KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

WINTER REVISION DOCUMENT

MATHEMATICS

GRADE 12

2026

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TOPICS		PAGE NUMBERS

TOPIC 1. Functions and inverses

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Straight Line	Parabola	Hyperbola	Exponential
$y = mx + c$ m … **gradient** and c … **y-intercept**	$y = a(x+p)^2 + q$ **Axis of symmetry with equation** $x = -p$ **Maximum** or **minimum value** $(-p;q)$ Turning point	$y = \frac{a}{x+p} + q$ **Vertical asymptote with equation** $x = -p$ **Horizontal asymptote with equation** $y = q$	$y = a.b^{x+p} + q$ $b > 0$ **and** $b \neq 1$ **Horizontal asymptote with equation** $y = q$
$m < 0$ **… graph is decreasing** $m > 0$ **… graph is increasing**	$a < 0$ **… graph faces downwards (concave down) and has a minimum turning point** $a > 0$ **… graph faces upwards (concave up) and has a maximum turning point**	$a < 0$ **… graph is on the second and the fourth quadrant** $a > 0$ **… graph is on the first and the third quadrant**	$a < 0$ **… graph is below the asymptote** $a > 0$ **… graph is above the asymptote**
Domain: $x \in R$ **Range:** $y \in R$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$	**Domain:** $x \in R$, $x \neq -p$ **Range:** $y \in R$, $y \neq q$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$
$y - y_1 = m(x - x_1)$	$y = ax^2 + bx + c$ **Axis of symmetry:** $x = \frac{-b}{2a}$ $y = a(x - x_1)(x - x_2)$ x_1 **and** x_2 **are x-intercepts**	**Axes of symmetry /Lines of symmetry:** $\left\{\begin{matrix} y = x + c \\ y = -x + c \end{matrix}\right\}$ **substitute point of intersection of asymptotes** **OR** $\left\{\begin{matrix} y = (x+p) + q \\ y = -(x+p) + q \end{matrix}\right\}$	

ACTIVITIES

KZN JUNE 2024

1.1 If it is given that the asymptotes of $f(x) = \frac{6}{x+p} + q$ intersect at $(4;3)$:

1.1.1 Write down the equation of f. (2) **L1**

1.1.2 Determine the intercepts of f with the axes. (3) **L2**

1.1.3	Sketch the graph of f, clearly showing all the intercepts with the axes and any asymptotes.	(3)	**L2**
1.1.4	g is one of the axes of symmetry of f, and it is a decreasing function. Determine the equation of g.	(3)	**L2**
1.1.5	$(-3;2)$ is a point on f. Determine the coordinates of the image of this point after reflection in g.	(2)	**L3**

SEDIBENG WEST DISTRICT MARCH 2024

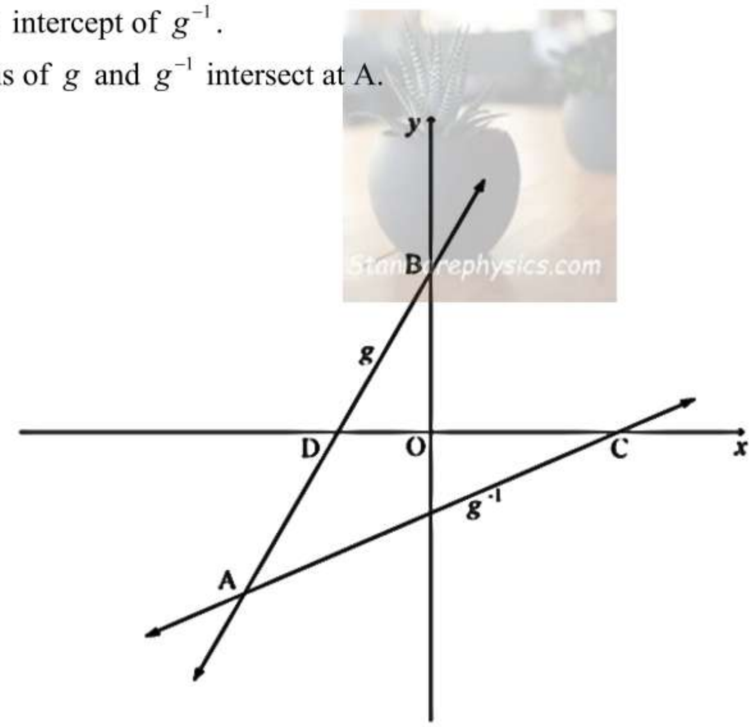
1.2 Given: $f(x)=\dfrac{3}{x-1}-2$

1.2.1	Write down the equations of the asymptotes of f.	(2)	**L1**
1.2.2	Calculate the $x-$ and $y-$ intercepts of the graph of f.	(3)	**L2**
1.2.3	Sketch the graph of f in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes.	(3)	**L2**
1.2.4	Write down the domain of $y=f(x-2)-3$.	(1)	**L2**
1.2.5	Use your graph to determine the value(s) of x for which: $f(x)\le -5$.	(2)	**L4**

DBE NOV 2022

1.3 The graphs of $g(x)=2x+6$ and g^{-1}, the inverse of g, are shown in the diagram below.

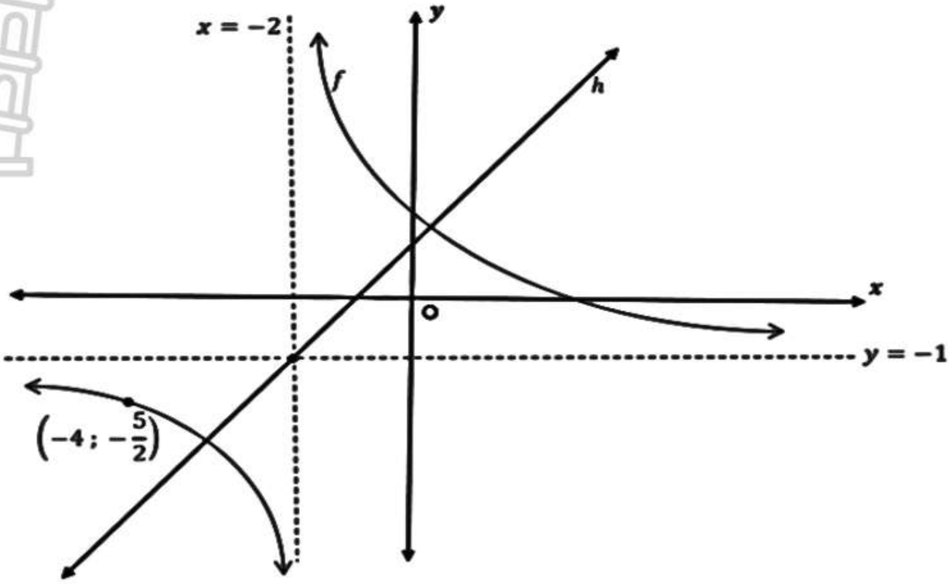
- D and B are $x-$ and $y-$ intercepts of g, respectively.
- C is the x intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



1.3.1	Write down the $y-$coordinate of B.	(1)	**L1**
1.3.2	Determine the equation of g^{-1} in the form $g^{-1}(x)=mx+n$	(2)	**L2**
1.3.3	Determine the coordinates of A.	(3)	**L2**
1.3.4	Calculate the length of AB.	(2)	**L2**
1.3.5	Calculate the area of ΔABC.	(5)	**L3**

GP SEPT 2025

1.4 The graphs of functions $f(x)=\dfrac{a}{x+p}+q$ and $h(x)=mx+c$ are sketched below.



1.4.1 Write down the values of p and q. (2) **L1**

1.4.2 Calculate the value of a. (1) **L1**

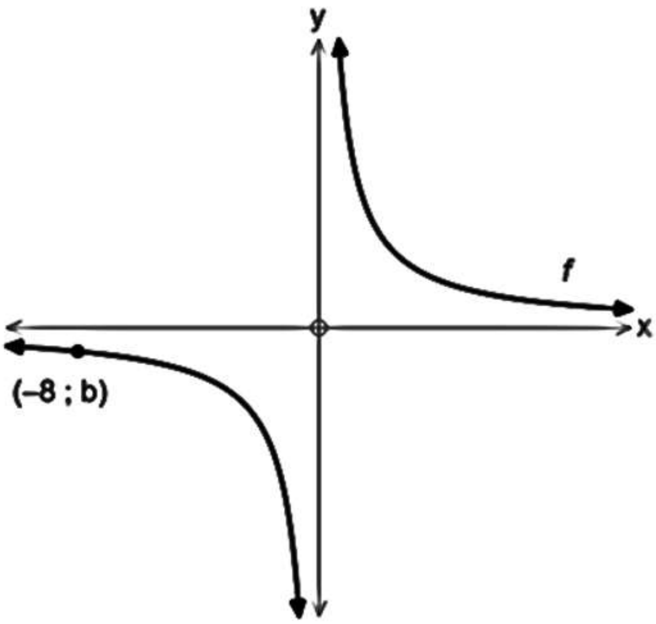
1.4.3 Write down the range of f. (1) **L1**

1.4.4 Determine the equation of the line of symmetry of f for $m<0$ in the form $y=...$ (3) **L2**

1.4.5 Write down the equations of the asymptotes of $f(x+4\frac{1}{2})$. (2) **L2**

IEB MAY 2025 (AMENDED)

1.5 In the diagram $f(x)=\dfrac{8}{x}$ has been drawn and the coordinates $(-8;b)$ are shown.



1.5.1 Determine the value of b. (1) **L1**

1.5.2 Write down the domain of f. (1) **L1**

1.5.3 Give the equations of the axes of symmetry of f. (2) **L2**

1.5.4 For which values of x will $f(x) \geq 0$? (1) **L2**

1.5.5 Write down the equation of the new graph in the form $y = ...$ when:

a) f is reflected about the $y-$axis. (1) **L1**

b) f is shifted 10 units down. (1) **L1**

c) f is reflected in the $y = x$ line. (2) **L2**

1.5.6 The graph of $h(x) = -x + k, k > 0$ is a tangent to f. Calculate the value of k. (5) **L4**

ADAPTED

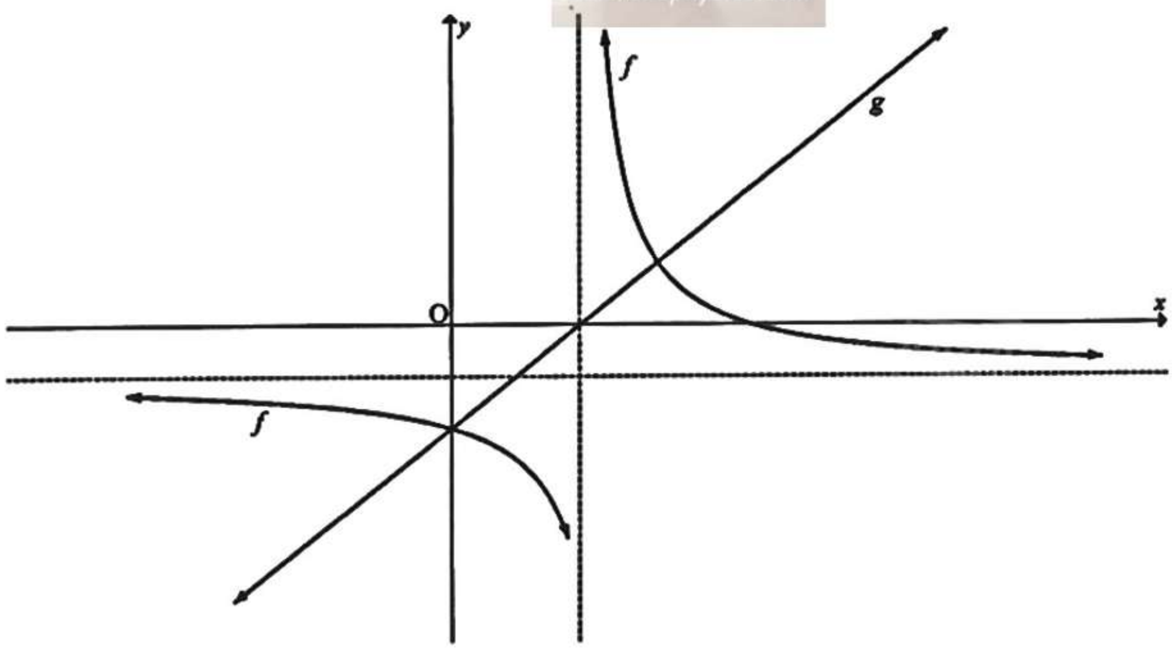
1.6 A linear function is defined as $g(x) = mx + c$. It is given that: (7) **L4**

- $g'(x) = 4$ for all $x \in R$.
- The line passes through the point $(k;12)$, where k is the $x-$intercept of the function $f(x) = 2x + 6$.
- A second line, $h(x)$, is perpendicular to $g(x)$ and passes through the point $(0;-2)$.

Determine the equation of the straight line $h(x)$ in the form $y = mx + c$.

DBE NOV 2025

1.7 The graphs of $g(x) = x + c$ and $f(x) = \dfrac{a}{x+p} + q$ are drawn below. Graph g and the vertical asymptote of f intersect at the $x-$axis.



1.7.1 Write down the coordinates of the $x-$intercept of g in terms of p. (1) **L1**

1.7.2 Graph g intersects the horizontal asymptote of f at $x = 1$ and the graph of f at $x = 3$. Graphs f and g also intersect on the $y-$axis. Determine the equation of f. (5) **L4**

1.7.3 Describe the transformation that g must undergo to become an axis of symmetry of f that cuts f at two points. (2) **L3**

ADAPTED

1.8 The function defined as $y = \dfrac{a}{x+p} + q$ has the following properties: (4) **L3**

- The domain is $x \in R, x \neq -2$.
- $y = -x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R, x \neq -2$.

Draw a neat sketch of the graph for this function. Include the asymptotes if any.

ADAPTED

1.9 Given: $f: x + y = 4$ and $g: x + 3y = 6$, where $x \geq 0$ and $y \geq 0$.

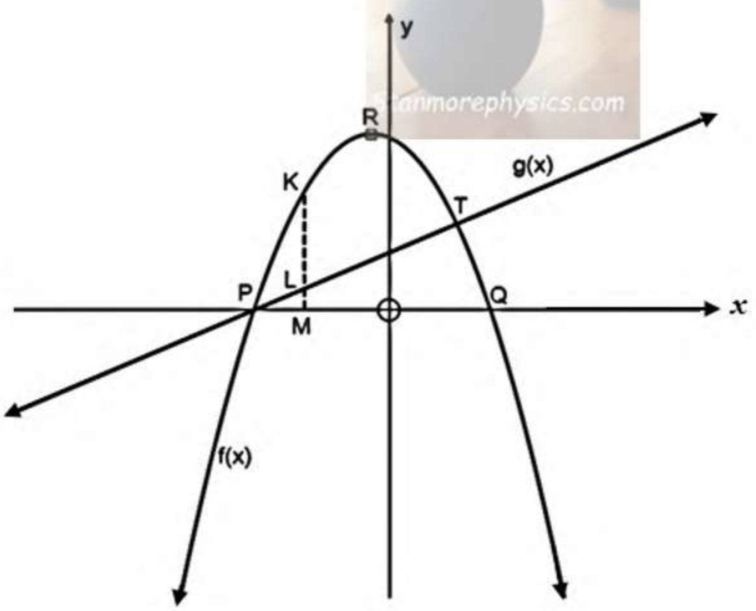
1.9.1 Sketch f and g on the same set of axes. (4) **L2**

1.9.2 Determine the value(s) of x for which $\dfrac{f(x)}{g(x)} \geq 1$. (3) **L3**

1.9.3 Determine the distance between $f(x)$ and $g(x)$ when $x = \dfrac{1}{2}$. (3) **L3**

ADAPTED

1.10 The sketch shows the graphs of the functions $f(x) = -x^2 - x + 12$ and $g(x) = x + 4$. P and Q are the $x-$intercepts of f and R is the turning point of f. The functions intersect at T and P. K is a point on f, L is a point on g and M lies on the $x-$axis so that KLM is a straight line parallel to the $y-$axis.



1.10.1 Determine the $x-$ coordinates at P and Q. (3) **L1**

1.10.2 Determine the coordinates of R. (3) **L2**

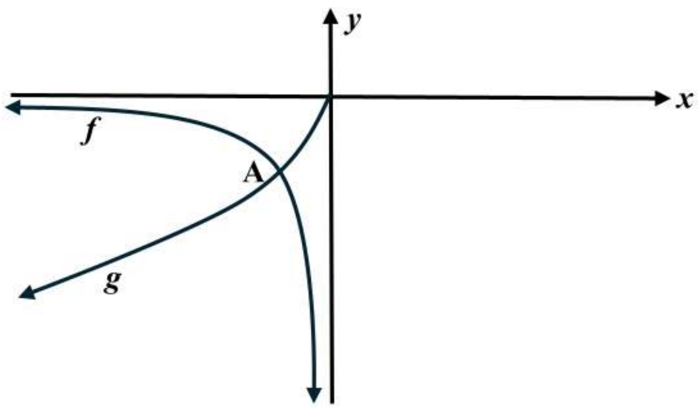
1.10.3 Determine the coordinates of M if $KL = 6\dfrac{3}{4}$ *units*. (5) **L3**

1.10.4 Determine the value(s) of k for which $-x^2 - x + 12 = k$ will have two negative, unequal roots. (2) **L2**

1.10.5 If h is the reflection of f in the straight line $x=1$, give the equation for h in the form $h(x)=a(x+p)^2+q$. (2) **L3**

RUSTENBURG GHS 2023

1.11 The graphs of $g(x)=-\sqrt{-x};\ x\leq 0$ and $f(x)=\dfrac{1}{x}$ for $x<0$ are given.



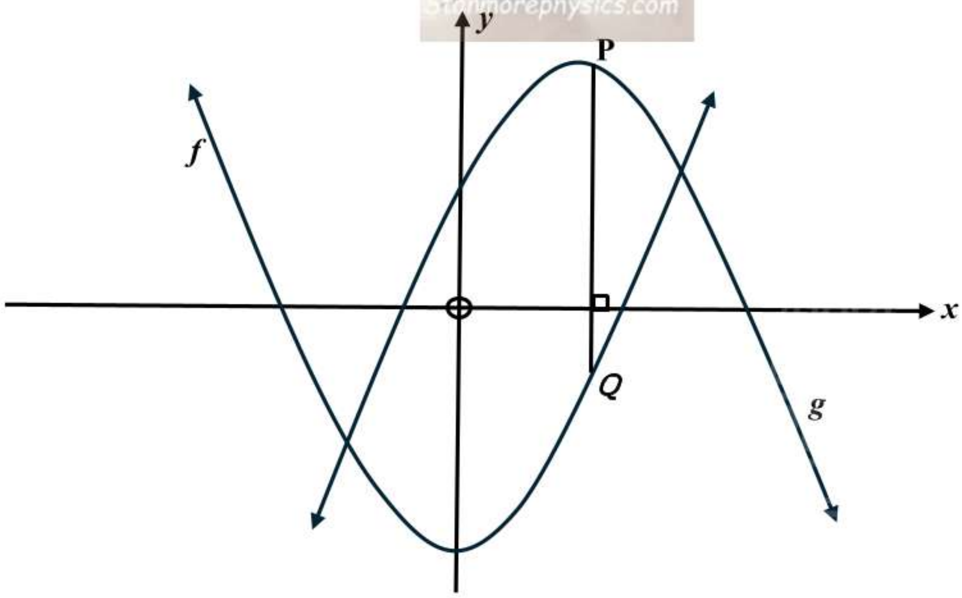
1.11.1 Determine the coordinates of A, the point of intersection between f and g. (4) **L2**

1.11.2 Determine the equation of g^{-1}. (3) **L2**

1.11.3 Give the equation of the graph which is obtained when $f(x)$ is reflected in the line $y=-x$. (2) **L2**

BISHOPS 2024

1.12 The diagram below represents the functions f and g with $f(x)=x^2-9$.
PQ is parallel to the $y-$axis with P on g and Q on f such that $y_P>y_Q$.



1.12.1 If the length of $PQ=-2x^2+3x+14$, determine the defining equation of g. (2) **L2**

1.12.2 For what value of x does the length of PQ have a maximum value? (4) **L2**

1.12.3 What is the maximum length of PQ? (3) **L3**

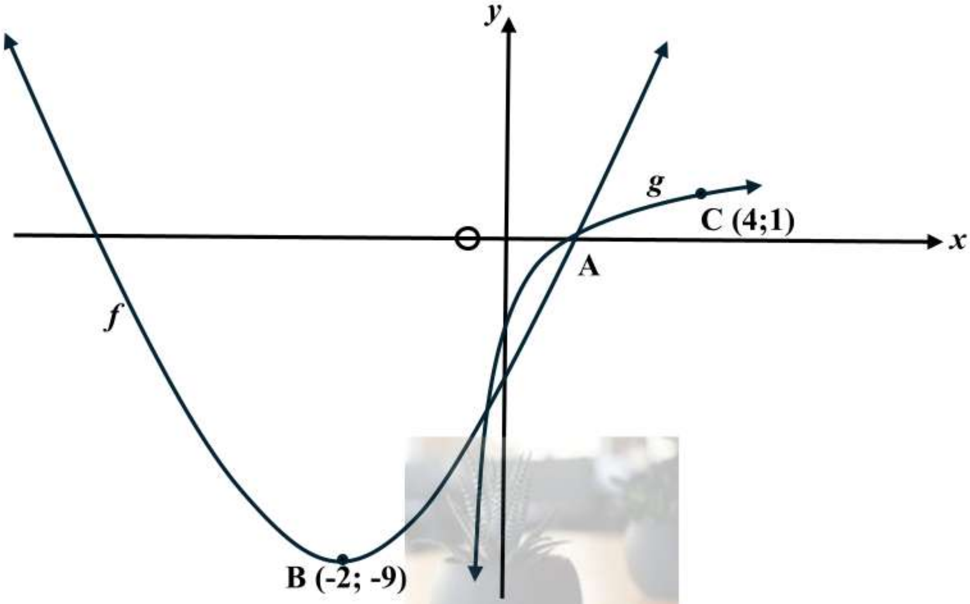
WESTERFORD HS 2024

1.13 Consider; $f(x)=\frac{1}{4}x^2, x \le 0$

1.13.1 Sketch f and f^{-1} showing any intercepts with the axes and ONE point on each graph. (4) **L2**

1.13.2 State the range of f^{-1}. (2) **L1**

1.14 The graphs of $f(x)$ and $g(x)=\log_n x$ are shown below. The graphs intersect at A on the x axis. $B(-2;-9)$ is the turning point of $f(x)$ and the point $C(4;1)$ lies on $g(x)$.



