KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL STANDARDISED ASSESSMENT

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2026

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
9. Write neatly and legibly.

QUESTION 1

A sports analyst wants to determine the relationship between the number of hours a professional soccer player spends in the gym per week (x) and the number of minutes they can play at maximum intensity (y) before their sprint performance drops. Data collected from 10 players at the 2026 FIFA World Cup is recorded in the table below:

Gym Hours per Week (x)	2	6	7	9	10	12	12	15	16	18
Max Intensity Minutes (y)	55	62	65	74	76	85	83	92	95	88

1.1 Calculate the mean number of gym hours spent per week by one of these soccer players. (2)

1.2 Determine the equation of the least squares regression line. (3)

1.3 Write down the value of the correlation coefficient for this data. (1)

1.4 Comment on the strength of the correlation between the number of gym hours per week and maximum intensity minutes. (1)

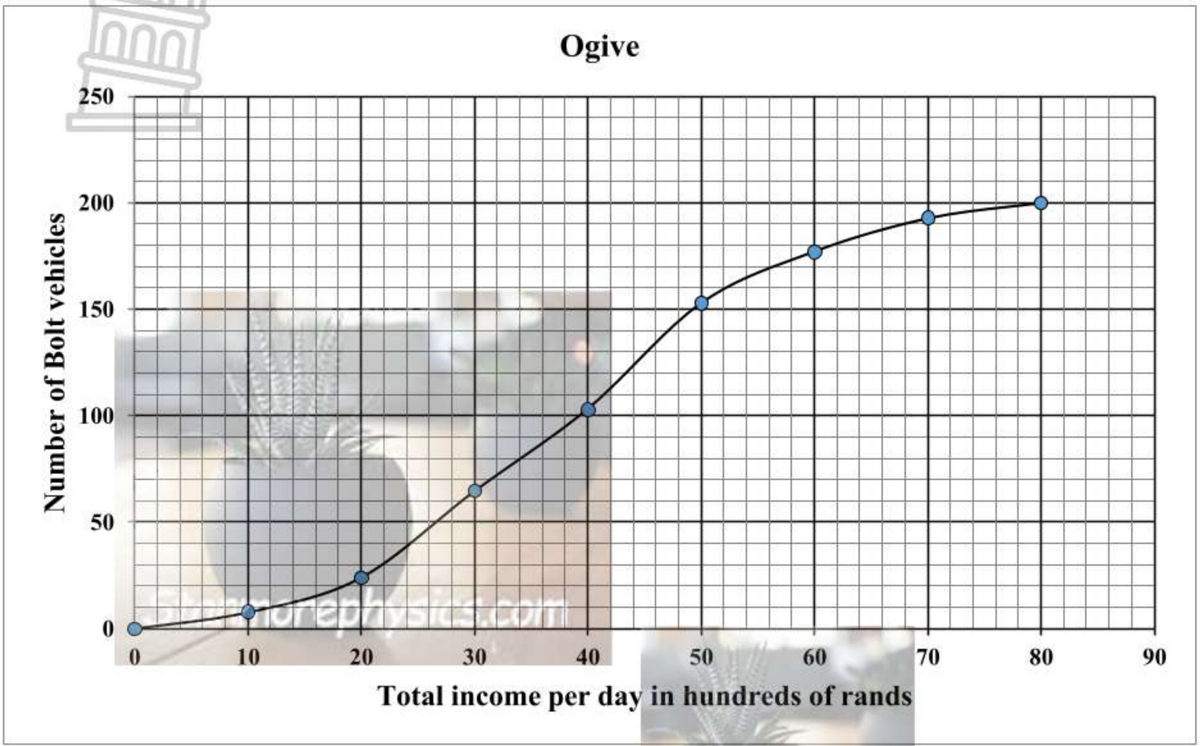
1.5 Predict the maximum intensity minutes of a soccer player who spends 4 hours per week in the gym. Round your answer off to TWO decimal places. (2)

1.6 The analyst discovers that the 9th player (16 hours ; 95 minutes) actually only trained for 10 hours to achieve his maximum intensity rating of 95 minutes. If this error is corrected in the data set, state whether the gradient of the regression line will increase, decrease or remain the same. (1)

[10]

QUESTION 2

In Durban there are a certain number of Bolt vehicles on call every day. The cumulative frequency curve (ogive) below shows the **total fares** (total income per vehicle per day), in hundreds of rands, taken per Bolt driver on a particular day. Stanmorephysics.com



2.1 How many Bolt vehicles were on call in Durban on this day? (1)

2.2 Use the cumulative frequency curve (ogive) to estimate:

2.2.1 the median of total fares per driver on this day. (1)

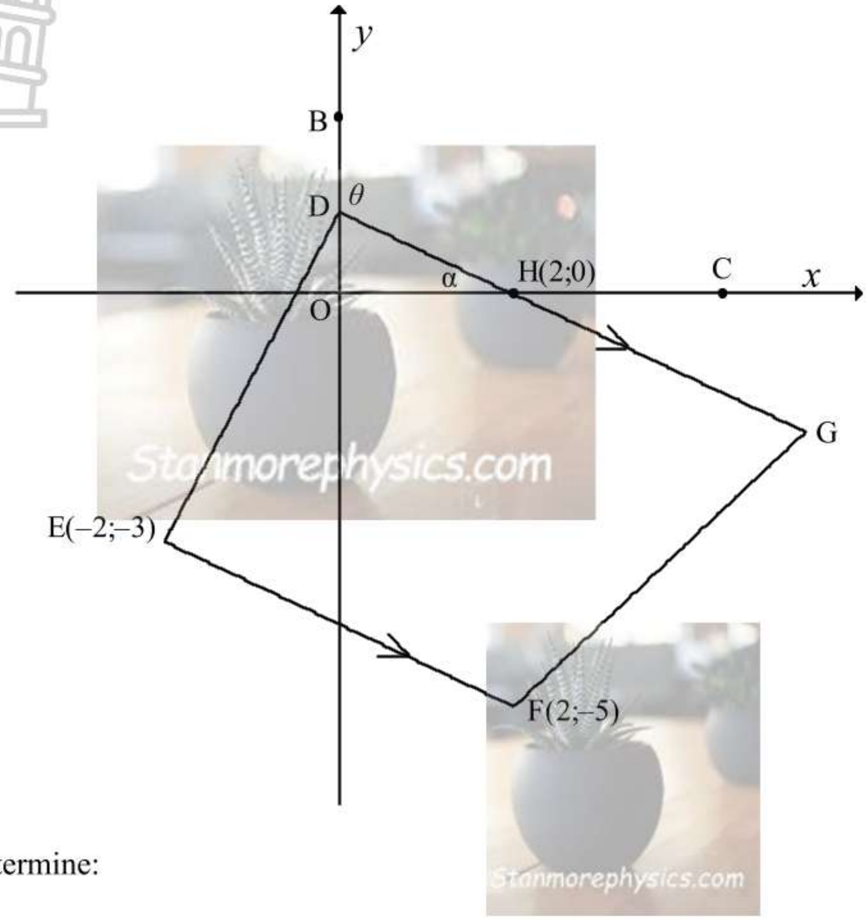
2.2.2 the number of Bolt vehicles for which the fares totalled to more than R5000. (3)

