DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

# MATHEMATICS

"# Break the 73% ceiling"

# MANUAL FOR WINTER CLASSES

## JUNE/JULY 2026

| TOPICS | |
|---|---|
| Inequalities | Trigonometry |
| Functions and inverses | Analytical Geometry |
| Differential Calculus | Euclidean Geometry |

# INEQUALITIES



**Graphical approach CAPS:**

Step 1: Write the inequality in a standard quadratic form:

$ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$

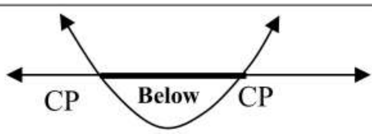
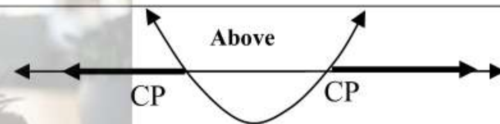
$ax^2 + bx + c \leq 0$ or $ax^2 + bx + c \geq 0$

Step 2: Ensure that the coefficient of $x^2$ is positive (remember the change of sign when multiplying or dividing by negative)

Step 3: Factorise to determine the critical points (zeroes of the inequality)

Step 4: Sketch the graph of the parabola, focusing on the $x$-intercepts,

Step 4: Determine the values of $x$ for which the inequality holds.

| | |
|---|---|
| For $y < 0$ and $y \leq 0$, the values are at the $x$-intercepts and below the $x$ – axis | For $y > 0$ and $y \geq 0$, the values are at the $x$-intercepts and above the $x$ – axis |
|   |   |

**EXAMPLES**

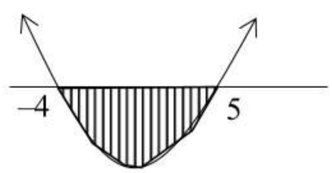
Solve for $x$:

1 $x^2 - x - 20 < 0$

**Solutions:**

$x^2 - x - 20 \leq 0$

$(x-5)(x+4) \leq 0$

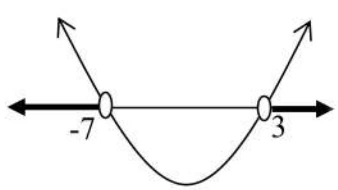



Critical values : $x = -4$ and $x = 5$ Solution: $-4 \leq x \leq 5$ or $x \in [-4;5]$

2. $x^2 + 4x > 21$

$x^2 + 4x - 21 > 0$

$(x+7)(x-3) > 0$



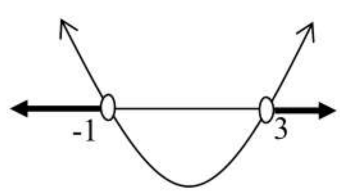

Critical values : $x = -7$ and $x = 3$ Solution: $x < -7$ or $x > 3$

3. $-x^2 + 2x + 3 < 0$

$(x-3)(x+1) > 0$

Critical values: $x = -1$ and $x = 3$

$x < -1$ or $x > 3$ OR $(-\infty;-1)$ or $(3;\infty)$



**ACTIVITIES**


Solve for $x$:

| | | | | | |
|---|---|---|---|---|---|
| 1 | $x^2 - 2x - 3 > 0$ | (Nov 2024) | 12 | $-3(x+7)(x-5) < 0$ | (Nov 2013) |
| 2 | $x^2 - 3 > 2x$ | (Nov 2023) | 13 | $(x+1)(x-3) > 12$ | (Feb/March 2012) |
| 3 | $x^2 - 90 > x$ | (Nov 2022) | 14 | $x^2 + 7x < 0$ | (Nov 2015) |
| 4 | $x^2 + 5x \leq -4$ | (Nov 2021) | 15 | $x(4 - x) < 0$ | (Feb/March 2016) |
| 5 | $(1-x)(x+2) < 0$ | (Nov 2020) | 16 | $(x+1)(4-x) > 0$ | (Feb/March 2013) |
| 6 | $4x^2 - 1 < 0$ | (Nov 2019) | 17 | $x^2 < -2x + 15$ | (May/June 2023) |
| 7 | $x^2 - 3x - 10 > 0$ | (Nov 2018) | 18 | $x^2 \leq 3x$ | (May/June 2022) |
| 8 | $8x^2 > 2x$ | (Nov 2025) | 19 | $-x^2 + 3x > -4$ | (May/June 2026) |
| 9 | $x^2(x+4) < 0$ | (NW Trial 2025) | 20 | $(x-8)(x+2) \leq 0$ | (May/June 2025) |
| 10 | $(x+4)(5-x) \geq 0$ | (FS Trial 2025) | 21 | $\left(\frac{1}{2}\right)^{-x}(3-x) \leq 0$ | (KZN Trial 2025) |
| 11 | $(3x-2)^2 > 3x$ | (L Trial 2025) | 22 | $x^2 + 5x \leq 0$ | (MP June 2025) |
| | | | | | |
| 15 | Given: $f(x) = x^2 + 8x + 16$ solve for $x$ if $f(x) > 0$ | | | | (Nov 2017) |

## CALCULUS:

### FIRST PRINCIPLE

First principle: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

| | | | | | |
|---|---|---|---|---|---|
| 1 | $f(x) = 2x + 1$ | 2 | $f(x) = -2x^2 - 3x + 2$ | 3 | $f(x) = \frac{2}{x}$ |
| 4 | $f(x) = 2x^3$ | 5 | $f(x) = -4x^2$ | 6 | $f(x) = 4 - 7x$ |

Using the rule $\frac{d}{dx}\left(ax^n\right) = anx^{n-1}, for\ n \in R$

**QUESTION 1** (Nov. 2014)

1.1 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$

1.2 If $y = (x^6 - 1)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$ if $x > 1$

**QUESTION 2** (FEB/MARCH 2015)

2.1 Differentiate: $f(x) = -3x^2 + 5\sqrt{x}$

2.2 Differentiate : $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$

**QUESTION 3** (Nov.2015)

Determine:

3.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$

3.2 $D_x\left[\frac{x^2 - 1}{x - 1}\right]$

**QUESTION 4** (Gauteng Sept.2016)

4.1 Determine $\frac{dy}{dx}$ if $y = \frac{2x - 3}{\sqrt[4]{x}}$

4.2 $D_x\left(x^4 - 2x + \frac{1}{x^2}\right)$

**QUESTION 5** (Nov.2022)

5.1 Determine $f'(x)$ if, $f(x) = 2x^5 - 3x^4 + 8x$

5.2 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1;-7)$. For which values of $x$ will g be concave up?

**QUESTION 6** (Nov.2021)

Determine:

6.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$

6.2 $D_x\left[\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$

**QUESTION 7** (Nov 2023)

Determine: 7.1 $f'(x)$ if $f(x) = 2x^2 - 3x$

7.2 $D_x\left[7.\sqrt[3]{x^2} + 2x^{-5}\right]$

**QUESTION 8** (Nov 2024)

Determine:

1. $\frac{d}{dx}\left[3x-5x^2\right]$
2. $g'(x)$ if $g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$

**QUESTION 9**

Determine:

| | | | |
|---|---|---|---|
| 1. $D_t[t^2+\sqrt{t}\,]$ | 2. $\frac{d}{dx}\left[\frac{1}{2x}+3\sqrt{x^3}+\frac{1}{4}\right]$ | 3. $\frac{dy}{dx}$ if $y=\sqrt{x^3}-\frac{1}{x}+\frac{1}{2}\pi$ | 4. $\frac{dy}{dx}$ if $y=\frac{5}{x}+x^2.\sqrt{x}$ |
| 5. $\frac{dy}{dx}$ if $\sqrt{y}=x-2$ | | 6. $\frac{dy}{da}$ if $y=ax^2+a$ | 7, $\frac{dy}{da}$ if $xy-y=2x^2-2x$ |

**QUESTION 10 [DBE NOV 2025]**

10.1 Differentiate $f(x)$ from the definition of derivative (i.e, first principles) if

$f(x)=-x^2+4x+2$ (5)

10.2 Find the derivatives of the following functions:

10.2.1 $f(x)=x^9$ (2)

10.2.2 $y=\frac{x^3+8}{x+2};\ x\neq-2$ (3)

10.2.3 $D_x\left[\frac{5}{\sqrt[4]{x^2}}+\left(\frac{2}{3x}\right)^{-2}\right]$ (4)

**QUESTION 11 [LP Trial 2025]**

11.1 Determine $f'(x)$ from first principles if $f(x)=x^2-3x$ (5)

11.2 Determine:

11.2.1 $D_x\left[(x+1)(5x+9)\right]$ (3)

11.2.2 $\frac{dy}{dx}$ *if* $y=\frac{-5}{\sqrt{x}}-x^5$ (3)

11.2.3 $f'(x)$ *if* $f(x)=\frac{x^3+x^2-2x}{x-1}$ (4)

**QUESTION 12 [May/June 2025]**

12.1 Determine $f'(x)$ from first principle if it is given that $f'(x)=x^2-2$ (5)

12.2 Determine:

12.2.1 $\frac{d}{dx}\left[3x^2-4x\right]$ (2)

12.2.2 $g'(x)$ if $g(x)=-2\sqrt{x}\,(x-1)^2$ (4)

**QUESTION 13 [KZN Trial 2025]**

13.1 Determine $f'(x)$ from first principle if it is given that $f'(x)=5x^2$ (5)

13.2 Determine the following:

13.2.1 $\frac{dy}{dx}$ if $y=\frac{x^3=27}{x-3}$ (3)

13.2.2 $D_x\left[x\left(4-x^{\frac{-1}{2}}\right)\right]$ (3)

**QUESTION 14 [NW TRIAL 2025]**

14.1 Given : $f(x)=x^2-x+3$ (5)

Determine $f'(x)$ from first principles

14.2 Determine $f'(x)$ if:

14.2.1 $f(x)=5x^3\ -7x+2$ (2)

14.2.2 $f(x)=\frac{2x+1}{\sqrt[3]{x}}$ (4)

**QUESTION 15 [GP TRIAL 2025]**

15.1 Given: $f(x)=-2x^2+1$

15.1.1 Determine $f'(x)$ from first principles. (4)

15.1.2 Hence , calculate the gradient of the tangent to *f* at $x=-\frac{1}{2}$. (1)

15.2 Determine the derivative of $f(x) = \sqrt[3]{x^2} - \frac{1}{4x^4}$ (3)

## FUNCTIONS AND INVERSES

**Definition:** A **function** is a relation between a set of input (domain) and a set of permissible (range) with the property that each input is related to exactly one output. In other words, it is a set of ordered pairs where, for every value of $x$ there is one and only one value for y. However, note that for the same value of y there may be different values for $x$.

**Functions may be a** one-to-one relation or many-to-one relation
**One-to-one relation:** A relation is one-to-one if for every input value there is only one output value.
**Many-to-one relation:** A relation is many-to-one if for more than one input value there is one output value.
**One-to-many relation:** A relation is one-to-many if for one input value there is more than one output value

**To test whether a given graph is a function or not, we use a vertical line test.** If any vertical line intersects the graph of $f$ only once, then $f$ is a function; and if any vertical line intersects the graph of $f$ more than once, then $f$ is not a function.

**Horizontal line test:** If the line cuts the graph once, the function is a one-to-one (the inverse will also be function). If the line cuts the graph at more than one point, the function is a many-to-one (the inverse will NOT be function)



### DEFINITIONS AND CONCEPTS

| | |
|---|---|
| **Function** | A relation for which each element of the domain corresponds to exactly one element of the range. For every x-value there is only one possible y-value<br>To test for a function on a graph, use the "vertical line test" run a ruler from left to right. If your ruler only ever touches the graph in one spot, it is a function |
| **Increasing function** | As $x$-values increase, the values of y also increase. It is a function that is going "uphill" when looking at it from left to right. |
| **Decreasing function** | As $x$-values increase, the values of y decrease. It is a function that is going "downhill" when looking at it from left to right. |
| **Horizontal shift** | A translation of the graph either to the left or the right |
| **Vertical shift** | A translation of the graph either up or down |
| **Asymptote** | A straight line that a curve graph gets closer and closer to but never touches |
| **Axis of symmetry** | A line that cuts the graph exactly in half<br>A parabola with $y = ax^2 + bx + c$ has a vertical line of symmetry<br>A hyperbola has 2 axes of symmetry |

### INTERPRETATION OF GRAPHS

Given: Graphs of $f(x)$ and $g(x)$. Tips on determining graphically the values of $x$ for which:

| | |
|---|---|
| $f(x) > 0$ | Here we need to determine the values of $x$ for which y-values of the parabola are positive. (where the parabola is above the $x$-axis) |
| $f(x) \leq 0$ | Here we need to determine the $x$-values for which the y-values of the parabola are zero or negative. (where the parabola cuts or is below the x-axis). |

| | |
|---|---|
| $f(x) = g(x)$. | The points of intersection between the graphs (e.g, the parabola and the line intersect each other. (solve simultaneously) |
| $f(x) \geq g(x)$. | $x$-values where $f$ is above $g$ (that is, $g$ is below he graph of $f$. Consider the points of intersection. |
| $f(x) < g(x)$ | $x$-values where $f$ is below $g$ (that is, the graph of $g$ is above that of $f$). Point of intersection not included |
| $f(x).\, g(x) > 0$ | $x$-values where both graphs are above the $x$-axis OR are both below the $x$-axis. |
| $f(x).\, g(x) \leq 0$ | $x$-values where one graph is above the $x$-axis and the other one is below the $x$-axis. |

| **Graph** | **Turning point** | **Asymptotes** | **Domain(x); Range(y)** | **Axis of symmetry** |
|---|---|---|---|---|
| $f(x) = ax^2 + bx + c$<br>$f(x) = a(x-p)^2 + q$<br>$f(x) = a(x-x_1)(x-x_2)$ | $(p\,;q)$ | None | $x \in R$<br>$y \geq p,\ a > 0$<br>$y \leq p;\ a < 0$ | $x = p$<br>$x = \dfrac{-b}{2a}$<br>$f'(x) = 0$ |
| $f(x) = \dfrac{a}{x-p} + q$ | None | y = $q$<br>y = $p$ | $x \in R, x \neq p$<br>$x \in R, y \neq q$ | $y = (x - p) + q$<br>$y = -(x - p) + q$ |
| $f(x) = ab^{x-p} + q$ | None | y = $q$ | $x \in R, y > q(\text{if } a > 0)\, y < q(\text{if } a < 0)$ | None |

## FUNCTIONS AND INVERSE FUNCTIONS

Use a vertical line test (which must cross the graph once) to determine whether a graph is function, but the definition of the function must be given as a reason for it being a function or not.

The inverse of a function is the graph obtained by reflecting the function about the line $y = x$.

This means swapping the $x$ and the $y$ values: $(x; y) \rightarrow (y; x)$

$f^{-1}$ is used to represent the inverse of $f(x)$.

The inverse of many to one function will not be function, unless its domain is restricted.

**Exponential and logarithmic functions**.

Exponential graphs have the form $y = a^x, a > 0, a \neq 1$.
The y-intercept is (0;1); The $x$-axis is a horizontal asymptote.
The domain is $x \in R$ the range is $y > 0$.
If $a > 1$, the graph increases and $0 < a < 1$, the graph decreases; It is a one-to-one function.

**The logarithmic function** is defined as $y = \log_a x;\ a > 0,\ a \neq 1,\ x \in R$

The $x$-intercept is (1; 0); The y-axis is a horizontal asymptote.
The domain is $x > 0$ and the range is $y \in R$.
If $a > 1$, the graph increases and $0 < a < 1$, the graph decreases.
The graph is a one-to-one function.

$y=\log_a x$ and $y=\log_{\frac{1}{a}} x$ you can see that the two graphs are reflections in the $x$-axis.

If $f(x)=y=a^x, a>0, a\neq 1$ then for $f^{-1}: x=a^y$ and $f^{-1}(x)=y=\log_a x$.

If $f(x)=y=\left(\frac{1}{a}\right)^x$, $0<a<1$ then for $f^{-1}: x=\left(\frac{1}{a}\right)^y$ and $f^{-1}(x)=y=\log_{\frac{1}{a}} x$.

## ACTIVITIES

**QUESTION 1 (NW Sept 2025)**

Given: $f(x)=\log_{\frac{1}{3}} x$

1.1 Write down the equation of the asymptote of $f$. (1)

1.2 Determine the equation of $f^{-1}$ in the form $y=\ldots$ (2)

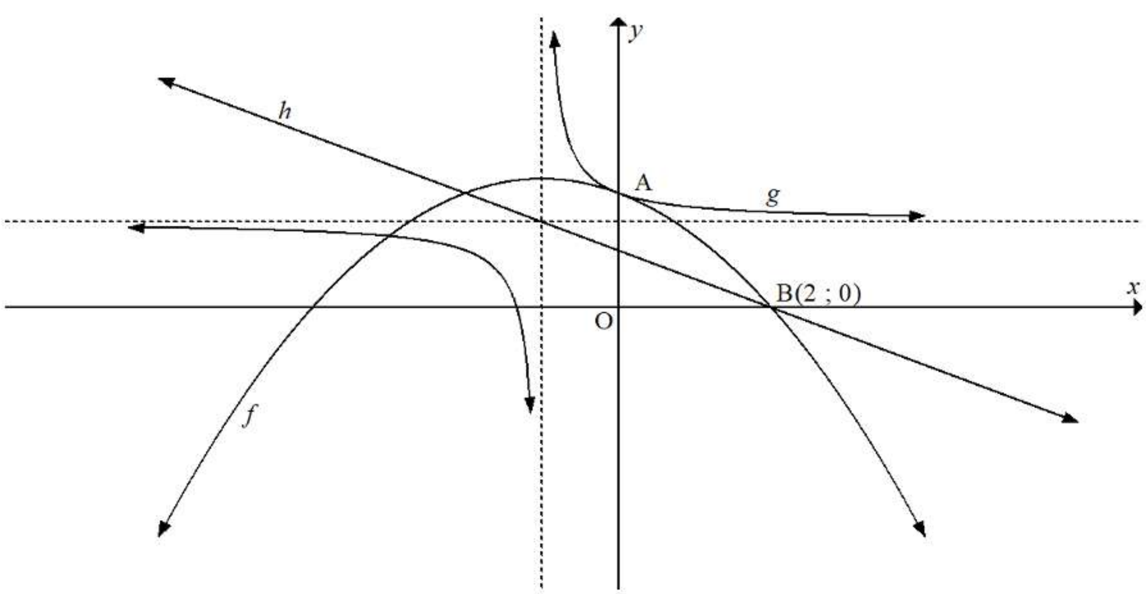
1.3 If $g(x)=f^{-1}(x-3)-2$, determine the x-intercept of $g$. (3)

1.4 Write down the equation of the asymptote of $g$. (1)

1.5 Sketch the graph of $g$. Clearly show ALL the intercepts with the axes and the asymptotes. (4)

**QUESTION 2**

Sketched below are the graphs of $f(x)=-\frac{1}{2}x^2+bx+c$ and $g(x)=\frac{a}{x+p}+q$. The axis of symmetry of $f$, $x=-1$, is the vertical asymptote of $g$. The line $h$ is an axis of symmetry of $g$. A is the $y$-intercept of $f$ and $g$. B(2 ; 0) is a $x$-intercept of $f$ and $h$.



