cos y-\cos x\sin y$	✓$\cos[90^0-(x-y)]$ ✓ Expansion / Uitbreiding ✓$\sin x\cos y-\cos x\sin y$ (3)
5.3	$\frac{\sin(x+10^0)\cos(x-20^0)-\cos(x+10^0)\sin(x-20^0)}{\frac{1}{2}(2\sin 22{,}5^0\cos 22{,}5^0)}$ $=\frac{\sin[(x+10^0)-(x-20^0)]}{\frac{1}{2}\sin 45^0}$ $=\frac{\sin 30^0}{\frac{1}{2}\sin 45^0}$ $=\frac{\frac{1}{2}}{\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)}=\sqrt{2}$	✓$\cos(x-20^0)$ ✓$\cos(x+10^0)$ ✓ $\frac{1}{2}(2\sin 22{,}5^0\cos 22{,}5^0)$ ✓$\frac{\sin[(x+10^0)-(x-20^0)]}{\frac{1}{2}\sin 45^0}$ ✓$\frac{\frac{1}{2}}{\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)}$ ✓ $\sqrt{2}$ (6)
5.4.1	$\cos 2\beta=\frac{a}{2}$ $\sin^2 2\beta=1-\cos^2 2\beta$ $\therefore \sin^2 2\beta=1-\left(\frac{a}{2}\right)^2$ $\therefore \sin^2 2\beta=\frac{4-a^2}{4}$ $\therefore \sin 2\beta=\frac{\sqrt{4-a^2}}{2}$	✓ $\sin^2 2\beta=1-\cos^2 2\beta$ ✓ $\sin 2\beta=\frac{\sqrt{4-a^2}}{2}$ (2)

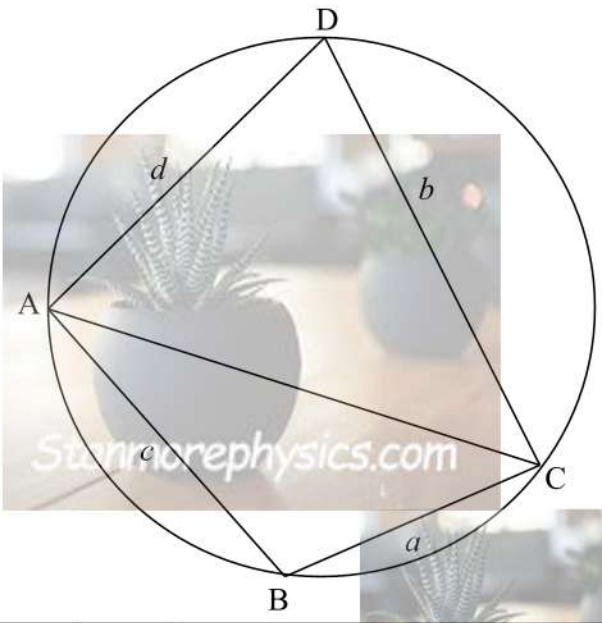
5.4.2	$\cos 2\beta = \frac{a}{2}$ $2\cos^2\beta - 1 = \frac{a}{2}$ $\therefore 2\cos^2\beta = \frac{a}{2} + 1$ $\therefore 2\cos^2\beta = \frac{a+2}{2}$ $\therefore \cos^2\beta = \frac{a+2}{4}$ $\therefore \cos\beta = \frac{\sqrt{a+2}}{2}$	✓ $2\cos^2\beta - 1$ ✓ $\cos^2\beta = \frac{a+2}{4}$ ✓ square rooting / vierkantswortel (3)
5.5.1	$x = -45°$ or/of $x = 135°$	✓ $-45°$ ✓ 135^0 (2)
5.5.2	$LHS = \tan(x-45°)$ $= \frac{\sin(x-45°)}{\cos(x-45°)}$ $= \frac{\sin x\cos 45° - \cos x\sin 45°}{\cos x\cos 45° + \sin x\sin 45°}$ $= \frac{(\sin x)\left(\frac{\sqrt{2}}{2}\right) - (\cos x)\left(\frac{\sqrt{2}}{2}\right)}{(\cos x)\left(\frac{\sqrt{2}}{2}\right) + (\sin x)\left(\frac{\sqrt{2}}{2}\right)}$ $= \frac{\left(\frac{\sqrt{2}}{2}\right)(\sin x - \cos x)}{\left(\frac{\sqrt{2}}{2}\right)(\cos x + \sin x)} = \frac{\sin x - \cos x}{\sin x + \cos x}$	✓ $\frac{\sin(x-45°)}{\cos(x-45°)}$ ✓ expanding top / brei teller uit ✓ expanding bottom / brei noemer uit ✓ $\frac{\sqrt{2}}{2}$ ✓ Common factor and answer/Gemeenskaplike faktor en antwoord (5)
5.5.3	$\tan 15°$ $= \tan(60° - 45°)$ $= \frac{\sin 60° - \cos 60°}{\sin 60° + \cos 60°}$ $= \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	✓ $\tan(60° - 45°)$ ✓ $\frac{\sin 60° - \cos 60°}{\sin 60° + \cos 60°}$ ✓ $\frac{\sqrt{3}}{2}$; $\frac{1}{2}$ ✓ simplifying (4)