2.3 The company charges R25 per kilometre for the distance travelled. There are no other charges. On that particular day, 20% of Bolt vehicles travelled less than x kilometres. Determine the value of x. Round your answer off to the nearest km. (4)

[9]

QUESTION 3

In the diagram below, D, $E(-2;-3)$, $F(2;-5)$ and G are the vertices of a trapezium with $EF \parallel DG$. $H(2;0)$ is the x-intercept of DG. C and B lie on the x- and y-axes respectively. $B\hat{D}H = \theta$ and $D\hat{H}O = \alpha$



3.1 Determine:

3.1.1 the midpoint of EH (2)

3.1.2 the gradient of EF (2)

3.1.3 the equation of DG in the form $y = mx + c$. (3)

3.1.4 the size of θ. (3)

3.2 Prove that $DE \perp DG$. (3)

3.3 The points E, D and H lie on the circumference of a circle. Determine:

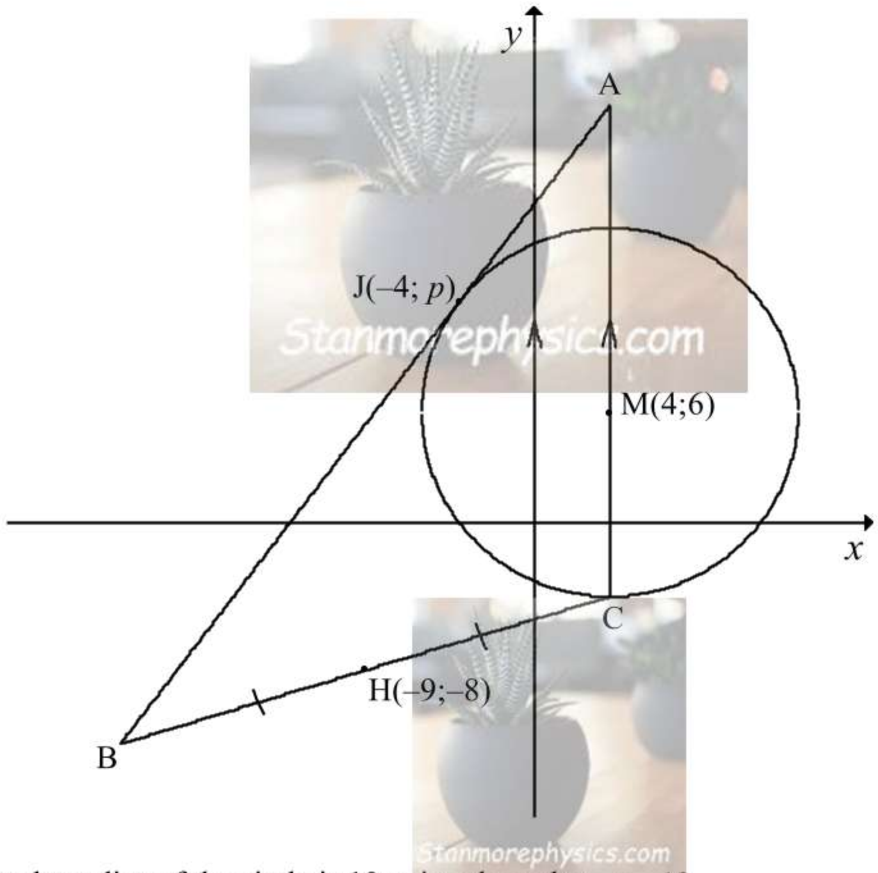
3.3.1 the centre of the circle. (1)

3.3.2 the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)

[18]

QUESTION 4

In the diagram, the circle is centred at $M(4;6)$. Radius CM is produced to A, a point outside the circle, such that CMA is parallel to the *y*-axis. AJB is a tangent to the circle, at $J(-4;p)$. $H(-9;-8)$ is the midpoint of BC.



4.1 Given that the radius of the circle is 10 units, show that $p=12$. (4)

4.2 Determine:

4.2.1 the coordinates of C. (2)

4.2.2 the equation of the tangent AJB in the form $y=mx+c$. (4)

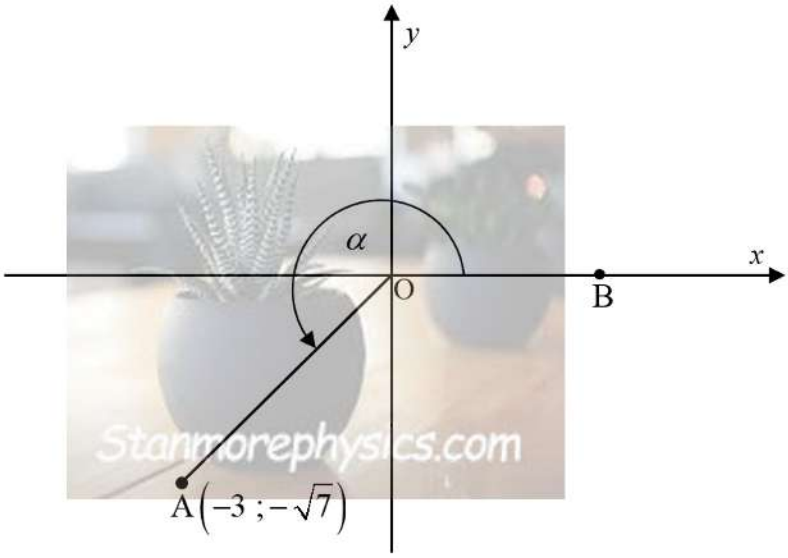
4.2.3 the area of ΔABC. (7)

4.3 Another circle with equation $(x-4)^2+(y-n)^2=100$ is drawn. Determine, with explanation, the values of n for which the two circles will touch each other externally. (4)

[21]

QUESTION 5

5.1 In the diagram, line OA is given with $A\left(-3:-\sqrt{7}\right)$. $B\hat{O}A=\alpha$.



Determine, WITHOUT the use of a calculator the value of:

5.1.1 $\cos\alpha$ (3)

5.1.2 $\sin 3\alpha\cos 2\alpha-\cos 3\alpha\sin 2\alpha$ (2)

5.2 Simplify the following, WITHOUT the use of a calculator:

$$\frac{\sin 290°.\sin\left(90°+x\right).\sin 160°}{\cos\left(-x-360°\right).\sin 220°}$$ (7)

5.3 Given: $$\frac{\frac{1}{4}-\cos\left(\theta-60°\right).\cos\left(\theta+60°\right)}{\sin 2\theta}=\frac{1}{2}\tan\theta$$

