



LIMPOPO

PROVINCIAL GOVERNMENT  
REPUBLIC OF SOUTH AFRICA



DEPARTMENT OF  
EDUCATION

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 12**

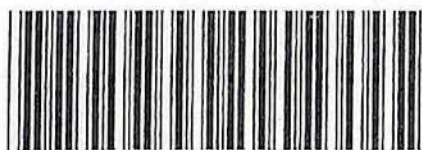
**MATHEMATICS PAPER 2**

**SEPTEMBER 2026**

Stanmorephysics.com

**MARKS: 150**

**TIME: 3 HOURS**



PEMATHP2

This question paper consists of 14 pages including an information sheet.

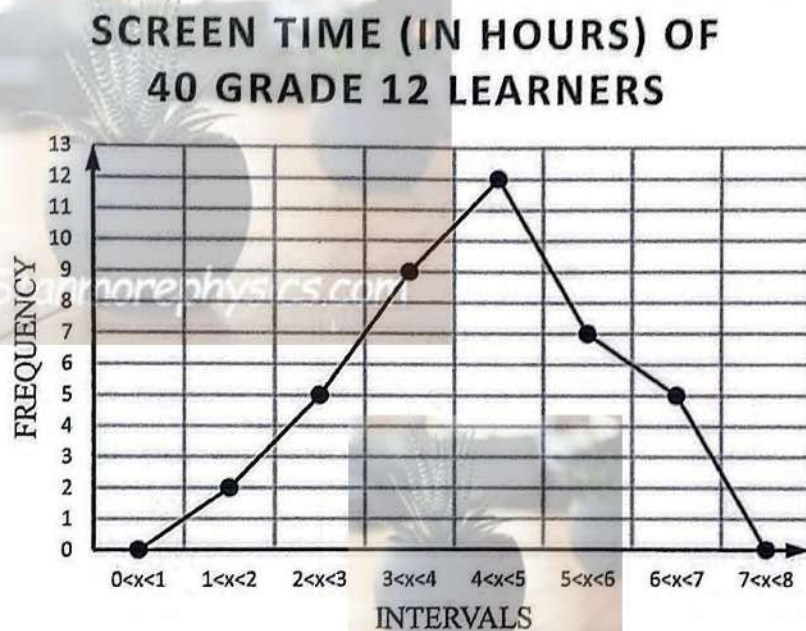
## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. ANSWERS ONLY will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper, if necessary.
9. Write legibly and present your work neatly.

**QUESTION 1**

The following diagram shows the daily screen time (in hours) of 40 Grade 12 learners during a weekday.



- 1.1 Complete the cumulative frequency table in the answer book. (2)
- 1.2 Draw the ogive in the answer book. (2)
- 1.3 Find the median value from the graph. (1)
- 1.4 If the average screen time is 4,04, comment on the skewness of the data.  
Motivate your answer with a reason. (2)
- 1.5 The parents of the community stick together and decide to reduce the screen time of the learners spending more than 4 hours to a maximum of 4 hours. What will the new average screen time be? (2)

[9]

**QUESTION 2**

The following dataset was collected at the G20 summit held in South Africa during the year 2025. It represents the Gross Domestic Product (GDP) per capita versus the per capita CO<sub>2</sub> (carbon dioxide) emissions of 8 of the 20 countries. GDP is the total value of all final goods and services produced within a country's geographical boundaries over a period.

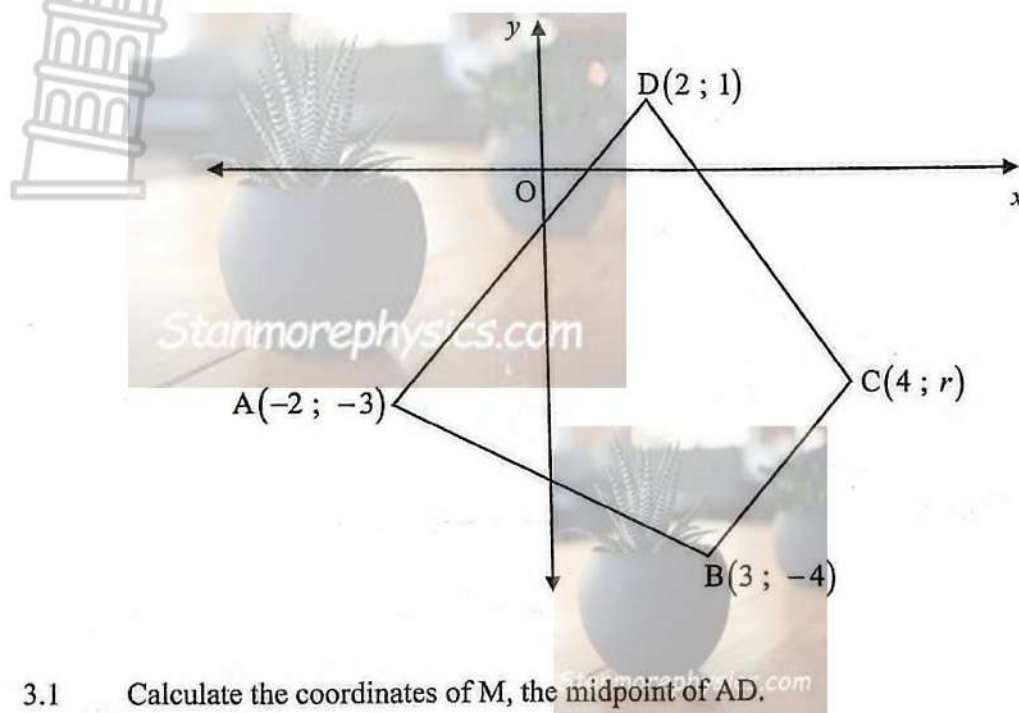
| Country       | GDP per capita<br>(USD) | CO <sub>2</sub> emissions per<br>capita<br>(tonnes/person) |
|---------------|-------------------------|------------------------------------------------------------|
| United States | 66 682                  | 13,9                                                       |
| Germany       | 44 109                  | 6,95                                                       |
| Australia     | 61 212                  | 14,02                                                      |
| India         | 2 397                   | 2,05                                                       |
| China         | 13 122                  | 9,4                                                        |
| South Korea   | 34 121                  | 11,09                                                      |
| Saudi Arabia  | 24 917                  | 18,48                                                      |
| Italy         | 34 398                  | 5,18                                                       |

- 2.1 Draw a scatterplot with GDP per capita on the  $x$ -axis and CO<sub>2</sub> emissions per capita on the  $y$ -axis for these 8 countries. (2)
- 2.2 Calculate the equation of the least-squares regression line in the form  $y = a + bx$ . All digits rounded to 4 decimal places. (3)
- 2.3 Calculate the correlation coefficient between GDP per capita and CO<sub>2</sub> emissions per capita for these 8 countries. (1)
- 2.4 Comment about the association between GDP and CO<sub>2</sub>. (1)
- 2.5 Use your regression line to predict what the CO<sub>2</sub> emissions per capita would be for a hypothetical G20 country with a GDP per capita of 40 000 USD. (2)
- 2.6 Do you think this is a good model for predicting emissions based on GDP per capita? Give a reason for your answer. (1)

**[10]**

**QUESTION 3**

In the diagram  $A(-2; -3)$ ,  $B(3; -4)$ ,  $C(4; r)$  and  $D(2; 1)$  are the vertices of quadrilateral ABCD.



- 3.1 Calculate the coordinates of M, the midpoint of AD. (2)
- 3.2 Determine the equation of BM, the median of  $\triangle ABD$ . (3)
- 3.3 Prove that  $BM \perp AD$ . (2)
- 3.4 Determine the value of  $r$  if  $AD \parallel BC$ . (3)
- 3.5 Calculate the size of the angle that BD forms with the positive  $x$ -axis. (4)
- 3.6 Calculate the coordinates of S, the point of intersection of AC and BD. (5)
- 3.7 Prove that  $\triangle ABD$  is isosceles. (4)
- 3.8 State, with a reason, what type of quadrilateral is ABCD. (2)

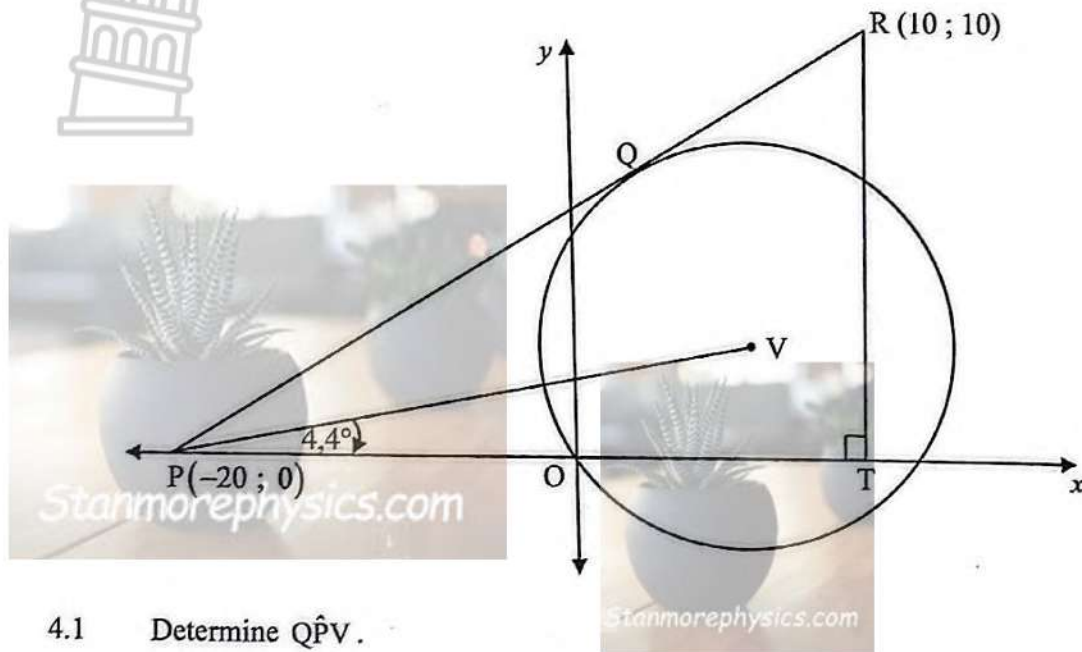
**[25]**

## QUESTION 4

In the diagram, PR is a tangent to circle, with centre V, at Q. The equation of PR is

$$y = \frac{1}{3}x + \frac{20}{3}. \text{ RT is parallel to the } y\text{-axis. The circle passes through the origin.}$$

$$\widehat{VPT} = 4,4^\circ \text{ and } PQ = 8\sqrt{10}.$$



- 4.1 Determine  $\widehat{QPV}$ . (4)
  - 4.2 Hence, calculate the length of the radius of the circle. (3)
  - 4.3 The point V is 4 units left of RT. Determine the coordinates of the centre of the circle, rounded off to the nearest whole number. (4)
  - 4.4 Determine the coordinates of S if PVT is a parallelogram. (2)
- [13]