1.14.1 Write down the range of f and the domain of g. (2) **L1**

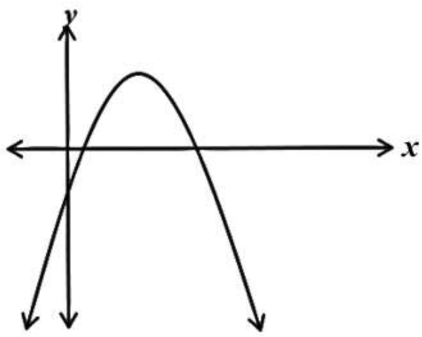
1.14.2 Write down the coordinates of A. (1) **L2**

1.14.3 Write down the values of x such that $f(x) \le 0$. (2) **L2**

1.14.4 Determine the equation of $f(x)$ in the form $y=ax^2+bx+c$ (5) **L3**

BERGVLIET 2023

1.15 The accompanying figure shows the sketch graph of $y=-x^2+8x-9$



1.15.1 Rewrite the equation in the form $y=a(x+p)^2+q$ (2) **L2**

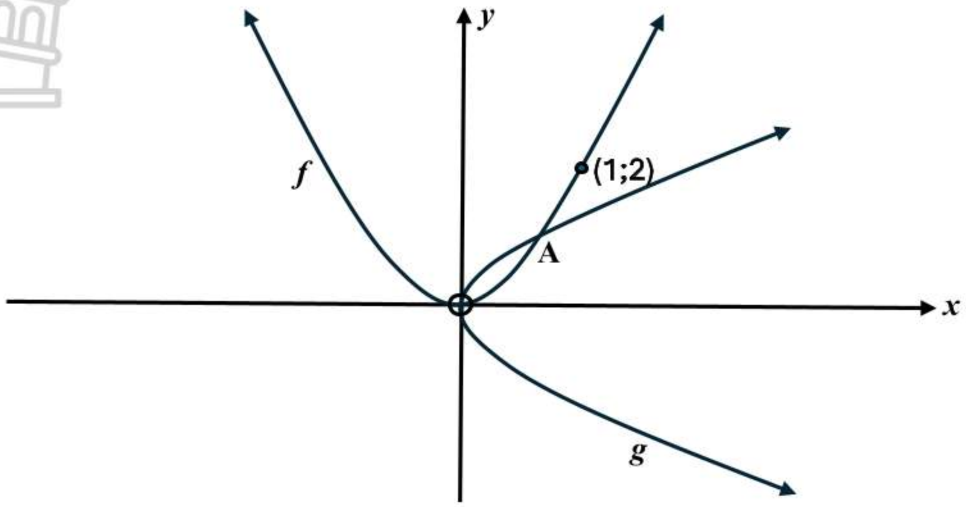
1.15.2 Determine with the aid of the graph, the values of k for which

$-x^2+8x-9=k$ has roots which are unequal, positive and real. (3) **L2**

1.15.3 Determine the equation if $y=-x^2+8x-9$ is shifted 3 units left and reflected about the x – axis. (2) **L3**

ADAPTED

1.16 The graphs of f and g intersect at the origin and at point A.



1.16.1 Which of the graphs, f or g, is not a function? Give a reason for your answer. (2) **L1**

1.16.2 If $f(x)=ax^2$ and the point $(1;2)$ is on f, show that $a = 2$. (1) **L1**

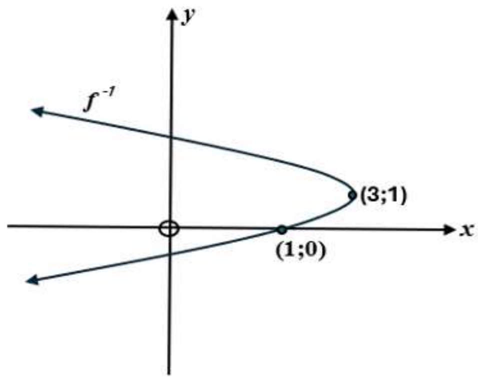
1.16.3 If $g(x)=f^{-1}$, write down the equation of g in the form $y=...$ (4) **L2**

1.16.4 If h is the reflection of f in the x-axis, write down the equation of h in the form $h(x) = \ldots$ (1) **L2**

1.16.5 If the graph of f is shifted down by 3 units and left by 1 unit, give the equation of the new graph in the form $y=ax^2+bx+c$ (4) **L3**

1.16.6 Determine the coordinates of A. (3) **L3**

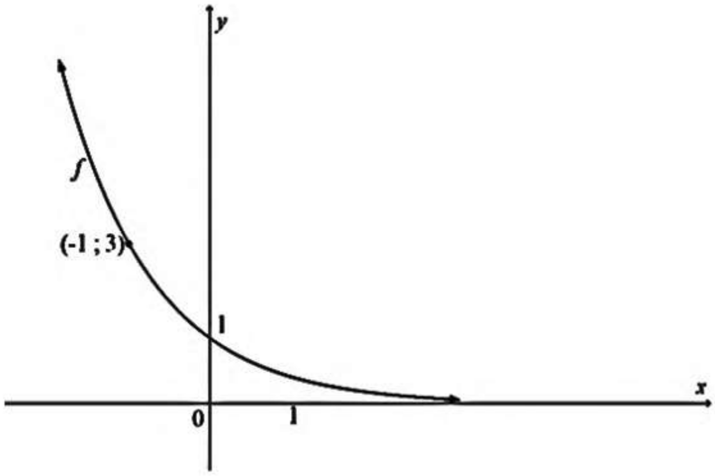
1.17 The inverse of a certain graph, f, is drawn below. Determine the equation of f, if it is known that f is a quadratic function.



(4) **L4**

EC MAR 2026

1.18 Given $f(x)=\left(\frac{1}{3}\right)^x$



1.18.1 Write down the domain of f. (1) **L1**

1.18.2 Write down the range of f. (1) **L2**

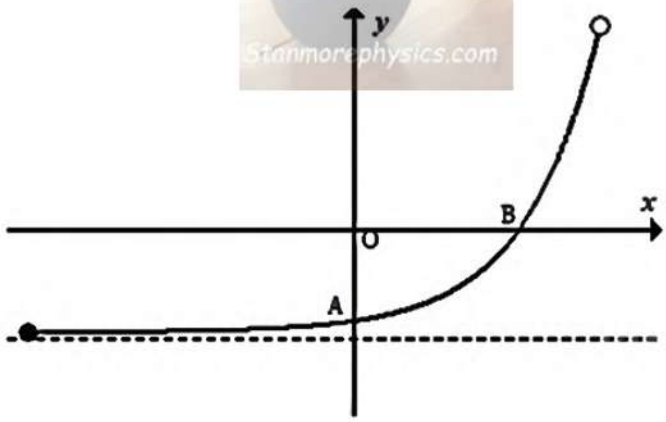
1.18.3 Determine the equation of the inverse of f in the form of $f^{-1}(x)=...$ (2) **L2**

1.18.4 Draw the graph of f^{-1} . Show clearly, the intercepts with the axes as well as the coordinate of the other point. (3) **L2**

1.18.5 Write down the equation $f^{-1}(x+2)$. (2) **L2**

NC MAR 2026

1.19 Sketched below is the graph of $h(x)=3^x-9$ for $x\in[-3;2)$.



1.19.1 Write down the equation of the asymptote of h. (1) **L1**

1.19.2 Determine the coordinates of A . (2) **L1**

1.19.3 Determine the equation of the straight line g ,if g is passing through A and B . (3) **L2**

1.19.4 Write down the equation of $m(x)=h(x)+9$. (1) **L2**

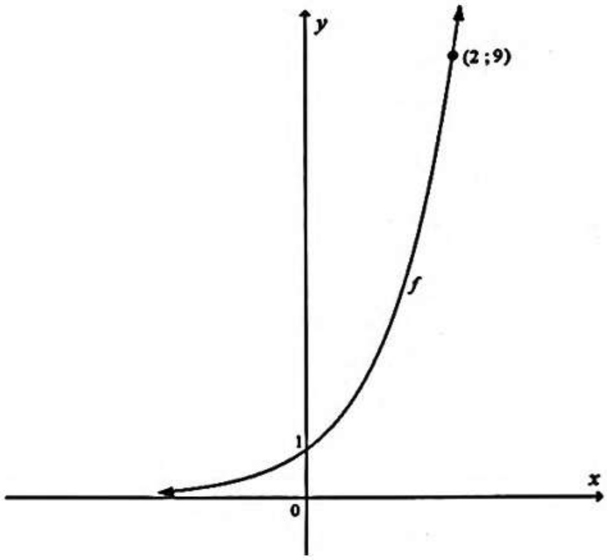
1.19.5 Write down the equation $m^{-1}(x)$ in the form $y=...$ (2) **L2**

1.19.6 Write down the range of $m^{-1}(x)$. (2) **L2**

1.19.7 Point S is the result of point A when reflected over the line $x=2$. Calculate the area of ΔABS. (4) **L3**

LIMP JUN 2025

1.20 The sketch below represents the graph of $f(x) = b^x + q$ which indicates the y-axis at $(0\ ;\ 1)$ and passes through point $(2\ ;\ 9)$.



1.20.1	Show that the equation of $f(x) = 3^x$.	(3)	**L1**
1.20.2	Determine the equation of $f^{-1}(x)$.	(2)	**L2**
1.20.3	Sketch the graph of $f^{-1}(x)$. Show all the intercepts with axes.	(3)	**L2**
1.20.4	Given $g(x) = 2x - 1$, calculate the value of P if: $P = \sum_{x=-2}^{5} f(x) - \sum_{x=1}^{10} g(x)$.	(5)	**L4**

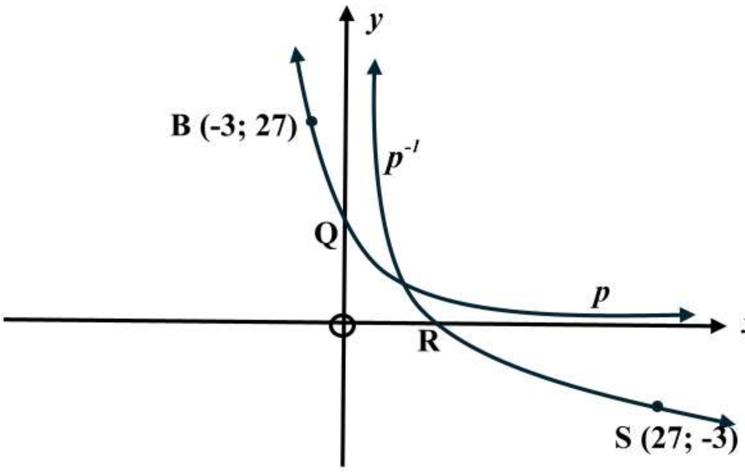
NW SEPT 2025

1.21 Consider the following function: $f(x) = \log_{\frac{4}{3}} x$.

1.21.1	Write down the coordinates of the x-intercept of f.	(1)	**L1**
1.21.2	Determine the equation of the inverse of f in the form of $f^{-1}(x) = \ldots$	(2)	**L2**
1.21.3	Sketch the graph of f and f^{-1} on the same set of axes. Clearly indicate which one if f and f^{-1}, as well as the intercepts with the axes and asymptotes. Also sketch the axis of symmetry of f and f^{-1} AND write its equation.	(5)	**L2**
1.21.4	If $A(e\ ;-2)$ is a point on f, calculate the value of e.	(2)	**L2**
1.21.5	Write down the domain of f.	(1)	**L1**
1.21.6	Given: $p(x) = \left(\frac{2}{3}\right)^x - 1$		
	a) Give the equation of t if $t(x)$ is the reflection of p in the y- axis.	(2)	**L1**
	b) Write down the range of p.	(1)	**L2**

NW JUN 2025

1.22 The graphs $p(x) = b^x$ and $p^{-1}(x)$ are sketched below. $K(-3\ ;\ 27)$ lies on the graph p and $S(27\ ;-3)$ lies on the graph p^{-1}. Q id the y-intercept of p and R is the x-intercept of p^{-1}.



1.22.1 Determine the value of b. (2) **L1**

1.22.2 Determine the equation of p^{-1} in the form of $y = ...$ (2) **L2**

1.22.3 Write down the coordinates of Q and R. (2) **L1**

1.22.4 For which value(s) of x will:

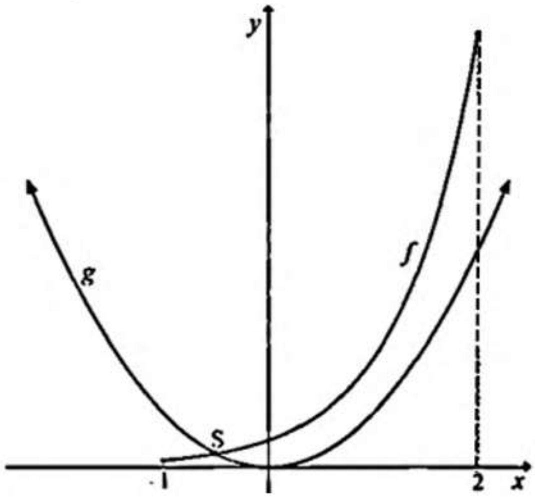
a) $0 < p(x) \leq 1$ (1) **L2**

b) $\log_{\frac{1}{3}} x < -3$ (1) **L2**

1.22.5 T is a point in the first quadrant where TQ is parallel to the x-axis and TS is parallel to the y-axis. Calculate the area of ΔQTS. (4) **L3**

KZN JUN 2025

1.23 The diagram below shows the graph of $f(x) = a^x$, for $x \in [-1; 2]$ and $g(x) = bx^2$. $S\left(-\frac{1}{2}; \frac{1}{2}\right)$ is the point if intersection between f and g.



1.23.1 Determine the values of a and b. (4) **L2**

1.23.2 Draw a sketch of graph of the inverse of g. Indicate the coordinates of one point on the graph. (2) **L2**

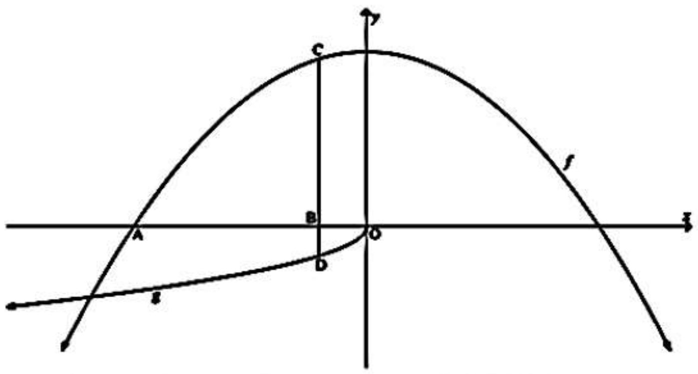
1.23.3 Is the inverse of g is a function? (1) **L1**

1.23.4 Determine the equation of f^{-1} in the form $y = ...$. (State the restriction of the domain) (4) **L2**

1.23.5 If $x < 0$, write the values of x for which $f(x) > g(x)$. (2) **L2**

GP JUN 2024

1.24 Given $f(x) = -4x^2 + 6$ and $g(x) = -2\sqrt{-x}; x \leq 0$. The graphs of f and g are sketched. A is an x-intercept of f and B is a point between O and A. The straight-line CBD, with C on f and D on g, is parallel to the y-axis.



1.24.1 Determine the x-coordinate of A, correct to TWO decimal places. (2) **L2**

1.24.2 Show that $f(x) \neq 8$. (2) **L2**

1.24.3 Write down the length of CBD in terms of x, where x is the x-coordinate of B. (2) **L1**

1.24.4 Determine the maximum length of CD. (6) **L3**

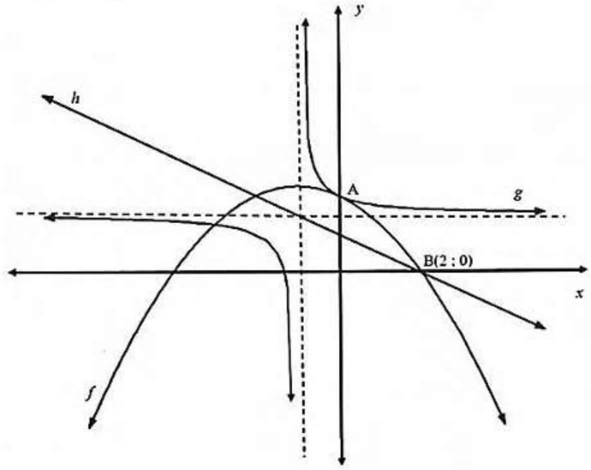
1.24.5 Write the equation of h, the inverse of g, in the form $y = ...$ (4) **L2**

1.24.6 Write the equation of k, the reflection of g about the x-axis, in the form $y = ...$ (1) **L1**

1.24.7 If $f(x) + k = 0$ has two distinct real roots, determine then value(s) of k. (2) **L3**

NW SEPT 2025

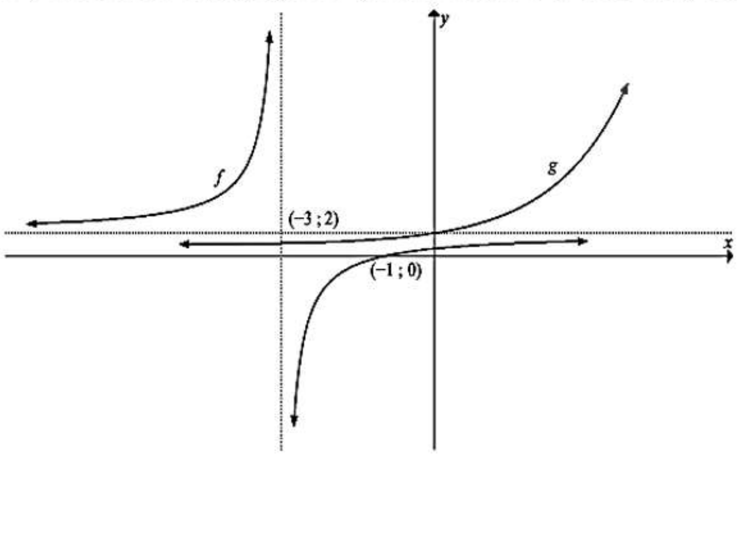
1.25 Sketched below are the graphs $f(x)=-\frac{1}{2}x^2+bx+c$ and $g(x)=\frac{a}{x+p}+q$. The axis of symmetry of f, $x=-1$, is the vertical asymptote of g. The line h is an axis of symmetry of g. A is the y-intercept of f and g. B$(2;0)$ is a x-intercept of f and h.



1.25.1	Show that the equation of f is $f(x)=-\frac{1}{2}x^2-x+4$.	(2)	**L2**
1.25.2	Write down the coordinates of the y-intercept of f.	(2)	**L1**
1.25.3	Determine the coordinates of the turning point of f.	(2)	**L1**
1.25.4	Determine the equation of h.	(3)	**L2**
1.25.5	Determine the equation of g.	(4)	**L3**
1.25.6	Determine the coordinates of the x-intercept of g.	(3)	**L2**
1.25.7	For which values of x will it be: $g(x).f'(x)\leq 0$?	(3)	**L3**
1.25.8	Determine the equation of $k(x)$ if k is the reflection of g about the line $x=4$.	(3)	**L3**
1.25.9	Determine values of p will both x-intercept of $f(x)+p$ be greater than -2?	(4)	**L4**

GP JUN 2024

1.26 The sketch below represents the graphs of $f(x)=\frac{a}{x+p}+q$ and $g(x)=2^x+1$. The point $A(-3;2)$ is where the asymptotes of f intersect. The graph of f intersect the x-axis at $(-1\ ;\ 0)$



1.26.1 Write down the equation of the asymptotes of f. (2) **L1**

1.26.2 Write down the coordinates of the y-intercept of g. (2) **L1**

1.26.3 Determine the equation of f. (4) **L2**

1.26.4 Write the equation of the axis symmetry of f in the form $y = mx + c$ if $m < 0$. (2) **L2**

1.26.5 Write down the equation of $4f(x-2)$. (2) **L2**

1.26.6 Write the equation of h, if h is the graph of g that is translated one unit down. (1) **L1**

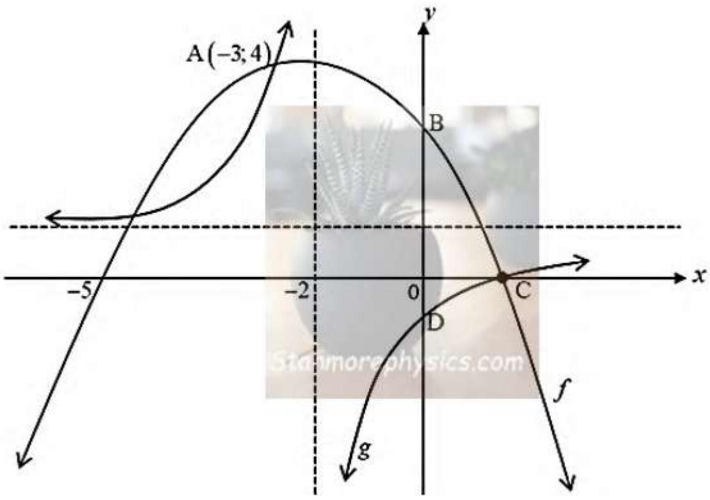
1.26.7 Determine the equation of h^{-1}, the inverse of h. (2) **L2**

1.26.8 Write down the range of h^{-1}. (1) **L1**

1.26.9 For which values of x if $f(x).g'(x) \le 0$. (2) **L3**

EC SEPT 2025

1.27 The graphs of $f(x) = -\frac{1}{2}(x+2)^2 + 4\frac{1}{2}$ and $g(x) = \frac{a}{x+p} + q$ are the drawn below. The graphs of f and g cut the y-axis at $2\frac{1}{2}$ and $-\frac{1}{2}$ respectively. One of the points of intersection of the graphs is $A(-3;4)$. Point C is the point of the intersection and x-intercept of f and g. The vertical asymptote of g is $x = -2$.



1.27.1 Write down the coordinates of B. (1) **L1**

1.27.2 Calculate the values of a, p and q. (6) **L3**

1.27.3 Determine the range of f. (2) **L1**

1.27.4 Determine an equation for the axis of symmetry of g that has gradient equal to -1. (2) **L2**

1.27.5 Determine the average gradient of f between B and C. (3) **L2**

1.27.6 For which values of x is:

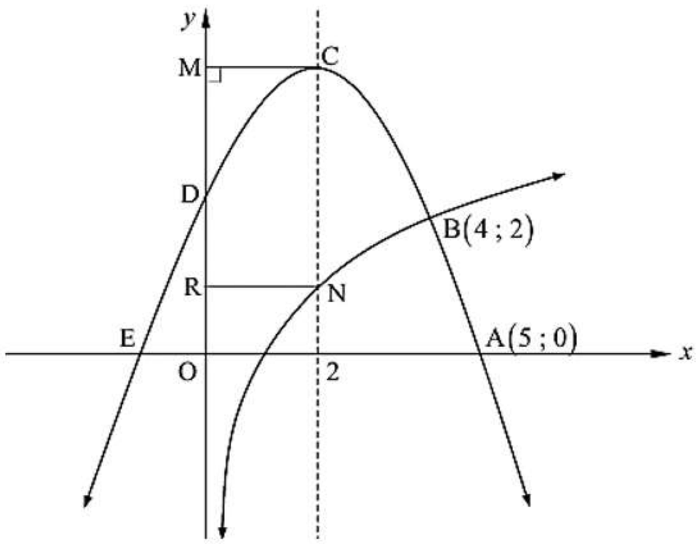
a) $f(x) \ge 0$? (2) **L2**

b) $g(x).g'(x) > 0$? (2) **L3**

1.27.7 If $h(x) = x$, determine the value(s) of k for which $f(x) = h(x) + k$ has two roots that have different signs. (2) **L3**

ACETP MAR 2026

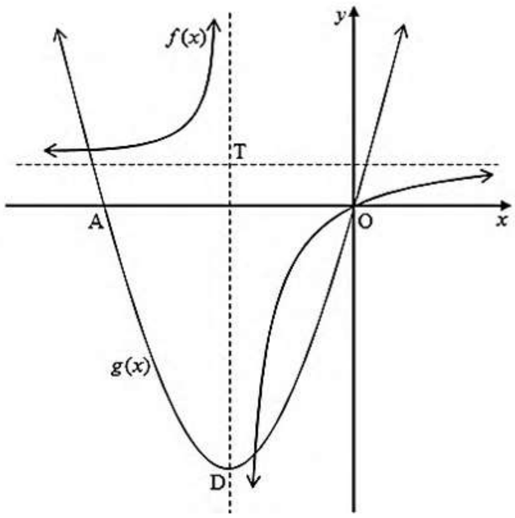
1.28 The diagram shows the graphs of $f(x)=ax^2+bx+c$ and $g(x)=\log_m x$. The graph of f cuts the x-axis at E and A$(5\,;0)$, the y-axis at D and has a turning at C. The graph of f has the axis of symmetry at $x=2$. B$(4\,;2)$ is the point of intersection of f and g. MCNR is a rectangle.



1.28.1	Determine the value of m.	(2)	**L1**
1.28.2	Write down the domain of g.	(1)	**L1**
1.28.3	Determine the equation of g^{-1}, in the form of $y=...$	(2)	**L2**
1.28.4	Write down the equation of h if h is obtained by shifting g^{-1} 2 units to the left.	(1)	**L2**
1.28.5	Show that $a=-\frac{2}{5}$; $b=1\frac{3}{5}$; $c=2$	(3)	**L3**
1.28.6	Hence, or otherwise calculate the area of MCNR.	(2)	**L3**
1.28.7	Use your answer to QUESTION 28.6 and the graph to explain why the equation $f(x)-4=0$ will have no real roots.	(1)	**L2**
1.28.8	For which values of x is $f(x).g(x)>0$?	(2)	**L2**

EC JUN 2024

1.29 The diagram below shows the graphs of $f(x)=\dfrac{2}{x+2}+1$ and $f(x)=a(x+2)^2-8$. Both graphs pass through the origin, O. The vertical asymptote of f pass through D, the turning point of g. The asymptote of f intersect at T. A is the other x-intercepts of g.