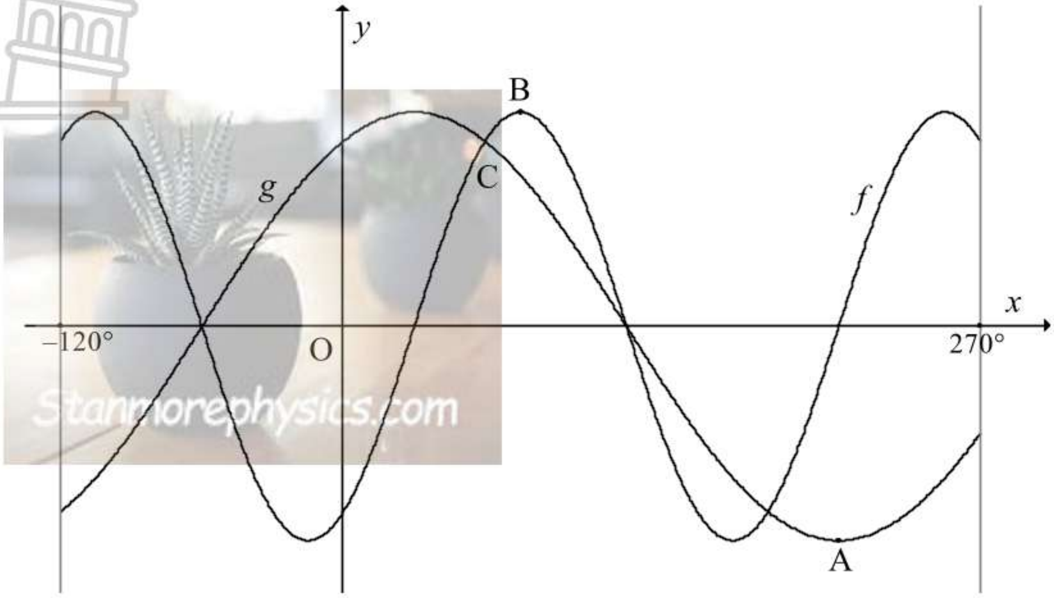
5.3.1 Prove the above identity, WITHOUT the use of a calculator. (7)

5.3.2 For which values of θ, in the interval $\theta\in\left[0°;180°\right]$, will the identity be undefined? (3)

[22]

QUESTION 6

In the diagram, the graphs of $f(x)=\sin(2x-60°)$ and $g(x)=\cos(x-30°)$ are drawn for the interval $x\in[-120°;270°]$. C is one of the points of intersection of the two graphs. A is a minimum turning point of g and B is a maximum turning point of f.



6.1 Determine the coordinates of:

6.1.1 A (2)

6.1.2 B (3)

6.2 Write down the period of g. (1)

6.3 Determine the range of $y=2^{\cos(x-30°)}$ (3)

6.4 Given: $\frac{1}{2}\sin 2x-\frac{\sqrt{3}}{2}\cos 2x=\cos(x-30°)$

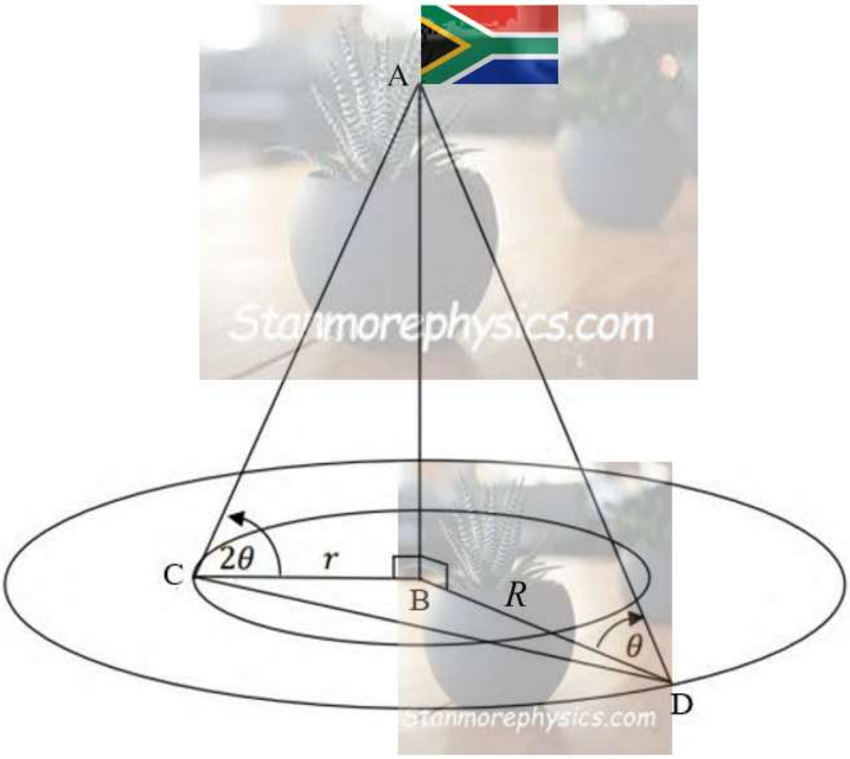
6.4.1 Determine the general solution of $\frac{1}{2}\sin 2x-\frac{\sqrt{3}}{2}\cos 2x=\cos(x-30°)$. Show all your calculations. downloaded from Stanmorephysics.com (6)

6.4.2 Hence, write down the x-coordinate of C. (1)

[16]

QUESTION 7

In support of Bafana Bafana during the 2026 FIFA World Cup, the South African flag was erected at the Moses Mabhida Stadium in Durban. In the diagram below, AB is a vertical flag pole positioned at the common centre B of two concentric circles which lie in the same horizontal plane. Point C lies on the inner circle and point D lies on the outer circle. The angle of elevation of A from C is 2θ, while the angle of elevation of A from D is θ. The radius of the inner circle is r units and the radius of the outer circle is R units.



7.1 Show that the radius of the outer circle is given by the expression $R = \dfrac{2r\cos^2\theta}{2\cos^2\theta - 1}$. (6)