5.6	$\cos(90° + 2x) = \tan^2 495°.\cos(-210°)$ $\therefore -\sin 2x = \tan^2 135°.\cos 210°$ $\therefore -\sin 2x = (-\tan 45°)^2(-\cos 30°)$ $\therefore -\sin 2x = (1)\left(-\frac{\sqrt{3}}{2}\right)$ $\therefore -\sin 2x = -\frac{\sqrt{3}}{2}$ $\therefore \sin 2x = \frac{\sqrt{3}}{2}$ $\therefore 2x = 60° + k.360°, k \in Z$ $\therefore x = 30° + k.180°$ or/of $2x = 120° + k.360°$ $\therefore x = 60° + k.180°, k \in \square$	✓ $-\sin 2x$ ✓ $\tan^2 135°$ ✓ $\tan^2 135° = 1$ ✓ $-\cos(30°) = -\frac{\sqrt{3}}{2}$ ✓ $x = 30° + k.180°$ ✓ $x = 60° + k.180°$ (6) Deduct 1 mark if $k \epsilon Z$ is omitted.
		[32]

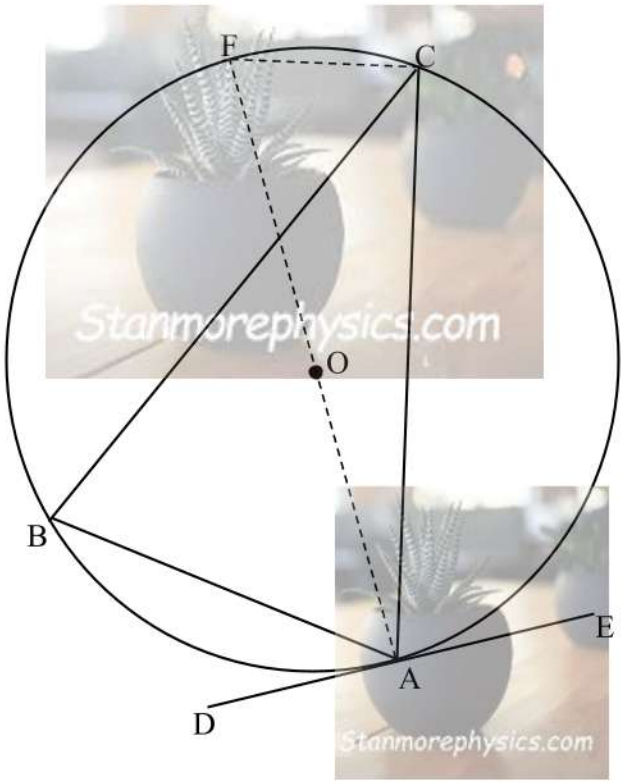
QUESTION/VRAAG 6

6.1	$120°$	✓ $120°$ (1)
6.2	$b = 3$	✓ $b = 3$ (1)
6.3	$x = -45°; x = -22,5°$ or $x = 67,5°$	✓ $x = -45°$ ✓ $x = -22,5°$ ✓ $x = 67,5°$ (3)
6.4	$-45° < x < -22,5°$ or $67,5° < x \leq 90°$	✓ first interval critical values / eerste interval kritieke punte ✓ notation / notasie ✓ 2nd interval critical values / 2e interval kritieke punte ✓ notation / notasie (4)
6.5	$y = \sin[3(x - 45°)] + 2$ $= \sin(3x - 135°) + 2$	✓ $3(x - 45°)$ ✓ $+2$ (2)
		[11]