1.29.1 Write down the coordinates of D, the turning point of g. (1) **L1**

1.29.2 Write down the equation of the asymptotes of f. (2) **L1**

1.29.3 Determine:

a) The value of a. (2) **L1**

b) The length of OA. (3) **L2**

c) The range of f. (1) **L1**

d) The equation of the axis of symmetry of f with a negative gradient. (2) **L2**

1.29.4 For which values of x will:

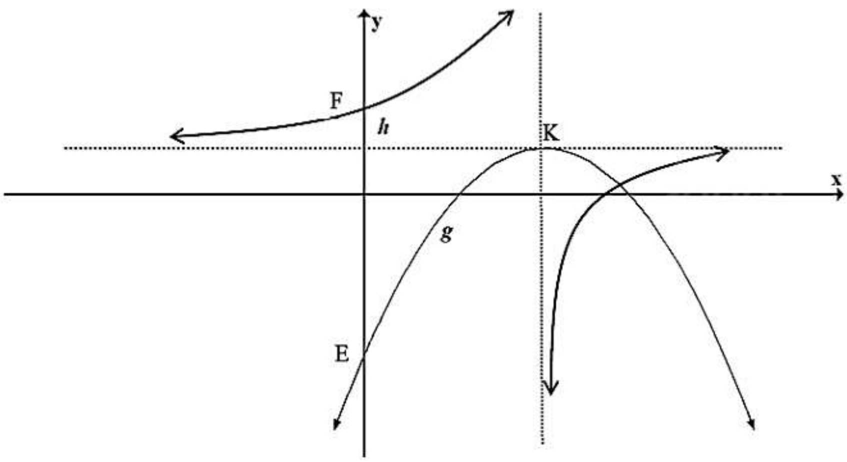
a) $g(x) < 0$? (2) **L2**

b) $g(x).f(x) \geq 0$? (2) **L2**

1.29.5 Determine the value(s) of k, for which $h(x) = -g(x) + k$ will have two distinct roots with the same sign. (3) **L3**

NC SEPT 2025

1.30 Sketched below are the graphs of $g(x) = ax^2 + bx - 7$ and $h(x) = -\frac{2}{x-3} + 2$. E and F are the y-intercepts of g and h respectively. The turning point K is also the point of intersection of the two asymptotes of h.



1.30.1 Write down the coordinates of K. (2) **L1**

1.30.2 Calculate the length of EF. (4) **L2**

1.30.3 Determine the values of a and b in $g(x)=ax^2+bx-7$. (4) **L2**

1.30.4 Write down the range of $h(x)+3$. (2) **L2**

1.30.5 For which values of p will $g(x)=p$ have TWO unequal positive roots? (2) **L2**

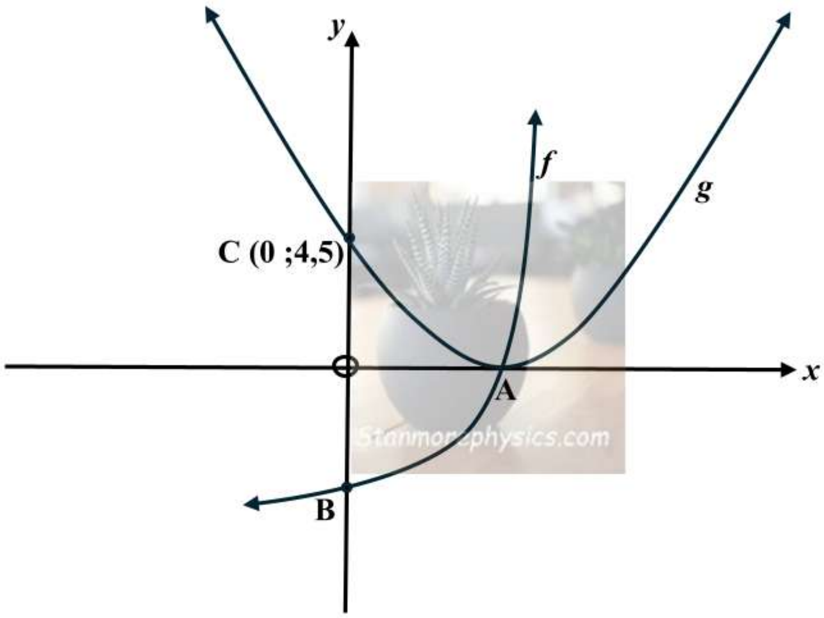
1.30.6 Determine the equation of the line of symmetry of h, $m<0$ in the form $y=mx+c$. (3) **L2**

1.30.7 Describe the transformation from graph g to the graph of t, where $t(x)=(x-3)^2$. (2) **L2**

1.30.8 It is given that $g(x)=f'(x)$. Determine the x-value(s) where the gradient of a tangent to f is equal to 1. (3) **L3**

DBE NOV 2011

1.31 The graphs of $f(x)=2^x-8$ and $g(x)=ax^2+bx+c$ are sketched below. B and C(0 ; 4,5) are the y-intercepts of the graphs of f and g respectively. The two graphs intersect at A, which is the turning point of the graph g and the x-intercept of the graphs of f and g.



1.31.1 Determine the coordinates of A and B. (4) **L1**

1.31.2 Write down an equation of the asymptote of the graph of f. (1) **L1**

1.31.3 Determine an equation of $h(x)=f(2x)+8$ (2) **L2**

1.31.4 Determine an equation of h^{-1} in the form $y=...$ (2) **L2**

1.31.5 Write down an equation of p is the reflection of h^{-1} about the h^{-1} about the x-axis. (1) **L1**

1.31.6 Calculate $\sum_{k=0}^{3} g(k)-\sum_{k=4}^{5} g(k)$. Show ALL your working. (4) **L3**

1.32

1.32.1 Draw a sketch graph of $y=ax^2+bx+c$, where $a<0; b<0; c<0$ and $ax^2+bx+c=0$ has only ONE solution. **(DBE NOV 2011)** (4) **L3**

1.32.2 A quadratic function, $f(x) = ax^2 + bx + c$, has the following properties:

- $f(6-p) = f(6+p)$ for all real values of p.
- $f\left(-\frac{b}{2a}\right) = 2$
- $b^2 < 4ac$

Draw a rough sketch of $f(x)$; labelling the turning point clearly. **(IEB NOV 2021)** (4) **L4**

1.32.3 A function $f(x) = -px^2 + 2px$ has $p > 1$

On the same set of axes, sketch the graphs of $f(x)$ and $f'(x)$. Indicate all the intercepts and turning points, expressing these in terms of p if necessary. **(IEB J21)** (4) **L4**

1.32.4 Quadratic function having the x that has the following properties:

- The function is symmetrical to $y = ax^2 - 7x + c$ about the y-axis
- The function is symmetrical about $x = 1$
- The function has a value of -20 when $x = 4$ **(IEB JUN 2022)**

Show that its equation is $y = -\frac{7}{2}x^2 + 7x + 8$ (4) **L4**

TOPIC 2. Trig Identities & Reduction Formulae [Trigonometry ±50 marks]

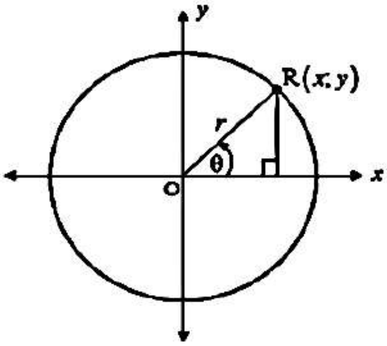
GUIDELINES, SUMMARY NOTES, & STRATEGIES

2.1 Definitions of trig ratios:

In a right-angled triangle: $\sin\theta = \frac{opposite}{hypotenuse}; \cos\theta = \frac{adjacent}{hypotenuse}; \tan\theta = \frac{opposite}{adjacent}$

SOH CAH TOA helps you to remember these definitions.

In a Cartesian plane:



$\sin\theta = \frac{y}{r}$;

$\cos\theta = \frac{x}{r}$;

$\tan\theta = \frac{y}{x}$;

and $r^2 = x^2 + y^2$;

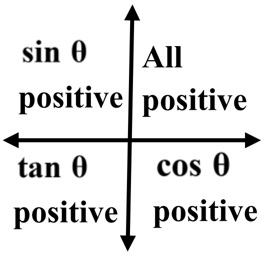
2.2 CAST Rule:

All trig ratios are positive in the 1st quadrant. **All**

Only $\sin\theta$ is positive in the 2nd quadrant. **Students**

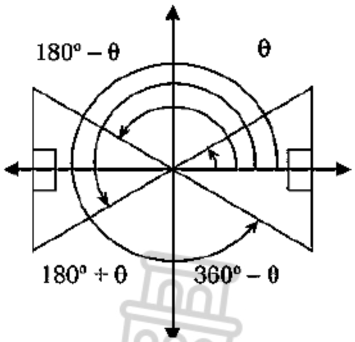
Only $\tan\theta$ is positive in the 3rd quadrant. **Take**

Only $\cos\theta$ is positive in the 4th quadrant. **Care**



2.3 Reduction Formulae:

If θ is an acute angle, i.e. θ is an angle in the 1st quadrant,
$180°-\theta$ will lie in the 2nd quadrant,
$180°+\theta$ will lie in the 3rd quadrant,
and $360°-\theta$ will lie in the 4th quadrant.



Example:

$\sin\theta = \sin(180°-\theta) = -\sin(180+\theta) = -\sin(360°-\theta)$

$\cos\theta = -\cos(180°-\theta) = -\cos(180+\theta) = \cos(360°-\theta)$

$\tan\theta = -\tan(180°-\theta) = \tan(180+\theta) = -\tan(360°-\theta)$

For $90°-\theta$ and $90°+\theta$ the ratio changes to its co-function

The co-function of $\sin\theta$ is $\cos\theta$, and the co-function of $\cos\theta$ is $\sin\theta$.

Example:

$\sin(90°-\theta) = \cos\theta$; $\cos(90°-\theta) = \sin\theta$; $\sin(90°+\theta) = \cos\theta$; $\cos(90°+\theta) = -\sin\theta$

2.4 Trigonometric Identities:

Square identity: $\sin^2\theta + \cos^2 = 1$

Quotient identity: $\frac{\sin\theta}{\cos\theta} = \tan\theta$

Compound Angle identities:

$\sin(\theta \pm \beta) = \sin\theta\cos\beta \pm \cos\theta\sin\beta$

$\cos(\theta \pm \beta) = \cos\theta\cos\beta \mp \sin\theta\sin\beta$

Double Angle identities:

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

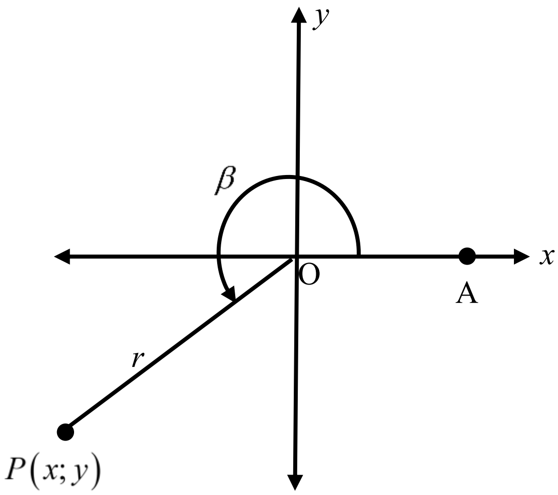
$$\cos 2\theta = 1 - 2\sin^2 \theta$$

ACTIVITIES

PYTHAGOREAN (CAST DIAGRAM) PROBLEMS:2.1-2.17

ADAPTED UNKNOWN SOURCE

2.1 In the diagram below, $P(x;y)$ is the point in the third quadrant such that $17\cos\beta + 15 = 0$. $A\hat{O}P = \beta$ as shown:



2.1.1 $\sin\beta$ (1) **L1**

2.1.2 $\cos 2\beta$ (2) **L1**

2.1.3 $\cos^2 300°.\tan\beta$ (3) **L2**

2.1.4 $\sin(\beta + 45°)$ (3) **L2**

ADAPTED UNKNOWN SOURCE

2.2 If $13\sin x + 12 = 0$, where $x \in [90°;270°]$ and $\cos y = \frac{5}{13}$, where $y \in (90°;360°)$, determine **without the use of a calculator** the value of:

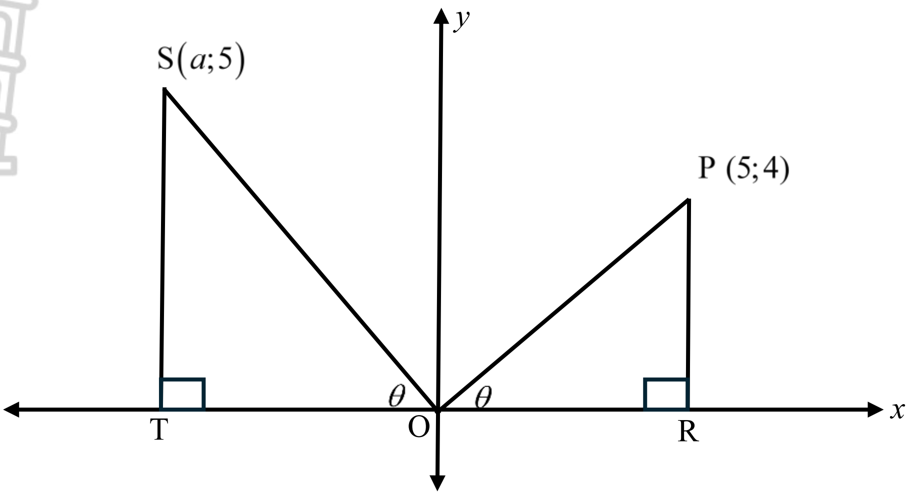
2.2.1 $\sin(x - y)$ (2) **L1**

2.2.2 $\sin 2x$ (1) **L1**

2.2.3 $\tan 2y$ (3) **L2**

ADAPTED WC 2018

2.3 In the diagram below similar triangles ΔOPR and ΔOST are presented. O is the origin. R and T are points on the $x-$axis .

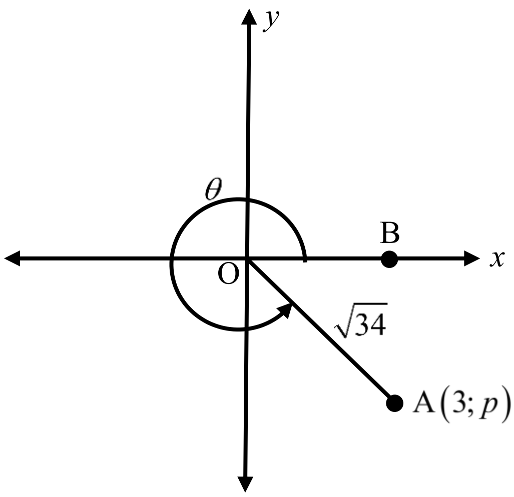


2.3.1 $\sin(90°-\theta)$ (2) **L1**

2.3.2 the value of a (3) **L1**

ADAPTED DBE NOVEMBER 2017

2.4 In the diagram below, $A(3\,;p)$ is a point in the Cartesian Plane $OA=\sqrt{34}$ and $B\hat{O}A=\theta$, which is a reflex angle.



Without using a calculator , determine :

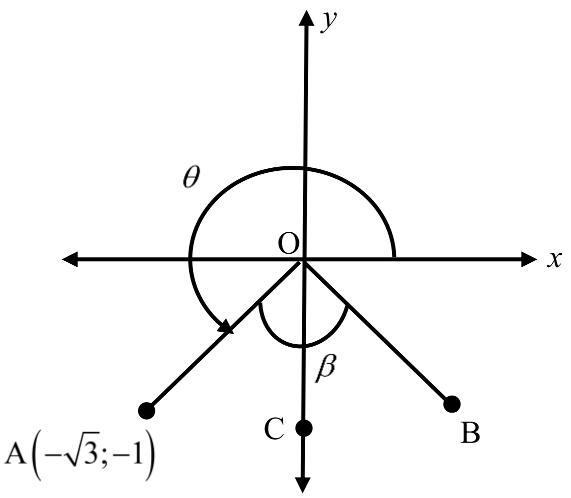
2.4.1 the value of p . (2) **L1**

2.4.2 $\cos(60°+\theta)$ (3) **L2**

2.4.3 $\tan(90°-\theta)$ (3) **L2**

ADAPTED UNKNOWN SOURCE

2.5 In the diagram below, A is the point $\left(-\sqrt{3}\ ;\ -1\right)$. **Without using a calculator**, determine the value of each of the following:



2.5.1 $\tan\theta$ (1) **L1**

2.5.2 $\cos\theta$ (1) **L1**

2.5.3 the size of β, if OB is a reflection of OA about the y-axis. (2) **L1**

ADAPTED UNKNOWN SOURCE

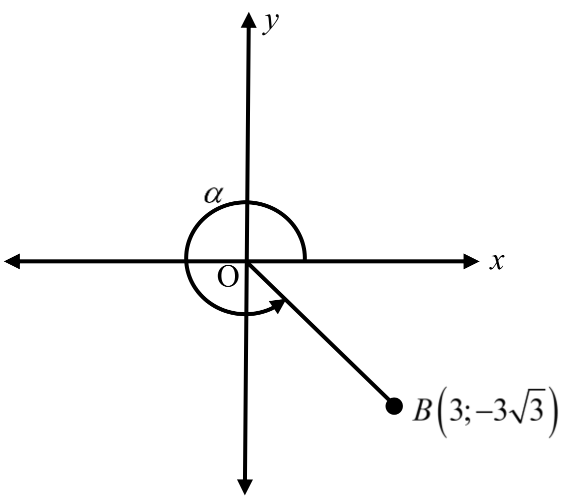
2.6 Given $\tan\theta = \dfrac{\sqrt{3}}{2}$ and $\cos\theta < 0$. Express each of the following in its simplest form **without the use of a calculator:**

2.6.1 $\sin(180° - 2\theta)$ (3) **L2**

2.6.2 $\sqrt{8}\sin\theta + \cos\theta$ (4) **L2**

ADAPTED FS 2019

2.7 Given $X\hat{O}B = \alpha$ and $B\left(3\ ;-3\sqrt{3}\right)$, determine the following:



2.7.1 OB (2) **L1**

2.7.2 $\sin(180°+\alpha)$ (2) **L2**

2.7.3 $\frac{-1}{\sqrt{3}}\sin\alpha+2$ (2) **L2**

ADAPTED UNKNOWN SOURCE

2.8 If $13\cos 2\theta=5$ and $2\theta\in[180°;360°]$, determine the values of the following **without the use of a calculator:**

2.8.1 $\sin 2\theta$ (2) **L2**

2.8.2 $\cos(2\theta+45°)$ (3) **L2**

2.8.3 $\sin\theta$ (2) **L2**

ADAPTED UNKNOWN SOURCE

2.9 Given: $7\cos\beta+5=0$ and $\tan\beta>0$. Use a suitable diagram to determine:

2.9.1 $\tan\beta$ (2) **L1**

2.9.2 $\sin(450°+\beta)$ (2) **L2**

2.9.3 $\sin 2\beta$ (2) **L2**

ADAPTED UNKNOWN SOURCE

2.10 Given that $\sin 10°=\sqrt{k}$, **without using a calculator,** write each of the following in terms of k.

2.10.1 $\sin 190°$ (2) **L2**

2.10.2 $\cos 20°$ (3) **L2**

2.10.3 $\cos 50°$ (4) **L3**

ADAPTED UNKNOWN SOURCE

2.11 Given: $\cos x=a$ and $0°<x<180°$, determine the following in terms of a.

2.11.1 $\sin x$ (2) **L1**

2.11.2 $\cos 2x$ (3) **L2**

2.11.3 $\tan(x-180°)$ (2) **L1**

2.11.4 $\sin(45°-x)$ (2) **L2**

ADAPTED UNKNOWN SOURCE

2.12 If $\cos 24°=t$, determine the following in terms of t **without using a calculator:**

2.12.1 $\cos 336°$ (2) **L1**

2.12.2 $\cos 48°$ (2) **L1**

2.12.3 $\cos 12°\cos 102°$ (3) **L3**

2.12.4 $\sin 12°$ (3) **L3**

ADAPTED UNKNOWN SOURCE

2.13 Given**:** $\sin 36°=p$ and $\cos 40°=q$, determine the following **without the use of a calculator**

2.13.1 $\cos 72°$ (2) **L2**

2.13.2 $\cos 4°$ (2) **L3**

2.13.3 $\sin 162°.\sin(-72°)$ (2) **L2**

ADAPTED UNKNOWN SOURCE

2.14 $\sin A=p$ and $\cos A=q$

2.14.1 Write down $\tan A$ in terms of p and q (2) **L2**

2.14.2 Simplify $p^4 - q^4$ to a single trigonometric ratio. (3) **L2**

ADAPTED UNKNOWN SOURCE

2.15 Given: $3\sin 38^\circ \sin 52^\circ = p$, determine the following in terms of p

2.15.1 $\tan 76^\circ$ (3) **L2**

2.15.2 $\cos 16^\circ$ (3) **L2**

2.15.3 $\sin^2 38^\circ$ (3) **L3**

ADAPTED UNKNOWN SOURCE

2.16 If $\cos 25^\circ = \sqrt{1-k^2}$, determine the following in terms of k:

2.16.1 $\sin 25^\circ$ (2) **L1**

2.16.2 $\sin 50^\circ$ (2) **L2**

2.16.3 $\cos 70^\circ$ (2) **L2**

ADAPTED UNKNOWN SOURCE

2.17 Given that: $\cos\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$, where $0 < b < a$ and $\sin\theta < 0$, determine the value of $\tan\theta$ in terms of a and b. (6) **L3**

REDUCTION FORMULAE PROBLEMS:2.18-2.22

ADAPTED UNKNOWN SOURCE

2.18 **Without the use of a calculator,** simplify the following:

2.18.1 $\dfrac{\cos(90^\circ - x)\sin(180^\circ + x)\cos(x - 180^\circ)}{\sin x \tan(180^\circ - x)\tan(90^\circ - x)}$ (5) **L2**

2.18.2 $\tan(-\theta).\sin(90^\circ + \theta) - \dfrac{\sin(180^\circ + 2\theta)}{2\cos(360^\circ + \theta)}$ (6) **L2**

2.18.3 $\tan(180^\circ + \theta).\sin^2(90^\circ + \theta) - \cos(-\theta^\circ)\sin(540^\circ - \theta)$ (5) **L2**

2.18.4 $\dfrac{\sin 2x + 2\cos(90^\circ - x)}{\sin^2(90^\circ - x) - \cos(x - 180^\circ)}$ (4) **L2**

2.18.5 $\dfrac{\cos(90^\circ + x)\sin(x - 180^\circ) - \cos^2(180^\circ - x)}{\cos(-2x)}$ (5) **L3**

2.18.6 $\dfrac{2\cos 105^\circ \cos 15^\circ}{\cos(45^\circ - x)\cos x - \sin(45^\circ - x)\sin x}$ (6) **L2**

2.18.7 $\dfrac{\cos^2 225^\circ.\tan(180^\circ + x).\cos(90^\circ + x)}{\sin(-x)}$ (5) **L2**

2.18.8 $\dfrac{\sin 780^\circ.\sin(90^\circ - x).\tan(x - 180^\circ)}{\cos(-30^\circ).\cos(90^\circ - x) - \cos 450^\circ}$ (5) **L2**

2.18.9 $\dfrac{\tan(360^\circ - x).\cos(x - 90^\circ) + \cos(540^\circ - x)}{\tan(x - 180^\circ)}$ (5) **L3**

2.18.10 $\dfrac{\sin(180^\circ - 2x)}{\cos 2x + 1}$ (4) **L2**

2.18.11 $\dfrac{\cos(\theta-x)}{\sin(x+450°-\theta)}$ (4) **L2**

2.18.12 $\dfrac{\cos(40°-x).\cos x-\sin(40°-x).\sin x}{\sin 205°.\cos 25°}$ (3) **L2**

2.18.13 $\sin(41°+\theta).\cos(11°+\theta)-\cos(41°+\theta).\sin(11°+\theta)$ (4) **L2**

2.18.14 $\sin 20°.\cos 320°+\cos(-20°).\sin 400°$ (4) **L2**

2.18.15 $\dfrac{1}{2}\sin 165°-\dfrac{\sqrt{3}\cos 15°}{2}$ (4) **L2**

2.18.16 $\dfrac{1-\cos^2 x}{\sin x}.\cos x\left(1+\tan^2 x\right)$ (5) **L2**

2.18.17 $\dfrac{1-\cos 2x}{\sin x\cos x}$ (4) **L2**

2.19 If $\sin 76°=x$ and $\cos 76°=y$, show that $x^2-y^2=\sin 62°$ (5) **L3**

2.20 If $\tan x=3k$ and $\tan y=2k$, determine $\dfrac{\sin(x-y)}{\cos x\cos y}$ in terms of k. (5) **L3**

2.21 Given: $\sin\hat{A}+\sin\hat{B}=\dfrac{3}{2}$ and $\hat{A}+\hat{B}=90°$ (2) **L2**

Without using a calculator, determine the value of $\sin 2A$

2.22 Calculate the value of the following expression, **without using a calculator** (3) **L3**

$(\tan 92°)\times(\tan 94°)\times(\tan 96°).......(\tan 176°)(\tan 178°)$

PROVING IDENTITIES PROBLEMS: 2.23-2.35

2.23 Prove the following identities **(without the use of a calculator)**

2.23.1 $\dfrac{\sin^3 x+\sin x\cos^2 x}{\cos(360°-x)}=\tan x$ (4) **L2**

2.23.2 $\dfrac{\sin 2x+\cos 2x+1}{\cos 2x}=\dfrac{2\cos x}{\cos x-\sin x}$ (5) **L2**