## QUESTION 5

- 5.1 Given:  $\tan \theta = -\frac{5}{4}$  and  $\theta \in (0^\circ; 180^\circ)$ . Calculate the following without using a calculator:

5.1.1  $\sin(180^\circ + \theta)$  (3)

5.1.2  $\sqrt{41} \cos \theta - 4 \sin(-150^\circ) \cdot \cos 180^\circ$  (4)

- 5.2 Complete the following: The equation  $\cos \theta = a$  is soluble if  $a$  is an element of ..... (1)

- 5.3 If  $\tan 36^\circ = t$ , express the following in terms of  $t$ :

5.3.1  $\frac{1}{\sin 306^\circ}$  (3)

5.3.2  $\tan 81^\circ$  (5)

[16]

## QUESTION 6

- 6.1.1 Prove the identity:  $2 \cos \theta \cdot \cos 2\theta + \frac{\sin^2 2\theta}{\cos \theta} = 2 \cos \theta$  (6)

- 6.1.2 For which value(s) of  $\theta$  will the identity be undefined, if  $\theta \in [0^\circ; 180^\circ]$ ? (1)

- 6.2 Find the general solution of  $\cos \theta = \frac{1}{\cos \theta} + \frac{5}{6}$ . (6)

- 6.3 If area  $\Delta PQR = \frac{1}{2} pq \sin R$ , show that the area  $\Delta PQR$  can also be

expressed as  $\frac{q^2 \sin P \sin R}{2 \sin Q}$ . (2)

[15]

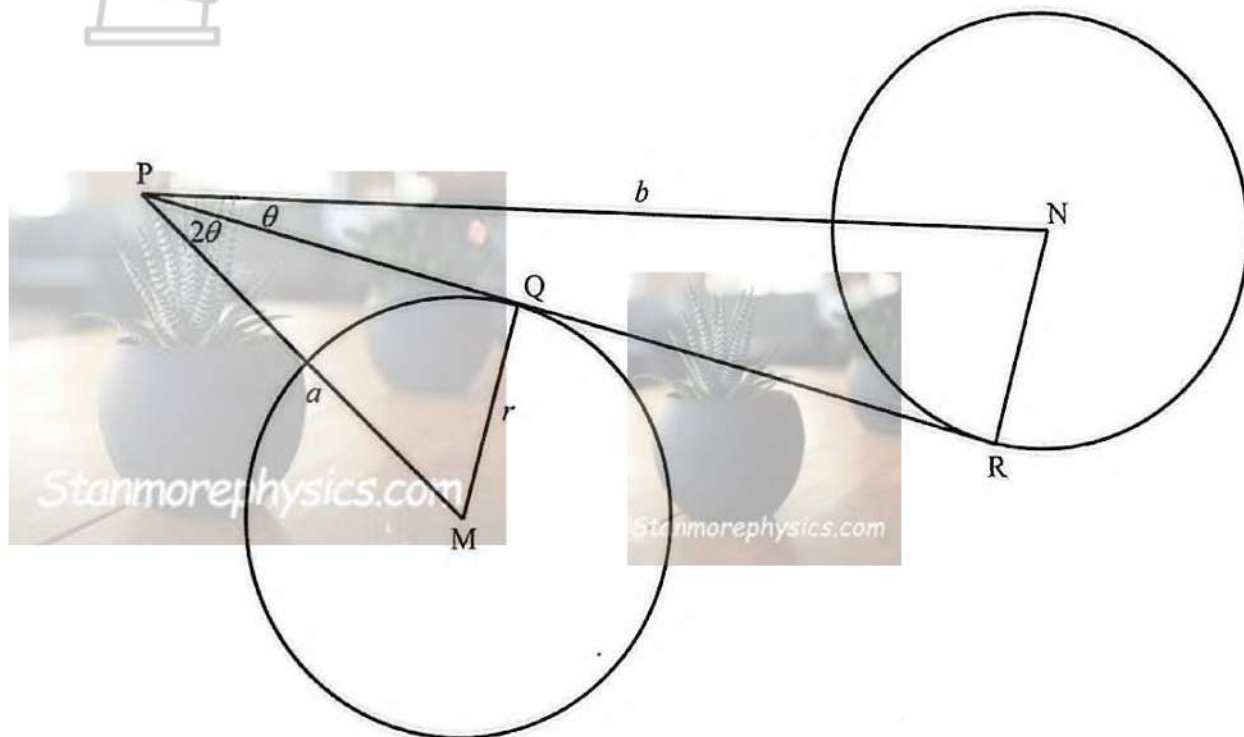
## QUESTION 7

7.1 7.1.1 Complete:  $\cos(\alpha - \beta) = \dots$  (1)

7.1.2 Hence, derive a formula for  $\sin 2\alpha$ . (3)

7.2 PQR is a common tangent at Q and R to circles with centres M and N.

Each circle has a radius of  $r$  units.  $\angle MPQ = 2\theta$ ,  $\angle RPN = \theta$ ,  $PM = a$  and  $PN = b$ .



7.2.1 Express  $r$  in terms of  $b$  and  $\theta$ . (2)

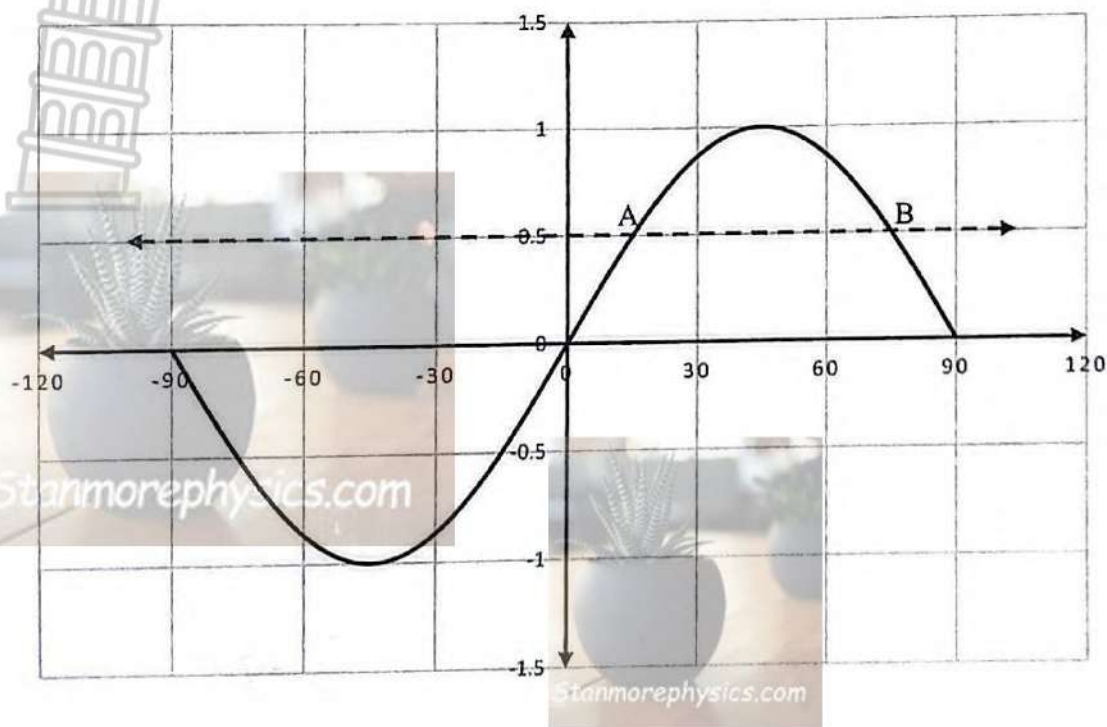
7.2.2 If it is also given that  $r = a \sin 2\theta$ , express  $b$  in terms of  $a$  and  $\theta$ . Leave the answer in the simplest form. (3)

7.2.3 Hence, prove that  $QR = a$ . (4)

[13]