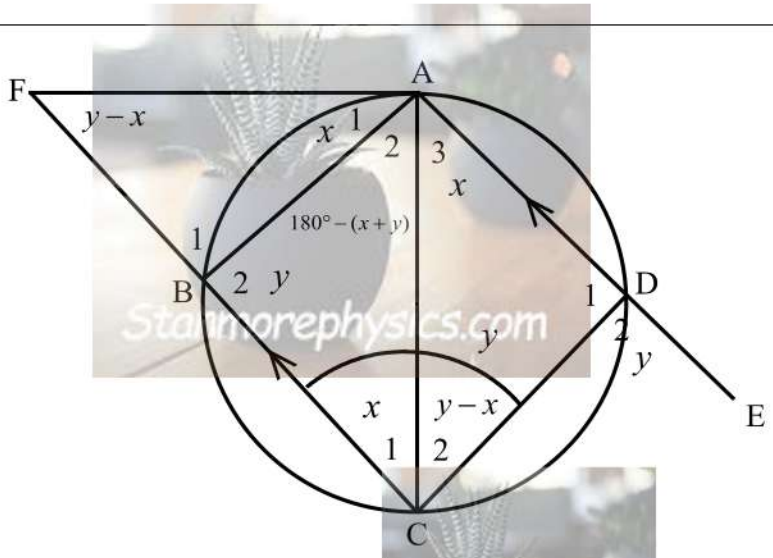
QUESTION/VRAAG 7

		
7.1	$AC^2 = AB^2 + BC^2 - 2(AB)(BC)CosB$	✓ Answer/Antwoord (1)
7.2	$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B$ $\therefore (\sqrt{a^2+ac+c^2})^2 = c^2 + a^2 - 2ac\cos B$ $\therefore \quad a^2 + ac + c^2 = a^2 + c^2 - 2ac\cos B$ $\therefore \quad ac = -2ac\cos B$ $\therefore \quad \cos B = -\frac{1}{2}$ $\therefore \quad \hat{B} = 120°$	✓ Substitution / Vervanging ✓ $(\sqrt{a^2+ac+c^2})^2$ $= a^2 + ac + c^2$ ✓ $\cos B = -\frac{1}{2}$ ✓ $\hat{B} = 120°$ (4)
7.3	Area ABCD $= \frac{1}{2}ac\sin B + \frac{1}{2}bd\sin D$ $= \frac{1}{2}ac\sin B + \frac{1}{2}bd\sin(180° - B)$ $= \frac{1}{2}ac\sin B + \frac{1}{2}bd\sin B$ $= \frac{1}{2}(\sin B)(ac + bd)$ $= \frac{1}{2}(ac + bd)\sin B$	✓ $\frac{1}{2}ac\sin B$ ✓ $\frac{1}{2}bd\sin D$ ✓ $\hat{D} = 180° - \hat{B}$ ✓ $\frac{1}{2}bd\sin B$ ✓ $\frac{1}{2}(ac + bd)\sin B$ (5) **[10]**

QUESTION/VRAAG 8

		
	R.T.P: $C\hat{A}E = A\hat{B}C$ Construction/Konstruksie: Draw a diameter AF then join FC / Teken middellyn AF en dan FC. $O\hat{A}C + C\hat{A}E = 90°$ [tan perpendicular radius/raaklyn loodreg op radius] $F\hat{C}A = 90°$ [angle in semi-circle/ angle subtended by diameter] [hoek in halwe sirkel/hoek onderspan deur middellyn] $O\hat{F}C + O\hat{A}C = 90°$ [sum of angles in Δ / som binnehoeke van Δ] $O\hat{A}C + C\hat{A}E = O\hat{F}C + O\hat{A}C$ [both / beide = 90°] $\therefore C\hat{A}E = O\hat{F}C$ But/maar $O\hat{F}C + A\hat{B}C$ [angles in the same segment / hoeke in dieselfde sierkelsegment] Then/dus $C\hat{A}E = A\hat{B}C$	✓ Construction / Konstruksie ✓ S/R ✓ S/R ✓ S ✓ R (5)
		[5]