2.23.3 $\dfrac{2\cos 2A.\cos(180°-A)}{\sin^2 A-\cos^2 A}+2\tan A.\sin A=\dfrac{2}{\cos A}$ (6) **L3**

2.23.4 If $M=\dfrac{2\sin^2(180°+\theta)-\sin(2\theta-180°)}{\cos 2\theta}$ and $P=\dfrac{2\sin\theta}{\cos\theta-\sin\theta}$ (6) **L3**

Prove that : $M=P$

2.23.5 $\cos(\alpha-30°)-\cos(\alpha+30°)=\sin\alpha$ (3) **L2**

2.23.6 $\sin(A-B)+\sin(A-B)=2\sin A.\cos B$ (3) **L2**

2.23.7 $\sin(45°+x)\sin(45°-x)=\dfrac{1}{2}\cos 2x$ (3) **L2**

2.23.8 $\sin(x+63°).\cos(x+378°)+\cos(x+63°).\cos(x+108°)=\dfrac{1}{\sqrt{2}}$ (4) **L2**

2.23.9 $\dfrac{\sin\theta}{\cos\theta-\sin\theta}-\dfrac{\sin(180°+\theta)}{\cos\theta+\sin\theta}=\tan 2\theta$ (5) **L3**

2.23.10 $\dfrac{1}{1-\cos(180°-x)}+\dfrac{1}{1-\sin(90°-x)}=\dfrac{2}{\sin^2 x}$ (5) **L2**

2.23.11 $\dfrac{\sin(180°-2x)}{\cos 2x+1}=\tan x$ (5) **L2**

2.23.12 $\dfrac{\sin 2x}{\cos 2x+\sin^2 x}=2\tan x$ (6) **L2**

2.23.13 Show that $\cos x\left(\dfrac{\cos 2x}{\cos x+\sin x}\right)=\dfrac{1}{2}$ can be simplified to $\cos 2x=\sin 2x$ (5) **L3**

2.23.14 $\dfrac{\cos 2x}{\cos x+\sin x}=\cos x-\sin x$ (3) **L2**

2.23.15 $(\sin x+\cos x)^2=\sin 2x+1$ (5) **L3**

2.23.16 $\dfrac{\sin 2x+1}{\cos 2x}=\dfrac{\sin x+\cos x}{\cos x-\sin x}$ (5) **L3**

2.23.17 $\cos 6x+\cos 2x=2\cos 4x.\cos 2x$ (7) **L3**

2.23.18 $\sqrt{3}\sin(x+60°)-\sin(x+30°)=\cos x$ (4) **L3**

2.23.19 $\cos(x+60°)=\dfrac{1}{2}\cos x-\dfrac{\sqrt{3}}{2}\sin x$ (5) **L3**

2.23.20 $\sin(\theta+30°)-\sqrt{3}\sin(\theta+60°)=-\cos\theta$ (5) **L3**

2.23.21 $\cos 4A = 8\cos 4A-8\cos 2A+1$ (5) **L3**

2.23.22 $\dfrac{(\cos^2 x-\sin^2 x)^2}{\cos^4 x-\sin^4 x}=\cos 2x$ (5) **L3**

2.23.23 $(1-\cos 2x)\left(1+\dfrac{1}{\tan^2 x}\right)=2$ (5) **L3**

2.23.24 $4\sin x\cos^3 x-4\cos x\sin^3 x=\sin 4x$ (5) **L3**

2.23.25 $\dfrac{\sin 2x+1}{\cos 2x}=\dfrac{\sin x+\cos x}{\cos x-\sin x}$ (5) **L3**

2.23.26 $\dfrac{\sin 2x-\sin x}{\cos 2x+\cos x}=\dfrac{\sin x}{1+\cos x}$ (5) **L3**

2.23.27 $\left(\dfrac{\cos 2A\sin 2A}{\cos A\sin A}\right)=4\cos^2 A$ (5) **L2**

2.23.28 $\dfrac{\cos 2A}{(\cos A+\sin A)^3}=\dfrac{\cos A-\sin A}{1+\sin 2A}$ (5) **L3**

2.23.29 $\dfrac{2+2\cos x}{\sin 2x}=\dfrac{\tan x}{1-\cos x}$ (5) **L3**

2.23.30 $\sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}}=\dfrac{2\cos\alpha+\sin 2\alpha}{\cos 2\alpha+1}$ (5) **L3**

2.23.31 $\dfrac{\sin 2\theta}{2\sin\theta+\cos 2\theta-1}=\dfrac{1+\sin\theta}{\cos\theta}$ (5) **L2**

2.23.32 $\dfrac{\sin\dfrac{3x}{2}\cos\dfrac{x}{2}+\cos\dfrac{3x}{2}\sin\dfrac{x}{2}}{\sin\dfrac{x}{2}\cos\dfrac{x}{2}}=4\cos x$ (5) **L3**

2.24 Consider the identity: $\dfrac{\cos\theta}{\sin\theta}-\dfrac{\cos 2\theta}{\sin\theta\cos\theta}=\tan\theta$

2.24.1 Prove the identity. (4) **L3**

2.24.2 For which value(s) of θ in the interval 0° < θ < 180° will the identity be undefined? (5) **L2**

2.25 Given the identity **:** $\dfrac{\cos x-\sin x\cos 2x}{\cos 2x}=\cos x$

2.25.1 Prove the identity. (4) **L3**

2.25.2 For which values of x is the identity undefined? (5) **L3**

2.26 Consider the identity : $\dfrac{1-\tan A}{1+\tan A}=\dfrac{\cos 2A}{1+\sin 2A}$

2.26.1 Prove that : $\dfrac{1-\tan A}{1+\tan A}=\dfrac{\cos 2A}{1+\sin 2A}$ (6) **L3**

2.26.2 Hence, calculate $\dfrac{1-\tan 22,5^\circ}{1+\tan 22,5^\circ}$ (5) **L3**

2.27 Given: $\cos(\alpha-\theta)=\cos\alpha\cos\theta+\sin\alpha\sin\theta$

2.27.1 Use the above identity to deduce that $\sin(\alpha-\theta)=\sin\alpha\cos\theta-\cos\alpha\sin\theta$ (3) **L2**

2.27.2 Hence or otherwise, simplify: $\sin 76^\circ.\sin 46^\circ-\sin 14^\circ.\sin 136^\circ$ to a single trigonometric ratio. (3) **L2**

2.28 If $T_n=\dfrac{1-\cos 2x}{\sin 2x}$, Show that $T_n=\tan x$ (4) **L2**

2.29 For what value(s) of $\theta\in[0^\circ;360^\circ]$ is $\sqrt{\cos 9\theta-2\cos 4\theta.\cos 5\theta}$ defined? (5) **L3**

2.30 Investigate whether the expression $\sqrt{\dfrac{1-\cos x}{1+\cos x}}$ is **real or undefined**, and state the **domain restrictions**: (4) **L3**

2.31 Determine all x values where the expression: $\sqrt{\dfrac{1-\cos 2x}{\sin x}}$ is **non-real**: (3) **L3**

2.32 Prove that $\sqrt{-\cos^2(90^\circ-K)\cos K\cos(-K)}$ is **non - real** for any real value of K. (5) **L3**

2.33 For which value(s) of x will $\dfrac{\sin(180^\circ-2x)}{\cos 2x+1}$ have **no real** solution if $0^\circ\le x\le 180^\circ$? (4) **L3**

2.34 If $\cos(A+B)=m$ and $\cos(A-B)=n$, prove that $\cos A.\cos B=\dfrac{m+n}{2}$ (5) **L4**

2.35 Express the following identity to a single trigonometric ratio without half angles: (4) **L3**

$\sin^2\left(\dfrac{x}{2}\right)-\cos^2\left(\dfrac{x}{2}\right)$

TOPIC 3. Trigonometric Equations and General Solutions

GUIDELINES, SUMMARY NOTES, & STRATEGIES

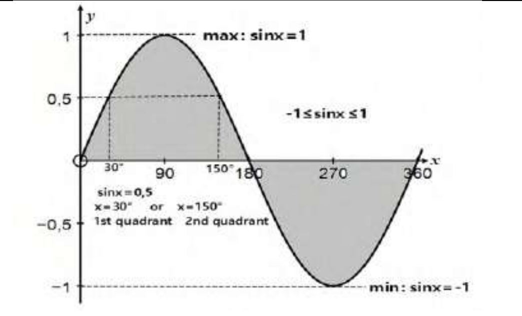
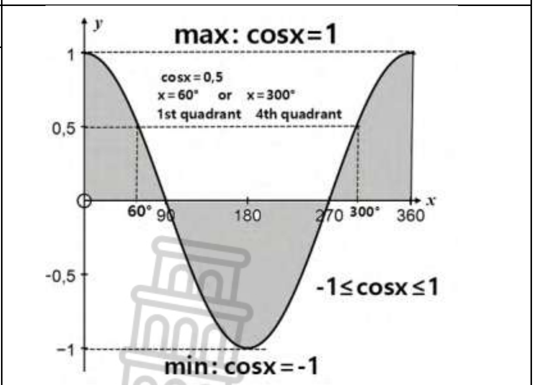
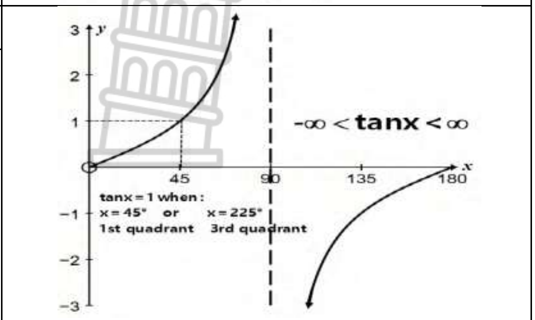
Trigonometric equations have **periodic solutions**, resulting in infinite possibilities. Therefore, **general solution** is required, by adding:

- $k.360°$ for sine and cosine because they repeat every $360°$.
- $k.180°$ for tangent because it repeats every $180°$.

STEPS TO SOLVE Trigonometric Equations

- Simplify the equation to a single trigonometric ratio by using algebraic methods and trigonometric identities (Trig. Ratio with angle = constant)
- Determine the reference angle (**DO NOT** simplify reference angle)
- Identify possible quadrants
- Apply restrictions where necessary.

NOTE:

sin x		
$\sin x = 1$ $0 < \sin x < 1$ $-1 < \sin x < 0$ $\sin x = -1$ $\sin x = 0$	$x = 90° + k.360°, k \in Z$ Quadrant 1 and Quadrant 2 Quadrant 3 and Quadrant 4 $x = 270° + k.360°, k \in Z$ $x = 0° + k.360°$ or $x = 180° + k.360°, k \in Z$	
cos x		
$\cos x = 1$ $0 < \cos x < 1$ $-1 < \cos x < 0$ $\cos x = -1$ $\cos x = 0$	$x = 0° + k.360°$ or $x = 180° + k.360°, k \in Z$ Quadrant 1 and Quadrant 4 Quadrant 2 and Quadrant 3 $x = 180° + k.360°, k \in Z$ $x = 90° + k.360°$ or $x = 270° + k.360°, k \in Z$	
tan x		
$\tan x > 0$ $\tan x = 0$ $\tan x < 0$ $\tan x$ is undefined	Quadrant 1 $x = 0° + k.180°, k \in Z$ Quadrant 2 $x = 90° + k.180°$	

EXAMPLES

Solve the following trigonometric equations:

3.1 $\sin 2x = \cos x$

$\sin 2x = \sin(90° - x)$ … co-ratio

$2x = 90° - x$ … Ref ∠ (do not simplify)

$2x = 90° - x + k.360°$ or $2x = 180° - (90° - x) + k.360°, k \in Z$

$x = 30° + k.120°$ or $x = 90° + k.360°$

Alternatively: $\sin 2x = \cos x$ (Inclusion of a double angle)

$2\sin x\cos x - \cos x = 0$ … double angle

$\cos x(2\sin x - 1) = 0$ … common factor

$\cos x = 0$ or $\sin x = \frac{1}{2}$

$x = 90° + k.360°$ $x = 30°$ Ref ∠

$x = 270° + k.360°$ $x = 30° + k.360°$

$x = 150° + k.360°\ , k \in Z$

3.2 $\cos^2 x + \cos x = \sin^2 x$

$\cos^2 x + \cos x = 1 - \cos^2 x$ … identity

$2\cos^2 x + \cos x - 1 = 0$ … quadratic trinomial

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = -1$

$x = 60°$ Ref ∠ $x = 180° + k.360°\ , k \in Z$

$x = 60° + k.360°$

OR $x = 360° - 60° + k.360°, k \in Z$

$= 300° + k.360°$

3.3 $2(\sin 20°\cos x - \sin x\cos 20°) = 1$

$2\sin(20° - x) = 1$

$\sin(20° - x) = \frac{1}{2}$ … compound angle

$20° - x = 30°$ … Ref ∠ **(do not simplify)**

$20° - x = 30° + k.360°$ or $20° - x = 180° - 30° + k.360°, k \in Z$

$x = -10° - k.360°$ or $x = -130° - k.360°$

3.4 $2\sin x\cos x - 4\cos x = \sin x - 2$ where $x \in [-180°; 360°]$

$2\cos x(\sin x - 2) - (\sin x - 2) = 0$ … factorise by grouping

$(\sin x - 2)(2\cos x - 1) = 0$ … common factor

$\sin x = 2$ or $\cos x = \frac{1}{2}$

No solution $x = 60°$ Ref ∠

It must be $-1 \le \sin x \le 1$ $x = 60° + k.360°$ or $x = 300° + k.360°, k \in Z$

$x = \pm 60°, \pm 300°$

ACTIVITIES

MAY/JUNE 2024

3.1 Given the expression: $\dfrac{\sin 150° + \cos^2 x - 1}{2}$

3.1.1 Simplify the expression to a single trigonometric term in terms of $\cos 2x$ (6) **L3**

3.1.2 Hence, determine the general solution of $\dfrac{\sin 150° + \cos^2 x - 1}{2} = \dfrac{1}{25}$ (5) **L2**

3.2 Without the use of a calculator, determine the number of solutions for the equation $\sqrt{2}\sin 2x = \sin x + \cos x$ in the interval $x \in [-90°; 180°]$ (4) **L3**

NOVEMBER 2024

3.3 Given: $f(x) = \sqrt{6\sin^2 x - 11\cos(90° + x) + 7}$ (6) **L3**

Solve for x in the interval: $x \in [0°; 360°]$ if $f(x) = 2$

MAY/JUNE 2025

3.4.1 Simplify $\sin 4x - \sin 10x$ (2) **L3**

3.4.2 Hence, determine the solution for: $\sin 4x - \sin 10x = \sin 3x$ for $x \in [0°; 30°]$ (5) **L2**

ADAPTED

3.5 Determine the general solution of the following equations:

3.5.1 $\sqrt{\tan x} = x + \dfrac{1}{x}$ if $x^2 + \dfrac{1}{x^2} = 1$ (3) **L3**

3.5.2 $\sin\theta\cos^2\theta = \sin^3\theta$ (5) **L2**

3.5.3 $\cos\dfrac{x}{2} = -\dfrac{1}{2}$ (3) **L2**

3.5.4 $\cos^2 x + 3\cos x = \sin^2 x - 2$ (5) **L2**

3.5.5 $3\cos^2 x - 3 = \sin^2 x - 1$ (4) **L3**

3.5.6 $2\cos^2\beta + \sin\beta = 1$ (4) **L2**

3.5.7 $\tan^2 x = \dfrac{2}{\tan^2 x} - 3$ (5) **L3**

3.5.8 $\sin^2 x - \cos^2 x + \sin x + 1 = 0$ (5) **L2**

3.5.9 $\sin^2 x + \cos 2x - \cos x = 0$ (6) **L2**

3.5.10 $6\cos A - 5 = \dfrac{4}{\cos A}$ (7) **L3**

3.5.11 $3\cos x = \sqrt{2}\sin x$ (5) **L3**

3.5.12 $\cos 2\theta = \sin(\theta - 30°)$ (5) **L2**

3.5.13 $2\cos x = \sin(x + 30°)$ (5) **L2**

3.5.14 $\cos x + \sin x = 1$ (6) **L3**

3.6 Given: $\dfrac{\cos x}{\sin 2x} - \dfrac{\cos 2x}{2\sin x} = \sin x$

3.6.1 Prove the identity (4) **L3**

3.6.2 Hence, solve for x where $x \in [0°;360°]$, if $1 + 2\cos 2x = \dfrac{\cos 2x}{2\sin x} - \dfrac{\cos x}{\sin 2x}$ (6) **L3**

3.7 Consider: $f(\theta) = \sin(2\theta - 15°)\cos(\theta - 30°) + \cos(2\theta - 15°)\sin(\theta - 30°)$ (7) **L3**

Determine the general solution of $f(\theta) = 0{,}8$

3.8 Determine the general solution of:

3.8.1 $\sin x + 2\cos(x + 30°) = -2\cos^2 x$ (6) **L3**

3.8.2 $\tan\theta = 2\sin 2\theta$, where $\cos\theta < 0$ (7) **L3**

3.8.3 $\sin 48 \cos x - \cos 48 \sin x = \cos 2x$ (5) **L3**

3.8.4 $3\tan 4\beta = -2\cos 4\beta$ (7) **L3**

3.8.5 $(4\sin 3x + 1)(\sin x - 5\cos x) = 0$ (6) **L2**

3.8.6 $\sin x \cos x + \sin x = 3\cos^2 x + 3\cos x$ (7) **L3**

3.8.7 $6\sin x \cos x + 3\cos x - 4\sin^2 x - 2\sin x = 0$ (7) **L3**

3.8.8 $\cos(\beta + 30°) = 2\sin\beta$ (4) **L3**

3.8.9 $1 - \tan x = \cos 2x$ (6) **L3**

3.8.10 $\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°) = \cos(2x - 20°)$ (6) **L3**

3.8.11 $2\sin\theta\cos\theta - 0{,}8 = 0$ (6) **L2**

3.8.12 $\cos(\text{A} - 60°) = -\sin 2\text{A}$ (7) **L3**

3.9 Solve for the following trigonometric equations in the specified domain

3.9.1 $\cos(x - 30°) = \sin 2x$; for $-90° \le x \le 180°$ (6) **L3**

3.9.2 $\sin 2A - \cos A = 0$; for $-360° \le A \le 180°$ (6) **L2**

3.9.3 $4\sin^2\theta - 3\sin\theta = 1$ for $\theta \in [0°;360°]$ (6) **L3**

3.9.4 $\cos 2x - 7\cos x = 3$; for $x \in [-180°;180°]$ (7) **L3**

3.9.5 $(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ for $-90° \le x \le 180°$ (6) **L3**

3.9.6 $1 - \cos 2\text{B} = \sin \text{B}$ for $\text{B} \in [-90°;360°]$ (6) **L2**

TOPIC 4. Trigonometric functions (graphs)

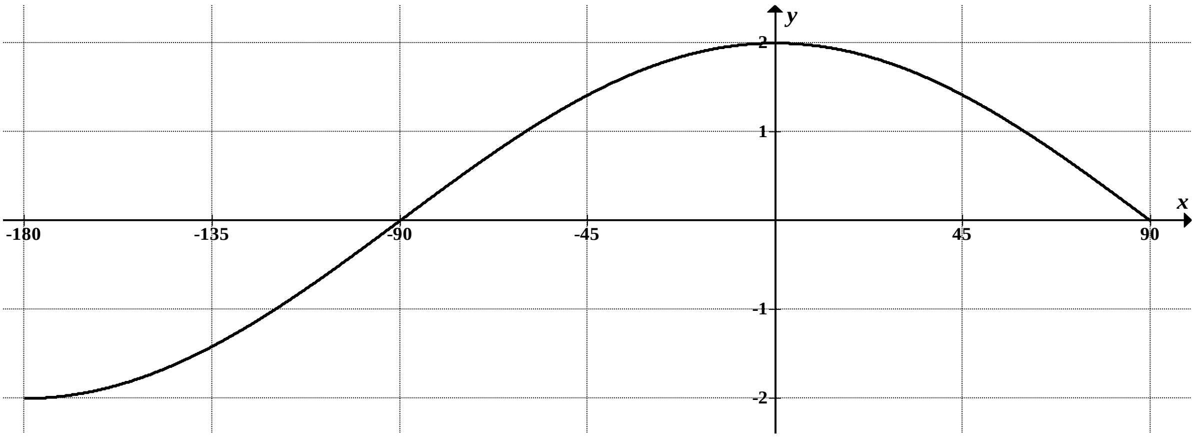
GUIDELINES, SUMMARY NOTES, & STRATEGIES

- The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.
- Candidates must be able to use and interpret functional notation. Learners must understand how has been transformed to generate $f(-x),\ -f(x),\ f(x+a),\ f(x)+a$ and $a.f(x)$ where $a \in \mathbb{R}$.

ACTIVITIES

4.1 **KZN NOV 2025 (GRADE 11)**

Below is a sketch of the graph of $f(x) = a\cos x$ for $x \in [-180°; 90°]$.



4.1.1	Write down the value of a.	(1)	**L1**
4.1.2	Write down the period of $g(x) = -\cos 2x$.	(1)	**L2**
4.1.3	Sketch the graph of $g(x) = -\cos 2x$ for $x \in [-180°; 90°]$ on the set of axes.	(3)	**L2**
4.1.4	Use the graphs to write down the value of x for which $f(x) - 1 = g(x) + 2$.	(2)	**L4**
4.1.5	Determine minimum value $1 - 2\left[\sin(90° - x)\right]$.	(3)	**L3**

4.2 **IEB MAY/JUNE 2024**

Given: $f(x) = \sin 3x$ and $g(x) = \tan\dfrac{3}{2}x$.

4.2.1	Determine the general solution for $f(x) = g(x)$.	(8)	**L4**
4.2.2	Sketch the graphs for $f(x)$ and $g(x)$ on the same set of axes for $0° \le x \le 120°$. Show all intercepts with axes, points of intersection between the graphs, and turning points.	(6)	**L2**
4.2.3	Determine the values of x for $0° \le x \le 120°$ for which $f(x) \ge g(x)$.	(2)	**L2**

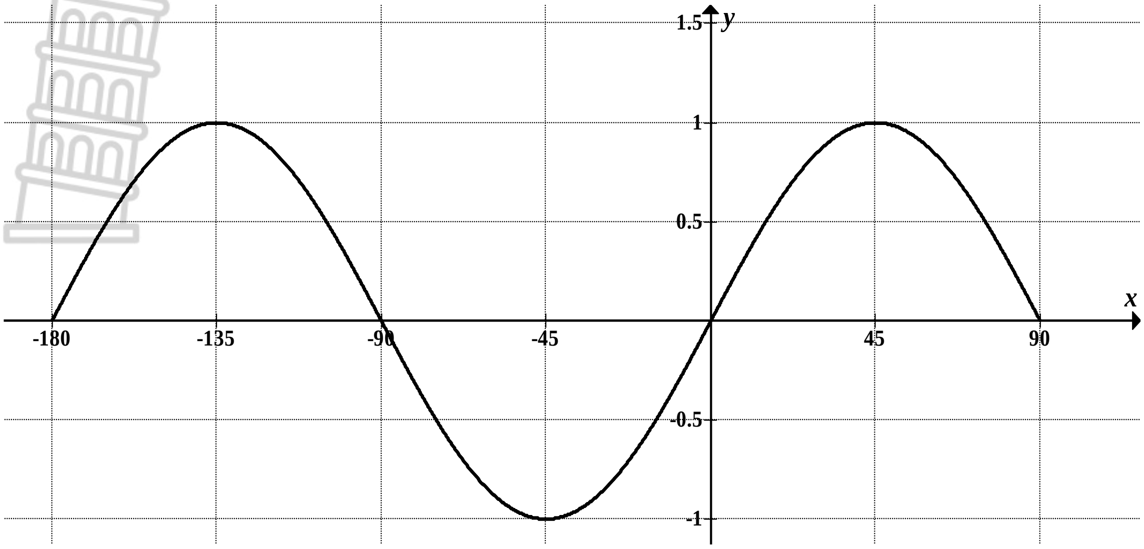
4.3 **IEB MAY/JUNE 2025**

Given $f(x) = \sin(x - 30°)$ and $g(x) = \cos 2x$.