## QUESTION 8

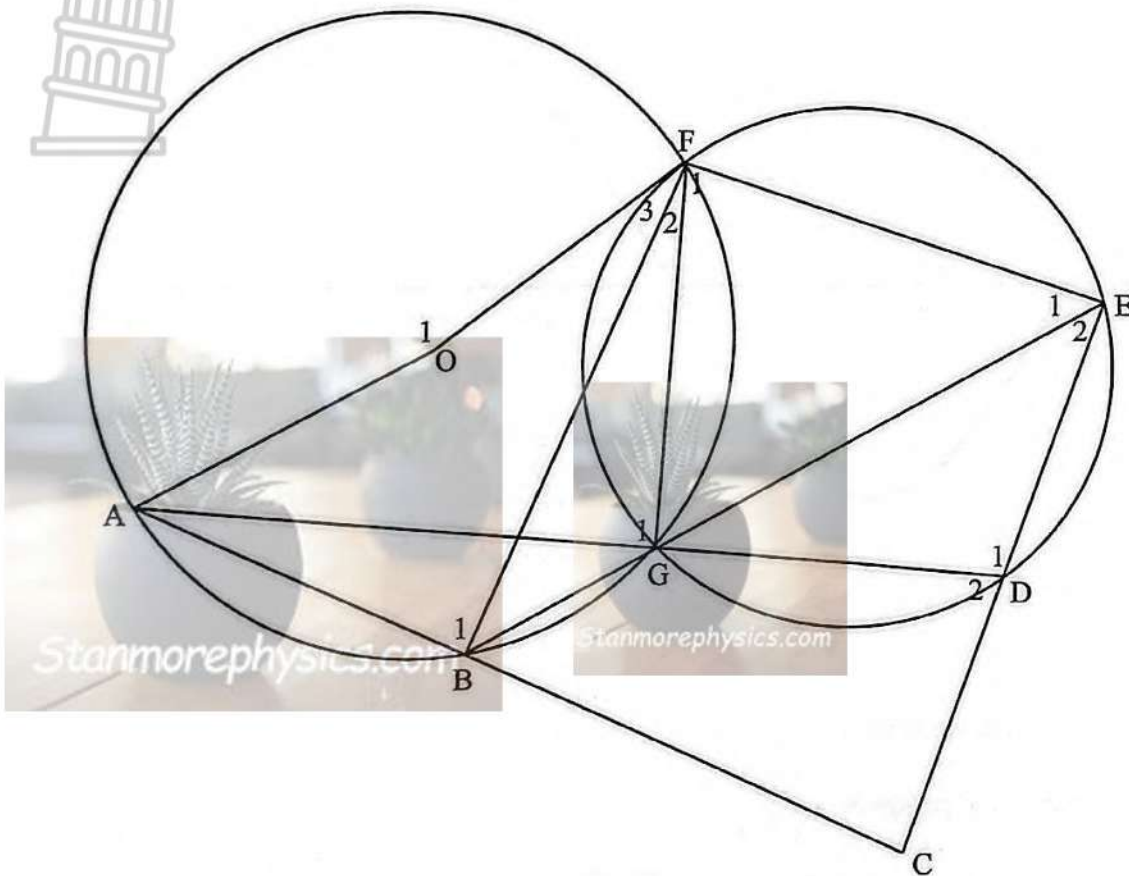
The diagram below shows the graph of  $f(x) = \sin ax$  for  $x \in [-90^\circ; 90^\circ]$  and  $y = \frac{1}{2}$ .



- 8.1 Determine the value of  $a$ . (1)
- 8.2 Write down the period of  $f\left(\frac{3x}{2}\right)$ . (2)
- 8.3 Write down the maximum value of  $\sqrt{\sin ax - 1}$ . (1)
- 8.4 If the  $x$ -axis is shifted to  $y = \frac{1}{2}$  to form a new graph  $h(x)$ , determine the equation of  $h(x)$ . (1)
- 8.5 The  $x$ -intercepts of  $h(x)$  is at A and B. Determine the coordinates of A and B without the use of a calculator. (2)
- [7]

## QUESTION 9

The diagram shows 2 circles. O is the centre of the circle drawn through ABGF, and the points E, F, G and D lies on the circumference of the other circle.



If  $\hat{O}_1 = 162^\circ$  and  $\hat{FED} = \hat{D}_1$ , calculate the following angles, providing reasons:

9.1  $\hat{B}_1$  (2)

9.2  $\hat{G}_1$  (2)

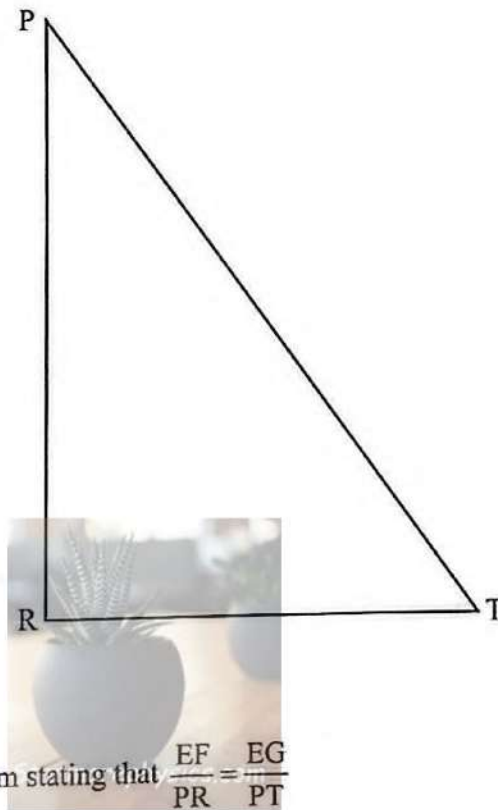
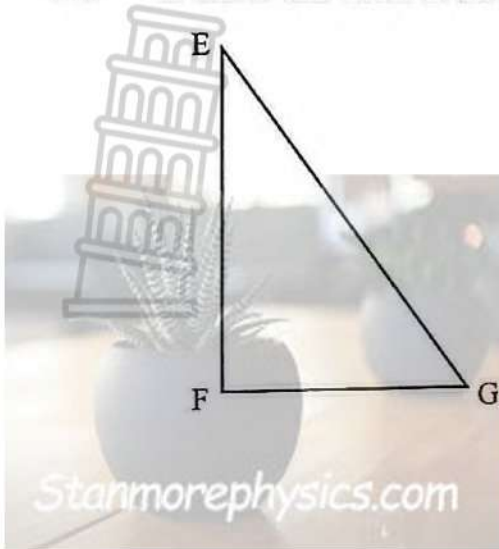
9.3  $\hat{D}_1$  (3)

[7]

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## QUESTION 10

10.1 In  $\triangle EFG$  and  $\triangle PRT$  in the diagram,  $\hat{E} = \hat{P}$  and  $\hat{F} = \hat{R}$ .



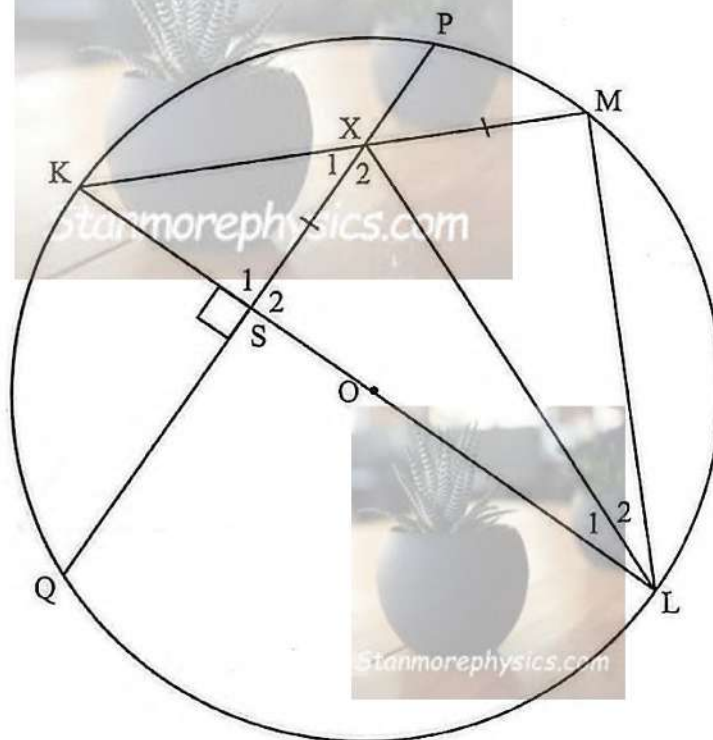
Use the diagram to prove the theorem stating that  $\frac{EF}{PR} = \frac{EG}{PT}$

(6)

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10.2 In the diagram KL is a diameter of the circle with centre O. The radius is 10 units, and M is a point on the circumference such that  $ML = 12$  units.

The bisector of  $\angle KLM$  meets KM at X.  $\triangle KSX \parallel \triangle KML$  and the chord PXQ cuts KL at S such that  $PXQ \perp KL$ .



10.2.1 Name 2 congruent triangles.

(1)

10.2.2 Calculate, with reasons:

(a)  $\frac{KM}{XS}$

(3)

(b)  $\frac{KX}{XS}$

(3)

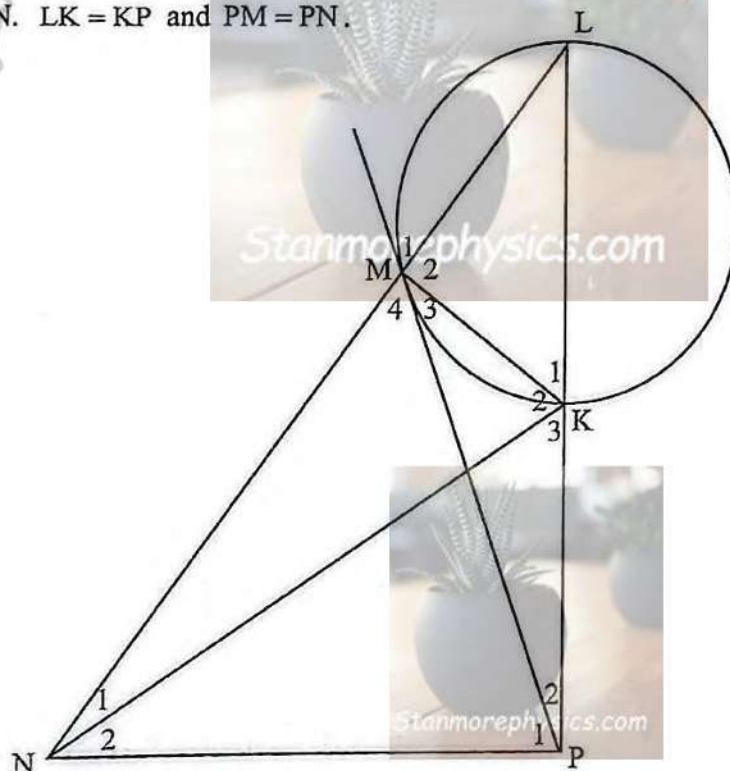
(c)  $\frac{XM}{XS}$

(5)

[18]

### QUESTION 11

In the diagram PM is a tangent to the circle through L, M and K.  $LP \perp NP$ . LM produced meets NP in N.  $LK = KP$  and  $PM = PN$ .