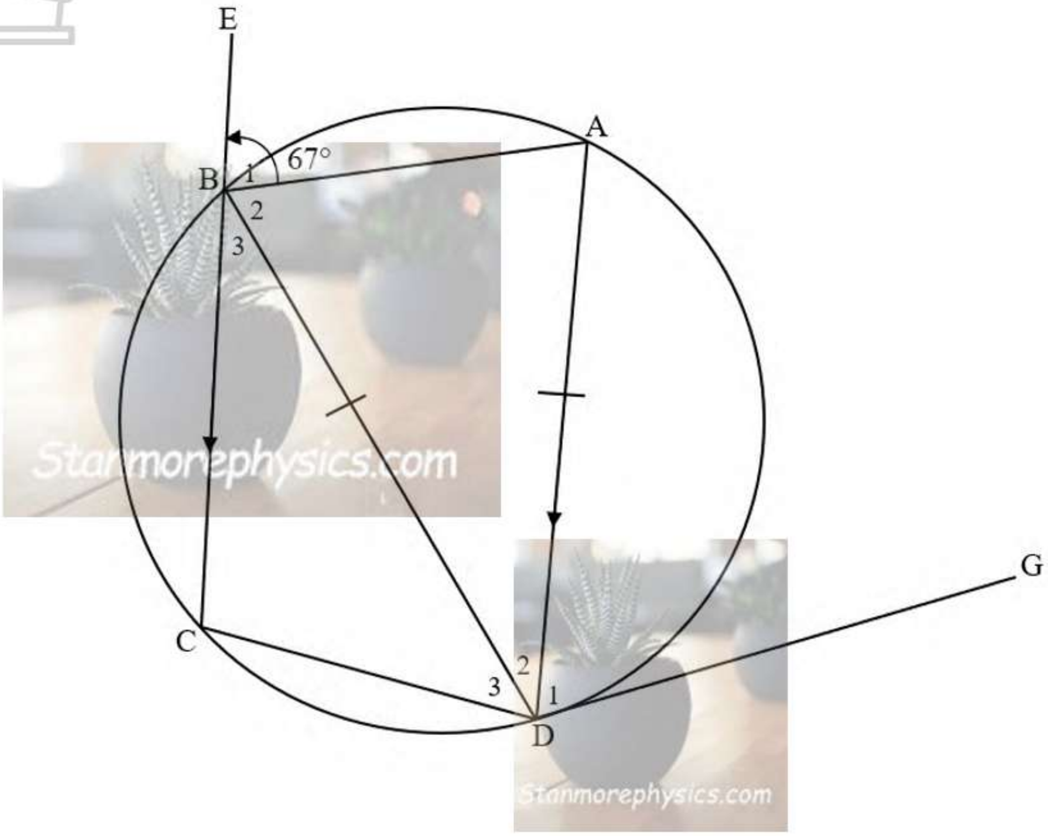
7.2 If $CD = 7$ m, $\theta = 30°$, $r = 2$ m, calculate the area of ΔBCD, correct to ONE decimal place. (6)

[12]

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 8, 9 AND 10.

QUESTION 8

In the diagram below, points A, B, C and D lie on the circumference of a circle with BC || AD. CB is produced to E and DG is a tangent to the circle at D. $AD = BD$ and $E\hat{B}A = 67°$.



8.1 Calculate, giving reasons, the size of the following angles:

8.1.1 $A\hat{D}C$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{C}$ (2)

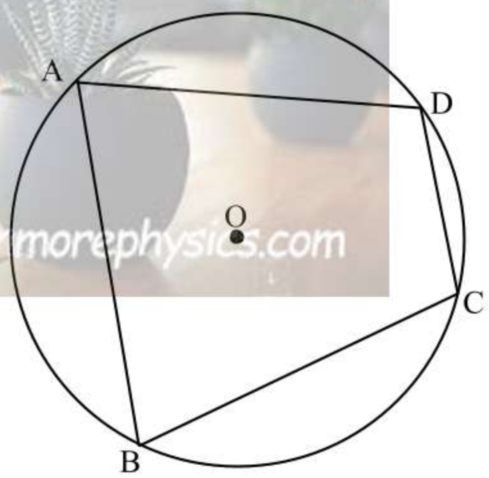
8.1.4 $\hat{D}_1$ (3)

8.2 Prove, giving reasons, that $AB = CD$. (3)

[12]

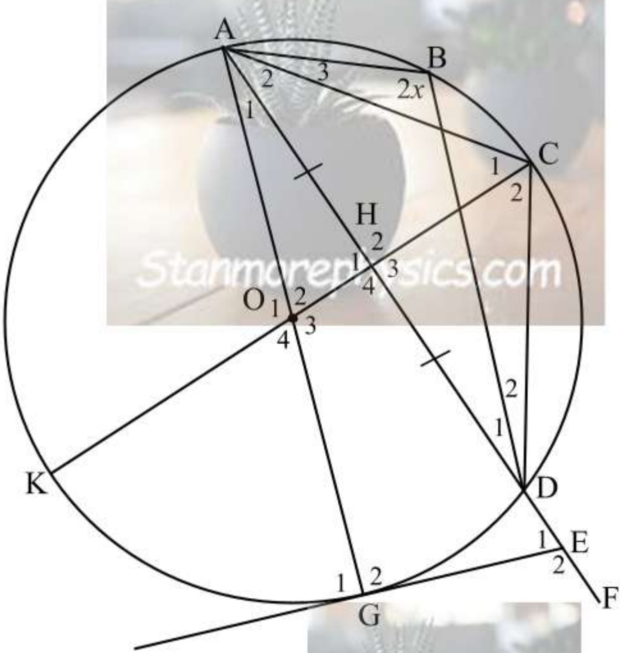
QUESTION 9

9.1 A, B, C and D are points on the circumference of the circle with centre O.



Use the diagram in the ANSWER BOOK to prove the theorem which states that $\hat{A} + \hat{C} = 180°$. (5)

9.2 In the diagram below, O is the centre of a circle. A, B, C, D, G and K are points on the circle. Diameter KC is drawn to bisect chord AD at H. AG is a diameter of the circle. A tangent is drawn to the circle at G. Chord AD is extended to F and meets the tangent at E. Let $\hat{B} = 2x$.



9.2.1 Prove that OGEH is a cyclic quadrilateral. (4)

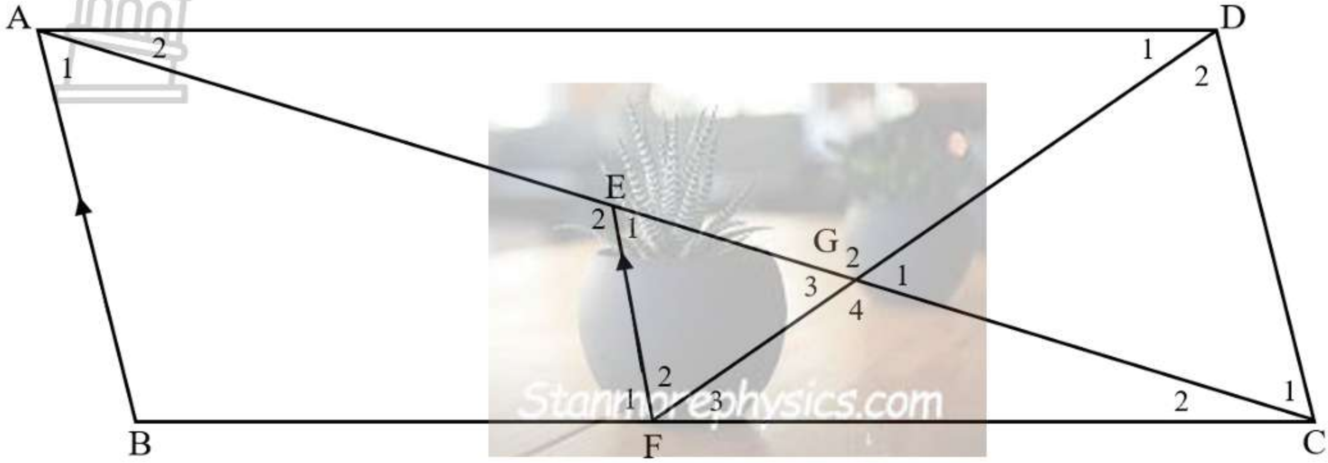
9.2.2 Prove that KC bisects $A\hat{C}D$. (3)

9.2.3 Determine $\hat{E}_1$ in terms of x. (6)

[18]

QUESTION 10

ABCD is a parallelogram. AC is joined and EF||AB, where E and F are points on AC and BC, respectively. FD cuts AC in G. $BF : FC = 2 : 3$.