4.3.1	Solve $f(x) = g(x)$ where $x \in [-180°\ ;\ 90°]$.	(6)	**L2**
4.3.2	Sketch the graphs of f and g on the same set of axes. Clearly label the endpoints, and y-intercepts.	(8)	**L2**
4.3.3	Using your graphs, determine the values of x in the same interval for which $\dfrac{f(x)}{g(x)} > 0$.	(3)	**L2**

KZN TOPIC TEST 2026

4.4 The graph of $f(x)=\sin 2x$, for $-180°\leq x\leq 90°$,is shown in the following sketch.

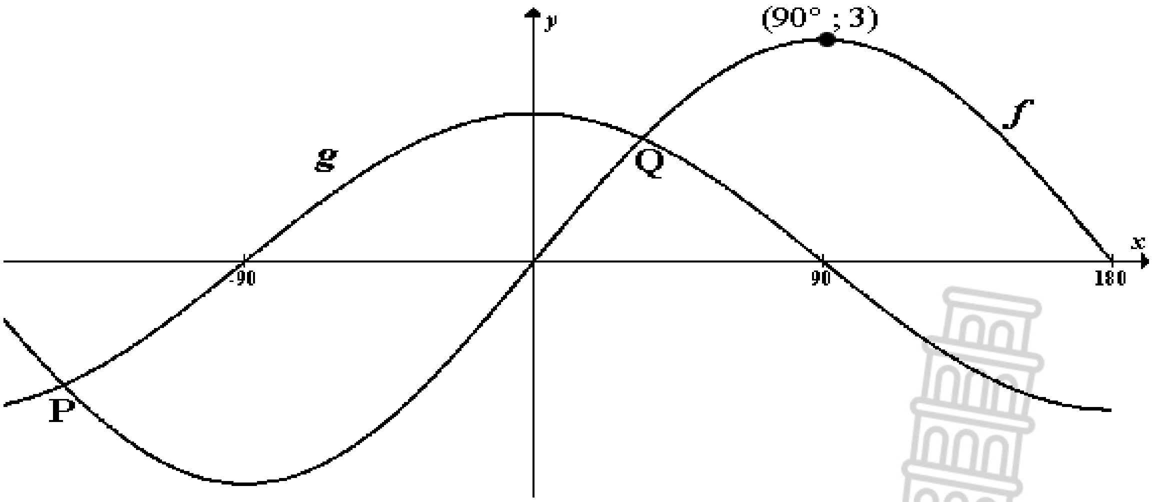


4.4.1 Determine the period of $f\left(\frac{3}{2}x\right)$ (2) **L3**

4.4.2 Draw the graph of $g(x)=\cos(x-30°)$ for $-180°\leq x\leq 90°$, on the same system of axes. Clearly label ALL the x-intercepts with the axes, turning points and coordinate of the end point. (4) **L2**

4.4.3 Describe the transformation that graph f has to undergo to form $y=\sin(2x+60°)$. (2) **L3**

4.5 **KZN MAR 2026**



In the diagram below, the graphs of $f(x)=a\sin x$ and $g(x)=2\cos bx$ are drawn for the interval $x\in[-180°;180°]$. g passes through $(90°;0)$, and $(90°;3)$ is a turning point of f.

4.5.1 Determine the values of a and b. (2) **L1**

4.5.2 Determine the values of x, in the interval $x\in[-180°;0°]$, for which $f(2x)\leq 0$. (2) **L2**

4.5.3 P and Q are two points of intersection of f and g.

(a) Calculate the x-coordinates of P and Q. (4) **L3**

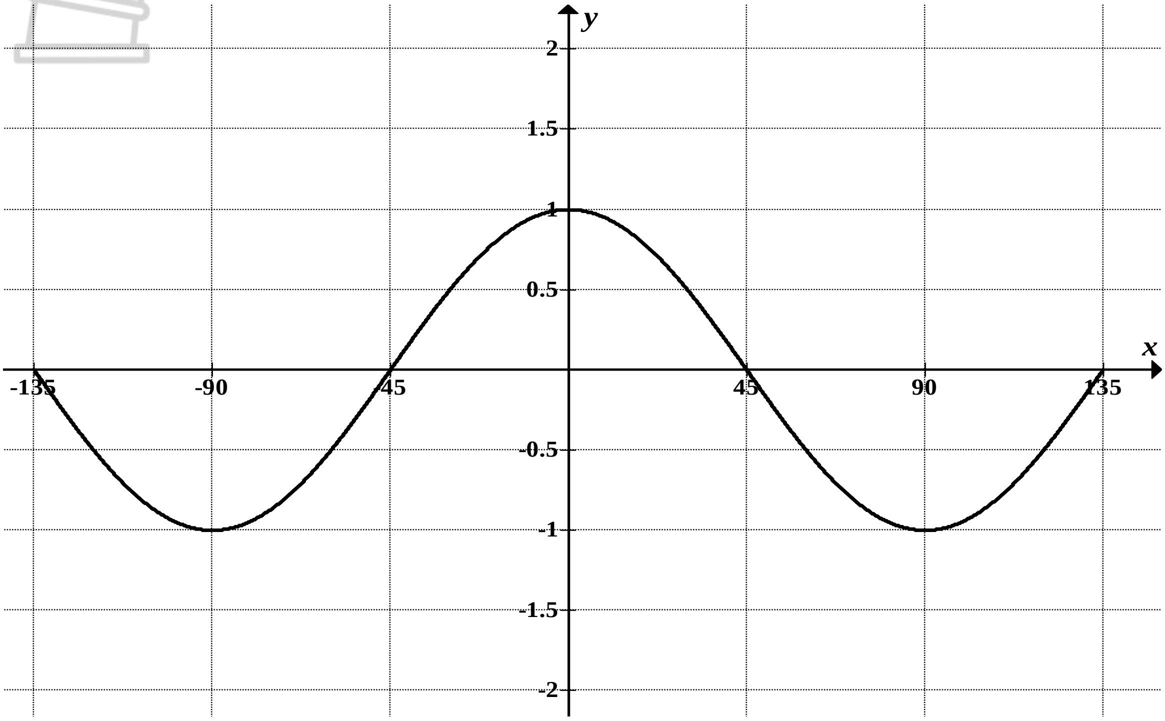
(b) Hence or otherwise determine the values of x in the interval $x \in [-180°;180°]$,

for which $\tan x > \frac{2}{3}$. (4) **L4**

4.5.4 What will the equation of g be after the y -axis has been shifted $45°$ to the left? (2) **L3**

4.6 **DBE NOV 2025**

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for $x \in [-135°;135°]$.



4.6.1 Write down the period of (1) **L1**

4.6.2 On the same set of axes as f , draw the graph of $g(x) = \tan 2x - 1$ for $x \in [-135°;135°]$. (3) **L2**

4.6.3 Graph f is translated $45°$ to the left to form graph h . (1) **L3**

4.6.4 Write down the range of h . (1) **L3**

4.6.5 Determine the values of x for which $(1 - \tan 2x)(\cos 2x) \geq 0$ in the interval $x \in [0°;135°]$. (4) **L4**

4.7 **FROM 2025 REVISION DOCUMENT**

Given: $f(x) = \cos(x + 45°)$ and $g(x) = -2\sin x$ where $-180° \leq x \leq 180°$

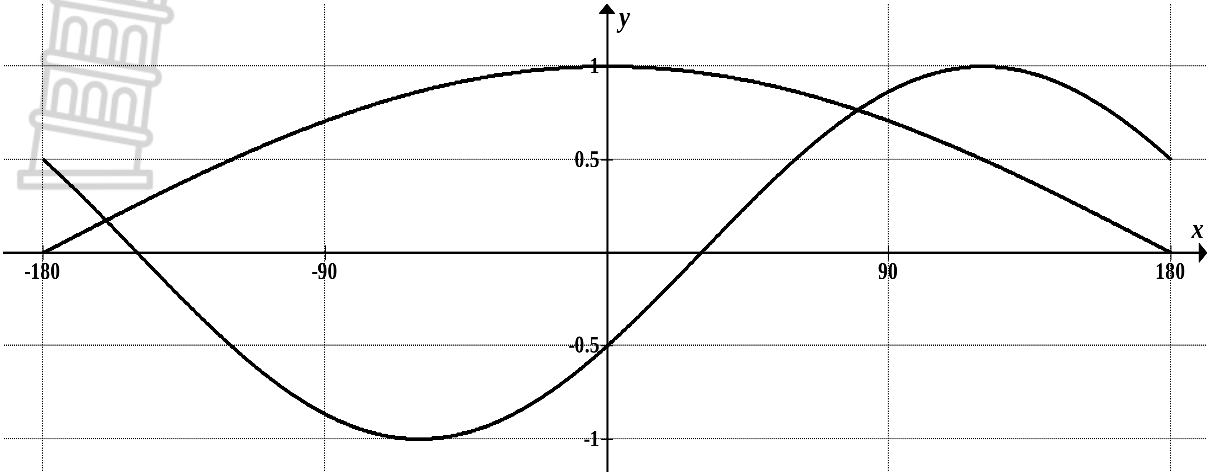
4.7.1 Draw the graph of f and g on the same set of axes, showing all the intercepts with the axes, turning points and endpoints. (2) **L1**

4.7.2 What is the period of g ? (3) **L2**

4.7.3 Determine by means of calculation, the values of x if $f(x) = g(x)$ in the interval above. (5) **L3**

4.8 **KZN PRE-EXAM 2025**

In the diagram are the graphs of the functions $f(x)=\cos 2x$ and $g(x)=\sin(x-30°)$ for $x\in[-180°;180°]$. The curve intersects at A and B.



4.8.1 Calculate the x - coordinates of the points A and B. (6) **L2**

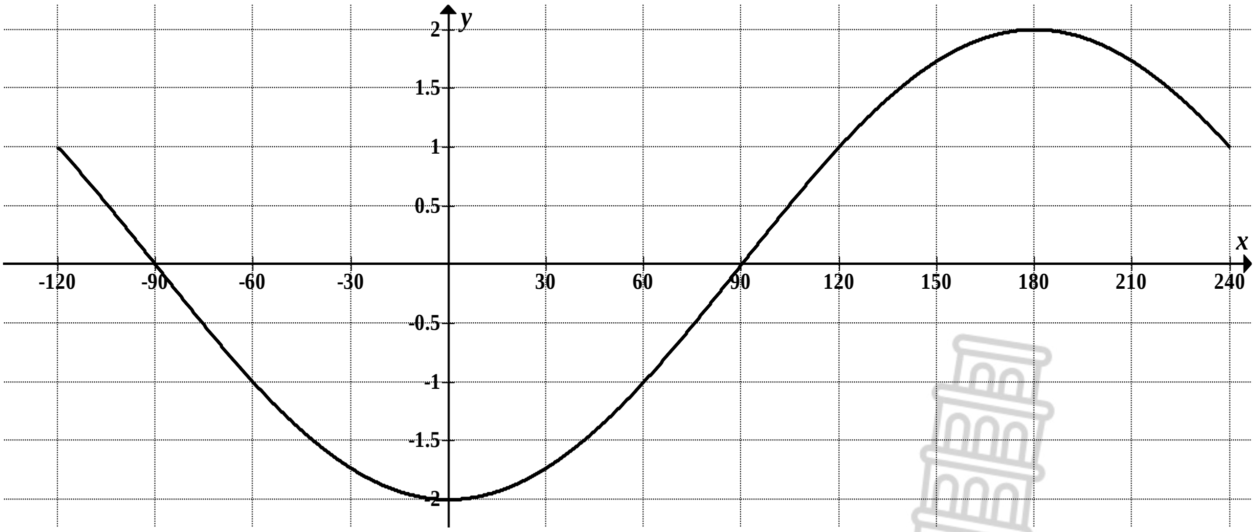
4.8.2 Determine the values of $x\in[-90°;180°]$ for which:

(a) $g(x)<0$ (2) **L2**

(b) $g(x)-f(x)<0$ (3) **L2**

4.9 **KZN TRIAL 2025**

In the diagram below, the graph of $f(x)=-2\cos x$ for $x\in[-120°;240°]$ is drawn.



4.9.1 On the same system of axes as f , sketch the graph of $g(x)=\sin(x+60°)$. Clearly indicate the intercepts with the axes, as well as the coordinates of the turning point and the endpoints. (4) **L2**

4.9.2 Write down the:

(a) period of g . (1) **L1**

(b) range of $f(x)-3$. (2) **L2**

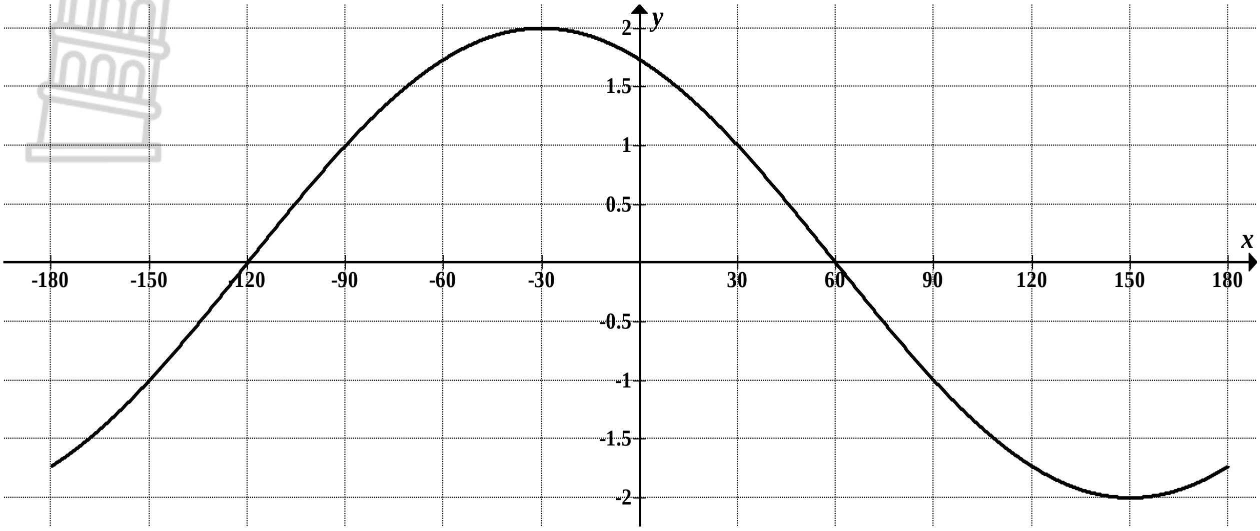
(c) number of solutions to $f(x)=g(x)$ in the interval $x\in[-120°;240°]$. (1) **L1**

4.9.3 For which value(s) of k will $g(x)-k=1$ have no real roots? (3) **L3**

4.9.4 The graph of h is obtained by reflecting g in the line $x = -30°$. Write down the equation of h in its simplest form. (2) **L3**

4.10 **NC TRIAL 2025**

In the diagram, the graph of $f(x) = a\cos(x + b)$ is drawn for the interval $x \in [-180°;180°]$.



4.10.1 Use the graph to determine the values of a and b. (2) **L1**

4.10.2 Draw the graph of $g(x) = \sin 2x + 1$ for the interval $x \in [-180°;180°]$ on the same axes as f. Clearly show the intercepts with axes as well as the coordinates of the turning points. (3) **L2**

4.10.3 Write down the period of g. (1) **L1**

4.10.4 Determine the range of $2g(x)$. (3) **L2**

4.10.5 Use the graphs to determine the value(s) of for which:

(a) $f(x) < g(x)$, in the interval $x \in [-180°;0°]$. (2) **L2**

(b) $\tan(x + b)$ is undefined in the interval $x \in [-180°;180°]$. (2) **L3**

4.10.6 The graph of g is shifted $45°$ to the left form the graph of p. Determine the equation of p in its simplest form. (2) **L3**

4.11 **MP SEPT 2024**

Given that $y = f(x) = 2\cos x$ and $y = g(x) = \sin(x + 30°)$:

4.11.1 Sketch the graphs of *f* and *g* on the same set of axes for $x \in [-180° ; 180°]$. (6) **L2**

4.11.2 Read the following from your graphs:

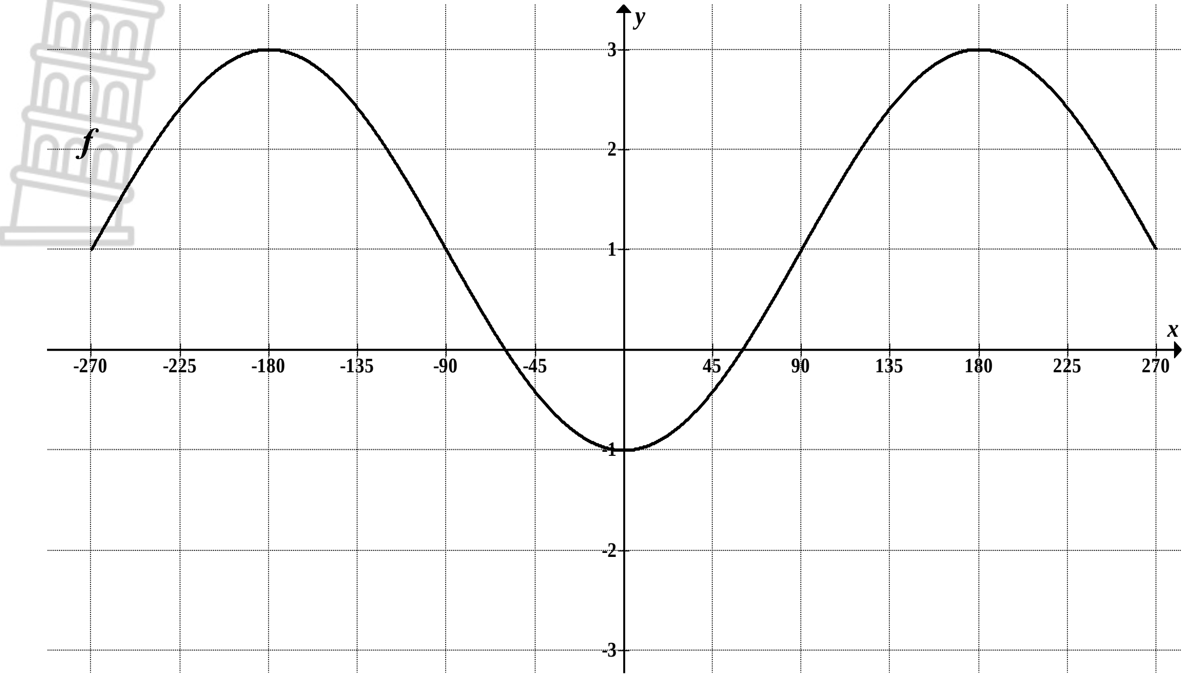
(a) Write down the period of *f*. (1) **L1**

(b) Determine one value of *x* for which $f(x) - g(x) = 1{,}5$. (1) **L2**

(c) Determine the positive values of *x* for which $2\sin(x + 30°).\cos x < 0$. (2) **L2**

4.12 **FS TRIAL 2025**

In the diagram below, the graph of $f(\theta) = p\cos + q$ is sketched for $-270° \leq \theta \leq 270°$.



4.12.1 Write down the amplitude of f. (1) **L1**

4.12.2 Write down the range of f. (1) **L1**

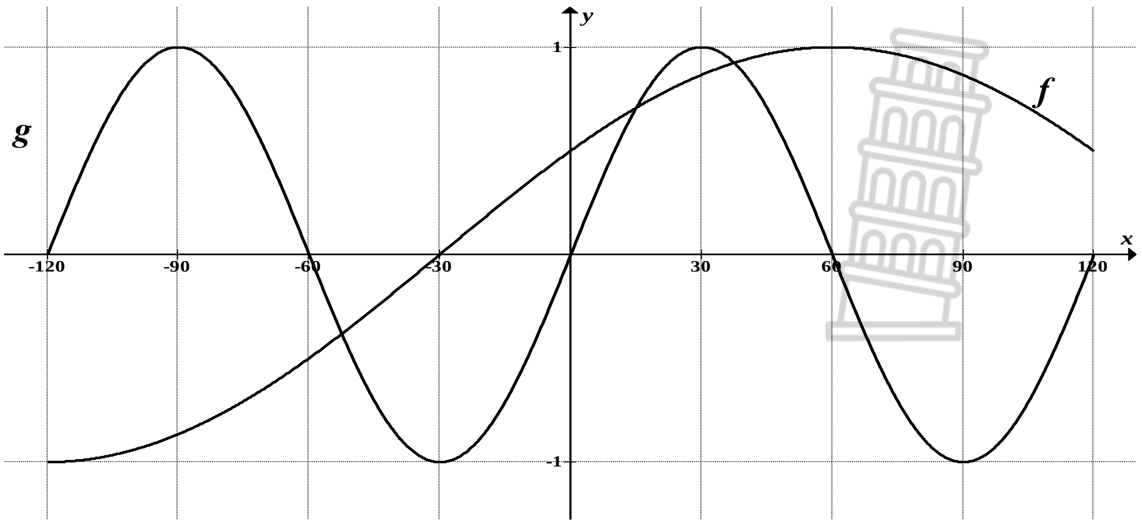
4.12.3 Write down the values of p and q. (2) **L2**

4.12.4 On the same set of axes, sketch the graph of $g(\theta) = -2\tan\theta$ for $\theta \in [-270°; 270°]$. (3) **L2**

4.12.5 Indicate on the graph using thickened lines the intervals on the horizontal θ axis, where $p\cos\theta + q \geq -2\tan\theta$ for $\theta \in [-270°; 270°]$. (3) **L2**

4.13 **WC TRIAL 2025**

In the diagram the functions of $f(x) = \cos(x - 60°)$ and $g(x) = \sin ax$ are given for $x \in [-120°; 120°]$.



4.13.1 Write down the period of g. (1) **L1**

4.13.2 Determine the value of a. (1) **L2**

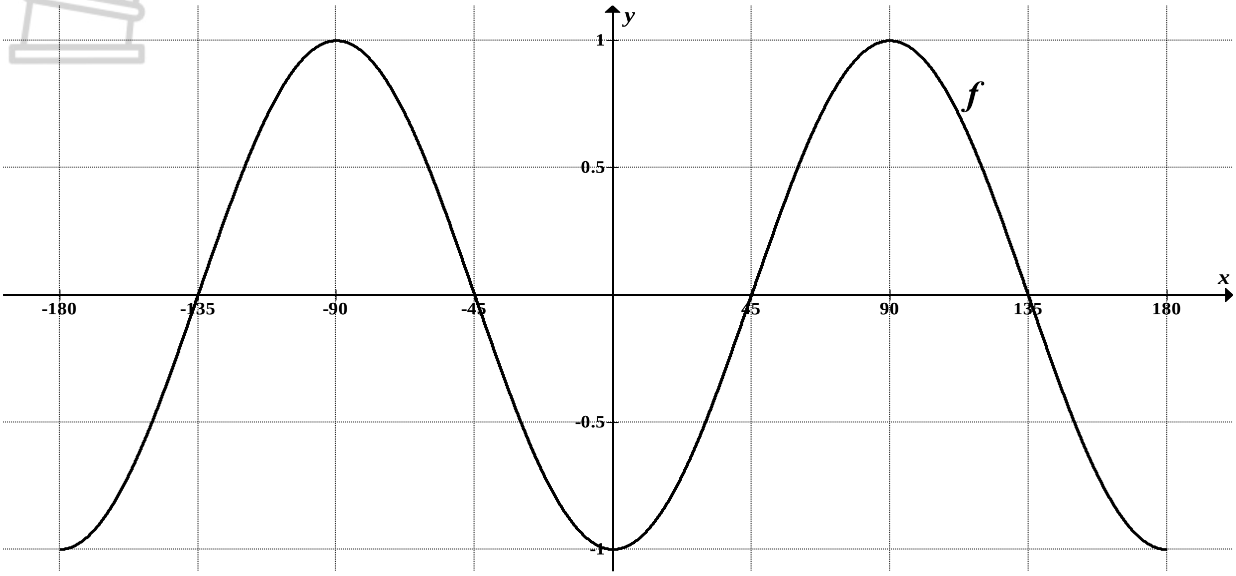
4.13.3 Write down the range of $f(x)-1$. (2) **L2**

4.13.4 If h is obtained by shifting f $30°$ to the right and reflecting it in the x-axis, write down the equation of h in its simplest form. (3) **L2**

4.13.5 For which values of x will $f'(x).f(x)>0$? (2) **L3**

4.14 **GP TRIAL 2025**

In the diagram below, the graph of $f(x)=-\cos 2x$ is drawn for the interval $x\in[-180°;180°]$.