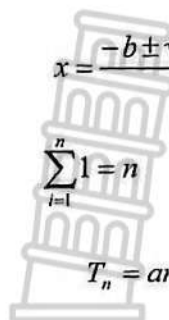


Prove that:

- 11.1 KMNP is a cyclic quadrilateral. (4)
- 11.2 LK is the diameter of the circle. (2)
- 11.3  $\triangle MKP \parallel \triangle LMP$  (3)
- 11.4  $PM^2 = PK \cdot PL$  (1)
- 11.5  $LN = \sqrt{6} \cdot PK$  (4)
- 11.6 If it is further given that  $\triangle NLP \parallel \triangle KLM$ , prove that  $PN = \sqrt{6} \cdot MK$  (3)

[17]

TOTAL: 150

**INFORMATION SHEET: MATHEMATICS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni)$$

$$A = P(1 - i)^n \quad A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta \quad (x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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**GRADE 12/GRAAD 12**

**MATHEMATICS P2/WISKUNDE V2**  
**SEPTEMBER 2026/SEPTEMBER 2026**  
**MARKING GUIDELINES/NASIENRIGLYNE**

**MARKS/PUNTE: 150**

**This marking guidelines consists of 18 pages/Hierdie nasienriglyne bestaan uit 18 bladsye**

**NOTE:**

If a candidate answers a question TWICE, only mark the FIRST attempt.

If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.

Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.

Assuming answers/values to solve a problem is NOT acceptable.

**LET WEL:**

*As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*

*As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*

*Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*

*Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

| GEOMETRY · MEETKUNDE |                                                                                                               |
|----------------------|---------------------------------------------------------------------------------------------------------------|
| S                    | A mark for a correct statement<br>(A statement mark is independent of a reason)                               |
|                      | <i>'n Punt vir 'n korrekte bewering<br/>( 'n Punt vir 'n bewering is onafhanklik van die rede)</i>            |
| R                    | A mark for the correct reason<br>(A reason mark may only be awarded if the statement is correct)              |
|                      | <i>'n Punt vir 'n korrekte rede<br/>( 'n Punt word slegs vir die rede toegeken as die bewering korrek is)</i> |
| S/R                  | Award a mark if statement AND reason are both correct                                                         |
|                      | <i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>                                                |

**QUESTION/VRAAG 1**

1.1

| Screen time (hours) | Frequency | Cumulative Frequency |
|---------------------|-----------|----------------------|
| $0 < x \leq 1$      | 0         | 0                    |
| $1 < x \leq 2$      | 2         | 2                    |
| $2 < x \leq 3$      | 5         | 7                    |
| $3 < x \leq 4$      | 9         | 16                   |
| $4 < x \leq 5$      | 12        | 28                   |
| $5 < x \leq 6$      | 7         | 35                   |
| $6 < x \leq 7$      | 5         | 40                   |
| $7 < x \leq 8$      | 0         | 40                   |

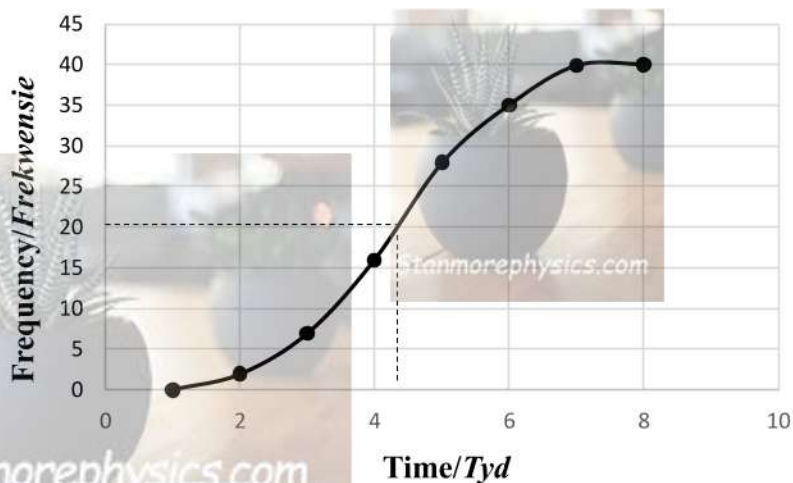
✓16

✓40

(2)

1.2

**Screen time(in hours)/Skermtyd (in ure)**



✓grounding/anker  
✓using cumulative values/gebruik van kumulatiewe waardes

(2)

1.3

$$Q_2 = 4,3$$

Accept/aanvaar 4,2 to 4,4

✓ answer/antwoord

(1)

1.4

$$\bar{x} < Q_2$$

Data skew to left/Data is skeef na links

$$\checkmark \bar{x} < Q_2$$

✓conclusion/  
gevolgtrekking

(2)

1.5

$$\begin{aligned} \text{new/nuwe } \bar{x} &= \frac{(1,5 \times 2) + (2,5 \times 5) + (3,5 \times 33)}{40} \\ &= \frac{131}{40} \\ &= 3,275 \end{aligned}$$

✓131

✓answer/antwoord

(2)

[9]

QUESTION/VRAAG 2

2.1

G20 SUMMIT/BERAAD 2025



✓4 points correct/4 punte korrek  
✓all points correct/alle punte korrek

(2)

2.2

$$a = 6,1289$$

$$b = 0,0001$$

$$y = 6,1289 + 0,0001x$$

✓value/waarde  $a$   
✓value/waarde  $b$   
✓equation/vergelijking

(3)

2.3

$$r = 0,47$$

✓answer/antwoord

(1)

2.4

Moderate correlation/Matige korrelasie

✓answer/antwoord

(1)

2.5

$$y = 6,1289 + 0,0001(40000)$$

$$y = 10,1289$$

✓subst/vervang  
✓answer/antwoord

(2)

2.6

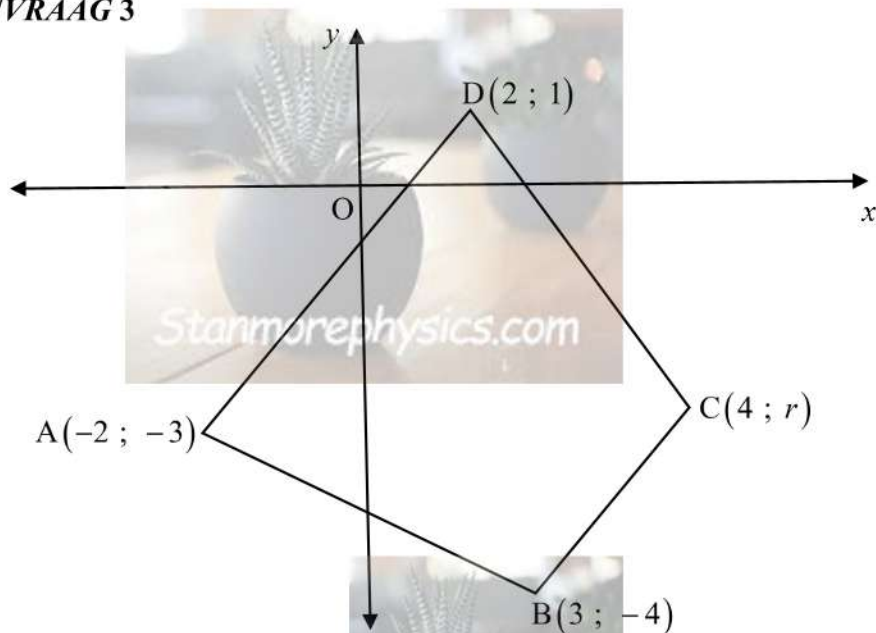
No, some countries are more developed than others and that can influence the Gross Domestic Product (GDP) per capita  
Or some countries are more densely populated than others.  
*Nee, sommige lande is meer ontwikkel as ander en dit kan die Bruto Binnelandse Produk (BBP) per capita beïnvloed*  
*Of sommige lande is digter bevolk as ander.*

✓answer/antwoord

(1)

[10]