2.1 Show that the equation of $f$ is $f(x) = -\frac{1}{2}x^2 - x + 4$ (2)

2.2 Write down the coordinates of the $y$-intercept $f$. (2)

2.3 Determine the coordinates of the turning point of $f$. (2)

2.4 Determine the equation of $h$. (3)

2.5 Determine the equation of $g$. (4)

2.6 Determine the coordinates of the $x$-intercept of $g$. (3)

2.7 For which values of $x$ will it be: $g(x)\ .\ f'(x) \leq 0$? (3)

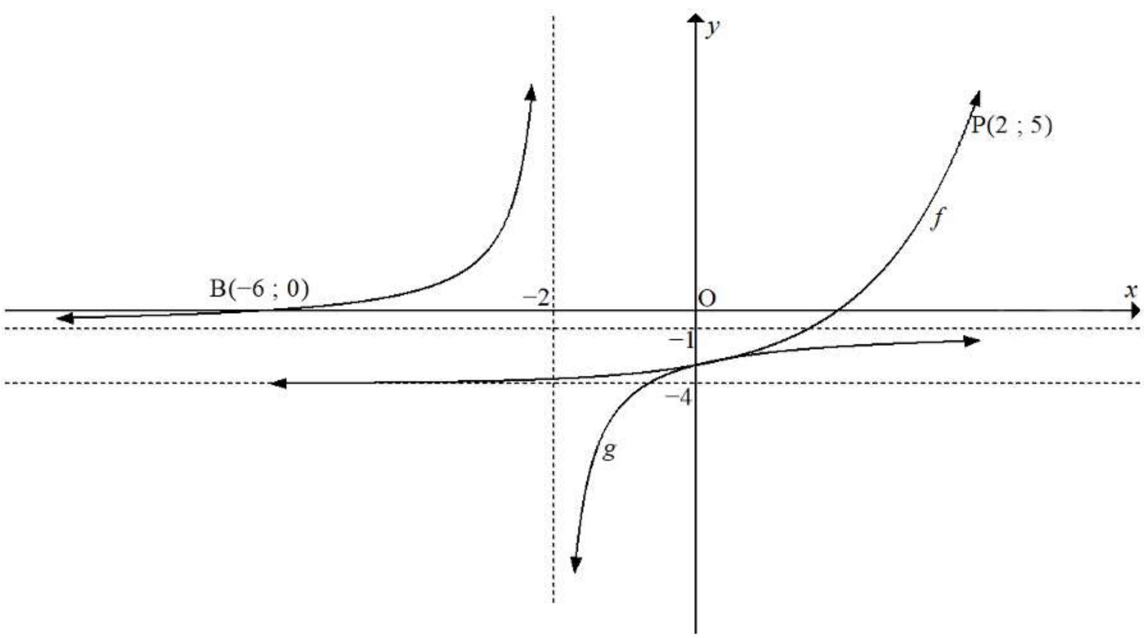
2.8 Determine the equation of $k(x)$ if k is the reflection of g about the line $x = 4$. (3)

2.9 For which values of $p$ will both $x$-intercepts of $f(x) + p$ be greater than $-2$? (4)

**QUESTION 3 (FS Sept 2025)**

The diagram below shows the graphs of the functions of $f(x) = b^x + c$ and $g(x) = \dfrac{a}{x+p} + q$

- $B(-6;0)$ is the $x$-intercept of $g$
- The graphs of $f$ and $g$ have a common $y$-intercept.
- $B(-6;0)$ is a point on $f$.




3.1 Write down the values of $p$ and $q$. (2)

3.2 Determine the equation of $g$. (3)



3.3 Determine the equation of the inverse of the axis of symmetry of $g$ for $m < 0$. (4)

3.4 Write down the equation of the asymptote of $f$. (1)

3.5 Determine the equation of $f$. (3)

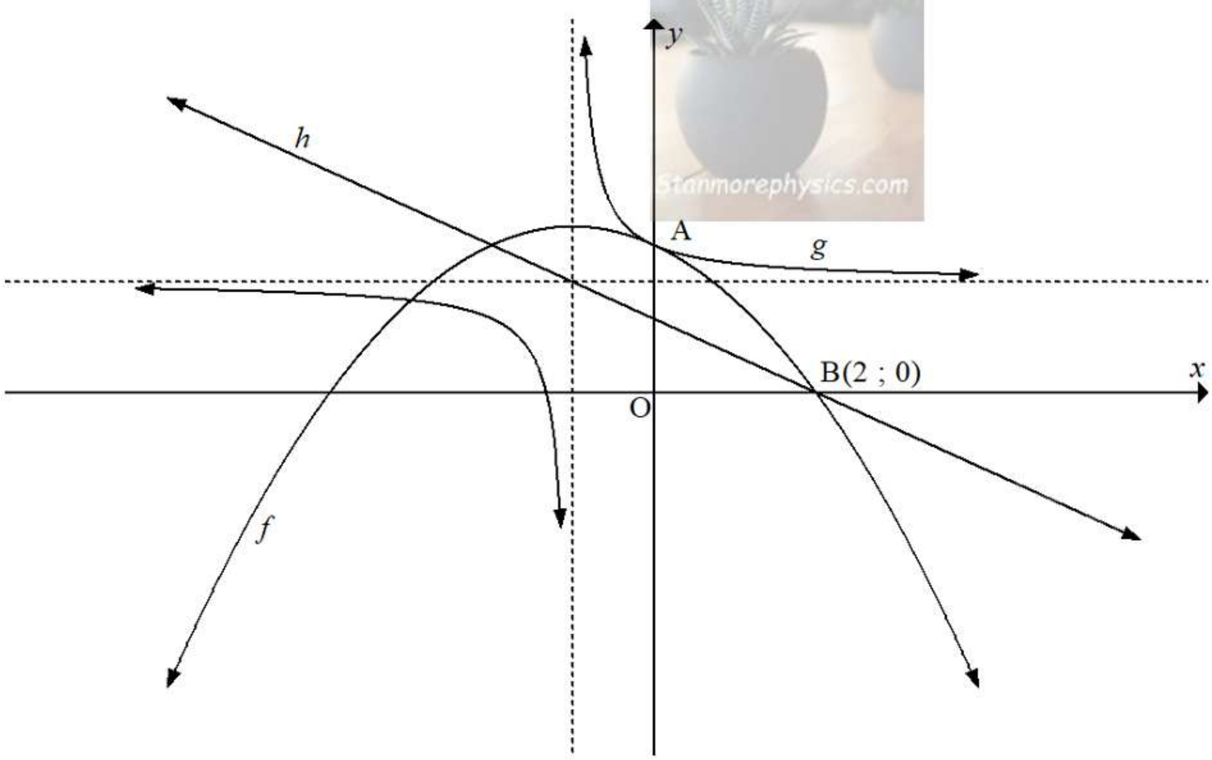
3.6 Determine the equation of $h$ if $h(x) = f(x) + 4$ (2)

3.7 Determine the equation of $h^{-1}$, the inverse of $h$, in the form $y = \ldots$ (2)

3.8 For which values of $x$ is $f(x) \geq g(x)$? (2)

**QUESTION 4 [NW Sept 2025]**

Sketched below are the graphs of $f(x) = -\frac{1}{2}x^2 + bx + c$ and $g(x) = \frac{a}{x+p} + q$. The axis of symmetry of $f$. $x = -1$, is the vertical asymptote of $g$. The line $h$ is an axis of symmetry of $g$. A is the $y-$intercept of $f$ and $g$. B$(2;0)$ is a $x-$intercept of $f$ and $h$.




4.1 Show that the equation of $f$ is $f(x) = -\frac{1}{2}x^2 - x + 4$. (2)

4.2 Write down the coordinates of the $y-$intercept of $f$. (2)

4.3 Determine the coordinates of the turning point of $f$. (2)

4.4 Determine the equation of $h$. (3)



4.5 Determine the equation of $g$. (4)

4.6 Determine the coordinates of the $x-$intercept of $g$. (3)

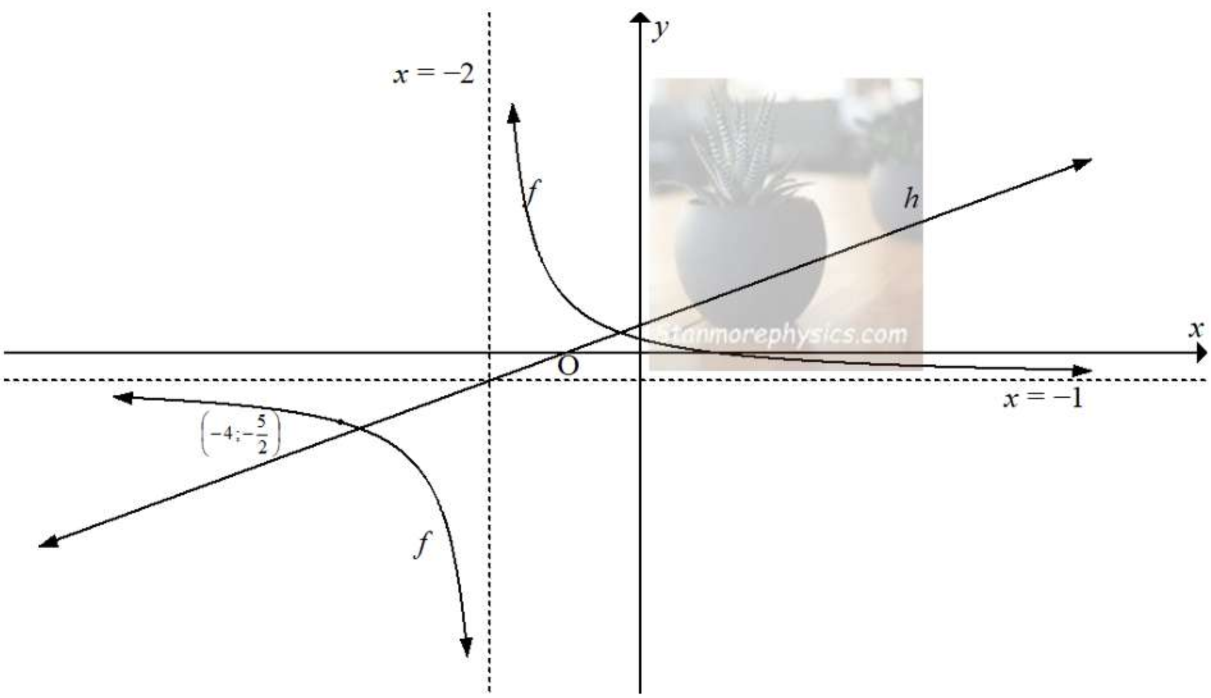
4.7 For which values of $x$ will it be: $g(x).f'(x) \leq 0$? (3)

4.8 Determine the equation of $k(x)$ if $k$ is the reflection of $g$ about the line $x = 4$. (3)

4.9 For which values of $p$ will both $x-$intercepts of $f(x)+p$ be greater than $-2$? (4)

**QUESTION 5 (GP Sept 2025)**

The graphs of the functions $f(x)=\dfrac{a}{x+p}+q$ and $h(x)=mx+c$ are sketched below.




5.1 Write down values of $p$ and $q$. (2)

5.2 Calculate the value of $a$. (1)

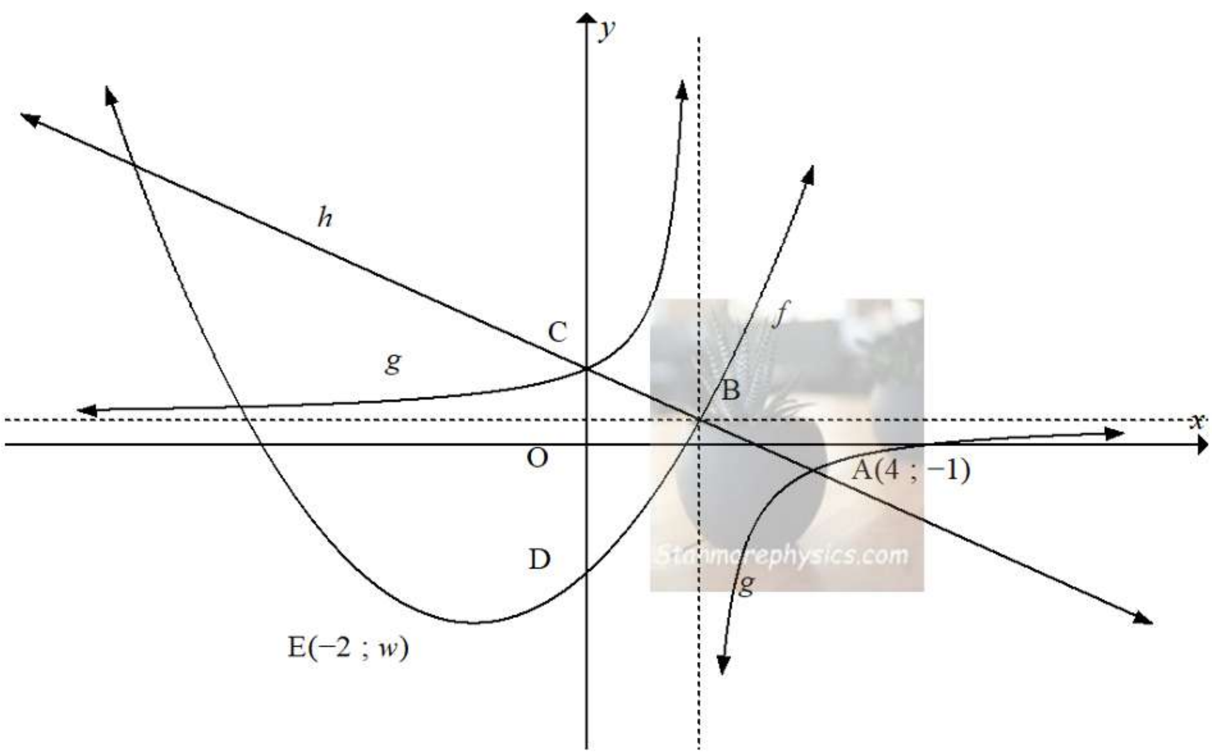
5.3 Write down the range of $f$. (1)

5.4 Determine the equation of the line of symmetry of $f$ for $m<0$ in the form $y=...$ (3)

5.5 Write down the equations of the asymptotes of $f\left(x+4\dfrac{1}{2}\right)$. (2)

**QUESTION 6 [KZN Sept 2025]**

The graphs of $f(x)=\frac{1}{2}(x+p)^2+q$ and $g(x)=\frac{a}{x+r}+t$ are sketched below. The line $h(x)=-x+3$ is an axis of symmetry of $g$. C is the $y-$intercept of both $g$ and $h$. $\text{E}(-2;w)$ is the turning point of $f$. B, is a point on $f$, is the point of intersection of the asymptotes of $g$.




6.1 Write down the coordinates of C. (1)

6.2 Show that the coordinates of B are $(2;1)$ (2)

6.3 Determine the values of $a$, $r$ and $t$. (4)

6.4 Determine the equations of the asymptotes of the graph of $j$ if $j(x)=g(x+3)-1$. (2)

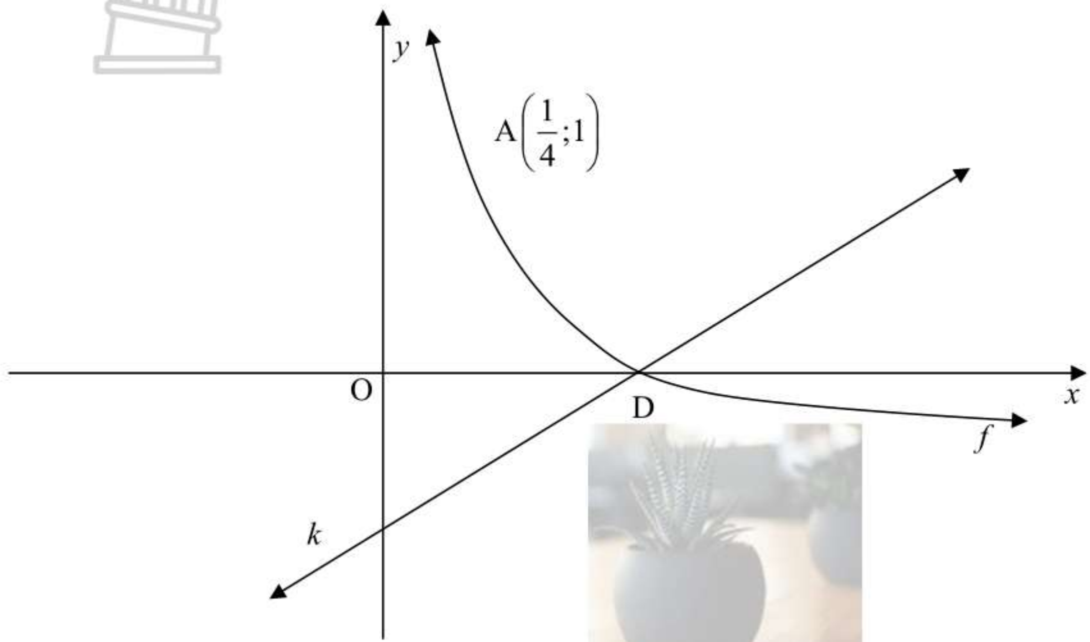
6.5 Show that the equation of $f$ is given by $f(x)=\frac{1}{2}x^2+2x-5$. (3)

6.6 If $f(x)=k$, determine the values if $k$ for $f(x)$ has TWO negative roots. (3)

6.7 Determine the values of $d$ such that $\frac{1}{2}(x+d)^2+2(x+d)-5=-(x+d)+3$ will have one positive and one negative root. (4)

**QUESTION 7 [KZN Sept 2025]**

The graphs of $f(x)=\log_b x$ and $k(x)=mx-3$ are drawn below. $A\left(\frac{1}{4};1\right)$ is a point on $f$ and D is the $x-$intercept of both $f$ and $k$.