4.14.1 Write down the period of f. (1) **L1**

4.14.2 Write down the range of f. (1) **L1**

4.14.3 On the same set of axes as f, draw the graph of $g(x)=\tan(x-45°)$ for the interval $x\in[-180°;180°]$. Clearly show the asymptotes and intercepts with the axes. (3) **L2**

4.14.4 For which value(s) of x is $f(x)\leq g(x)$ for the interval $x\in[-180°;0°]$. (2) **L2**

4.14.5 What is the maximum value of $h(x)=4^{2\sin^2 x-1}$ for $x\in\mathbb{R}$. (2) **L4**

4.15 **KZN SEPT 2023**

4.15.1 Sketch the graphs of $f(x)=2\sin x$ and $g(x)=\cos(x-30°)$ for $x\in[-180°\,;\,180°]$ on the same set of axis. Indicate the intercepts with the axes and also the turning points. (6) **L2**

4.15.2 Use your graphs to answer the following questions:

(a) Write down the period of g. (1) **L1**

(b) Write down the amplitude of $f(x)+1$. (1) **L2**

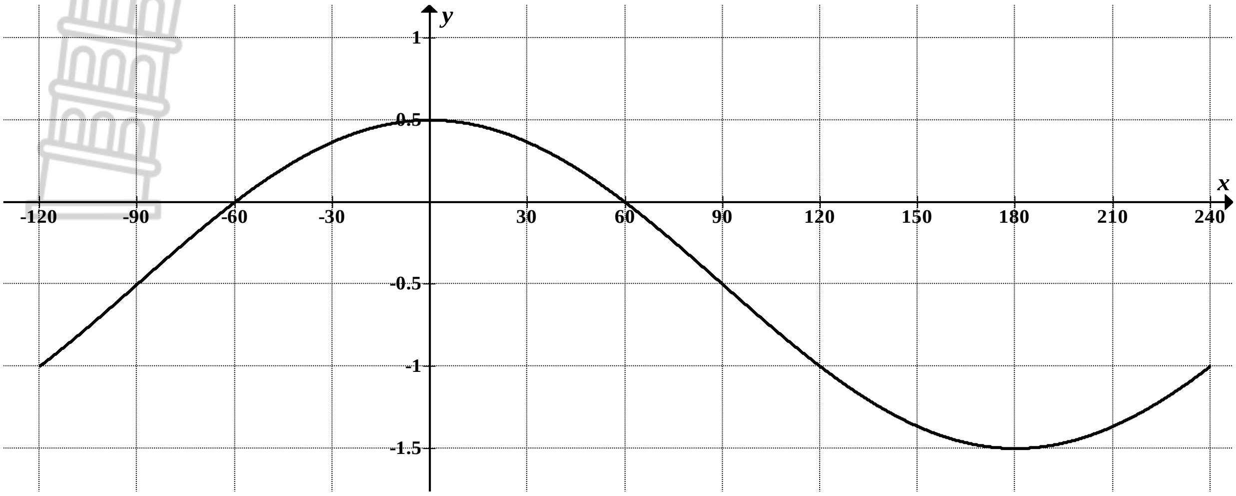
(c) Determine the values of x for which $f(x)>g(x)$. (4) **L2**

(d) Write down the values of x for which $f(x)=1,5+g(x)$. (2) **L2**

(e) Describe the transformation of g to h if $h(x)=-\sin x$. (2) **L4**

4.16 **EC TRIAL 2025**

Sketched below is the diagram of $f(x)=\cos x-\frac{1}{2}$ in the interval of $x\in[-120°;240°]$.



4.16.1 Determine the range of $f(x)+1$ (2) **L2**

4.16.2 On the same set of axes as f, sketch the graph $g(x)=\sin(x+30°)$ in the interval of $x\in[-120°;240°]$. Clearly indicate intercepts with axis. (3) **L2**

4.16.3 Write down the value(s) of x where g has a minimum value. (2) **L2**

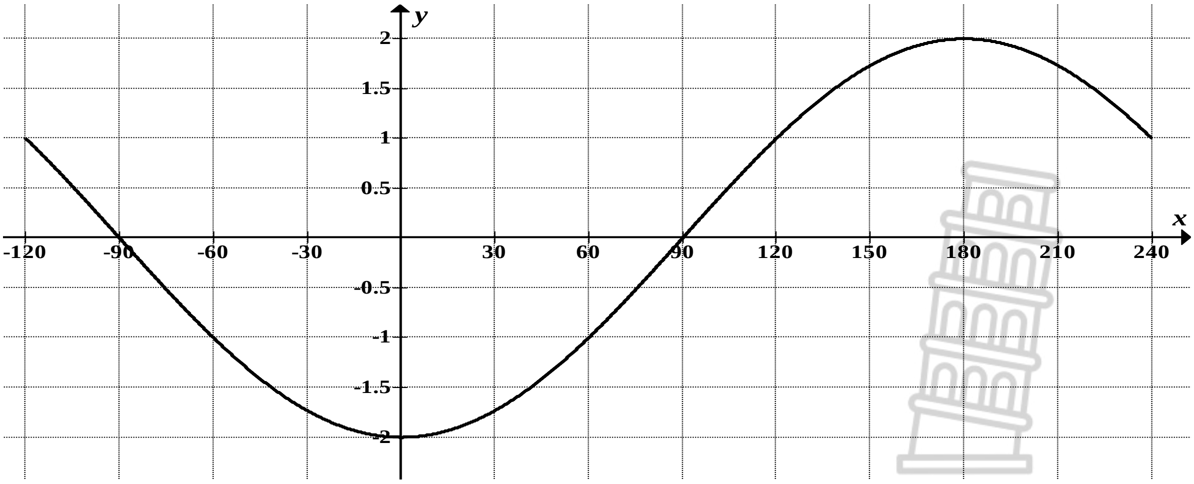
4.16.4 For which values of x is $f'(x)<0$. (2) **L2**

4.16.5 Write down the amplitude of f. (1) **L1**

4.16.6 Describe the transformation of f to $h(x)=-\cos x$. (2) **L2**

4.17 **MP TRIAL 2025**

In the diagram below, the graph of $f(x)=-2\cos x$ is drawn for $-120\leq x\leq 240°$.



4.17.1 Write down the amplitude of f. (1) **L1**

4.17.2 Write down the range of $f(x)+3$. (2) **L2**

4.17.3 Draw the graph of $g(x)=\sin(x+60°)$ for $-120\leq x\leq 240°$ on the same set of axes as f. (3) **L2**

4.17.4 For which values of k will $f(x)=k$ have no real roots? (2) **L3**

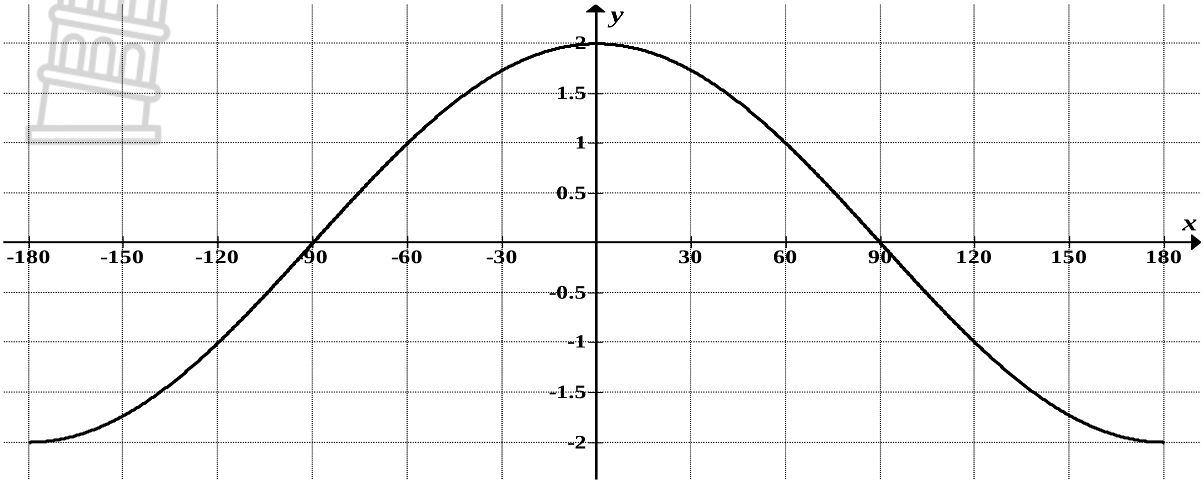
4.17.5 Determine the values of x in the interval $-120\leq x\leq 240°$ for which $x.g'(x)<0$. (3) **L4**

4.18 **LP TRIAL 2025**

4.18.1 Show that $2\cos x = \sin(x+30°)$ can be written as $\sqrt{3}\sin x = 3\cos x$. (3) **L3**

4.18.2 Hence solve the equation $2\cos x = \sin(x+30°)$ for $x \in (-180°\ ;\ 180°]$. (4) **L4**

The graph of $f(x) = 2\cos x$ for $x \in [-180°;180°]$ is drawn in the diagram below.



4.18.3 Draw the graph of $y = \sin(x+30°)$ for $x \in [-180°;180°]$ on the same set of axes as f.

Clearly indicate the intercepts and turning points. (3) **L2**

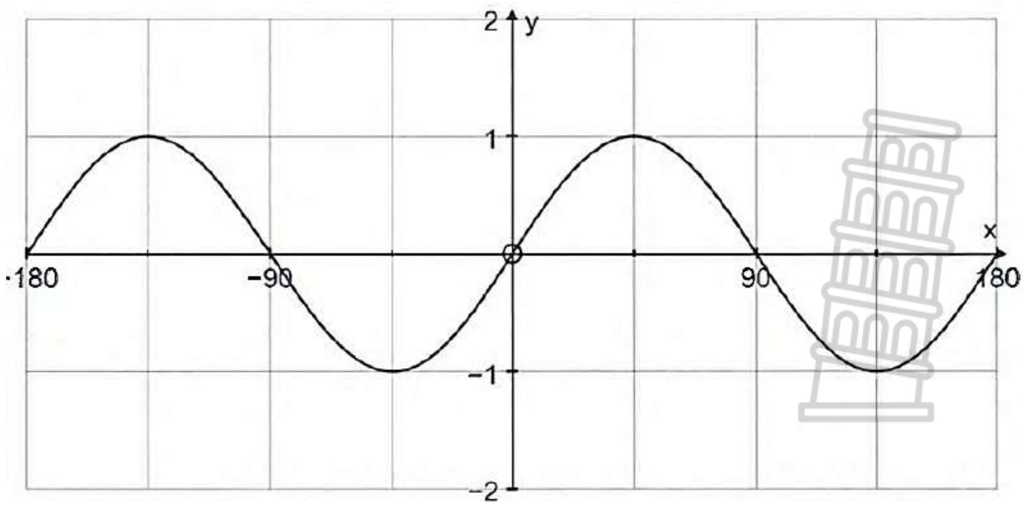
4.18.4 Use the graph and the information you have, to solve the following:

(a) $2\cos x > \sin(x+30°)$, $x \in (-180°;180°]$. (2) **L2**

(b) $\dfrac{2\cos x}{\sin(x+30°)} \geq 0$, $x \in (-180°;0°]$. (3) **L2**

4.19 **IEB NOVEMBER 2024**

Sketched is a graph of $f(x) = \sin bx$ for $x \in [-180°\ ;\ 180°]$.



4.19.1 Determine the value of *b*. (1) **L2**

4.19.2 State the period of *f*. (1) **L1**

4.19.3 Sketch the graph of $g(x) = -2\cos x$ on the same set of axes as *f*. (3) **L2**

4.19.4 State the amplitude of *g*. (1) **L1**

4.19.5 State the general solution for $f(x) = g(x)$. (2) **L3**

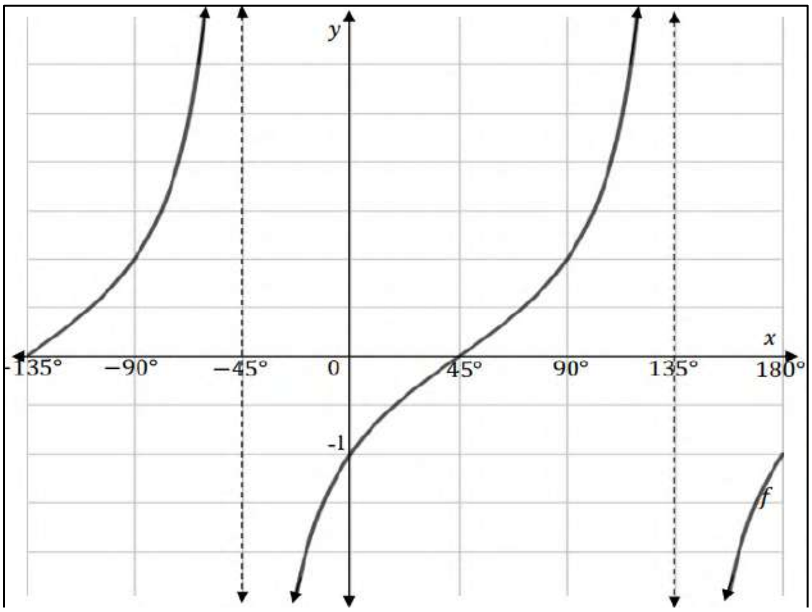
4.19.6 If $x \in [-180° ; 180°]$, determine the values of x where:

(a) $f(x) \geq g(x)$ (1) **L2** (b) $\frac{f(x)}{g(x)} < 0$ (2) **L2** (c) $g'(x) > 0$ (2) **L2**

4.19.7 If g is shifted 2 units up and $45°$ to the right, write down the new equation for g in the form $y = \ldots$ (2) **L2**

4.20 **CAPE WINELANDS SEPT 2024**

In the diagram, the graph of $f(x) = \tan(x + p°)$ is drawn for the interval $x \in [-135°; 180°]$ with asymptotes at $x = -45°$ and $x = 135°$.



4.20.1 Write down the value of p. (1) **L2**

4.20.2 Draw the graph of $g(x) = \sin 2x$ for the interval $x \in [-135° ; 180°]$ on the grid above. Show ALL intercepts with axes, as well as the minimum and maximum points. (3) **L2**

4.20.3 Write down the period of g. (1) **L1**

4.20.4 The graph of g is shifted 45° to the left to form the graph of h. Determine the equation of h in the simplest form. (2) **L2**

4.20.5 Use the graph to determine the values of x in the interval $x \in [-135° ; 0°]$ forwhich:

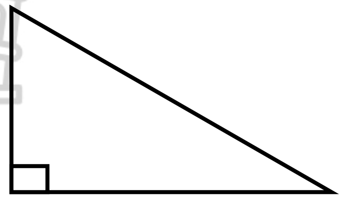
(a) $f(x) \leq -1$ (2) **L2**

(b) $\sin x \cos x + 2 < 2$ (3) **L3**

TOPIC 5. Problems in two and three dimensions

GUIDELINES, SUMMARY NOTES, & STRATEGIES

For Right-Angled Δ use:



- **Basic trig ratios (** $\sin\theta, \cos\theta, \tan\theta$ **)**
- **Pythagoras theorem** $r^2 = x^2 + y^2$
- Area $\Delta ABC = \frac{1}{2}bh$

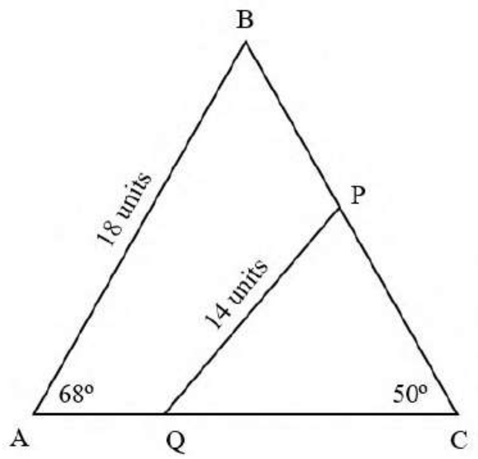
For Non-Right-angled Δ use:

Rule	Formula	When to use
Sine rule	$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$ or $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c}$	➢ Given two sides and the angle opposite one side ➢ One side and any two angles
Cosine rule	$a^2 = b^2 + c^2 - 2bc.\cos \hat{A}$ $b^2 = a^2 + c^2 - 2ac.\cos \hat{B}$ $c^2 = a^2 + b^2 - 2ab.\cos \hat{C}$	➢ Given two sides and the included angle ➢ Three sides
Area rule	Area of Δ ABC $= \frac{1}{2}bc.\sin \hat{A}$ $= \frac{1}{2}ac.\sin \hat{B}$ $= \frac{1}{2}ab.\sin \hat{C}$	➢ In order to use the formula for Area, two sides and the included angle is required

STRATEGIES

Note: When solving 3D problems separate all the triangles so that they will be in 2D and easy to solve. It is also advisable that you write all your findings back to the diagrams to help you with the next sub-question(s).

IEB 2017



5.1 In the diagram alongside an acute-angle Δ ABC is drawn.

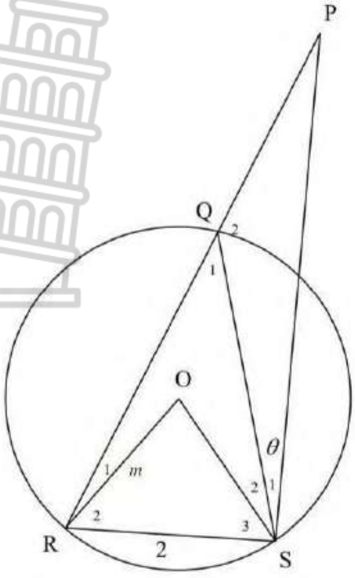
- A line PQ is drawn where P lies on the line BC and Q lies on the line AC
- The length of PQ is 14 units and the length of BC is 18 units.

$\hat{A} = 68°$ and $\hat{C} = 50°$

If the ratio of BP:PC is 2:3 Determine the size of $P\hat{Q}C$.

(6) **L3**

GP SEPT 2024

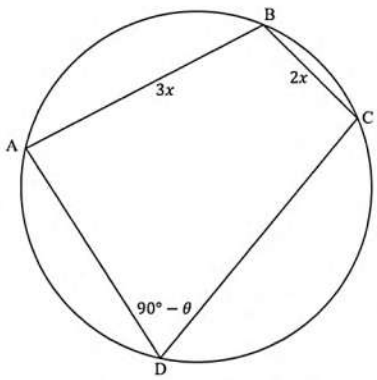


5.2 In the diagram, O is the centre of the circle with RS = 2cm, OR = *m* cm, $\hat{S}_1 = \theta$ and $\hat{P} = 2\hat{S}_1$

5.2.1 Determine the size of $R\hat{O}S$ in terms of θ, giving reasons. (2) **L2**

5.2.2 Prove that $m = \dfrac{1}{\sin 3\theta}$ (4) **L2**

NC SEPT 2024

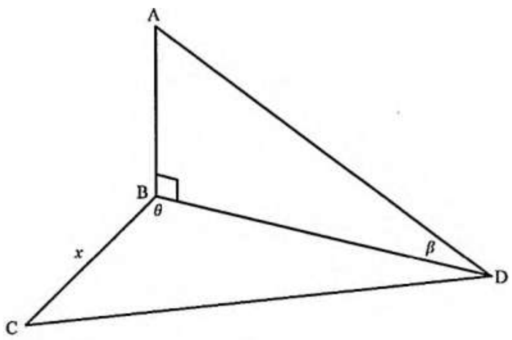


5.3 ABCD is a cyclic quadrilateral. $AB = 3x$, $BC = 2x$ and $\hat{D} = 90° - \theta$

Show that: $AC = x\sqrt{13 + 12\sin\theta}$

(6) **L3**

NC SEPT 2024

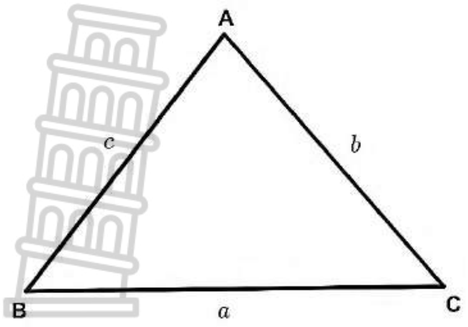


5.4 B, C and D are on the same horizontal plane. AB is a vertical tower. $CB = CD = x$, $C\hat{B}D = \theta$ and the angle of elevation of A from D is β

5.4.1 Show that $BD = 2x\cos\theta$ (4) **L2**

5.4.2 Determine an expression of AB in terms of x, θ and β (2) **L2**

GP SEPT 2024

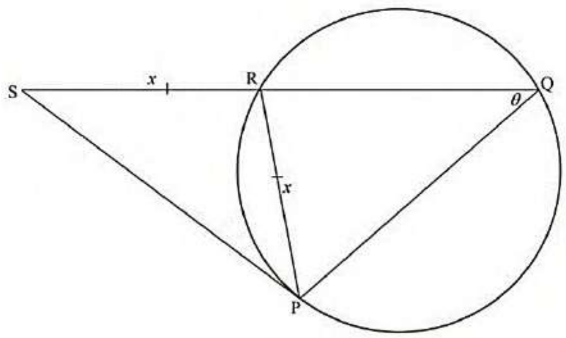


5.5 Triangle ABC is an isosceles with AB = BC

Prove that: (4) **L2**

$$\cos\hat{B} = 1 - \frac{b^2}{2a^2}$$

NC SEPT 2024

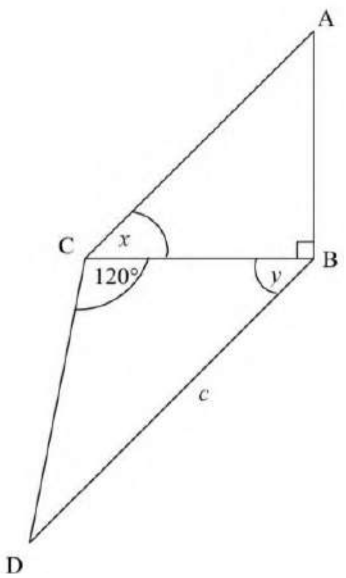


5.6 In the diagram alongside, PS is a tangent to the circle through P, Q and R. QRS is a straight line

$PS = RS = x$, $P\hat{Q}R = \theta$

Prove that $PS = 2x\cos\theta$. (6) **L3**

NC SEPT 2024

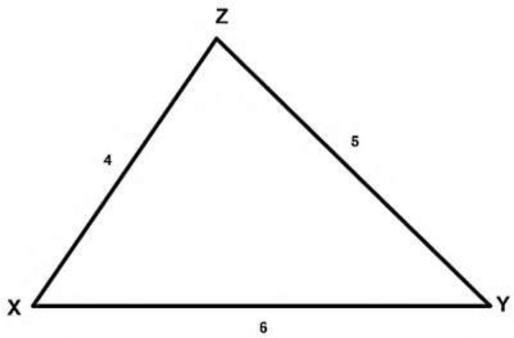


5.7 In the diagram alongside, AB is a building. B, the foot of the building and the point C and D are on the same horizontal plane. From C, the angle of elevation to the top of the building is x.