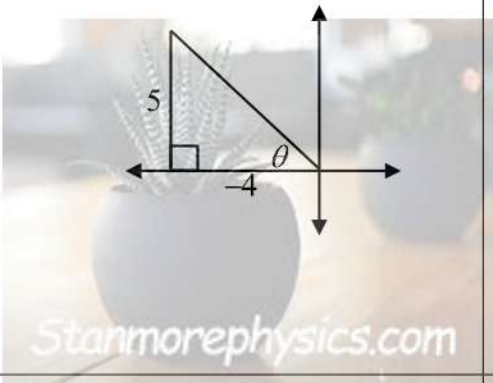
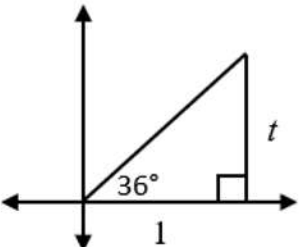
QUESTION/VRAAG 3



|     |                                                                                                                                     |                                                                                           |
|-----|-------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| 3.1 | $x_M = \frac{-2+2}{2} = 0$ $y_M = \frac{-3+1}{2} = -1$                                                                              | ✓ value $x$ / waarde $x$<br>✓ value $y$ / waarde $y$<br>(2)                               |
| 3.2 | $y - y_1 = m(x - x_1)$ $y - (-4) = -1(x - 3) \quad \text{or/of} \quad y - (-1) = -1(x - 0)$ $y + 4 = -x + 3$ $y = -x - 1$           | ✓ $m(\text{AP})$<br>✓ subst in formula / vervang in formule<br>✓ answer / antwoord<br>(3) |
| 3.3 | $m(\text{AD}) = \frac{-3-1}{-2-2} = 1$ $m(\text{AD}) \times m(\text{BP}) = 1 \times -1 = -1$ $\therefore \text{BP} \perp \text{AD}$ | ✓ $m(\text{AD})$<br>✓ subst in formula / vervang in formule<br>(2)                        |
| 3.4 | $m(\text{AD}) = m(\text{BC})$ $1 = \frac{-4-r}{3-4}$ $-1 = -4 - r$ $r = -3$                                                         | ✓ $m(\text{BC})$<br>✓ equate gradients / gelykstel<br>✓ answer / antwoord<br>(3)          |

|     |                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                 |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3.5 | $\tan \theta = m(\text{BD})$<br>$\tan \theta = \frac{-4-1}{3-2}$<br>$\tan \theta = -5$<br>$\theta = 180^\circ - 78,69^\circ$<br>$\theta = 101,31^\circ$                                                                                                                                                                          | $\checkmark$ subst in formula/ vervang in formule<br>$\checkmark -5$<br>$\checkmark 180^\circ - 78,69^\circ$<br>$\checkmark$ answer/antwoord<br>(4)                                             |
| 3.6 | AC: $y = -3 \dots\dots ①$<br>BD: $y - y_1 = m(x - x_1)$ $m(\text{BD}) = -5$<br>$y - (-4) = -5(x - 3)$ or/of $y - 1 = -5(x - 2)$<br>$y + 4 = -5x + 15$ $y - 1 = -5x + 10$<br>$y = -5x + 11$ $y = -5x + 11 \dots\dots ②$<br>Subst ① into ②: $-3 = -5x + 11$<br>$5x = 14$<br>$x = \frac{14}{5}$<br>$S\left(\frac{14}{5}; -3\right)$ | $\checkmark$ equation AC/vergelyking AC<br>$\checkmark m(\text{BD})$<br>$\checkmark$ equation BD/vergelyking BD<br>$\checkmark$ equating/gelykstel<br>$\checkmark$ value $x$ /waarde $x$<br>(5) |
| 3.7 | $AB = \sqrt{(-2-3)^2 + (-3-(-4))^2}$<br>$= \sqrt{26}$<br>$BD = \sqrt{(3-2)^2 + (-4-1)^2} = \sqrt{26}$<br>$AB = BD$<br>$\therefore \triangle ABD$ is isosceles/gelykbenig                                                                                                                                                         | $\checkmark$ subst in distance formula/ vervang in afstand formule<br>$\checkmark$ length AB/lengte AB<br>$\checkmark$ length BD/lengte BD<br>$\checkmark$ conclusion/gevolgtrekking<br>(4)     |
| 3.8 | Trapezium      one pair of sides parallel<br>Trapesium      een paar sye ewewydig                                                                                                                                                                                                                                                | $\checkmark$ answer/antwoord<br>$\checkmark$ reason/rede<br>(2)                                                                                                                                 |
|     |                                                                                                                                                                                                                                                                                                                                  | [25]                                                                                                                                                                                            |

| QUESTION/VRAAG 4 |                                                                                                                                                                   |                                                                                                                                       |
|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| 4.1              | $\tan P = \frac{1}{3}$<br>$\hat{P} = 18,43^\circ$<br>$Q\hat{P}V = 18,43^\circ - 4,4^\circ$<br>$Q\hat{P}V = 14,03^\circ$                                           | ✓ subst in tan def/vervang in tan def<br>✓ size $\hat{P}$ /grootte $\hat{P}$<br>✓ $18,43^\circ - 4,4^\circ$<br>✓ $14,03^\circ$<br>(4) |
| 4.2              | $\hat{Q} = 90^\circ$ rad $\perp$ tan<br>In $\Delta PQV$ : $\tan 14,03^\circ = \frac{QV}{8\sqrt{10}}$<br>$QV = 6,32$<br>$\therefore \text{rad} = 6,32$             | ✓ state $90^\circ$ /stel $90^\circ$<br>✓ tan def/vervang in tan def<br>✓ radius<br>(3)                                                |
| 4.3              | $(x-a)^2 + (y-b)^2 = r^2$ through/deur $(0; 0)$<br>$(0-6)^2 + (0-b)^2 = (6,32)^2$<br>$36 + b^2 = 39,9424$<br>$b^2 = 3,9424$<br>$b = 2$<br>Centre/midpunt $(6; 2)$ | ✓ a – value/a – waarde<br>✓ substitution $(0, 0)$ /vervang<br>✓ subst radius/vervang rad<br>✓ b – value/b – waarde<br>(4)             |
| 4.4              | $S(-16; -2)$                                                                                                                                                      | ✓ value $x$ /waarde $x$<br>✓ value $y$ /waarde $y$<br>(2)                                                                             |
|                  |                                                                                                                                                                   | [13]                                                                                                                                  |

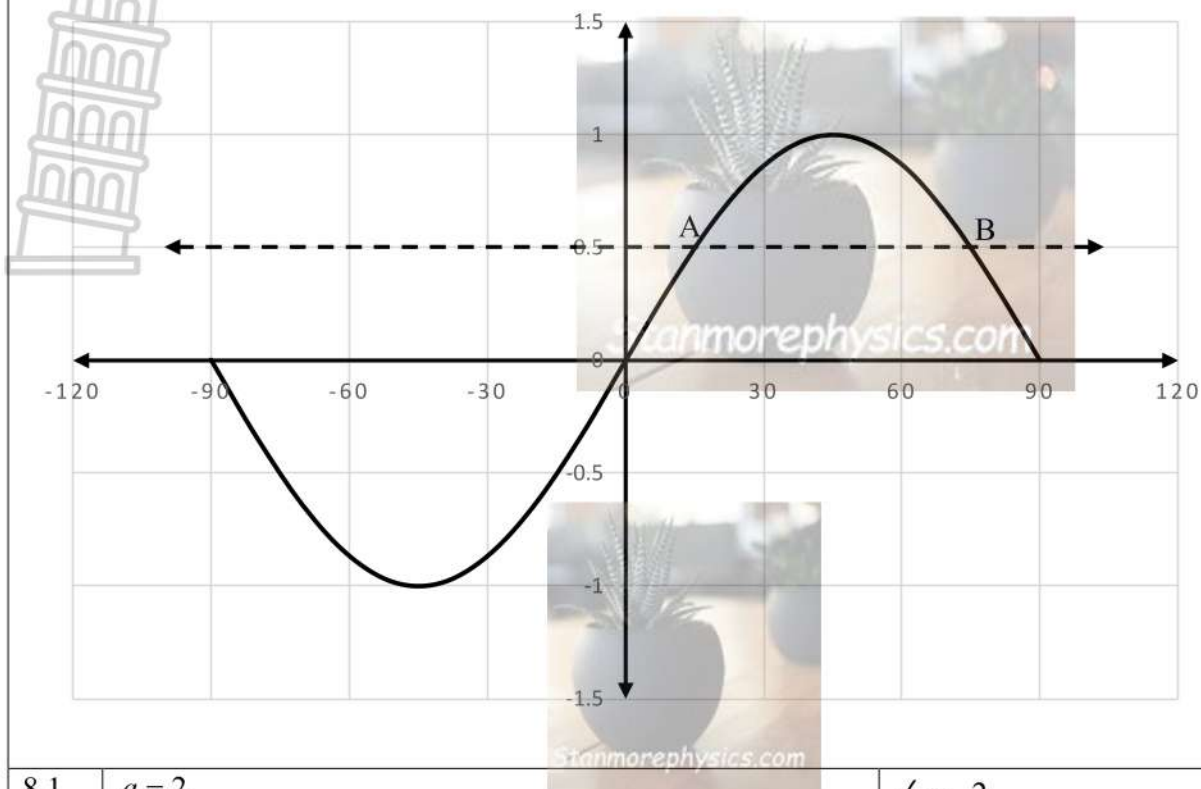
| QUESTION/VRAAG 5 |                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                     |
|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.1.1            | $\tan \theta = -\frac{5}{4}$ $r = \sqrt{41}$ $\sin(180^\circ + \theta)$ $= -\sin \theta$ $= -\frac{5}{\sqrt{41}}$                                                                                                                                                                                             | <p>✓ value r/waarde r</p> <p>✓ reduction/reduksie</p> <p>✓ answer/antwoord</p> <p>(3)</p>                                                                                                                           |
| 5.1.2            | $\sqrt{41} \cos \theta - 4 \sin(-150^\circ) \cdot \cos 180^\circ$ $= \sqrt{41} \cos \theta + 4 \sin 150^\circ (-1)$ $= \sqrt{41} \cos \theta + 4 (\sin 30^\circ) (-1)$ $= \sqrt{41} \left( -\frac{4}{\sqrt{41}} \right) + 4 \left( \frac{1}{2} \right) (-1)$ $= -4 - 2$ $= -6$                               | <p>✓ <math>\sin 30^\circ</math></p> <p>✓ <math>\frac{1}{2}</math> and/en <math>-1</math>    ✓ <math>-\frac{4}{\sqrt{41}}</math></p> <p>✓ answer/antwoord</p> <p>(4)</p>                                             |
| 5.2              | $a \in [-1 ; 1]$                                                                                                                                                                                                                                                                                                                                                                                | <p>✓ answer/antwoord</p> <p>(1)</p>                                                                                                                                                                                 |
| 5.3.1            | $r = \sqrt{t^2 + 1}$ $\frac{1}{\sin 306^\circ}$ $= \frac{1}{-\sin 54^\circ}$ $= 1 \div -\left( \frac{1}{\sqrt{t^2 + 1}} \right)$ $= -\sqrt{t^2 + 1}$ <p><b>OR/OF</b></p> $= \frac{1}{-\sin 54^\circ}$ $= \frac{1}{-\cos 36^\circ}$ $= 1 \div -\left( \frac{1}{\sqrt{t^2 + 1}} \right)$ $= -\sqrt{t^2 + 1}$  | <p>✓ <math>-\sin 54^\circ</math></p> <p>✓ substitution/vervanging</p> <p>✓ answer/antwoord</p> <p>(3)</p> <p>✓ <math>-\sin 54^\circ</math></p> <p>✓ substitution/vervanging</p> <p>✓ answer/antwoord</p> <p>(3)</p> |