QUESTION/VRAAG 9

9.1	Opposite interior angle / teenoorstaande binnehoek		✓ Answer /Antwoord (1)
			
9.2.1	$\hat{C}_1 = x$ $\hat{A}_3 = x$	tan-chord theorem /raaklyn koord stelling alt /verw ∠s ; AD || FC	✓ S ✓ R ✓ S ✓ R (4)
9.2.2	$\hat{B}_2 = y$ $\hat{C}_1 + \hat{C}_2 = y$	ext ∠ of cyclic quad / buite ∠ koordevierhoek alt / verw ∠s ; AD || FC	✓ S ✓ R ✓ S ✓ R (4)
9.2.3	$\hat{C}_1 = x$ $\hat{C}_1 + \hat{C}_2 = y$ $\therefore \hat{C}_2 = y - x$ $\hat{B}_2 = y$ $\hat{A}_1 = x$ $\therefore \hat{F} = y - x$ $\therefore \hat{F} = \hat{C}_2$	proved / bewys proved / bewys proved / bewys given / gegee ext ∠ of Δ /buite ∠ van Δ	 ✓ S ✓ S + R ✓ S (3)
9.2.4	$\hat{A}_2 = 180° - (x+y)$	int ∠s of Δ / binne ∠e van Δ	✓ S ✓ R (2)
9.2.5	If AC is a tangent to the circle passing through the points A, B and F, then $\hat{A}_2 = \hat{F}$. $\therefore 180° - (x+y) = y - x$ $\therefore \quad 180° - x - y = y - x$ $\therefore \qquad -2y = -180°$ $\therefore \qquad y = 90°$	tan-chord theorem /raaklyn koord stelling	✓ $\hat{A}_2 = \hat{F}$ ✓ $180° - (x+y) = y - x$ ✓ $y = 90°$ (3)
			[17]

QUESTION/VRAAG 10

10.1	Line from centre circle to midpoint of chord / Lyn van middelpunt van sirkel na middelpunt van koord	✓ Answer/Antwoord (1)
10.2.1	$VO = OR$ equal radii / gelyke radiusse $SP = PS$ given / gegee $\therefore VS = 2OP$ midpoint theorem/middelpuntstelling	✓ S ✓ R ✓ R (3)
10.2.2	In ΔROP and/en ΔRVS: (a) $\hat{R} = \hat{R}$ common/gemeenskaplik (b) $\hat{P}_2 = 90°$ proved/bewys $\hat{S} = 90°$ $\angle$ in semi-circle/$\angle$ in halfsirkel $\therefore \hat{P}_2 = \hat{S}$ (c) $\hat{O}_2 = \hat{V}$ int $\angle$s of Δ / binne $\angle$e van Δ $\therefore \Delta ROP \| \| \| \Delta RVS$ AAA / HHH	✓ $\hat{R} = \hat{R}$ ✓ $\hat{S} = 90°$ ✓ R ✓ $\hat{P}_2 = \hat{S}$ ✓ $\hat{O}_2 = \hat{V}$ (5) Deduct 1 mark for any incorrect Reason/Trek 1 punt af vir enige verkeerde rede OR / OF Reason omitted/Redes uitgelaat
10.2.3	In ΔRVS and/en ΔRST: (a) $\hat{R} = \hat{R}$ common / gemeenskaplik (b) $\hat{T}_1 = 90°$ $\angle$ in semi-circle/$\angle$ in halfsirkel $\hat{S} = 90°$ $\angle$ in semi-circle/$\angle$ in halfsirkel $\therefore \hat{S} = \hat{T}_1$ (c) $\hat{V} = \hat{S}_2$ int $\angle$s of Δ / binne $\angle$e van Δ $\therefore \Delta RVS \| \| \| \Delta RST$ AAA / HHH	✓ $\hat{T}_1 = 90°$; $\hat{S} = 90°$ ✓ $\hat{S} = \hat{T}_1$ ✓ $\hat{V} = \hat{S}_2$ (3) Deduct 1 mark for any incorrect reason/ Trek 1 punt af vir enige verkeerde rede OR / OF Reason omitted/Redes uitgelaat
10.2.4	$\frac{RV}{RS} = \frac{VS}{ST} = \frac{RS}{RT}$ since $\Delta RVS \| \| \| \Delta RST$ $\therefore RV.RT = RS^2$ $\therefore RS^2 = RV.RT$ But $RS = 2PS$ $\therefore (2PS)^2 = RV.RT$ $\therefore 4PS^2 = RV.RT$	✓ $\frac{RV}{RS} = \frac{VS}{ST} = \frac{RS}{RT}$ ✓ $RS^2 = RV.RT$ ✓ $RS = 2PS$ ✓ $(2PS)^2 = RV.RT$ (4) **[16]**
		TOTAL/TOTAAL : 150