$B\hat{C}D = 120°$, $C\hat{B}D = y$ and the distance of B and D is c meters. Prove without the use of a calculator, that:

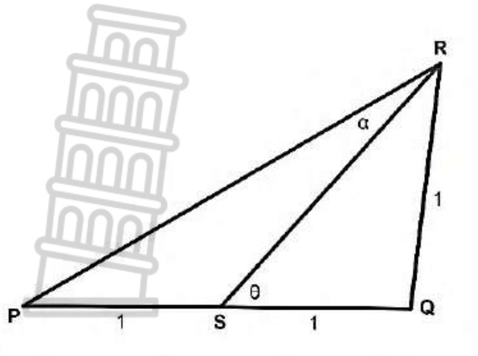
$$AB = \frac{2c\tan x\sin(60° - y)}{\sqrt{3}}$$ (6) **L3**

IEB 2019



5.8 The diagram alongside ΔXYZ with length $XY = 6$ units, $XZ = 4$ units and $YZ = 5$ units, using cosine rule, show that :

$$\cos\hat{Y} + \cos\hat{Z} = \frac{7}{8}$$ (6) **L2**

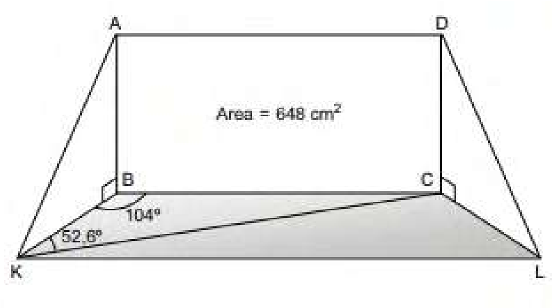


IEB NOV 2013

5.9 In the diagram alongside, ΔPQR is drawn with PQ = 2 units and QR = 1 unit. S is the midpoint of PQ. $P\hat{R}S = \alpha$ and $R\hat{S}Q = \theta$

5.9.1 Determine $\hat{P}$ in terms of θ and α (3) **L1**

5.9.2 Show that $\tan\theta = 3\tan\alpha$ (6) **L3**



DBE NOV 2025

5.10 As part of the school project, learner are required to design a portable stage for a puppet show, as shown in the diagram. The design must fulfil the following requirements:

- BKLC is a horizontal base having $K\hat{B}C = 104°$ and $B\hat{K}C = 52,6°$
- The rectangular backdrop, ABCD, is vertical to the horizontal base and must have an area of 648 cm^2.
- The sides of ABCD must be in a ratio $AB : BC = 1 : 2$
- The stage must be partly enclosed with triangular sides ABK and DCL

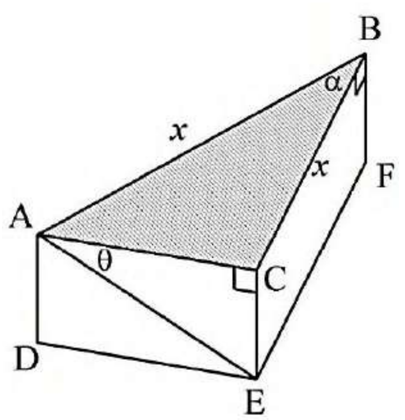
5.10.1 Show that AB = 18cm (2) **L1**

5.10.2 Calculate the length of AC (2) **L2**

5.10.3 Calculate the length of the diagonal KC (2) **L2**

5.10.4 If AB = BK, calculate the size of $K\hat{A}C$ (4) **L3**

HERSCHEL GIRLS SCHOOL SEP 2020



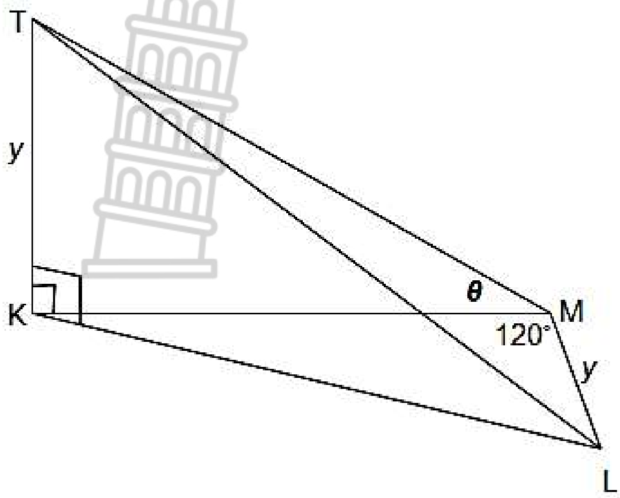
5.11 The upper surface of a right prism is an isosceles triangle, with $AB = BC = x$, $A\hat{B}C = \alpha$ and $C\hat{A}E = \theta$

5.11.1 Show that the length of EC is given by:

$EC = x\tan\theta\sqrt{2(1-\cos\alpha)}$ (5) **L3**

5.11.2 If $\theta = 35°$, $\alpha = 20°$ and $x = 9$ cm, determine the volume of the prism (5) **L3**

ST JOHN'S COLLEGE SEP 2023



5.12 In the figure, K, M and L are three points in the horizontal plane so that $K\hat{M}L = 120°$ T represents the position of a point directly above K, $TK = ML = y$. The angle of elevation of T from M is θ.

5.12.1 Show that:

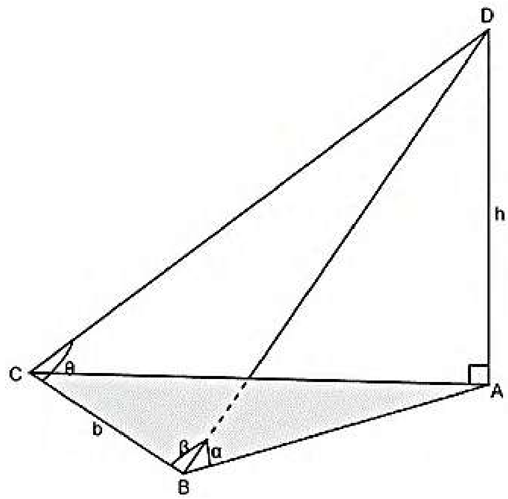
$$KL = y\sqrt{\frac{1}{\tan^2\theta} + \frac{1}{\tan\theta} + 1}$$ (7) **L4**

5.12.2 If $y = 15$ m and $\theta = 22°$ calculate:

(a) Length of KL (2) **L1**

(b) Size of $K\hat{T}L$ (3) **L2**

EVERYTHING MATHS TEXTBOOK



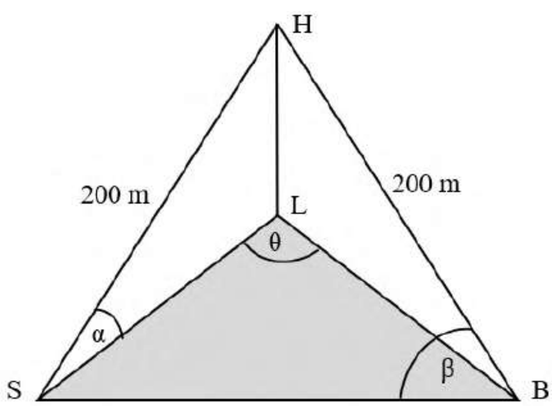
5.14 D is the top of a building of height h The base of the building is at A and the height of ΔABC lies on the ground (a horizontalplane) $BC = b$,

$D\hat{B}A = \alpha$, $D\hat{B}C = \beta$ and

$D\hat{C}B = \theta$

Show that: $h = \dfrac{b\sin\alpha\sin\theta}{\sin(\beta+\theta)}$ (5) **L3**

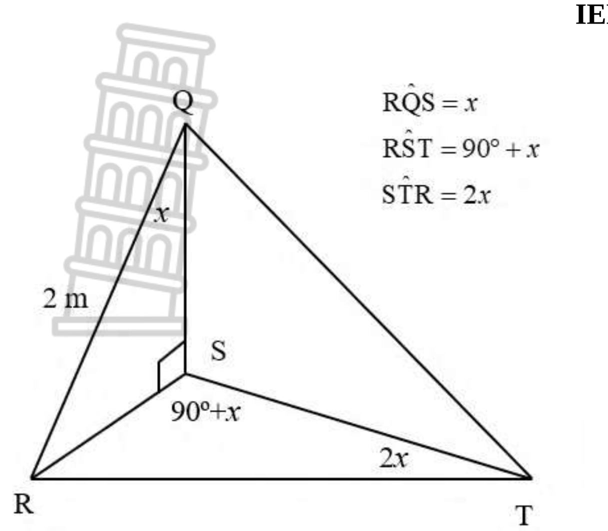
IEB NOV 2023



5.15 Two ships at the sea can see a lighthouse on the shore. The distance from top of the lighthouse H to ship S and to ship B is 200m. the angle of elevation from S to H is α, $H\hat{B}S = \beta$ and $S\hat{L}B = \theta$

Show that the distance between the two ship is given by $SB = 400\cos\beta$

(5) **L3**



IEB NOV 2015

5.16 Two boys (Phila and Thabani) are standing at point R and T. each boy is looking up at point Q, which is 2 metres from the bottom of the pole. S, R and T are in the same horizontal plane.

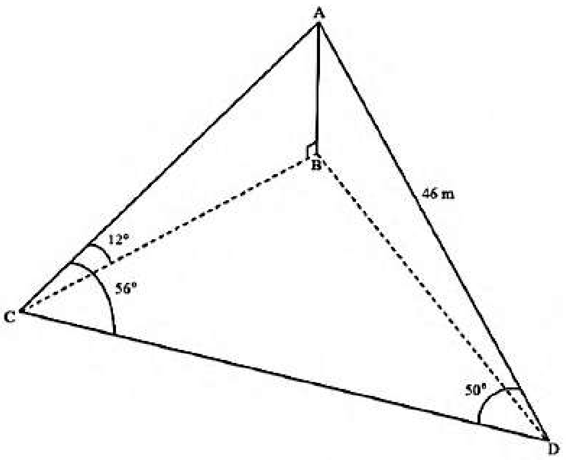
5.16.1 Prove that (Phila and Thabani) are standing one metre apart (4) **L2**

5.16.2 Determine $\hat{SRT}$ in terms of x. (2) **L2**

5.16.3 Prove that: $ST = 2\cos 2x - 1$. (7) **L4**

5.16.4 Show that $\cos x = \tan y$ (4) **L2**

5.16.5 If $x = 60°$, calculate the size of y. (2) **L1**



KZN MAR 2026

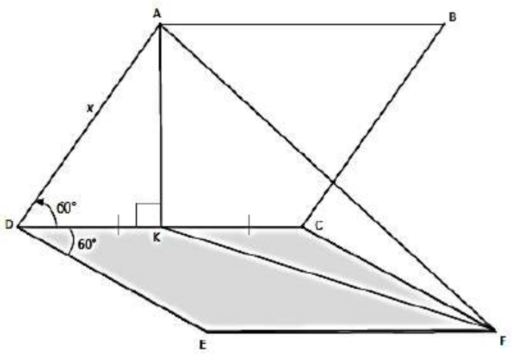
5.17 A vertical steel pole AB is show in the diagram alongside. AC and AD are steel cables, anchoring the pole to the ground at C and D. B, C and D are in the same horizontal plane. The length of AD is 46 m. the angle of elevation of A from C is $12°$. $\hat{ACD} = 56°$ and $\hat{ADC} = 50°$

5.17.1 Write down the size of $\hat{CAD}$ (1) **L1**

5.17.2 Calculate the length of AC (3) **L2**

5.17.3 Calculate the height of the pole AB (3) **L2**

5.17.4 Calculate the area of triangle ACD (3) **L2**



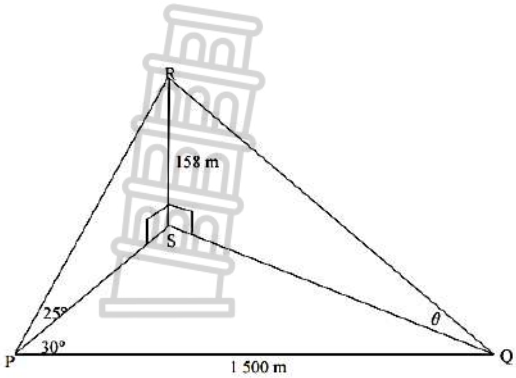
DBE NOV 2019

5.18 The diagram alongside shows a solar panel, ABCD, which is fixed to a flat piece of concrete slab EFCD. ABCD and EFCD are two identical rhombus. K is a point on DC such that DK= KC and $AK \perp DC$. AF and KF are drawn $\hat{ADC} = \hat{CDE} = 60°$ and $AD = x$ *units* .

5.18.1 Determine AK in terms of x (2) **L1**

5.18.2 Write down the size of $\hat{KCF}$ (1) **L1**

5.18.3 It is further given that $\hat{AKF}$, the angle between the solar panel and concrete slab, is y. Determine the area of ΔAKF in terms of x and y (7) **L4**

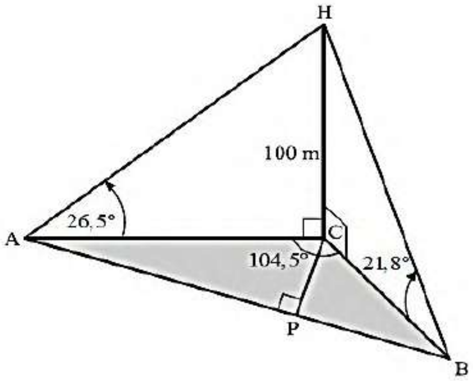


ADAPTED

5.19 In the diagram alongside PQ is a straight line 1500 m long. RS is a vertical tower 158 m high with P, Q and S points in the same horizontal plane. The angle of elevation of R from P and Q are $25°$ and θ. $S\hat{P}Q = 30°$

5.19.1	Determine the length of PS	(3)	**L2**
5.19.2	Determine the length of SQ	(3)	**L2**
5.19.3	Hence, find the value of θ.	(3)	**L2**
5.19.4	Determine the area of ΔSPQ	(4)	**L2**

MIND THE GAP STUDY GUIDE



5.20 In the figure alongside, C is the foot of a vertical tower HC, while A and B are two points in the same horizontal plane as C. The angle of elevation of H, as measured from A is $26,5°$ and the angle of elevation of H, as measured from B, is $21,8°$. The height of the tower HC from the ground 100 metres.

5.20.1	Calculate the length of AB	(5)	**L2**
5.20.2	Calculate the area of ΔABC	(3)	**L2**
5.20.3	If $CP \perp AB$ calculate the length of CP	(3)	**L2**

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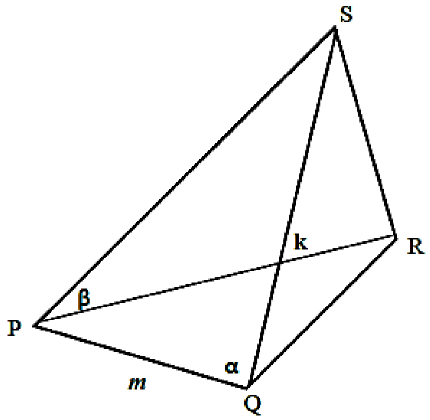

5.21 In the figure SR is a vertical mast. P, Q and R are 3 points in the same horizontal plane. PS and QS are stay ropes. $PQ = m$, $SQ = k$ and $P\hat{Q}S = \alpha$. The angle of elevation of S from P is β If $K = 2m$ show that $PS = m\sqrt{5 - 4\cos\alpha}$ (5) **L3**

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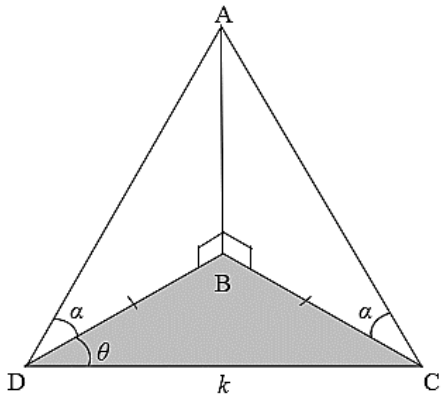

5.22 In the diagram, points B, C and D lie in the same horizontal plane. $A\hat{D}B = A\hat{C}B = \alpha$, $C\hat{D}B = \theta$ and $DC = k$ units. $BD = BC$

5.22.1	Prove that AD = AC	(2)	**L2**
5.22.2	Prove that $BD = \frac{k}{2\cos\theta}$	(3)	**L3**
5.22.3	Determine the area of ΔBCD in terms of k and a single trigonometric ratio of θ	(3)	**L3**

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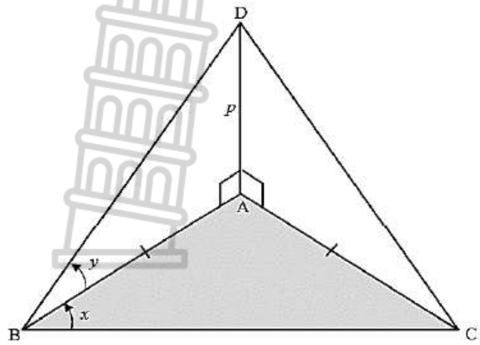


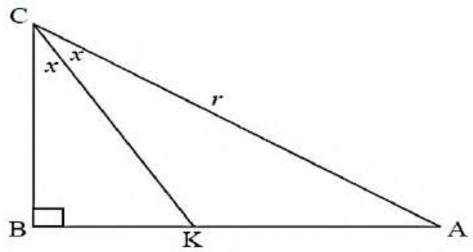
5.23 In the diagram, A, B and C lie in the same horizontal plane with $AB = AC$. D is directly above A such that $2AD = BC$. Also, $AD = p$, $A\hat{B}C = x$ and $D\hat{B}A = y$

5.23.1 Determine AB in terms of p and y. (2) **L1**

5.23.2 Show that $\cos x = \tan y$ (4) **L3**

5.23.3 If $x = 60°$, calculate the size of y (2) **L1**

NW FEB/MARCH 2013



5.24 In the diagram alongside, ABC is a right-angled triangle. KC is the bisector of $A\hat{C}B$. $AC = r$ units and $B\hat{C}K = x$

5.24.1 Write down AB in terms of x and r (2) **L1**

5.24.2 Give the size of $A\hat{K}C$ in terms of x. (1) **L1**

5.24.3 If it is given that $\frac{AK}{AB} = \frac{2}{3}$, calculate the value of x. (8) **L4**

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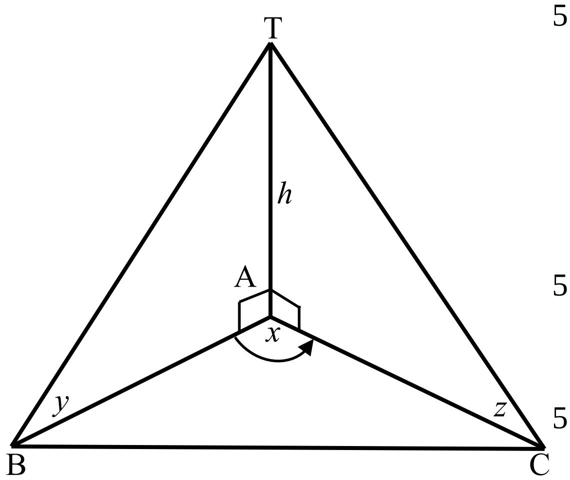


5.25 In the diagram alongside TA represent the vertical pole height h erected in the horizontal plane ABC

- $A\hat{B}T = y$
- $B\hat{A}C = x$
- $A\hat{C}T = z$

5.25.1 Prove that Area of $\Delta ABC = \frac{h^2 \sin x}{2 \tan y . \tan z}$ (5) **L3**

5.25.2 Calculate the value of h if the area of $\Delta ABC = 51{,}8 \text{ m}^2$, $x = 123{,}7°$, $y = 37{,}2°$ and $z = 61{,}6°$ (4) **L2**

ANSWERS

TOPIC 1: Algebraic functions and Inverses

1.1

1.1.1 $f(x)=\frac{6}{x-4}+3$

1.1.2 $\left(0;\frac{3}{2}\right)$ and $(2;0)$

1.1.3 Graph

1.1.4 $y=-x+7$

1.1.5 $(5;10)$

1.2

1.2.1 $x=1$ and $y=-2$

1.2.2 $(0;-5)$ and $\left(\frac{5}{2};0\right)$

1.2.3 Graph

1.2.4 $x\in R, x\neq 3$

1.2.5 $0<x\leq 1$

1.3

1.3.1 $B(0;6)$

1.3.2 $g^{-1}(x)=\frac{1}{2}x-3$

1.3.3 $A(-6;-6)$

1.3.4 $AB=6\sqrt{5}$ units

1.3.5 Area $\Delta ABC=54unit^2$

1.4

1.4.1 $p=2$ and $q=-1$

1.4.2 $a=3$

1.4.3 $y\in R, y\neq -1$

1.4.4 $y=-x-3$

1.4.5 $y=-1$ and $x=\frac{13}{2}$

1.5

1.5.1 $b=-1$

1.5.2 $x\in R, x\neq 0$

1.5.3 $y=\pm x$

1.5.4 $x>0$

1.5.5a $y=\frac{-8}{x}$

1.5.5b $y=\frac{8}{x}-10$

1.5.5c $y=\frac{8}{x}$

1.5.6 $k=4\sqrt{2}$

1.6 $y=-\frac{1}{4}x-2$

1.7

1.7.1 $(-p;0)$

1.7.2 $f(x)=\frac{2}{x-2}-1$

1.7.3 1 unit down **OR** 1 unit right

1.8 Graph

1.9

1.9.1 Graph

1.9.2 $0<x\leq 3$

1.9.3 $\frac{4}{3}units$

1.10

1.10.1 $x=-4$ or $x=3$

1.10.2 $R\left(\frac{-1}{2};\frac{-49}{4}\right)$

1.10.3 $\left(\frac{-5}{2};\frac{3}{2}\right)$

1.10.4 $12<k<\frac{49}{4}$

1.10.5 $y=-1\left(x+\frac{5}{2}\right)^2+\frac{49}{4}$

1.11.1 $(-1;-1)$

1.11.2 $y=-x^2$

1.11.3 $y=-\frac{1}{x}$

1.12.1 $g(x)=-x^2+3x+5$

1.12.2 $x=\frac{3}{4}$

1.12.3 $\frac{121}{8}$

1.13.1 graph

1.13.2 $y\leq 0$

1.14.1 $y\geq -9$

$x>0$

1.14.2 $A(1;0)$

1.14.3 $-5<x<1$

1.14.4 $y=(x+2)^2-9$

1.15.1 $y=-(x-4)^2+7$

1.15.2 $-9<k<7$

1.16.1 g

1.16.2 proof

1.16.3 $y=\pm\sqrt{\frac{x}{2}}$

1.16.4 $y=-2x^2$

1.16.5 $y=-2(x+1)^2-3$

1.16.6 $A\left(\frac{1}{2};\frac{1}{2}\right)$

1.17 $y=-2(x-1)^2+3$

1.18

1.18.1 $x\in R$

1.18.2 $y>0$

1.18.3 $f^{-1}(x)=\log_{\frac{1}{3}} x$

1.18.4 Graph

1.18.5 $x=-2$

1.19

1.19.1 $y=-9$

1.19.2 $A(0;-8)$

1.19.3 $y=\frac{1}{4}x-8$

1.19.4 $m(x)=3^x$

1.19.5 $y=\log_3 x$

1.19.6 $y\in[-3;2)$

1.19.7 Area of $\Delta ABC=16\ units^2$

1.20

1.20.1 Show

1.20.2 $f^{-1}(x)=\log_3 x$

1.20.3 Graph

1.20.4 P=264,44

1.21

1.21.1 (1;0)

1.21.2 $f^{-1}=\left(\frac{4}{3}\right)^x$

1.21.3 Graph

1.21.4 $e=\frac{9}{16}$

1.21.5 $x>0$

1.21.6

(a) $t(x)=\left(\frac{2}{3}\right)^{-x}-1$ or $t(x)=\left(\frac{3}{2}\right)^x-1$

(b) $y>-1$

1.22

1.22.1 $b=\frac{1}{3}$

1.22.2 $f^{-1}(x)=\log_{\frac{1}{3}} x$

1.22.3 Q(0;1) and R(1;0)

1.22.4 a) $x\geq 0$

b) $x>27$

1.22.5 area of $\Delta QTS=54\ units^2$

1.23

1.23.1 a=4 and b=2

1.23.2 Graph

1.23.3 No

1.23.4 $y=\log_4 x\ ;x\in\left[\frac{1}{4};16\right]$

1.23.5 $-\frac{1}{2}<x<0$

1.24

1.24.1 $x=-\sqrt{\frac{3}{2}}=-1,22$

1.24.2 TP(0;6)

1.24.3 $f(x)\leq 6$

1.24.4 Max length = $\frac{27}{4}$

1.24.5 $y=-\frac{x^2}{4};x\leq 0$

1.24.6 $k(x)=2\sqrt{-x};\ x\leq 0$

1.24.7 $-6<k<0$

1.25

1.25.1 Show

1.25.2 (0;4)

1.25.3 (-1; 4,5)

1.25.4 $h(x)=-x+2$

1.25.5 $g(x)=\frac{1}{x+1}+3$

1.25.6 $x=-\frac{4}{3}$

1.25.7 $x\in\left[-\frac{4}{3};-1\right)\cup(-1;\infty)$

$or\ x\in\left[-\frac{4}{3};\infty\right);x\neq -1$

1.25.8 $k(x)=\frac{-1}{x-9}+3$

1.25.9 $-4,5\leq p<-4$

1.26

1.26.1 $x=3$

$y=2$

1.26.2 (0;2)

1.26.3 $f(x)=-\frac{4}{x+3}+2$

1.26.4 $y=-x-1$

1.26.5 $x\in R;\ x\neq -1$

1.26.6 $h(x)=2^x$

1.26.7 $f^{-1}(x)=\log_2 x$

1.26.8 $y \in R$

1.26.9 $-3 < x \leq -1$

1.27

1.27.1 $B\left(0; 2\frac{1}{2}\right)$

1.27.2 $p = 2,\ q = 1$ and $a = -3$

1.27.3 $y \in \left(-\infty; 4\frac{1}{2}\right]$

1.27.4 $y = -x - 1$

1.27.5 $m = -\frac{5}{2}$

1.27.6 a) $-5 \leq x \leq 1$

b) $x < -2$ *or* $x > 1$

1.27.7 $k < \frac{5}{2}$

1.28 1.28.1 $m = 2$

1.28.2 $x > 0$

1.28.3 $y = 2^x$

1.28.4 $y = 2^{x+2}$

1.28.5 Show

1.28.6 Area= $\frac{26}{5}$ *units*^{2}

1.28.7 $y \leq \frac{18}{5}$

1.28.8 $1 < x < 5$

1.29

1.29.1 $D(-2;-8)$

1.29.2 $x = -2$ *and* $y = 1$

1.29.3 a) $a = 2$

b) OA=4 units

c) $y \in R; y \neq 1$

d) $y = -x - 1$

1.29.4 a) $x \in (-4;0)$

b) $x \leq -4$ *or* $x \geq -2$

1.29.5 $-8 < k < 0$

1.30

1.30.1 (3;2)