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                          |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.3.2 | $\begin{aligned} & \tan 81^\circ \\ &= \frac{\sin 81^\circ}{\cos 81^\circ} \\ &= \frac{\sin(45^\circ + 36^\circ)}{\cos(45^\circ + 36^\circ)} \\ &= \frac{\sin 45^\circ \cos 36^\circ + \cos 45^\circ \sin 36^\circ}{\cos 45^\circ \cos 36^\circ - \sin 45^\circ \sin 36^\circ} \\ &= \frac{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{\sqrt{t^2+1}}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{t}{\sqrt{t^2+1}}\right)}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{\sqrt{t^2+1}}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{t}{\sqrt{t^2+1}}\right)} \\ &= \frac{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{\sqrt{t^2+1}}\right)(1+t)}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{\sqrt{t^2+1}}\right)(1-t)} \end{aligned}$ | <p>✓ quotient identity/kwosiënt identiteit</p> <p>✓ <math>45^\circ + 36^\circ</math></p> <p>✓ expansion/uitbreiding</p> <p>✓ substitution numerator/vervanging teller</p> <p>✓ substitution denominator/vervanging noemer</p> <p>(5)</p> |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | [15]                                                                                                                                                                                                                                     |

| QUESTION/VRAAG 6 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                        |
|------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6.1.1            | $\text{LHS} = 2 \cos \theta \cdot \cos 2\theta + \frac{\sin^2 2\theta}{\cos \theta}$ $\text{LHS} = 2 \cos \theta (2 \cos^2 \theta - 1) + \frac{(2 \sin \theta \cos \theta)^2}{\cos \theta}$ $\text{LHS} = 4 \cos^3 \theta - 2 \cos \theta + \frac{4 \sin^2 \theta \cos^2 \theta}{\cos \theta}$ $\text{LHS} = 4 \cos^3 \theta - 2 \cos \theta + 4 \sin^2 \theta \cos \theta$ $\text{LHS} = 2 \cos \theta (2 \cos^2 \theta - 1 + 2 \sin^2 \theta)$ $\text{LHS} = 2 \cos \theta (2 - 1)$ $\text{LHS} = 2 \cos \theta$                                                                                                    | <p>✓ cos double angle/cos dubbelhoek</p> <p>✓ sin double angle/sin dubbelhoek</p> <p>✓ simplification/vereenv</p> <p>✓ simplification/vereenv</p> <p>✓ common factor/gemeensk faktor</p> <p>✓ 2</p> <p>(6)</p>                                         |
| 6.1.2            | $x = \{90^\circ\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | <p>✓ <math>90^\circ</math></p> <p>(1)</p>                                                                                                                                                                                                              |
| 6.2              | $\cos \theta = \frac{1}{\cos \theta} + \frac{5}{6}$ $6 \cos^2 \theta = 6 + 5 \cos \theta$ $6 \cos^2 \theta - 5 \cos \theta - 6 = 0$ $(3 \cos \theta + 2)(2 \cos \theta - 3) = 0$ $3 \cos \theta + 2 = 0 \quad \text{or/of} \quad 2 \cos \theta - 3 = 0$ $\cos \theta = -\frac{2}{3} \quad \text{or/of} \quad \cos \theta = \frac{3}{2}$ <p>NA</p> $\theta = 180^\circ - 48,19^\circ + k \cdot 360^\circ \text{ or/of } \theta = 180^\circ + 48,19^\circ + k \cdot 360^\circ$ $\theta = 131,81^\circ + k \cdot 360^\circ, k \in \mathbb{Z} \text{ or/of } \theta = 228,19^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ | <p>✓ LCM/KGV</p> <p>✓ standard form/std vorm</p> <p>✓ factors/faktore</p> <p>✓ both equations/albei vgl's</p> <p>✓ <math>131,81^\circ</math> and/en <math>228,19^\circ</math></p> <p>✓ <math>k \cdot 360^\circ, k \in \mathbb{Z}</math></p> <p>(6)</p> |
| 6.3              | $\text{area/opv } \Delta PQR = \frac{1}{2} pq \sin R \dots\dots \textcircled{1}$ $\frac{p}{\sin P} = \frac{q}{\sin Q}$ $p = \frac{q \sin P}{\sin Q} \dots\dots \textcircled{2}$ <p>Subs <math>\textcircled{2}</math> into <math>\textcircled{1}</math>: <math>\text{area/opv } \Delta PQR = \frac{1}{2} \left( \frac{q \sin P}{\sin Q} \right) q \sin R</math></p> $\text{area/opv } \Delta PQR = \frac{q^2 \sin P \sin R}{2 \sin Q}$                                                                                                                                                                                 | <p>✓ <math>p = \frac{q \sin P}{\sin Q}</math></p> <p>✓ substitution/vervang</p> <p>(2)</p>                                                                                                                                                             |
|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | [15]                                                                                                                                                                                                                                                   |

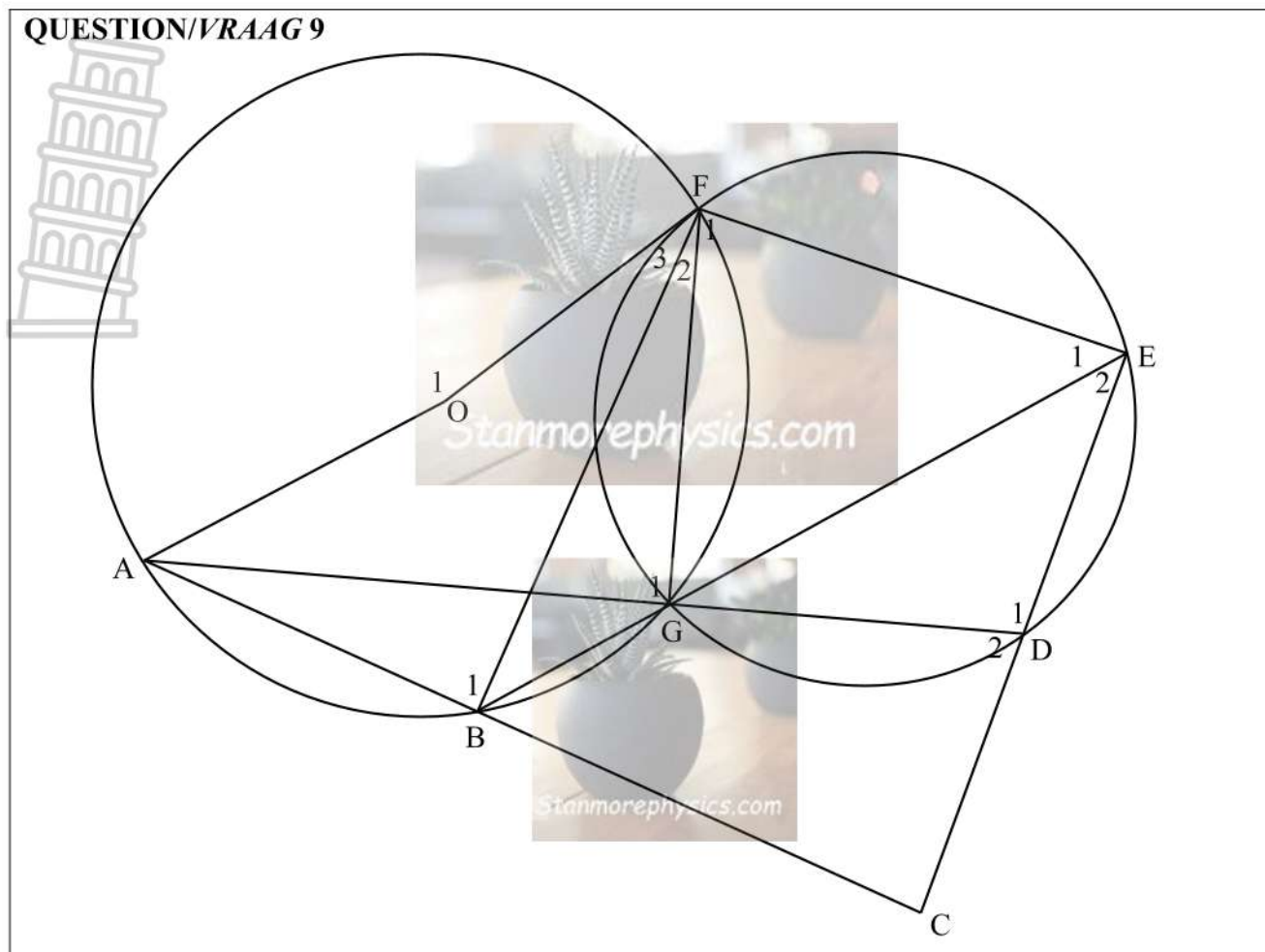
| QUESTION/VRAAG 7 |                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                |
|------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7.1.1            | $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$                                                                                                                                                                                                                                              | ✓ answer/antwoord<br>(1)                                                                                                                                                       |
| 7.1.2            | $\sin 2\alpha$<br>$= \cos(90^\circ - (\alpha + \alpha))$<br>$= \cos(90^\circ - \alpha - \alpha)$<br>$= \cos((90^\circ - \alpha) - \alpha)$<br>$= \cos(90^\circ - \alpha) \cos \alpha + \sin(90^\circ - \alpha) \sin \alpha$<br>$= \sin \alpha \cos \alpha + \cos \alpha \sin \alpha$<br>$= 2 \sin \alpha \cos \alpha$ | $\checkmark 90^\circ - (\alpha + \alpha)$<br><br>$\checkmark$ expansion/uitbreiding<br>$\checkmark$ co-functions/ ko-funksies<br>(3)                                           |
| 7.2.1            | In $\triangle PNR$ : $\sin \theta = \frac{r}{b}$<br>$r = b \sin \theta \dots\dots \textcircled{1}$                                                                                                                                                                                                                    | $\checkmark$ sin def<br><br>$\checkmark$ r subject/r onderwerp<br>(2)                                                                                                          |
| 7.2.2            | $r = b \sin \theta \dots\dots \textcircled{1}$<br>$r = a \sin 2\theta \dots\dots \textcircled{2}$<br>From $\textcircled{1}$ and $\textcircled{2}$ : $b \sin \theta = a \sin 2\theta$<br>$b = \frac{a \sin 2\theta}{\sin \theta}$<br>$b = \frac{a(2 \sin \theta \cos \theta)}{\sin \theta}$<br>$b = 2a \cos \theta$    | $\checkmark$ equating/gelykstel<br><br>$\checkmark$ b subject/b onderwerp<br><br>$\checkmark$ sin double angle/sin dubbelhoek<br>(3)                                           |
| 7.2.3            | $\cos \theta = \frac{PR}{b}$<br>$PR = b \cos \theta$<br>$\cos 2\theta = \frac{PQ}{a}$<br>$PQ = a \cos 2\theta$<br>$\therefore QR = PR - PQ$<br>$QR = b \cos \theta - a \cos 2\theta$<br>$QR = (2a \cos \theta) \cos \theta - a(2 \cos^2 \theta - 1)$<br>$QR = 2a \cos^2 \theta - 2a \cos^2 \theta + a$<br>$QR = a$    | $\checkmark PR = b \cos \theta$<br><br>$\checkmark PQ = a \cos 2\theta$<br><br>$\checkmark QR = b \cos \theta - a \cos 2\theta$<br>$\checkmark$ substitution/vervanging<br>(4) |
|                  |                                                                                                                                                                                                                                                                                                                       | [13]                                                                                                                                                                           |