7.1 Write down the coordinates of D. (1)

7.2 Determine the value of $b$. (2)

7.3 Determine the value $m$. (2)

7.4 Write down the domain of $f$. (1)

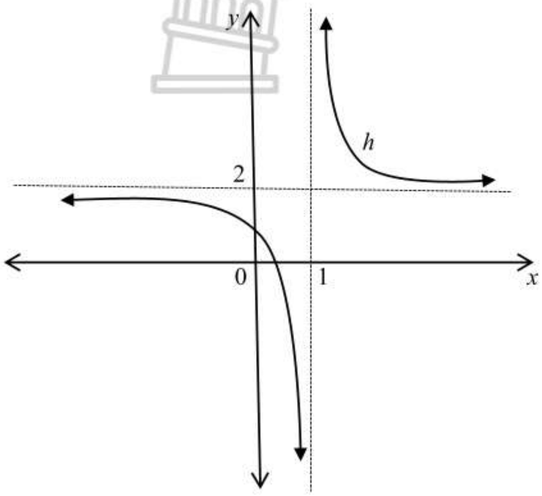
7.5 Determine the equation of $f^{-1}$, the inverse of $f$ in the form of $y=...$ (2)

7.6 Determine the values of $x$ for which $\frac{1}{4}\leq f^{-1}(x)\leq 16$. (3)

7.7 Sketch the graphs of $f^{-1}$ and $k^{-1}$ on the same system of axes. Clearly indicate intercepts with the axes. (4)

**QUESTION 8 [DBE NOV 2022]**

8.1 Sketched is the graph of $h(x)=\frac{1}{x+p}+q$. The asymptotes of $h$ intersect at $(1;2)$.




8.1.1 Write down the values of $p$ and $q$.

8.1.2 Calculate the coordinates of the $x$-intercept of $h$.

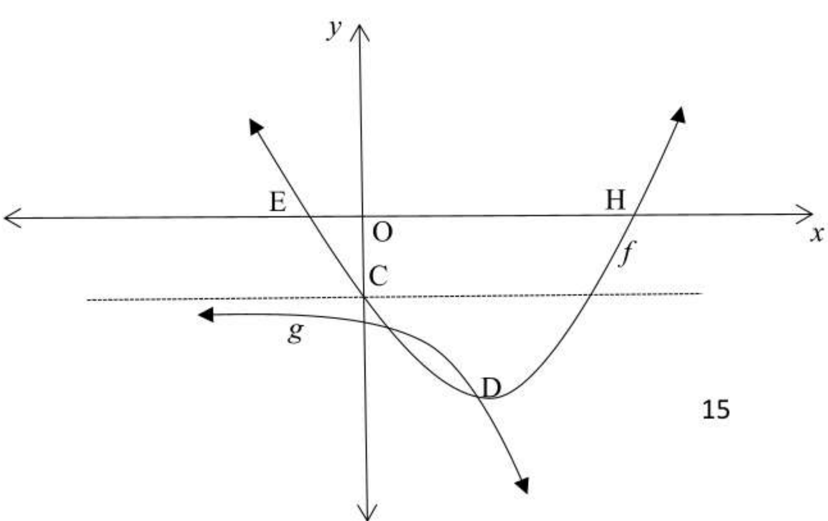
8.1.3 Write down the $x$-coordinate of the $x$-intercept of $g$ if $g(x)=h(x+3)$.

8.1.4 The equation of an axis of symmetry of $h$ is $y=x+t$. Determine the value of $t$.

8.1.5 Determine the values of $x$ for which $-2\leq\frac{1}{x-1}$

8.2 The graphs of $f(x)=x^2-4x-5$ and $g(x)=a.2^x+q$ are sketched below.

- E and H are the $x$-intercepts of $f$.
- C is the $y$-intercept of $f$ and lies on the asymptote of $g$.
- The two graphs intersect at D, the turning point of $f$.




8.2.1 Write down the $y$-coordinate of C.

8.2.2 Determine the coordinates of D.

8.2.3 Determine the values of $a$ and $q$.

8.2.4 Write down the range of $g$.

8.2.5 Determine the values of $k$ for which the value of $f(x) - k$ will always be positive.

**QUESTION 9 (DBE MAY/JUNE 2022)**

The graph of $g(x) = a\left(\frac{1}{3}\right)^{x} + 7$ passes through point $E(-2; 10)$.

9.1 Calculate the value of $a$. (3)

9.2 Calculate the coordinates of the $y$-intercept of $g$. (2)

9.3 Consider: $h(x) = \left(\frac{1}{3}\right)^{x}$

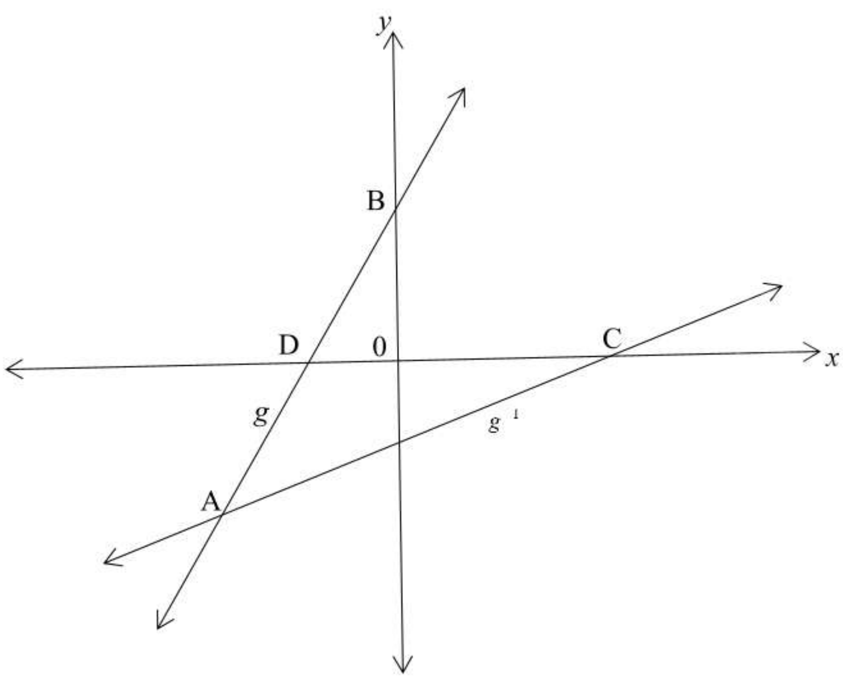
9.3.1 Describe the translation from $g$ to $h$. (2)

9.3.2 Determine the equation of the inverse of $h$, in the form $y = \ldots\ldots$ (2)

**QUESTION 10 (DBE NOV 2022)**

The graph of $g(x) = 2x + 6$ and $g^{-1}$, the inverse of $g$, are shown in the diagram below.

- D and B are the $x$- and $y$-intercepts respectively of $g$.
- C is the $x$-intercept of $g^{-1}$
- The graphs of $g$ and $g^{-1}$ intersect at A.

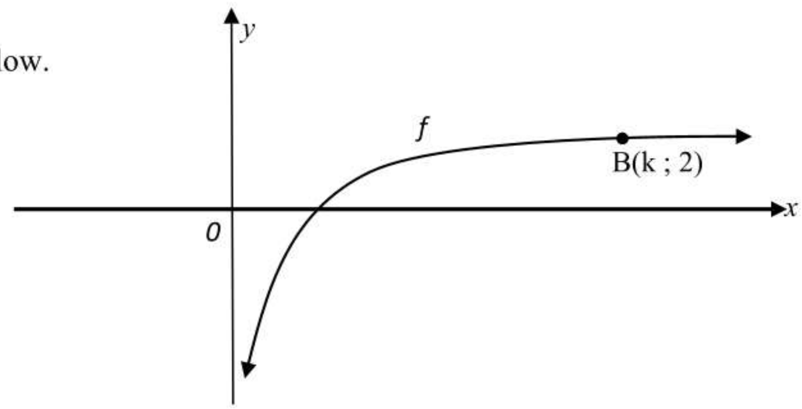


10.1 Write down the $y$-coordinate of B. (1)
10.2 Determine the equation of $g^{-1}$ in the form $g^{-1}(x) = mx + n$ . (2)
10.3 Determine the coordinates of A. (3)
10.4 Calculate the length AB. (2)
10.5 Calculate the area of $\Delta ABC$ (5)

**QUESTION 11 (DBE Nov 2021)**

The graph of $f(x) = \log_4 x$ is drawn below.

$B(k;2)$ is a point on $f$ .




11.1 Calculate the value of $k$

11.2 Determine the value of $x$ for which $-1 \leq f(x) \leq 2$

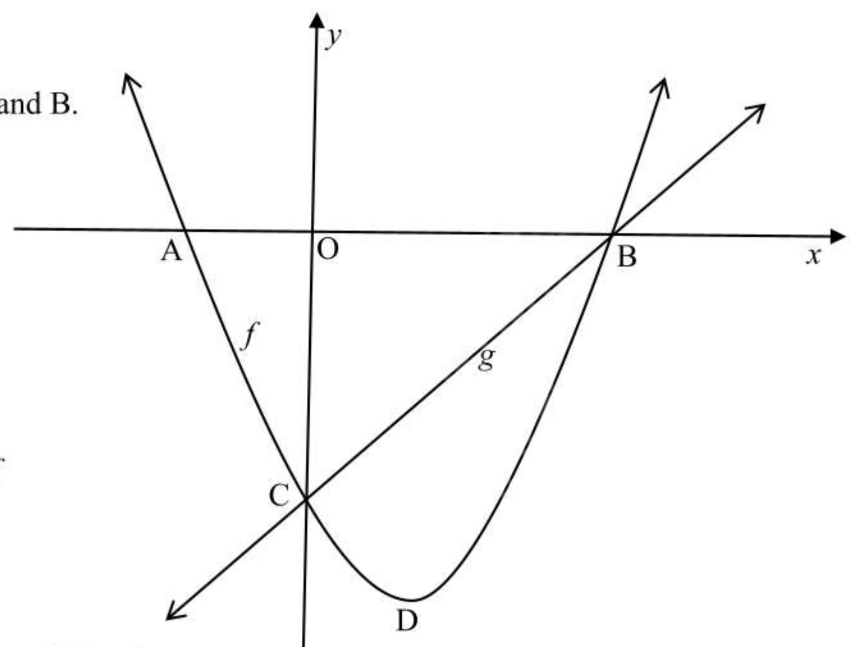
11.3 Write down the equation of $f^{-1}$, the inverse of $f$ , in the form $y = .....$

11.4 For which values of $x$ will $x.f^{-1}(x) < 0$ ?


**QUESTION 12 [DBE FEB/MARCH 2017]**

12.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$ .

- A and B are the $x-$intercepts of $f$.
- The graphs of $f$ and $g$ intersect at C and B.
- D is the turning point of $f$.
- 




12.1.1 Determine the coordinates of C.

12.1.2 Calculate the length of AB.

12.1.3 Determine the coordinates of D.

12.1.4 Calculate the average gradient of $f$ between C and D.

12.1.5 Calculate the size of $O\hat{C}B$

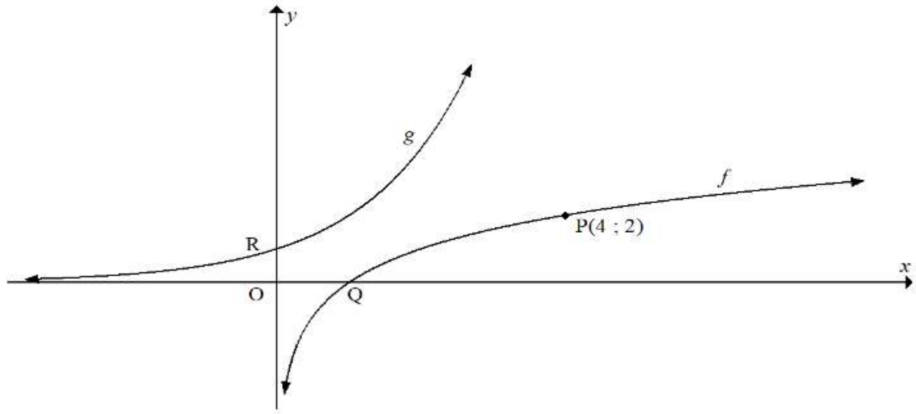
12.1.6 Determine the values of $k$ for which $f(x) = k$ will have two unequal positive roots.

12.1.7 For which values of $x$ will $f'(x).f''(x) > 0$?

12.2 The graph of a parabola $f$ has $x$-intercepts at $x = 1$ and $x = 5$ . $g(x) = 4$ is a tangent to $f$ at P, the turning point of $f$. Sketch the graph of $f$, clearly showing the intercepts with axes and the coordinates of the turning point. (5)

**QUESTION 13**

In the diagram, the graphs of $f(x)=\log_a x$ and $g$ are drawn. Graph $g$ is the reflection of $f$ in the line $y=x$. Graph $f$ passes through the point P(4 ; 2). Q is the $x$-intercept of $f$ and R is the $y$-intercept of $g$.




13.1 Write down the coordinates of P′, the image of P on g.

13.2 Show that $a=2$.

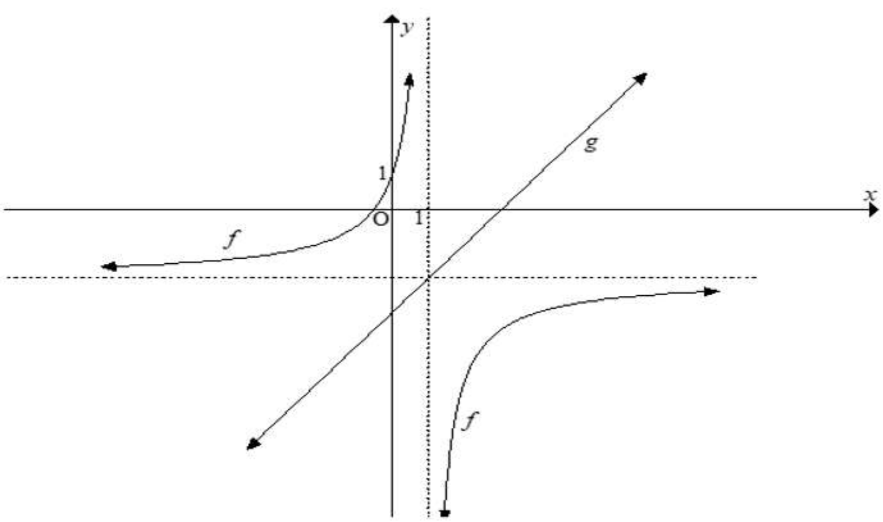
13.3 Write down the equation of $g$ in the form $y=...$

13.4 T is a point on $f$ in the first quadrant where TR is parallel to $x$-axis. Calculate the area of $\Delta RTP'$.

**QUESTION 14 (DBE NOV 2024)**

Sketched below is the graph of $f(x)=\dfrac{a}{x+p}+q$ having the domain $(-\infty;1)\cup(1;\infty)$.

The graph of $f$ cuts the $y$-axis at (0 ; 1). A line of symmetry of $f$ is given by $g(x)=x-3$.



14.1 Write down the value of $p$.

14.2 Determine the equation of the horizontal asymptote of $f$.

14.3 Calculate the value of $a$.

14.4 For which values of $x$ is $f(x) \geq 0$?

14.5 Graph $f$ undergoes a transformation to $h$ where:

- The domain and range of $h$ are the same as that of $f$
- $h'(x)$, the derivative of $h$, is negative on its domain

Describe a possible transformation that $f$ could have undergone to result in $h$.

**QUESTION 15**

$f(x) = ax^3 + bx^2 + cx + d$ has the following properties:

| | |
|---|---|
| $a < 0$ | $d = 0$ |
| $f(-3) = 0$ *en* $f(8) = 0$ | $f'(-1) = 0$ *en* $f'(5) = 0$ |



Draw a sketch graph of $f$ using the information given above. (5)

**QUESTION 16**

The following information about a cubic polynomial, $y = f(x)$ is given:

- $f(2) = 0$
- $f(1) = -4$
- $f(0) = -2$
- $f'(-1) = f'(1) = 0$
- If $x < -1$ then $f'(x) > 0$
- If $x > 1$ then $f'(x) > 0$

Use this information to draw a neat sketch graph of $f$.. (5)

16.1 For which value(s) of $x$ is f strictly decreasing? (2)

16.2 Use your graph to determine the x-coordinate of point of inflection (2)

16.3 For which value(s) of $x$ is $f$ concave up? (2)

**QUESTION 17**

Given: $f(x) = x^3 - 4x^2 - 11x + 30$

17.1 Determine the y-intercept of $f$. (1)

17.2 Calculate the $x$-intercepts of $f$. (4)

17.3 Calculate the coordinates of the turning points of $f$. (5)

17.4 Draw a neat sketch graph of $f$, showing clearly all intercepts with the axes and turning points (3)

17.5 Determine the $x$-coordinate of the point of inflection of the graph of $f$. (2)

**QUESTION 18**

Given $f(x) = ax^3 + bx^2 + cx + d$ :

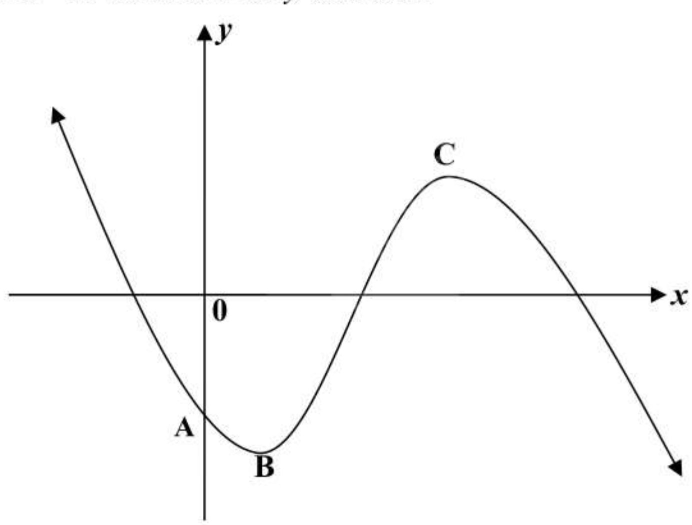
The gradient at any point $(x; f(x))$ is given by $(18x^2 + 14x - 8)$, and $f(0) = -7$ .

Determine the values of $a, b, c$ and $d$. (5)

**QUESTION 19**

In the diagram, the graph of $f(x) = -x^3 + 10x^2 - 17x - 28$ intersects the $y$-axis at A.

B and C are the turning points of $f$.




19.1 Write down the coordinates of A.

19.2 Calculate the coordinates of B and C.

19.3 For which value(s) of $x$ is $f$ concave up?

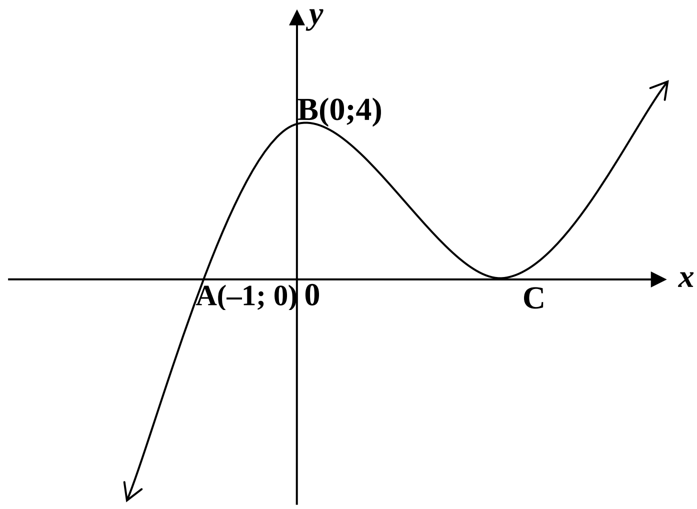
19.4 Determine the value(s) of $p$ for which $f(x) = p$ has only one positive root.