1.30.2 $FE = \frac{29}{3}$

1.30.3 $a = -1$ *and* $b = 6$

1.30.4 $y \in R; y \neq 5$

1.30.5 $-7 < p < 2$

1.30.6 $y = -x + 5$

1.30.7 Reflect about x-axis and shift 2units up

1.30.8 $x = 2$ or $x = 4$

1.31 1.31.1 $A(3;0)\ B(0;-7)$

1.31.2 $y = -8$

1.31.3 $h(x) = 4^x$ or $h(x) = 2^{2x}$

1.31.4 $y = \log_4 x$

1.31.5 $y = -\log_4 x$ or $y = \log_{\frac{1}{4}} x$

1.31.6 Answer = 4,5

1.32

1.32.1 Graph

1.32.1 Graph

1.32.2 Graph

1.32.3 Show

ANSWERS

TOPIC 2: Trig Identities and Reduction formulae

2.1

2.1.1 $\frac{-8}{17}$

2.1.2 $\frac{161}{289}$

2.1.3 $\frac{2}{15}$

2.1.4 $\frac{-23\sqrt{2}}{34}$

2.2

2.2.1 $\frac{-120}{169}$

2.2.2 $\frac{120}{169}$

2.2.3 $\frac{120}{119}$

2.3

2.3.1 $\frac{5}{\sqrt{41}}$

2.3.2 $a = \frac{-25}{4}$

2.4

2.4.1 $p = -5$

2.4.2 $\frac{3\sqrt{34} + 5\sqrt{102}}{68}$

2.4.3 $\frac{-3}{5}$

2.5

2.5.1 $\frac{\sqrt{3}}{3}$

2.5.2 $\frac{-\sqrt{3}}{2}$

2.5.3 $\beta = 30°$

2.6

2.6.1 $\frac{4\sqrt{3}}{7}$

2.6.2 $\frac{-2\sqrt{6} - 2}{\sqrt{7}}$

2.7

2.7.1 $OB = 6$

2.7.2 $\frac{\sqrt{3}}{2}$

2.7.3 $\frac{5}{2}$

2.8

2.8.1 $-\frac{12}{13}$

2.8.2 $\frac{17\sqrt{2}}{26}$

2.8.3 $\frac{2\sqrt{13}}{13}$

2.9

2.9.1 $\frac{2\sqrt{6}}{5}$

2.9.2 $-\frac{5}{7}$

2.9.3 $\frac{20\sqrt{6}}{49}$

2.10

2.10.1 $-\sqrt{k}$

2.10.2 $1-2k$

2.10.3 $4\sqrt{k(1-k)}(1-2k)$

2.11

2.11.1 $\sqrt{1-a^2}$

2.11.2 $2a^2 - 1$

2.11.3 $\frac{\sqrt{1-a^2}}{a}$

2.11.4 $\frac{\sqrt{2}}{2}\left(a - \sqrt{1-a^2}\right)$

2.12

2.12.1 t

2.12.2 $2t^2 - 1$

2.12.3 $-\frac{1}{2}\sqrt{1-t^2}$

2.12.4 $\sqrt{\frac{1-t}{2}}$

2.13

2.13.1 $1 - 2p^2$

2.13.2 $q\sqrt{1-p^2} + p\sqrt{1-q^2}$

2.13.3 $-\frac{p}{2}$

2.14
2.14.1 $\frac{p}{q}$
2.14.2 $-\cos 2A$

2.15
2.15.1 $\frac{2p}{\sqrt{9-4p^2}}$
2.15.2 $\frac{\sqrt{9-4p^2}+2p\sqrt{3}}{6}$
2.15.3 $\frac{3-\sqrt{9-4p^2}}{6}$

2.16
2.16.1 k
2.16.2 $2k\sqrt{1-k^2}$
2.16.3 $\frac{\sqrt{2}}{2}\left(\sqrt{1-k^2}-k\right)$

2.17 $\frac{-2ab}{a^2-b^2}$

2.18
2.18.1 $-\sin x.\cos x$ or $-\frac{1}{2}\sin 2x$
2.18.2 0
2.18.3 0
2.18.4 $2\tan x$
2.18.5 -1
2.18.6 $\frac{-\sqrt{2}}{2}$
2.18.7 $\frac{1}{2}\tan x$
2.18.8 1
2.18.9 $\frac{-1}{\sin x}$
2.18.10 $\tan x$
2.18.11 1
2.18.12 -2
2.18.13 $\frac{1}{2}$
2.18.14 $\frac{\sqrt{3}}{2}$
2.18.15 $\frac{-\sqrt{2}}{2}$
2.18.16 $\tan x$
2.18.17 $2\tan x$

2.19 **Proof**

2.20 k

2.21 $\frac{5}{4}$

2.22 -1

2.23
2.23.1 **2.23.1 to 2.23.32 Proofs**

2.24
2.24.1 **Proof**
2.24.2 $\theta = 90°$

2.25
2.25.1 **Proof**
2.25.2 $x = 45° + k.90°; k \in Z$

2.26
2.26.1 **Proof**
2.26.2 $\sqrt{2}-1$

2.27
2.27.1 **Proof**
2.27.2 $\sin 32°$

2.28 **Proof**

2.29 $\theta \in [90°; 270°]$

2.30 Real for all x, where
$1+\cos x \neq 0$
Domain Restriction:
$x \neq 180° + k.360°; k \in Z$

2.31 $180° < \theta < 360°$

2.32 non-real
for all k
$\sin 2k \neq 0$

2.33 $x = 90°$

2.34 **Proof**
2.35 $-\cos x$

ANSWERS
TOPIC 3: Trig Equations & General Solutions

3.1

3.1.1 $\frac{\cos 2x}{2}$

3.1.2 $x = 40.40° + k.180°$
Or $x = 139.60° + k.180°;\ k \in Z$

3.2 $x = 45° + k.120°$
Or $x = 45° + k.360°;\ k \in Z$

3.3 $x = 199.47°$ or $x = 340.53°$

3.4

3.4.1 $-2\cos 7x \sin 3x$

3.4.2 $x = 0°$
Or $x = 17.14°$, $x = 34.29°$ (N/A)

3.5

3.5.1 $\theta = 71.57° + k.180°;\ k \in Z$

3.5.2 $\theta = 0° + k.360°$
Or $\theta = 180° + k.360°;\ k \in Z$
$\theta = 45° + k.360°$
Or $\theta = 135° + k.360°;\ k \in Z$
$\theta = 225° + k.360°$
Or $\theta = 315° + k.360°;\ k \in Z$

3.5.3 $x = 240° + k.720°$
Or $x = 480° + k.720°;\ k \in Z$

3.5.4 $x = 180° + k.360°;\ k \in Z$
$x = 120° + k.360°$
Or $x = 240° + k.360°;\ k \in Z$

3.5.5 $x = 30° + k.360°$
Or $x = 330° + k.360°;\ k \in Z$
$x = 150° + k.360°$
Or $x = 210° + k.360°;\ k \in Z$

3.5.6 $x = 90° + k.360°;\ k \in Z$
$x = 210° + k.360°$
Or $x = 330° + k.360°;\ k \in Z$

3.5.7 $x = 36{,}83° + k.180°; k \in Z$
Or $x = 143{,}17° + k.180°;\ k \in Z$

3.5.8 $x = 0° + k.360°$ **or**
$x = 180° + k.360°, k \in Z$
$x = 210° + k.360°$ **or**
$x = 330° + k.360°, k \in Z$

3.5.9 $x = 90° + k.360°$ **or**
$x = 270° + k.360°, k \in Z$
$x = 0° + k.360°$ **or**
$x = 360° + k.360°, k \in Z$

3.5.10 $x = 120° + k.360°$ **or**
$x = 240° + k.360°, k \in Z$

3.5.11 $x = 64{,}76° + k.180°, k \in Z$

3.5.12 $x = 40° + k.120°$ **or**
$x = 240° + k.360°, k \in Z$

3.5.13 $x = 60° + k.180°, k \in Z$

3.5.14 $x = 0° + k.180°$ **or**
$x = 90° + k.180°, k \in Z$

3.6

3.6.2 $x = 228{,}54° + k.360°$ **or**
$x = 311{,}41° + k.360°, k \in Z$
$x = 90° + k.360°, k \in Z$

3.7 $\theta = 32{,}71° + k.120°$ **or**
$x = 57{,}29° + k.120°, k \in Z$

3.8

3.8.1 $x = 90° + k.360°$ **or**
$x = 270° + k.360°$
$x = 150° + k.360°$ **or**
$x = 210° + k.360°, k \in Z$

3.8.2 $x = 0° + k.360°$ **or**
$x = 180° + k.360°, k \in Z$
$x = 60° + k.360°$ **or**
$x = 300° + k.360°, k \in Z$

3.8.3 $x = 42° + k.360°$ **or**
$x = -14° - k.120°, k \in Z$
OR
$x = 46° + k.120°$ **or**
$x = -138° - k.360°, k \in Z$

3.8.4 $x = 52{,}5° + k.90°$ **or**
$x = 82{,}5° + k.360°, k \in Z$

3.8.5 $x = 180° + k.360°, k \in Z$
$x = 71{,}56° + k.360°, k \in Z$

3.8.6 $x = 210° + k.360°$ **or**
$x = 330° + k.360°, k \in Z$
$x = 56{,}31° + k.180°, k \in Z$

3.8.7 $x = 30° + k.180°, k \in Z$

3.8.8 $\beta = 19{,}11° + k.180°;\ k \in Z$

3.8.9 $x = 0° / 26{,}57° + k.180°;\ k \in Z$

3.8.10 $x = 60° + k.360°;\ k \in Z$
$x = 33{,}3° + k.120°;\ k \in Z$

3.8.11 $x = 26,57° / 63,43° + k.180°;\ k \in Z$

3.8.12 $x = -150° + k.360°;\ k \in Z$

$x = -10° + k.120°;\ k \in Z$

3.9

3.9.1 $x = 40° / 60°$

3.9.2 A= $-330°\ ; -270°\ ; -90°\ ; 30°\ ; 90°\ ; 150°$

3.9.3 θ= 90° ;194,5° ;345,5°

3.9.4 $x = \pm 120°$

3.9.5 $x = -26,6°\ ; -9,74°\ ;\ 80,26°\ ; 153,4°$

3.9.6 B = 0° ;30° ;150° ;180° ;360°

ANSWERS

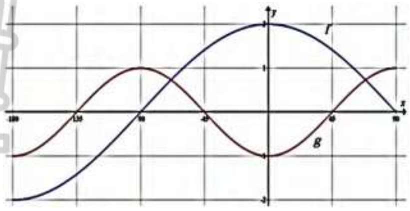
TOPIC 1: Trigonometric Functions (Graphs)

4.1

4.1.1 $a = 2$

4.1.2 Period $= 180°$

4.1.3



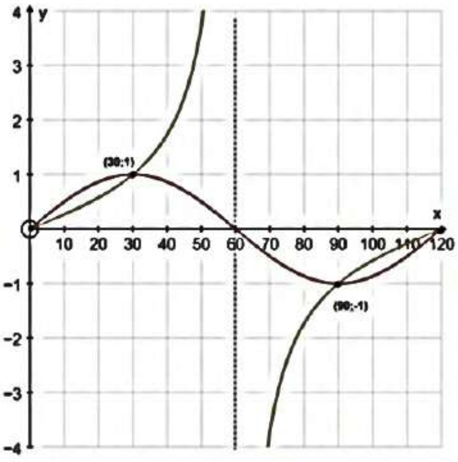
4.1.4 $x = 0°$

4.1.5 Minimum value of $= -1$

4.2

4.2.1 $\therefore x = \pm 30° + k120°\ k \in z$

4.2.2



4.2.3 $0° \leq x \leq 30° \cup 60° < x \leq 90°$

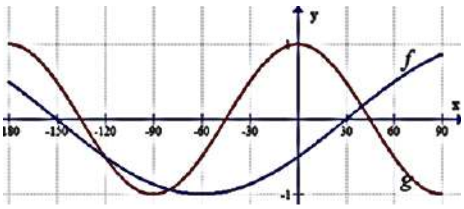
4.3

4.3.1 $x = 40° + k120°\ , k \in z$

OR

$x = -120° + k360°\ , k \in z$

4.3.2

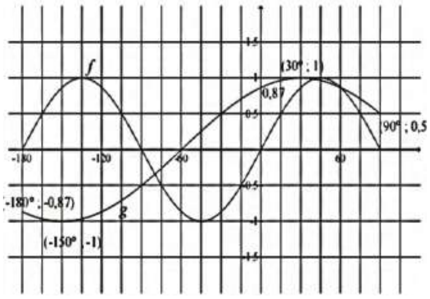


4.3.3 $x \in (-180°; -150°) \cup (-135°; -45°) \cup (30°; 45°)$

4.4

4.4.1 $\therefore \text{Period} = \dfrac{360°}{3} = 120°$

4.4.2



4.4.3 $y = \sin 2(x + 30°)$

$\therefore$ translation of $30°$ to the left

4.5

4.5.1 $a = 3$

$b = 1$

4.5.2 $-90° \leq x \leq 0°$ OR $x \in [-90°; 0°]$

4.5.3 $x -$ coord of P $= -146,31°$

(a) $x -$ cood of Q $= 33,69°$

(b) $-146,31° < x < -90°$ *or* $33,69° < x < 90°$

OR

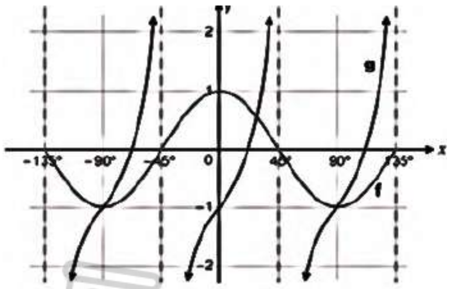
$x \in (-146,31°; -90°)$ or $x \in (33,69°; 90°)$

4.5.4 $y = 2\cos(x - 45°)$

4.6

4.6.1 $180°$

4.6.2



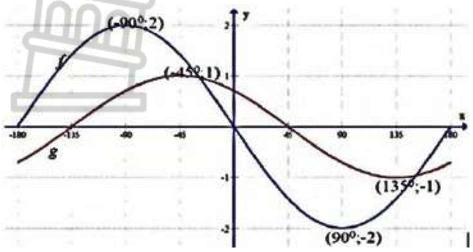
4.6.3 $h(x) = -\sin 2x$

4.6.4 $y \in [-1; 1]$ or $-1 \leq y \leq 1$

4.6.5 $0° \leq x \leq 22,5°$ or $112,5° \leq x \leq 135°$

4.7

4.7.1



4.7.2 $360°$

4.7.3 $x = 151,32°$ or $x = -28,68°$

4.8

4.8.1 A: $x = -160°$

B: $x = 80°$

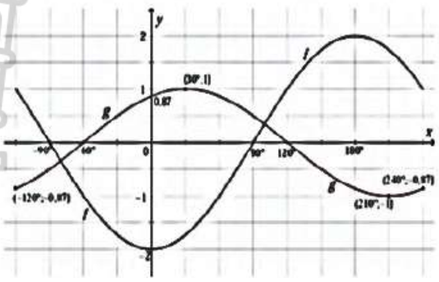
4.8.2

(a) $x \in (-150°;30°)$

(b) $x \in (-160°;80°)$

4.9

4.9.1



4.9.2

(a) 360°

(b) $y \in [-5;-1]$ or $-5 \le y \le -1$

(c) 2 solutions

4.9.3 $k > 0$ and $k < -2$

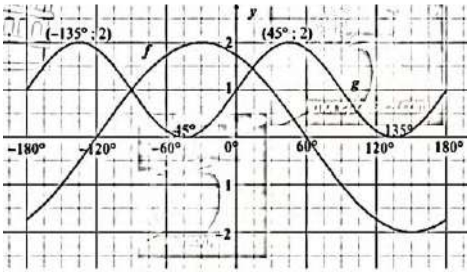
4.9.4 $h(x) = -\sin x$

4.10

4.10.1 $a = 2$

$b = 30°$

4.10.2



4.10.3 180°

4.10.4 $2g(x) = 2\sin 2x + 2$

$\therefore 0 \le y \le 4$ OR $y \in [0;4]$

4.10.5 (a) $x \in [-180°;-90°]$

OR

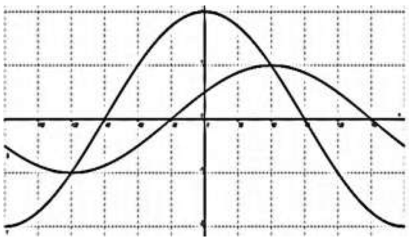
$-180° \le x < -90°$

(b) where $\cos(x+b) = 0$

$x = -120°$ or $60°$

4.10.6 $p(x) = \cos 2x + 1$

4.11 4.11.1



4.11.2 (a) 360°

(b) 0°

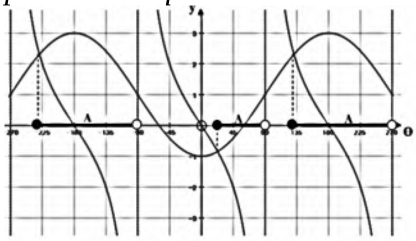
(c) $90° < x < 150°$

4.12 4.12.1 Amplitude = 2

4.12.2 y $[-1\ ;\ 3]$ or $y \in -1 \le y \le 3$

4.12.3 $p = -2$ and $q = 1$

4.12.4



4.12.5 See graph

4.13 4.13.1 120°

4.13.2 $a = 3$

4.13.3 $-2 \le y \le 2$ or

$y \in [-2\ ;\ 2]$

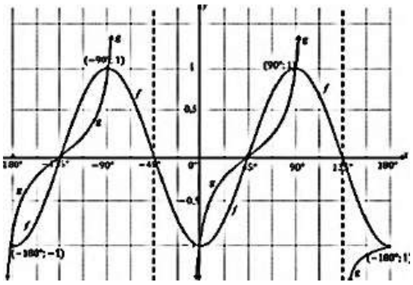
4.13.4 $h(x) = -\sin x$

4.13.5 $-30° < x < 60°$

4.14 4.14.1 180°

4.14.2 $y \in [-1\ ;\ 1]$

4.14.3

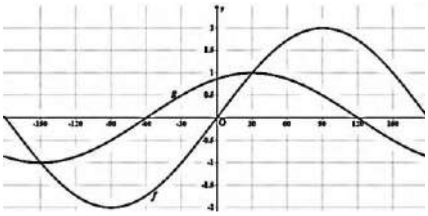


4.14.4 $-180° \le x \le -135$ or

$-90° \le x < -45°$

4.14.5 $Maximum = 4$

4.15 4.15.1



4.15.2 (a) 360°

(b) Amplitude = 2

(c) $x \in [-180°\ ;\ -150°) \cup$

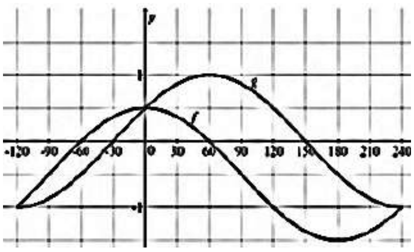
$(30°\ ;\ 180°]$

(d) $x = 90°$ or $x = 150°$

(e) Horizontal shift by 120° to the left.

4.16 4.16.1 $y \in \left[-\frac{3}{2}\ ;\ \frac{1}{2}\right]$ or $-\frac{3}{2} \le y \le \frac{1}{2}$

4.16.2



4.16.3 $x = -120°$ and $x = 240°$

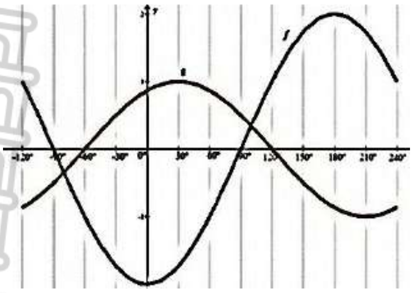
4.16.4 $0° < x < 180°$

4.16.5 Amplitude = 1

4.17 4.17.1 Amplitude = 2

4.17.2 $y \in [1\ ;\ 5]$

4.17.3



4.17.4 $k < -2$ or $k > 2$

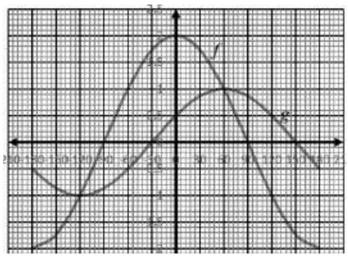
4.17.5 $-120° \leq x < 0°$

$30° < x < 210°$

4.18 4.18.1 Proof

4.18.2 $x = \{-120°\ ;\ 60°\}$

4.18.3 (a)



4.18.3 (b) (i) $x \in (-120°\ ;\ 60°)$

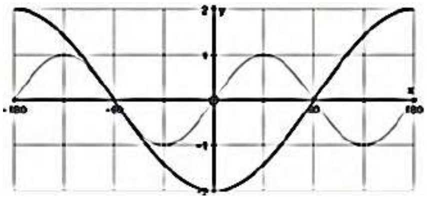
4.18.5 (ii) $x \in (-120°\ ;\ -90°]$

and $x = 0°$

4.19 4.19.1 $b = 2$

4.19.2

4.19.3



4.19.4 2

4.19.5 $\theta = 90° + k.180°$

4.19.6 (a) $x \in [-90°\ ;\ 90°]$

(b) $x \in (0°\ ;\ 90°) \cup$

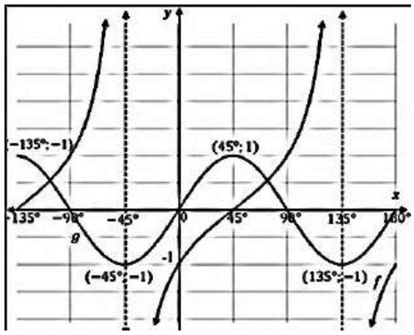
$(90°\ ;\ 180°)$

(c) $x \in (0\ ;\ 180°)$

4.19.7 $y = -2\cos(x - 45°) + 2$

4.20 4.20.1 $p = -45°$

4.20.2



4.20.3 180°

4.20.4 $h(x) = \cos 2x$

4.20.5 (a) $-45° < x \leq 45°$

(b) $-90° < x < 0°$

ANSWERS

TOPIC 5: Trigonometry 2D and 3Ds

5.1 $P\hat{Q}C = 45,7°$

5.2.1 $R\hat{O}S = \frac{3\theta}{2}$ (angle at centre)

5.2.2 Proof

5.3. Proof

5.4.1 Proof

5.4.2 $2x\cos\theta\tan\beta$

5.5 Proof

5.6 Proof

5.7 Proof

5.8 Proof

5.9.1 $\hat{P} = \theta - \alpha$

5.9.2 Proof

5.10.1 Proof

5.10.2 $AC = 18\sqrt{5}$ cm

5.10.3 $KC = 43,97$ cm

5.10.4 $K\hat{A}C = 108,4°$

5.11.1 Proof

5.11.2 $V = 30,3\ cm^3$

5.12.1 Proof

(a) KL= 3,1 unit

(b) $11,7°$

5.14 Proof

5.15 Proof

5.16.1 Proof

5.16.2 $\hat{R} = 90° - 3x$

5.16.3 Proof

5.16.4 Proof

5.16.5 $y = 26,6°$

5.17.1 $74°$

5.17.2 $AC = 42,50$ m

5.17.3 $AB = 8,84$ m

5.17.4 Area of $\Delta ACD = 939,63\ m^2$

5.18.1 $AK = \frac{\sqrt{3}}{2}x$

5.18.2 $K\hat{C}F = 120°$

5.18.3 Area $= \frac{x^2\sqrt{21}\sin y}{8}$

5.19.1 $PS = 333,8$ m

5.19.2 $SQ = 122,7$ m

5.19.3 $\hat{Q} = 7,4°$

5.19.4 $102043\ m^2$

5.20.1 $AB = 375,2$

5.20.2 Area of $\Delta ABC = 271,20\ m^2$

5.20.3 $CP = 135,78$ m

5.21 Proof

5.22.1 Proof

5.22.2 Proof

5.22.3 $\frac{k\sin\theta}{4}$

5.23.1 $AB = \frac{p}{\tan y}$

5.23.2 Proof

5.23.3 $y = 26,6°$

5.24.1 $AB = r\sin 2x$

5.24.2 $A\hat{K}C = 90° - x$

5.24.3 $x = 30°$

5.25.1 Proof

5.25.2 $h = 13.22$