QUESTION/VRAAG 8



|     |                                                                                                                             |                                                      |     |
|-----|-----------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|-----|
| 8.1 | $a = 2$                                                                                                                     | $\checkmark a = 2$                                   | (1) |
| 8.2 | $f(x) = \sin 2x$<br>$f\left(\frac{3x}{2}\right) = \sin 2\left(\frac{3x}{2}\right) = \sin 3x$<br>Period/Periode: $120^\circ$ | $\checkmark \sin 3x$<br>$\checkmark$ answer/antwoord | (2) |
| 8.3 | $\sqrt{\sin 2x - 1}$<br>Max/maks = 0                                                                                        | $\checkmark$ answer/antwoord                         | (1) |
| 8.4 | $f(x) = \sin 2x$ shifted down/afgeskuif $\frac{1}{2}$ units/eenhede<br>$h(x) = \sin 2x - \frac{1}{2}$                       | $\checkmark$ answer/antwoord                         | (1) |
| 8.5 | $x = 15^\circ$ and/en $x = 75^\circ$                                                                                        | $\checkmark x = 15^\circ$ $\checkmark x = 75^\circ$  | (2) |
|     |                                                                                                                             |                                                      | [7] |

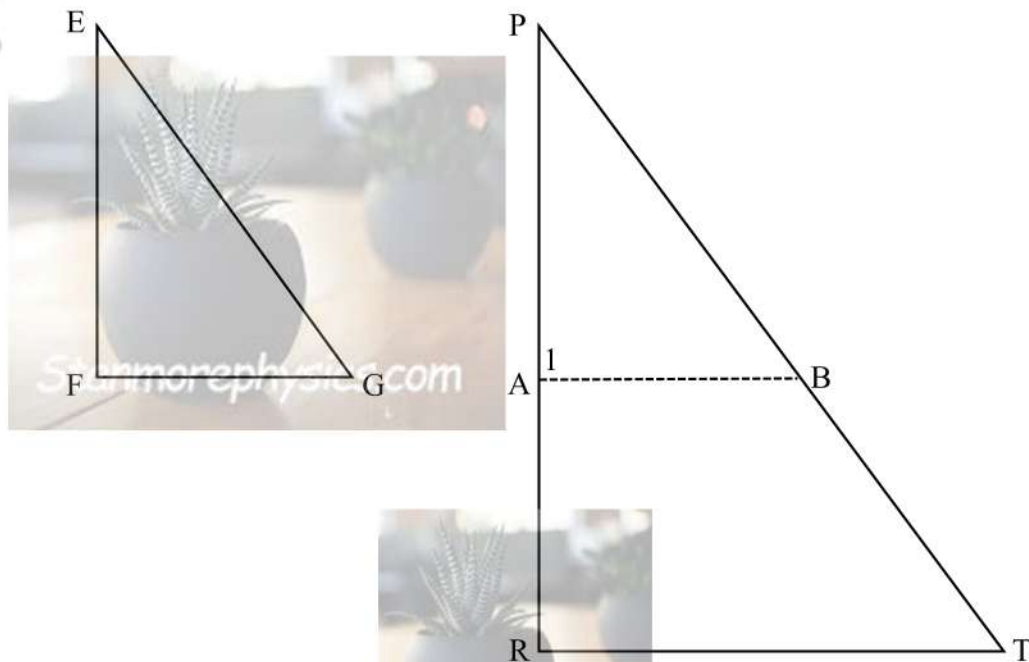
QUESTION/VRAAG 9



|     |                                                                                                                                       |                    |
|-----|---------------------------------------------------------------------------------------------------------------------------------------|--------------------|
| 9.1 | $\hat{B}_1 = 81^\circ$ $\angle$ at centre is $2 \times \angle$ at circumf/midpt $\angle = 2 \times \angle$ by omtrek                  | ✓S ✓R<br>(2)       |
| 9.2 | $\hat{G}_1 = 81^\circ$ $\angle$ 's in same segment/ $\angle$ 'e in dies segment                                                       | ✓S ✓R<br>(2)       |
| 9.3 | $\hat{G}_1 = \hat{FED} = 81^\circ$ ext $\angle$ cyclic quad ext /buite $\angle$ kvh<br>$\hat{FED} = \hat{D}_1 = 81^\circ$ given/gegee | ✓S ✓R<br>✓S<br>(3) |
|     |                                                                                                                                       | [7]                |

QUESTION/VRAAG 10

10.1



Draw AB with A on PR and B on PT so that  $EF = PA$  and  $EG = PB$   
 /Trek AB met A op PR en B op PT sodat  $EF = PA$  en  $EG = PB$

In  $\triangle EFG$  and/en  $\triangle PAB$ :

- (i)  $EF = PA$  constr/konstr
- (ii)  $EG = PB$  constr/konstr
- (iii)  $\hat{E} = \hat{P}$  given/gegee

$\triangle EFG \equiv \triangle PAB$  SAS/SHS

$$\therefore \hat{F} = \hat{A}_1 \quad \equiv \Delta$$

But/maar:  $\hat{F} = \hat{R}$  given/gegee

$$\therefore \hat{A}_1 = \hat{R}$$

$\therefore AB \parallel RT$  corresp  $\angle$ 's equal/ooreenk  $\angle$ 'e gelyk

$$\therefore \frac{PA}{PR} = \frac{PB}{PT} \quad \text{prop theorem } AB \parallel RT \text{ /eweredigh st } AB \parallel RT$$