**QUESTION 20**

The diagram shows the graph of $f(x) = x^3 + ax^2 + bx + 4$.

B and C are turning points of $f(x)$




| | | |
|---|---|---|
| 20.1 | Show that the values of $a$ and $b$ are –3 and 0 respectively. | (4) |
| 20.2 | Determine the coordinates of C. | (3) |
| 20.3 | Determine the equation of the tangent to the curve at A. | (5) |
| 20.4 | For which values of $k$ will $x^3 - 3x^2 + 4 = k$ have 3 distinct roots. | (2) |
| 20.5 | Write the values of $x$ for which $f'(x) > 0$ | (2) |

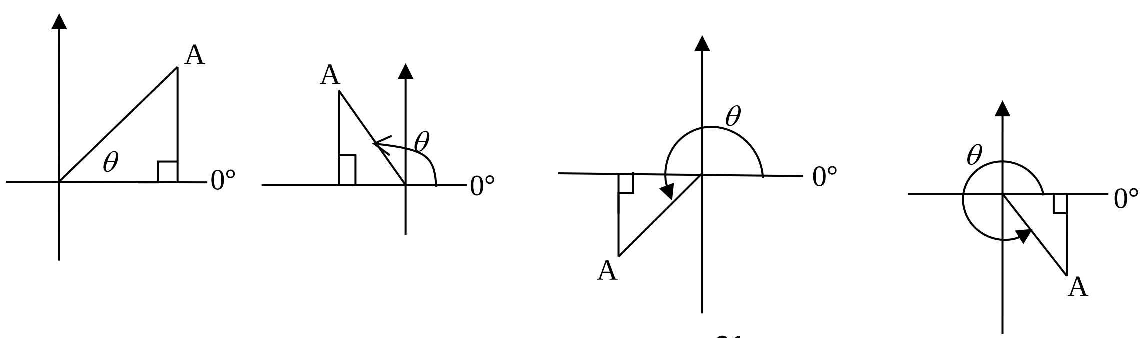
## TRIGONOMETRY

**USING DIAGRAM TO DETERMINE THE NUMERICAL VALUES OF RATIOS FOR ANGLES**

1. Write the given equation in a form of a simple trig ratio, for example, if $5\cos x + 3 = 0$, then:

$$5\cos x = -3 \therefore \cos x = -\frac{3}{5}$$

2. Draw the sketch in the correct quadrant, using the interval/restriction given and also the CAST rule for example



3. Fill in the known details in the diagram (drawn in the correct quadrant.)
4. Use Pythagoras theorem to calculate the value of the unknown side. Decide whether it is positive or negative.
5. Use the diagram to answer the questions

**ACTIVITIES**

1. If $\tan\beta = -\frac{3}{4}$ and $\beta \in [180^\circ; 360]$ Calculate without the use of a calculator and with the aid of a diagram the value of:

   1.1 $\frac{1}{\cos\beta} + \sin 2\beta$ 1.2 $\cos(180^\circ - \beta)$ 1.3 $\cos 2\beta$

2.1 If 4tan A = – 3 where 0° < A < 180° and 13cos B – 12 = 0 where 180° < B < 360°

Calculate the value of

2.1.1 $\sin A \cos B$

2.1.2. $\sin 2A \cos B$

2.2 If $90^0 < A < 360^0$ and tan A = $\frac{2}{3}$, determine without the use of a calculator.

2.2.1 sin A

2.2.2 cos 2A – sin 2A

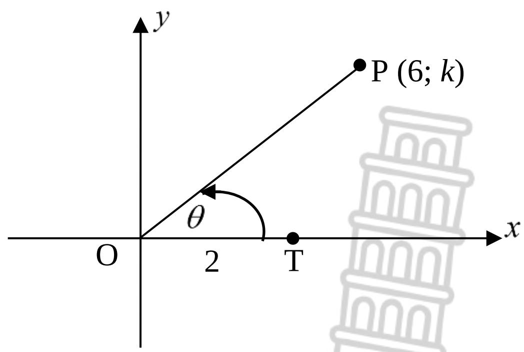
3. In the diagram, P (6 ; $k$) is a point in the first quadrant. $P\hat{O}T = \theta$ and OT = 2

   It is further given that $\sqrt{5}\cos\theta - 2 = 0$

   Determine, without the use of a calculator:

   3.1 $\tan\theta$ in terms of $k$

   3.2 The value of $k$




4. Given that $\sqrt{13}\sin x + 3 = 0,$ *where* $x \in (90°;\ 270°)$

   Without using a calculator, determine the value of:

   4.1 $\sin(360^\circ + x)$ 4.2 $\tan x$ 4.3 $\cos(180^\circ + x)$

5. If $\cos\alpha = \sqrt{t}$, where α is an acute angle, express each of the following in terms of $t$:

   5.1 $\tan\alpha$ 5.2 $\sin(180^\circ - \alpha)$ 5.3 $\sin 2\alpha$

6. It is known that $13\sin\alpha - 5 = 0$ and $4\tan\beta = -3$ where $\alpha \in (90^\circ; 270^\circ)$ and

$\beta \in (90°;270°)$



Determine, without using a calculator, the values of the following:

6.1 $\cos \alpha$ 6.2 $\cos(\alpha + \beta)$

7. If $\sin 36^0 = p$, express $\sin 66^0$ in terms of $p$

8. Given: $\sin 36^0 = \sqrt{1 - p^2}$ ,

Determine, without using a calculator, the values of the following in terms of *p*:

8.1 $\tan\ 36^0$

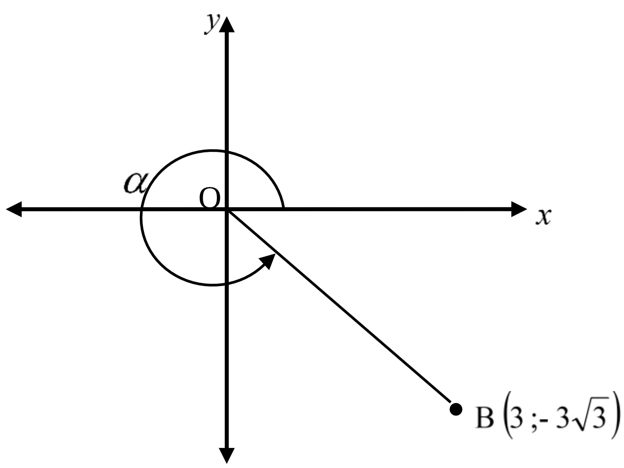
8.2 $\cos 108^0$

9. In the Cartesian plane alongside, the point $B\left(3\,;-3\sqrt{3}\right)$ and the reflex angle, $\alpha$, are shown.

Determine (without using a calculator) the value of:

9.1 OB

9.2 $\cos\left(\alpha + 30^{o}\right)$




10. If $\sin 34° = k$, determine the value of the following in terms of *k*:

10.1 $\cos 68°$

10.2 $\cos 34°.\cos(-22°) + \sin 34°.\sin 202°$

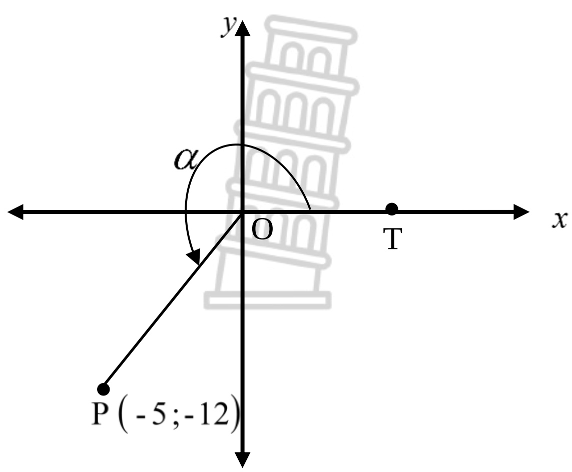
11. In the diagram, reflex $T\hat{O}P = \alpha$ and P(–5; –12) is a point on the Cartesian plane.

Determine the value of each of the following trigonometric ratio WITHOUT using a calculator

11.1 $\cos \alpha$ (3)

11.2 $\tan\left(180^{o} - \alpha\right)$ (2)

11.3 $\sin\left(30^{o} - \alpha\right)$ (3)



12 In the diagram, $P(3;t)$ is a point in the Cartesian plane ;

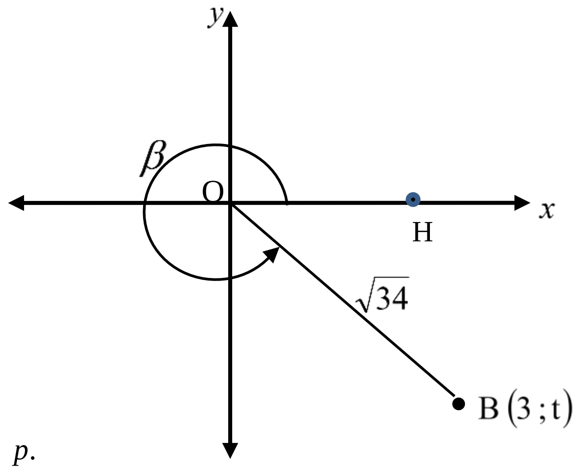
$OP=\sqrt{34}$ and $H\hat{O}P=\beta$ is reflex angle.

Without using a calculator, determine the value of

12.1 $t$

12.2 $\tan\beta$

12.3 $\cos 2\beta$




13. If $\sin 40° = p$, write EACH of the following in terms of $p$.

13.1 $\sin 220°$

13.2 $\cos^2 50°$

13.3 $\cos(-80°)$

14. If $\sin\beta=\frac{1}{3}$ and $\beta\in(90°;270°)$ Calculate without the use of a calculator

and with the aid of a diagram the value of:

14.1 $\cos\beta$

14.2 $\sin 2\beta$

14.3 $\cos(450°-\beta)$

15. It is given that $\tan 50° = k$. Express EACH of the following in terms of $k$ :

15.1.1 $\cos 40°$

15.1.2 $\dfrac{2\sin 25°.\cos 25°}{-2+4\sin^2 25°}$

15.1.3 $\sin 10°$

16. If $\cos\theta=\frac{-5}{13}$ where $180°<\theta<360°$, determine, without using a calculator,

the value of:

16.1.1 $\sin^2\theta$

16.1.2 $\tan(360°-\theta)$

16.1.3 $\cos(\theta-135°)$

17. Given: $\sin\beta = \frac{12}{13}$, where $\tan\beta < 0$.

With the aid of a diagram, and without the use of a calculator, determine the value of $\sin 2\beta$.

**IDENTITIES**

| | | |
|---|---|---|
| **GRADE 11 CONTENT** | | |
| $\sin^2\theta + \cos^2\theta = 1$ | $\sin^2\theta = 1 - \cos^2\theta$ | $\cos^2\theta = 1 - \sin^2\theta$ |
| $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\theta \neq k.90^0$, $k$ an odd integer | | $\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ |
| **GRADE 12 CONTENT** | | |
| $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ | $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ | |
| $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ | $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ | |
| $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ | $\cos 2\alpha = 2\cos^2\alpha - 1$ | |
| $\cos 2\alpha = 1 - 2\sin^2\alpha$ | $\sin 2\alpha = 2\sin\alpha\cos\beta$ | |

**EXAMPLE:**

Prove that: $\frac{\sin\theta + \sin 2\theta}{\cos 2\theta - \cos\theta} = \frac{\sin\theta}{\cos\theta - 1}$

$$LHS = \frac{\sin\theta + \sin 2\theta}{\cos 2\theta - \cos\theta}$$

$$= \frac{\sin\theta + 2\sin\theta\cos\theta}{2\cos^2\theta - 1 - \cos\theta}$$ (use $\sin 2\alpha = 2\sin\alpha\cos\beta$ and $\cos 2\alpha = 2\cos^2\alpha - 1$)

$$= \frac{\sin\theta(1 + 2\cos\theta)}{(2\cos\theta + 1)(\cos\theta - 1)}$$ (factorise and simplify)

$$= \frac{\sin\theta}{\cos\theta - 1}$$

**Exercise**

Prove that

1. $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1 + \sin x} = 1 - \sin x$

2. $\frac{1}{\sin^2 2x} - \frac{1}{\tan^2 2x} = 1$

3. $\cos 2x+\cos^2 x+3\sin^2 x=2$

## REDUCTION FORMULA

The *trigonometric reduction formulas* help us to "reduce" a ***trigonometric*** ratio to a ratio of an acute angle.

Any angle in a form of $90^\circ \pm\theta$, $180^\circ \pm\theta$ and $360^\circ \pm\theta$ can be written simply in terms of $\theta$

| $(90^\circ-\theta)$ | $(180^\circ-\theta)$ | $(180^\circ+\theta)$ |
|---|---|---|
| $\sin(90^\circ-\theta)=+\cos\theta$<br>$\cos(90^\circ-\theta)=+\sin\theta$<br>$\sin(90^\circ+\theta)=+\cos\theta$<br>$\cos(90^\circ+\theta)=-\sin\theta$ | $\sin(180^\circ-\theta)=+\sin\theta$<br>$\cos(180^\circ-\theta)=-\cos\theta$<br>$\tan(180^\circ-\theta)=-\tan\theta$ | $\sin(180^\circ+\theta)=-\sin\theta$<br>$\cos(180^\circ+\theta)=-\cos\theta$<br>$\tan(180^\circ+\theta)=+\tan\theta$ |
| $(360^\circ-\theta)$ | $(360^\circ+\theta)$ | $(-\theta)$ |
| $\sin(360^\circ-\theta)=-\sin\theta$<br>$\cos(360^\circ-\theta)=+\cos\theta$<br>$\tan(360^\circ-\theta)=-\tan\theta$ | $\sin(360^\circ+\theta)=\sin\theta$<br>$\cos(360^\circ+\theta)=\cos\theta$<br>$\tan(360^\circ+\theta)=\tan\theta$ | $\sin(-\theta)=-\sin\theta$<br>$\cos(-\theta)=+\cos\theta$<br>$\tan(-\theta)=-\tan\theta$ |

## ACTIVITIES

### QUESTION 1

Prove each of the following:

1.1 $\dfrac{\sin\theta}{1-\cos\theta}-\dfrac{\cos\theta}{\sin\theta}=\dfrac{1}{\sin\theta}$

1.2 $\dfrac{\sin 2x}{\cos 2x+\sin^2 x}=2\tan x$

1.3 $\dfrac{-2\sin^2 x+\cos x+1}{1-\cos(540^\circ-x)}=2\cos x-1$

1.4 $\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}=\tan x$

1.5 $\dfrac{\sin 2x}{\sin x}=\dfrac{\cos 2x}{\cos x}=\dfrac{1}{\cos x}$

1.6 $\dfrac{\cos 2x+\cos^2 x+3\sin^2 x}{2-2\sin^2 x}=\dfrac{1}{\cos^2 x}$

1.7 $\dfrac{\sin 2x+\cos 2x+1}{\sin x+\cos x}=2\cos x$

1.8 $\dfrac{\cos 2x+\cos^2 x+3\sin^2 x}{2-2\sin^2 x}=\dfrac{1}{\cos^2 x}$

1.9 $\dfrac{\sin x-\sin x\cos x}{\cos x-1+\sin^2 x}=\tan x$

1.10 $\dfrac{\sin 2x+\cos 2x+1}{\sin x+\cos x}=2\cos x$

1.11 $\sin(30^\circ+x)+\sin(30^\circ-x)=\cos x$

1.12 $\cos(90^\circ-2x)\tan(180^\circ+x)+\sin^2(360^\circ-x)=3\sin^2 x$

1.13 $\dfrac{1-\tan A}{1+\tan A}=\dfrac{\cos 2A}{1+\sin 2A}$

1.14 $2\cos^2(45^\circ+x)=1-\sin 2x.$

**QUESTION 2**



2.1 Consider the following identity: $\dfrac{1-2\sin A\cos A}{\sin A-\cos A}=\sin A-\cos A$

(i) Prove the above identity

(ii) For which values of A will the above identity not be defined?