10.1 Determine, with reasons, the value of:

10.1.1 $\frac{CE}{EA}$ (2)

10.1.2 $\frac{EF}{AB}$ (5)

10.2 Prove that $EG = \frac{CG.EF}{AB}$ (5)

[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

InΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

FINAL

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PREPARATORY EXAMINATION

MARKING GUIDELINES

MARKS: 150

These marking guidelines consist of 15 pages.

QUESTION 1

1.1	$\bar{x}=\frac{107}{10}$ $=10,7$ Answer only: Full marks	✓A $\bar{x}=\frac{107}{10}$ ✓CA answer (2)
1.2	$a=49,51$ $b=2,62$ $\hat{y}=49,51+2,62x$	✓A value of a ✓A value of b ✓CA equation (3)
1.3	$r=0,96$ Penalty for incorrect rounding: Only in this question.	✓A answer (1)
1.4	There is a very strong correlation between gym hours and maximum performance minutes.	✓CA very strong (1)
1.5	$\hat{y}=49,51+2,62(4)$ $=59,99$ minutes **OR** 59,97 minutes (from calculator)	✓CA substitution ✓CA answer (2) **OR** ✓✓A A answer (2)
1.6	Increase	✓A answer (1)
		[10]

QUESTION 2

2.1	200	✓A answer (1)
2.2.1	R3920	✓A answer (accept: R3850 – R4000) (1)
2.2.2	No. of Bolt vehicles for which fares are R5000 or less = 153 No of Bolt vehicles for which fares are more than R5000 $=200-153$ $=47$	✓A 153 (accept: 150 – 155) ✓CA subtraction ✓CA answer (accept: 45 – 50) (3)
2.3	20% of $200=40$ Reading from the graph this equates to R2425 in fares. distance travelled $=\frac{2425}{25}$ $=97$ km $\therefore x=97$	✓A 40 ✓CA amount in fares (accept: R2400 – R2450) ✓CA dividing amount by 25 ✓CA answer (accept: 96 to 98 km) (4)
		[9]

QUESTION 3

3.1.1	Midpoint of EH $=\left(\frac{-2+2}{2}\ ;\ \frac{-3+0}{2}\right)$ $=\left(0\ ;\ \frac{-3}{2}\right)$ Answer only: Full marks	✓A x-value ✓A y-value (2)
3.1.2	$m_{EF}=\frac{-5-(-3)}{2-(-2)}$ $=\frac{-1}{2}$ Answer only: Full marks	✓A substitution ✓CA answer (2)
3.1.3	$m_{DG}=m_{EF}=\frac{-1}{2}$ [DG ǁ EF] $y=mx+c$ Substitute $(2\ ;\ 0)$: $0=\left(\frac{-1}{2}\right)(2)+c$ $c=1$ $\therefore y=\frac{-1}{2}x+1$	✓CA $m_{DG}=\frac{-1}{2}$ ✓CA substitution Stanmorephysics ✓CA answer (3)
3.1.4	$m_{DG}=\tan D\hat{H}C=-\frac{1}{2}$ $D\hat{H}C=153,43^\circ$ $\alpha=180^\circ-153,43^\circ=26,57^\circ$ [∠s on a straight line] $\theta=90^\circ+26,57^\circ=116,57^\circ$ [ext ∠ of ΔDOH]	✓CA $\tan D\hat{H}C=-\frac{1}{2}$ ✓CA size of $D\hat{H}C$ ✓CA size of θ (3)
3.2	$D(0;1)$ $m_{DE}=\frac{-3-1}{-2-0}=2$ $m_{DE}\times m_{DG}=2\times-\frac{1}{2}=-1$ $\therefore DE\perp DG$	✓CA coordinates of D ✓CA $m_{DE}=2$ ✓A showing that product of gradients $=-1$ (3)
3.3.1	$\left(0\ ;\ \frac{-3}{2}\right)$	✓CA answer (1)

3.3.2	$(x-0)^2+\left[y-\left(-\frac{3}{2}\right)\right]^2=r^2$ $x^2+\left(y+\frac{3}{2}\right)^2=r^2$ $r^2=(2-0)^2+\left[0-\left(-\frac{3}{2}\right)\right]^2$ $=4+\frac{9}{4}=\frac{25}{4}$ $\therefore x^2+\left(y+\frac{3}{2}\right)^2=\frac{25}{4}$ Answer only: 3/4	✓CA substitution of coordinates of centre stanmorephysics.com ✓CA substitution of $H(2\ ;\ 0)$ ✓CA value of r^2 ✓CA equation of circle (4)
		[18]

QUESTION 4

4.1	$(x-4)^2+(y-6)^2=10^2$ $(-4-4)^2+(p-6)^2=10^2$ $(p-6)^2=36$ $p-6=\pm\sqrt{36}$ $p-6=6$ or $p-6=-6$ $p=12$ or $p\neq 0$ $\therefore p=12$ **OR** $p^2-12p+36=36$ $p(p-12)=0$ $p=12$ or $p\neq 0$ $\therefore p=12$	✓A equation of the circle/distance formula ✓CA substitution of point J ✓CA simplification ✓A $p-6=\pm\sqrt{36}$ **OR** $p(p-12)=0$ (4)
4.2.1	$C(4\ ;\ 6-10)$ $\therefore C(4\ ;\ -4)$ Answer only: Full marks	✓A x-coordinate ✓A y-coordinate (2)
4.2.2	$m_{MJ}=\frac{12-6}{-4-4}=-\frac{3}{4}$ $\therefore m_{AJB}=\frac{4}{3}$ [radius $\perp$ tangent] $y=\frac{4}{3}x+c$ Subst. point J: $12=\frac{4}{3}(-4)+c$ $c=\frac{52}{3}$ $y=\frac{4}{3}x+\frac{52}{3}$	✓A gradient of radius ✓CA gradient of tangent ✓CA substitution of gradient and point J ✓CA equation of tangent (4)