and/en  $EF = PA$  constr/konstr

$EG = PB$  constr/konstr

$$\therefore \frac{EF}{PR} = \frac{EG}{PT}$$

✓ constr/konstr

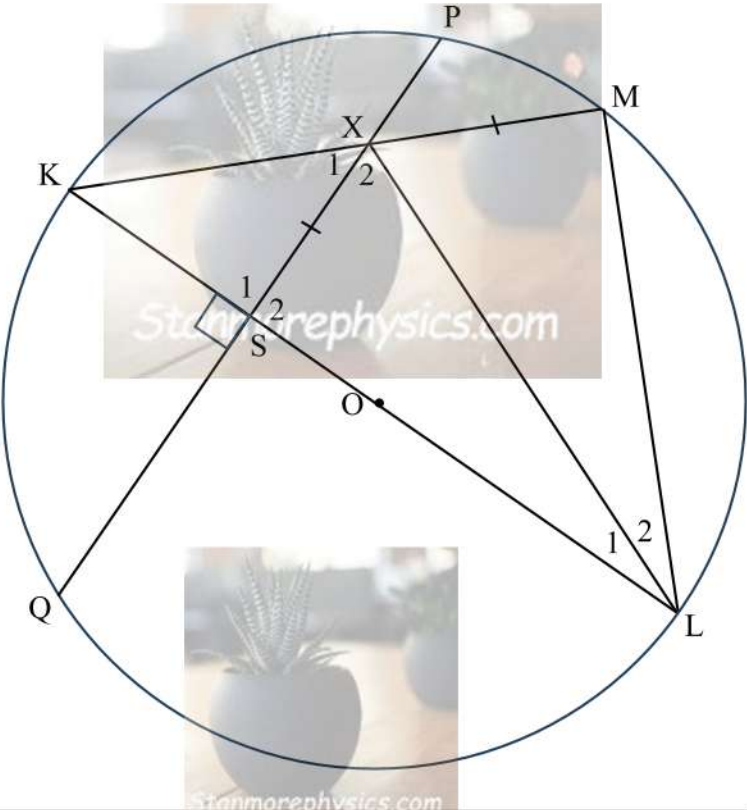
✓  $\equiv$  proof/bewys

✓ R

✓ S ✓ R

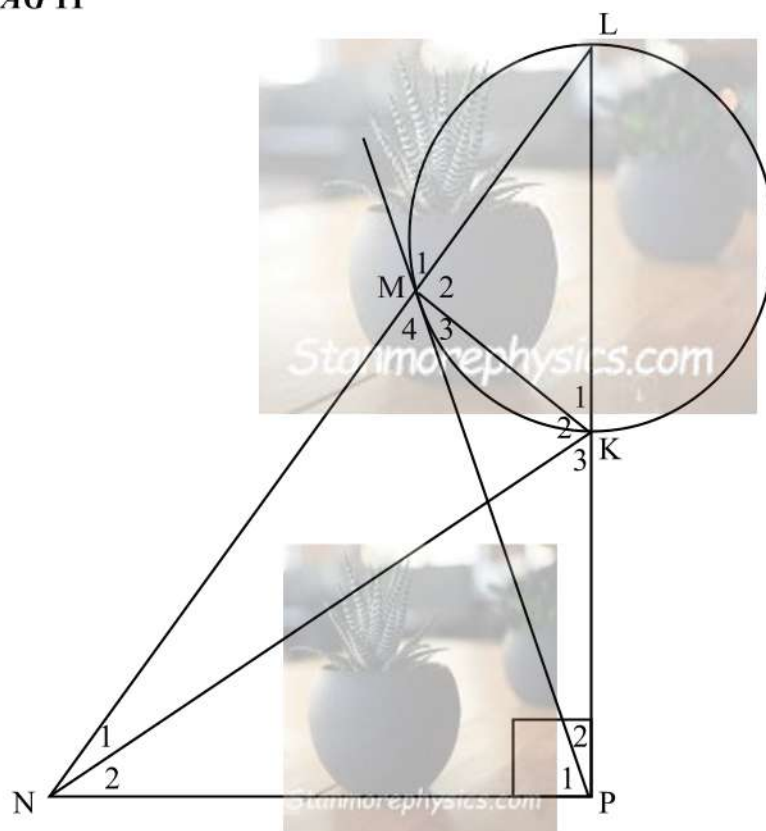
✓ S

(6)

|               |                                                                                                                                                                                                                         |                                                            |                                                                                        |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|----------------------------------------------------------------------------------------|
| 10.2          |                                                                                                                                      |                                                            |                                                                                        |
| 10.2.1        | $\Delta SXL \equiv \Delta MXL$                                                                                                                                                                                          | $\angle \angle S$                                          | ✓S<br>(1)                                                                              |
| 10.2.2<br>(a) | $KL^2 = KM^2 + ML^2$<br>$20^2 = KM^2 + 12^2$<br>$KM^2 = 256$<br>$KM = 16$                                                                                                                                               | Pythagoras                                                 | ✓S<br>✓subst/verv<br>✓answer/<br>antwoord<br>(3)                                       |
| (b)           | $\frac{KX}{SX} = \frac{KL}{ML}$<br>$\frac{KX}{SX} = \frac{20}{12} = \frac{5}{3}$<br><b>OR/OF</b><br>$\frac{KX}{KL} = \frac{SX}{ML}$<br>$\frac{KX}{20} = \frac{SX}{12}$<br>$\frac{KX}{SX} = \frac{20}{12} = \frac{5}{3}$ | $\parallel \Delta$ 's<br><br><br><br>$\parallel \Delta$ 's | ✓S    ✓R<br>$\sqrt{\frac{5}{3}}$<br>(3)<br><br>✓S    ✓R<br>$\sqrt{\frac{5}{3}}$<br>(3) |

|                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                              |
|----------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(c)</p>  | $ML = SL = 12$<br>$\therefore KS = 8$<br>$\frac{KS}{KM} = \frac{SX}{ML}$<br>$\frac{8}{KX + XM} = \frac{XM}{12}$<br>$XM(KX + XM) = 96$<br>$XM\left(\frac{5}{3}XS + XM\right) = 96$ from/uit 10.2.2 (b)<br>$XM\left(\frac{5}{3}XM + XM\right) = 96$<br>$XM\left(\frac{8}{3}XM\right) = 96$<br>$XM^2 = 36$<br>$XM = 6$<br><br><b>OR/OF</b><br><br>In $\Delta KML$ :<br>$\cos L = \frac{20^2 + 12^2 - 16^2}{2(20)(12)}$<br>$\cos L = \frac{3}{5}$<br>$\hat{L} = 53,13^\circ$<br>$\therefore \hat{L}_1 = \hat{L}_2 = 26,57^\circ$<br>In $\Delta XML$ :<br><br>$M = 90^\circ$ $\angle$ in semi circle<br>$\tan 26,57^\circ = \frac{XM}{12}$<br>$XM = 6$ | $\Delta SXL \equiv \Delta MXL$<br><br>$\parallel \Delta$ 's<br><br>$KX + XM$ and/en $SX = XM$<br><br><br><br><br><br><br><br><br><br> | $\checkmark S$<br><br>$\checkmark S$<br><br>$\checkmark S$<br><br><br><br>$\checkmark$ subst/verv<br><br><br><br><br><br><br><br><br><br>$\checkmark$ answer/<br>antwoord<br>(5)<br><br><br><br><br>$\checkmark$ Subs cos rule<br><br>$\checkmark \frac{3}{5}$<br><br>$\checkmark 26,57^\circ$<br><br><br>$\checkmark S/R$<br><br>$\checkmark$ answer/<br>antwoord<br>(5)<br><br><b>[18]</b> |
|----------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

QUESTION/VRAAG 11



|      |                                                                                                                                                                                             |                                                                                                                                                                                    |                                                                              |     |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|-----|
| 11.1 | $\hat{N} = \hat{M}_4$<br>$\hat{M}_4 = \hat{M}_1$<br>$\hat{M}_1 = \hat{K}_1$<br>$\therefore \hat{N} = \hat{K}_1$<br>$\therefore$ KMNP is cyclic quad/kvh                                     | $\angle$ 's opp equal sides/ $\angle$ 'e teenoor gelyke sye<br>vertically opp $\angle$ 's/regoorst $\angle$ 'e<br>tan-chord theorem/rklyn krd stelling                             | $\checkmark$ S/R<br>$\checkmark$ S<br>$\checkmark$ S/R<br>$\checkmark$ R     | (4) |
| 11.2 | $\hat{M}_2 = \hat{P} = 90^\circ$<br>$\therefore$ LK is diameter of the circle<br>$\therefore$ LK is middellyn van die sirkel                                                                | ext $\angle$ cyclic quad/buite $\angle$ kvh<br>chord subtend $90^\circ \angle$ /converse $\angle$ in semi-circle<br>koord onderspan $90^\circ \angle$ /omg $\angle$ in semi sirkel | $\checkmark$ S<br>$\checkmark$ R                                             | (2) |
| 11.3 | In $\triangle MKP$ and/en $\triangle LMP$ :<br>(i) $\hat{P} = \hat{P}$<br>(ii) $\hat{M}_3 = \hat{L}$<br>(iii) $\hat{MKP} = \hat{LMP}$<br>$\therefore \triangle MKP \parallel \triangle LMP$ | common/gemeensk<br>tan-chord theorem/rklyn krd stelling<br>int $\angle$ 's $\triangle$ /binne $\angle$ 'e $\triangle$<br>$\angle \angle \angle$                                    | $\checkmark$ S<br>$\checkmark$ S/R<br>$\checkmark$ S or/of<br>$\checkmark$ R | (3) |
| 11.4 | $\frac{KP}{MP} = \frac{MP}{LP}$<br>$\therefore MP^2 = KP \cdot LP$                                                                                                                          | $\parallel \triangle$ 's                                                                                                                                                           | $\checkmark$ S/R                                                             | (1) |

|      |                                                                                                                                                                                                                 |                                                                                              |                                                                                                                   |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| 11.5 | $LN^2 = NP^2 + LP^2$<br>$\therefore PL = 2 \times PK$<br>and/en $NP = MP$<br>$\therefore LN^2 = MP^2 + (2PK)^2$<br>$LN^2 = PK.PL + (2PK)^2$<br>$LN^2 = PK.2PK + (2PK)^2$<br>$LN^2 = 6PK^2$<br>$LN = \sqrt{6}PK$ | Pythagoras<br>given/gegee<br>given/gegee<br><br><br>$MP^2 = KP.LP$ and/en $PL = 2 \times PK$ | $\checkmark S$<br><br>$\checkmark S$<br><br>$\checkmark$ Subst/Verv<br>$\checkmark$ Simpl/<br>vereenvoudig<br>(4) |
| 11.6 | $\triangle NLP \parallel \triangle KLM$<br>$\therefore \frac{NL}{KL} = \frac{NP}{KM}$<br>$NP = \frac{KM.NL}{KL}$<br>$NP = \frac{KM.\sqrt{6}PK}{KL}$<br>$PK = KL$<br>$NP = \sqrt{6}KM$                           | $\parallel \triangle$ 's<br><br>$LN = \sqrt{6}PK$ proven /bewys<br>given/gegee               | $\checkmark S/R$<br><br>$\checkmark S$<br><br>$\checkmark S$<br>(3)                                               |
|      |                                                                                                                                                                                                                 |                                                                                              | [17]                                                                                                              |

**TOTAL/ TOTAAL: 150**