2.2 Prove that:

2.2.1 $\cos(x-y)-\cos(x+y)=2\sin x\sin y$

2.2.2 Hence find without the use of a calculator the value of $\cos 15°-\cos 75°$

2.3 Prove that:

2.3.1 $\sin(45°+\theta).\sin(45°-\theta)=\dfrac{1}{2}\cos 2\theta$

2.3.2 Hence determine the value of $\sin 75°.\sin 15°$

2.4 Prove that:

2.4.1 $\cos(x+y).\cos(x-y)=1-\sin^2 x-\sin^2 y$

2.4.2 Hence determine the value of $1-\sin^2 45°-\sin^2 15°$

2.5 Simplify to a single term: $\cos(x-360°).\sin(90°+x)+\cos^2(-x)-1$

2.6 Consider $\sin(2x+40°), \cos(x+30°)-\cos(2x+40°)\sin(x+30°)$

Write as a single trigonometric term in its simplest form

**QUESTION 3**

3.1 Prove the following identity: $\dfrac{(1-\sin\beta)(1+\sin\beta)}{\cos^2\beta}=1$

3.2 Prove the following identity: $\dfrac{\sin\alpha-\sin\alpha\cos\alpha}{\cos\alpha-(1-\sin^2\alpha)}=\tan^2\alpha$

3.3 Prove that: $\dfrac{8}{\sin^2 \mathrm{A}}-\dfrac{4}{1+\cos \mathrm{A}}=\dfrac{4}{1-\cos \mathrm{A}}$

3.4 Using the identity in Question 3.3, determine the values of $A$ in the interval $0^{o}\le A\le 360^{o}$ for which the identity is undefined

3.5 Given: $\tan x\left(1-\cos^2 x\right)+\cos^2 x=\dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$

3.5.1 Prove the above identity. (5)

3.5.2 For which values of $x$, in the interval $x\in[-180°;180°]$, will the identity be undefined? (3)

3.6 Given the expression: $\dfrac{\sin 150^\circ + \cos^2 x - 1}{2}$

3.6.1 **Without using a calculator**, simplify the expression given above to a single trigonometric term in terms of $\cos 2x$. (6)

3.6.2 Hence, determine the general solution of $\dfrac{\sin 150^\circ + \cos^2 x - 1}{2} = \dfrac{1}{25}$ (5)

**QUESTION 4**

4.1 Determine the value of the following expression:

4.1.1 $\dfrac{\cos(360^\circ - \theta)(\sin(90^\circ + \theta)\sin(-\theta)}{\sin(\theta + 180^\circ)}$

4.1.2 $\dfrac{\sin(180^\circ + \theta)\tan(720^\circ + \theta)\cos(-\theta)}{\cos(90^\circ + \theta)}$

4.1.3 $\dfrac{\tan(180^\circ + x)\cos(360^\circ - x)}{\sin(x - 180^\circ)\cos(90^\circ + x) + \cos(720^\circ + x)\cos(-x)}$

4.1.4 $\dfrac{\cos(-250^\circ)\cos 120^\circ}{\sin 20^\circ} + \dfrac{\sin 315^\circ \tan^2 240^\circ}{\cos 405^\circ}$

4.1.5 $\dfrac{4\cos(-x).\cos(90^\circ + x)}{\sin(30^\circ - x).\cos x + \cos(30^\circ - x).\sin x}$

4.1.6 $\sin(180^\circ - x).\cos(-x) + \cos(90^\circ + x).\cos(x - 180^\circ)$

4.1.7 $\dfrac{2\cos(90^\circ - x)}{\sin(180^\circ - 2x)} \times \dfrac{\cos(60^\circ - x)\cos x - \sin(60^\circ - x)\sin x}{\tan(-x)}$

4.1.8 $\dfrac{\sin 104^\circ(2\cos^2 15^\circ - 1)}{\tan 38^\circ.\sin^2 412^\circ}$

4.1.9 $\dfrac{\cos(-180^\circ - x)\tan(360^\circ - x)\cos^2(90^\circ - x)}{\sin(180^\circ - x).\sin(-x)}$

4.1.10 $\dfrac{\cos\theta.\tan(180^\circ - \theta)\sin\theta}{\sin(540^\circ + \theta).\cos(90^\circ - \theta)}$

4.2 Prove that:

4.2.1 $\dfrac{\tan(180^\circ + x)\cos(360^\circ - x)}{\sin(x - 180^\circ)\cos(90^\circ + x) + \cos(720^\circ + x)\cos(-x)} = \sin x$

4.2.2 $\dfrac{\sin 190^\circ.\cos 225^\circ.\tan 390^\circ}{\sin 135^\circ.\cos 100^\circ} = -\dfrac{1}{\sqrt{3}}$

4.3. Without using a calculator, determine the value of $\dfrac{\sin^2 35° - \cos^2 35°}{4\sin 10° \cos 10°}$ **[DBE NOV 2019]**

**QUESTION 5**

5.1 Determine, without using a calculator, the value of

$\cos(A+55°).\cos(A+10°)+\sin(A+55°).\sin(A+10°)$

5.2 Simplify: $\dfrac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$

**QUESTION 6 [DBE NOV 2025]**

6.1 Prove that: $[\tan(180+x)](1-\cos^2 x)+\cos^2 x = \dfrac{(\sin x-\cos x)(1+\sin x.\cos x)}{-\cos x}$ (6)

6.2 It is given that $\sin^2 x;\ \cos^2 x$ and $\dfrac{1}{2}\sin 2x$ are the first three terms of an arithmetic sequence. The constant difference of the arithmetic sequence is NOT zero. (7)

**QUESTION 7**

Calculate without the use of a calculator: $\dfrac{\cos^2 208°}{\tan 118°.\sin 124°}$

**QUESTION 8 [DBE NOV 2025]**

8.1 Given: $\dfrac{\sin(540°+x).\cos(90°+x)}{\sin(-x)}$

8.1.1 Simplify the expression above fully to a single trigonometric ratio. (4)

8.1.2 Hence, determine the values of $x$ in the interval $x \in [0°;360°]$ for which

$\sqrt{\dfrac{\sin(540°+x).\cos(90°+x)}{\sin(-x)}}$ will be real. (2)

**QUESTION 9 [DBE May/June 2025]**

9.1 Simplify the expression to a single trigonometric term: $\dfrac{2\cos(180°-x)\sin(-x)}{1-2\cos^2(90°-x)}$ (6)

9.2 Calculate the value of the following expression **without using a calculator**:

$(\tan 92°)(\tan 94°)(\tan 96°)...(\tan 176°)(\tan 178°)$ (4)

**QUESTION 10 [May/June 2025]**

Consider the expression: $\sin(A-B)-\sin(A+B)$

10.1 Prove that $\sin(A-B)-\sin(A+B)=-2\cos A\sin B$.

10.2 Simplify the following expression to a single term: $\sin 4x-\sin 10x$

10.3 Hence, determine the solution for $x\in[0°;30°]$.

**QUESTION 11 [GP Trial 2025]**

11.1 Given: $\cos(\alpha-\theta)=\cos\alpha\cos\theta+\sin\alpha\sin\theta$

11.1.1 Use the above identity to deduce that $\sin(\alpha-\theta)=\sin\alpha\cos\beta-\cos\alpha\sin\theta$.

11.1.2 Hence , or otherwise, evaluate $\sin 76°.\sin 44°-\sin 14°.\sin 46°$.

11.2 Given: $f(x)=\sin x$

Show that $\dfrac{f(x+h)-f(x)}{h}$ can be written as $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)$.

**QUESTION 12 [GP Trial 2025]**

12.1 Simplify the following to a single trigonometric term, without the use of a calculator:

$$\frac{\tan(180°-x)\cos(180°-x)}{\cos 240°\left(\tan^2 y-\dfrac{1}{\cos^2 y}\right)}$$ (7)

12.2 Prove the identity: $\dfrac{\sin 3x}{\sin x\cos x}=\dfrac{4\cos^2 x-1}{\cos x}$ (5)

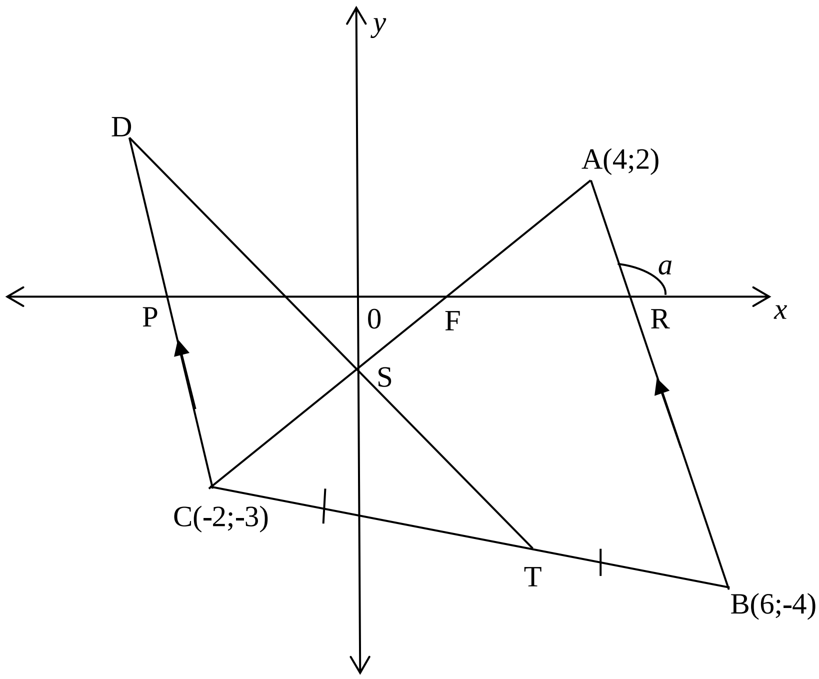
12.3 Determine the general solution of $\cos x+1=\sin x$ (7)

**QUESTION 1 [DBE Nov 2022]**

In the diagram, A(4 ; 2), B(6 ; -4) and C(-2 ; -3) are vertices of $\Delta ABC$. T is the midpoint of CB. The equation of line AC is $5x - 6y = 8$. The angle of inclination of AB is *a*.

$\Delta DCT$ is drawn such that CD//BA. The lines AC and DT intersect at S, the *y*-intercept of AC.

P, F and R are the *x*-intercepts of DC, AC and AB respectively.




1.1 Calculate the:

1.1.1 Gradient of AB (2)

1.1.2 Size of *a* (2)

1.1.3 Coordinates of T (2)

1.1.4 Coordinates of S (2)

1.2 Determine the equation of CD in the form $y = mx + c$. (3)

1.3 Calculate the:

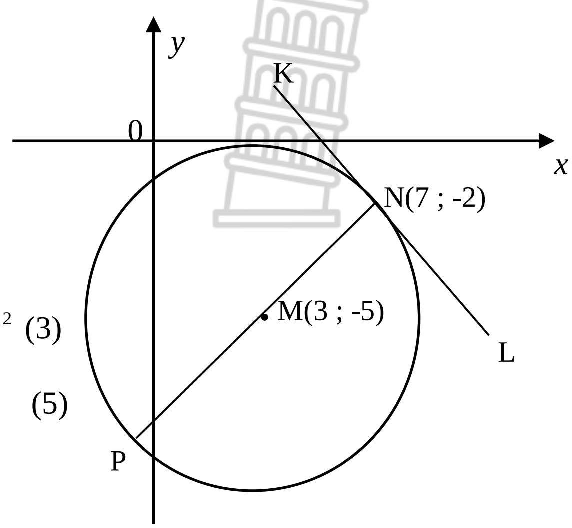
1.3.1 Size of $D\hat{C}A$ (4)

**[15]**

**QUESTION 2 [DBE Nov 2022]**

In the diagram, M(3 ; -5) is the centre of the circle having PN as its diameter.

KL is a tangent to the circle at N(7 ; -2).




2.1 Calculate the coordinates of P. (2)

2.2 Determine the equation of:

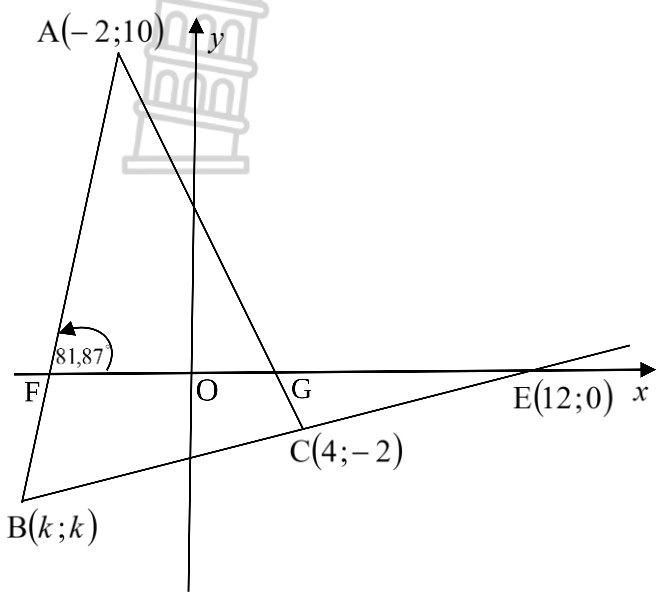
2.2.1 The circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (3)

2.2.2 KL in the form $y = mx + c$ (5)

**QUESTION 3 [DBE SC 2021]**



In the Diagram, $A(-2;10)$, $B(k;k)$ and $C(4;-2)$ are the vertices of $\Delta$ ABC. Line BC is produced to H and cuts the $x-$axis at $E(12;0)$. AB and AC intersect the $x-$axis at F and G respectively. The angle of inclination of line AB is $81,87^{\circ}$.




3.1 Calculate the gradient of :

3.1.1 BE (2)

3.1.2 AB (2)

3.2 Determine the equation of BE in the form $y = mx + c$ (2)

3.3 Calculate the:

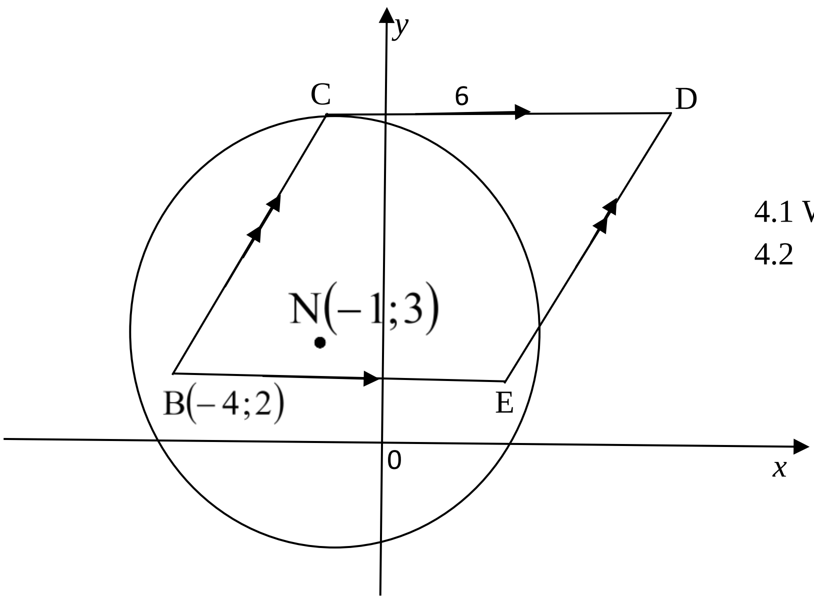
3.3.1 Coordinates of B, where $k < 0$ (2)

3.3.2 Size of $\hat{A}$ (4)

**[12]**

**QUESTION 4 [DBE SC 2021]**

In the diagram, the circle centred at $N(-1;3)$ passes through $A(-1;-1)$ and $B(-4;2)$, C, D and E are joined to form a parallelogram such that BE is parallel to the $x-$axis. CD is a tangent to ne circle at C and CD = 6 units.




4.1 Write the length of the radius of the circle.

4.2 Calculate the:

4.2.1 Coordinates of C.

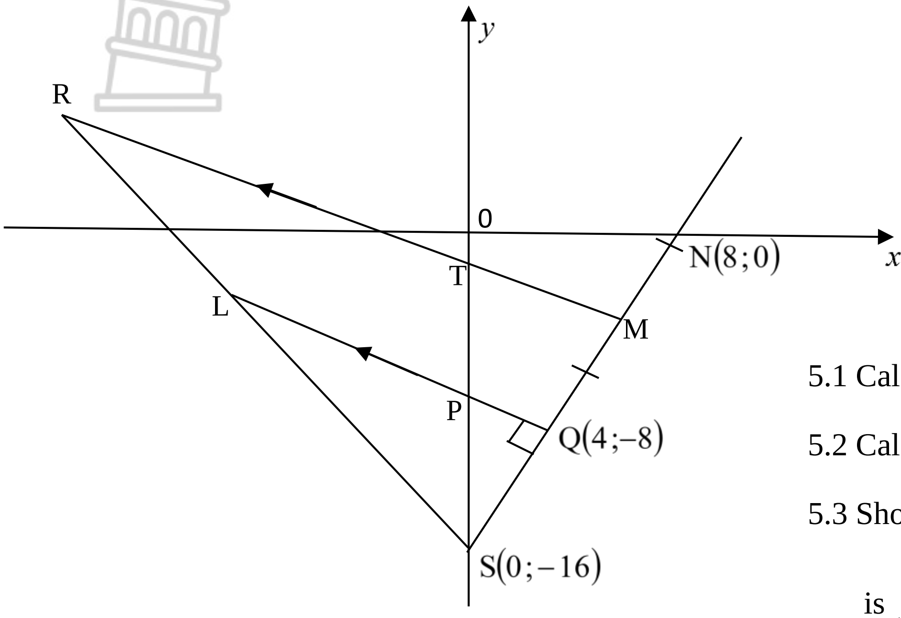
4.2.2 Coordinates of D.

4.2.3 Area of $\Delta$BCD.

**QUESTION 5 [DBE SC 2021]**


In the diagram, S(0 ; 16), L and $Q(4;-8)$ are the vertices of $\Delta$SLQ having LQ perpendicular to SQ. SL and SQ are produced to points R and M respectively such that RM||LQ. SM produced cuts the $x$ – axis at N(8 ; 0).

QM = MN.T and P are the $y$ – intercepts of RM and LQ respectively.




5.1 Calculate the coordinates of M. (2)

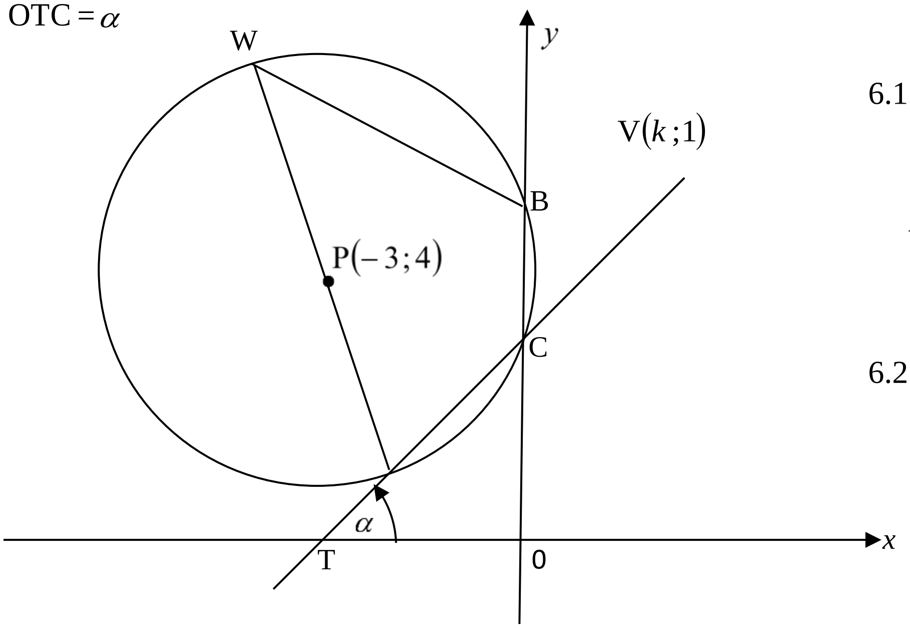
5.2 Calculate the gradient of NS. (2)

5.3 Show that the equation of line LQ is $y=-\frac{1}{2}x-6$ (3)

**QUESTION 6 [DBE SC 2021]**

In the diagram, P( –3 ; 4) is the centre of the circle. V($k$ ; 1) and W are the endpoints of a diameter. The circle intersects the $y$ – axis at B and C.BCVW is a cyclic quadrilateral.CV is produced to intersect the $x$ – axis at T.