4.2.3	$y_A = \frac{4}{3}(4) + \frac{52}{3} = \frac{68}{3}$	✓CA calculation of y_A
	$AC = y_A - y_C = \frac{68}{3} - (-4) = \frac{80}{3}$	✓CA length of AC
	Calculate x-coordinate of B: $\frac{x_B + 4}{2} = -9$	✓CA calculation of x_B
	$\therefore x_B = -22$	✓CA value of x-coordinate of B
	height of $\Delta ABC \perp AC = 4 - (-22) = 26$ units	✓CA height of ΔABC
	$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$	
	$= \frac{1}{2} \times \frac{80}{3} \times 26$	✓CA substitution into area formula
	$= \frac{1040}{3} = 346,67$ units^2	✓CA answer (7)
	OR Coordinates of A: $y_A = \frac{4}{3}(4) + \frac{52}{3} = \frac{68}{3}$	**OR** ✓CA calculation of y_A
	$AC = y_A - y_C = \frac{68}{3} - (-4) = \frac{80}{3}$	✓CA length of AC
	Coordinates of B: $\frac{x_B + 4}{2} = -9$ $\frac{-4 + y_B}{2} = -8$ $\therefore x_B = -22$ $\therefore y_B = -12$ $\therefore B(-22; -12)$	✓CA values of coordinates of B
	Length of AB $= \sqrt{(-22-4)^2 + \left(-12 - \frac{68}{3}\right)^2} = \frac{130}{3}$	✓CA length of AB
	$\tan^{-1}\left(\frac{4}{3}\right) = 53,13°$ $\therefore \hat{A} = 180° - 90° - 53,13° = 36,87°$	✓CA size of $\hat{A}$
	$\therefore$ Area of $\Delta ABC = \frac{1}{2} AB.AC \sin A$	
	$= \frac{1}{2}\left(\frac{130}{3}\right)\left(\frac{80}{3}\right)\sin 36,87°$	✓CA substitution into area formula
	$= 346,67$ units^2	✓CA answer (7)

4.3	For circles touching externally, the distance between centres: $d = r_1 + r_2$ $(4-4)^2 + (6-n)^2 = (10+10)^2$ $36 - 12n + n^2 = 400$ $n^2 - 12n - 364 = 0$ $(n+14)(n-26) = 0$ $\therefore n = -14$ or $n = 26$	✓A $d = r_1 + r_2$ ✓A substitution ✓A $n = -14$ ✓A $n = 26$ (4)
	OR $n = 6 - 2(r)$ $= 6 - 2(10)$ $= -14$ or $n = 6 + 2(r)$ $= 6 + 2(10)$ $= 26$	**OR** ✓A $n = 6 - 2(r)$ ✓A $n = 6 + 2(r)$ ✓A $n = -14$ ✓A $n = 26$ (4)
		[21]

QUESTION 5

5.1.1	$x = -3\ ;\ y = -\sqrt{7}$ $(-3)^2 + (-\sqrt{7})^2 = r^2$ [Pythagoras] $r^2 = 16$ $r = 4$ $\therefore \cos\alpha = \frac{-3}{4}$	✓A substitution ✓CA value of r ✓CA answer (3)
5.1.2	$\sin 3\alpha \cos 2\alpha - \cos 3\alpha \sin 2\alpha$ $= \sin(3\alpha - 2\alpha)$ $= \sin\alpha$ $= \frac{-\sqrt{7}}{4}$	✓A $\sin\alpha$ ✓CA answer (2)
5.2	$\frac{\sin 290°.\sin(90°+x).\sin 160°}{\cos(-x-360°).\sin 220°}$ $= \frac{(-\sin 70°).(\cos x).\sin 20°}{\cos x.(-\sin 40°)}$ $= \frac{-\cos 20°.\sin 20°}{-\sin 40°}$ $= \frac{\sin 20° \cos 20°}{2\sin 20° \cos 20°}$ $= \frac{1}{2}$	✓A $-\sin 70°$ and $\sin 20°$ ✓A $\cos x$ (in numerator) ✓A $\cos x$ (in denominator) ✓A $-\sin 40°$ ✓CA $\sin 70° = \cos 20°$ ✓CA double angle expansion ✓CA answer (7)

5.3.1	$\dfrac{\frac{1}{4}-\cos(\theta-60^\circ).\cos(\theta+60^\circ)}{\sin 2\theta}$ $=\dfrac{\frac{1}{4}-\left[(\cos\theta\cos 60^\circ+\sin\theta\sin 60^\circ)(\cos\theta\cos 60^\circ-\sin\theta\sin 60^\circ)\right]}{2\sin\theta\cos\theta}$ $=\dfrac{\frac{1}{4}-\left[\cos^2\theta\cos^2 60^\circ-\sin^2\theta\sin^2 60^\circ\right]}{2\sin\theta\cos\theta}$ $=\dfrac{\frac{1}{4}-\cos^2\theta\left(\frac{1}{2}\right)^2+\sin^2\theta\left(\frac{\sqrt{3}}{2}\right)^2}{2\sin\theta\cos\theta}$ $=\dfrac{\frac{1}{4}-\frac{1}{4}\cos^2\theta+\frac{3}{4}\sin^2\theta}{2\sin\theta\cos\theta}$ $=\dfrac{\frac{1}{4}(1-\cos^2\theta)+\frac{3}{4}\sin^2\theta}{2\sin\theta\cos\theta}$ $=\dfrac{\frac{1}{4}\sin^2\theta+\frac{3}{4}\sin^2\theta}{2\sin\theta\cos\theta}$ $=\dfrac{\sin^2\theta}{2\sin\theta\cos\theta}$ $=\dfrac{1}{2}\tan\theta$ $=$ RHS	✓A compound ∠ expansions ✓A double ∠ expansion ✓A simplifying numerator ✓A special angle values ✓A common factor ✓A using square identity ✓A $\dfrac{\sin^2\theta}{2\sin\theta\cos\theta}$ (7)
5.3.2	Undefined at $\theta=90^\circ$, because $\tan 90^\circ$ is undefined. Also undefined when $\sin 2\theta=0$ $2\theta=0^\circ+k.180^\circ$ $\therefore\theta=0^\circ+k.90^\circ$ Therefore, in the interval $\theta\in[0^\circ;180^\circ]$, the identity is undefined at: $\theta=0^\circ;\ 90^\circ;\ 180^\circ$.	✓A 0° ✓A 90° ✓A 180° (3)
		[22]