$O\hat{T}C=\alpha$




6.1 The radius of the circle is $\sqrt{10}$ . Calculate the value of $k$ if point V is to the right of point P. Clearly show ALL calculations. (5)

6.2 The equation of the circle is given as $x^2+6x+y^2-8y+15=0$ .Calculate the length of BC. (4)

6.3 If $k = -2$, calculate the size of:

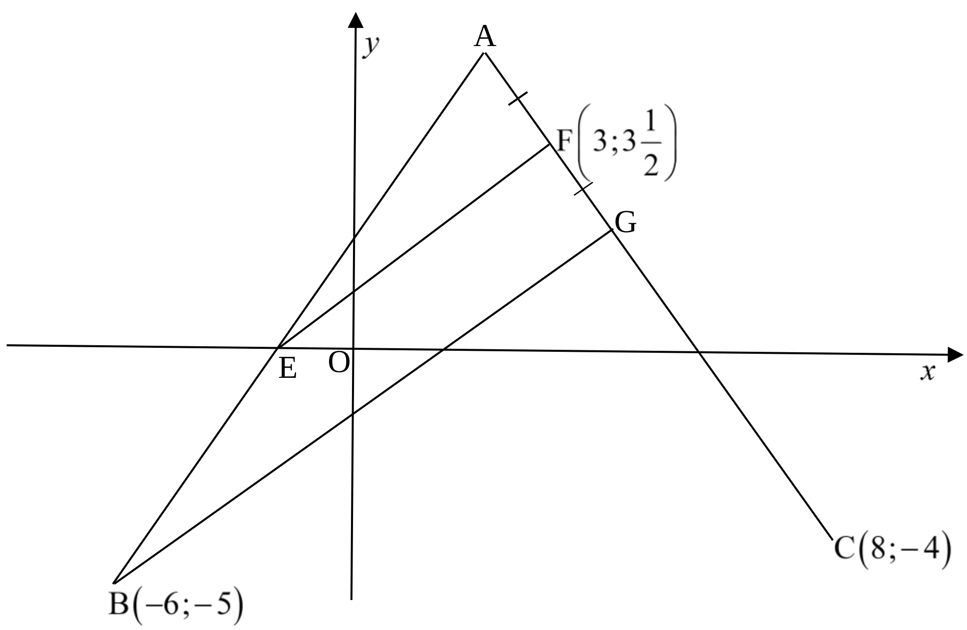
6.3.1 $\alpha$ (2)

6.3.2 $V\hat{W}B$ (2)

**[13]**

**QUESTION 7 [DBE NOV 2017]**

In the diagram, A, $B(-6;-5)$ and $C(8;-4)$ are points in the Cartesian plane. $F\left(3;3\frac{1}{2}\right)$ and G are points online AC such that AF = FG. E is the $x$-intercept of AB.




7.1 Calculate:

7.1.1 The equation of AC in the form $y = mx + c$ (4)

7.1.2 The coordinates of G if the equation of BG is $7x - 10 = 8$ (3)

7.2 Show by calculation that the coordinates of A is $(2;5)$ (2)

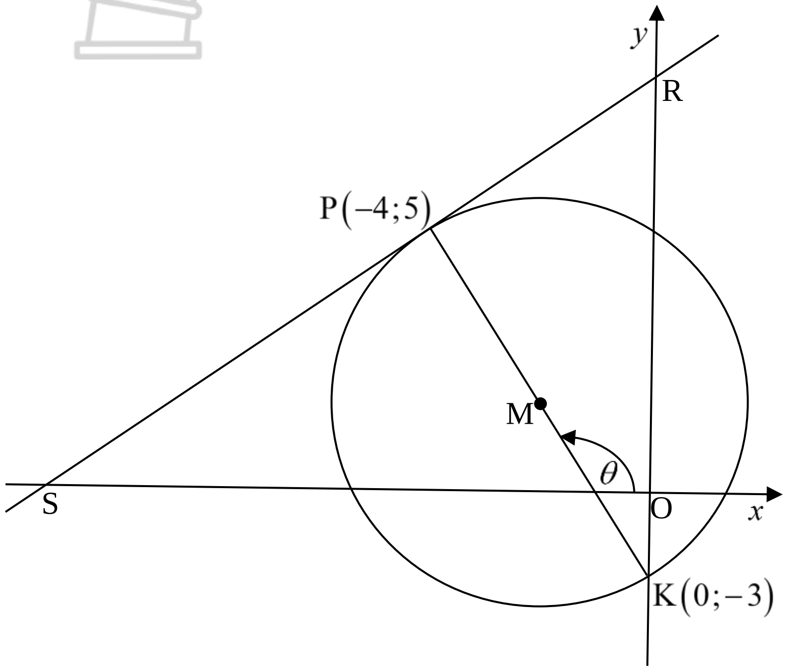
7.3 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)

**[13]**

**QUESTION 8 [DBE NOV 2017]** 

In the diagram, $P(-4;5)$ and $K(0;-3)$ are the end points of the diameter of a circle with centre M. S and R are respectively the $x$- and $y$-intercept of the tangent to the circle at P.

$\theta$ is the inclination of PK with the positive $x$-axis.




8.1 Determine:

8.1.1 The gradient of SR (4)

8.1.2 The equation of SR in the form $y = mx + c$ (2)

8.1.3 The equation of circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

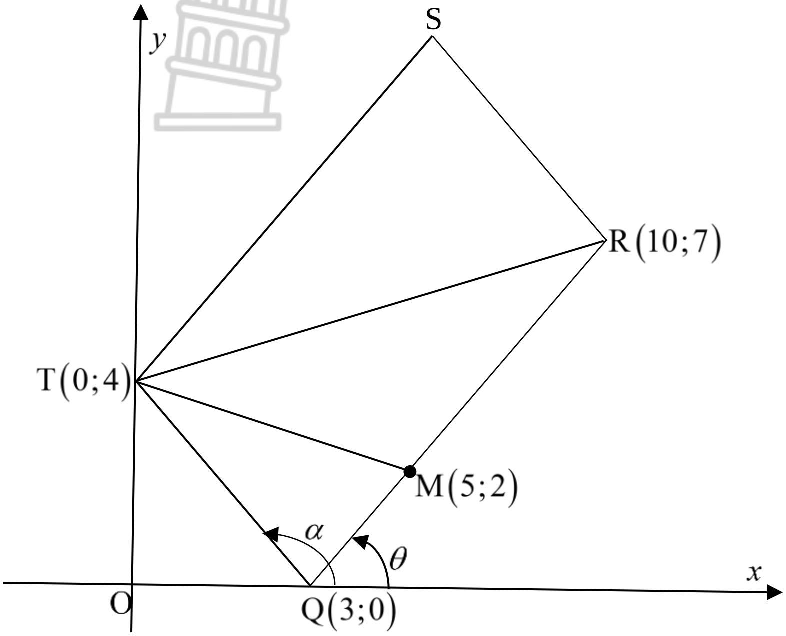
8.1.4 The size of $P\hat{K}R$ (3)

8.1.5 The equation of the tangent to the circle at K in form $y = mx + c$ (2)

**[15]**

**QUESTION 9 [DBE FEB/MARCH 2017]** 

In the diagram, $Q(3;0), R(10;7), S$ and $T(0;4)$ are vertices of parallelogram QRST. From T a straight line drawn meet QR at $M(5;2)$. The angles of inclination of TQ and RQ are $\alpha$ and $\beta$ respectively.




9.1 Calculate the gradient of TQ (1)

9.2 Calculate the length of RQ. Leave your answer in surd from. (2)

9.3 $F(k;-8)$ is a point in the Cartesian plan such that T, Q and F are collinear. Calculate the value of $k$. (4)

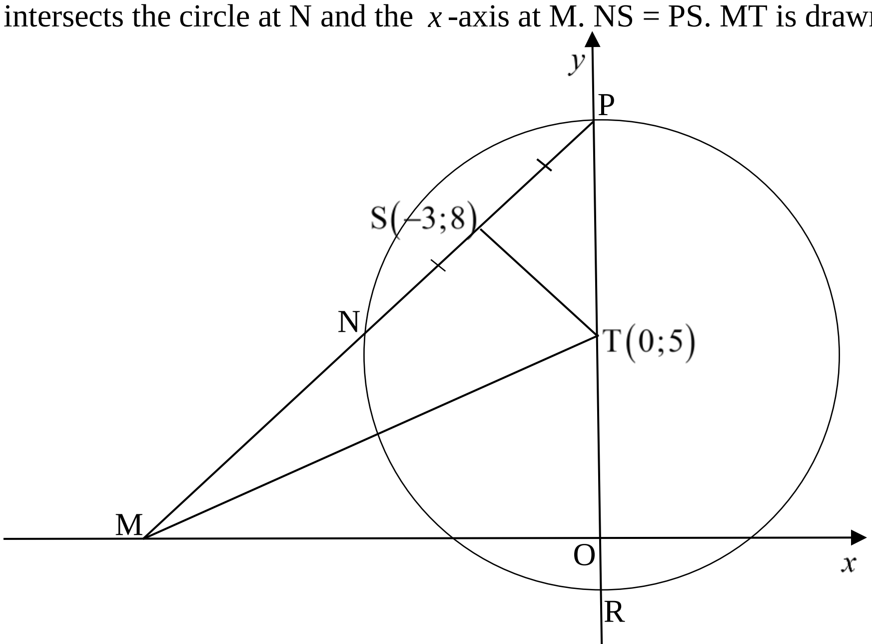
9.4 Calculate the coordinates of S. (4)

9.5 Calculate the size of $T\hat{S}R$. (6)

**[17]**

**QUESTION 10 [DBE FEB/MARCH 2017]**

In the diagram, the circle, having centre $T(0;5)$, cuts the $y$-axis at P and R. The line through P and $S(-3;8)$ intersects the circle at N and the $x$-axis at M. NS = PS. MT is drawn.




10.1 Give a reason why $TS \perp NP$.

10.2 Determine the equation of the line passing through N and P in the form $y = mx + c$

10.3 Determine the equations of the tangents to the circle that are parallel to the $x$-axis. (4)

10.4 Determine the length of MT.

**NOTE:**

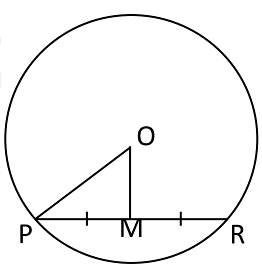
The following proofs of theorems are examinable:

- The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
- The line drawn from the centre of a circle that bisects a chord is perpendicular to the chord.
- The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
- The opposite angles of a cyclic quadrilateral are supplementary;
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
- A line drawn parallel to one side of a triangle divides the other two sides proportionally;
- Equiangular triangles are similar.

**WHAT SHOULD NOT BE DONE WHEN SOLVING RIDERS**

- Never attempt to prove a theorem without a construction (if a construction is required).
- Do not assume information, e.g. that lines are tangents unless given or proved. The examiner may ask candidates to prove that the line is a tangent
- When solving a rider, never presume the results and use the reason required in the conclusion as a reason for the statement(s) that are written for the proof. That is, do not use means that assume that the proof in concluded. For example, the reasons: opp ∠s quad supplementary or ext ∠ = int opp ∠ or line subtends = ∠s cannot be used when proving the same cyclic quad.
- Do not write random, correct or incorrect statements that do not lead to the solution
- Do not confuse similarity (///) with congruency (≡). The conditions are not the same although all congruent triangles are also similar

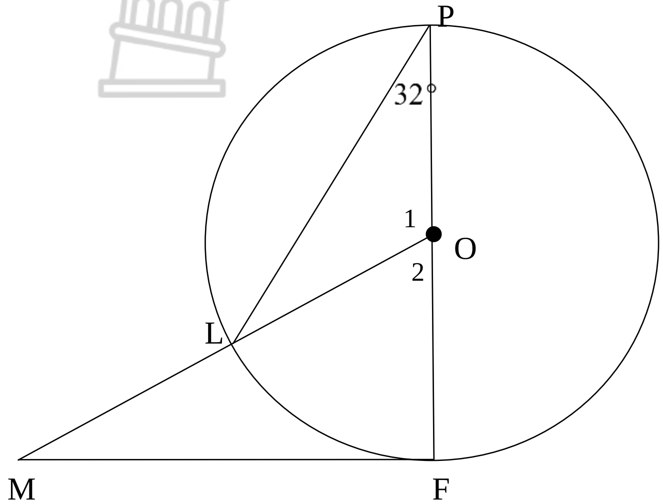
**BASIC HINT**




- Read the statement first. Useful information is in the statement, if used correctly, most questions will be easily answered
- When writing a reason for a statement, WRITE based on what **IS GIVEN** using The correct theorem, refer to the acceptable reason. For example, In the diagram, OM bisects PR or M is the midpoint of PR.
  We deduce that $OM \perp PR$ , the reason is: Line from centre to midpt of chord.
  We do not write perpendicular to chord!
- Do not write random, correct or incorrect statements that do not lead to the solution
- Know what is associated with each key word, for example, Centre, tangent, parallel, diameter, etc.

**QUESTION 1 [DBE Nov 2025]**

In the diagram, O is the Centre of the circle of the circle. POF is the diameter of the circle and MF is a tangent to the circle at F. OM cuts the circle at L. $\hat{P} = 32°$



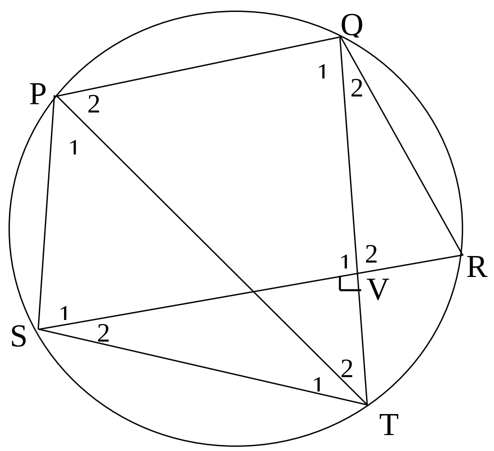

Calculate, with reason, the size of:

1.1 $\hat{O}_2$ (2)

1.2 $\hat{M}$ (3)

**QUESTION 2 [DBE Nov 2025]**

In the diagram, PQRS is a cyclic quadrilateral. T is a point on the circle such that QT is perpendicular to SR at V. PT and ST are drawn. $\hat{Q} = 35°$ And $\hat{R} = \hat{S}_1$



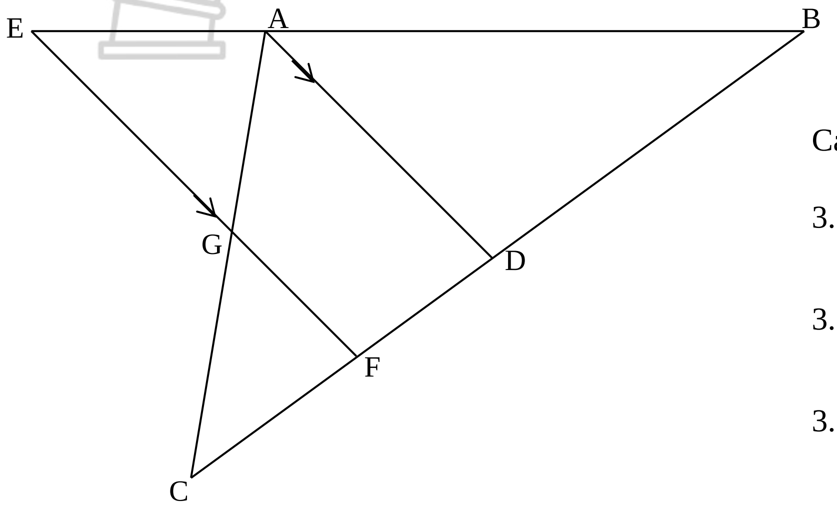

2.1 Calculate, with reason, the size of $Q\hat{T}S$. (3)

2.2 Prove that $PQ \parallel SR$. (3)

2.3 Prove that PT is a diameter of the circle (2)

**QUESTION 3 [DBE Nov 2025]**

3.1 In the diagram, $\Delta ABC$ is drawn. BA is produced to E. F and D are points on BC such that AC and EF intersect at G. $\frac{CF}{FB}=\frac{2}{5}$ and $\frac{CG}{GA}=\frac{3}{2}$.



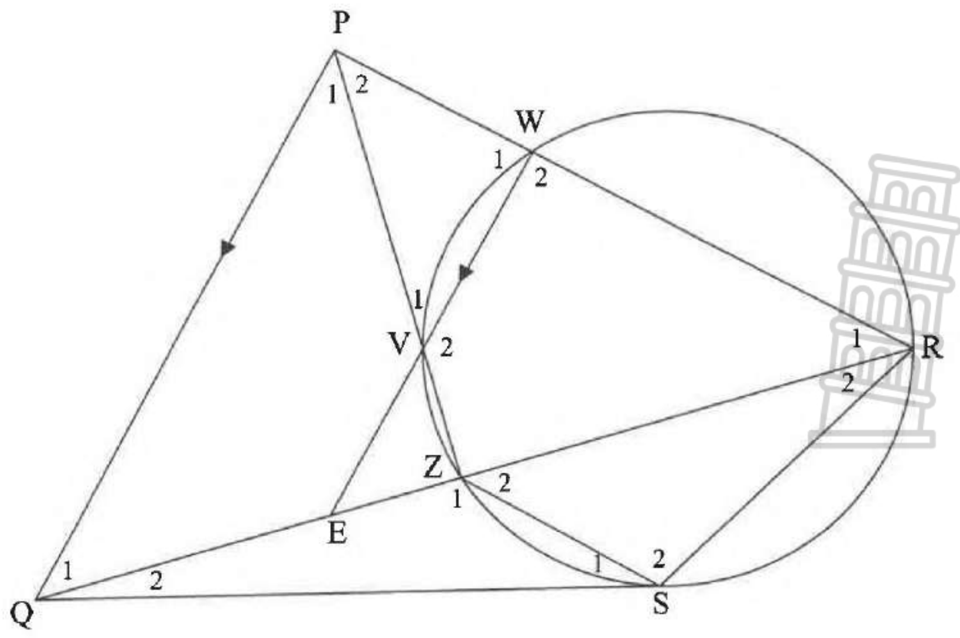

Calculate, with reasons, the value of

3.1.1 $\frac{FD}{CF}$ (2)

3.1.2 $\frac{BA}{EA}$ (4)

3.1.3 $\frac{\text{Area of } \Delta GCF}{\text{Area of } \Delta GFDA}$ (4)

3.2 In the diagram, WVZR is a cyclic quadrilateral. RZ is produced to Q. A tangent is drawn from Q to touch the circle at S. WV is produced to E, a point on ZQ. RW produced meets ZV produced in P. PQ||WE. RS and ZS are drawn.



Prove with reason that:

3.2.1 $PR = \frac{PW.QR}{QE}$ (2)

3.2.2 If $\Delta PQZ \parallel\!| \Delta RQP$, then $PQ^2 = RQ.QZ$ (1)

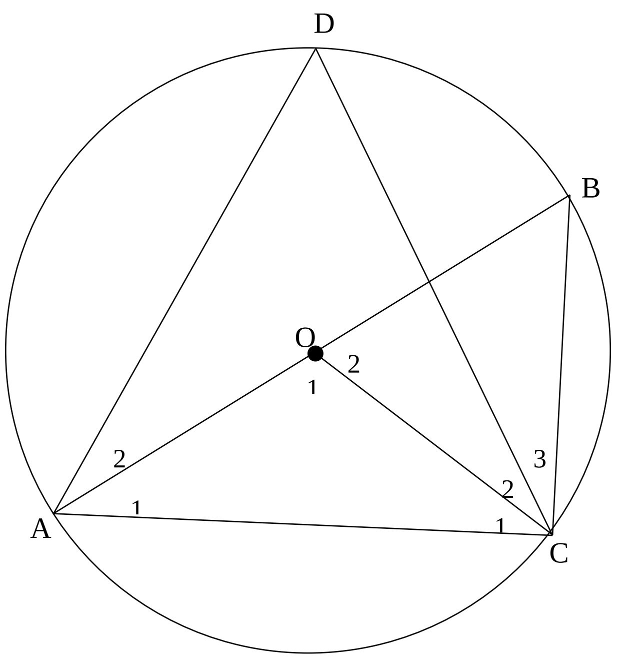
3.2.3 $\Delta QSZ \parallel\!| \Delta QRS$ (3)

3.2.4 $PQ = QS$ (3)

3.2.5 $PW = \frac{QE.PZ}{\sqrt{QR.QZ}}$ (4)

**QUESTION 4 [MP SEPT. 2026]**

4.1 In the diagram below, O is the centre of the circle. AB is a diameter. $A\hat{O}C = 104°$ and $D\hat{A}B = 32°$




Calculate the size of:

4.1.1 $\hat{D}$ (2)

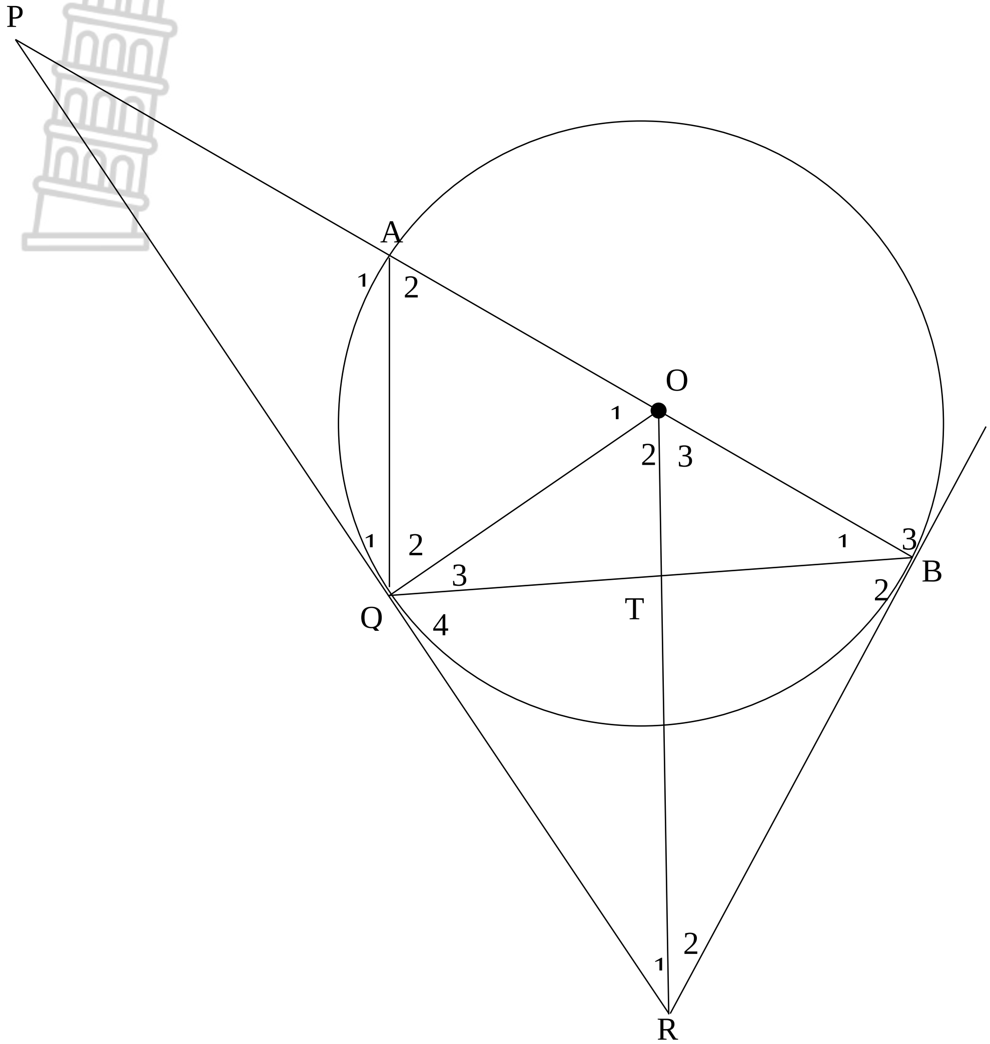
4.1.2 $\hat{C}_3$ (1)

4.1.3 $\hat{A}_1$ (3)

4.1.4 $\hat{C}_2$ (3)

4.2 In the diagram below, RQ and RB are tangents at the points Q and B respectively to the circle with centre O and TQ=TB. The radius BO produced meets the circle at A and RQ produce at P.






4.2.1 Prove that RBOQ is a cyclic quadrilateral. (4)

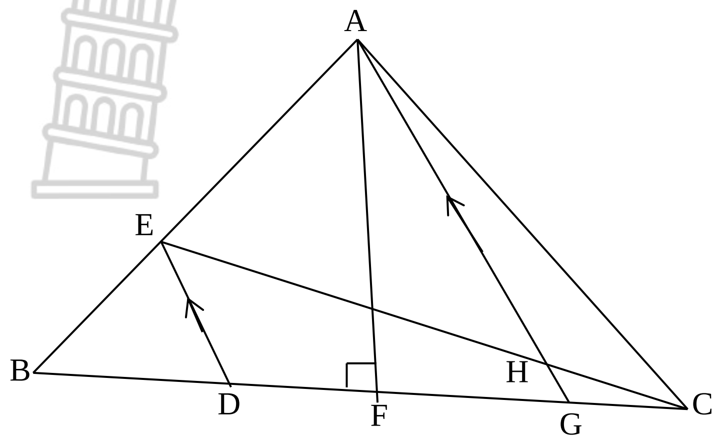
4.2.2 Prove that RB is a tangent to the circle passing through T, O and B (4)

4.2.3 Express $\hat{P}$ in terms of $\hat{B}_1$ (3)

**QUESTION 5 [MP SEPT. 2026]**



In the diagram below $\frac{DG}{GC}=\frac{3}{2}$, $\frac{BE}{EA}=\frac{1}{2}$ and $AG \parallel DE$.



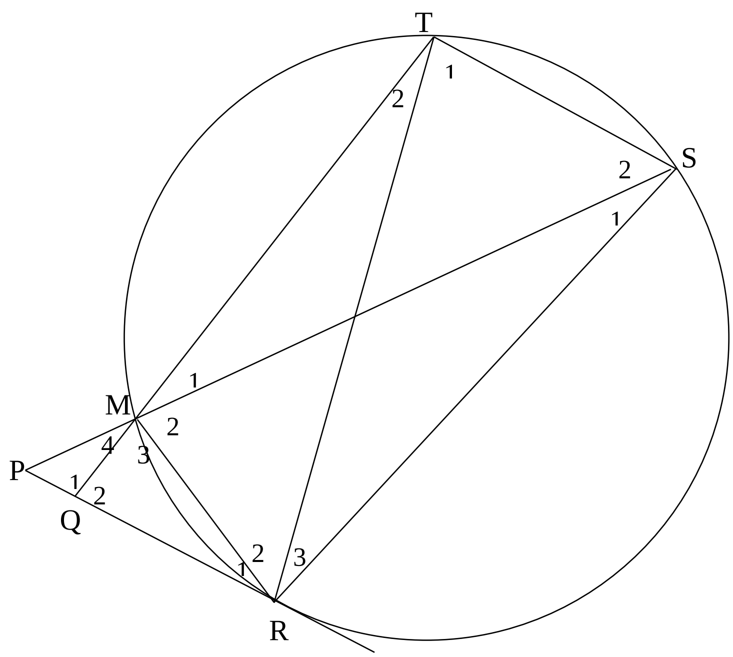

Determine, with reason, the value of:

5.1 $\frac{EH}{HC}$ (2)

5.2 $\frac{\text{Area of } \Delta BAG}{\text{Area of } \Delta CAG}$ (3)

**QUESTION 6 [MP SEPT. 2026]**

In the diagram alongside, RS∥QT and PR is a tangent to the circle at R. SMP is a straight line.




6.1 Why is $\hat{R}_1 = \hat{T}_2$? (1)

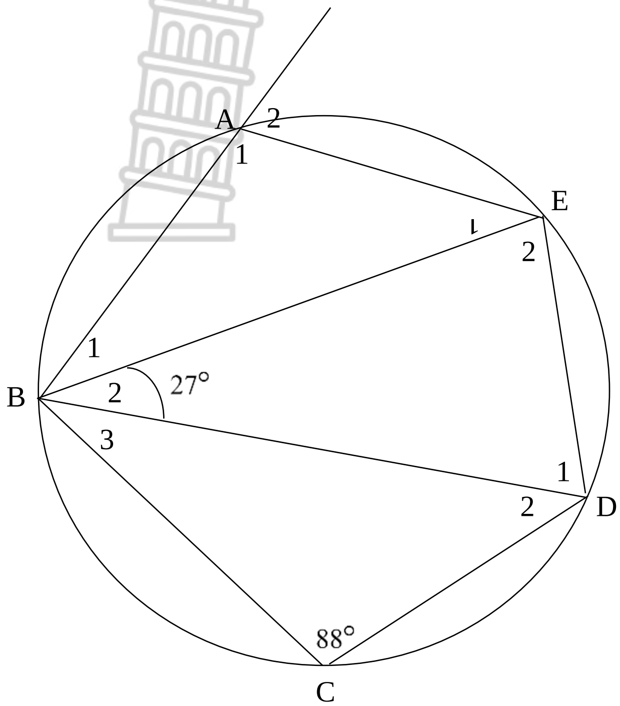
6.2 Prove that $\Delta RTS \,|||\, \Delta RQM$ (4)

6.3 Prove that $RQ.TS = QM.RT$ (1)

6.4 Calculate the numerical value of $\frac{PQ}{RT}$ if it is given that $\frac{PM}{ST}=\frac{2}{3}$ and $\frac{QM}{SM}=\frac{1}{2}$ (4)

**QUESTION 7 [DBE MAY/JUNE 2026]**

In the diagram below A, B, C, D, and E lies on the circle. BA is produced to F. $\hat{B} = 27°$ and $\hat{C} = 88°$. AE, ED, DC, BC, BE and BD are drawn.



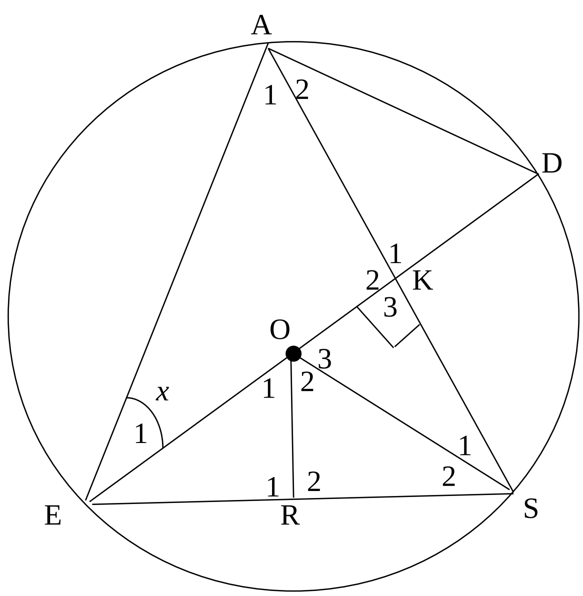

7.1 Calculate, with a reason, the size of $\hat{E}_2$. (2)

7.2 If it is given that ED=DC, write down, with a reason, the size $\hat{B}_3$. (2)

7.3 Calculate, with reasons, the size of $\hat{A}_2$. (3)

**QUESTION 8 [DBE MAY/JUNE 2026]**

In the diagram, O is the centre of the circle. Chord AS is perpendicular to diameter BOD at . OR is drawn with R on BS. AB, AD and OS are drawn. $\hat{B}_1 = x$




8.1 Calculate the following theorem statement:
The angle subtended by a chord at the circumference of the circle, on the same side of the chord are……

8.2 Determine, with, the size of the following angles in terms of $x$: (3)

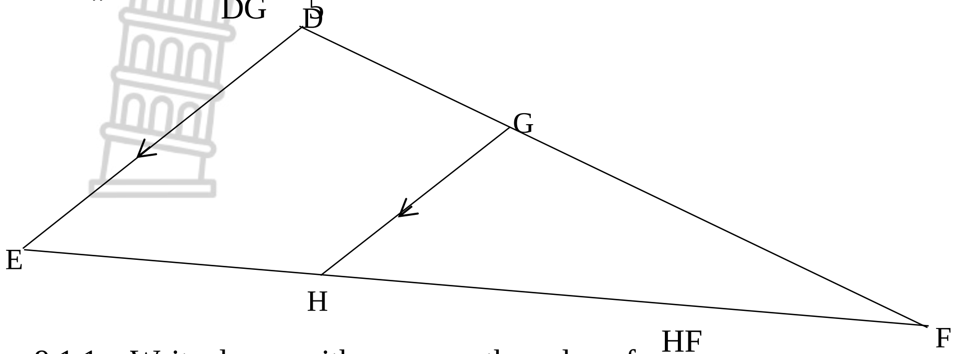
8.2.1 $\hat{B}_2$ (3)

8.2.1 $\hat{S}_1$ (2)

8.3 It is further given that BR=RS. Prove that OKSR is a cyclic quadrilateral (3)

**QUESTION 9 [DBE MAY/JUNE 2025]**


9.1 In the diagram below, $\Delta$ ADE is drawn. Line GH intersect DF and EF at G and H respectively such that GH || DE and $\frac{GF}{DG} = \frac{2}{5}$.



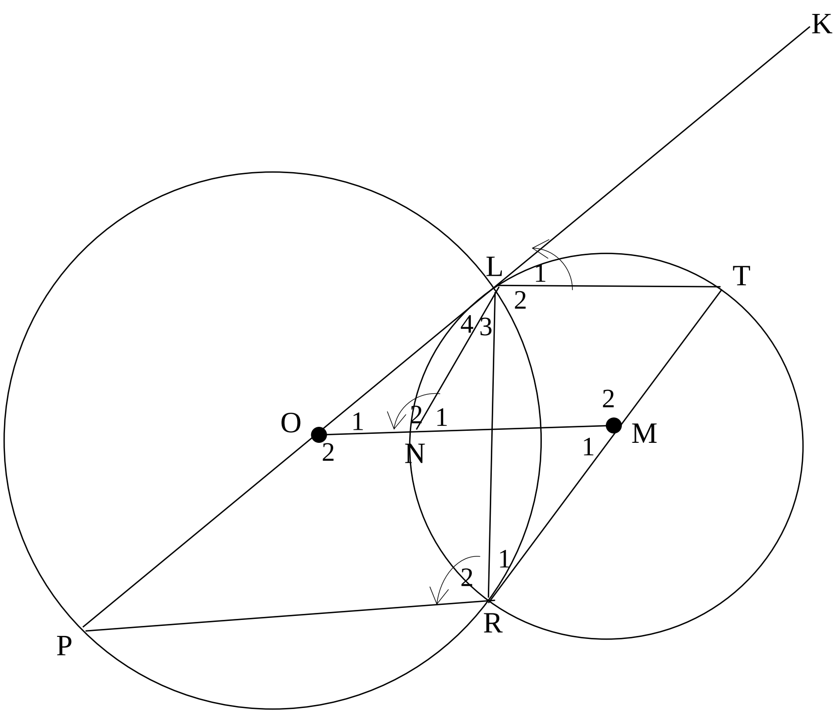

9.1.1 Write down, with a reason, the value of $\frac{HF}{EH}$. (2)

9.1.2 If $EF = 21cm$, calculate the length of EH. (2)

9.1.3 Write down a triangle which is similar to $\Delta FGH$. (1)

9.1.4 Hence, calculate the value of $\frac{GH}{EH}$. (2)

9.2 In the diagram, POL is a diameter of the larger circle with centre O. TRM is a diameter of the smaller circle with centre M. The two circles intersect at L and R respectively. PLK is a tangent to the smaller circle at L and TR is a diameter to the larger circle at R. OM intersect the smaller circle at N. Straight lines LT, LR, LN and PR are drawn




Prove, giving reasons that

9.2.1 LT || PR (4)

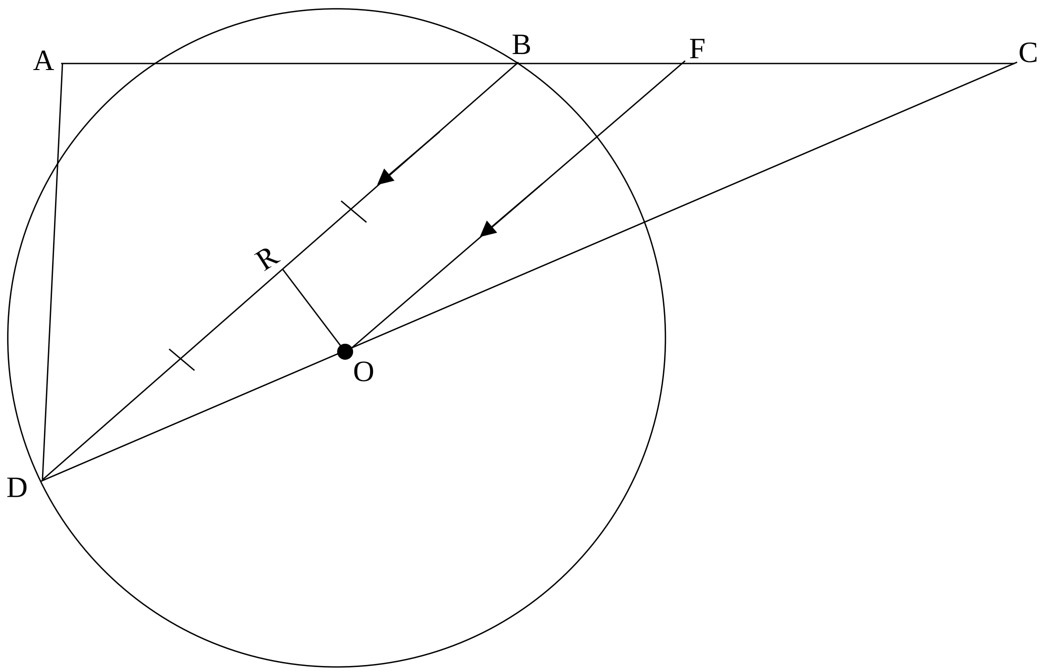
9.2.2 LORM is a cyclic quadrilateral, if it is also given that $LT \parallel OM$ (5)

9.2.3 LN bisects $O\hat{L}R$ (4)

**QUESTION 10 [DBE MAY/JUNE 2025]**

10.1 In the diagram below, O is the centre of the circle. Points D and B lie on the circle. Points A and C lie outside the circle such that side AC of $\Delta ADC$ passes through B.

F is a point on BC such that $FO \parallel BD$. DR = RB and RO is drawn.