QUESTION 6

6.1.1	$A(210°;-1)$	✓A 210°; ✓A −1 (2)
6.1.2	$\sin(2x-60°)=1$ $2x-60°=90$ $2x=150°$ $x=75°$ $\therefore B(75°;1)$ **OR** $y=\sin(2x-60°)=\sin 2(x-30°)$ The graph of $y=\sin 2(x-30°)$ is obtained by translating the graph of $y=\sin 2x$ to the right by 30°. $y=\sin 2x$ has a maximum turning point at $x=45°$. Therefore, the graph of $y=\sin(2x-60°)$ will have a maximum turning point at $x=45°+30°=75°$. Therefore, the coordinates of B will be $(75°;1)$.	✓A setting up equation ✓A x-coordinate ✓A y-coordinate (3) **OR** ✓A translation of 30° to the right ✓A x-coordinate ✓A y-coordinate (3)
6.2	360°	✓A answer (1)
6.3	Range of $y=\cos(x-30°)$ is $-1\le y\le 1$ $\therefore$ Range of $y=2^{\cos(x-30°)}$ is $\frac{1}{2}\le y\le 2$ Answer only: Full marks	✓A range of $\cos(x-30°)$ ✓✓CA CA answer (3)
6.4.1	$\frac{1}{2}\sin 2x-\frac{\sqrt{3}}{2}\cos 2x=\cos(x-30°)$ $\sin 2x\cos 60°-\cos 2x\sin 60°=\cos(x-30°)$ $\sin(2x-60°)=\cos(x-30°)$ $\sin(2x-60°)=\sin(90-(x-30°))$ $\sin(2x-60°)=\sin(120°-x)$ $2x-60°=120°-x+k.360°$ or $2x-60°=180°-(120°-x)+k.360°$ $3x=180°+k.360°$ $x=120°+k.360°,\ k\in Z$ $x=60°+k.120°,\ k\in Z$ $k\in Z$: at least once. **OR**	✓A special angles ✓A compound angle identity used ✓CA co-ratio ✓CA both equations ✓CA $x=120°+k.360°$ ✓CA $x=60°+k.120°$ (6) **OR**

	$\frac{1}{2}\sin 2x - \frac{\sqrt{3}}{2}\cos 2x = \cos(x-30°)$ $\sin 30° \sin 2x - \cos 30° \cos 2x = \cos(x-30°)$ $-(\cos 30° \cos 2x - \sin 30° \sin 2x) = \cos(x-30°)$ $-\cos(2x+30°) = \cos(x-30°)$ $2x+30° = 180° - (x-30°) + k.360°$ or $2x+30° = 180° + (x-30°) + k.360°$ $3x = 180° + k.360°$ or $x = 120° + k.360°,\ k \in Z$ $x = 60° + k.120°,\ k \in Z$	✓A special angles ✓A compound angle identity used ✓CA co-ratio ✓CA both equations ✓CA $x = 120° + k.360°$ ✓CA $x = 60° + k.120°$ (6)
	OR	**OR**
	$\frac{1}{2}\sin 2x - \frac{\sqrt{3}}{2}\cos 2x = \cos(x-30°)$ $\cos 60° \sin 2x - \sin 60° \cos 2x = \cos(x-30°)$ $\sin 2x \cos 60° - \cos 2x \sin 60° = \cos(x-30°)$ $\sin(2x-60°) = \cos(x-30°)$ $\sin 2(x-30°) = \cos(x-30°)$ $2\sin(x-30°)\cos(x-30°) = \cos(x-30°)$ $\cos(x-30°)[2\sin(x-30°)-1] = 0$ $\cos(x-30°) = 0$ or $\sin(x-30°) = \frac{1}{2}$ $x-30° = 90° + k.180°$ or $x-30° = 30° + k.360°$ or $x-30° = 150° + k.360°$ $x = 120° + k.180°\ k \in Z$ $x = 60° + k.360°,\ k \in Z$ $x = 180° + k.360°,\ k \in Z$	✓A special angles ✓A compound angle identity used ✓CA factorisation ✓CA all 3 equations ✓CA $x = 120° + k.180°$ ✓CA $x = 60° + k.360°$ and $x = 180° + k.360°$ (6)
6.4.2	$60°$	✓CA answer (1)
		[16]

QUESTION 7

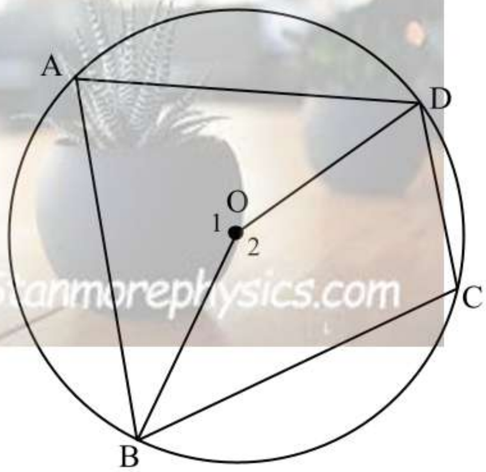
7.1	In ΔABC: $\frac{AB}{r} = \tan 2\theta$ $\therefore AB = r\tan 2\theta$ In ΔABD: $\frac{AB}{R} = \tan\theta$ $\therefore AB = R\tan\theta$ $\therefore r\tan 2\theta = R\tan\theta$ $R = \frac{r\tan 2\theta}{\tan\theta}$ $= \frac{(r)\frac{\sin 2\theta}{\cos 2\theta}}{\frac{\sin\theta}{\cos\theta}}$ $= \left(\frac{r.2\sin\theta\cos\theta}{2\cos^2\theta - 1}\right) \times \frac{\cos\theta}{\sin\theta}$ $= \frac{2r\cos^2\theta}{2\cos^2\theta - 1}$	✓A $\frac{AB}{r} = \tan 2\theta$ ✓A $\frac{AB}{R} = \tan\theta$ ✓A equating ✓A quotient identity ✓A double angle expansion ✓A double angle expansion (6)
7.2	$BD = \frac{2r\cos^2\theta}{2\cos^2\theta - 1} = \frac{2(2)\cos^2 30°}{2\cos^2 30° - 1} = 6 \text{ m}$ In ΔBCD: $CD^2 = BD^2 + BC^2 - 2BD.BC.\cos C\hat{B}D$ $7^2 = 6^2 + 2^2 - 2(6)(2)\cos C\hat{B}D$ $\cos C\hat{B}D = -\frac{9}{24}$ $C\hat{B}D = 112,02°$ Area of $\Delta BCD = \frac{1}{2}.BC.BD.\sin C\hat{B}D$ $= \frac{1}{2}(2)(6)\sin 112,02°$ $= 5,6 \text{ m}^2$ **OR**	✓A length of BD ✓CA substitution into cosine rule ✓CA value of $\cos C\hat{B}D$ ✓CA size of $C\hat{B}D$ ✓CA substitution into area rule ✓CA answer (6) **OR**

	$BD=\dfrac{2r\cos^2\theta}{2\cos^2\theta-1}=\dfrac{2(2)\cos^2 30°}{2\cos^2 30°-1}=6\text{ m}$	✓A length of BD
	In ΔBCD: $BD^2=BC^2+CD^2-2BC.CD.\cos B\hat{C}D$	
	$6^2=2^2+7^2-2(2)(7)\cos B\hat{C}D$	✓CA substitution into cosine rule
	$\cos B\hat{C}D=\dfrac{17}{28}$	✓CA value of $\cos B\hat{C}D$
	$B\hat{C}D=52,62°$	✓CA size of $B\hat{C}D$
	Area of $\Delta BCD=\dfrac{1}{2}.BC.CD.\sin B\hat{C}D$	
	$=\dfrac{1}{2}(2)(7)\sin 52,62°$	✓CA substitution into area rule
	$=5,6\text{ m}^2$	✓CA answer (6)
	OR	**OR**
	$BD=\dfrac{2r\cos^2\theta}{2\cos^2\theta-1}=\dfrac{2(2)\cos^2 30°}{2\cos^2 30°-1}=6\text{ m}$	✓A length of BD
	In ΔBCD: $BC^2=BD^2+CD^2-2BD.CD.\cos B\hat{D}C$	
	$2^2=6^2+7^2-2(6)(7)\cos B\hat{D}C$	✓CA substitution into cosine rule
	$\cos B\hat{D}C=\dfrac{81}{84}$	✓CA value of $\cos B\hat{D}C$
	$B\hat{D}C=15,36°$	✓CA size of $B\hat{D}C$
	Area of $\Delta BCD=\dfrac{1}{2}.BD.CD.\sin B\hat{D}C$	
	$=\dfrac{1}{2}(6)(7)\sin 15,36°$	✓CA substitution into area rule
	$=5,6\text{ m}^2$	✓CA answer (6)
		[12]