10.1.1 Prove, with reasons that $\Delta CFO \parallel\!| \Delta CBD$ (3)

10.1.2 If it is given that $R\hat{D}O = F\hat{C}O$, show, with reasons, that OF.CD = CO.BC (2)

10.1.3 If it is further given that $DC = 19{,}2$ units, $BD = 12$ units and $\frac{RO}{RD} = \frac{3}{4}$.

Prove, with reasons, that $BF = \frac{75}{16}$. (6)

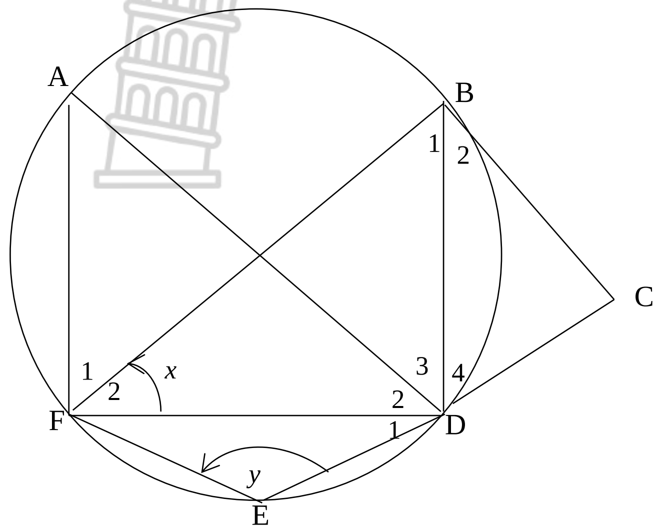
10.1.4 Calculate the size of $A\hat{B}D$ (3)

**QUESTION 11 [FS SEPT 2025]**



In the diagram below, a circle is drawn passing through A, B, D, E and F. The tangents at B and D meet at C

$B\hat{F}D = x$ and $\hat{E} = y$



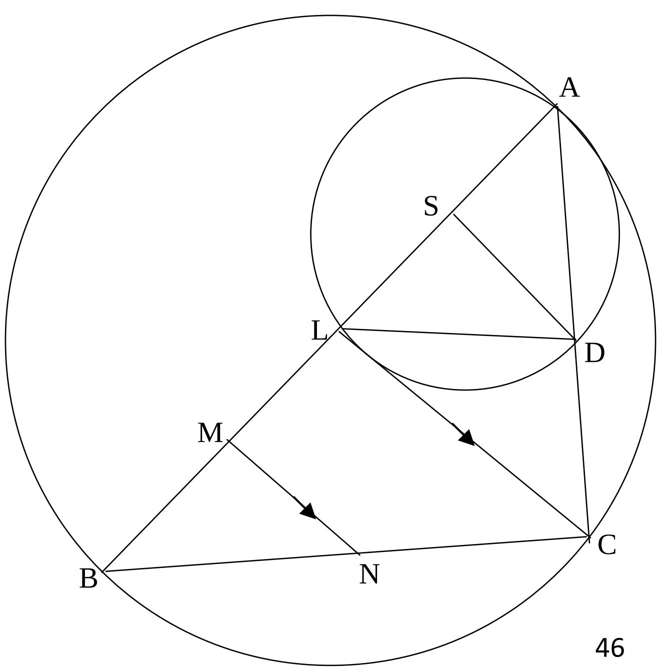

Express the following in terms of *x* or *y*, giving reasons:

| | | |
|---|---|---|
| 11.1 | $\hat{B}_2$ | (2) |
| 11.2 | $\hat{D}_2$ | (2) |
| 11.3 | $\hat{D}$ | (2) |
| 11.4 | $\hat{A}$ | (2) |
| 11.5 | $\hat{B}_2$ | (2) |

**QUESTION 12 [FS SEPT 2025]**

In the diagram below, two circles touch internally at A.

- AB is the diameter of the larger circle, and AL is the diameter of the smaller circle.
- S and L are the centres of the circles.
- D is the point on the smaller circle, and C is a point on the larger circle. ADC is a straight line.
- M is a point on LB such that MN ∥LC.




12.1 Prove DL∥CB. (4)

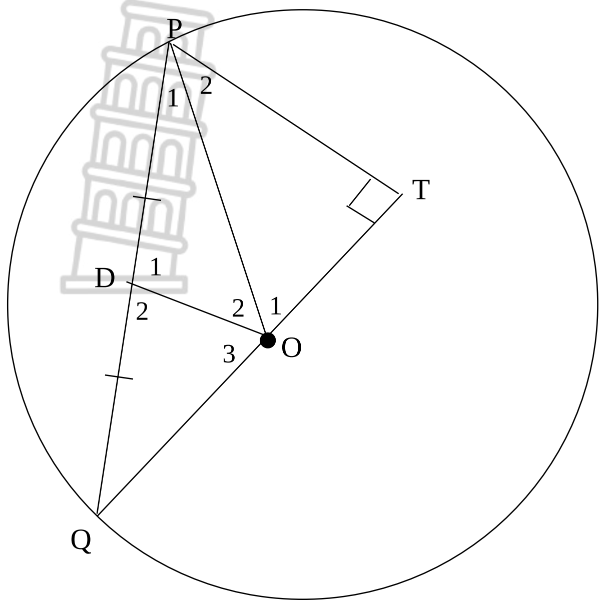
12.2 Prove that 2SD = LC (3)

12.3 Determine the value of $\frac{SL}{AB}$ (2)

12.4 Determine the length of LM, if AB is 30 units and $\frac{BN}{NC} = \frac{7}{9}$ (3)

**QUESTION 13 [FS SEPT 2025]**

In the diagram below $\Delta$ PQT is drawn. O is the centre of the circle, and OD bisects PQ. PT $\perp$ QT.




Prove the following:

13.1 $\hat{O}_3 = Q\hat{P}T$ (5)

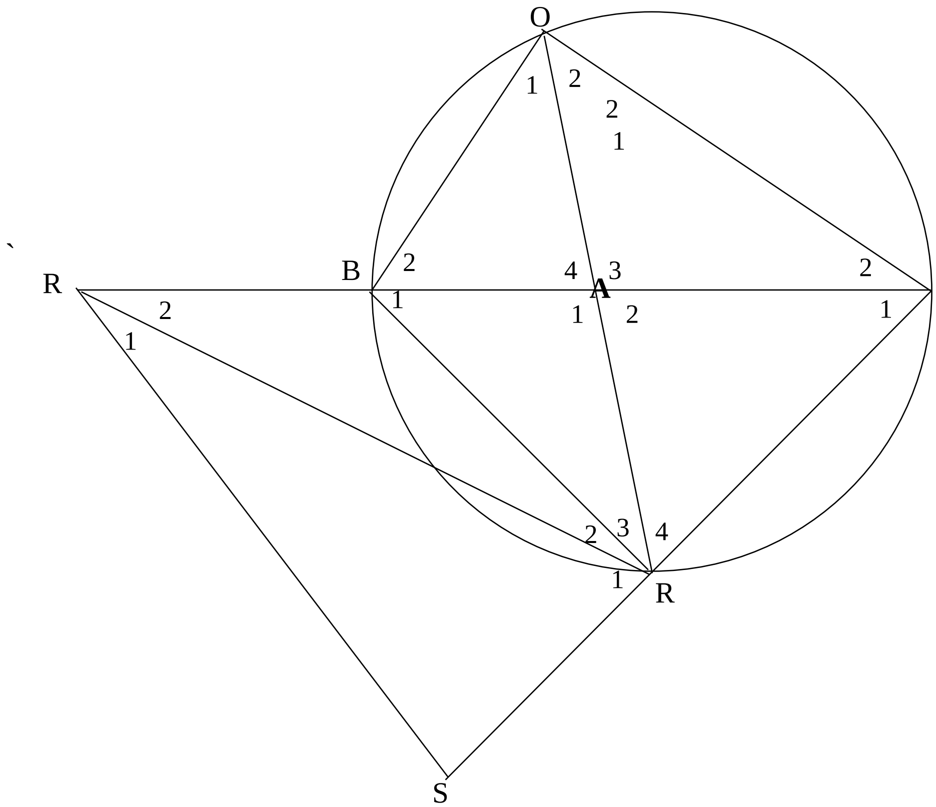
13.2 $\Delta OPD \;///\; \Delta PQT$ (4)

13.3 $OQ.QT = 2PD^2$ (6)

**QUESTION 14 [NW SEPT 2025]** 

In the diagram below, PQBR is a cyclic quadrilateral

- BP is a diameter of the circle. PR is produced to S, and PB to T.
- PBT intersect QR at A and the circle at B
- RT bisect $Q\hat{R}S$, $TS \perp SP$
- $PQ = PR$




14.1 If $\hat{Q}_2 = 55°$, calculate with reasons the size of:

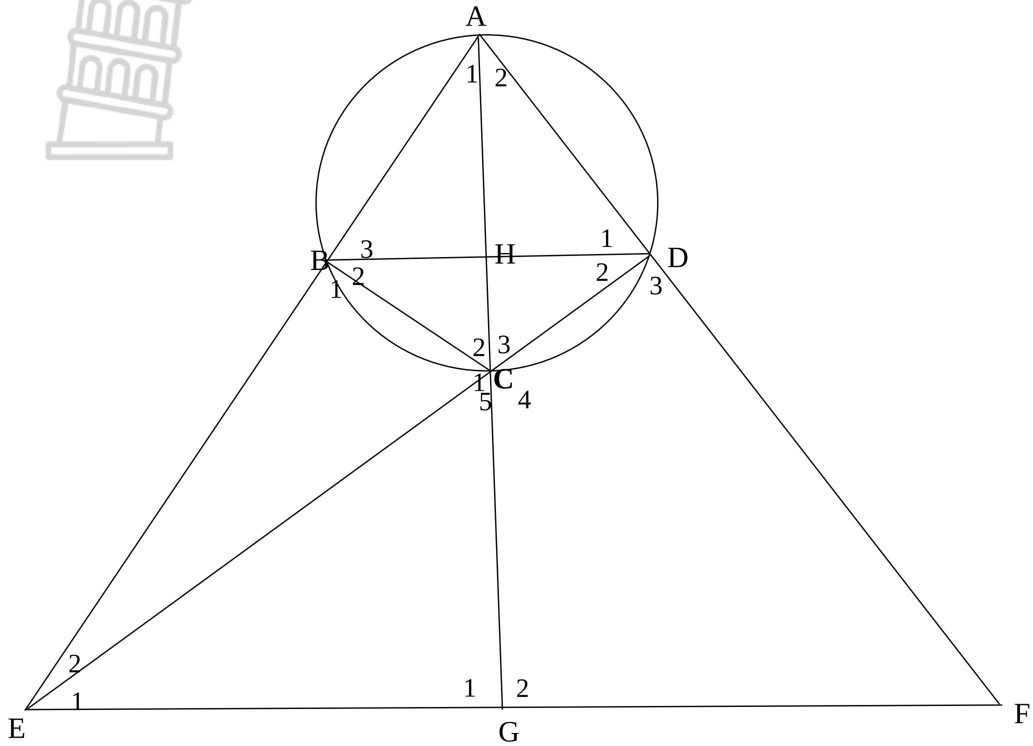
| | | |
|---|---|---|
| 14.1.1 | $\hat{R}_4$ | (2) |
| 14.1.2 | $\hat{B}_3$ | (2) |
| 14.1.3 | $B\hat{R}P$ | (2) |
| 14.1.4 | $\hat{A}_1$ | (3) |

14.2 Give a reason why $AQ = AR$

14.3 Prove that $AT = TS$ (4)

**QUESTION 15 [NW SEPT 2025]**

In the diagram below, ABCD is a cyclic quadrilateral. EF || BD . ACG and DCE are straight lines




Prove that:

15.1 $\hat{E}_1 = \hat{A}_1$ (3)

15.2 EF is a tangent to the circle EAC. (1)

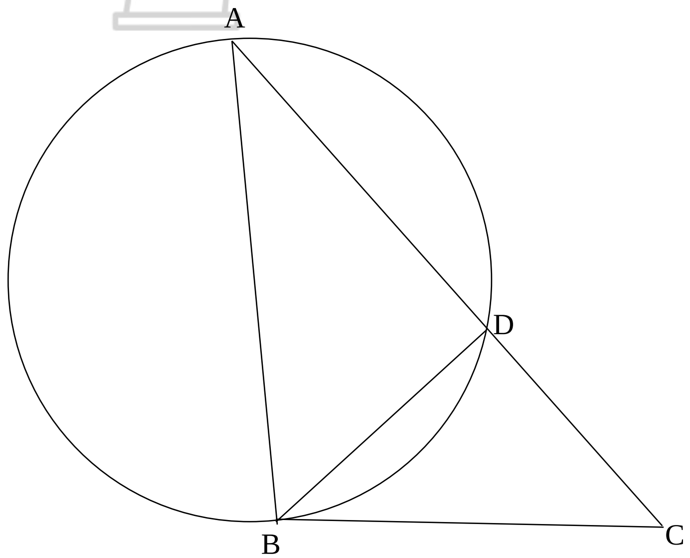
15.3 Δ ABC ||| ΔEDF (4)

**QUESTION 16 [NW SEPT 2025]**


In the diagram below, AB is a diameter of the circle. CB is a tangent to the circle at B.

AC intersect the circle at D

- DC = $x$
- DC = $\frac{1}{2}AD$




16.1 Prove that $\frac{BD}{DC} = \sqrt{2}$ (4)

16.2 Calculate the perimeter of $\Delta ABC$ if $AD = 10cm$

# ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

| THEOREM STATEMENT | ACCEPTABLE REASON(S) |
|---|---|
| **LINES** | |
| The adjacent angles on a straight line are supplementary. | ∠s on a str line |
| If the adjacent angles are supplementary, the outer arms of these angles form a straight line | adj ∠s supp |
| The adjacent angles in a revolution add up to 360°. | ∠s round a pt OR ∠s in a rev |
| Vertically opposite angles are equal. | vert opp ∠s = |
| If AB // CD, then the alternate angles are equal. | alt ∠s; AB ‖ CD |
| If AB ‖ CD, then the corresponding angles are equal | corresp ∠s; AB ‖ CD |
| If AB ‖ CD, then the co-interior angles are supplementary. | co-int ∠s; AB ‖ CD |
| If the alternate angles between two lines are equal, then the lines are parallel. | Alt ∠s = |
| If the corresponding angles between two lines are equal, then the lines are parallel. | corresp ∠s = |
| If the co-interior angles between two lines are supplementary, then the lines are parallel. | coint ∠s supp |
| **TRIANGLES** | |
| The interior angles of a triangle are supplementary | ∠ sum in Δ OR sum of ∠s in Δ OR Int ∠s Δ |
| The exterior angle of a triangle is equal to the sum of the interior opposite angles | Ext ∠ of Δ |
| The angles opposite the equal sides in an isosceles triangle are equal. | ∠s opp equal sides |
| The sides opposite the equal angles in an isosceles triangle are equal | sides opp equal ∠s |
| In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. | Pythagoras OR Theorem of Pythagoras |
| If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled. | Converse Pythagoras OR Converse Theorem of Pythagoras |
| If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent | SSS |
| If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent. | SAS OR S∠S |
| If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent. | AAS OR ∠∠S |
| If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent | RHS OR 90°HS |
| The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side | Midpt Theorem |
| The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side. | line through midpt ‖ to 2nd side |
| A line drawn parallel to one side of a triangle divides the other two sides proportionally. | line ‖ one side of Δ OR prop theorem; name ‖ lines |
| If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side. | line divides two sides of Δ in prop |
| If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar) | /// Δs OR equiangular Δs |
| If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar) | Sides of Δ in prop |
| If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas. | same base; same height OR equal bases; equal height |

| CIRCLES | |
|---|---|
| The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact. | tan ⊥ radius OR tan ⊥ diameter |
| If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle. | line ⊥ radius OR converse tan ⊥ radius OR converse tan ⊥ diameter |
| The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord | line from centre to midpt of chord |
| The line drawn from the centre of a circle perpendicular to a chord bisects the chord | line from centre ⊥ to chord |
| The perpendicular bisector of a chord passes through the centre of the circle; | perp bisector of chord |
| The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre) | ∠ at centre = 2 ×∠ at circumference |
| The angle subtended by the diameter at the circumference of the circle is 90°. | ∠s in semi circle OR diameter subtends right angle OR ∠ in $\frac{1}{2}$⊙ |
| If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter. | chord subtends 90° OR converse ∠s in semi circle |
| Angles subtended by a chord of the circle, on the same side of the chord, are equal | ∠s in the same seg |
| If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic. | line subtends equal ∠s OR converse ∠s in the same seg |
| Equal chords subtend equal angles at the circumference of the circle | equal chords; equal ∠s |
| Equal chords subtend equal angles at the centre of the circle. | equal chords; equal ∠s |
| Equal chords in equal circles subtend equal angles at the circumference of the circles | equal circles; equal chords; equal ∠s |
| Equal chords in equal circles subtend equal angles at the centre of the circles. | equal circles; equal chords; equal ∠s |
| The opposite angles of a cyclic quadrilateral are supplementary | opp ∠s of cyclic quad |
| If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic | opp ∠s quad supp OR converse opp ∠s of cyclic quad |
| The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. | ext ∠ of cyclic quad |
| If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic. | ext ∠ = int opp ∠ OR converse ext ∠ of cyclic quad |
| Two tangents drawn to a circle from the same point outside the circle are equal in length | Tans from common pt OR Tans from same pt |
| The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. | tan chord theorem |
| If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. | converse tan chord theorem OR ∠ between line and chord |
| **QUADRILATERALS** | |
| The interior angles of a quadrilateral add up to 360°. | sum of ∠s in quad |
| The opposite sides of a parallelogram are parallel. | opp sides of ‖m |
| If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram | opp sides of quad are ‖ |
| The opposite sides of a parallelogram are equal in length. | opp sides of ‖m |
| If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram | opp sides of quad are = OR converse opp sides of a parm |
| The opposite angles of a parallelogram are equal. | opp ∠s of ‖m |

| | |
|---|---|
| If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram | opp ∠s of quad are = OR converse opp angles of a parm |
| The diagonals of a parallelogram bisect each other. | diag of \|\|m |
| If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. | pair of opp sides = and \|\| |
| If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram. | pair of opp sides = and \|\| |
| The diagonals of a parallelogram bisect its area. | diag bisect area of \|\|m |
| The diagonals of a rhombus bisect at right angles. | diags of rhombus |
| The diagonals of a rhombus bisect the interior angles | diags of rhombus |
| All four sides of a rhombus are equal in length. | sides of rhombus |
| All four sides of a square are equal in length | sides of square |
| The diagonals of a rectangle are equal in length | diags of rect |
| The diagonals of a kite intersect at right-angles | diag of kite |
| A diagonal of a kite bisects the other diagonal | diag of kite |
| A diagonal of a kite bisects the opposite angles | diag of kite |