QUESTION 8

8.1.1	$\hat{ADC} = 67°$ [ext $\angle$ of cyclic quad]	✓S ✓R (2)
8.1.2	$\hat{A} = 67°$ [alt $\angle$s; BC ∥ AD]	✓S ✓R (2)
8.1.3	$\hat{C} = 180° - 67°$ [opp $\angle$s of cyclic quad] $= 113°$	✓ R ✓ S (2)
8.1.4	$\hat{B}_2 = \hat{A} = 67°$ [$\angle$s opp = sides] $\hat{D}_1 = \hat{B}_2 = 67°$ [tan-chord th]	✓S/R ✓S ✓R (3)
8.2	$\hat{D}_2 = \hat{B}_3$ [alt. $\angle$s; BC ∥ AD] $\therefore AB = CD$ [converse: equal chords subtend equal angles] **OR** $\hat{D}_2 = 180° - (67° + 67°)$ [sum of $\angle$s in Δ] $= 46°$ $\hat{B}_3 = 180° - (67° + 67°)$ [$\angle$s on a str line] $= 46°$ $\hat{D}_2 = \hat{B}_3$ $\therefore AB = CD$ [converse: equal chords subtend equal angles]	✓S ✓R ✓R (3) **OR** ✓S/R ✓S $\hat{B}_3 = 46°$ ✓R (3)
		[12]

QUESTION 9

9.1		
	Construct radii OD and OB. $\hat{O}_1 = 2\hat{C}$ [$\angle$ at centre $=$ $2\times\angle$ at circumf] $\hat{O}_2 = 2\hat{A}$ [$\angle$ at centre $=$ $2\times\angle$ at circumf] $\hat{O}_1 + \hat{O}_2 = 360°$ [$\angle$s around a point] $\therefore 2\hat{C} + 2\hat{A} = 360°$ $\hat{C} + \hat{A} = 180°$ or: $\hat{A} + \hat{C} = 180°$	✓ construction ✓S ✓R ✓S/R ✓S (5)
9.2.1	$\hat{G}_2 = 90°$ [radius $\perp$ tangent] $\hat{H}_4 = 90°$ [line from centre to midpoint of chord] $\hat{G}_2 + \hat{H}_4 = 180°$ $\therefore$ OGEH is a cyclic quadrilateral [converse opp $\angle$s of cyclic quad] **OR** $\hat{G}_2 = 90°$ [radius $\perp$ tangent] $\hat{H}_1 = 90°$ [line from centre to midpoint of chord] $\hat{G}_2 + \hat{H}_1 = 180°$ $\therefore$ OGEH is a cyclic quadrilateral [converse: exterior $\angle$ of cyclic quad]	✓S/R ✓S ✓R ✓R (4) **OR** S/R ✓S ✓R ✓R (4)
9.2.2	In ΔAHC and ΔDHC 1. AH = DH [given] 2. HC = HC [common] 3. $\hat{H}_2 = \hat{H}_3$ [both $= 90°$] $\therefore \Delta AHC \equiv \Delta DHC$ [SAS] $\therefore \hat{C}_1 = \hat{C}_2$ [congruent Δs]	 ✓S ✓R ✓R (3)

9.2.3	$\hat{B} = A\hat{C}D = 2x$	[$\angle$s in the same segment]	✓S ✓R
	$\therefore \hat{C}_1 = \hat{C}_2 = x$	[proved]	
	$OA = OC$	[equal radii]	
	$G\hat{A}C = \hat{C}_1 = x$	[$\angle$s opp = sides]	✓S/R
	$\hat{O}_3 = 2x$	[ext $\angle$ of Δ]	✓S/R
	$\therefore \hat{E}_1 = 180° - 2x$	[opp $\angle$s of cyclic quad]	✓S ✓R
			(6)
	OR		**OR**
	$\hat{B} = A\hat{C}D = 2x$	[$\angle$s in the same segment]	✓S ✓R
	$\therefore \hat{C}_1 = \hat{C}_2 = x$	[proved]	
	$OA = OC$	[radii]	
	$G\hat{A}C = \hat{C}_1 = x$	[$\angle$s opp = sides]	✓S/R
	$\hat{A}_2 = 90° - x$	[sum of $\angle s$ of Δ]	
	$\hat{A}_1 = x - \hat{A}_2 = 2x - 90°$		✓S
	$\therefore \hat{E}_1 = 180° - (90° + 2x - 90°)$ $= 180° - 2x$	[sum of $\angle$s of Δ]	✓S ✓R
			(6)
			[18]

QUESTION 10

10.1.1	$\frac{CE}{EA}=\frac{FC}{BF}$ [prop th , EF ∥ AB] $\therefore \frac{CE}{EA}=\frac{3}{2}$	✓R ✓ answer (2)
10.1.2	In ΔEFC and ΔABC 1) $\hat{C}_2=\hat{C}_2$ [common] 2) $\hat{E}_1=\hat{A}_1$ [corr ∠s , AB ∥ EF] 3) $E\hat{F}C=\hat{B}$ [sum of ∠s in Δ] $\therefore \Delta EFC \mathbin{\vert\vert\vert} \Delta ABC$ [∠∠∠] $\frac{EF}{AB}=\frac{FC}{BC}$ [||| Δs] $\therefore \frac{EF}{AB}=\frac{3}{5}$	✓S ✓S/R ✓R ✓S/R ✓S (5)
10.2	In ΔEGF and ΔCGD 1) $\hat{E}_1=\hat{C}_1$ [alt ∠s , EF ∥ CD] 2) $\hat{F}_2=\hat{D}_2$ [alt ∠s , EF ∥ CD] 3) $\hat{G}_3=\hat{G}_1$ [vert opp ∠s] $\therefore \Delta EGF \mathbin{\vert\vert\vert} \Delta CGD$ [∠∠∠] $\frac{EG}{CG}=\frac{EF}{CD}$ [||| Δs] But, $CD = AB$ [opp sides of parm ABCD] $\therefore \frac{EG}{CG}=\frac{EF}{AB}$ $\therefore EG=\frac{CG.EF}{AB}$	✓S/R ✓S/R ✓S/R ✓S/R ✓S (5)
		[12]

TOTAL: 150 MARKS